Financing Bidders in Bankruptcy and Takeover Auctions
(Job Market Paper)

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December 1, 2011

Abstract

A firm is sold in a cash auction in which cash-constrained bidders must raise external financing if they win the takeover contest. This paper shows that a bidder's expected payoff after paying the seller and repaying his financier does not depend on the financier's ability to extract rent. For any given security type, an increase in the cost of financing (e.g., a higher interest rate) is fully passed on to the seller. The type of financing (e.g., debt or equity) depends on whether bidders can raise capital at competitive terms or are locked in to an investor. Finally, the seller can induce *all* bidders to bid more aggressively by offering alternative financing, even if bidders ultimately raise financing from outside investors.

Journal of Economic Literature Classification Codes: D44, G32, G33, G34.

Keywords: Auctions, financing auction bids, financially constrained bidders, bankruptcy auctions, takeover contests.

1 Introduction

Firms or large-scale projects are sometimes completely sold off by their current owners to buyers who are privately informed about their valuation. Most takeovers, both in and outside bankruptcy, present a typical example. Recent evidence shows that cash auctions have become one of the predominant choices for such transactions. Boone and Mulherin (2007) report that close to half of the takeovers from their sample involve multiple bidders – much more than what becomes publicly known. In the majority of takeover auctions,

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the winner needs to raise external financing to make his payment to the seller.[1] In such cases, it is known that bidding behavior when capital is easily available differs from when it is scarce and expensive. It also differs depending on whether bidders use debt, equity, or some other security to finance their cash payment in the auction (see DeMarzo et al., 2005; Rhodes-Kropf and Viswanathan, 2005).

What is not so well understood is precisely what determines the shape of the equilibrium financing contracts when bidders need to raise cash from *outside* financiers and how bidders' incentives to participate in the auction change when financiers impose tougher financing conditions. In bankruptcy auctions, in particular, there are widespread concerns that entry in the auction will be deterred by the inability of bidders to raise cash at favorable terms, making such auctions illiquid and leading to fire sales (e.g., Shleifer and Vishny, 1992; Hart, 2000). However, Baird and Rasmussen (2003) and Hotchkiss and Mooradian (1998) find that 30 to 50 percent of successful reorganizations under Chapter 11 involve multiple bidders. The size of the deals varies from small to large and the way cash bids are financed differs from case to case. For example, CenterSpan bought Scour Inc. for $9 million, which were raised from a private equity investor. In contrast Sungard financed its $850 million cash bid for Comdisco with debt, and Columbia Sussex's $2.7 billion bid for Aztar was backed by a $2.9 billion line of credit. The participation in these auctions was reasonably high – there were, on average, three bidders. Furthermore, winners seemed to make good deals at the time: On the day of the announcement, Sungard's stock jumped up by 3.8 percent, while CenterSpan's stock jumped up by 18 percent (Columbia Sussex is a private company). One objective of the following analysis is to propose an explanation for this evidence.

The paper solves a model in which cash-constrained bidder-managers, henceforth "bidders", must secure financing from outside financiers, henceforth "investors", to bid in a cash auction of a firm. Bidders are privately informed about the profitability of this firm under their management and their ability to generate synergies and cash flows. If they win the auction, the payment is raised according to the financing contract and investors are repaid out of the eventually realized cash flows.

The paper's main contribution lies in explicitly solving the equilibrium of the financing

[1] BHP Billiton raised $5.5 billion new debt to finance its $7.2 billion acquisition of WMC Resources. Similarly, TomTom entered a new €1.6 billion credit facility to pay its €2.9 billion bid for Tele Atlas.

game, thereby shedding light on the use of different security contracts and on the payoff and revenue effects for bidders, investors, and the seller. The first result is that the extent to which bidders can pass on their costs of financing to the seller depends only on the shape of the financing contract (e.g., debt or equity) and on how cash-constrained they are. In particular, it does *not* depend on whether the costs of a given type of security contract increase, e.g. a higher interest rate in the case of debt financing. Such costs are fully passed on to the seller. Second, the shape of the equilibrium financing contract depends on whether bidders or investors have more bargaining power. The pecking order theory is confirmed if outside investors compete to provide capital: Auction payments are financed with debt. This prediction is reversed, however, if outside investors can dictate the terms of financing: Auction payments are financed with (levered) equity. The final result is that by committing to provide alternative financing, the seller can increase her expected revenue, even if she sacrifices surplus when bidders take her offer. The type of security used by the seller will differ from that offered on a competitive market (debt).

The intuition why an increase in the cost of financing for a given security contract is fully passed on to the seller is the building block for solving the equilibrium security design problem. It relies on the following insight. A bidder's net payment in the auction is effectively determined by the security contract signed with the investor who finances him, as the price paid in the auction is just raised and channeled through to the seller. Therefore, one can present the bidder's decision as *first* choosing the optimal level of his expected security repayment to the investor, and *then* "reverse engineering" the cash bid he needs to make to achieve this payment by taking into account the agreed-upon terms in the financing contract. Stipulating that the financing game is part of the equilibrium of the overall game provides a sufficient condition that this strategy leads to the same probability of winning as choosing the cash bid directly. Therefore, a bidder's optimization problem can be presented as the same problem he would have in a security-bid auction.[2] His expected payoff is, thus, also the same, implying that an increase in the cost of financing is fully passed on to the seller.

This intuition can be used to show that from all contracts that can be signed with investors at $t = 1$, financing all bidders with debt (levered equity) will result in their

[2] This is an auction in which there are no outside investors and bidders compete by offering the seller securities (e.g. debt or equity), backed by the future cash flows of the firm. Note that bidders' payoffs in such auctions depend on the type of security in which they make their bid (e.g., DeMarzo et al., 2005).

lowest (highest) equilibrium net payments. Suppose, for an illustration, that bidders were split in two pools, such that all bidders above a certain valuation were financed with a different type of security contract than the bidders below this threshold. The bidders from the low pool have exactly the same expected net payments as when they are not separated from the high-valuation buyers, as any increase in their cost of financing is passed on to the seller. What changes are the security payments of the bidders from the high pool. Incentive compatibility causes their payments to shift upwards or downwards from what they would have been in an auction in which they are not separated from the low-valuation bidders.

The direction of the shift depends on the type of security used by the second pool. Financing with securities such as (levered) equity, which depend more strongly on a bidder's true valuation, induce higher payments: a *promise* to repay in the future is worth less to a low-valuation type, making him bid more aggressively (DeMarzo et al., 2005). Debt has the opposite effect, as it is the least information sensitive security. The logic with the two pools can be generalized to show that financing all types of bidders with debt (levered equity) will indeed give the lower (upper) bound for the equilibrium expected payments. Given this building block, the shape of the *equilibrium* security used in the financing contract can be shown now to depend on whether bidders or investors have more bargaining power.

If a bidder is locked in to an investor, so that the bargaining power is in the hands of the investor, financing is provided against (levered) equity. The reason is that by controlling the financing terms of a given security, the investor effectively controls the cash bid. Hence, he also controls the seller's share of surplus. What he cannot control is the bidder's share of surplus for a given security type, as an increase in the cost of financing is passed on to the seller. Therefore, he will choose the security type that minimizes this share. Offering financing against a security, which depends more strongly on the bidder's true type, makes bidders more aggressive in their bidding behavior and achieves this goal.

This result is overturned in a competitive market for capital in which bidders have more bargaining power than investors. Then, the equilibrium contract follows Myers and Majluf's (1984) pecking order theory. Payments are financed with debt, as the value of a debt claim depends least on a bidder's true valuation. It is, thus, the cheapest security for high valuation types when they receive the same financing contract as low types. Specific features of this financing game are the ability of bidders to pass on their cost of financing and the fact that the investor effectively receives two signals: one when the contract offer is made and one when he observes the auction payment.

Finally, if the seller can commit to offer alternative financing, he effectively provides a so-called type-dependent outside option for bidders when they negotiate with outside investors. Offering financing against securities such as (levered) equity can increase the seller's revenue, as they induce all bidders to bid more aggressively, even if they eventually take financing from outside investors.

These results give rise to rich empirical implications. One is that the efficiency of auctions should not decline when financing is scarce and expensive. Given that the cost of financing is fully passed on to the seller, tougher financing conditions do not make an auction less attractive to bidders. Recent empirical findings support these predictions: Bankruptcy auctions attract considerable interest, they appear to be efficient, there is no evidence for fire-sales, and bidders profit in expectation (e.g., Eckbo and Thorburn, 2008 and 2009; Hotchkiss and Mooradian, 1998). Evidence on bankruptcy auctions also supports the implication that the seller, i.e., the firm's main creditor, can increase her expected revenue by providing financing to bidders (Eckbo and Thorburn, 2009). A similar finding emerges from the literature on takeover contests (Povel and Singh, 2010). The present paper explains how this can happen in equilibrium and that it raises the seller's revenue even when the market for capital is competitive and bidders can play off the seller against outside investors. Moreover, it predicts that debt should become less popular if bargaining power is in the hands of investors, thereby suggesting an explanation for the use of different securities observed in practice when financing takeover bids. The main text below discusses these implications in more detail and how they compare to existing findings.

Related Literature The literature most closely related to this paper is on auctions in which bids are in securities. Hansen (1985), Crémer (1987), and Samuelson (1987) are the first to illustrate that security-bid auctions can increase the seller's revenue, but that they can lead to adverse selection and moral hazard issues. The literature following these three papers has generalized and formalized these ideas (e.g., Rhodes-Kropf and Viswanathan, 2000; DeMarzo et al., 2005). Also closely related are Board (2007) and Zheng (2001).

The bidders in these papers bid in cash, but can default on their payments to the seller, making the cash bids equivalent to bidding in debt. The most general treatment so far is in DeMarzo et al. (2005). They provide a framework for comparing different security types and show that bidding becomes more aggressive if the value of the security depends more strongly on the bidder's true type.[3] Not surprisingly, the same intuition also describes the effect of security design on bidders' aggressiveness in the setup of this paper. The reason is that from the bidder's point of view his payment is determined by the security contract with the investor.

The crucial difference between the present paper and the above literature is the presence of an outside investor. Hence, if a bidder defaults, it is no longer with respect to the seller, but to this new third party. So, the focus is on how payoffs, revenues, and the shape of the equilibrium financing contract depend on the game between bidders, outside investors, and the seller. The only paper to my knowledge that explicitly considers the game with an outside investor is Rhodes-Kropf and Viswanathan (2005). However, they analyze the existence of an efficient equilibrium in a competitive market and do not discuss payoff and revenue effects as well as equilibrium security design.

In a recent paper, Povel and Singh (2010) also show that the seller can increase her expected revenue by offering to finance the winner in the auction. Just as with the literature on bidding in securities, the main difference from this paper is the presence of outside investors with whom bidders can try to negotiate better financing terms. As a consequence, *all* bidders become more aggressive, though only some types take the seller's offer. In addition, Povel and Singh (2010) do not consider equilibrium security design.

In terms of corporate finance theory, the present paper relates to the literature on raising capital under adverse selection. As in this literature, debt is the equilibrium security when a privately informed bidder makes an offer to the investor (e.g., Nachman and Noe, 1994; DeMarzo and Duffie, 1999). The novel aspect here is that bidders' expected payoffs do not depend on an increase in the cost of financing. In fact, all bidders are better off signing the same financing contract even if separation is possible. Another novel aspect is that these security design predictions are overturned if a bidder is locked in to an investor. Then, the optimal security contract is (levered) equity. Inderst and Mueller

[3] Che and Kim (2010) obtain the opposite results when higher absolute returns require higher investment costs. Their results are driven, however, by the assumption that the return on investment *de*creases in the valuation of a bidder. From this perspective, the intuition is similar to that in DeMarzo et al.

(2006) also find that levered equity may be optimal when investors have more bargaining power. The intuition for their result, however, is that this security reduces a privately informed investor's aggressiveness by exposing him more to the upside. In contrast, the intuition here is that financing with levered equity induces a bidder to give up more of his share of expected surplus. Related, Axelson et al. (2009) find that levered equity may be optimal in an adverse selection setup, but the reason in their setting is that it mitigates risk shifting incentives, stemming from issuing debt.

The paper proceeds as follows. Section 2 introduces the model. Section 3 discusses in detail how financing costs are passed on to the seller. Section 4 solves for the equilibrium of the financing game when the bargaining power is in the hands of outside investors and the game when the bargaining power lies with the bidders. It also analyzes the case in which the seller can offer alternative financing. A conclusion follows in Section 5. Appendix A collects the proofs of all lemmas and propositions reported in the main text. Appendix B presents a detailed discussion of the first-price auction.

2 The Model

The model has three time periods. At $t = 1$, $N \geq 2$ bidder-managers, "bidders", secure financing from outside financiers, "investors", to participate in an auction for a firm, which is sold off fully as a going concern. For simplicity, it is assumed that the only asset of the firm is a project that generates stochastic cash flows in the future. At $t = 2$, an all-cash auction takes place. The bidders who must make a payment raise the money according to the contract signed at $t = 1$. Finally, at $t = 3$, the firm's cash flows are realized and the bidders repay the investors. All parties are risk neutral and there is no discounting.

The firm generates stochastic cash flows X, which are verifiable at $t = 3$. The distribution of X depends on the winning bidder's type $\theta \in [\underline{\theta}, \overline{\theta}]$. One can think of θ as his ability to to generate cash flows, which reflect the potential for synergies and his ability as a manager. Conditional on θ, the density of X, $g(x|\theta)$, has full support $[0, \infty)$. It is assumed that the conditional cumulative distribution functions are ordered in terms of first order stochastic dominance

$$G(x|\theta') > G(x|\theta) \text{ for } \theta > \theta',\ x \in X, \tag{1}$$

where $G(\cdot)$ is the cumulative density function. For simplicity, the conditional density

$g(x|\theta)$ is assumed to be continuously differentiable in x and θ. Further, it is assumed that $xg(x|\theta)$ and $xg_2(x|\theta)$ are integrable on $x \in (0, \infty)$.[4] The subscript denotes the derivative with respect to the second argument.

Each bidder privately learns his type θ at $t = 1$ before the contract is signed.[5] What is commonly known is that θ is independently drawn from the distribution function F with density f. Bidders further have the same liquid assets in place w, which they use to co-finance their bids. Most of these assumptions are relaxed in Section 3.4, 3.5, and 4.4. Finally, it is assumed that the investor observes only the payment in the auction, and not the individual bids. The seller ("she") has no private information and her outside option is zero.

External Financing To secure financing for his bid, a bidder negotiates a security R. It may be conditioned on the cash flows X realized by the asset and the payment $y \in \mathbb{R}_+$ to the seller at $t = 2$. The analysis considers both cases when bidders face a non-competitive or a competitive market for capital, so that the resulting security design problem will be to maximize the ex ante expected value of the investor's or bidders' claim respectively. To reiterate, it is assumed that only bidders who make a payment to the seller raise money from the investor at $t = 2$. By standard security design arguments, $R(x, \cdot)$ and $x - R(x, \cdot)$ are non-decreasing in x and $0 \leq R(x, \cdot) \leq x$.[6] Securities that satisfy these conditions are referred to as *feasible* securities. It is straightforward to show that the expected payoff of a feasible security increases in the bidder's type.

Lemma 1 *For any given security $R(x, \cdot)$, the expected security payments increase in the bidder's type: $\frac{\partial}{\partial \theta} \int_X R(x, \cdot)\, dG(x|\theta) > 0$ and $\frac{\partial}{\partial \theta} \int_X (x - R(x, \cdot))\, dG(x|\theta) > 0$.*

Proof. See Appendix.

3 Passing-on Financing Cost to Seller

The first set of results in the paper shows that the extent to which bidders pass on their costs of financing to the seller depends only on how cash-constrained they are and on the

[4] This weak assumption allows us to take the derivative through expectation operators.

[5] It will be shown that it does not matter whether the financing contract is signed at $t = 1$ or $t = 2$. Section 4.4 discusses how the analysis changes if it is signed before the bidders learn their type.

[6] See, e.g., Nachman and Noe (1994) and DeMarzo and Duffie (1999).

shape of the security contract (e.g., debt or equity). In particular, it does *not* depend on outside investors' ability to extract rent. These results constitute the basis for solving the security design problem at $t = 1$. They also have powerful empirical implications. This section builds up the problem by discussing initially the case in which all bidders sign the *same* financing contract. It is left open who makes the financing offer at $t = 1$ and it is only required that the financing contract is incentive-compatible. Upon showing the intuition for the pass-on result for this special case (the intuition will be quite general), this section also discusses the effect of bidders' cash constraint and that of security design. Stepping on this basis, Section 4 solves then the full equilibrium of the financing game.

3.1 Second-Price Auction and Debt Financing

The central issue in what follows is how the financing terms affect the bidders' payoffs and the seller's revenue. The difficulty in addressing this is that bidding strategies often lack a closed-form solution and are difficult to interpret. To highlight the main issues, it is assumed initially that all bidders are financed with debt and that they participate in a second-price auction [SPA]. Analyzing this setup is useful for streamlining the intuition before extending the results to general security types and a more general auction setting in Section 3.4.

Solving the bidding game requires giving some structure to the contract signed with outside investors at $t = 1$. The financing terms of a debt contract can be captured by the interest rate r or, equivalently, by the promised debt repayment $D(y, r(y)) := (y - w)(1 + r(y))$. Note that r can depend on the auction payment y and that $D(\cdot)$ can be used to uniquely "index" the debt contract for any given y:

$$R(X, D(y, r(y))) := \min[X, D(y, r(y))].$$

Generally, there are not many restrictions on the functional form of r. Allowing the interest rate to change in the investor's interim beliefs about the bidder's type after observing y, implies that r can be a decreasing function of y. That is, if a high payment convinces the investor that he is facing a high type, he may agree to cheaper financing terms. What cannot happen in the equilibrium of the overall game, however, is that a bidder's promised debt repayment D to the investor decreases in his cash payment y to the seller. Any financing game that is part of the equilibrium of the overall game must satisfy the following condition.

Lemma 2 *(i) Incentive-compatibility of the financing game implies that the promised debt repayment $D(y, r(y))$ must increase in y. (ii) If this monotonicity is weak at some y, the allocation rule must also remain the same.*[7]

Proof. See Appendix.

Even if the investor is convinced that he is facing a very high type and sets a lower interest rate, he rationally expects the bidders' behavior induced by the terms of the financing contract. Intuitively, if D would decrease for some y, there would always be a bidder who is strictly better off deviating upwards from his equilibrium bid. Thereby, he would make an additional profit by winning over additional types, while not increasing his security payments when winning over types he outbids also on the equilibrium path. As is illustrated next, this equilibrium condition on the financing contracts is sufficient for proving the main result in this section.

Bidding Strategies and Financing Terms In an SPA, in which bidders are not financially constrained, it is a weakly dominant strategy for every bidder to bid his valuation. That is, he should just break even when the price paid in the auction equals his bid. Whether this is an equilibrium when financing is provided from an outside investor depends on the financing contract, however. Such bidding can lead to a different allocation rule, as the bidder's valuation depends on the terms of this contract. Consider the following example with some exogenously given debt contract:

Suppose that some type's expected valuation of the asset is 120. According to the standard characterization of an SPA, his net payment must be 120 when the auction price is equal to his bid.[8] Suppose, however that the interest rate $r(y)$ decreases in y, so that his expected net payment is 120 when y is 100, but it is 119 when y is 101. A deviation to bidding 101 can now be strictly profitable. It increases the probability of winning and leaves the bidder with a strictly positive payoff in at least one additional instance in which he wins.

[7] It is important to note that the restriction is on y and not on the type θ. It may be that the equilibrium security payment $D(\cdot)$ decreases at some θ if y decreases at this θ.

[8] The bidder's net payment is equal to the price paid to the seller plus the contractual repayment to the invetor, minus the cash that he raises from this investor.

Lemma 2 makes sure that such a setting cannot occur. Requiring that the financing contract must be part of the equilibrium of the whole game implies that the equilibrium of the SPA can be characterized in the standard fashion. It is a weakly dominant strategy for every type θ_i to bid, such that he just breaks even when the auction price is equal to his bid.

Lemma 3 *Suppose that each bidder's valuation of the asset* $\int_X xdG(x|\theta_i)$ *exceeds his cash* w *and that all bidders issue debt to finance their payment if they win the auction. There is a unique, efficient equilibrium in weakly undominated strategies in the second-price cash auction, in which the equilibrium bidding strategy* $\beta(\theta_i)$ *is the solution to*

$$\int_X R(x, D(\beta(\theta_i), r(\beta(\theta_i)))) dG(x|\theta_i) = \int_X xdG(x|\theta_i) - w. \tag{2}$$

Proof. See Appendix.

One can see from (2) that the interest rate affects the equilibrium bidding strategies. However, it is not clear at first sight how bidders' expected payoffs will depend on it. The issue of making a general statement regarding payoff and revenue effects becomes even more involved outside of the simple framework of the SPA. (See Lemma B.1 in the Appendix for a discussion on the FPA.) The following analysis turns explicitly to the question of how the different bidding strategies are reflected in the bidders' expected payoffs. It shows that any increase in the cost of financing is fully passed on to the seller.

Payoff and Revenue Comparison Continuing the exposition with the SPA, suppose that there is a change in the interest rate function. This may be due to the seller gaining more bargaining power, for instance. Suppose further that the new financing contract is also part of the equilibrium of the overall game. Lemma 2 is key for analyzing the equilibrium effects on the bidder's expected payoff. It ensures that under both the new and the old contracts ranking equilibrium cash payments leads to the same allocation rule as ranking equilibrium debt payments. The second crucial observation comes from Lemma 3. It shows that the equilibrium debt repayment $D(\cdot)$, when the bidder actually has to pay his cash bid, is the *same* regardless of how the interest rate is set (cf. (2)).

Together, these two results imply that the bidder's problem can be rewritten as choosing the maximum debt repayment he is willing to make and then "reverse engineering" the financing contract to derive the cash bid that corresponds to this debt payment. Stipulating that the financing game is part of the equilibrium of the whole game, guarantees that this alternative strategy leads to the same probability of winning as choosing the cash bid directly (Lemma 2). The bidder's problem is, thus, the same as in a security-bid auction, in which bidders compete by offering the seller a debt claim, backed by the firm's cash flows, instead of resorting to outside financing (as in the present model) and making pure cash bids. Thus, his payoff is also the same as in such an auction, implying that it cannot depend on the particular functional form of $r(y)$.

Proposition 1 *If all bidders are financed with debt, their expected payoffs do not depend on the interest rate. The cost of financing are fully passed on to the seller.*

Proof. See Appendix.

Example. Suppose there are only two states of the world at $t = 3$ with $X = \{x, x + \Delta x\}$, where $x, \Delta x > 0$. The bidder's type $\theta \in (0, 1]$ is his probability of being in the high cash flow state. Assuming that the bidder defaults in the low cash flow state, the state-dependent payoffs of the debt contract are $R = \{x, (1 + r(y))(y - w)\}$.[9] By Lemma 3, it is a weakly dominant strategy for every bidder to bid such that he just breaks even when the auction price is equal to his bid; i.e., when $y = \beta(\theta_i)$:

$$x + \theta_i \left[(1 + r(\beta(\theta_i)))(\beta(\theta_i) - w) - x\right] = (x + \theta_i \Delta x) - w$$

The equilibrium bidding strategy is, thus, implicitly defined in:

$$\beta(\theta_i) = w + \frac{1}{1 + r(\beta(\theta_i))}\left(x + \Delta x - \frac{w}{\theta_i}\right). \tag{3}$$

Recalling that the payment y in an SPA is the second-highest bid and that the auction is efficient (Lemma 3), it is possible to express the bidder's expected payoff without explicitly solving for $\beta(\theta_i)$. Defining $F_1(\theta) := F^{N-1}(\theta)$, this payoff is:

$$\begin{aligned} & \int_{\underline{\theta}}^{\theta_i} \left(x + \theta_i \Delta x - w - x - \theta_i \left[(1 + r(\beta(t)))\frac{x + \Delta x - \frac{w}{t}}{1 + r(\beta(t))}\right] - x\right) dF_1(t) \\ = \;& w \int_{\underline{\theta}}^{\theta_i} \left(\frac{\theta_i - t}{t}\right) dF_1(t), \end{aligned} \tag{4}$$

[9] A sufficient condition for default in the low state is that $w < \underline{\theta}\Delta x$.

which is independent of $r(y)$.[10] ▲

It is important to note that Proposition 1 does not claim that the individual cash bids do not depend on how the interest rate is set. As the example shows, exactly the opposite is true. The reason is that the valuation of a bidder depends on the terms at which he obtains financing. However, the same holds for the valuations and equilibrium bidding strategies of all other bidders. It is, therefore, intuitive that, facing the same strategic considerations, the bidders adjust their equilibrium bidding strategy in such a way that they bid away the financing advantage or disadvantage they may have.[11] Note, further, that the payoff equivalence from Proposition 1 holds only within the same auction format and security type. For instance, bidders' expected payoffs would be different in a first-price auction [FPA] and different if they were to raise money by issuing equity. After generalizing the results in Section 3.4, Section 3.5 discusses these issues in detail.

So far, nothing has been said about the seller's expected revenue. If the investor just breaks even, it also does not depend on how the financing terms are set. This is no longer true if the investor can extract rent, as the full costs of this are borne by the seller (Proposition 1). Thus, rather than letting the bidders finance their payment from outside investors, the seller could provide financing herself by agreeing to accept bids in the form of security claims on the future cash flows. By doing so, she can obtain the same payoffs as in the case when the investor offers financing for which he just breaks even (Proposition 1).

Corollary 1 *Suppose that all bidders are financed with debt. The seller's expected revenue*

[10] It is straightforward to show that this is the same expected payoff as in an SPA where the highest bid $w + D$ wins. Bidder's equilibrium debt bid $D(\theta_i) = x + \Delta x - \frac{w}{\theta_i}$ can be derived in the usual fashion by setting his expected payoff conditional on paying his bid equal to zero

$$0 = x + \theta_i \Delta x - (x + \theta_i (D(\theta_i) - x)) - w.$$

[11] This is not a general result in auctions with type-dependent costs. Consider an SPA with type-dependent costs $c(\theta)$ in which the cash flows are as in the example. The example preceding Lemma 3 shows that such an auction need not even be efficient. Suppose, however, that it is. The equilibrium bidding strategy is then $\beta(\theta) = x + \theta \Delta x - c(\theta)$ and bidder θ's expected payoff is

$$\int_{\underline{\theta}}^{\theta} [(\theta - t)\Delta x - c(\theta) + c(t)] \, dF_1(t).$$

Thus, one can see that bidders' expected payoffs depend non-trivially on the type-dependent cost function c. The contribution of Proposition 1 is to show that $\int_{\underline{\theta}}^{\theta} [c(\theta) - c(t)] \, dF_1(t)$ is constant for debt.

does not depend on the how the interest rate is set as long as the investor breaks even. If the investor extracts rent, the seller can increase her revenue by offering financing herself.

The second statement in the corollary implicitly assumes that outside investors and bidders remain passive in the financing game and the seller offers the same security as outside investors. Section 4.3 contains a detailed treatment of how the seller can increase her expected revenue by offering appropriate financing herself and of the equilibrium response of bidders and outside investors in such a case.

3.2 Effect of Cash Constraint and Financing Fees

The Introduction argued that a bidder's expected payoff depends only on how cash-constrained he is and on the shape of his financing contract. Given the result from Proposition 1, we can now make the first part of this statement more precise. Suppose that w decreases uniformly for all types. The bidder's equilibrium bidding strategy β continues to be implicitly defined in (2). He bids such that he just breaks even, conditional on actually paying his bid. However, decreasing the bidder's co-investment increases the amount he needs to raise from outside investors to make such a payment. As the example above shows, this has a non-trivial effect on his equilibrium cash bids and his expected payoffs (cf. (3) and (4))

Proposition 2 *Holding the investor's expected payoff fixed, uniformly decreasing bidders' cash participation decreases their expected payoffs and increases the seller's expected revenue.*[12]

Proof. See Appendix.

The reason that bidders bid less aggressively when they pay a higher portion of their bid with cash follows an intuition similar to Milgrom and Weber's (1989) Linkage Principle. For an illustration in the context of the SPA, observe that a payment of \$100 is worth exactly \$100 to each bidder no matter what his type is. In contrast, if a promise to pay later has an expected value of \$100 to a type who defaults on his payments half of the

[12] Changing w only for one bidder, respectively asymmetric auctions, are more difficult to analyze. See Milgrom and Roberts (1990) for a discussion of monotone comparative statics in Bayesian games and Reny and Zamir (2004) for a treatment in the context of asymmetric first price auctions.

time, the value of this promise is higher for a type who never defaults. Hence, raising the \$100 bid of a low type is more expensive for a high type when he needs to finance this payment externally. Thus, his expected payoff is lower compared to the case when he is not financially constrained. In the case in which the outside investor always breaks even, this implies that the seller's expected revenue must increase. The larger the portion of the payment for which the bidder must raise outside financing, the stronger this effect is.

As an example of a uniform decrease in w, suppose that the investor requires a fee regardless of whether a bidder eventually raises financing. Holding the investor's ex ante expected payoffs fixed, the interest rate required from the winning bidder will then decrease. From Proposition 1, however, bidders' expected payoffs are independent of the financing terms for any given w. Even though the investor does not make an additional profit, Proposition 2 implies that their expected payoffs will decrease.

Corollary 2 *Introducing a financing fee $\varphi \in (0, w]$, decreases bidders' expected payoffs and increases the seller's expected revenue.*

3.3 Empirical Implications I

Before continuing with a more general setup and solving the full equilibrium of the financing game, several empirical implications can be derived already through this preliminary analysis. The first novel prediction of the model is that bidders fully pass on any increase in their cost of financing to the seller (Proposition 1). As a result, their interest in participating in an auction should not depend on the market conditions and the ability of investors to extract rent. In the cross section, when the cost of financing increase:

Implication 1. *Auction participation should not decline.*

Implication 2. *Auctions do not become less efficient.*

Implication 3. *Assets are auctioned at a discount despite competition.*

Implication 4. *Bidders make an expected profit.*

Bankruptcy auctions seem an ideal environment for testing these predictions. One traditional argument against selling bankrupt firms in auctions has been precisely that bidders would find it difficult to obtain financing. The fear is that a resulting lack of competition would lead to fire-sales and allocative inefficiencies. In practice, however, Baird and Rasmussen (2003) report that more than half of all large Chapter 11 cases resolved in 2002 use some form of an auction mechanism. Similarly, Hotchkiss and Mooradian (1998) find that one third of the acquired firms in their sample have been sold in an auction with multiple bidders. The results for Sweden, where there is a mandatory bankruptcy procedure, also support the claims. Eckbo and Thorburn (2008) find that 63 percent of the firms auctioned as going concern involve multiple bidders. These figures are actually higher than the number of bidders in takeover contests of non-bankrupt firms found by Boone and Mulherin (2007).

The evidence from bankruptcy auctions supports also Implications 2-4. Hotchkiss and Mooradian (1998) find significant positive abnormal returns both for the target and the bidder for the days surrounding the announcement of the acquisition. Like Eckbo and Thorburn (2008), they further observe a significant discount on the bankrupt firms' assets, but no evidence of fire-sales. Moreover, they find that the post-bankruptcy operating performance is on par with that of industry rivals. All of this indicates that bidders in bankruptcy auctions indeed pass on their cost of financing to the seller and ultimately create value.

Though the seller's expected revenue decreases as bidders pass on their cost of financing, this effect may be somewhat ameliorated in practice. Proposition 2 implies that cash-constrained bidders bid more aggressively the more capital they need to raise from outside investors. This also may help explain why the evidence for fire-sales is weak even in bankruptcy auctions. Summarizing, one has the following cross-sectional prediction:

Implication 5. Holding the cost of financing fixed, the seller's expected revenue increases in the bidders' cash constraints.

The model can be naturally applied not only to takeovers, but also to any auctions of assets with stochastic cash flows. Section 4.4 contains further examples. Based on the results of the financing game from the next section, it also discusses how the seller can increase her expected revenue by providing financing herself.

3.4 The General Case

The intuition for the passing-on an increase in the cost of financing discussed in Proposition 1 is that bidder's payoffs are always the same as in a security-bid auction. The driving force for this intuition is that the financing contract signed in the financing game must be incentive compatible when being part of the overall game. This game has not been solved so far. The next step before doing so is to show that the results from the previous section extend to much more general auction setups and security types. In particular, this subsection shows that they obtain for all common security types as well as for common values and interdependent types. The discussion is kept deliberately short, as the intuition is the same as before. Appendix B extends the results to the FPA.

Defining financing terms in a general security design context consists of two steps. First, it is necessary to define how securities can be compared in terms of their expected repayment within the same security type. For example, in the previous subsection debt was indexed by the promised debt repayment D. Similarly, equity can be indexed by the equity share and call and put options by the strike price. DeMarzo et al. (2005) capture this idea by introducing the notion of an ordered set of securities.

Definition 1 *A function $R(x, s_R)$ defines an ordered set of securities indexed by s_R if R is feasible and if the expected security payment increases in the order s_R for every type θ: $\int_X \frac{d}{ds_R} R(x, s_R)\, dG(x|\theta) > 0$.*

Introducing the notion of ordered securities allows us to say that two debt contracts that promise a repayment of \$120 have the same order s_R. It does not say, however, which contract is financed at better terms. For instance, if to raise \$100, the interest rate is 20 percent, then these terms are clearly worse than raising \$120 at zero percent. The second step is, thus, to define how the *financing terms* are included in s_R.

For this purpose, the analysis closely follows the same intuition as in the previous section. The financing terms for an ordered set of securities are defined by the dependence of the order function s_R on y (analogously to the dependence of D on y). The functional form of $s_R(y)$ results from the financing game between the bidder and the investor at $t = 1$. As in the previous section, requiring that the financing contract is part of the equilibrium of the whole game restricts the shape of $s_R(y)$.

Lemma 4 *(i) Incentive-compatibility of the financing game implies that the order s_R must increase in y. (ii) If this monotonicity is weak at some y, the allocation rule must also remain the same.*

Proof. See Appendix.

The intuition behind Lemma 4 is the same as before and is therefore omitted. In particular, it does not depend on whether the values are common or the types are independent. With the help of Lemma 4 one can now derive the main result of this section.

Proposition 3 *Suppose that all bidders are financed with the same security type. Also in a setup with common values and interdependent types, bidders fully pass on an increase in their cost of financing to the seller.*

Proof. See Appendix.

Similar to Proposition 1, the core of the proof is to show that bidders always bid such that they obtain a certain level of expected payment for a given security type. The reason is that from their perspective, what they pay in the cash auction is effectively a security to the investor. One can, therefore, rewrite their optimization problem as one of choosing the optimal security payment– i.e. the order s_R. Then, the cash bid needed to achieve this s_R can be derived by "reverse engineering" the financing contract signed with the outside investor. Lemma 4 provides a sufficient condition that such a strategy leads to the same probability of winning as choosing the cash bid directly. Therefore, the expected payoffs are also the same as in a security-bid auction, implying that the cost of financing are fully passed on to the seller.

3.5 Effect of Security Design and Discussion

The final step before solving the equilibrium of the financing game is to discuss the effect of security design on the extent to which the costs of financing are passed on to the seller. Unlike bargaining power, which has no effect on a bidder's payoff for a given security type, security design and the cash constraint are the primary factors affecting this pass-on. The following analysis introduces again the private values assumption and subsequently discusses the robustness of the results if this assumption is relaxed (cf. Section 4.4).

Recall from Proposition 2 that a bidder's net expected payment in an externally financed auction depends on his type. Hence, a promise for a future repayment is worth less to low types than for high types, making the former types bid more aggressively. The literature on bidding in securities has used essentially the same intuition to compare bidding in different security types by comparing how strongly a security's value depends on a bidder's type. The following analysis uses the definition of "steepness" introduced by DeMarzo et al. (2005).[13]

[13]It is *not* surprising that the effect of security design in an externally financed auction is the same as in a security-bid auction. From the point of view of the bidder, his overall payment is determined by the

A security R is steeper than $\widetilde{R}$ if $E[R(X)|\theta] = E[\widetilde{R}(X)|\theta]$ implies $\frac{\partial}{\partial\theta}E[R(X)|\theta] > \frac{\partial}{\partial\theta}E[\widetilde{R}(X)|\theta]$. Hence, levered equity is steeper than equity, which is steeper than debt. In what follows, "financing with a steeper/flatter security" will refer to this security design aspect and *not* to the specific financing terms of the financing instrument. The latter are captured by $s_R(y)$. It is straightforward to extend DeMarzo et al.'s main result to externally financed auctions.

Corollary 3 *Holding the expected payoff of outside investors fixed, the seller's expected revenue is higher if bidders finance their bids with steeper securities.*

Proof. See Appendix.

One can further extend this analogy to compare different auction formats. The results of DeMarzo et al. extend also in this context. In particular, there is generally no revenue equivalence among different auction formats.[14]

Discussion Payoff equivalence within the same security type is the first main result in this paper. Unlike Myerson's (1981) classical payoff equivalence, which compares payoffs across auction formats, Proposition 3 holds only for a given auction format, security type, and cash constraint w. The assumptions behind it are, however, quite weak. Unlike Myerson's result, it is independent of whether the auction is symmetric in that each bidder is drawn from the same distribution F. Furthermore, it does not matter whether types are interdependent and/or the values are common, as long as the financing game is an equilibrium of the overall game. In fact, Proposition 3 can be extended to any auction format, for which Lemma 4 is satisfied. The critical assumptions are that there are at least two bidders and that all bidders have the same cash participation w.

Indeed, if there is only one bidder, he will offer the minimum price for which the seller will sell the asset. Let this price be $\overline{p} > w$, so that raising financing is an issue. The

security contract promised to the outside investor. One of the contributions of this paper is to show that his expected payoff is exactly the same as in a security-bid auction. The latter fact implies then that the costs of financing are fully passed on to the seller.

[14] It is not possible to generalize Corollary 3 to any auction format. The reason is that the winner's expected payment may also depend on the losing bids. Thus, taking the expectation over θ_{-i} yields an "expected security" that may have different properties compared to the original. In particular, debt may no longer be the "flattest" security.

bidder's optimal offer is always $\overline{p}$ and the cost of financing play no role in this offer. They only determine whether the bidder will be able to afford to pay $\overline{p}$.

In addition, if bidders have different cash holdings w and the investor can extract rent, financially unconstrained low types may outbid higher types. This leads to a different allocation rule compared to the case when the bids are in securities. A bidder's ex ante expected payoff after paying the seller and repaying the investor will, thus, depend non-trivially on the cost of financing. Even if the investor does not extract rent, the allocation rule can depend on factors such as whether the financing terms are set before or after the auction (Rhodes-Kropf and Viswanathan, 2005). Introducing asymmetries in the distribution from which types are drawn has a similar effect.

4 Financing Game

The previous section builds up the intuition for the cost-pass-through result for the case in which all bidders are financed with the same security contract. In what follows, it is shown that focusing on this case is without loss of generality. Coexistence of different financing contracts will not be observed in equilibrium when bidders are financed by outside investors. One exception is when the seller can also offer financing. Section 4.3 discusses in detail how the analysis changes in this case.[15]

Though financing with different contracts is feasible when it can be obtained only from outside investors, it comes at a cost, which makes the bidders and the investor prefer financing with the same contract (albeit different financing contracts). In particular, financing with the flattest (steepest) security will give the lower (upper) bound for a bidder's overall net payment in the auction. The following example illustrates the intuition. Suppose that there are two pools of types. Bidders from the lower pool $[\underline{\theta}, \theta')$ finance their payment with the debt contract $R(\cdot, s_R(y))$, while bidders from the higher pool $[\theta', \overline{\theta}]$ finance their payment with the steeper security $\widetilde{R}\left(\cdot, s_{\widetilde{R}}(y)\right)$. Observe that "pool" can be somewhat misleading in this context. Even if bidders are financed with the same contract at $t = 1$, they still make different type-dependent bids at $t = 2$.

The expected payoffs of the types from the low pool do not depend on the terms at which they receive financing. The reason is the same as in Proposition 3. Hence,

[15] Note that while the difficulty for the seller in a security-bid auction is how to compare bids if they are in different securities, this is not an issue in an externally-financed cash auction.

irrespective of their financing terms (i.e., the functional form of $s_R(y)$), the first pool in the separating equilibrium finds the debt contract R just as expensive as when all types finance their payments with debt. Things look different for the types from the higher pool, however. Analogous to Corollary 3, financing in a steeper security makes the security payment more strongly dependent on their true type. Hence, the types in this second pool are induced to bid more aggressively, so that their expected equilibrium security payments increase relative to the case in which all bidders are financed with debt.

The following lemma formalizes this intuition. Whatever contract is offered at $t = 1$, it must be incentive-compatible, implying a result analogous to Lemma 4: the expected security payment promised to an investor must increase in y. The difference is that bidders can now deviate along two dimensions: along the security contract and along the cash bid. The lemma shows that debt financing gives the lowest incentives for low types to mimic higher types when optimally choosing the financing contract and their bids. That is, by relaxing the incentive constraint, debt financing minimizes the potential for mispricing of the financing contract, which is the cause for overbidding. As a result, bidders' equilibrium bids are least aggressive when all types are financed with debt. By incentive compatibility, this must be the same contract for all types. An analogous intuition explains why offering the steepest security (levered equity) to all bidders leads to the highest equilibrium expected payments.

Lemma 5 *The bidders' overall expected payments are lowest (highest) when all bidders are financed with the flattest (steepest) security.*

Proof. See Appendix.

The implications for the seller's expected revenue are straightforward. It is lowest if all types finance their bids with the flattest security (debt). It is highest if they finance their bids by issuing the steepest security (levered equity). All other equilibria yield an expected revenue that is in between these two extremes. This insight is key for showing that financing with different securities will not be observed in equilibrium. In what follows, the analysis discusses, in turn, the two orthogonal cases when the investor can screen the bidders and when the bidders can make an offer to the investor.

4.1 Non-Competitive Market for Capital

Suppose that the market for capital is not competitive, so that the investor can make a take-it-or-leave-it offer to a bidder. The investor's objective is to choose the security R and its financing terms $s_R(y)$ so as to maximize his expected payoff

$$E_{\theta_i,\theta_{-i}}\left[\left(\int_X \left[R\left(x, s_R(y(\theta_i,\theta_{-i}))-(y(\theta_i,\theta_{-i})-w)\right] dG\left(x|\theta_i\right)\right) P(\theta_i,\theta_{-i})\right]$$

subject to the restrictions that the contract is feasible, incentive-compatible, and individually rational. Consistent with the Appendix, $E_{\theta_i,\theta_{-i}}[\cdot]$ is the expectation operator over all possible realizations of the N types and P is the corresponding allocation rule.

To illustrate the intuition, consider first the case in which there is only one investor who is prepared to finance the winner in the auction. His expected payoff is the residual surplus from the auction after subtracting the bidders' expected payoffs and the seller's expected revenue. By designing the financing contract, the investor effectively controls the cash payments in the auction (cf. (3)). In particular, he can choose the financing terms such that the seller's expected revenue approaches zero.[16] What he cannot control is the bidders' expected payoff for a given security type. Lemma 5 implies that he should, therefore, offer the steepest security to all types. With such financing, bidders compete away more of their information rent. By effectively controlling the bid prices, the investor can then extract this additional rent from the seller.[17] Recall, thereby, that steepness refers merely to the security's type (e.g., debt, equity, etc.) and not the concrete financing terms (captured by $s_R(y)$).

Suppose now that there are multiple investors, but there is one bidder who is locked in to one of these investors. One way to endogenize such a setting is by adding an additional layer of information asymmetry between an incumbent and outside investors. Then, a refusal to provide financing may make it impossible for the bidder to raise cash elsewhere, as it sends a negative signal about his type (e.g., Rajan, 1992).

The main difference from the case above is that the incumbent investor stays in indirect competition to other outside investors. Thus, a financing contract that extracts the

[16]For the seller to be able to rank the bidders' cash bids, they must be strictly positive. Furthermore, bids by different types may need to differ by a non-trivial amount so that adding pure white noise to the bids does not destroy their ordering (Samuelson, 1987).

[17]By offering the same security to all bidders, the investor provides an opportunity for all types to enter the auction. This may not be optimal for the seller. By setting a reserve price, she may try to prevent the investor from expropriating (close to) her full rent.

maximum surplus from the locked in bidder and the seller, conditional on winning, induces an equilibrium cash bid with a very low probability of winning. Thus, it may be optimal to relax the financing conditions by taking into account the other bidders' contracts. The main security-design trade-offs remain unchanged, however. Financing in steeper securities makes the locked in bidder's expected security payment more strongly dependent on his type and induces him to bid more aggressively. This can be used by the investor to extract a higher portion of the bidder's surplus for every realization of y, while not affecting his probability of winning. The following proposition summarizes these insights.

Proposition 4 *(i) A monopolistic investor offers the same financing contract to all bidders. Financing is provided against the steepest security.*
(ii) A bidder who is locked in to an investor receives financing against the steepest security, independent of the other bidders' equilibrium contracts.

Proof. See Appendix.

4.2 Competitive Market for Capital

Consider next the case when bidders have all the bargaining power at $t = 1$. This is modeled in analogy to the previous section by stipulating that they make a take-it-or-leave-it offer to the investor. The result is a game of signaling, as bidders are privately informed about their types. One interpretation for this setup is that of a competitive market for capital in which no investor has privileged information.

An equilibrium candidate of the financing game is a quintuple of functions $(R(X, s_R(y)), \phi, \mu, \pi, \beta(\theta))$: $R(X, s_R(y))$ is the security offered by the bidder, which sets the order $s_R(y)$ for every cash payment y; ϕ is the investor's updated belief at $t = 1$, which maps the proposed security contract into the set of probability distributions over the type set $\theta \in [\underline{\theta}, \overline{\theta}]$; μ is the investor's interim belief at $t = 2$, which maps the observed auction payment y over the same type set; $\pi : R(\cdot) \rightarrow [0, 1]$ represents the investor's decision to refinance the project, where $\pi = 1$ corresponds to accepting, while $\pi = 0$ to rejecting; finally, $\beta(\theta) : (R(\cdot), \pi) \rightarrow \mathbb{R}_+$ is the investor's equilibrium bidding strategy. It is assumed that the bidder makes an offer to the investor for which the latter breaks even at the *interim* stage for every realization of y. Appendix B shows that the same arguments can be extended to the more general case in which the investor requires only to break even at $t = 1$. As above, the presentation in the main text focuses on the SPA. Appendix B contains an extension to the FPA.

As is standard, the equilibrium concept is that of Perfect Bayesian Equilibrium. To rule out equilibria supported by arbitrary off-equilibrium beliefs, the equilibrium set is refined with D1 (Cho and Kreps, 1987; Ramey, 1996).[18] A formal definition is given in the Appendix, but the intuition is simple: D1 requires the investor to restrict his beliefs to the types who are most likely to deviate. In the context of this game, "most likely" refers to the probability that the deviation is more profitable than the equilibrium contract also after the auction payment becomes known at $t = 2$.

In what follows, it is argued that the unique equilibrium of the signaling game is for all bidders to issue the flattest security type (debt). Lemma 5 already suggests the intuition. Debt is the least information sensitive security. As a result, it maximally relaxes the bidders' incentive constraint and minimizes mispricing of the financing contract. Hence, financing with debt is the only equilibrium from which the highest type will not deviate. Moreover, any type who offers a different contract will signal that he is a low type. This intuition is made precise in what follows.

The first step is to show that there is no equilibrium in which some type fully separates from all other types and, thus, obtains better financing terms than lower types. As is common in such signaling games, such separation is not incentive-compatible. Thus, any financing contract issued in equilibrium is offered by more than one type.

A consequence of having pooling contracts, is that whether financing with a steep or a flat security is cheaper or more expensive to the winning bidder depends on the second-highest bid. If this bid is very close to the winner's offer, the interim expectation of the investor regarding the winner's type is higher than the actual type. The bidder benefits more from this "mistake" when the financing contract is in steeper securities, as such securities are *more* sensitive to the true type. In contrast, if the second-highest bid is much lower than the winner's offer, the expectation of the investor is lower than the true type. The winner prefers financing with a flatter security in this case. As the security repayment is *less* sensitive to the true type, he suffers less from the investor's misjudgment

[18] D1 has become, by now, a standard refinement in the security design literature (e.g., Nachman and Noe, 1994; DeMarzo and Duffie, 1999).

of his type.

The key in the second step of the proof is to show that the highest type (in a pool) will break any equilibrium candidate in which he is not financed with debt. Take, for instance, the highest type $\overline{\theta}$ and suppose that his payment is not financed with debt. He can deviate to offering debt financing to the investor, for which the latter would break even at the interim stage for some non-degenerate beliefs. With such a contract, the highest type submits the *same* bid as with the equilibrium contract. The reason is that the investor correctly infers $\overline{\theta}$ when he observes the highest payment that can be rationally expected.[19] Hence, deviating does not change the highest type's probability of winning. Moreover, by the arguments in the previous paragraph, he is the only type for whom a deviation to this flatter contract is profitable with probability one also at $t = 2$. D1 implies, therefore, that the investor should place probability one on the deviation coming from this type. By construction, he makes an expected profit accepting for such beliefs. The only equilibrium candidate is, thus, that all types issue the flattest security.

To show that such financing can be sustained in equilibrium, one must only rule out deviations to steeper contracts. It is straightforward to show that there exist beliefs for which the following strategies constitute an equilibrium: All types issue debt $R(\cdot, s_R(y))$, for which the investor breaks even at the interim stage. The investor accepts $R(\cdot, s_R(y))$. Deviations are accepted only if he at least breaks even in the same stage as in equilibrium (i.e., at the interim stage). His out-of-equilibrium beliefs are refined with D1.

Proposition 5 *Debt financing is the unique equilibrium of the financing game if the bidders make an offer to the investor(s).*

Proof. See Appendix.

Somewhat related, the literature on bidding with securities obtains that the only equilibrium satisfying D1 is for all bidders to bid with debt. What makes the two signaling games different is that the investor effectively observes two signals: one when the financing offer is made and one when he observes the auction payment. As a result, he can infer the deviating bidder's type more precisely also off the equilibrium path. In the case of the

[19] Analogously to (2), the highest type's bid is defined by the case when he actually has to pay his bid. Since the investor infers the highest type correctly upon observing the highest bid, the security contract is also priced correctly. Thus, outside financing has no distorting effect on the cash bid.

first-price auction, he can even perfectly infer this type and no equilibrium refinements are needed to show that debt financing emerges as the unique equilibrium of the financing game (s. Appendix B).

4.3 Seller Financing

Consider the same setup as in the previous subsection, in which bidders make an offer to outside investors. Even though the investor makes no expected profit, the seller can do better compared to the case in which all bidders are financed with debt (Lemma 5). It is immediate that by committing to extremely cheap levered equity financing– i.e., financing for which an outside investor would never break even– she could attract all bidders to take her offer. This would increase her expected revenue (Lemma 5). Her expected losses from providing very cheap financing are, thereby, exactly offset by the higher bids.

Corollary 4 *The seller is always at least weakly better off offering financing herself.*

In some cases, however, the seller may want to minimize the probability that a bidder actually accepts her offer. A typical example would be a case in which she strictly prefers liquidity at $t = 2$ compared to $t = 3$. It is still possible to construct an offer, which raises her expected revenue.

Suppose, as an illustration, that the seller can commit to an offer for which an outside investor would just break even at the interim stage if all types were financed with this contract.[20] If there were no outside investors, this form of financing would lead to the highest expected revenues irrespective of the financing terms (Lemma 5). With outside investors, the bidders will always choose the ex post cheaper financing alternative. This alternative will, therefore, define the equilibrium cash bid.[21] Suppose the winning bidder can make an offer to outside investors after the payment y is known. In what follows, it is argued that for any given realization of y, the unique security offered by the winning bidder to outside investors is debt. It is shown then that, though only some bidders will be financed by the seller, *all* will use her offer to determine their equilibrium cash bids. This pushes these bids higher by Lemma 5.

[20] See Gorbenko and Malenko (2011) for a model, in which sellers compete for bidders in security-bid auctions.

[21] Otherwise, a bidder would potentially lose in cases in which he would have still made an expected profit winning the auction.

Lemma 6 *The unique equilibrium security offered by the winning bidder to outside investors is debt.*

Proof. See Appendix.

The Lemma follows by standard security design arguments. Outside investors know that bidders resort to outside financing only if it is the cheaper alternative. In this typical adverse selection problem, the unique equilibrium for all types who apply for outside financing is to offer a pooling debt contract. The intuition is that debt is the least information-sensitive security. As such, it minimizes the amount of underpricing for high types and is, thus, the only security contract from which these types will not deviate (Nachman and Noe, 1994).

The main difference from the standard analysis is that financing from the seller provides a type-dependent outside option to the auction winner. As a result, not all winning types will prefer outside financing. Just as in the previous section, bidders whose payment in the auction is very close to their true type would always choose the seller's offer. The reason is that more information sensitive contracts are more profitable to a bidder if the financing contract overvalues his true type. The opposite is true if the second highest bid is much lower than the winning bid. Then, the effect of a contract that undervalues a bidder's true type is smallest for debt financing. In equilibrium, therefore, only "overvalued" types take the seller's steeper financing contract. Since the relevant contract for a bidder when designing his equilibrium bids is the one he takes when he actually needs to pay his bid (and all types take the seller's contract in this case) it will be the seller's offer that will determine the equilibrium bids.

Proposition 6 *The seller can increase her expected revenue if she can commit to offering alternative financing in steeper securities. She is, however, less likely to break even on the financing contract than are outside investors.*

Together, Proposition 6 and Corollary 4 imply that the seller can always find a levered equity contract, which induces a higher equilibrium expected revenue, while minimizing the probability that a bidder actually takes the offer.

Finally, suppose that the seller cannot commit to her financing offer. Offering the same debt contract to both the seller and outside investors now becomes an equilibrium. On the one hand, offering debt to outside investors follows the same intuition as in Proposition 5. On the other hand, the seller will never accept a deviation to a different offer. Since the payment is the second highest bid, she is strictly worse off if deviating makes a bidder more aggressive, so that he, instead of a more efficient type, wins the auction. In this case, the expected value of her security claim would be worth strictly less than on the equilibrium path.

4.4 Discussion and Empirical Implications II

Unlike the results from Section 3, it is more difficult to extend the results from Section 4.2 beyond the first and second-price auction. This is best illustrated with an example. Suppose that all bidders are financed with debt. Take a winner-pays-auction, in which the payment y is the average of all bids. When solving the bidder's problem, one needs to consider the "expected" security that the bidder has to repay, depending on the realization of the other $N-1$ bids. The problem is that this expected security does not look like debt any longer. Thus, it may be that there is a separating equilibrium in which the expected securities are flatter than the ones in a pooling equilibrium with debt. Lemma 5 and the propositions that follow it may, then, fail for some auction formats.

The results for the first and second-price auctions are robust to introducing correlated types or common values, however. As shown in DeMarzo et al. (2005), the use of steeper securities also leads to lower expected payoffs for the seller under these settings.[22] It is, thus, straightforward to modify Lemma 5 and Proposition 4, as their proofs do not critically depend on the private values assumption. Though more involved, one can also modify the signaling game by assuming that each bidder is financed by a different investor. This assumption ensures that the outside investor has no informational advantage over the bidders and that there are no further adverse selection problems.[23]

It has been stipulated so far that all bidders invest their entire cash holdings when paying their bid. Allowing for the possibility that they separate with the amount they are willing to co-invest does not change the results in Section 4.2. Proposition 2 implies that the lowest bound for the bidders' payments is when all types finance their bids with debt

[22] Axelson (2007) shows that selling debt minimizes underpricing when the seller auctions securities in a common-value multi-unit auction. In this case the seller is left with a levered equity claim on the firm's cash flows, which implies qualitatively the same ranking of securities as in DeMarzo et al. (2005).

[23] Note that with affiliated types and common values, an investor who observes more than one offer will have an informational advantage over the bidder. This may change the equilibrium set.

and co-invest their full cash holdings w. One can show, along the lines of Proposition 5, that this is the only equilibrium from which the highest type has no incentive to deviate. Things become more complicated if bargaining power is in the hands of the investor. The results from Section 4 continue to hold if the investor can choose the amount of co-financing.[24] Otherwise, the steepest security continues to be optimal, but there is a conflict about how much bidders should co-invest. Allowing for heterogeneous cash holdings, which are uncorrelated with valuations, makes the problem even more involved.

Finally, suppose that bidders learn their types only after signing the contract with an outside investor at $t = 1$, but before they bid in the auction at $t = 2$. This case is easy to analyze given the above results. Given the ex ante symmetry, all bidders offer the investor the same contract. To be precise, by Lemma 5, no menu is optimal. The bidders' (investor's) ex ante expected payoff is maximized by offering financing in the flattest (steepest) security for which the investor just breaks even.

Empirical Implications A number of empirical implications emerge from the results of this Section. Several important ones are listed below using the examples of bankruptcy auctions and takeover contests. Similar conclusions can be drawn, however, for any auction that puts up for sale an asset with stochastic cash flows. Starting with the results from the last section, it should hold that:

Implication 6. *The seller can increase her revenues in a cash auction by offering financing to the bidders.*

In a sample of bankrupt firms sold in an auction, Eckbo and Thorburn (2009) find that the lead creditor, who is effectively the seller of the firm, often provides financing to the bidders. This results in higher cash bids and there is no sign of allocative inefficiency. Related, Povel and Singh (2010) argue that providing the option of seller financing in all-cash takeover contests has become increasingly popular in recent years. The authors explain that this practice, known as stapled finance, can significantly increase the seller's revenue when her financing offer effectively gives a subsidy to low-valuation bidders. Proposition 6 adds to this argument by showing that increasing expected revenues is possible even when the bidders can negotiate a better financing contract with outside investors.[25] The next

[24] The investor will choose the smallest amount, which still allows for separation of the bids. Note that (2) implies that all types bid the same if they do not co-invest from their own cash.

[25] In contrast to Povel and Singh (2010), the resulting subsidy to bidders who accept the seller's offer is

two implications are concerned with the type of financing in externally financed auctions

Implication 7. *Seller financing should be in the form of steeper securities.*

Indeed, stapled finance is in the form of a non-recourse loan. That is, unlike typical bank credit, it is secured only by the assets of the target company and, thus, represents financing in a steeper security. Furthermore, there is vast evidence that payment in stock is often used as an additional method of payment in takeover contests (see Betton et al. (2008) for a recent survey). It is not surprising that the seller readily accepts such bids. Prior work has shown that bidding in securities increases the seller's revenue (Hansen, 1985; DeMarzo et al., 2005). The novel result of this paper is to show that bidding in non-debt securities can be an *equilibrium* even if bidders can often negotiate a better deal with outside investors.

The results from Sections 4.1 and 4.2 show that the securities used to finance auction bids differ depending on the distribution of bargaining power when raising capital. The last implication summarizes the predictions from Propositions 4-5.

Implication 8. *The use of debt to finance bids in cash auctions decreases with investors' ability to extract rent.*

As the examples in the introduction show, equity seems to be used when bidders are smaller, but debt financing seems prevalent when bidders are large and have easy access to capital (see also Ghosh and Jain, 2000; Morellec and Zhdanov, 2008). Though these findings correspond to the model's predictions, more research is needed to test the effect of bargaining power on bidders' financing contracts.

5 Conclusion

Financing is a critical aspect when a bidder's free cash on hand is insufficient to pay for his bid in large-scale auctions. With the increasing popularity of such auctions for the sale of assets and companies both in and outside bankruptcy, it is important to understand how their outcome is driven by financial contracting. Previous work has focused mainly on comparing different auction formats and security types in security-bid auctions (e.g., DeMarzo et al., 2005). Though closely related, it makes no predictions on how payoffs and revenues as well as financial contracting depend on the financing game between bidders,

not fully compensated by higher bids.

outside investors, and the seller. The motivation for the present paper is that the existing literature on externally financed cash auctions also provides no such analysis.

The first main result is that bidders' ability to pass on their cost of financing to the seller depends only on the shape of their financing contract and their cash constraint. It does not depend, however, on their financier's ability to extract rent. The critical assumption is that the financing game is part of the equilibrium of the overall game. This guarantees that there is a monotonic relation between equilibrium cash and security payments. As a result, ranking cash bids and ranking security-bids leads to the same allocation rule. Therefore, one can present the bidder's problem as one of choosing the optimal security-bid. His corresponding cash bid can be derived, then, from the specific financing contract. This is intuitive, as the bidder's payment in the auction is effectively determined not by the cash raised to pay the seller, but by his security repayment to the investor. Bidders' payoffs in an externally-financed cash auction are, therefore, the same as in a security-bid auction.

The presence of a game with a third party makes cash auctions very different form security-bid auctions, however. The second contribution of the paper is to show that financial contracting depends on whether bidders or outside investors have more bargaining power. In the latter case, investors require higher participation on the upside to provide financing for the winning bid. Intuitively, the investor cannot control the bidder's surplus for a given security type, as any increase in the cost of financing is passed on to the seller. The bidder's share of the surplus is, however, minimal if the security contract is more information-sensitive, as this induces him to bid more aggressively. In the former case, when bidders have more bargaining power, they offer debt financing. Similar to the predictions of the pecking order, financing with debt is the only equilibrium in which the highest-valuation bidders have no incentive to deviate when pooled with lower types.

The final result discusses how seller financing alters the equilibrium of the financing game. It shows that the seller can benefit from committing to provide alternative financing in securities steeper than debt. On the one hand, this may reduce her expected revenue. Such financing is in effect a type-dependent outside option to the bidders when they negotiate with outside investors. Hence, the seller knows that only overvalued types would accept it. On the other hand, this alternative financing opportunity causes more aggressive bidding, thereby making up for this expected subsidy.

There are several implications of these results. One is that many standard arguments against bankruptcy auctions have little theoretical justification. Bidders' incentives to participate in auctions are not reduced by expensive financing, as they pass on an increase in their cost of financing to the seller. This explains recent findings that participation in bankruptcy auctions is high, that these auctions seem to be efficient in that they do not underperform industry peers. Moreover, assets are sold at a discount (though not at fire-sale prices), and there is a positive market reaction on the winner's stock price, indicating that bidders indeed pass on their costs and make a profit. Another implication is that the seller can benefit from committing to provide financing in the form of equity or non-recourse loans. This prediction squares up with the empirical evidence that such financing is often provided in bankruptcy and takeover auctions. Finally, the prediction that debt is the preferred financing instruments in a competitive market, but not when bidders are locked in to their investors, also seems to find some confirmation in practice, but more evidence is needed on how financial contracting depends on the distribution of bargaining power.

Appendix A Omitted Proofs

Proof of Lemma 1. Let $\theta' > \theta$. Since R is a nondecreasing function on a compact set, it is differentiable a.e. and $R'(X, \cdot) \geq 0$. Using then that $G(X|\theta')$ dominates $G(X|\theta)$ in terms of first order stochastic dominance, it holds

$$\int_X R(x, \cdot)\left(g(x|\theta') - g(x|\theta)\right) dx = -\int_X R'(x, \cdot)\left(G(x|\theta') - G(x|\theta)\right) dx > 0,$$

where the equality follows from integration by parts. Dividing by $\theta' - \theta$ and taking the limit, yields the result. The result for the bidder's claim can be shown analogously. **Q.E.D.**

The following notation is used for the proofs below. Let $\beta(\theta_i)$ be the equilibrium bid of type θ_i and $\beta(\theta_{-i})$ the vector of bids of the other $N-1$ types. The *allocation rule* P in a second-price auction is defined as

$$P(\beta(\theta_i), \beta(\theta_{-i})) = \begin{cases} 1 & \text{if } \beta(\theta_i) > \max_{\theta_j \in \theta_{-i}} \beta(\theta_j) \\ 0 & \text{if } \beta(\theta_i) < \max_{\theta_j \in \theta_{-i}} \beta(\theta_j) \end{cases}.$$

If $\beta(\theta_i) = \max_{\theta_j \in \theta_{-i}} \beta(\theta_j)$, P is determined by some tie-breaking rule. For general use below, let $F_1(\theta_i) := F^{N-1}(\theta_i)$ and as usual $f_1(\theta_i) := \frac{\partial}{\partial\theta_i} F_1(\theta_i)$.[26] The payment of bidder θ_i conditional on winning is the minimum of his bid $\beta(\theta_i)$ and the highest bid from $\beta(\theta_{-i})$. One can, thus, write $y(\beta(\theta_i), \beta(\theta_{-i}))$ to make explicit this dependence. We are now ready to prove Lemma 2.

Proof of Lemma 2. The first part of the proof shows that $D(\cdot)$ is weakly increasing in y. The second part argues that strict monotonicity can fail only if this does not lead to a change in the allocation rule.

(i) Monotonicity. Suppose that the claim were false and denote for brevity $D(y) := D(y, r(y))$. Take the lowest payment y' for which $D(\cdot)$ decreases at y': $D(y') > D(y'')$ where $y' < y''$ and $y'' \to y'$. Let types θ_i' and θ_i'' be the types who bid $\beta(\theta_i') = y'$ and $\beta(\theta_i'') = y''$ in equilibrium. Further, let

$$\begin{aligned} A &: = \{\theta_{-i} : \max_{\theta_j \in \theta_{-i}} \beta(\theta_j) \leq \beta(\theta_i')\} \\ B &: = \{\theta_{-i} : \max_{\theta_j \in \theta_{-i}} \beta(\theta_j) \in (\beta(\theta_i'), \beta(\theta_i'')]\} \end{aligned}$$

be the realizations of θ_{-i} such that the second highest bid is below $\beta(\theta')$ and between $\beta(\theta_i')$ and $\beta(\theta_i'')$ respectively. This more general notation is needed, as no restrictions have been imposed on the shape of $\beta(\cdot)$. Incentive compatibility for type θ_i' can be written as

$$\begin{aligned} & E_{\theta_{-i}}\left[\left(\int_X [x - R(x, D(y(\beta(\theta_i'), \beta(\theta_{-i}))))]\, dG(x|\theta_i') - w\right) P(\beta(\theta_i'), \beta(\theta_{-i}))\right] \quad \text{(A.1)} \\ \geq\ & E_{\theta_{-i}}\left[\left(\int_X [x - R(x, D(y(\beta(\theta_i''), \beta(\theta_{-i}))))]\, dG(x|\theta_i') - w\right) P(\beta(\theta_i''), \beta(\theta_{-i}))\middle| A\right] \\ & + E_{\theta_{-i}}\left[\left(\int_X [x - R(x, D(y(\beta(\theta_i''), \beta(\theta_{-i}))))]\, dG(x|\theta_i') - w\right) P(\beta(\theta_i''), \beta(\theta_{-i}))\middle| B\right], \end{aligned}$$

where $E_{\theta_{-i}}$ is the conditional expectation over the realization of θ_{-i} given type θ_i'. Note that the expressions in the first and the second lines are equal. Since in both cases $y \leq \beta(\theta_i')$, both the payments $y(\cdot)$ and the allocation rules $P(\cdot)$ are the same. Sufficient for a contradiction is, thus, that the third line is strictly positive. This is shown next.

Observe that by submitting $\beta(\theta') = y'$ in equilibrium, type θ' must have a weakly positive expected payoff when he actually has to pay his bid. He is, otherwise, strictly better off deviating to a lower bid, since by assumption $D(\cdot)$ increases in y for all $y \leq y'$,

[26] Note that with symmetric bidders and independent types the probability of winning in an efficient auction for this allocation rule becomes simply $F_1(\theta_i)$.

and y increases in $\beta(\cdot)$.[27] The third line must be, therefore, strictly positive, as by the contradiction assumption, the debt repayment is strictly less than $D(y')$ for $\theta_{-i} \in B$.

(ii) Weak monotonicity. Suppose that $D(y') = D(y'')$ where $y' < y''$ and $y'' \to y'$. Let θ_i' and θ_i'' be the types who bid $\beta(\theta_i') = y'$ and $\beta(\theta_i'') = y''$ in equilibrium. Clearly, type θ' will deviate to $\beta(\theta'')$, as this increases his probability of winning without changing his equilibrium security payment conditional on winning. Hence, the equilibrium security payment $D(y')$ can be equal to $D(y'')$ only if bidding y' and y'' leads to the same allocation rule. **Q.E.D.**

Proof of Lemma 3. Consider type θ_i and suppose that all other types bid as implied by (2). Observe first that a bidder is indifferent between any two cash bids that lead to the same allocation rule and the same promised debt repayment. It is, thus, without loss of generality to assume that he prefers the higher bid in such a case. This allows to concentrate only on bids that lead to different allocation rules. Lemma 2 implies then that for any realization of θ_{-i}

$$y(\beta(\theta_i), \beta(\theta_{-i})) > y(\beta(\widehat{\theta}_i), \beta(\theta_{-i})) \Longleftrightarrow D(y(\beta(\theta_i), \beta(\theta_{-i}))) > D(y(\beta(\widehat{\theta}_i), \beta(\theta_{-i}))). \tag{A.2}$$

Hence, a higher cash payment increases a bidder's security payment to outside investors. It is, thus, easy to check that the standard characterization of the SPA applies (Krishna, 2002): For every type θ_i it is a weakly dominant strategy to submit a cash bid for which he would just break even conditionally on paying this bid $y = \beta(\theta_i)$:[28]

$$0 = \int_X x dG(x|\theta_i) + (\beta(\theta_i) - w) - \beta(\theta_i) - \int_X R(x, D(\beta(\theta_i))) dG(x|\theta_i)$$

where the first term is the bidder's expected valuation of the asset. The second and the third terms say that, upon winning, the bidder must raise the money he doesn't have to pay his bid and then pay it to the seller. Finally, the remaining term stands for the expected security payment to the investor. **Q.E.D.**

[27] If y' were the lowest equilibrium payment, a deviation to a higher bid would be strictly profitable.

[28] Winning with a higher bid is weakly dominated, as in all additional cases that the bidder wins, he must make a security payment for which he makes an expected loss. Similarly, bidding less is weakly dominated, as the bidder loses in cases in which his security payment would have been sufficiently low to make an expected profit.

Proof of Proposition 1. The proof follows straightforwardly from Lemma 2 and 3 and the arguments from the main text. It only remains to argue more formally that the allocation rule in an externally financed auction is the same as in a security-bid auction.

Let $D(\theta_i, \theta_{-i})$ be the equilibrium promised debt repayment of type θ_i given types θ_{-i}. Before defining the allocation rule in a security-bid auction, recall that for any given realization of θ_{-i}, $P(\beta(\theta_i), \beta(\theta_{-i}))$ is uniquely determined by the "rank" of the cash bid $\beta(\theta_i)$ among $\beta(\theta_{-i})$. Let, therefore, analogously $D(\theta_i)$ be the equilibrium promised debt repayment determined alone by the bid of type θ_i. This is the analogue to a debt-bid in a security-bid auction. For any realization of θ_{-i} the allocation rule $P(D(\theta_i, \theta_{-i}))$ is determined by the "rank" of $D(\theta_i)$:

$$P(D(\theta_i, \theta_{-i})) = \begin{cases} 1 \text{ if } D(\theta_i) > \max_{\theta_j \in \theta_{-i}} D(\theta_j) \\ 0 \text{ if } D(\theta_i) < \max_{\theta_j \in \theta_{-i}} D(\theta_j) \end{cases}.$$

Ties are resolved with the same rule as in the cash auction. The relation in (A.2) implies now that for any two types θ_i and $\widehat{\theta}_i$, $\beta(\theta_i) > \beta(\widehat{\theta}_i)$ if and only if $D(\theta_i) > D(\widehat{\theta}_i)$. Hence, $P(D(\theta_i, \theta_{-i})) = P(\beta(\theta_i), \beta(\theta_{-i}))$ for every θ_i and θ_{-i}. A more general proof follows in Proposition 3 below. **Q.E.D.**

Proof of Proposition 2. For use below, note that the bidding strategy absent financial constraints $\beta_c(\cdot)$ is given by the first equality in:

$$\int_X x dG(x|\theta_i) = \beta_c(\theta_i) = \int_X R(x, D(\beta(\theta_i))) dG(x|\theta_i) + w \tag{A.3}$$

while the second equality defines the bidding strategy in an externally financed auction (cf. (2)).[29] The proof starts by showing that the equilibrium cash bids given outside financing are higher compared to the case in which bidders are not cash-constrained, i.e. $\beta(\theta_i) > \beta_c(\theta_i)$. It shows then that this effect increases as the co-investment w decreases.

The first key step in the proof is to use the result from Proposition 1. As long as all bidders are financed with the same contract, their expected payoff does not depend on how the interest rate is set. To analyze the bidder's payoffs, it is, therefore, without loss of generality to assume that the investor requires to break even at the interim stage for every realization of y. Suppose that in such a game, the investor observes $y = \beta(\theta_i)$. He

[29]Note that the case absent external financing is equivalent to the case when there is no information asymmetry between the investor and the bidders at $t = 1$ (cf. Section 4).

knows that the lowest type who may have won with such payment is type θ_i and so his interim beliefs are distributed on the support $\left[\theta_i, \overline{\theta}\right]$. Recalling that the auction payment is the second highest bid, his interim participation constraint for financing $\beta(\theta_i) - w$ can be written as

$$\int_{\theta_i}^{\overline{\theta}} \int_X R\left(x, D(\beta(\theta_i))\right) dG\left(x|t\right) d\mu\left(t|\beta(\theta_i)\right) + w = \beta(\theta_i), \tag{A.4}$$

where μ are his interim beliefs conditional on observing $y = \beta(\theta_i)$. Since the auction is efficient (Lemma 3) and the equilibrium security payment D increases in θ (Lemma 2 and 3), the RHS of (A.3) is less than the LHS of (A.4). It follows that $\beta(\theta_i) \geq \beta_c(\theta_i)$ with the inequality being strict for all types $\theta_i < \overline{\theta}$. More precisely

$$\begin{aligned} & \beta(\theta_i) - \beta_c(\theta_i) \\ = & \int_{\theta}^{\overline{\theta}} \int_X R\left(x, D(\beta(\theta_i))\right) dG\left(x|t\right) d\mu\left(t|\beta(\theta_i)\right) - \int_X R\left(x, D(\beta(\theta_i))\right) dG\left(x|\theta_i\right) \\ = & \int_{\theta}^{\overline{\theta}} \int_X R\left(x, D(\beta(\theta_i))\right) \left(dG\left(x|t\right) - dG\left(x|\theta_i\right)\right) d\mu\left(t|\beta(\theta_i)\right). \end{aligned}$$

This difference increases in the bidder's equilibrium debt repayment $D(\beta(\theta_i))$ when he has to pay his bid. From (2), this debt repayment decreases in the bidder's own cash participation w. Hence, the seller's expected revenue also decreases in w. Given that the investor just breaks even and that the overall surplus remains unchanged, it follows that the bidder's expected payoff increases in w. The proof of the FPA is presented in Appendix B. **Q.E.D.**

Proof of Lemma 4. Extending Lemma 2 to any ordered set of securities indexed by $s_R(y)$ is straightforward. One only needs to replace $D(y, r(y))$ with $s_R(y)$ in $R(\cdot)$. The proof holds also for common values and interdependent types, as it makes no reference to the assumptions of symmetry or independent types.[30] Appendix B contains the proof for the FPA. **Q.E.D.**

Proof of Proposition 3. The following proof applies to a wider set of auction formats, and not only the SPA. In particular, it covers also the FPA in Appendix B. It proceeds

[30] Common values in this setup can be represented by making the distribution function over X dependent also on θ_{-i}: $G(X|\theta_i, \theta_{-i})$.

in three steps. It shows first that there is a monotonic relation between equilibrium cash and security payments. This is used to argue that choosing the equilibrium cash payment leads to the same allocation rule as choosing the equilibrium security payment. The last step shows that the bidder's equilibrium problem can be rewritten as one of choosing the optimal security-bid instead of choosing the optimal cash bid.

Claim 1. *For any two types θ_i and $\widehat{\theta}_i$ and any realization of θ_{-i}*

$$y\left(\beta\left(\theta_i\right), \beta\left(\theta_{-i}\right)\right) > y(\beta(\widehat{\theta}_i), \beta\left(\theta_{-i}\right)) \Longleftrightarrow s_R\left(y\left(\beta\left(\theta_i\right), \beta\left(\theta_{-i}\right)\right)\right) > s_R(y(\beta(\widehat{\theta}_i), \beta\left(\theta_{-i}\right)))$$

Proof. Since a bidder is indifferent between any two cash bids that lead to the same allocation rule and the same order $s_R\left(y\right)$, it is without loss of generality to assume that the higher bid is preferred in such a case. This allows to concentrate only on bids that lead to different allocation rules. The claim is then a straightforward application of Lemma 4. **Q.E.D.**

Claim 2. *The allocation rule in an externally financed cash auction is the same as in a security-bid auction.*

Proof. Define $s_R\left(\theta_i, \theta_{-i}\right)$, $s_R\left(\theta_i\right)$, and $P(s_R(\theta_i, \theta_{-i}))$ analogously to $D\left(\theta_i, \theta_{-i}\right)$, $D\left(\theta_i\right)$, and $P(D(\theta_i, \theta_{-i}))$ in the proof of Proposition 1. The same arguments as in this proof imply together with Claim 1 that for any two types θ_i and $\widehat{\theta}_i$, $\beta\left(\theta_i\right) > \beta(\widehat{\theta}_i)$ if and only if $s_R\left(\theta_i\right) > s_R(\widehat{\theta}_i)$. Hence, $P(s_R(\theta_i, \theta_{-i})) = P(\beta(\theta_i), \beta\left(\theta_{-i}\right))$ for every θ_i and θ_{-i}. **Q.E.D.**

Claim 3. *A bidder in an externally financed cash auction has the same expected payoff as a bidder in a security-bid auction.*

Proof. Take any joint equilibrium of the bidding and the financing game. The financing game sets the equilibrium security repayment $s_R(\theta_i, \theta_{-i})$ for any realization of types θ_i and θ_{-i} given the equilibrium bidding strategies induced by the financing contract. Accordingly, from the bidding game we know that if all types θ_{-i} report truthfully, bidder θ_i also reports truthfully. Hence, for every realization of types, $\beta\left(\theta_i\right)$ satisfies

$$\begin{aligned}\theta_i \in \arg\max_{\widehat{\theta}_i} E_{\theta_{-i}} \Bigg[\Bigg(\int_X \left(x - R\left(x, s_R(y(\beta(\widehat{\theta}_i), \beta\left(\theta_{-i}\right)))\right)\right) dG\left(x|\theta_i, \theta_{-i}\right) - w \Bigg) \\ \times P(\beta(\widehat{\theta}_i), \beta\left(\theta_{-i}\right)) \Bigg]\end{aligned}$$

Note that for every $\widehat{\theta}_i$ and θ_{-i}, the equilibrium security repayment $s_R(\cdot)$ is determined as a "state variable" given the equilibrium contract from the financing game. The key step now is to rewrite the above problem as one of choosing the order $s_R(\widehat{\theta}_i, \cdot)$ instead of the cash bid $\beta(\widehat{\theta}_i)$. Using that there is a one-to-one correspondence between $s(\theta_i, \theta_{-i})$ and $y(\theta_i, \theta_{-i})$ (Lemma 4) and that $P(s_R(\theta_i, \theta_{-i})) = P(\beta(\theta_i), \beta(\theta_{-i}))$ (Claim 2), this problem can be stated as

$$\theta_i \in \arg\max_{\widehat{\theta}_i} E_{\theta_{-i}} \left[\left(\int_X \left(x - R\left(x, s_R(\widehat{\theta}_i, \theta_{-i})\right)\right) dG\left(x|\theta_i, \theta_{-i}\right) - w \right) P(s_R(\widehat{\theta}_i, \theta_{-i})) \right] \tag{A.5}$$

where crucially, for every given $\widehat{\theta}_i$ and θ_{-i}, y can be derived from the equilibrium of the financing game as the cash payment corresponding to the security repayment indexed by $s_R(\widehat{\theta}_i, \theta_{-i})$. (A.5) is, however, the same problem that bidders solve in a security-bid auction. Hence, their expected payoffs are also the same as in such an auction, implying that any increase in the cost of financing is fully passed on to the seller. **Q.E.D.**

Proof of Corollary 3. The Corollary is a straightforward extension of DeMarzo et al. (2005). Suppose $\widetilde{R}$ is flatter than R. The bidding strategies in the SPA are defined by

$$\int_X \widetilde{R}\left(x, s_{\widetilde{R}}(\widetilde{\beta}(\theta_i))\right) dG(x|\theta_i) = \int_X R(x, s_R(\beta(\theta_i))) \, dG(x|\theta_i) = \int_X x dG(x|\theta_i) - w$$

and so by the definition of steepness

$$\int_X \widetilde{R}\left(x, s_{\widetilde{R}}(\widetilde{\beta}(\theta_i))\right) dG(x|\theta) < \int_X R(x, s_R(\beta(\theta_i))) \, dG(x|\theta) \text{ for } \theta > \theta_i.$$

Hence raising the bid of a lower type is strictly more expensive for bidders if the payments in the auction are financed with a steeper security. Since both auctions are efficient and the investor just breaks even in both cases, the seller's expected revenue must be higher when the bidder finances his payments with R. The proof for the FPA is presented in Appendix B below. **Q.E.D.**

Proof of Lemma 5. In what follows $P(\beta(\theta_i'), \beta(\theta_{-i}))$ and $y(\beta(\theta_i'), \beta(\theta_{-i}))$ are written as $P(\theta_i', \theta_{-i})$ and $y(\theta_i', \theta_{-i})$. The proof shows only that financing with the flattest security yields the lowest expected payments. That financing with the steepest security leads to the highest payments can be shown analogously.

Observe first that in any equilibrium of the financing game, the expected security payment of every type θ_i

$$\int_X R\left(x, s_R(y(\theta_i, \theta_{-i}))\right) dG\left(x|\theta_i\right)$$

must increases in y also when the financing contract R (including its shape– e.g., debt/equity) is type- and payment-dependent. The proof of this claim follows the same argument as that of Lemma 2 and 4. A bidder's equilibrium strategy in the SPA is, thus, defined analogously to (2) and his cash and security payments are an increasing function of his type. Hence, irrespective of whether bidders sign the same financing contract at $t = 1$, they separate with their bids at $t = 2$. By stating and reformulating the incentive constraint, it is shown in what follows that debt financing most easily implements such separation. It reduces the incentives to overbid and, thus, leads to lower equilibrium security payments.

Step 1: *Reformulating the incentive constraint.* Suppose that bidders sign non-debt, possibly type- and payment-dependent contracts. Let

$$v\left(y, \theta_i\right) := \int_X R\left(x, s_R\left(y\right)\right) dG\left(x|\theta_i\right) - \left(y - w\right) \tag{A.6}$$

be the type-dependent difference between the expected true value of R in $t = 2$ and the amount raised from the investor (on the equilibrium path). Note that this difference reflects the investors's interim gain/loss from providing financing for y. Using the same notation as in Lemma 2, one can state the incentive constraint for type θ_i' who considers deviating to security contract $\widehat{R}$, issued in equilibrium by type $\widehat{\theta}_i$, and then bidding as some higher type θ_i'' as[31]

$$\begin{aligned} & E_{\theta_{-i}}\left[\left(\int_X \left[\widehat{R}\left(x, s_{\widehat{R}}(y(\theta_i', \theta_{-i}))\right) - R\left(x, s_R(y(\theta_i', \theta_{-i}))\right)\right] dG\left(x|\theta_i'\right)\right) P(\theta_i', \theta_{-i}) | A\right] \\ \geq \; & E_{\theta_{-i}}\left[\left(\int_X \left[x - \widehat{R}\left(x, s_{\widehat{R}}(y(\theta_i'', \theta_{-i}))\right)\right] dG\left(x|\theta_i'\right) - w\right) P(\theta_i'', \theta_{-i}) \middle| B\right]. \end{aligned}$$

Note that we have used that the highest bid is by definition less than $\min\left(\beta\left(\theta'\right), \beta\left(\theta''\right)\right)$ for $\theta_{-i} \in A$, implying that $y(\theta_i', \theta_{-i}) = y(\theta_i'', \theta_{-i})$ and $P(\theta_i', \theta_{-i}) = P(\theta_i'', \theta_{-i})$ for $\theta_{-i} \in A$. Using (A.6) to plug in for $R\left(\cdot\right)$ and $\widehat{R}\left(\cdot\right)$, the integral inside the expectation operator in the first line can be rewritten as

$$v(y, \widehat{\theta}_i) - v\left(y, \theta_i'\right) - \int_X \widehat{R}\left(x, s_{\widehat{R}}(y)\right) \left(dG(x|\widehat{\theta}_i) - dG\left(x|\theta_i'\right)\right)$$

[31] The argument for deviating to a lower bid is analogous.

for any given y. Analogously rewriting the second line, the incentive constraint can be stated only in terms of $\widehat{R}$, y, and the "mispricing" terms $v\left(y,\cdot\right)$:

$$
\begin{aligned}
& E_{\theta_{-i}}\Bigg[-\int_X \widehat{R}\left(x, s_{\widehat{R}}(y(\theta_i', \theta_{-i}))\right)\left(dG(x|\widehat{\theta}_i) - dG\left(x|\theta_i'\right)\right) \\
& \qquad\qquad + v(y(\theta_i', \theta_{-i}), \widehat{\theta}_i) - v\left(y(\theta_i', \theta_{-i}), \theta_i'\right)\Big) P(\theta_i', \theta_{-i})|A\Bigg] \\
\geq \ & E_{\theta_{-i}}\Bigg[\Bigg(\int_X \widehat{R}\left(x, s_{\widehat{R}}(y(\theta_i'', \theta_{-i}))\right)\left(dG(x|\widehat{\theta}_i) - dG\left(x|\theta_i'\right)\right) \\
& \qquad\qquad + \int_X x dG\left(x|\theta_i'\right) - y(\theta_i'', \theta_{-i}) - v(y(\theta_i'', \theta_{-i}), \widehat{\theta}_i)\Bigg) P(\theta_i'', \theta_{-i})\Bigg| B\Bigg].
\end{aligned}
$$

Step 2: *Debt financing relaxes the incentive constraint.* Suppose $\widehat{R}$ is non-debt for some y. Consider a debt security $\widetilde{R}$ such that type $\widehat{\theta}_i$ would be indifferent between $\widehat{R}$ and $\widetilde{R}$ for every y:

$$
\int_X \widehat{R}\left(x, s_{\widehat{R}}(y)\right) dG(x|\widehat{\theta}_i) = \int_X \widetilde{R}\left(x, s_{\widetilde{R}}(y)\right) dG(x|\widehat{\theta}_i),
$$

By the definition of steepness it holds then that

$$
\int_X \widehat{R}\left(x, s_{\widehat{R}}(y)\right)\left(dG(x|\widehat{\theta}_i) - dG\left(x|\theta_i'\right)\right) > \int_X \widetilde{R}\left(x, s_{\widetilde{R}}(y)\right)\left(dG(x|\widehat{\theta}_i) - dG\left(x|\theta_i'\right)\right).
$$

for $\widehat{\theta}_i > \theta_i'$. Hence, for any given y and $v\left(y,\theta\right)$, debt financing relaxes the "upward" incentive constraint requiring that a bidder should have no incentive to deviate to the security contract of a higher type (cf. Step 1).

Step 3. *In the equilibrium in which bidders have the lowest expected net payments all types must be financed with debt.* Finally, suppose that type $\widehat{\theta}_i$ is the highest type. Step 2 implies bidders can be made (weakly) better off if this type were financed with debt. (Note that since there are no higher types, there is no need to consider how the "downward" incentive constraint of higher types would be changed.) Hence, one can assume that in the equilibrium in which bidders have the lowest expected payments, this type must be financed with debt. By repeating this argument for every highest type, who is not financed with debt for some y, one can see that bidders' expected payoffs can be maximized by financing all types with debt. Clearly, incentive compatibility implies that this must be the same contract for all types. Recall thereby that bidders' expected debt payments are independent of the concrete financing terms of this contract (Proposition 3). **Q.E.D.**

Proof of Proposition 4. (i) Consider a monopolistic investor who chooses the security contract R and the financing terms $s_R(y)$ such that they maximize his expected return from providing financing. The proof follows by two simple observations. First, by appropriately choosing the financing terms, the investor can achieve any given equilibrium payments y. Note that he does not need to be concerned about the probability of winning as he always finances the winning bidder. Second, for any given payment y, the investor's expected payoff is maximized by choosing the security, which minimizes the bidders' share of surplus. Using the steepest security achieves this goal (Lemma 5).

(ii) Suppose that type θ_i is locked in to his incumbent investor and receives a contract $R(\cdot, s_R(\cdot))$, which is not in the steepest security. Let $v(y)$ be the difference between the expected value of the security and the money raised from the investor from his perspective at $t = 2$ after the auction payment is known:

$$\int_{\beta^{-1}(y)}^{\overline{\theta}} \int_X R(x, s_R(y))\, dG(x|t)\, d\mu\left(t|\beta^{-1}(y)\right) - (y - w) = v(y) \tag{A.7}$$

where $\beta^{-1}(y)$ is the lowest types who bids y in equilibrium and where μ are the investor's interim beliefs.[32] Every contract can be rewritten in this "interim" form. The existence of a profitable deviation is shown in two steps.

First, consider a deviation to the steepest security $\widetilde{R}(\cdot, s_{\widetilde{R}}(\cdot))$ defined such that the investor has the same interim expected rent $v(y)$ for every y. The proof of Lemma 5 shows that the standard characterization of the SPA applies.[33] On- and off-equilibrium bidding strategies in the SPA are implicitly defined analogously to (2) and are strictly increasing in θ. It holds:

$$\begin{aligned}
& \int_X x dG(x|\theta_i) - w \\
= \; & \int_X R(x, s_R(\beta(\theta_i)))\, dG(x|\theta_i) = \int_X \widetilde{R}\left(x, s_{\widetilde{R}}(\widetilde{\beta}(\theta_i))\right) dG(x|\theta_i) . \\
< \; & \int_{\theta_i}^{\overline{\theta}} \int_X R(x, s_R(\beta(\theta_i)))\, dG(x|t)\, d\mu(t|\beta(\theta_i)) = \beta(\theta_i) - w + v(\beta(\theta_i)) \\
< \; & \int_{\theta_i}^{\overline{\theta}} \int_X \widetilde{R}\left(x, s_{\widetilde{R}}(\widetilde{\beta}(\theta_i))\right) dG(x|t)\, d\mu(t|\beta(\theta_i)) = \widetilde{\beta}(\theta_i) - w + v(\beta(\theta_i))
\end{aligned}$$

[32] Though $v(y)$ is referred to as "rent" below, it can also be non-positive. Note $v(\cdot)$ is a function of y, and not θ, as the investor only observes the payment y at $t = 2$.

[33] Note that a special feature of the SPA is that a bidder's bidding strategies is independent of the financing contract offered to other bidders.

where the first inequality follows by Lemma 1: For any security contract, the expected repayment increases in θ. The second inequality follows from the definition of steepness: Since $\widetilde{R}(\cdot, s_{\widetilde{R}}(\cdot))$ is steeper than $R(\cdot, s_R)$, it intersects it (in θ) at most once from below. From the first line, this intersection is at type θ_i. Note thereby that $\beta^{-1}(\beta(\theta_i)) = \widetilde{\beta}^{-1}(\widetilde{\beta}(\theta_i)) = \theta_i$, so that upon observing bid $\beta(\theta_i)$ or $\widetilde{\beta}(\theta_i)$, the seller rationally believes that the bidder comes from $[\theta_i, \overline{\theta}]$. This explains the third and fourth equalities. Hence, by offering a steeper security, the investor increases the probability that the locked in bidder wins the auction: $\widetilde{\beta}(\theta_i) > \beta(\theta_i)$. But then, as the second step in constructing a profitable deviation, one can also construct a financing offer in the steepest security $\widehat{R}(\cdot, s_{\widehat{R}}(y))$, such that every type is induced to bid the same as in equilibrium $\widehat{\beta}(\theta_i) = \beta(\theta_i)$, but the investor extracts $(\widetilde{\beta}(\theta_i) - \beta(\theta_i))$-more expected rent on every realization $y = \beta(\theta_i)$. **Q.E.D.**

D1 is defined now more formally.[34] Let $u(y, R(\cdot), \theta)$ be the equilibrium expected payoff of a bidder after the auction payment is known at $t = 2$. Similarly, let $\widetilde{u}(y, \widetilde{R}(\cdot), \theta)$ be his expected payoff at $t = 2$ upon deviating to a security $\widetilde{R}(\cdot)$. For each type θ, determine the probability that he is better off deviating also from the perspective of $t = 2$:

$$\Pi(\theta|\widetilde{R}(\cdot)) = \Pr\left(u(y, R(\cdot), \theta) > \widetilde{u}(y, \widetilde{R}(\cdot), \theta)\right).$$

Then, provided that this leads to a non-empty set, D1 restricts the support of the investor's beliefs to those types that would find $\widetilde{R}$ attractive with the highest probability also after the payment becomes known at $t = 2$

$$\Theta^{dev}(\widetilde{R}(\cdot)) = \left\{\theta \in [\underline{\theta}, \overline{\theta}] \mid \Pi(\theta|\widetilde{R}(\cdot)) = \max_{\theta} \Pi(\theta|\widetilde{R}(\cdot))\right\}.$$

Proof of Proposition 5. The proof proceeds in several steps. It is shown first that a fully separating equilibrium does not exist. Based on this, it is shown then that financing with debt is the unique candidate for an equilibrium in the SPA when the investor breaks even at the interim stage. The final step is to show that such an equilibrium can be supported. Appendix B shows that the argument can be extended also to the case when the investor does not break even at the interim stage for every realization of y.

[34] D1, as discussed in Cho and Kreps (1987), was originally defined for discrete type spaces. The extension to continuous types follows, e.g., Ramey (1996) or DeMarzo et al. (2005).

Step 1. *An equilibrium in which a type separates from all other types does not exist.* Suppose the highest type $\overline{\theta}$ separates from all other types. Since the investor breaks even, the financing contract must satisfy

$$\int_X R\left(x, s_R(y)\, dG\left(x|\overline{\theta}\right) = y - w \tag{A.8}$$

for any y. Note that this type's bidding strategy is then the same as in the case in which he is not cash-constrained.[35] Consider now a deviation by type $\theta_i < \overline{\theta}$, $\theta_i \to \overline{\theta}$ who takes the contract of type $\overline{\theta}$ and bids $\beta\left(\overline{\theta}\right)$. His expected payoff is

$$\begin{aligned}
& \int_{\underline{\theta}}^{\overline{\theta}} \left(\int_X \left[x - R\left(x, s_R(\beta\left(t\right))\right)\right] dG\left(x|\theta_i\right) - w \right) dF_1\left(t\right) \\
= & \int_{\underline{\theta}}^{\overline{\theta}} \left(\int_X x dG\left(x|\theta_i\right) - \beta\left(t\right) + \int_X R\left(x, s_R(\beta\left(t\right))\right)\left(dG\left(x|\overline{\theta}\right) - dG\left(x|\theta_i\right)\right)\right) dF_1\left(t\right)
\end{aligned} \tag{A.9}$$

where the equality follows from the previous equation. Cross dividing by $\overline{\theta} - \theta_i$ gives the marginal change in this profit by selecting the contract and bid of type $\overline{\theta}$ instead of those tailored for his own type. Observe now that the sum of the first two terms is zero. The reason is that $\beta\left(\overline{\theta}\right)$ is optimally chosen to be the same as in the case in which the highest type is not cash-constrained. The remaining term is positive by Lemma 1.

Step 2. *Eliminating non-debt equilibria.* Suppose the highest type issues a non-debt contract $R\left(\cdot, s_R\left(y\right)\right)$ for which the investor breaks even at the interim stage. (The argument is for every highest type in a pool is analogous.) Consider a deviation to a debt security $\widetilde{R}(\cdot, s_{\widetilde{R}}\left(y\right))$. The deviation contract is such that the investor would break even at the interim stage for some non-degenerate beliefs $\widetilde{\mu}$.[36] That is, $s_R\left(\cdot\right)$ and $s_{\widetilde{R}}\left(\cdot\right)$ are implicitly defined in

$$\begin{aligned}
y - w &= \int_{\beta^{-1}(y)}^{\overline{\theta}} \int_X R\left(x, s_R\right) dG\left(x|t\right) d\mu\left(t|y\right) \\
&= \int_{\widetilde{\beta}^{-1}(y)}^{\overline{\theta}} \int_X \widetilde{R}\left(x, s_{\widetilde{R}}\right) dG\left(x|t\right) d\widetilde{\mu}\left(t|y\right).
\end{aligned} \tag{A.10}$$

[35] If he is not cash constrained, this bidder solves

$$\max_{\theta'} \int_{\underline{\theta}}^{\theta'} \left(\int_X \left(x - \beta\left(t\right)\right) dG\left(x|\overline{\theta}\right)\right) dF_1\left(t\right).$$

By plugging (A.8) into the bidder's problem when he is cash constrained, one obtains the same maximization problem.

[36] For example, if the bidder retains his ex ante beliefs, i.e. $\phi = F$, then $\widetilde{\mu}\left(\theta|y\right) = \frac{F(\theta|\widetilde{\beta}^{-1}(y))}{1-F(\theta|\widetilde{\beta}^{-1}(y))}$.

The bidding strategies are implicitly defined analogously to (2)

$$\int_X R\left(x, s_R\left(\beta\left(\theta_i\right)\right)\right) dG\left(x|\theta_i\right) = \int_X \widetilde{R}\left(x, s_{\widetilde{R}}(\widetilde{\beta}\left(\theta_i\right))\right) dG\left(x|\theta_i\right) = \int_X x dG\left(x|\theta_i\right) - w.$$

Note that the highest type bids the same as in a cash auction in which bidders are not cash-constrained, so that his probability of winning is the same with both contracts. It is, thus, sufficient to show that his expected deviation payoff conditional on winning is higher and that the seller accepts the deviation. This is done next by formalizing the argument from the main text when the winner in an SPA prefers financing in steeper and when in flatter securities.

Since $R\left(\cdot, s_R\left(y\right)\right)$ is steeper than $\widetilde{R}(\cdot, s_{\widetilde{R}}\left(y\right))$ and s_R and $s_{\widetilde{R}}$ are set such that the investor breaks even at the interim stage, (A.10) implies that for any payment y there is a type $\theta' \geq \max[\beta^{-1}\left(y\right), \widetilde{\beta}^{-1}\left(y\right)]$ for which

$$\int_X \left(x - \widetilde{R}\left(x, s_{\widetilde{R}}(y)\right)\right) dG\left(x|\theta'\right) = \int_X \left(x - R\left(x, s_R(y)\right)\right) dG\left(x|\theta'\right).$$

Then, from R being steeper than $\widetilde{R}$, it follows that for $\theta > \theta'$

$$\int_X \left(x - \widetilde{R}\left(x, s_{\widetilde{R}}(y)\right)\right) dG\left(x|\theta\right) > \int_X \left(x - R\left(x, s_R(y)\right)\right) dG\left(x|\theta\right)$$

with the inequality being reversed for $\theta < \theta'$. This implies that the highest type is the only one who is always better off deviating to a flatter security also at $t = 2$- irrespective of how close the second highest bid is to his own bid. Hence, by D1, the investor should place probability one on a deviation to a flatter security coming from the highest type. The investor makes an expected profit accepting for such beliefs, as by construction he breaks even for $\widetilde{\mu}$ (which is non-degenerate). Hence, the highest type must be financed with debt in equilibrium. It is straightforward to extend the same arguments to show that the same must be true for every highest type (in a pool) issuing a certain security contract. Hence, the only candidate for an equilibrium is financing all bidder types with debt.

Step 3. *Existence of a debt equilibrium.* The following strategies constitute an equilibrium. Suppose all bidders issue debt $R\left(\cdot, s_R\left(y\right)\right)$, for which the investor breaks even at the interim stage. The investor accepts $R\left(\cdot, s_R\left(y\right)\right)$, i.e. $\pi = 1$. If he observes a deviation, he requires to break even at the same stage as in equilibrium (i.e. not only ex ante, but also at the interim stage for each realization of y).[37] His out-of-equilibrium beliefs must satisfy

[37] Restricting the deviations in this fashion is essential, as there is otherwise no pure-strategy equilibrium.

D1. To verify that this is an equilibrium, it only remains to show that there is no profitable deviation to a steeper security. This is straightforward. For any auction payment y, the type most likely to profit from the deviation is the lowest type θ, for whom it is optimal to bid $\widetilde{\beta}(\theta) = y$ when financed with $\widetilde{R}(\cdot, s_{\widetilde{R}}(y))$. Suppose the investor places probability one on this type. Not deviating is strictly more profitable for every type for these beliefs. First, a bidder is held for a weakly lower type than would be the case in equilibrium. Second, financing the second highest bid with a steeper security is more expensive than with a flat one (just as financing with a steeper security is always more expensive for the highest type above). Hence, these beliefs cannot be eliminated by D1 and the equilibrium can be supported. **Q.E.D.**

Proof of Lemma 6. This is a standard security design problem under adverse selection in which a privately informed party raises a fixed amount $(y - w)$. The proof is, therefore, omitted and the reader is referred to Nachman and Noe (1994) for a detailed analysis. They show that, first, a separating equilibrium in which the investor breaks even does not exist. Second, the only equilibrium candidate from which no type (in particular the highest type) has an incentive to deviate is financing in the flattest security. A small modification to Nachman and Noe's analysis is that the financing contract offered by the seller presents a type-dependent outside option for the winning bidder. In particular, let β be the equilibrium bidding strategy and let R and $\widetilde{R}$ be the financing contract offered by the seller and the contract offered to outside investors respectively. Upon observing the payment y, outside investors rationally believe that the winner is distributed on the interval $\left[\beta^{-1}(y), \overline{\theta}\right]$. Since R is the steepest security, it follows that $\int_X (x - R(x, s_R(y)))\, dG(x|\theta)$ intersects $\int_X \left(x - \widetilde{R}(x, s_{\widetilde{R}}(y))\right) dG(x|\theta)$ at most once from above. Let θ' be this intersection if it exists. Only types $\theta \geq \theta'$ prefer $\widetilde{R}$. Types $\theta < \theta'$ strictly prefer R. Hence, the investor's expectation over who makes this offer should be over types $\left[\theta', \overline{\theta}\right]$ and not $\left[\beta^{-1}(y), \overline{\theta}\right]$. With this minor change, the standard analysis goes through. **Q.E.D.**

Appendix B First Price Auction

Equilibrium in the FPA

We first show Lemma 4 in the context of an FPA.

Proof of Lemma 4 for FPA. Arguing analogously to the SPA, incentive compatibility in the FPA requires

$$
\begin{aligned}
& E_{\theta_{-i}}\left[\left(\int_X \left(x - R\left(x, s_R(\beta(\theta_i'))\right)\right) dG\left(x|\theta_i', \theta_{-i}\right) - w\right) P(\beta(\theta_i'), \beta\left(\theta_{-i}\right))\right] \\
\geq\ & E_{\theta_{-i}}\left[\left(\int_X \left(x - R\left(x, s_R(\beta(\theta_i''))\right)\right) dG\left(x|\theta_i', \theta_{-i}\right) - w\right) P(\beta(\theta_i''), \beta\left(\theta_{-i}\right))\right],
\end{aligned}
$$

which leads again to a contradiction: By deviating to $\beta\left(\theta_i''\right)$ type θ_i' can increase his probability of winning, while reducing his expected security payment conditional on winning. **Q.E.D.**

Showing existence and uniqueness in the first-price auction follows standard arguments.

Lemma B.1 *A symmetric equilibrium of the FPA in increasing, differentiable bidding strategies exists. It is the unique solution to the following differential equation*

$$
\beta'(\theta) = \frac{\left(\int_X \left(x - R(x, s_R(\beta\left(\theta\right)))\right) dG\left(x|\theta\right) - w\right)}{\int_X \frac{\partial}{\partial s_R} R(x, s_R(\beta\left(\theta\right))) \frac{ds_R}{d\beta(\theta)} dG\left(x|\theta\right)} \frac{f_1\left(\theta\right)}{F_1\left(\theta\right)} \tag{B.1}
$$

together with the boundary condition

$$
\int_X \left(x - R(x, s_R(\beta\left(\underline{\theta}\right)))\right) dG\left(x|\underline{\theta}\right) = w.
$$

Proof of Lemma B.1. As in Lemma 3, the critical step is to use that s_R increases monotonically in y by Lemma 4. Hence, if the investor's payoff (conditional on winning)

$$
u\left(y, \theta\right) := \int_X \left(x - R(x, s_R(y))\right) dG\left(x|\theta\right) - w
$$

is log-supermodular in s and θ, it is also log-supermodular in y and θ. By standard arguments, the latter condition guarantees uniqueness and existence of the bidders' strategies (e.g., Lemma 3 in DeMarzo et al., 2005). **Q.E.D.**

Using this characterization of the equilibrium bidding strategies and Proposition 3, it is straightforward to extend Lemma 2 also to the FPA.

Proof of Proposition 2 for FPA. Suppose w decreases uniformly for all types. By Proposition 3 the bidder's equilibrium payment can be equivalently analyzed by looking at a security-bid auction in which the bidder bids in security R. The equilibrium bidding strategy is given analogously to Lemma B.1 by the following differential equation

$$s_R'(\theta) = \frac{\int_x (x - R(x, s_R(\theta)) - w)\, dG(x|\theta)}{\int_X \frac{\partial}{\partial s} R(x, s_R)\, dG(x|\theta)} \frac{f_1(\theta)}{F_1(\theta)}$$

with the boundary condition $\int_x R(x, s_R(\underline{\theta}))\, dG(x|\underline{\theta}) + w = \int_x x dG(x|\underline{\theta})$. Observe now that, while $s_R'(\theta)$ decreases in w, the overall cash and security payment of the lowest type remains the same irrespective of w. Hence, the overall cash and security payment of all types $\theta \in (\underline{\theta}, \overline{\theta}]$ decreases if w increases. **Q.E.D.**

The next proof extends the result from Corollary 3 that financing in steeper securities makes the bidders more aggressive.

Proof of Corollary 3 for FPA. Let $U(\theta, R)$ denote the maximized expected payoff of bidder θ financed with a non-debt security R and suppose that $\widetilde{R}$ is debt. Further let β and $\widetilde{\beta}$ be the equilibrium bidding strategies when the bidders are financed with these securities. In an FPA, the lowest type makes zero expected profits: $U(\underline{\theta}, R) - U(\underline{\theta}, \widetilde{R}) = 0$. Suppose, there is a second intersection at some type θ_i, $U(\theta_i, R) = U(\theta_i, \widetilde{R})$. To see now that this leads to a contradiction, one can apply the envelope theorem to obtain

$$\begin{aligned} U'(\theta_i, R) &= \int_X [x - R(x, s_R(\beta(\theta_i))) - w]\, g_2(x|\theta_i)\, dx F_1(\theta_i) \\ &< \int_X \left[x - \widetilde{R}\left(x, s_{\widetilde{R}}(\widetilde{\beta}(\theta_i))\right) - w\right] g_2(x|\theta_i)\, dx F_1(\theta_i) = U'(\theta_i, \widetilde{R}), \end{aligned}$$

where the inequality follows by the definition of steepness. But $U(\theta, \widetilde{R})$ cannot intersect $U(\theta, R)$ both times from below, since both functions are absolutely continuous (by incentive compatibility). Hence, $U(\theta_i, \widetilde{R}) > U(\theta_i, R)$ for all $\theta_i > \underline{\theta}$. **Q.E.D.**

Showing that financing with the flattest and the steepest security set the lower and the upper bound for the bidders' expected payments in the auction can be shown also for the FPA.

Proof of Lemma 5 for FPA. The proof for the FPA is analogous and is, thus, omitted. **Q.E.D.**

Extending the results from the signaling game to the FPA is interesting, as the proof does not rely on equilibrium refinements. It neatly illustrates the critical role of the assumption that the investor is different from the seller. In particular, in the FPA the investor can infer the bidder's type from his optimal bid both on and off the equilibrium path.

To see the intuition, suppose that $R(\cdot)$ is a non-debt security. Then, there is a deviation to a flatter security such that it is optimal for any type making the deviation to bid lower than in equilibrium while keeping the investor at least at break even. The reason is that by issuing a flatter contract, low bidders have less of an incentive to bid as high types, as their residual claim becomes more strongly dependent on their true type (Corollary 3). This relaxes their optimization problem and submitting a lower bid becomes more profitable than the equilibrium alternative. With debt being the flattest security, this deviation leaves only debt financing as a potential equilibrium candidate.

Consider, thus, the following equilibrium strategies. All types finance their bids with debt R, for which the investor breaks even at the interim stage. The investor accepts R, i.e. $\pi = 1$, and accepts a deviation $\widetilde{R}$ only if he at least breaks even at the same stage as in equilibrium (i.e. at the *interim stage)*. Importantly, this strategy makes it unnecessary to define out-of-equilibrium beliefs, as the investor can perfectly infer every bidder's type from his optimal bidding strategy $\beta(\theta)$ (given R) or $\widetilde{\beta}(\theta)$ (given $\widetilde{R}$).

Since debt is the flattest security, there is no deviation to a flatter security, for which the investor at least breaks even at $t = 2$ and which makes at least some bidders strictly better off. Analogously to Corollary 3 there is also no profitable deviation to a steeper security $\widetilde{R}$: All bidder types using $\widetilde{R}$ will make higher bids than in equilibrium. This is because the dependence of their residual claims on their true types decreases with the steepness of the security offered to the investor. Hence, their incentives to bid as higher types increase, tightening their optimization problem and leaving them with a lower expected payoff compared to equilibrium. The following proposition formalizes this intuition by extending Proposition 5 also to the FPA.

Proposition B.1 *Debt financing is the unique equilibrium of the financing game when the financing offer is made by the bidders.*

Proof of Proposition B.1. Suppose that not all types are financed with debt. Let $R(\cdot, s_R(y))$ be the equilibrium security contract, which may be type- and payment-dependent. (It will be without loss of generality to omit the type-dependence in the notation.) The contract(s) is such that the investor breaks even at the interim stage. The proof starts by showing that there is always a profitable deviation for any non-debt financing contract. Thereby, it is assumed that only the investor observes a deviation by a bidder. It is shown then that financing with debt can be supported as equilibrium.

Consider a deviation to a contract $\widetilde{R}(\cdot, s_{\widetilde{R}}(y))$ for which the investor just breaks even at the interim stage.[38] For any financing contract used in the FPA the investor can perfectly infer the winner's type from his bidding strategy $\widetilde{\beta}(\theta)$. Naturally, this strategy should be a best response to the reaction of the investor and the equilibrium strategies of the other bidders:

$$\max_{\widehat{\theta}_i} \int_{\underline{\theta}}^{\beta^{-1}(\widetilde{\beta}(\widehat{\theta}_i))} \left(\int_X \left(x - \widetilde{R}(x, s_{\widetilde{R}}(\widetilde{\beta}(\widehat{\theta}_i))) \right) dG(x|\theta_i) - w \right) dF_1(t)$$

where for any payment y, $s_{\widetilde{R}}$ is determined from the investor's interim participation constraint:

$$\int_X \widetilde{R}(x, s_{\widetilde{R}}) dG(x|\widetilde{\beta}^{-1}(y))) = y - w.$$

Observe now that $\beta(\theta_i) \neq \widetilde{\beta}(\theta_i)$. If not, then evaluating the FOC at $\widehat{\theta}_i = \theta_i$ yields

$$\begin{aligned} & \left(\int_X \left(x - \widetilde{R}(x, s_{\widetilde{R}}(\beta(\theta_i))) \right) dG(x|\theta_i) - w \right) \frac{f_1(\theta_i)}{F_1(\theta_i)} \\ = \; & \left(\int_X \frac{d}{ds_{\widetilde{R}}} \widetilde{R}(x, s_{\widetilde{R}}(\beta(\theta_i))) \frac{ds_{\widetilde{R}}}{d\beta} \beta'(\theta_i) \, dG(x|\theta_i) \right), \end{aligned}$$

which is the same problem the bidders solve when all types are financed with $\widetilde{R}(\cdot, s_{\widetilde{R}}(y))$. (Note that the boundary condition for type $\underline{\theta}$ is the same for all types irrespective of $R(\cdot, s_R(y))$.) By Corollary 3 and Lemma 5, the cash bids must be different in this case, leading to a contradiction. Hence, analogously to the same two results, it is optimal for every deviating type to bid higher than in equilibrium if $\widetilde{R}(\cdot, s_{\widetilde{R}}(y))$ is steeper and to bid lower if $\widetilde{R}(\cdot, s_{\widetilde{R}}(y))$ is flatter than $R(\cdot, s_R(y))$.

[38] Note that the bidder can always break the investor's indifference by offering him a rent of $\varepsilon \to 0$ on every bid.

Suppose now that the deviating contract $\widetilde{R}(\cdot, s_{\widetilde{R}}(y))$ uses the flattest security type (debt). Then, financing the bid $\beta(\theta_i) > \widetilde{\beta}(\theta_i)$ with $\widetilde{R}(\cdot, s_{\widetilde{R}}(y))$ is cheaper for type θ_i than financing it with $R(\cdot, s_R(y))$. The reason is that upon observing $\beta(\theta_i)$ given financing with $\widetilde{R}(\cdot, s_{\widetilde{R}}(y))$, the investor is led to believe that he is facing a higher type. By optimality, bidding $\widetilde{\beta}(\theta_i)$ and financing it with $\widetilde{R}(\cdot, s_{\widetilde{R}}(y))$ is, thus, even more profitable than conforming to the equilibrium strategy.

The only equilibrium candidate is, therefore, for all types to issue debt. Consider debt financing $R(\cdot, s_R(y))$, for which the investor breaks even at the interim stage. The strategy of the investor is to accept $R(\cdot, s_R(y))$. A deviation is accepted only if he breaks even at the same stage as in equilibrium, i.e. here the *interim stage*. As a deviation to a flatter security does not exist, it remains to verify that there is no profitable deviation to a non-debt contract $\widetilde{R}(\cdot, s_{\widetilde{R}}(y))$. By Lemma 5, any such contract effectively represents a deviation to a steeper security.

Because he would be effectively mimicking a higher type, a bidder who deviates to the larger bid $\widetilde{\beta}(\theta_i) > \beta(\theta_i)$ would obtain strictly cheaper financing terms if he financed this bid with the equilibrium security $R(\cdot, s_R(y))$ than with $\widetilde{R}(\cdot, s_{\widetilde{R}}(y))$. The reason is that in the latter case the investor correctly infers the true type θ_i. But by optimality of $\beta(\theta_i)$ the investor receives an even higher expected payoff on the equilibrium path, confirming that a deviation is not profitable.

Finally, observe that there can be no equilibrium in which the investor does not break even at the interim stage, but breaks even ex ante. In any such equilibrium there are types for whom raising capital is more expensive compared to the case in which the investor breaks even also at $t = 2$. Since, the latter can perfectly infer the bidder's type from his bidding strategy, it is straightforward to construct a profitable deviation. **Q.E.D.**

Finally, we show the claim from the main text that the proof of Proposition 5 extends also to the case in which the investor breaks even ex ante, but does not break even at the interim stage for every realization of y.

Ex Ante Equilibrium SPA The proof is almost identical to the case in which the investor breaks even at the interim stage. The following analysis shows only the corresponding Step 2 from this proof. It is shown below that financing with debt is the only candidate for an equilibrium of the financing game. To see the analogy to the proof in Proposition 5, observe that any ex ante equilibrium can also be rewritten as an equilibrium in which the investor obtains a net payoff of $v(y)$ from the perspective of $t=2$ (cf. (A.7)). Then, any non-debt equilibrium can be broken by the highest type offering debt financing. Precisely, consider a deviation to a debt contract such that the investor would have the same payoff $v(y)$ for every realization of y for some non-degenerate posterior beliefs $\widetilde{\mu}$:

$$\begin{aligned} v(y) &= \int_{\beta^{-1}(y)}^{\overline{\theta}} \int_X R(x, s_R)\, dG(x|t)\, d\mu(t|y) - (y-w) \\ &= \int_{\widetilde{\beta}^{-1}(y)}^{\overline{\theta}} \int_X \widetilde{R}\left(x, s_{\widetilde{R}}\right) dG(x|t)\, d\widetilde{\mu}(t|y) - (y-w), \end{aligned} \tag{B.2}$$

Note that by (2) and (B.2), the highest type makes the same cash bid in both cases, so his probability of winning remains the same. By the same arguments as above, he is the only type for whom such a deviation is always profitable also at $t=2$. By D1, the investor should place probability one on the highest type making this deviation. It remains to show that the investor will accept. Given that the investor breaks even ex ante, it is sufficient to argue that

$$\begin{aligned} 0 &= \int_Y \left(\int_{\widetilde{\beta}^{-1}(y)}^{\overline{\theta}} \int_X \widetilde{R}\left(x, s_{\widetilde{R}}(y)\right) dG(x|t)\, d\widetilde{\mu}(t|y) - (y-w) \right) dH(y) \\ &\leq \int_Y \left(\int_X \widetilde{R}\left(x, s_{\widetilde{R}}(y)\right) dG\left(x|\overline{\theta}\right) - (y-w) \right) dH(y) \\ &< \int_Y \left(\int_X \widetilde{R}\left(x, s_{\widetilde{R}}(y)\right) dG\left(x|\overline{\theta}\right) - (y-w) \right) dH\left(y|\overline{\theta}\right), \end{aligned}$$

where $H(y)$ is the unconditional distribution of the second highest bid and $H\left(y|\overline{\theta}\right)$ is the distribution conditional on the investor's belief that the deviating type is type $\overline{\theta}$. The equality follows by construction. The first inequality follows from Lemma 1. To see the second inequality, observe first that $H\left(y|\overline{\theta}\right)$ stochastically dominates $H(y)$.[39] Hence, it is sufficient to argue that the term in brackets in the second line increases monotonically in y. Constructing such a debt contract $\widetilde{R}\left(\cdot, s_{\widetilde{R}}(y)\right)$ is indeed possible, even if $v(y)$ (and so

[39] The second highest bid in a symmetric auction is distributed as the second highest type

$$\begin{aligned} H(y) &= F\left(\beta^{-1}(y)\right)^N + NF\left(\beta^{-1}(y)\right)^{N-1}\left(1 - F\left(\beta^{-1}(y)\right)\right) \\ &> F\left(\beta^{-1}(y)\right)^{N-1} = H\left(y|\overline{\theta}\right). \end{aligned}$$

the bracketed term in the first line) is non-monotonic in y. One only needs to choose the functional form of $\widetilde{\mu}$ appropriately. **Q.E.D.**

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