

# Jumps and Information Flow in Financial Markets

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## Abstract

I propose a new two-stage semi-parametric test to investigate the predictability of stochastic jump arrivals in asset prices. The test allows us to pin down relevant information for jump prediction up to the intraday level. Based on the test, I find that systematic jumps in U.S. individual equity markets are likely to occur shortly after macroeconomic information release such as Fed's announcements, market jumps, employment reports, or initial jobless claims. I also present firm-specific jump predictors of earnings release, analyst recommendation, and dividend dates along with the jump clustering effect. Evidence suggests systematic jump intensity has increased in recent years.

*JEL classification:* G10, C14

*Key words:* mixed unobservability, jumps predictor tests, systematic risk, partial likelihood, high frequency data

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The predictability of asset return characteristics and the impact of information flow on asset prices have been two of the most enduring questions in financial economics and econometrics. There is a large stream of literature devoted to the predictability of the first and second moments of asset returns in relation to market information, using simple regression and various types of time-varying variance models.<sup>1</sup> In contrast to the assumptions imposed in these studies, researchers found evidence of jumps that better explain higher moments of returns, and they widely incorporate them in many asset pricing models.<sup>2</sup> My goal in this paper is to extend the prediction analysis for jumps by providing an empirical method and present empirical evidence based on the new method.

Given the importance and strong impact of jump risk, econometricians have worked on how to distinguish jump risk from volatility risk, using discrete observations from continuous-time models.<sup>3</sup> While this literature is focusing on testing the presence of jumps itself, what is important in financial decision making is for us to be able to pin down market information driving those jump risk so that we could predict *ex ante* whether and when these jumps are more likely to occur. Accordingly, I am motivated to propose a technique for performing a regression-type analysis to select or sort predictors for the purpose of developing jump intensity model.

The first contribution of this paper over prior work is to recognize the problem of identifying

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<sup>1</sup>Examples include ARCH, GARCH, EGARCH, stochastic volatility, and forward-looking implied volatility models. See Keim and Stambaugh (1986), Fama and French (1988 and 1990), among others, for prediction of the expected returns, and Bollerslev, Chou, and Kroner (1992), with an excellent survey on the ARCH/GARCH literature for prediction of the second moments. The relation between stochastic volatility and information flow is investigated by Andersen (1996). The impact of mean return predictability on option pricing was first studied by Lo and Wang (1995), and many investigators such as Heston (1993), Hull and White (1987), and Stein and Stein (1991) have studied the impact of stochastic volatility but without jumps.

<sup>2</sup>Bates (1996), Bakshi, Cao, and Chen (1997), Bakshi, Kapadia, and Madan (2003), Aït-Sahalia (2002), Andersen, Benzoni, and Lund (2002), Pan (2002), Johannes (2004), and Carr and Wu (2007), among others, have shown the presence of jumps and their impact on option pricing and other financial applications.

<sup>3</sup>See Aït-Sahalia (2004), Aït-Sahalia and Jacod (2007), Huang and Tauchen (2005), Barndorff-Nielsen and Shephard (2005), Andersen, Bollerslev, and Dobrev (2007), Jiang and Oomen (2008), Lee and Mykland (2008b), and the references therein.

jump predictors in jump diffusion processes as a new econometric problem and to provide a solution. I name it the *mixed unobservability* problem in this paper because this problem arises from the simultaneous presence of two unobservability problems as follows. The first is due to the problem we usually face when making inference for a continuous-time jump counting process (without diffusion) using discrete observations. The second is due to the presence of the diffusion process. Because of the mixture of these, jumps in continuous-time models become unobservable and this makes the identification of jump predictors difficult. As a resolution, I suggest a test called Jump Predictor Test (JPT), which has two stages. The idea underlying this JPT is simple. I first extract jumps from the return series over time by the multiple nonparametric jump tests in Stage I. Using these detected jumps, I suggest the likelihood tests to determine the jump predictors in Stage II. I prove that this procedure asymptotically makes the effect of this mixed unobservability problem negligible, and hence achieve the goal of determining jump predictors. I discuss a theory of inference justifying this approach.

Using my proposed test, I perform an extensive empirical study. My analysis in this paper is focused on searching for effective macroeconomic and firm-specific jump predictors in U.S. individual equity markets and assessing their relative importance and precision. In order to generate unique and broad intra-day evidence, I use high frequency data for 16 years from January 4, 1993 to December 31, 2008 for individual component stocks of the Dow Jones Industrial Average (DJIA) index, collected from the Trade and Quote (TAQ) database. For macroeconomic predictors, I consider various intra-day macroeconomic information available from government agencies along with a Bloomberg search and market jumps detected in the S&P 500 index. For firm-specific jump predictors, I consider real-time stock-specific news events and the publication of analyst recommendation and dividend and stock split dates, and individual stock jumps for

clustering effect.<sup>4</sup>

My empirical results confirm evidence of stochastic jump arrivals. I find that if there are jumps on a trading day, they tend to come in the morning (about 86% before 11:00am) around market opening. However, the average number of jumps a year for each company is about 21, much less than the total number of trading days a year, which leads me to search for jump triggering information.

After controlling for the intra-day seasonality of jump arrivals, I find four most important macroeconomic jump predictors related to U.S. Federal Open Market Committee (FOMC) decisions, overall market jumps detected in the S&P 500 market index, U.S. Nonfarm payroll employment reports, and initial unemployment claims. Except for overall market jumps, this macroeconomic market information is mostly prescheduled, and hence, can be used for long-horizon predictions, although I study short-term jump prediction in this paper. My parsimonious jump intensity model suggests that the likelihood of jump arrivals is significantly increased within a short time horizon such as 30 minutes after these information arrivals. The indicator of times within 30 minutes after FOMC announcements turns out to be the most effective predictor of U.S. individual stock jumps, among all factors considered. Macroeconomic information is in general a better jump predictor than firm-specific information.

I also show that four most important firm-specific jump predictors for large-cap equities are indicators of times within one day before earnings release, within 30 minutes after analyst recommendation, within 3 hours after arrival of previous jumps of own stock, and in the morning hours (9:30am to 11:00am) of dividend dates. It is interesting to observe that we have increased jump probability within 1 day before earnings release, which suggests the possibility of information

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<sup>4</sup>All the information data also satisfy the requirement for jump predictors. See Table 4 for details of data, sample period, frequencies, source, and the requirement for predictors in subsection 1.1.

leakage before the pre-scheduled announcements.

Based on the new estimation results on the sensitivity of jumps to both macroeconomic and firm-specific predictors, I extracted jumps that are related to only macroeconomic jump predictors during my sample period in order to distinguish systematic jump components. I find that on average systematic jumps account for more than 14% of the total jump intensity. Furthermore, I show that all the companies under consideration experienced more systematic jumps during the second half of my sample period after January 2001, indicating that we have been exposed to greater systematic risk through large jump risk in recent years.

I expect to broadly contribute to the literature by providing an efficient method for solving similar problems as I considered in this paper. For example, the proposed method can be applied to individual stock prices, which I used in this study, as well as to all kinds of other financial time series such as bond prices, exchange traded funds, exchange rates, interest rates, volatility, international market indices, etc., in relation to market information.<sup>5</sup> Parts of my empirical evidence contributes to the other literature that studies the impact of macroeconomic information in financial markets.<sup>6</sup> Specifically, using the powerful jump predictor test, which enables us to study jump dynamics within a day, this paper uncovers a new evidence on how systematic jump risk in U.S. individual equity markets is affected by macroeconomic information release.

The remainder of the paper is organized as follows. Section 1 sets up a general theoretical framework. After discussing the intuition behind the development, Section 2 introduces the test,

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<sup>5</sup>A related study by Duffie, Saita, and Wang (2007) investigates corporate default prediction using stochastic covariates. Though theirs is not within a jump diffusion framework, it indirectly demonstrates that identifying and utilizing predictors of dramatic financial market events could enhance relevant models in financial applications.

<sup>6</sup>There are studies on the impact of macroeconomic fundamentals such as Andersen, Bollerslev, Diebold, and Vega (2003), Wongswan (2006), Andersen, Bollerslev, and Huang (2007), among others, for foreign currency exchange markets, futures markets, treasury (bond) markets, and international stock markets. These studies are not particularly designed for pinning down predicting information related to intra-day jumps. As far as I know, the evidence I provide is new for jumps in U.S. individual equity markets.

explains the theory of inference, and discusses finite sample performance. Then, empirical evidence found in U.S. equity markets is presented in Section 3 and its implication for systematic jumps is discussed in Section 4. Finally, I conclude in Section 5.

## 1 A Theoretical Model

I employ a one-dimensional asset return process with a complete probability space  $(\Omega, \mathcal{F}_t, \mathcal{P})$ , where  $\Omega$  is the set of market events,  $\{\mathcal{F}_t : t \in [0, T]\}$  is an information filtration<sup>7</sup> for market participants up to time  $t$ , and  $\mathcal{P}$  is a data-generating measure in continuous time. Let the continuously compounded return be written as  $d \log S(t)$  for  $t \geq 0$ , where  $S(t)$  is the asset price at  $t$  under  $\mathcal{P}$ . For simplicity, I illustrate here a univariate model of individual asset returns.

According to empirical evidence of jumps and relevant models from the literature, I assume that a log return process  $d \log S(t)$  is represented by the following stochastic differential equation (SDE):

$$d \log S(t) = \mu(t)dt + \sigma(t)dW(t) + Y(t)dJ(t), \quad (1)$$

where  $W(t)$  is an  $\mathcal{F}_t$ -adapted standard Brownian motion and the drift  $\mu(t)$  and spot volatility  $\sigma(t)$  are  $\mathcal{F}_t$ -adapted and bounded processes. Assumptions on the drift and volatility are stated in the Appendix for readers' convenience. These assumptions allow for stochastic drift and volatility, each of which can depend on itself.<sup>8</sup> This model without its jump component describes diffusive risk in returns due to normal randomness in markets. So long as drift and volatility satisfy the assumptions, I do not impose any further specifications on them.

For the jump risk, I include  $Y(t)dJ(t)$ , which describes more dramatic, irregular, and unusually

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<sup>7</sup>See Protter (2004) for the usual technical conditions this filtration satisfies.

<sup>8</sup>I do not assume the presence of jumps in volatility. One possible way to improve the tests in this respect would be to modify the tests with backward windows, which use the last  $K - 1$  observations to estimate local volatility and compare it to the first return in the window. I am currently developing the theoretical justification for this alternative method.

large risk in returns. In particular,  $Y(t)$  represents jump size, and has its mean  $\mu_y(t)$  and standard deviation  $\sigma_y(t)$ . I assume that jump sizes  $Y(t)$  are independent of each other and identically distributed and independent of other random components. Hence, the jump counting process  $J(t)$  and the diffusion  $W(t)$  are independent from one another. Empirical evidence of dependence in volatility risk is usually accommodated through stochastic volatility. I aim to model dependence in jump risk similarly as follows.

### 1.1 A Sub-Model for Stochastic Jumps

Early research based on jump diffusion models assumed the rate of jump arrival (jump intensity) to be constant so that jumps occur regularly. However, recent studies by Andersen, Benzoni, and Lund (2002), Chernov, Gallant, Ghysels, and Tauchen (2003), Pan (2002), Johannes, Kumar, and Polson (1999), and Maheu and McCurdy (2004), among others, recognize the fact that the process governing jump arrivals can be dynamic and heterogeneous with respect to the type of news and other relevant market information. Accordingly, I frame a structure that allows the jump arrival mechanism to incorporate any heterogeneous information flow over time.

I set  $J(t) = \int_0^t dJ(s)$  to be a doubly stochastic Poisson process, that is a nonhomogeneous Poisson process with an integrated stochastic intensity  $\Lambda_\theta(t) = \int_0^t d\Lambda_\theta(s)ds$ .<sup>9</sup> The instantaneous intensity process with respect to filtration up to time  $t$  is  $d\Lambda_\theta(t) = E(dJ(t)|\mathcal{F}_{t-})$ . Its integrated intensity process  $\Lambda_\theta(t)$  is specified by a  $q$ -dimensional parameter  $\theta = (\theta_1, \dots, \theta_q) \in \Theta$ , a subset of the  $q$ -dimensional Euclidean space. Thus, I write

$$\Lambda_\theta(t) = \int_0^t d\Lambda_\theta(s)ds = \gamma(t, X(t); \theta), \quad (2)$$

where  $X(t)$  denotes the conditional information predictors that affect the likelihood of jump

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<sup>9</sup>This doubly stochastic Poisson process is also known as a Cox process and applied in modeling corporate default events in recent studies by Duffie, Saita, and Wang (2007) and Das, Duffie, Kapadia, and Saita (2007), among others.

arrivals, and  $\gamma$  is a general function of time and the predictors. The counting process I consider in this paper is assumed to be nonexplosive with finite jump intensity. This assumption excludes models with infinite-activity jumps.<sup>10</sup>

Here, I make an important note on the minimal assumption imposed on  $X(t)$ . I require  $X(t)$  to be a  $\mathcal{F}_t$ -predictable process. In other words, the values of  $X(t)$ 's components are supposed to be determined according to information observable at anytime up to  $t$ . Therefore,  $X(t)$  can be deterministic variables such as time (time of the day or day of the week), exogenous information variables available before  $t$ , jump indicators observed at anytime up to  $t$ , waiting time since the last jump time, jump indicators from other markets observed at anytime up to  $t$ , or other state variables forecasted using a conditional expectation based on dynamic (time-series) models. For the formation of the expectation, we are not restricted by any type of static or dynamic model specification or estimation procedures. This allows the procedure to accommodate dynamic jump model developments. I only impose assumptions on this general function  $\gamma$ , as stated in the Appendix. The integrated intensity function  $\Lambda_\theta(t)$  is only required to be continuous and differentiable so that the martingale central limit theorem can hold and the solution for the corresponding score function exists and is consistent.

I assume a time horizon  $T$  and a number of observations  $n$ . The observation of asset prices  $S(t)$  and informational predictor  $X(t)$  occurs only at discrete times  $0 = t_0 < t_1 < \dots < t_n = T$ . For simplicity, I set observation times as equally spaced:  $\Delta t = t_i - t_{i-1} = \frac{T}{n}$ . This simplified assumption can easily be generalized to non-equidistant cases by letting  $\max_i |t_i - t_{i-1}| \rightarrow 0$ .

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<sup>10</sup>There is some evidence of likely presence of extremely small jumps (see Ait-Sahalia and Jacod (2007), Todorov and Tauchen (2008), among others). Although it could be interesting to characterize this type of extremely small jumps, it is beyond the scope of this current paper.

## 2 Inference for Stochastic Jump Model

In this section, I explain a new econometric problem in the analysis of jump risk dynamics in jump diffusion frameworks and describe the relevant theory of inference.

In making inference for sub-jump models within jump diffusion processes using discrete observations, econometricians face two different unobservability problems simultaneously. The first is the problem which we usually face when making inference for a continuous-time counting process (without diffusion) using discrete observations. The second problem is due to the presence of a diffusion process. The mixture of these two problems makes jumps in continuous time unobservable, hence they become latent variables. Since this problem has never been recognized in the literature, I first name it in this paper the *mixed unobservability* problem. Figure 2 illustrates graphically how this mixed unobservability problem is resolved by my proposed test. I suggest relying on the partial likelihood (which will be defined later in subsection 2.1) by showing that it converges to the true likelihood in continuous time in the limit.

To approximate latent jumps in continuous time, jump arrivals first need to be distinguished in the jump diffusion models. A few researchers have proposed methodologies for such purpose. Examples include Aït-Sahalia and Jacod (2009), Barndorff-Nielsen and Shephard (2006), Huang and Tauchen (2005), Andersen, Bollerslev, and Dobrev (2007), Jiang and Oomen (2008), and Lee and Mykland (2008b), among others.<sup>11</sup> In this paper, I first discuss what kind of tests are admissible and define a class of jump tests which facilitate the theoretical justification. Building upon the jump detection, I propose a new approach, namely a two-stage, semi-parametric JPT

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<sup>11</sup>There are a few important studies for the analysis of diffusion risk,  $\sigma(t)$ . [See Aït-Sahalia (2004), Aït-Sahalia and Jacod (2007), Barndorff-Nielsen and Shephard (2004), and Barndorff-Nielsen, Shephard, and Winkel (2005) for more detail.] It has been proven therein that we can consistently estimate the instantaneous stochastic volatility by the realized bi-power (more generally, truncated power or multi-power) variation. Hence, application of the realized multi-power or truncated-power variation is sufficient for the separate analysis on dynamics of diffusive risk part of the assumed model.

described below.

The idea underlying this JPT is simple. Jumps are first extracted from the return series over time by the multiple nonparametric jump tests. The detected jumps become partial observations of the continuous-time jump counting process  $J(t)$ . Using these detected jumps, I suggest the likelihood tests to determine the jump predictors. I graphically illustrate the idea of these two stages in panels A and B of Figure 1. Each step is explained in the following subsections and provide a guide to selecting good jump predictors and prediction error distribution.

## 2.1 Two-Stage Semi-Parametric JPT

In this subsection, I provide a detailed discussion of the JPT.

### 2.1.1 Stage I: Application of Nonparametric Jump Detection Test

In order to use the likelihood approach undertaken in Stage II, it is important to note the requirements for the jump test in Stage I. The test should be able to detect a jump arrival in a time interval which shrinks to zero as we increase the frequency of observations. For simplicity, I define a class of nonparametric jump detection tests which satisfy the requirements as follows and on which my discussion is based.

#### Definition 1. Jump Detection Tests Admissible for the Stage I of JPT

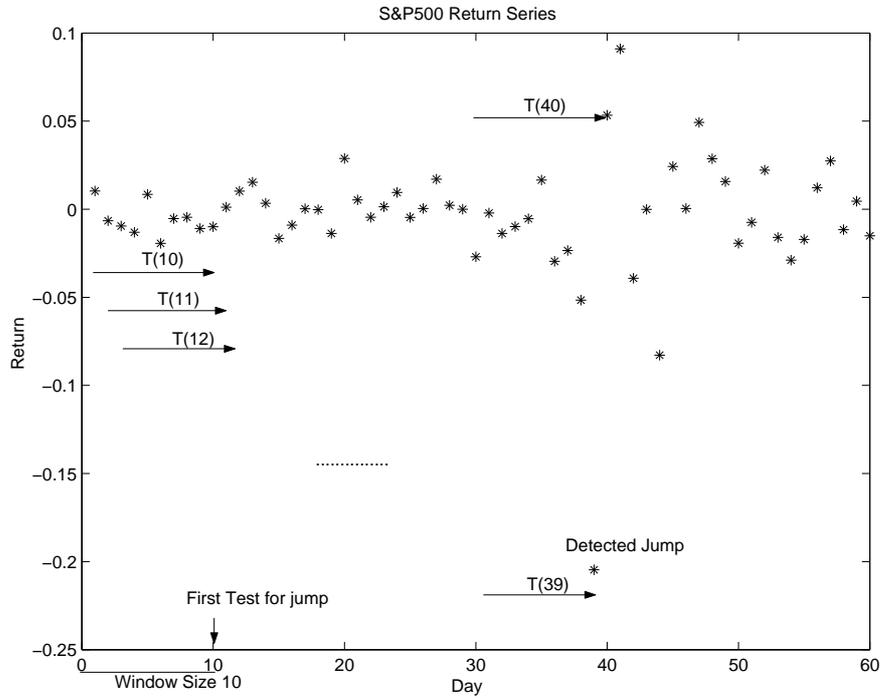
The statistic  $\mathcal{L}(i)$ , which tests at time  $t_i$  whether there was a jump from  $t_{i-1}$  to  $t_i$ , is defined as

$$\mathcal{L}(i) \equiv \frac{\log S(t_i)/S(t_{i-1})}{\widehat{\sigma}(t_i)\sqrt{\Delta t}}, \quad (3)$$

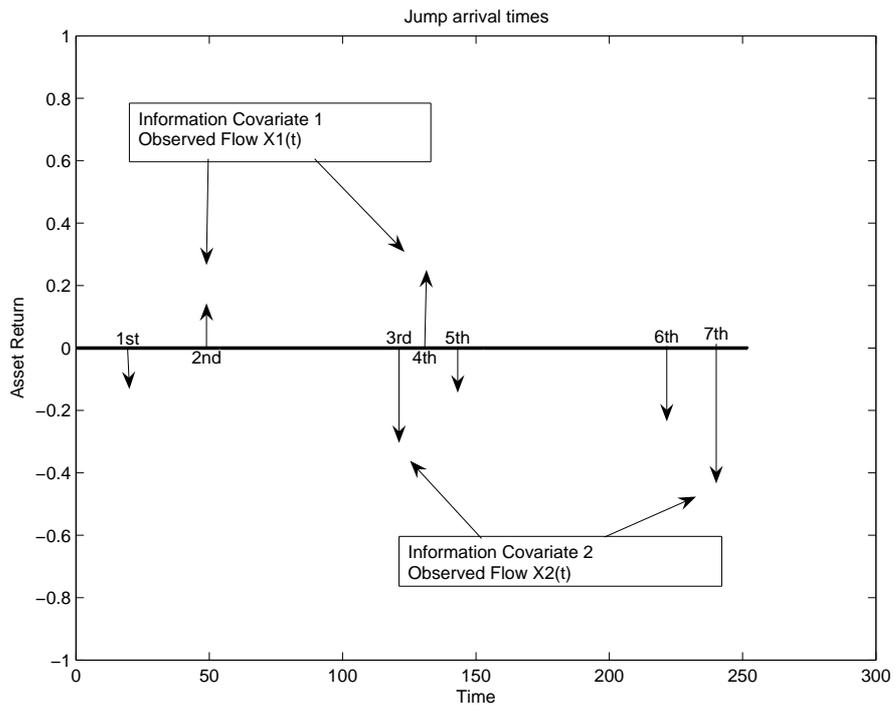
where  $\widehat{\sigma}(t_i)$  is an instantaneous volatility estimator, which can be chosen from one of the following:

Figure 1: Intuition of Jump Predictor Test (JPT)

Panel A: Stage I by nonparametric jump detection test



Panel B: Stage II by the predictor test through likelihood



A. *Estimator based on bipower variation.*

$$\widehat{\sigma}(t_i)^2 \equiv \frac{1}{(K-2)c^2} \sum_{j=i-K+2}^{i-1} |\log S(t_j)/S(t_{j-1})| |\log S(t_{j-1})/S(t_{j-2})|, \quad (4)$$

where  $K = b\Delta t^a$  with  $-1 < a < -1/2$  for some constant  $b$ , and  $c = E|u| \approx 0.7979$  with  $u$  being a standard normal random variable.

B. *Estimator based on truncated power variation.* For any  $g > 0$  and  $0 < \tilde{\omega} < 1/2$ ,

$$\widehat{\sigma}(t_i)^2 \equiv \frac{\Delta t^{-1}}{K} \sum_{j=i-K}^{i-1} (\log S(t_j)/S(t_{j-1}))^2 I_{\{|\log S(t_j)/S(t_{j-1})| \leq g\Delta t^{\tilde{\omega}}\}}, \quad (5)$$

where  $K = b\Delta t^a$  with  $-1 < a < 0$ , for some constant  $b$ .

$K$  in the definition is a window size within which a local movement of the process is considered.

**Definition 1.A** and **1.B** are studied in Lee and Mykland (2008b) and Lee and Hannig (2009). For the asymptotic arguments,  $K$  needs to satisfy slightly different conditions as stated depending on the choice of volatility estimator. All the conditions, however, are imposed for the same purpose to make the effect of jumps in volatility estimation negligible in theory. These tests satisfy the properties stated in **Proposition 1** as follows.

### Proposition 1. Properties of Admissible Tests

Let  $\mathcal{L}(i)$  be as in **Definition 1** and **Assumption C** is satisfied. Then, the following statements hold, as  $\Delta t \rightarrow 0$ .

A. If there is no jump in  $(t_{i-1}, t_i]$ , i.e.  $dJ(t_i) = J(t_i) - J(t_{i-1}) = 0$ , then,

$$\mathcal{L}(i) \xrightarrow{\mathcal{D}} \mathcal{N}(0, 1),$$

where  $\mathcal{N}(0, 1)$  denotes a standard normal random variable.

B. If there is a jump at  $\tau$  within  $(t_{i-1}, t_i]$ , i.e.  $dJ(t_i) = J(t_i) - J(t_{i-1}) = 1$ , then,  $\mathcal{L}(i) \rightarrow \infty$ .

C. Let the rejection region for a chosen test be  $\mathcal{R}_n(\alpha_n) = (-\infty, -q_{\alpha_n}S_n + C_n) \cup (q_{\alpha_n}S_n + C_n, \infty)$ , where  $q_{\alpha_n}$  is the  $(1 - \alpha_n)$ th percentile of a standard gumbel distribution with  $\alpha_n$  being the significance level,  $C_n = (2 \log n)^{1/2} - (\log \pi + \log(\log n))/(2(2 \log n)^{1/2})$ , and  $S_n = 1/(2 \log n)^{1/2}$  with  $n$  being the number of observations. Then,

$$d\hat{J}(t_i) = \hat{J}(t_i) - \hat{J}(t_{i-1}) = I(\mathcal{L}(i) \in \mathcal{R}_n(\alpha_n)) \xrightarrow{P} dJ(t_i) = 1,$$

for any  $(t_{i-1}, t_i]$  with jump and

$$d\hat{J}(t_i) = \hat{J}(t_i) - \hat{J}(t_{i-1}) = I(\mathcal{L}(i) \in \mathcal{R}_n(\alpha_n)) \xrightarrow{P} dJ(t_i) = 0,$$

for any  $(t_{i-1}, t_i]$  without jump.

Notice in particular part C of the above proposition. This is a necessary property for meeting the requirement for admissible tests: for every set of discrete time interval during which we do (or do not) have a jump, we do (or do not) detect the jump by the tests used in Stage I.<sup>12</sup> I assume that when we observe a jump in an interval, the order of magnitude of the jump in the realized returns dominates that of the diffusion part of the model. Hence, I use the return itself as the jump size.

### 2.1.2 Stage II: Likelihood Estimation and Tests

In this subsection, I explain why one can naively use “usual” maximum likelihood estimation and related tests on jumps detected in Stage I in order to search for jump predictors.

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<sup>12</sup>This requirement is the same as the absolute continuity of two different probability measures on the same measurable sets. In this case, two equivalent measures are the discrete data-generating measure  $\mathcal{P}_{L_n}$  for the full likelihood and the discrete data-generating measure  $\mathcal{P}_{PL_n}$  for the partial likelihood. These are different from the continuous data generating measure  $\mathcal{P}$  for the true likelihood. See the definition of the three likelihoods in **Definition 3**. The global misclassification of one of the tests in the class of admissible test (**Definition 1.A**) was mentioned in Lee and Mykland (2008b). However, this local property as stated needs to be satisfied.

Since my jump model is specified by a continuous-time process but the data are sampled only at discrete times, the likelihood function of the jump model has to be approximated. To illustrate the approximation, I need to use a notion of *product integration* as follows.

## Definition 2. Product Integration

The product integration  $\widetilde{\prod}$  over  $[0, T]$  of any cadlag (left continuous and right limit) function with  $t_i \in [0, T]$  is defined as

$$\widetilde{\prod}_{s \in [0, T]} (c_1 + c_2 dg(s))^{c_3 + c_4 dh(s)} = \lim_{\Delta t \rightarrow 0} \prod_{1 \leq i \leq n} (c_1 + c_2 dg(t_i))^{c_3 + c_4 dh(t_i)}, \quad (6)$$

where  $c_1, c_2, c_3,$  and  $c_4$  are some constants,  $dg(t_i) = g(t_i) - g(t_{i-1}), dh(t_i) = h(t_i) - h(t_{i-1}),$  and  $\Delta t = |t_{i+1} - t_i|,$  when  $t_0 = 0 < t_1 < t_2 < \dots < t_n = T$  are discrete times to make a partition of  $[0, T].$

This concept of product integration can be understood as a product in continuous time. Using this notation, we have the following three likelihood functions that are involved in continuous-time jump model inference.

## Definition 3. Three Likelihoods

### A. True Likelihood of Continuous-time Jump Model

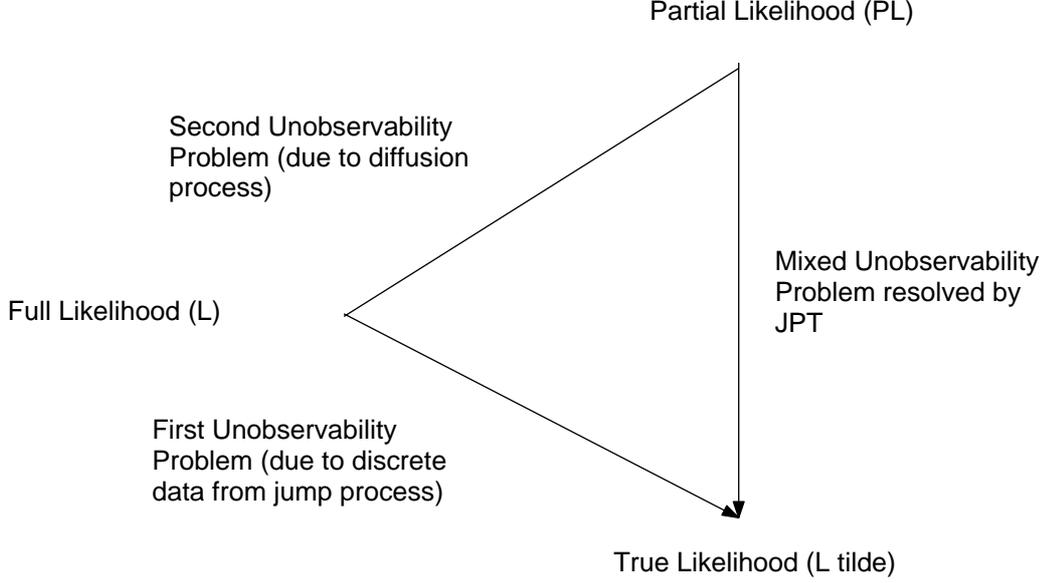
$$\widetilde{L}(\theta | \mathcal{F}_T) = \widetilde{\prod}_{s \in [0, T]} d\Lambda_\theta(s)^{dJ(s)} \widetilde{\prod}_{s \in [0, T]} (1 - d\Lambda_\theta(s))^{1 - dJ(s)}, \quad (7)$$

where  $d\Lambda_\theta(t)$  satisfies the equation  $\Lambda_\theta(t) = \int_0^t d\Lambda_\theta(s) = \gamma(t, X(t); \theta)$  and  $X(t)$  is a  $\mathcal{F}_t$ -predictable process.

### B. Full Likelihood of Pure Jump Models without Diffusion

$$L_n(\theta | \mathcal{F}_T) = \prod_{1 \leq i \leq n} d\Lambda_\theta(t_i)^{dJ(t_i)} \prod_{1 \leq i \leq n} (1 - d\Lambda_\theta(t_i))^{1 - dJ(t_i)}, \quad (8)$$

Figure 2: **How the Mixed Unobservability Problem is Resolved**



where  $dJ(t_i) = J(t_i) - J(t_{i-1})$  and  $d\Lambda_\theta(t_i) = \Lambda_\theta(t_i) - \Lambda_\theta(t_{i-1})$ .

### C. Partial Likelihood of Sub-Jump Models in Jump-Diffusion

$$PL_n(\theta|\mathcal{F}_T) = \prod_{1 \leq i \leq n} d\hat{\Lambda}_\theta(t_i)^{d\hat{J}(t_i)} \prod_{1 \leq i \leq n} (1 - d\hat{\Lambda}_\theta(t_i))^{1-d\hat{J}(t_i)}, \quad (9)$$

where  $d\hat{\Lambda}_\theta(t_i) = E[I_{\{\mathcal{L}(i) \in \mathcal{R}_n(\alpha_n)\}}]$  and  $d\hat{J}(t_i) = I_{\{\mathcal{L}(i) \in \mathcal{R}_n(\alpha_n)\}}$ , with  $\mathcal{L}(i)$ ,  $\mathcal{R}_n(\alpha_n)$ , and  $\alpha_n$  as in

#### Proposition 1.

For the continuous-time jump model, we have the well-defined continuous-time (conditional) likelihood function  $\widetilde{L}(\theta|\mathcal{F}_T)$  as defined in **Definition 3.A**. The definition of product integration and the (conditional) likelihood function suggest that we can approximate the likelihood function by replacing the instantaneous changes by the increments of  $J(t)$  and  $\Lambda_\theta(t)$  over intervals from  $t_{i-1}$

to  $t_i$ , and forming the corresponding finite products. Hence, if there is no diffusion term, the actual data analysis can be done by the full likelihood as in **Definition 3.B**. Since we do not have complete data for the full likelihood function, we should depend on test results from Stage I using the partial likelihood as in **Definition 3.C**. Currently, there is no theoretical ground in the literature for us to simply use **Definition 3.C** for significance tests to determine jump predictors. I explain here that this partial likelihood based on the detected jumps in Stage I is sufficient as the objective function to be maximized.

As the first step for the argument, I show in the following proposition how the second unobservability problem due to the presence of diffusion process is resolved, as  $\Delta t \rightarrow 0$ .

**Proposition 2. Asymptotic Equivalence of Partial Likelihood to Full Likelihood**

*Suppose that Assumptions C and D hold. Let  $L_n(\theta|\mathcal{F}_T)$ , and  $PL_n(\theta|\mathcal{F}_T)$  be as in Definition 3.B and 3.C with  $\mathcal{F}_T$  being the information filtration up to time  $T$ . The test used in Stage I satisfies the properties of admissible tests in Proposition 1. Then, as  $\Delta t \rightarrow 0$  and  $\alpha_n \rightarrow 0$ ,*

$$\frac{PL_n(\theta|\mathcal{F}_T)}{L_n(\theta|\mathcal{F}_T)} \xrightarrow{P} 1, \tag{10}$$

*when there is a finite number of jumps during the time horizon  $[0, T]$ .*

This proposition only tells us that the probability of the full likelihood and partial likelihood being different from each other becomes negligible as we increase the frequency of observations. In other words, it justifies performing likelihood inference based on detected jumps as if they are from pure jump models in the absence of a diffusive part.<sup>13</sup> However, it is not about the

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<sup>13</sup>This result can essentially be achieved by the application of jump tests suggested in Stage I, which enables us to separate jumps from the jump diffusion models. See Lee and Mykland (2008b) and Lee and Hannig (2009) for more details on this issue.

relationship between  $PL_n(\theta|\mathcal{F}_T)$  and  $L(\theta|\widetilde{\mathcal{F}}_T)$ . Therefore, this result itself does not guarantee in theory that the likelihood inference using detected jumps would hold in continuous time. Here, we need the following important proposition to resolve the first unobservability problem. This connects the partial likelihood which we can use for my actual analysis and the true likelihood.

**Proposition 3. Partial Likelihood is Sufficient.**

Suppose **Assumptions C** and **D** hold. Let  $L(\theta|\widetilde{\mathcal{F}}_T)$  and  $PL_n(\theta|\mathcal{F}_T)$  be as in **Definition 3.A** and **3.C** with  $\mathcal{F}_T$  being the information filtration up to time  $T$ . The test used in Stage I satisfies the properties of admissible tests in **Proposition 1**. Then, as  $\Delta t \rightarrow 0$  and  $\alpha_n \rightarrow 0$ ,

$$\frac{PL_n(\theta|\mathcal{F}_T)}{L(\theta|\widetilde{\mathcal{F}}_T)} \xrightarrow{P} 1, \tag{11}$$

when there is a finite number of jumps during the time horizon  $[0, T]$ .

Although this simple result with the likelihood ratios appears subtle, it is in fact a crucial step in enabling us to provide the asymptotic distributions of jump predictor tests because now the limiting behavior between the partial likelihood which can be used for actual analysis and the true likelihood in continuous time becomes clear. This likelihood approximation technique has never been used for making inference on jump processes, and it can be applied in other contexts.<sup>14</sup>

Once the above convergence is established, the main results can be obtained for how to select jump predictors for the purpose of this study and the prediction error distribution as stated in **Theorem 1** below.

**Theorem 1. Jump Predictor Test (JPT)**

Suppose that **Assumptions C** and **D** hold. Let  $X(t) = [X_1(t), X_2(t), \dots, X_p(t)]$  be the vector of

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<sup>14</sup>A similar technique is applied for volatility or leverage effect estimation in Mykland and Zhang (2009).

information predictors that affect  $\Lambda_\theta(t)$  and  $\hat{\theta} = [\hat{\theta}_1, \dots, \hat{\theta}_p]$  be the maximum likelihood estimate for the effect parameter  $\theta$  based on  $PL_n(\theta|\mathcal{F}_T)$  function as in **Definition 3.C**. Then, the following results hold, as  $\Delta t \rightarrow 0$ .

A.  $X_k(t)$  is a good jump predictor if  $\text{Prob}(z > \frac{\hat{\theta}_k}{SE(\hat{\theta}_k)}) < \beta$ , where  $\beta$  is the significance level and  $z$  is a standard normal random variable.  $SE(\hat{\theta}_k)$  can be found in the usual manner from the covariance matrix of  $\mathcal{Z}^{-1}(\hat{\theta})$ , with  $-\mathcal{Z}(\theta)$  being the matrix of second-order partial derivatives of the log- $PL_n(\theta|\mathcal{F}_T)$ .<sup>15</sup>

B. The prediction error for jump intensity,  $d\hat{\Lambda}_\theta(t) - d\Lambda_\theta(t)$ , is asymptotically normal with its mean 0 and variance  $\nabla d\Lambda'_\theta \mathcal{Z}^{-1}(\theta) \nabla d\Lambda_\theta$ , where  $\nabla d\Lambda_\theta$  is the partial derivatives of  $d\Lambda_\theta(t)$  with respect to  $\theta$ .<sup>16</sup>

My final solution appears similar to the usual MLE methods. However, I emphasize that this work is distinguished from that of others in that I solve the newly introduced “mixed unobservability” problem in the model framework and discuss necessary requirements for admissible tests in the analysis. I also illustrate a theoretical justification for the naively applied likelihood approach. Finally, I remark that the term “partial likelihood” is also used in the statistics literature for continuous-time counting process inference using the full likelihood, as in **Definition 3.B**. I emphasize that the partial likelihood in this paper is different from the existing approach and is specific to the aforementioned mixed unobservability problem in the jump diffusion framework.

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<sup>15</sup>The formula for  $-\mathcal{Z}(\theta)$ , the matrix of second-order partial derivatives of the log-partial likelihood function, is

$$\mathcal{Z}(\theta) = - \sum_{1 \leq i \leq n} \frac{\partial^2}{\partial \theta_j \partial \theta_l} \log d\hat{\Lambda}_\theta(t_i) d\hat{J}(t_i) - \sum_{1 \leq i \leq n} \frac{\partial^2}{\partial \theta_j \partial \theta_l} \log(1 - d\hat{\Lambda}_\theta(t_i))(1 - d\hat{J}(t_i)). \quad (12)$$

<sup>16</sup>As usual,  $\nabla d\Lambda_\theta$  can be estimated by replacing  $\theta$  with  $\hat{\theta}$ .  $\hat{\theta}$  is asymptotically normal under the null hypothesis around its mean  $\theta_0$  with its covariance matrix  $-\mathcal{Z}^{-1}(\theta_0)$ .

## 2.2 Simulation Study

In this subsection, I examine the effectiveness of my jump predictor test using a Monte Carlo simulation. This study shows the finite sample performance of the asymptotic results. In summary, overall results prove that the JPTs do perform well in distinguishing the effects of predictors. For series generation, I use the Euler-Maruyama Stochastic Differential Equation (SDE) discretization scheme [see Kloeden and Platen (1992)], an explicit order 0.5 strong and order 1.0 weak scheme. To avoid the starting value effect on series generation, I discard the burn-in period – the first part of the whole series (1 month) – every time I generate time series and keep observations over 1-year time horizon.

I consider two models from the general model stated in Section 1, with constant volatility and stochastic volatility along with one information predictor to affect the stochastic jump intensity. The encompassing model is

$$d \log S(t) = \mu(t)dt + \sigma(t)dW(t) + Y(t)dJ(t), \quad (13)$$

where the constant volatility model sets  $\sigma(t) = \sigma = 30\%$  and the stochastic volatility model assumes the Affine model of Heston (1993), specified as

$$d\sigma^2(t) = \kappa (\theta - \sigma^2(t)) dt + \omega\sigma(t)dB(t), \quad (14)$$

where  $B(t)$  denotes a Brownian Motion. The parameter values used for stochastic volatility simulation are the estimates from equity markets reported in the empirical study by Li, Wells, and Yu (2008), Table 4. These are  $\kappa = 0.0162$ ,  $\theta = 0.8465$ , and  $\omega = 0.1170$ . For the stochastic jump intensity, I assume the following functional form

$$d\Lambda_\theta(t) = \frac{1}{1 + \exp(-\theta_0 - \theta_1 X(t))}, \quad (15)$$

where  $X(t)$  is set to be an indicator of monthly information release, which becomes equal to 1 once every month (12 times a year) and equals 0, otherwise. I choose this generalized linear model for the jump intensity in order to ensure that the intensity (probability) is within the admissible range of  $[0,1]$ . I suppose that an analyst tests his or her hypothesis of whether the monthly information release is a significant jump predictor by the proposed test. Three thousand series of returns over 1 year ( $T = 1$ ) are simulated at an intraday 15-minute frequency. I assume that the number of trading day per year is 252 with 24 trading hours per day. Hence, the total number of observations per year is 24,192(= 252  $\times$  96). The usage of sampling with 15-minute frequency is recommended for Stage I [see Lee and Mykland (2008b)].  $\theta_0$  and  $\theta_1$  are set at  $-4$  and  $6$ , so that the probability of jump arrival when there is no news release is 1.8% and the probability of jump arrival when there is news release is 88%. The standard deviations of jump size distribution are assumed at three different levels. In particular, I consider the cases with  $\sigma_y = 1\sigma, 2\sigma,$  and  $3\sigma$  for the constant volatility models and the cases with  $1\tilde{\sigma}, 2\tilde{\sigma},$  and  $3\tilde{\sigma}$  where  $\tilde{\sigma}$  is the mean of volatility  $\tilde{\sigma} = E(\sigma(t))$  for the stochastic volatility models.

Table 1 reports simulation results averaged over three thousand simulation runs. Parameter estimates, their standard errors, and p-values from the second stage analysis after using **Definition 1.A** and **Definition 1.B** in Stage I are listed in the table. The simulation results indicate that as long as we consider finite activity jumps in the finite samples, the jump size change does not appear to strongly affect the conclusion on the jump predictors.

I also find that the bias in parameter estimates are slightly greater when using **Definition 1.A** than when using **Definition 1.B**. On the other hand, standard errors of parameter estimates are lower, hence, p-values are smaller when using **Definition 1.A** than when using **Definition 1.B**. This is because although **Definition 1.B** is more likely to detect smaller jumps than **Definition**

**1.A**, the results from **Definition 1.B** are less stable than those from **Definition 1.A**. Despite the trade-offs in bias and variance, however, ultimate conclusions from the two tests are qualitatively similar and I report my results based on **Definition 1.A** for the empirical study because this gives greater precision in recognizing the good predictors.

### **3 Empirical Analysis for U.S. Individual Equity Markets**

In this section, I perform data analysis on the major U.S. individual equity markets. After describing data for individual equity jumps detected in Stage I and data for information variables used to create jump predictors in Stage II, I describe the specific model I chose and discuss the empirical evidence I found. I am particularly interested in understanding the sensitivity of individual equity jump intensity to macroeconomic jump predictors as well as firm-specific jump predictors to assess their relative importance and in distinguishing systematic jumps.

#### **3.1 Data for Individual Equity Jumps**

I select the most actively traded U.S. large-cap component stocks in the Dow Jones Industrial Average (DJIA) traded on the New York Stock Exchange (NYSE). Data are collected from the TAQ database. The TAQ database contains tick-by-tick data for trading information such as transaction times, prices, exchanges, and volume information beginning with the year 1993. My sample extends from January 4, 1993 to December 31, 2008, for a total of 4,017 trading days over 16 years. It is based on price data from 9:30am to 4:00pm, the normal trading hours on the NYSE. I select transactions on the NYSE in order to maintain a sufficient degrees of liquidity and to maintain a similar organization of trading mechanisms and trading hours across different stocks. For this reason, two among the 30 stocks are excluded because they are traded on the

NASDAQ. I also exclude another five stocks due to either significant missing data problem or unusual name changes, which could create significant bias in empirical results. I list the names of the remaining 23 stocks under my consideration in Table 1, along with their ticker symbols.<sup>17</sup> The table also includes the total number of tests undertaken, number of detected jumps over my sample period, and daily, monthly, and yearly frequency of jumps on average.

Since the simulation study shows that a 15-minute frequency is enough for the tests to achieve sufficient power, I choose to use 15-minute stock returns by taking the differences of log transaction prices at 15-minute intervals and multiplying all returns by 100 to present them as percentages. A 15-minute frequency is also chosen to mitigate the impact of market microstructure noise. When dealing with high-frequency intraday returns, as noted in Andersen, Bollerslev, Diebold, and Ebens (2001), market microstructure effects can be avoided with 5-minute or longer frequency. Therefore, the evidence presented in this paper is robust to the effect of market microstructure noise.

To avoid unnecessary data recording errors, I also preprocess the raw data as follows. All stocks I selected from the DJIA index in this study are assured to pass the active trade filter (50 trades per day).<sup>18</sup> For transactions that happen at the same time, I take the first transaction price recorded in the database. I exclude all recording errors such as zero prices. As noted in Ait-Sahalia, Mykland, and Zhang (2006), high frequency data may contain *bounce-back* type data errors caused by extreme round trips of recorded prices to unreasonably different price levels. If returns from a stock are followed by returns with opposite signs and similar magnitudes and if the magnitudes of any jumps in the stock are significantly different from those without the

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<sup>17</sup>I listed in the table the symbols being used as of December 31, 2008. For the data collection, we first checked if there were changes in the symbol and made sure the observations are from the same company before and after the change.

<sup>18</sup>This filtering rule was used in Easley, O'Hara, and Srinivas (1998), Chan, Chung, and Fong (2002), and Tookes (2006) for their high frequency data analysis.

bounce-back effect, I exclude those returns from my consideration.

Because the trading on the NYSE is interrupted overnight (after 4pm and before 9:30am on next days), one may regard instantaneous volatility are not observable for overnight returns and hence, one should omit all the overnight return data. However, my initial analysis with 15-minute returns shows that overnight returns do not necessarily include jumps. In other words, there are many days with overnight returns that do not include any unusual jumps in high frequency data. If they appear, it is more likely to be in the morning. Hence, in this paper, I take a different view of those returns. Instead of simply removing all the overnight returns, I keep them in time series data. If overnight returns are detected, I report them as jumps. In the second stage, I further control for this in selecting jump predictors. See subsection 3.4 for more detail.<sup>19</sup>

### 3.2 Empirical Results of Detected Jumps from Stage I

In this subsection, I discuss results from Stage I presented in Tables 2 and 3. The significance level for the first stage nonparametric jump detection test is 5%. The outcomes of Stage I are each jump size, jump arrival date, and time.

Table 2 shows the descriptive statistics of detected jumps. In particular, it includes the average number of jumps detected over 1 year, 1 month, and 1 day. Each year, stocks in the sample experience about 21 jumps on average, from 15 for XOM to 25 for AA, BA, or HPQ. Every month, one to two jumps occur on average in each stock. The daily average rate of jump arrival is 8%. This rate is calculated with the assumption that the jump arrival rate is constant over time. I observe, however, that jumps do not occur regularly. Therefore, models with constant jump intensities are not appropriate. Table 3 presents more specifically when in a day these jumps

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<sup>19</sup>As the robustness check, I also applied the jump test with the backward rolling window and I found similar evidence. By backward rolling window, I mean estimating the volatility level using the last  $K-1$  observations in the window and compare the volatility estimate with the first return in the window by taking the ratio.

arrive. It reports the percentages of detected jumps during specific time intervals in a trading day among all realized jumps. I divide the trading hours of the NYSE, 9:30am to 4:00pm, into seven categories. I find that more than 86% of jumps arrive before 11:00am, around the time of market opening. In order to control for the opening market effect in my analysis, I include time of the day indicators in the second stage inference.<sup>20</sup>

Summarizing Tables 2 and 3, I conclude that if jumps occur, they tend to arrive in the morning, but every overnight return does not necessarily includes jumps. In fact, there are far fewer jumps than the number of trading days. The NYSE trading mechanism for opening markets provides a naturally controlled experiment framework to study whether the market interruption itself is the cause of jumps in stock prices. Based on my results, I conclude that without information waiting to be reflected in prices, the interruption itself does not trigger jumps. At this stage, I hypothesize that jumps are triggered when investor's demand for trading increases due to the information flow in a relatively illiquid market. To study which information is more important for jump prediction, I now select information variables in the next subsections.

### **3.3 Data for Jump Predictors $\mathbf{X}(t)$**

In this subsection, I describe raw data used to create jump predictors for Stage II of my analysis. Table 4 lists details on information variables related to 4 macroeconomic and 4 firm-specific jump predictors I focus on in this study. These 8 variables are selected based on how broadly each variable is significant when it is used for jump predictors of the U.S. individual equity markets. The table contains the names of the information variables, their mnemonic abbreviations, total number of raw data, all dates and times for each variable, data source, and sample period which

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<sup>20</sup>To mitigate the noisy data problem, I removed all the observations which might be driven by the bounce-back effect from the sample. I could further improve by modeling the noise explicitly in the analysis. Though this is interesting, it is beyond the scope of this paper and I am currently developing new methods for this.

is matched exactly to the sample period for the jump data shown in Table 2. The frequency of all information data is set at 15 minutes to exactly match the frequency of jump data from Stage I.

### 3.3.1 Macroeconomic Information Variables

Macroeconomic variables I consider to create jump predictors are U.S. market jumps (MARKET) in the S&P 500 index, Federal Open Market Committee’s news release (FOMC), nonfarm payroll employment report release (NONFARM), and initial unemployment claims news release (JOBLESS). I set 4 different time series of indicators for arrival times of these information. For example, U.S. market jump variable,  $\text{MARKET}(t)$ , is set to be a time series of indicators for arrival times of jumps in the S&P 500 index detected by the nonparametric jump test statistic defined in **Definition 1.A**. The significance level  $\alpha$  applied for detecting U.S. market jumps is 5% and the total number of detected market jumps during the sample period is 446, hence,  $\int_0^T \text{MARKET}(s) = 446$ . FOMC announcements occur every 6 weeks and I have 134 observations, hence,  $\int_0^T \text{FOMC}(s) = 134$ . Nonfarm payroll employment information is released monthly and I have 191 observations, hence,  $\int_0^T \text{NONFARM}(s) = 191$ . Jobless claims information is released weekly, hence there are many more observations for this variable than the other variables:  $\int_0^T \text{JOBLESS}(s) = 834$ . Since NONFARM and JOBLESS information are released outside trading hours at 8:30am in the morning, I set the indicator of  $\text{NONFARM}(t)$  and  $\text{JOBLESS}(t)$  to become 1 at the earliest possible time in which the information would be reflected. In this particular case, the earliest time is 9:30am. Except for U.S. market jumps, these macroeconomic variables are released regularly at a prescheduled time as noted in Table 4 for the most part of the sample period.<sup>21</sup>

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<sup>21</sup>Before April 1995, FOMC news were not released regularly at the time specified in Table 3. I follow Andersen, Bollerslev, Diebold, and Vega (2003) for the irregular release time as in the table note and regular release times of 11:30am for the year before 1994.

In choosing the aforementioned macroeconomic variables, I initiated my investigation with 25 U.S. macroeconomic news information variables and jumps in the S&P 500 index and I chose the 4 most effective variables. In the presence of the listed variables, the following variables were also considered but rejected for lack of significance: GDP advance, GDP preliminary, GDP final, retail sales, industrial production, capacity utilization, personal income, consumer credit, personal consumption expenditures, new home sales, durable goods orders, construction spending, factory orders, business inventory, government budget deficit, trade balance, producer price index, consumer price index, consumer confidence, housing starts, NAPM Index, and leading indicator.<sup>22</sup> These macroeconomic news information variables were selected to capture real-time information release regarding real activity of overall economy, consumption, investment, price, exports, or monetary policy. Some of these variables are found to be significant jump predictors for some companies but they are not as broadly robust or significant as the 4 variables I selected.

### 3.3.2 Firm-specific Information Variable

In the presence of the aforementioned macroeconomic variables, for each company, I consider in my model the following firm-specific variables to create jump predictors: earnings announcement (EARNINGS), analyst recommendation (ANALYST), individual stocks' own past jumps (CLUSTER), and dividend related dates (DIVIDEND). Similar to macroeconomic variables, for a company, say  $c$ , I first set time series of indicators for arrival times of these variables and denote them by  $\text{EARNINGS}_c(t)$ ,  $\text{ANALYST}_c(t)$ ,  $\text{CLUSTER}_c(t)$ , and  $\text{DIVIDEND}_c(t)$ .

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<sup>22</sup>Data source for these data are government agencies such as Bureau of Labor Statistics, Bureau of the Census, Bureau of Economic Analysis, Federal Reserve Board, Conference Board, and Bloomberg News depending on data. Most of release times were regular, either quarterly or monthly, at a specific time of the day such as 8:30am, 9:15am, etc. These macroeconomic news release variables are also considered in studies by Andersen, Bollerslev, Diebold, and Vega (2003) for foreign currency markets to find high frequency exchange rate dynamics that are linked to fundamentals. Wongswan (2006) also applied these variables for the study on international stock markets, using high frequency data analysis.

For earnings announcements, I collected release times and dates from the First Call Historical Database. To minimize data error, release dates were compared between the First Call Historical Database and I/B/E/S databases. If the dates from these sources were different, I used the timing information from the Factiva search due to possible recording errors, also mentioned in Dubinsky and Johannes (2006). For those earnings that are released after trading hours, I set the indicator of  $\text{EARNINGS}_c(t)$  for a company  $c$  to become 1 in the earliest possible time at which the information would be reflected. As noted in Table 4, I include all the quarterly earnings announcements and revision (if any) by companies over the sample period. I have the cross-sectional average number of the announcement and revision  $\frac{1}{23} \sum_{c=1}^{23} \int_0^T \text{EARNINGS}_c(s) = 70$  for 23 companies listed in Table 2 and its standard error is 10.14.

For analyst recommendation, I collected the comprehensive real-time release history again from the First Call, a subsidiary of Thomson Corporation, which most brokerage firms and institutional investors depend on in order to disseminate their research reports electronically to their clients through a news wire service. This provides the exact dates and time-stamps of analyst recommendation updates, measured within 1 minute, which allows us to learn when the information becomes available to investors and whether it affects jump arrivals. To reduce bias due to sample selection, I include all types of recommendations changes by all analysts reported in the database, unlike Womack (1996), who examines immediate market reactions to dramatic recommendation changes [added or removed to buy (sell) recommendation] made by the highest rated U.S. brokerage research departments. Hence, my analysis includes those not only added to buy (sell) recommendations or removed from buy (sell) recommendations but also changes from buy to strong buy or sell and other kinds. Each recommendation record from the database contains the ticker symbol of the corresponding company, date and time of update (up to minutes),

and one-to-five point recommendation scales, with one being most favorable and five being least favorable. For those analyst recommendation released during nontrading hours, I again set the indicator of  $\text{ANALYST}_c(t)$  for a company  $c$  to become 1 at the earliest possible time when the information to be reflected. As noted in Table 4, I have the cross-sectional average number of recommendations  $\frac{1}{23} \sum_{c=1}^{23} \int_0^T \text{ANALYST}_c(s) = 519$  for 23 companies over the sample period and its standard error is 129.85.

I also aim to examine whether past jump arrivals in a specific stock increase investor’s demand for trading that equity and hence, change the likelihood of future jump arrivals in the same company in normal trading hours. In short, I test jump clustering evidence. The term “volatility clustering” has been used to describe the market phenomenon whereby volatility shows positive and significant autocorrelation.<sup>23</sup> I use the notion of “clustering” similarly for my study. More precisely, I mean by “jump clustering” that jump arrivals tend to follow previous jump arrivals. Since my test distinguishes jumps from volatility, clustering evidence for jumps is also separate from volatility clustering evidence. To incorporate this jump clustering effect, I use its own jump arrival times detected in Stage I and create the time series of jump time indicator variable  $\text{CLUSTER}_c(t)$  for a company  $c$ . As noted in Table 4, I have the cross-sectional average number of jumps  $\frac{1}{23} \sum_{c=1}^{23} \int_0^T \text{CLUSTER}_c(s) = 348$  for 23 companies over the sample period and its standard error is 45.30. Further detailed description of the jump data used for this  $\text{CLUSTER}$  variable can be found in Table 2.

For dividend related dates, I collected data from the CRSP database. There are four major dates related to dividend payments available: dividend announcement date when the board of directors announces to shareholders and the market that the company will pay a dividend, ex-dividend date on (or after) which a stock holder can sell the stock and still receive the declared

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<sup>23</sup>Small changes in returns tend to follow small changes, and large changes in returns tend to follow large changes.

dividend payments, date of record when investors must be listed as a holder to ensure the right of dividend payout, and date of payment when the company mails the dividend to the listed holders. I found the ex-dividend date significant for the majority of companies and used these for my analysis. For a company  $c$ , I create a time series of indicator  $\text{DIVIDEND}_c(t)$  that becomes 1 on those dates. As noted in Table 4, I have the cross-sectional average number of dividend related dates  $\frac{1}{23} \sum_{c=1}^{23} \int_0^T \text{DIVIDEND}_c(s) / 26 = 190$  for 23 companies over the sample period and its standard error is 23.79. (Since I set the information variable  $\text{DIVIDEND}_c(s)$  is measured at every 15 minutes, I had divisor of 26 (number of 15 minute observations per trading day) in this case in order to report the average number of dates.)

For choosing the firm-specific variables, I also initiated my investigation with other firm specific information variables for each company. In the presence of all the aforementioned 8 information variables (both macroeconomic and firm-specific) for each company, the following variables were also considered but rejected for lack of significance: earnings estimates issued by brokers contributing to the First Call Historical Database, and dividend announcement dates, dates of record, and payout dates and stock split related dates from the CRSP database. Again, some of these variables are found to be significant for some companies but they are not as broadly robust or significant as the variables I use in my analysis.

### 3.4 Model for Stage II and Empirical Results

In this subsection, I specify a parametric jump intensity model for Stage II and discuss the estimation results presented in Table 5 and 6. I derive my jump predictors  $X(t)$  from the original indicators,  $\text{MARKET}(t)$ ,  $\text{FOMC}(t)$ ,  $\text{NONFARM}(t)$ ,  $\text{JOBLESS}(t)$ ,  $\text{EARNINGS}_c(t)$ ,  $\text{ANALYTST}_c(t)$ ,  $\text{CLUSTER}_c(t)$ , and  $\text{DIVIDEND}_c(t)$  for a company  $c$ , which I set up in the previous section. Then, I choose one simple parsimonious model for each company  $c$  that links the instantaneous jump

intensity with predictors  $X(t)$  in the following fashion:

$$d\Lambda_\theta(t) = \frac{1}{1 + \exp(-\theta_0 - \sum_{j=1}^{10} \theta_j X_j(t))} \quad (16)$$

where  $X_1(t) = I(9 : 30 \leq h(t) < 10 : 00)$  is the time-of-the-day indicator for times between 9:30am and 10:00am, with  $h(t)$  being the hour:minute of the time  $t$ ,

$X_2(t) = I(10 : 00 \leq h(t) < 11 : 00)$  is the time-of-the-day indicator for times between 10:00am and 11:00am,

$X_3(t) = I(\int_{t-30min}^t \text{MARKET}(s) > 0)$  is the indicator of MARKET for jumps in the S&P 500 index being 1 within 30 minutes prior to  $t$ ,

$X_4(t) = I(\int_{t-30min}^t \text{FOMC}(s) > 0)$  is the indicator of FOMC being 1 within 30 minutes prior to  $t$ ,

$X_5(t) = I(\int_{t-30min}^t \text{NONFARM}(s) > 0)$  is the indicator of NONFARM being 1 within 30 minutes prior to  $t$ ,

$X_6(t) = I(\int_{t-30min}^t \text{JOBLESS}(s) > 0)$  is the indicator of JOBLESS being 1 within 30 minutes prior to  $t$ ,

$X_7(t) = I(\int_t^{t+1day} \text{EARNINGS}_c(s) > 0)$  is the indicator of EARNINGS for the company  $c$  being 1 within 1 day after  $t$ ,

$X_8(t) = I(\int_{t-30min}^t \text{ANALYST}_c(s) > 0)$  is the indicator of ANALYST for the company  $c$  being 1 within 30 minutes prior to  $t$ ,

$X_9(t) = I(\int_{t-3hour}^t \text{CLUSTER}_c(s) > 0)$  is the indicator of CLUSTER for the company  $c$  being 1 within 3 hours prior to  $t$ , and

$X_{10}(t) = I(\text{DIVIDEND}_c(t) \times (X_1(t) + X_2(t)) > 0)$  is the indicator of morning hours between 9:30am and 11:00am on DIVIDEND dates of the company  $c$ . I choose the linking function  $\gamma$  as in equation (16) in order for the intensity (probability) to be within admissible range  $[0, 1]$ .

Table 5 contains the parameter estimates for the all companies' jump model as specified above to assess the relative importance of the jump predictors. Results are based on those applied to all companies listed in Table 2. Coefficients on controls for intraday seasonality patterns of jump arrivals (in particular, morning hours) appear in the left two columns after the coefficient for intercept. Then, I list coefficients on 4 macroeconomic jump predictors and finally coefficients on 4 firm-specific jump predictors in the subsequent columns. \*,\*\*,\*\*\* indicate the significance of coefficients at the 10%, 5%, and 1% level, respectively. As recognized earlier in Table 3, the significance of coefficients for  $X_1(t)$  and  $X_2(t)$  on the time-of-the-day between 9:30am and 11:00am is confirming evidence that jumps often tend to occur early in the morning.<sup>24</sup>

The significance of the coefficients for predictors depending on market jump arrivals ( $X_3(t)$ ) suggests strong evidence that overall market jump arrivals increase the likelihood of individual equity jumps within 30 minutes. All the coefficients are significant at the 1% level except for one case with GE significant at 5% level. Another noteworthy macroeconomic variable is the FOMC announcement on federal fund rate changes ( $X_4(t)$ ). Results indicate that the FOMC announcements are likely to lead individual equity jump arrivals within 30 minutes. Since this information is usually released in the afternoon around 14:15, this result means it is very likely that the jumps would arrive between 14:15 and 14:45 on these announcement dates. Considering the magnitude of the coefficient of this predictor, this is the most influential predictor of U.S. individual equity jumps among all considered. The other two effective macroeconomic predictors ( $X_5(t)$  and  $X_6(t)$ ) are indicators of times shortly after release of employment and unemployment reports. Given their actual release times, which is 8:30am for both cases, the results show that jumps are likely to occur during the first half hour of NYSE trading from 9:30am to 10:00am.

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<sup>24</sup>I also considered the other time-of-the-day variables for times beyond 11:00am till 4:00pm but they were found to be insignificant and hence omitted in the model.

Except for the case of CVX with Nonfarm payroll reports and the two cases of DIS and GE with Jobless claims, these two jump predictors are significant mostly at the 1% significance level.<sup>25</sup> Although there are previous studies that show the impact of macroeconomic fundamentals on various other financial markets [see Ederington and Lee (1993), Andersen, Bollerslev, Diebold, and Vega (2003), Piazzesi (2003), Wongswan (2006), and Andersen, Bollerslev, and Huang (2007), and the references therein], this paper refines our understanding on the short-term predictability of jumps in U.S. individual stock markets at intra-day level, which hasn't been uncovered by other existing methodology. The powerful jump predictor test introduced in this paper allows us to perform more precise timing analysis using real-time information available. Macroeconomic fundamentals are found to be important in intra-day jump predictions and turn out to be in general better predictors than firm-specific information.

In addition to the macroeconomic variables, I also consider four firm-specific jump predictors that are related to my chosen information variables. The best firm-specific jump predictor I find is  $X_7(t)$ , which indicates times within 1 day before earnings release, and it is the second most influential predictor after  $X_4(t)$ , related to FOMC announcements. Earnings announcement information is the only information that tends to lead jump arrivals *before* their release time. It could be both because of the possible information leakage and because firms do not sometimes release information at prescheduled release times. All the other prescheduled variables such as FOMC, NONFARM, and JOBLESS tend to lead jump arrivals within 30 minutes *after* the news release.

Another effective firm-specific jump predictor is  $X_5(t)$  that indicates times within 30 minutes after analysts publish their recommendation. As can be seen in Table 5, this is the second best

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<sup>25</sup>I also considered the other time horizons such as 15 minutes, 60 minutes, 90 minutes, 120 minutes, 180 minutes, etc. for these variables and I found that 30 minutes is the most effective across all companies under my consideration.

firm-specific jump predictor in terms of impact magnitude and significance (they are all significant at the 1% level for all companies except XOM). Controlling for all the aforementioned predictors, I find the third and fourth best predictors are morning hours (from 9:30am to 11:00am) of ex-dividend dates (significant at the 10% level for 14 out of 23 companies) and arrivals of own jumps within the previous 3 trading hours (significant at the 5% level for 15 out of 23 companies), which shows jump clustering evidence. The last two firm-specific predictors are not as important as the other macroeconomic and firm-specific factors. But, the effect is fairly strong for some companies and broad enough. Hence, I include them to improve the performance of the model.

Furthermore, I report in Table 6 the associated p-values for the coefficients to show the relative precision of those predictors. The degree of significance can be taken as the degree of precision of the jump predictor: if the p-values are lower, the precision is higher. At the bottom of the table, I present the average rankings of jump predictors in terms of precision. For that summary measure, I rank the 8 predictors for each company and then take the cross-sectional average of those rankings for each predictor. I find that among all 8 predictors I considered,  $X_7(t)$ , related to EARNINGS is the most precise predictor. The next three most precise predictors are  $X_3(t)$  related to overall market jumps,  $X_8(t)$  related to analyst recommendation, and  $X_4(t)$  related to Fed's announcements. This evidence shows that there is a room for improving the individual equity option pricing model in Dubinsky and Johannes (2006), which only incorporates jump events conditional on scheduled earnings announcement dates, by adding other influential predictors found in this study.

## 4 Implications: Distinguishing Systematic Jump Risk

Being able to select jump predictors and set up dynamic models for jump risk is expected to be useful in many contexts. For example, predictors that are selected by the proposed procedure could be used as market-timing tool for intra-day program trading using high frequency data, which has been popular in hedge fund industry in recent years. Alternatively, one can incorporate both macroeconomic and firm-specific information to increase their predictability for jump risk, which should allow us to reduce the uncertainty involved in such discontinuity in asset prices and enhance asset pricing models.<sup>26</sup> Although I only investigate jump predictors for U.S. individual equity markets in the previous section, the proposed test can be applied to all sorts of financial time series for other financial markets so long as high frequency data are available. Hence, one can investigate other significant jump predictors for relevant markets and make clear the relative importance or precision of these predictors, as discussed.

In this subsection, I present one of the important implications of the proposed method, that is, distinguishing systematic jump intensity over time. In particular, I show implied empirical evidence that systematic jump intensity has increased in recent years. By systematic jumps, I mean jumps that are associated with the macroeconomic predictors discussed in the previous section. Based on the estimated jump model, I simply extract the systematic jump intensity over the interval  $[0, T]$  by subtracting terms that are not related to macroeconomic information as in the following method:

$$\widehat{\Lambda}_{\theta}^{systematic}(T) = \int_0^T d\Lambda_{\theta}(s)|_{\theta_i=\hat{\theta}_i} \text{ for all } i - \int_0^T d\Lambda_{\theta}(s)|_{\theta_i=0} \text{ for } i=3,4,5,6, \text{ and } \theta_i=\hat{\theta}_i \text{ for } i=1,2,7,8,9,10, \quad (17)$$

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<sup>26</sup>This benefit would be stronger for any financial management problems that are involved with derivative securities because they are more sensitive to the extreme movements of underlying asset prices as emphasized in Bates (2003).

where  $\hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, \dots, \hat{\theta}_{10}]$  is the parameter estimates based on the partial likelihood function as defined in **Definition 3.C** and the instantaneous intensity is same as in equation (16), that is

$$d\Lambda_{\theta}(t) = \frac{1}{1 + \exp(-\theta_0 - \sum_{j=1}^{10} \theta_j X_j(t))}.$$

In Table 7, I list the proportion of the systematic jump component relative to the total jumps and that of remaining idiosyncratic jumps. The results are presented for all 23 companies over the total sample period of 16 years. In order to see if there was a change in the level of systematic jump intensity, I split the sample period into 2 sub-sample periods of 8 years and present the results in the same table. Over the whole sample period, I find that on average systematic jumps account for more than 14% of the total jumps, ranging from 2.74% for GE to 20.28% for WMT. Using the sub-sample period analysis, I show that the systematic jump risk has increased more recently. Specifically, the proportions of systematic jumps for all companies under my consideration are always found to be greater during the second half of the sample period from 2001 to 2008 than that for the first half of the sample period from 1993 to 2000. This coherent evidence represents the fact that jump risk, which has traditionally been regarded as idiosyncratic risk component, should be not only modeled separately from usual diffusive risk, but also one should take into account this increased large systematic risk in portfolio or risk management.

## 5 Concluding Remarks

Motivated by recent efforts to make better inference for asset pricing models with both diffusive and jump risk, I propose a new test to identify market information that tend to increase the likelihood of jump arrivals and justify why the naively applied likelihood inference to set up dynamic jump models using filtered jumps is valid. This allows us to improve the predictability of higher moments of asset return distributions through jumps in relation to market information.

I explain the mixed unobservability problem in making inference for jump models within jump diffusion models.

The theoretical results presented are expected to provide a foundation and support for other studies which utilize jump detection tests for jump dynamics analysis. As long as high frequency observations are available for target returns and proposed jump predictors during a sufficiently long time period, this likelihood results can be applied to any type of financial data from various markets, including equity, bond, foreign currency as well as their corresponding options, and other derivatives markets.

Empirical evidence using my new test indicates that macroeconomic information such as Fed's announcements, overall market jumps, nonfarm payroll report, and initial jobless claim tend to increase the likelihood of large-cap individual equity jump arrivals within a short time horizon such as 30 minutes. Firm-specific jump predictors discovered are indicators for times within 1 day before earnings release, within 30 minutes after publication of analyst recommendation, and during the morning hours in ex-dividend dates. It is also found that jumps tend to cluster during normal trading hours of equity markets. Overall results show that macroeconomic fundamentals tend to have greater impact and precision than firm-specific variables in predicting future jumps. This evidence is after controlling for intra-day seasonality pattern in jump arrivals and is unique evidence in the literature thanks to the newly developed test on intra-day jump dynamics. Based on the estimation results, I further prove that systematic jumps have increased for U.S. individual equity markets in recent years. This is important because it immediately implies different diversification schemes for portfolio and risk management.

One intriguing problem is whether the overnight effect due to trading mechanisms may have an impact on my results. Improvements can be made in Stage I. However, no matter how we change

the jump detection test in Stage I, as long as the employed jump test satisfies the requirement discussed in **Proposition 1**, my proposed inference method using partial likelihood will be still valid. Though interesting, this is beyond the scope of this paper, and one possible solution to this problem is offered in Lee and Mykland (2008a). Finally, regarding applications, it would be interesting to investigate multiple markets such as the bond and credit markets or multiple competitive markets and to learn about the cross-sectional jump dynamics between those markets. The new test developed in this paper is expected to easily accommodate such studies.

## Appendix

### A.1. Assumption C on $\mu(t)$ and $\sigma(t)$ in equation (1)

The following assumptions are imposed on the drift  $\mu(t)$  and spot volatility  $\sigma(t)$  in the stochastic differential equation (equation (1)) for the asset price  $S(t)$ . For any  $\epsilon > 0$  and  $\Delta t = t_{i+1} - t_i$ ,

$$\mathbf{C.1.} \sup_i \sup_{t_i \leq u \leq t_{i+1}} |\mu(u) - \mu(t_i)| = O_p(\Delta t^{\frac{1}{2}-\epsilon}) \quad (18)$$

$$\mathbf{C.2.} \sup_i \sup_{t_i \leq u \leq t_{i+1}} |\log \sigma(u) - \log \sigma(t_i)| = O_p(\Delta t^{\frac{1}{2}-\epsilon}) \quad (19)$$

### A.2. Assumption D on $\Lambda_\theta(t)$ in equation (2)

The following assumptions are imposed on  $\Lambda_\theta(t)$ , which is a modified version of Condition VI.1.1. of Andersen, Borgan, Gill, and Keiding (1992). Denote by  $\theta_0$  the true value of parameter and  $\theta$  the free parameter. Let  $T$  be a given terminal time,  $0 < T \leq \infty$  and  $n$  be the number of observations within the terminal time  $T$ .

**D.1.** There exists a neighborhood  $\Theta_0$  of  $\theta_0$  such that for all  $n$  and  $\theta \in \Theta_0$ ,  $\log d\Lambda_\theta(t)$  and  $d\Lambda_\theta(t)$  are three times differentiable with respect to  $\theta \in \Theta_0$ .

**D.2.** There exist finite functions  $\sigma_{jl}(\theta)$  defined on  $\Theta_0$  such that for all  $j, l$ ,

$$\frac{1}{n} \int_0^T \left\{ \frac{\partial}{\partial \theta_j} \log d\Lambda_{\theta_0}(t) \right\} \left\{ \frac{\partial}{\partial \theta_l} \log d\Lambda_{\theta_0}(t) \right\} d\Lambda_{\theta_0}(t) dt \xrightarrow{p} \sigma_{jl}(\theta_0),$$

as  $n \rightarrow \infty$ . Moreover, the matrix  $\Sigma = \{\sigma_{jl}(\theta_0)\}$  is positive definite.

**D.3.** For all  $j$  and  $\epsilon > 0$ , we have

$$\frac{1}{n} \int_0^T \left\{ \frac{\partial}{\partial \theta_j} \log d\Lambda_{\theta_0}(s) \right\}^2 I \left( \left| \frac{1}{\sqrt{n}} \frac{\partial}{\partial \theta_j} \log d\Lambda_{\theta_0}(s) \right| > \epsilon \right) d\Lambda_{\theta_0}(s) ds \xrightarrow{p} 0,$$

as  $n \rightarrow \infty$ .

**D.4.** For any  $n$ , there exist  $G_n$  and  $H_n$  such that

$$\sup_{\theta \in \Theta_0} \left| \frac{\partial^3}{\partial \theta_j \partial \theta_l \partial \theta_m} d\Lambda_{\theta}(t) \right| \leq G_n(t)$$

and

$$\sup_{\theta \in \Theta_0} \left| \frac{\partial^3}{\partial \theta_j \partial \theta_l \partial \theta_m} \log d\Lambda_{\theta}(t) \right| \leq H_n(t)$$

for all  $j, l, m$ . And

$$\frac{1}{n} \int_0^T G_n(t) dt, \quad \frac{1}{n} \int_0^T H_n(t) d\Lambda_{\theta_0}(t) dt, \quad \frac{1}{n} \int_0^T \left\{ \frac{\partial^2}{\partial \theta_j \partial \theta_l} \log d\Lambda_{\theta_0}(t) \right\}^2 d\Lambda_{\theta_0}(t) dt$$

all converge in probability to finite quantities as  $n \rightarrow \infty$ , and for all  $\epsilon > 0$ ,

$$\frac{1}{n} \int_0^T H_n(t) I \left( \sqrt{\frac{H_n(t)}{n}} > \epsilon \right) d\Lambda_{\theta_0}(t) dt \xrightarrow{p} 0.$$

### A.3. Proof of Proposition 1

Proofs of parts A and B for **Definition 1.A** and **1.B** are in Lee and Mykland (2008b) and Lee and Hannig (2009). For part C, with the rejection region  $\mathcal{R}_n(\alpha_n) = (-\infty, -q_{\alpha_n} S_n - C_n, q_{\alpha_n} S_n + C_n, \infty)$ , if  $dJ(t_i) = 0$  for each single interval  $(t_{i-1}, t_i]$ ,

$$P(d\hat{J}(t_i) = 0 = dJ(t_i)) = 1 - P(\mathcal{L}(i) \in \mathcal{R}_n(\alpha_n))$$

$$= 1 - 2(1 - \Phi(q_{\alpha_n} S_n + C_n)) \approx 1 - 2(1 - \Phi(\sqrt{2 \log n})) \sim 1 - \frac{1}{\sqrt{\pi n \sqrt{2 \log n}}} \rightarrow 1, \quad (20)$$

as  $n \rightarrow \infty$  i.e.  $\Delta t \rightarrow 0$ .  $\Phi(x)$  is the standard normal cumulative distribution function. The last approximation is due to the asymptotic expression for  $1 - \Phi(x)$  as  $x \rightarrow \infty$ , which is  $\lim_{x \rightarrow \infty} x(1 - \Phi(x))e^{x^2/2} = (2\pi)^{-1/2}$ . See Galambos (1978) for its derivation.

If  $dJ(t_i) = 1$  for an interval with its jump time  $\tau \in (t_{i-1}, t_i]$ ,

$$\begin{aligned} P(d\hat{J}(t_i) = 1 = dJ(t_i)) &= P(\mathcal{L}(i) \in \mathcal{R}_n(\alpha_n)) \approx P(|Y(\tau)| > (q_{\alpha_n} S_n + C_n)\sigma(\tau)\sqrt{\Delta t}) \\ &\approx 1 - F_{|Y|} \left( \sigma(\tau)\sqrt{-2\Delta t \log(\Delta t)} \right) \sim 1 - \frac{2}{\sqrt{2\pi}} \sigma(\tau)\sqrt{-2\Delta t \log(\Delta t)} \rightarrow 1, \end{aligned} \quad (21)$$

as  $\Delta t \rightarrow 0$ , hence,  $\sigma(\tau)\sqrt{-2\Delta t \log(\Delta t)} \rightarrow 0$ .  $F_{|Y|}(y)$  is the distribution function of absolute jump sizes  $|Y|$  and  $\sigma(\tau)$  denotes the local volatility level at the jump time.

#### A.4. Proof of Proposition 2

I decompose the approximate full likelihood function into two different mutually exclusive parts for actual jump times and non-jump times as follows:

$$\begin{aligned} L_n(\theta | \mathcal{F}_T) &= \underbrace{\prod_{1 \leq i \leq n, dJ(t_i)=1} d\Lambda_\theta(t_i)^{dJ(t_i)}}_{(16.1)} \underbrace{\prod_{1 \leq i \leq n, dJ(t_i)=1} (1 - d\Lambda_\theta(t_i))^{1-dJ(t_i)}}_{(16.2)} \\ &\times \underbrace{\prod_{1 \leq i \leq n, dJ(t_i)=0} d\Lambda_\theta(t_i)^{dJ(t_i)}}_{(16.3)} \underbrace{\prod_{1 \leq i \leq n, dJ(t_i)=0} (1 - d\Lambda_\theta(t_i))^{1-dJ(t_i)}}_{(16.4)} \end{aligned} \quad (22)$$

where  $\Lambda_\theta(t) = \gamma(t, X(t); \theta)$ .

The second (16.2) and third (16.3) products are 1 under the full observation of jumps. Hence, it is enough to show that both of these two products, (16.2) and (16.3), based on partial observations, become 1, with probability 1, as  $\Delta t \rightarrow 0$ , so that the other two products based on partial observations match the corresponding ones, (16.1) and (16.4).

For the term (16.2), let  $H$  be the finite number of jumps during the time horizon and  $\tau_h$  be the jump times in  $[0, T]$  with  $h = 1, \dots, H$ . Then, from Proposition 2, as  $\Delta t \rightarrow 0$ ,

$$\begin{aligned} P \left( \prod_{1 \leq i \leq n, dJ(t_i)=1} \left( 1 - d\hat{\Lambda}_\theta(t_i) \right)^{1-d\hat{J}(t_i)} = 1 | H \right) &= P \left( \text{for all } i \text{ s.t. } dJ(t_i) = 1, d\hat{J}(t_i) = 1 | H \right) \\ &\approx \prod_{1 \leq h \leq H} \left[ 1 - F_{|Y|} \left( \sigma(\tau_h) \sqrt{-2\Delta t \log(\Delta t)} \right) \right] \sim 1 - \frac{2}{\sqrt{2\pi}} \sum_{h=1}^H \sigma(\tau_h) \sqrt{-2\Delta t \log(\Delta t)} \rightarrow 1, \end{aligned} \quad (23)$$

where  $F_{|Y|}(y)$  is the distribution function of absolute jump sizes  $|Y|$  and  $\sigma(\tau_h)$  denotes the local (bounded) volatility level at the  $h$ th jump time. Notice here that I only allow the finite activity jumps with finite number  $H$  of jumps to obtain this result.

For the term (16.3),

$$\begin{aligned} P \left( \prod_{1 \leq i \leq n, dJ(t_i)=0} d\hat{\Lambda}_\theta(t_i)^{d\hat{J}(t_i)} = 1 | H \right) &= P \left( \text{for all } i \text{ s.t. } dJ(t_i) = 0, d\hat{J}(t_i) = 0 | H \right) \\ &\sim P \left( \max_{1 \leq i \leq n, dJ(i)=0} |\mathcal{L}(i)| \in \mathcal{R}(\alpha_n)^c \right) = G(q_{\alpha_n}) = 1 - \alpha_n \rightarrow 1, \end{aligned} \quad (24)$$

as  $q_{\alpha_n} \rightarrow \infty$  and  $\alpha_n \rightarrow 0$ , with the distribution function of a standard Gumbel variable  $G(q_{\alpha_n})$ .

Therefore, the result holds, because

$$\begin{aligned} &P \left( \frac{PL_n(\theta | \mathcal{F}_T)}{L_n(\theta | \mathcal{F}_T)} = 1 | H \right) \\ &= P \left( \prod_{1 \leq i \leq n, dJ(t_i)=1} \left( 1 - d\hat{\Lambda}_\theta(t_i) \right)^{1-d\hat{J}(t_i)} = 1 | H \right) \times P \left( \prod_{1 \leq i \leq n, dJ(t_i)=0} d\hat{\Lambda}_\theta(t_i)^{d\hat{J}(t_i)} = 1 | H \right) \\ &\sim \left( 1 - \frac{2}{\sqrt{2\pi}} \sum_{h=1}^H \sigma(\tau_h) \sqrt{-2\Delta t \log(\Delta t)} \right) \times (1 - \alpha_n) \rightarrow 1, \end{aligned} \quad (25)$$

as  $\Delta t \rightarrow 0$  and  $\alpha_n \rightarrow 0$ . Note that this pointwise convergence in probability combined with Corollary 4.2 of Newey (1991) implies uniform convergence in probability in a compact subset of  $\Theta$  due to Condition D.

### A.5. Proof of Proposition 3

By the definition of product integration,

$$\frac{L_n(\theta|\mathcal{F}_T)}{\widetilde{L(\theta|\mathcal{F}_T)}} \xrightarrow{a.s.} 1, \text{ which implies } \frac{L_n(\theta|\mathcal{F}_T)}{\widetilde{L(\theta|\mathcal{F}_T)}} \xrightarrow{P} 1. \quad (26)$$

Then, due to **Proposition 2**,

$$\frac{PL_n(\theta|\mathcal{F}_T)}{\widetilde{L(\theta|\mathcal{F}_T)}} = \frac{PL_n(\theta|\mathcal{F}_T)}{L_n(\theta|\mathcal{F}_T)} \times \frac{L_n(\theta|\mathcal{F}_T)}{\widetilde{L(\theta|\mathcal{F}_T)}} \xrightarrow{P} 1. \quad (27)$$

### A.6. Proof of Theorem 1

Given **Assumption C**, we know that as  $\Delta t \rightarrow 0$ , for any  $\theta$ ,  $\log(L_n(\theta|\mathcal{F}_T)) - \log(PL_n(\theta|\mathcal{F}_T)) \xrightarrow{P} 0$ , which also implies the uniform convergence in probability from **Proposition 2**. Here, let  $\mathcal{U}_L(\theta)$  and  $\mathcal{U}_{PL}(\theta)$  be the score functions based on  $\log(L_n(\theta|\mathcal{F}_T))$  and  $\log(PL_n(\theta|\mathcal{F}_T))$ . Then, the two estimators,  $\hat{\theta}_{L,n}$  and  $\hat{\theta}_{PL,n}$  such that  $\mathcal{U}_L(\hat{\theta}_{L,n}) = 0$  and  $\mathcal{U}_{PL}(\hat{\theta}_{PL,n}) = 0$  are asymptotically equivalent. In other words, as  $\Delta t \rightarrow 0$  (as  $n \rightarrow \infty$ ),  $\hat{\theta}_{L,n} - \hat{\theta}_{PL,n} \rightarrow 0$  in probability: this is proved by contradiction. Now, according to the Slutsky Theorem as in Ferguson (1996), it is enough to show that the estimator based on  $L_n(\theta|\mathcal{F}_T)$ ,  $\hat{\theta}_{L,n}$ , is consistent and converges in law to a normal distribution around its mean  $\theta_0$ . For this part, I apply a modified version of proofs for Theorem VI.1.1. and VI.1.2 in Andersen, Borgan, Gill, and Keiding (1992). Due to a Taylor expansion,  $1 - d\Lambda_\theta(t) = \exp(-d\Lambda_\theta(t))$ ,  $\mathcal{U}_L(\theta)$  can be written as

$$\mathcal{U}_L(\theta) = \int_0^\cdot \frac{\partial}{\partial \theta} \log d\Lambda_\theta(s) dM(s),$$

where  $M(t) = J(t) - \int_0^t d\Lambda_\theta(s) ds$  and is a local square integrable martingale. Here, I first apply Lengart's inequality to establish the existence of a consistent estimator that is the solution for the score function. Second, I use the Martingale Central Limit Theorem to establish the convergence

of estimators in distribution to normal. Lastly, it is obvious that the last result can be obtained due to the delta method.

An alternative to the proof given above is to consider two equivalent probability measures  $\mathcal{P}$  and  $\mathcal{P}_{PL_n}$ .  $\mathcal{P}$  is the true (latent) data-generating measure for  $\widetilde{L}(\theta|\mathcal{F}_T)$  in continuous time as in **Definition 3.A** and  $\mathcal{P}_{PL_n}$  is the observable data-generating measure for  $PL_n(\theta|\mathcal{F}_T)$  in discrete time as in **Definition 3.C**. Instead of going through  $L_n(\theta|\mathcal{F}_T)$ , the above weak convergence proof can be directly applied on  $\mathcal{U}_{PL}(\theta)$  because of the convergence of  $\mathcal{P}_{PL_n}$  to  $\mathcal{P}$ , as shown in **Proposition 3**.

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Table 1: **Simulation Results from the Jump Predictor Test (JPT)<sup>†</sup>**

Constant Volatility								
Stage I by <b>Definition 1.A</b>					Stage I by <b>Definition 1.B</b>			
$\sigma_y$	$\hat{\theta}_0$	$SE(\hat{\theta}_0)$	$z_{\theta_0}$	p-value	$\hat{\theta}_0$	$SE(\hat{\theta}_0)$	$z_{\theta_0}$	p-value
$3\sigma$	-4.0479	0.0494	-81.9413	0.0000	-4.0165	0.0486	-82.6440	0.0000
$2\sigma$	-4.0558	0.0496	-81.7702	0.0000	-4.0238	0.0488	-82.4549	0.0000
$1\sigma$	-4.0764	0.0501	-81.3653	0.0000	-4.0419	0.0492	-82.1524	0.0000
$\sigma_y$	$\hat{\theta}_1$	$SE(\hat{\theta}_1)$	$z_{\theta_1}$	p-value	$\hat{\theta}_1$	$SE(\hat{\theta}_1)$	$z_{\theta_1}$	p-value
$3\sigma$	5.6928	0.8231	6.9163	2.3532e-010	5.8343	0.8679	6.7223	3.8844e-010
$2\sigma$	5.6779	0.8184	6.9378	2.2597e-010	5.8009	0.8568	6.7704	3.4773e-010
$1\sigma$	5.5858	0.7925	7.0483	1.8240e-010	5.7409	0.8390	6.8426	2.8307e-010
Stochastic Volatility								
Stage I by <b>Definition 1.A</b>					Stage I by <b>Definition 1.B</b>			
$\sigma_y$	$\hat{\theta}_0$	$SE(\hat{\theta}_0)$	$z_{\theta_0}$	p-value	$\hat{\theta}_0$	$SE(\hat{\theta}_0)$	$z_{\theta_0}$	p-value
$3\tilde{\sigma}$	-4.0479	0.0494	-81.9413	0.0000	-4.0170	0.0486	-82.6543	0.0000
$2\tilde{\sigma}$	-4.0582	0.0496	-81.8185	0.0000	-4.0234	0.0488	-82.4467	0.0000
$1\tilde{\sigma}$	-4.0762	0.0501	-81.3613	0.0000	-4.0368	0.0491	-82.2159	0.0000
$\sigma_y$	$\hat{\theta}_1$	$SE(\hat{\theta}_1)$	$z_{\theta_1}$	p-value	$\hat{\theta}_1$	$SE(\hat{\theta}_1)$	$z_{\theta_1}$	p-value
$3\tilde{\sigma}$	5.6857	0.8251	6.8909	2.5821e-010	5.8168	0.8632	6.7386	3.7465e-010
$2\tilde{\sigma}$	5.6965	0.8235	6.9174	2.2980e-010	5.8021	0.8583	6.7600	3.5877e-010
$1\tilde{\sigma}$	5.6248	0.8005	7.0266	2.1608e-010	5.7495	0.8420	6.8284	3.0509e-010

<sup>†</sup> This table contains averaged simulated results from the two stage semi-parametric procedure described in Section 2 using the two nonparametric test as defined in **Definition 1.A** and **1.B** for the Stage I. All the figures in this table are results averaged over 3,000 simulation runs. I simulate returns from the model  $d \log S(t) = \mu(t)dt + \sigma(t)dW(t) + Y(t)dJ(t)$ , where the constant volatility model sets  $\sigma(t) = \sigma = 30\%$  and the stochastic volatility model assumes the Affine model of Heston (1993), specified as  $d\sigma^2(t) = \kappa(\theta - \sigma^2(t))dt + \omega\sigma(t)dB(t)$ , where  $B(t)$  denotes a Brownian Motion. The parameter values used for stochastic volatility simulation are the estimates from equity markets reported in the empirical study by Li, Wells, and Yu (2008), Table 4. They are  $\kappa = 0.0162$ ,  $\theta = 0.8465$ , and  $\omega = 0.1170$ . For the jump intensity model, we assume  $d\Lambda_\theta(t) = \frac{1}{1 + \exp(-\theta_0 - \theta_1 X(t))}$ . Jump sizes are set in comparison with volatility level. The standard deviation  $\sigma_y$  of the jump size distribution is set at three different levels as noted in the table. I denote  $\tilde{\sigma} = E(\sigma(t))$  for the stochastic volatility model.  $\theta_0$  and  $\theta_1$  are set to be  $-4$  and  $6$ , so that when  $X(t) = 0$ ,  $d\Lambda_\theta(t) = 0.0180$  and when  $X(t) = 1$ ,  $d\Lambda_\theta(t) = 0.8808$ . 15-minute return data are used. The significance level  $\alpha$  is set equal to 5% in Stage I. The statistic  $z_{\theta_i} = \frac{\hat{\theta}_i}{SE(\hat{\theta}_i)}$  for  $i = 0, 1$

Table 2: Descriptive Statistics for Jump Counts in DJIA Individual Equities <sup>†</sup>

Name (Ticker)	# of tests	# of jumps	per year	per month	per day
ALCOA (AA)	106299	409	25.56	2.13	0.11
AMERICAN EXPRESS (AXP)	106406	338	21.12	1.76	0.08
BOEING (BA)	106756	409	25.56	2.13	0.10
CATERPILLA (CAT)	105840	391	24.43	2.03	0.10
CHEVRON CORPORATION(CVX)	105872	284	17.75	1.48	0.07
DU PONT (DD)	106741	313	19.56	1.63	0.08
WALT DISNEY (DIS)	106527	368	23.00	1.91	0.09
GEN ELECTRIC (GE)	107577	291	18.18	1.51	0.07
HOME DEPOT (HD)	106529	392	24.50	2.04	0.10
HEWLETT PACKARD(HPQ)	106588	411	25.68	2.41	0.10
INTL BUSINESS MACH (IBM)	107335	363	22.68	1.89	0.09
JOHNSON & JOHNSON (JNJ)	106368	358	22.37	1.86	0.09
JP MORGAN & CHASE (JPM)	106624	346	21.62	1.80	0.09
COCA COLA (KO)	106420	281	17.56	1.46	0.07
MCDONALD (MCD)	106418	374	23.37	1.94	0.09
3M (MMM)	105901	320	20.00	1.66	0.08
MERCK & CO INC(MRK)	105878	388	24.25	2.02	0.10
PFIZER (PFE)	105955	387	24.18	2.01	0.10
PROCTOR & GAMBEL (PG)	106316	276	17.25	1.43	0.07
AT & T (T)	106531	293	18.31	1.52	0.07
UNITED TECHNOLOGIES (UTX)	106005	365	22.81	1.90	0.09
WAL MART STORES (WMT)	106178	354	22.12	1.84	0.09
EXXON MOBILE (XOM)	106640	253	15.81	1.31	0.06
AVERAGE	106447	348	21.79	1.82	0.08
Standard Error	(418)	(45.30)	(2.83)	(0.25)	(0.012)

<sup>†</sup> This table includes the total number of tests and the total number of jumps detected in Stage I over my sample period of 16 years. The average numbers of detected jumps per year, per month, and per day are also listed for each individual component stock of the DJIA (as of December 31, 2008) with their cross-sectional averages at the bottom. The transaction price data from the TAQ database are used. Observations are based on trades in the New York Stock Exchange (NYSE) from January 4, 1993 to December 31, 2008 with a total of 4,017 trading days. The component stocks traded on the Nasdaq are excluded to keep consistent trading mechanisms across different securities for comparison purposes. Kraft (KFT) is excluded due to significant missing data because it only started to be traded in 2001. CITI GROUP (C), BANK OF AMERICA (BAC), and VERIZON (VZ) are also excluded because of unusual company name changes over the sample period, to avoid any complication due to the change. The 15-minute returns from transaction prices are applied. The test statistic based on **Definition 1.A** is applied in Stage I for this table. The significance level  $\alpha$  in Stage I is 5%. Ticker denotes the ticker symbol of each company as of December 31, 2008.

Table 3: **At What Times do Jumps Occur more often?**<sup>†</sup>

Ticker	9:30am	10:00am	11:00am	12:00pm	1:00pm	2:00pm	3:00pm
AA	0.7237	0.1320	0.0367	0.0367	0.0147	0.0269	0.0269
AXP	0.7337	0.0828	0.0355	0.0089	0.0325	0.0385	0.0562
BA	0.7408	0.1369	0.0318	0.0147	0.0244	0.0196	0.0269
CAT	0.7621	0.1202	0.0205	0.0256	0.0153	0.0205	0.0281
CVX	0.7746	0.0775	0.0141	0.0106	0.0141	0.0317	0.0669
DD	0.7923	0.1086	0.0288	0.0128	0.0128	0.0160	0.0288
DIS	0.7618	0.1118	0.0206	0.0118	0.0206	0.0059	0.0412
GE	0.7320	0.1031	0.0550	0.0103	0.0206	0.0172	0.0412
HD	0.7832	0.0944	0.0230	0.0153	0.0255	0.0408	0.0179
HPQ	0.7981	0.0925	0.0268	0.0122	0.0195	0.0195	0.0243
IBM	0.7906	0.0909	0.0303	0.0193	0.0138	0.0193	0.0165
JNJ	0.7570	0.0978	0.0223	0.0140	0.0223	0.0223	0.0475
JPM	0.7341	0.0983	0.0462	0.0145	0.0173	0.0289	0.0491
KO	0.8648	0.0605	0.0142	0.0142	0.0142	0.0071	0.0214
MCD	0.7620	0.0722	0.0401	0.0187	0.0241	0.0348	0.0455
MMM	0.6813	0.1375	0.0406	0.0219	0.0219	0.0125	0.0688
MRK	0.7448	0.0876	0.0258	0.0232	0.0129	0.0284	0.0696
PFE	0.7804	0.0827	0.0465	0.0207	0.0181	0.0155	0.0336
PG	0.7971	0.1196	0.0000	0.0181	0.0072	0.0145	0.0326
T	0.7577	0.0853	0.0239	0.0239	0.0171	0.0239	0.0512
UTX	0.6932	0.1425	0.0575	0.0219	0.0219	0.0137	0.0411
WMT	0.7712	0.0876	0.0198	0.0226	0.0226	0.0169	0.0508
XOM	0.7708	0.0949	0.0198	0.0158	0.0198	0.0316	0.0356
Average	0.7628	0.1002	0.0289	0.0176	0.0183	0.0229	0.0398

<sup>†</sup> The table reports the percentages of jumps in individual equities detected during specific time intervals in a trading day among all detected during my sample period from January 4, 1993 to December 31, 2008 for a total of 4,017 trading days. I divided the NYSE trading day (9:30am to 4:00pm) into 7 time intervals. Column names are the starting points of the time intervals. For example, the first column 9:30am includes the percentages of jumps occurred during the time interval starting at 9:30am and ending at 10:00am. To save space, except for the first column, which is for a 30-minute interval, all the columns are 1-hour intervals.

Table 4: Description of Data for Jump Predictors in U.S. Individual Equity Markets<sup>†</sup>

Macroeconomic Variable Names	Total	Times	Dates	Data Source
U.S. Market Jump (MARKET)	446	Irregular	1993.01.04-2008.12.31	Trade and Quote (TAQ)
Federal Open Market Committee Meeting (FOMC)	134	14:15 $\diamond$	1993.01.04-2008.12.31	Federal Reserve Board & Bloomberg
Nonfarm Payroll Employment (NONFARM)	192	8:30	1993.01.04-2008.12.31	Bureau of Labor Statistics & Bloomberg
Initial Unemployment Claims (JOBLESS)	834	8:30	1993.01.04-2008.12.31	Employment and Training Administration & Bloomberg
Firm-specific Variable Names	Total <sup>§</sup>	Times	Dates	Data Source
Earnings Announcements (EARNINGS)	70 (10.14)	Irregular	1993.01.04-2008.12.31	First Call Historical Database
Analyst Recommendation Release (ANALYST)	519 (129.85)	Irregular	1993.01.04-2008.12.31	First Call Historical Database
Past Jumps (CLUSTER)	348 (45.30)	Irregular	1993.01.04-2008.12.31	Trade and Quote (TAQ)
Dividend Dates (DIVIDEND)	190 (23.79) <sup>§§</sup>	Irregular	1993.01.04-2008.12.31	Center for Research in Security Prices (CRSP)

<sup>†</sup> This table shows the source, time, and number of observations of each information variable used in our analysis. Total denotes the total number of times in which the corresponding event happened over the sample period as stated in Dates column. <sup>§</sup> For the firm-specific variables, I report the averages of total numbers across all the companies I consider in my study, along with their standard error in parenthesis. The companies I choose to study are listed in Table 2. Times are local U.S. Eastern times. U.S. market jumps and past jump denote variables based on the jumps in the S&P 500 and own individual equity returns, respectively. Jumps used are detected by the nonparametric jump test statistic as defined in **Definition 1.A** using the 15-minute returns from transaction prices. The significance level  $\alpha$  applied for detecting both U.S. market jumps and individual equity jumps is 5%. All the observations are in real time up to 1 minute, except Dividend Dates, for which daily observations are available. GENERAL MOTORS (GM) is excluded due to significant missing data in the First Call Historical Database. <sup>§§</sup> Since the observations for DIVIDEND are available in daily frequency, I report the average of number of daily observations and the standard deviation.  $\diamond$  Before April 1995, FOMC news were not released regularly at the specified time. We follow Andersen, Bollerslev, Diebold, and Vega (2003) for the irregular release times of FOMC news release, which are 11:05am on 1994.2.4, 2:20pm on 1994.3.22 and 1994.7.6, 2:30pm on 1994.11.15, 2:26pm on 1994.5.17, 2:23pm on 1994.12.20, 1:17pm on 1994.8.16, 2:22pm on 1994.9.27, and 2:24pm on 1995.2.1. Before 1994, we use 11:30am. See the detailed description of data selection in subsection 3.3.

Table 5: Jump Predictor Test Results for U.S. Individual Equity Markets<sup>†</sup>

Ticker	Intercept	$X_1(t)$	$X_2(t)$	$X_3(t)$	$X_4(t)$	$X_5(t)$	$X_6(t)$	$X_7(t)$	$X_8(t)$	$X_9(t)$	$X_{10}(t)$
AA	-7.8144***	3.6418***	1.8084***	1.5326***	3.6508***	1.2330***	0.9882***	1.4039***	1.2981***	0.95378***	-0.5298
AXP	-7.7531***	3.4042***	1.2469***	1.9561***	3.1663***	1.4879***	0.7655***	1.4932***	1.1224***	0.92842***	1.1995***
BA	-7.9999***	3.9836***	1.9775***	1.1086***	2.1912***	1.0610***	0.5417***	2.2272***	1.1460***	0.7856***	0.1997
CAT	-8.0818***	3.8428***	2.0611***	1.6874***	3.8204***	1.2347***	0.9712***	2.0695***	1.2969***	0.6643***	0.4924*
CVX	-8.1292***	3.6083***	1.5752***	1.8762***	2.9845***	0.5167	1.0633***	1.0199***	1.0021***	1.0327***	2.0428***
DD	-8.4827***	4.0620***	2.1965***	1.2496***	3.9256***	1.6242***	1.0490***	2.3641***	1.1901***	0.3525	1.2100***
DIS	-8.0831***	3.7731***	1.7896***	1.3690***	1.6904***	0.9829***	1.4441	1.3044***	1.4369***	1.4733***	0.6911
GE	-8.0210***	3.8356***	1.2015***	0.8198**	3.6902***	0.7399**	-0.4940	1.6548***	0.8496***	1.0862***	1.2008***
HD	-8.0724***	3.9520***	1.9986***	1.4200***	4.0128***	1.3721***	0.9442***	2.3218***	0.8497***	0.2880	1.0947***
HPQ	-8.0351***	4.0713***	1.8896***	1.1256***	3.3822***	0.8238***	0.7690***	1.4297***	1.1971***	0.64140***	-0.3814
IBM	-8.0761***	4.0309***	1.6760***	0.7944***	3.3143***	1.1395***	0.5880***	1.7858**	1.4604***	0.0690	0.2185
JNJ	-7.9060***	3.6711***	1.5072***	1.5687***	3.3266***	1.1265***	1.1194***	2.1832***	0.9213***	0.5559**	1.0080***
JPM	-7.8185***	3.5438***	1.3923***	1.6798***	3.8022***	1.2384***	0.9482***	1.9435***	0.9370***	0.2926	1.2376***
KO	-8.8063***	4.4579***	1.9059***	1.6545***	1.9381***	1.3850***	0.6545***	2.2853***	1.4942***	0.1379	0.9913**
MCD	-7.6797***	3.5861***	1.1577***	1.5203***	2.6346***	1.0066***	0.5497***	1.5543***	1.0218***	1.1421***	0.8517*
MMM	-7.8196***	3.2633***	1.7792***	1.7631***	2.3914***	1.6108***	0.4482**	2.3990***	1.4641***	0.1769	0.6819
MRK	-7.6606***	3.3771***	1.5106***	1.8177***	2.4264***	1.2788***	1.1487***	1.8247***	0.9554***	0.5873***	0.9024**
PFE	-7.8430***	3.7069***	1.5421***	1.3149***	2.5535***	1.4551***	0.9195***	1.7231***	0.8047***	1.0330***	1.4674***
PG	-8.7633***	4.3661***	2.2297***	1.6371***	2.7987***	0.6228	0.6867***	2.3072***	0.9834***	0.7624***	0.4922
T	-8.0451***	3.5796***	1.2137***	2.1107***	2.0850***	0.9886***	0.8000***	1.7980***	0.9498***	0.9444***	1.8488***
UTX	-7.7814***	3.4414***	1.7798***	1.7532***	2.7354***	1.4421***	0.8144***	2.2252***	1.3406***	0.4870**	0.5611
WMT	-8.0105***	3.6988***	1.6869***	1.6252***	3.5814***	1.4561***	1.2996***	2.5700***	1.1536***	-0.4644	0.0629
XOM	-8.4024***	3.9781***	1.5942***	1.9345***	3.8401***	0.9399***	0.5122**	1.9373***	0.6535**	0.1565	1.5759***
Average	-8.0472	3.7772	1.6835	1.5356	3.0410	1.1142	0.7644	1.9055	1.1099	0.5686	0.7445

<sup>†</sup> This table contains the parameter estimates for jump predictor tests applied to the individual equity price data to assess the relative importance of predictors. Company names associated with ticker symbols are listed in Table 2 and description of relevant jump data used can also be found therein. The jump results from Stage I are based on NYSE transaction price data sampled every 15 minutes with a significance level  $\alpha$  of 5%. The jump predictors are assumed to be related to doubly stochastic Poisson jump arrivals as in the following fashion:  $d\Lambda_\theta(t) = \frac{1}{1 + \exp(-\theta_0 - \sum_{j=1}^{10} \theta_j X_j(t))}$ , where the definitions of  $X_j(t)$ 's are in subsection 3.4. The sample period extends 16 years from January 4, 1993 to December 31, 2008. \*, \*\*, and \*\*\* indicate the significance at the 10%, 5%, and 1% level, respectively. The averages in the last row are taken after setting the insignificant parameter values equal to zero.

Table 6: Precision of Macroeconomic and Firm-specific Jump Predictors in U.S. Individual Equity Markets<sup>†</sup>

Ticker	$X_3(t)$	$X_4(t)$	$X_5(t)$	$X_6(t)$	$X_7(t)$	$X_8(t)$	$X_9(t)$	$X_{10}(t)$
AA	1.20e-013	4.81e-018	4.61e-007	2.04e-010	1.46e-009	4.65e-019	4.48e-007	4.60e-001
AXP	2.72e-021	4.83e-011	9.71e-009	4.01e-005	5.74e-008	5.73e-010	2.26e-006	7.73e-004
BA	1.02e-005	3.63e-003	2.01e-004	1.98e-003	8.07e-033	1.22e-012	1.91e-006	6.95e-001
CAT	1.14e-016	2.60e-018	5.34e-006	9.56e-010	3.54e-027	4.13e-016	7.74e-004	1.02e-001
CVX	2.645e-016	1.33e-006	1.94e-001	2.44e-009	1.91e-003	6.61e-008	1.84e-006	4.53e-015
DD	1.90e-006	1.90e-014	3.75e-010	2.52e-009	1.00e-028	3.80e-012	2.24e-001	1.04e-003
DIS	1.24e-008	4.63e-002	7.26e-004	5.14e-001	1.09e-006	4.76e-026	2.60e-022	2.45e-001
GE	1.48e-002	3.23e-015	4.67e-002	1.03e-001	1.57e-012	4.42e-004	8.57e-007	1.32e-003
HD	4.44e-010	1.62e-026	9.68e-008	9.91e-009	1.08e-032	2.54e-006	2.37e-001	9.82e-004
HPQ	6.14e-006	1.26e-011	7.10e-003	1.27e-006	4.10e-009	2.22e-018	2.63e-003	5.24e-001
IBM	1.08e-002	1.62e-009	7.24e-005	1.18e-003	9.09e-014	1.35e-021	8.11e-001	6.39e-001
JNJ	7.16e-012	3.81e-011	1.58e-004	4.08e-012	1.38e-024	2.14e-007	1.99e-002	6.35e-003
JPM	6.42e-014	1.13e-021	1.46e-005	4.80e-008	4.83e-017	4.86e-007	2.30e-001	1.62e-004
KO	1.58e-010	6.56e-002	1.94e-006	6.15e-004	1.87e-024	5.21e-019	6.84e-001	2.02e-002
MCD	6.31e-011	1.56e-005	7.21e-004	2.47e-003	7.70e-012	6.01e-009	1.73e-012	6.75e-002
MMM	5.56e-015	1.06e-003	9.88e-010	3.85e-002	9.14e-035	1.00e-015	4.77e-001	1.81e-001
MRK	4.03e-019	9.79e-005	1.45e-006	1.00e-013	4.88e-018	4.81e-010	7.39e-003	1.45e-002
PFE	1.70e-008	4.37e-005	6.62e-009	1.74e-008	1.18e-014	5.46e-007	3.02e-008	7.38e-007
PG	1.20e-010	2.27e-004	1.14e-001	4.39e-004	6.18e-033	8.11e-007	1.67e-003	2.89e-001
T	1.16e-022	9.68e-003	1.68e-003	1.83e-005	2.87e-012	3.38e-008	1.07e-005	1.00e-011
UTX	8.87e-017	6.79e-006	2.61e-008	3.45e-006	3.79e-030	2.24e-012	3.38e-002	2.70e-001
WMT	1.17e-012	6.66e-015	4.34e-008	4.75e-017	4.78e-039	4.15e-011	1.74e-001	9.14e-001
XOM	2.88e-015	1.30e-014	5.90e-003	1.73e-002	1.68e-013	1.74e-002	6.62e-001	1.09e-006
Average Ranking	3.04	4.04	5.39	4.96	2.17	3.47	6.13	6.78

<sup>†</sup> This table contains the p-values associated with the parameter estimates for macroeconomic and firm-specific jump predictors presented in Table 5. For each company, I rank the 8 jump predictors and then for each jump predictor, I report the averaged ranking at the bottom of the table. The corresponding company name for each ticker symbol and jump data description can be found in Table 2. The jump results from Stage I are based on NYSE transaction price data sampled every 15 minutes with a significance level  $\alpha$  of 5%. The description of macroeconomic and firm-specific information related to our jump predictors can be found in Table 3 and in subsection 3.3. I assume that jump predictors are related to doubly stochastic Poisson jump arrivals as in the following fashion:  $d\Lambda_\theta(t) = \frac{1}{1 + \exp(-\theta_0 - \sum_{j=1}^{10} \theta_j X_j(t))}$ , where the definition of  $X_j(t)$ 's are in subsection 3.4. The sample period is 16 years from January 4, 1993 to December 31, 2008.

Table 7: **Distinguishing Systematic Jumps**<sup>†</sup>

Sample Period	1993-2008		1993-2000		2001-2008	
Ticker	Systematic	Idiosyncratic	Systematic	Idiosyncratic	Systematic	Idiosyncratic
AA	0.1719	0.8281	0.1659	0.8341	0.1777	0.8223
AXP	0.1788	0.8212	0.1648	0.8352	0.1920	0.8080
BA	0.0898	0.9102	0.0873	0.9127	0.0921	0.9079
CAT	0.1763	0.8237	0.1662	0.8338	0.1861	0.8139
CVX	0.1695	0.8305	0.1636	0.8364	0.1753	0.8247
DD	0.1738	0.8262	0.1641	0.8359	0.1832	0.8168
DIS	0.0794	0.9206	0.0704	0.9296	0.0885	0.9115
GE	0.0274	0.9726	0.0170	0.9830	0.0369	0.9631
HD	0.1609	0.8391	0.1542	0.8458	0.1673	0.8327
HPQ	0.1155	0.8845	0.1067	0.8933	0.1241	0.8759
IBM	0.0932	0.9068	0.0872	0.9128	0.0989	0.9011
JNJ	0.1693	0.8307	0.1589	0.8411	0.1794	0.8206
JPM	0.1729	0.8271	0.1561	0.8439	0.1889	0.8111
KO	0.1382	0.8618	0.1288	0.8712	0.1476	0.8524
MCD	0.1082	0.8918	0.1036	0.8964	0.1127	0.8873
MMM	0.1294	0.8706	0.1258	0.8742	0.1329	0.8671
MRK	0.1822	0.8178	0.1785	0.8215	0.1860	0.8140
PFE	0.1465	0.8535	0.1402	0.8598	0.1527	0.8473
PG	0.1206	0.8794	0.1188	0.8812	0.1224	0.8776
T	0.1700	0.8300	0.1604	0.8396	0.1789	0.8211
UTX	0.1539	0.8461	0.1470	0.8530	0.1606	0.8394
WMT	0.2028	0.7972	0.1882	0.8118	0.2167	0.7833
XOM	0.1424	0.8576	0.1290	0.8710	0.1551	0.8449
Average	0.1423	0.8577	0.1340	0.8660	0.1503	0.8497

<sup>†</sup> This table includes the proportion of systematic jump components that is extracted from total jumps by the following method:  $\widehat{\Lambda}_\theta^{systematic}(T) = \int_0^T d\Lambda_\theta(s)|_{\theta_i=\hat{\theta}_i}$  for all  $i$  -  $\int_0^T d\Lambda_\theta(s)|_{\theta_i=0}$  for  $i=3,4,5,6$ , and  $\theta_i=\hat{\theta}_i$  for  $i=1,2,7,8,9,10$ . The results are based on estimates reported in Table 5 for 8 jump predictors in the model:  $d\Lambda_\theta(t) = \frac{1}{1+\exp(-\theta_0 - \sum_{j=1}^{10} \theta_j X_j(t))}$ , where the definitions of  $X_j(t)$ 's are in subsection 3.4. This model is used to estimate parameters and extract the systematic jumps separately using data sampled every 15 minutes for the whole sample period and for the sub-sample periods.