# Endogenous Liquidity Cycles

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September 2011

## Abstract

This paper presents a theory of liquidity cycles based on endogenous fluctuations in economic activity and the availability of informed capital. Risky assets are illiquid due to adverse selection. The degree of adverse selection and hence the liquidity of these assets depends on the endogenous information structure in the market. Liquidity provision is modeled as a repeated game with imperfect public monitoring. We construct a trigger-strategy equilibrium along the lines of Green and Porter (1984) that is characterized by stochastic fluctuations in liquidity. Liquidity is procyclical in our economy. Periods of economic growth are associated with more liquid asset markets. However, unlike other explanations in the literature, our results do not rely on exogenous shocks to the economy. Rather, fluctuations in liquidity arise endogenously from the need to create incentives for investors to engage in costly information production.

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# 1  Introduction

Market liquidity appears to fluctuate over time. Recent research suggests that these fluctuations are related to the state of the economy.[1] Spreads between liquid and illiquid assets are larger in recessions and liquidity crises are typically associated with economic downturns (e.g., Næs, Skjeltorp, and Ødegaard, 2011). During the recent economic crisis of 2008-2009, many markets—especially those for securities backed by subprime mortgages—suffered sudden liquidity dry-ups.

The origin of this empirical regularity remains largely a puzzle. This paper attempts to shed light on this puzzle by developing a dynamic model of liquidity based on adverse selection. The degree of adverse selection and hence the level of liquidity is determined by the endogenous information structure in the market. We construct an equilibrium in which liquidity is procyclical. Periods of economic growth are associated with more liquid asset markets. In this equilibrium, fluctuations in liquidity and economic activity arise endogenously and are not driven by exogenous shocks to the economy.

Our model economy is based on the premise that informed capital is a scarce resource. Agents who have the ability to identify profitable investment opportunities may lack the resources to cover the cost of investment. This applies to entrepreneurs in our model, who must therefore resort to outside funding. We assume that the investment return is not verifiable by a third party and hence restrict our attention to debt financing. This creates an agency problem: since entrepreneurs are protected by limited liability, they have an incentive to take excessive risks, even at the cost of a lower expected return. This cost is ultimately borne by entrepreneurs themselves, since in equilibrium lenders correctly anticipate the entrepreneurs' risk-shifting incentives and adjust the terms of their loan agreement accordingly.

The only way for entrepreneurs to avoid this moral hazard problem in our economy is

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[1]Chordia, Roll, and Subrahmanyam (2000), Hasbrouck and Seppi (2001), and Huberman and Halka (2001) provide evidence of a systematic, time-varying component of liquidity. Chordia, Roll, and Subrahmanyam (2001) and Huberman and Halka (2001) document that liquidity drops significantly in down markets. Watanabe (2004) reports that liquidity is lower when the economy is in a recession.

to sell their assets before risk can be added. Asset sales are efficient if the new owners have a sufficiently high equity stake in the assets that they acquire. However, these asset sales are characterized by an adverse selection problem. Because of their involvement in the early stages of the investment, entrepreneurs have an informational advantage over potential buyers, which enables them to better forecast future returns. This informational asymmetry between buyers and sellers makes assets illiquid. In the absence of informed buyers, prices reflect the average quality of assets sold. In equilibrium, rather than selling high-quality assets at a discount, entrepreneurs may be better off keeping them and bearing the agency cost due to the moral hazard problem. Of course, this inefficient outcome can be avoided when informed buyers are present in the market. If these informed investors are well capitalized, their valuation of the assets exceeds that of entrepreneurs and the two parties can agree on a sales price that makes them both strictly better off. We refer to such a market as "liquid" and use the difference between the true asset value and the secondary market price as a measure of liquidity. A smaller difference implies higher liquidity.

In our economy, market liquidity both affects and reflects the entrepreneurs' investment decisions. Clearly, as market liquidity improves, the expected return on investment goes up, which increases the number of profitable investment opportunities. Thus, in equilibrium, market liquidity positively affects the aggregate amount of investment. However, the causality also runs in the opposite direction. This follows from an analysis of the investors' incentive to acquire information. Since acquiring information is costly, investors will only do so, if they can use this information to identify undervalued assets in the secondary market. This implies that their decision to collect information depends on the entrepreneurs' investment policy. The more projects entrepreneurs invest in, the more they have to sell in the secondary market, and hence the better are the investors' chances to benefit from their information. Thus, more investment attracts more informed capital, which in turn attracts even more investment.

The positive feedback loop between market liquidity and investment activity suggests that

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investors internalize some of the welfare gains associated with more information production in the form of a higher trading profit. This indirect effect does, however, not influence the investors' information acquisition decision, because the amount of information collected by investors is not observed by entrepreneurs. Not surprisingly, we thus find that the equilibrium amount of information production is lower than the amount that investors would optimally choose if their choice were publicly observable. In other words, investors face a time consistency problem: ex ante they prefer to announce a high amount of information production in order to encourage more investment by entrepreneurs, but ex post they choose a low amount that renders the entrepreneurs' investment policy suboptimal. We show that the ability of investors to commit to an information production strategy before entrepreneurs make their investment decisions leads to a Pareto improvement: it not only benefits investors, but makes entrepreneurs better off as well.

In a static setting, such a commitment is not feasible. In particular, we show that the unique equilibrium of the one-shot game features a low level of market liquidity and little investment activity. A more favorable outcome can be achieved though in a dynamic setting in which long-lived investors repeatedly interact with (a series of short-lived) entrepreneurs. If entrepreneurs were to observe the investors' information choice at the end of each period, an equilibrium with a high level of market liquidity could be sustained through credible threats by entrepreneurs to "punish" investors in future periods by permanently reducing investment if any deviation from equilibrium play is observed. Such a trigger strategy would ensure a high level of information production in every period, since with perfect monitoring the threat is never carried out in equilibrium. This is not the case, however, with imperfect monitoring. Since the observed level of market liquidity does not fully reveal the investors' information choice, deviations cannot be unambiguously detected. Thus, in order to create intertemporal incentives supporting a high level of information production, low realizations of liquidity that are likely to arise in the event of a deviation must be followed by "punishments" (i.e., low continuation values for investors). In contrast to the perfect-monitoring case, imperfect

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monitoring ensures that these punishments will sometimes occur on the equilibrium path.

Our analysis of the repeated game focuses on trigger-strategy equilibria along the lines of Green and Porter (1984). These equilibria are characterized by stochastic fluctuations in market liquidity and economic activity. Periods of high liquidity and high economic growth are followed by periods of relative illiquidity and reduced economic activity. The length of these cycles depends on the performance of past investments. The low-liquidity regime is triggered by the failure of an entrepreneur to sell a successful project at a "reasonable" price. The associated reduction in the number of investment projects serves as a "punishment" to discipline the behavior of investors. Unlike other explanations in the literature, our theory does not rely on exogenous shocks to the economy (e.g., in the form of productivity or preference shocks). Rather, fluctuations in liquidity arise endogenously from the need to create incentives for investors to engage in costly information production.

There are many different notions of "liquidity" in the literature. Our notion is closely related to that developed in the market microstructure literature with privately informed agents.[2] In this literature, risky assets are illiquid due to adverse selection: uninformed agents demand a premium to purchase assets of high quality to offset their losses from acquiring "lemons" (Akerlof, 1970).[3] However, in contrast to most microstructure models, our model does not rely on the presence of "noise traders" or "liquidity traders" who are forced to buy or sell assets for exogenous reasons. Rather, the amount of liquidity-motivated trades is endogenous. In this respect, our approach is closer to Eisfeldt (2004), who presents a general equilibrium model of adverse selection-driven liquidity. In her model, agents are motivated to trade by changes in productivity: when productivity is high, investors initiate larger scale projects and sell more claims to high-quality projects in order to supplement their current income for use in consumption and new investment.

A number of papers have studied models in which market liquidity is a self-fulfilling

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[2]For recent surveys of this literature, see Madhavan (2000) and Biais, Glosten, and Spatt (2005).

[3]The effect of adverse selection on equity issues is studied by Myers and Majluf (1984), Lucas and McDonald (1990), and Korajczyk, Lucas, and McDonald (1992), among others.

phenomenon. A partial list includes Pagano (1989a,b), Allen and Gale (1994), Dow (2004), and Plantin (2009). In these models, low-liquidity equilibria are the result of a coordination failure. However, none of these papers explains why liquidity fluctuates over time and how these fluctuations are related to the business cycle. An important exception is Carlin, Lobo, and Viswanathan (2007), who develop a dynamic model in which episodic illiquidity results from a breakdown in cooperation between market participants. Similar to our model, traders cooperate most of the time through repeated interaction, providing liquidity to one another. However, episodically cooperation breaks down, especially when the stakes are high. Our setting differs from theirs in that fluctuations in liquidity are related to the state of the economy. Moreover, in our model the need to liquidate assets arises from an agency problem, rather than being driven by an exogenous liquidity shock.

Our paper is also related to the recent literature that links market liquidity and funding liquidity. Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009) present models in which collateral requirements limit the positions that arbitrageurs can take, and study the implications of these financial constraints for asset prices and welfare. In these models, market liquidity and funding liquidity are mutually reinforcing, leading to liquidity spirals. A similar result obtains in our model if the investors' cost of acquiring information is interpreted as a restriction on the availability of informed capital. However, the actual mechanism is rather different and works via complementarities between real investment and informed trading.

The remainder of this paper is organized as follows. Section 2 presents the economic setting. Section 3 describes the equilibrium of the stage game and discusses the investor's commitment problem. Section 4 constructs a trigger-strategy equilibrium of the repeated game in which market liquidity fluctuates over time. Section 5 summarizes our contribution and concludes. All proofs not provided in the main text are contained in the Appendix.

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## 2    The Model

We consider an infinite-horizon economy with three types of risk neutral agents: (i) *entrepreneurs*, who have access to a private investment technology; (ii) *financiers*, who loan capital to entrepreneurs and acquire assets from them; and (iii) *investors*, who can produce information about the value of the entrepreneurs' assets and use this information to identify undervalued assets. In the rest of this section, we describe the choices faced by these different types of agents in detail and discuss the relevant assumptions.

### 2.1    Entrepreneurs

In each period $t = 0, 1, \ldots$, a new entrepreneur is born who lives for one period.[4] The entrepreneur can invest in a risky project that requires an investment of $1 at the beginning of the period (i.e., at date $t_0$) and yields a terminal cash flow of $v_t$ at the end of the period (i.e., at date $t_2$). If the project succeeds, which happens with probability $\theta_t$, the cash flow is equal to $R > 1$; if it fails, the payoff is zero. The success probability $\theta_t$ is drawn from a uniform distribution over the interval $[0, \bar{\theta}]$ and is the entrepreneur's private information. The payoffs as well as the success probabilities are assumed to be independent across entrepreneurs.

The uncertainty about the project's cash flow is resolved at the intermediate date $t_1$. At this time, the entrepreneur observes the future payoff $v_t$. After the value of the project has been established, the entrepreneur's human capital, which is crucial in the initial phase, is no longer required. Specifically, we assume that once the payoff $v_t$ has been determined, outside investors can run the project as efficiently as the entrepreneur. Thus, it is not the specificity of the investment project that makes it costly for the entrepreneur to sell it to outside investors, but rather the informational advantage about $v_t$ that she has over potential buyers.[5]

Entrepreneurs have no resources that can be used toward covering the cost of invest-

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[4]In the remainder of this article, we refer to entrepreneurs as women, while we take investors to be men.

[5]The notion that firms whose assets are "specific" in the sense that they cannot easily be redeployed by firms outside of the industry tend to experience lower liquidation values was first introduced in the corporate finance literature by Williamson (1988) and Shleifer and Vishny (1992).

ment. However, they can borrow funds from financiers. The credit market is assumed to be competitive. Thus, entrepreneurs receive the entire expected surplus and lenders earn zero expected profits. Entrepreneurs are protected by limited liability, and so their income cannot be negative. For simplicity, we set the (intra-period) risk-free rate to zero. We rule out equity financing by assuming that project payoffs are not verifiable by a third party.

**Assumption 1.** *Neither the states at date $t_1$ nor the cash flows realized at date $t_2$ are contractible. Thus, outside equity financing is not feasible.*

The entrepreneur's investment is characterized by a moral hazard problem. After observing the project payoff $v_t$ at date $t_1$, the entrepreneur can engage in asset substitution. Specifically, we assume that by adding risk, one unit of certain value generates either $X > 1$ units with probability $\lambda$, or zero units with probability $1 - \lambda$. The switch to a riskier technology is unobservable to outside investors.

**Assumption 2.** $\lambda X < 1$. *That is, adding risk to the project is inefficient.*

**Assumption 3.** $(\lambda X)(\bar{\theta}R) > 1$. *This implies that if a project's success probability $\theta_t$ is sufficiently high, the project has a positive expected NPV even if risk is being added.*

The entrepreneur's risk-shifting incentives depend on the face value of her debt contract, $F_t$. Suppose that she learns at date $t_1$ that her cash flow is $R$.[6] Then her payoff is equal to $R - F_t$, if she does not add risk. If she adds risk, her expected payoff is equal to $\lambda(XR - F_t)$. Thus, the entrepreneur does not find it optimal to add risk if and only if $R - F_t \geq \lambda(XR - F_t)$, that is, if and only if the face value of debt does not exceed $\kappa R$, where:

$$\kappa = \frac{1 - \lambda X}{1 - \lambda}. \tag{1}$$

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[6]Clearly, adding risk cannot increase the entrepreneur's payoff when $v_t = 0$. This follows from the specification of our risk-adding technology and the assumption that a failed project has zero value. Giving entrepreneurs' an incentive to add risk to a failed project would not change our results, since they could always sell the project at full value to outside financiers in this state of the world.

Since the project is entirely debt financed, the face value has to be at least \$1. The following assumption ensures that entrepreneurs find it always (i.e., for any $\theta_t \in [0, \bar{\theta}]$) optimal to add risk to their projects when $v_t = R$.[7]

**Assumption 4.** $\kappa R < 1$. *This implies that the cash flow $R$ is not large enough to prevent entrepreneurs from adding risk.*

We want to emphasize that although entrepreneurs have an incentive to engage in asset substitution at date $t_1$, *ex ante* they want to prevent it because they get the entire expected surplus and thus bear the entire loss associated with it. In general, there are two possible ways to avoid the moral hazard problem. First, entrepreneurs can try to renegotiate their debt contract with the lender at date $t_1$. In particular, if the lender is willing to forgive some of the debt by lowering the face value $F_t$ to $\kappa R$, the entrepreneur has no longer an incentive to add risk to her project. However, such a reduction in $F_t$ necessarily means that lenders lose money in expectation, since $\kappa R < 1$ (Assumption 4). Thus, *debt forgiveness* is not feasible under our assumptions.

Alternatively, entrepreneurs can eliminate their risk-shifting incentives by selling their projects at date $t_1$.[8] Selling a high-value project is optimal if the sales price is higher than the continuation value with risk being added. Clearly, this will only be the case if the new owner has enough equity in the project so that he does not have an incentive to add risk after buying the project. We discuss the mechanism through which projects are sold in Section 2.4.

## 2.2 Investor

There is a single investor in the economy who has the ability to collect information about the value of the entrepreneurs' investment projects. By incurring a cost of $\phi(\alpha)$ at the beginning

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[7]This assumption simplifies our analysis without changing our results qualitatively.

[8]The idea that asset sales provide a mechanism for financially constrained borrowers to avoid a risk-shifting moral hazard problem has also been explored by Gorton and Huang (2004).

of a period, he learns the project payoff $v_t$ at the intermediate date $t_1$ with probability $\alpha$. With probability $1-\alpha$, he does not receive any useful information. For expositional purposes, we refer to $\alpha$ as the amount of information acquired by the investor. The investor has to choose the amount $\alpha$ before he observes the entrepreneur's investment decision.

**Assumption 5.** *The cost function $\phi : [0,1] \to \mathbb{R}$ is increasing, strictly convex, and twice continuously differentiable. It satisfies the following properties:*

$$\phi(0) = 0, \qquad \phi'(0) = 0, \qquad and \quad \lim_{\alpha \to 1} \phi'(\alpha) = +\infty. \tag{2}$$

The investor is long-lived and discounts future payoffs with the discount factor $\delta \in [0,1)$, so that his average discounted payoff from the infinite sequence of payoffs $(\pi_0, \pi_1, \ldots)$ is given by:

$$(1 - \delta) \sum_{t=0}^{\infty} \delta^t \pi_t. \tag{3}$$

The investor has sufficient capital to finance the purchase of a project entirely with equity. Thus, he has no incentive to engage in asset substitution upon buying a project.

Alternatively, rather than limiting the investor's access to information, we could assume that he has limited wealth. In this alternative model specification, the investor has to decide whether to hold liquid assets, which yield a low return, or to invest in illiquid assets with a higher return. Liquid assets have the advantage that they can easily be sold at the intermediate date $t_1$ should the investor have the opportunity to acquire an undervalued project (i.e., a project that has a value of $R$ to a buyer with sufficient capital to prevent the moral hazard problem, but that is sold for less than $R$). The parameter $\alpha$ can then be interpreted as the probability with which the investor has liquid funds available at date $t_1$ and $\phi(\alpha)$ as the opportunity cost of holding liquid funds.

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## 2.3  Financiers

Financiers have unlimited funds. They are willing to finance any investment project and to buy any asset that allows them to at least break even in expectation. In contrast to the investor, financiers do not have the ability to produce information about the value of assets offered for sale at the intermediate date $t_1$. They can, however, observe a project's success probability $\theta_t$ before lending capital to an entrepreneur at date $t_0$.

## 2.4  Asset Sales

At the intermediate date $t_1$, entrepreneurs can sell their projects either to the potentially informed investor or to uninformed financiers. The following assumption ensures that no successful project will be sold to the latter. Of course, there is then no reason for entrepreneurs to offer failed projects to financiers either, since in equilibrium uninformed buyers would know that all offered projects are worthless.

**Assumption 6.** $\bar{\theta} < \lambda X$. *This assumption guarantees that the maximum amount that uninformed investors are willing to pay for a project, which cannot exceed $\bar{\theta}R$, is lower than $\lambda X R$, the continuation value of a successful project when risk is being added.*

We model the interaction between an entrepreneur and the investor as a two-stage bargaining game of alternating offers with the risk of breakdown. First, the entrepreneur makes an offer by specifying a price at which she is willing to sell her project to the investor. If the investor accepts the offer, the sale is carried out and the bargaining process is over. If the investor rejects the offer, then, with probability $\beta \in (0,1)$, bargaining breaks down and no sale takes place. With probability $1 - \beta$, however, bargaining continues and the investor gets to make an offer to the entrepreneur. If the entrepreneur accepts the offer, the project is sold at the offered price. Otherwise, bargaining breaks down and no sale takes place.

# 3 Equilibria in the Stage Game

Before studying the infinite-horizon repeated game, we first analyze the stage game, a one-period version of the model. We show that the investor would benefit from the ability to commit to an amount of information production before the entrepreneur makes her investment decision.

The equilibrium concept we adopt is that of a Perfect Bayesian Equilibrium (PBE). Formally, a PBE of the stage game is defined by an investment strategy of the entrepreneur (characterized by a cutoff value $\theta_c$, as we will show below), a face value $F$ of the debt contract that financiers offer the entrepreneur, an information acquisition strategy $\alpha$ of the investor, an offer price at which the entrepreneur is willing to sell her project, a decision by the investor on whether to accept the offer, and, if rejected, a counter-offer price suggested by the investor and a decision by the entrepreneur to accept or reject the counteroffer, such that:

(i) the entrepreneur's investment and bargaining strategy maximizes her expected utility, subject to her budget constraint;

(ii) the investor's information acquisition and bargaining strategy maximizes his expected utility, conditional on the available information; and

(iii) financiers break even in expectation.

## 3.1 Bargaining

We first solve for the equilibrium of the bargaining subgame. We show that under our assumptions about the structure of the bargaining process and the exogenous parameters of the model, the payoffs to the different parties are uniquely determined. However, there are multiple equilibria that generate these payoffs.

We begin our analysis with the counteroffer. Suppose that the investor rejected the entrepreneur's offer, bargaining continued, and now the investor gets to make an offer. The

optimal offer price maximizes the investor's expected profit subject to the entrepreneur's payoff exceeding her outside option. If the project succeeds and pays a cash flow of $R$, the lowest price that the entrepreneur is willing to accept is $\lambda X R$, the continuation value with risk being added. Clearly, this is the optimal price that the investor offers when he is informed (that is, when he received an informative signal or when the entrepreneur's offer revealed the project value). If the investor does not have any information about the project payoff, the maximum price he is willing to pay cannot exceed $\bar{\theta} R$, which is lower than the entrepreneur's continuation value (Assumption 6). Hence, the investor's offer will be rejected. This, in fact, implies that the optimal offer price of an uninformed investor is zero, the value of a failed project. Any positive price would be accepted by unsuccessful entrepreneurs, causing a loss to the investor. Clearly, zero is also the price that an informed investor offers to pay for a failed project. The entrepreneur is indifferent between accepting and rejecting the offer in this case.

We now turn to the first stage of the bargaining game in which the entrepreneur makes an offer. At this stage, the entrepreneur does not know whether the investor is informed about the cash flow generated by her investment project or not. An informed investor will accept an offer to buy a successful project with a cash flow of $R$ only if the offer price $P_e$ satisfies the condition:

$$R - P_e \geq (1 - \beta)(1 - \lambda X) R \tag{4}$$

The left-hand side of this inequality is the investor's profit when he accepts the offer. The right-hand side is his expected profit when he rejects the offer and makes a counteroffer with a price of $\lambda X R$ in the subsequent round of bargaining. Since the entrepreneur will accept such an offer, the investor's profit is equal to $(1 - \lambda X) R$ in this case. However, there is a chance of $\beta$ that bargaining breaks down after the investor rejects the entrepreneur's offer, which reduces his expected profit to $(1 - \beta)(1 - \lambda X) R$. In equilibrium, the above constraint has to hold with equality and the entrepreneur's optimal offer price when $v = R$ is given by

$P_e = \beta R + (1 - \beta) \lambda X R.$

Whether an uninformed investor will accept an offer price of $P_e$ as well depends on the price at which failed projects are offered for sale. If the entrepreneur chooses a different offer price when $v = 0$, the offer price fully reveals the value of the project. In this case, the investor will accept the offer to buy the project at a price of $P_e$ even if he did not receive an informative signal about its cash flow, and reject the offer to buy it at any other price. Such a separating equilibrium cannot exist, however, since a failed entrepreneur clearly has an incentive to deviate and mimic the strategy of a successful entrepreneur by setting the offer price to $P_e$. Thus, the equilibrium strategy necessarily pools the two types of entrepreneurs as each prefers a higher sales price. It follows immediately from Assumption 6 that the investor is willing to accept this pooling offer only if he knows that the project has a cash flow of $R$.

We conclude our discussion of the bargaining game by pointing out that $P_e$ is the unique offer price in a pooling equilibrium. Clearly, a price that exceeds $P_e$ will not be accepted by the investor, and thus lowers the entrepreneur's payoff. Further, there are no out-of-equilibrium beliefs that can support an equilibrium with an offer price below $P_e$. In order to see this, suppose that uninformed investors believe that a project has zero value whenever an offer price other than $P < P_e$ is observed. Even with these extreme beliefs successful entrepreneurs are better off setting the price to $P_e$, since in equilibrium it is only informed investors who accept the offer. The following proposition summarizes the relevant features of our bargaining solution.

**Proposition 1.** *The outcome of the bargaining subgame is characterized by the following properties: (i) the entrepreneur offers her project for sale at a price of $P_e = \beta R + (1 - \beta) \lambda X R$ that is independent of its fundamental value; (ii) the investor accepts the offer only if he knows that the project's cash flow is equal to $R$; in all other instances, he rejects the offer and makes a counteroffer with a price of zero.*

Proposition 1 implies that the seller of a successful project receives a fraction $\beta$ of the

surplus $(1 - \lambda X)R$ that is generated by avoiding the moral hazard problem, and the informed buyer receives a fraction $1 - \beta$. It is important to note that the outcome of the bargaining game does not depend on the terms of the entrepreneur's debt contract. As long as the sales price is greater than the continuation value, the entrepreneur can always reach an agreement with her borrowers to sell the project and split the surplus.

## 3.2   Investment Choice

Having solved the bargaining game, we now turn to an analysis of the entrepreneur's optimal investment policy, taking as given the investor's decision to acquire information. Although entrepreneurs cannot observe the investor's information choice, they have rational expectations about his strategy and correctly anticipate the equilibrium level of $\alpha$.

Entrepreneurs are protected by limited liability and have no wealth when they are born. Thus, they prefer to invest in the risky project whenever they can borrow the necessary funds. Since the credit market is competitive, this will be the case if the project has a positive expected NPV so that lenders can break even in expectation. As argued above, the terms of the loan agreement are inconsequential for our analysis, since they do not affect the entrepreneur's liquidation decision at date $t_1$.

Taking the outcome of the bargaining process between the entrepreneur and the investor as given, the expected NPV of a risky project with success probability $\theta$ is given by:

$$\rho = \theta \big( \lambda X R + \alpha \beta \left( 1 - \lambda X \right) R \big) - 1. \tag{5}$$

The expression for $\rho$ is intuitive. If the project succeeds, which happens with probability $\theta$, and if it cannot be sold, risk will be added, which reduces its expected payoff to $\lambda X R$. If it can be sold to an informed investor at a price of $P_e$, which happens with probability $\alpha$, the seller receives an additional payoff of $\beta(1 - \lambda X)R$, that is, a fraction $\beta$ of the surplus generated by avoiding the asset substitution problem. The lender's zero-profit condition implies that $\rho$

is also the expected profit of the entrepreneur.

Using the above expression for $\rho$, we can rewrite the positive-NPV condition in terms of the project's success probability as follows: the entrepreneur invests in the risky project if and only if $\theta \geq \theta_c(\alpha)$, where the cutoff value $\theta_c(\alpha)$ is defined as:[9]

$$\theta_c(\alpha) = \left(\lambda X R + \alpha \beta \left(1 - \lambda X\right) R\right)^{-1}. \tag{6}$$

In other words, only entrepreneurs with a sufficiently high success probability are able to raise the funds necessary to undertake the investment project. We have written the cutoff value as a function of $\alpha$ to emphasize its dependence on the investor's information acquisition decision. Equation 6 shows that $\theta_c$ is decreasing in $\alpha$. This is intuitive. The more likely it is that there will be an informed buyer, the higher is the expected liquidation value of a successful project, and hence the lower the success probability has to be to make the investment profitable.

## 3.3 Information Acquisition

The investor's optimal information acquisition decision trades off the cost of acquiring information against the benefit of being able to identify an "undervalued" project (i.e., a project that has a value of $R$ to the investor, but that is sold for less than $R$). Clearly, if the entrepreneur does not invest in the risky project, there is no benefit to producing information. If, on the other hand, the entrepreneur chooses to invest, there is a chance that the investor can buy the project at a discounted price. This will be the case if the project succeeds (which happens with probability $\theta$) and the investor receives an informative signal about its value (probability $\alpha$). Formally, the investor's expected profit is given by:

$$\pi(\alpha, \theta_c) = \int_{\theta_c}^{\bar{\theta}} \frac{\alpha \theta \left(R - P_e\right)}{\bar{\theta}} \, d\theta - \phi(\alpha), \tag{7}$$

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[9]Assumption 3 ensures that $\theta_c(\alpha) < \bar{\theta}$ for all $\alpha \in [0, 1]$.

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where $\theta_c$ denotes the cutoff value of the success probability below which the entrepreneur will not be able to raise the funds required for the investment.

The optimal amount of information produced by the investor can then be found by solving the following maximization problem:

$$\max_{\alpha \in [0,1]} \pi(\alpha, \theta_c(\hat{\alpha})) \tag{8}$$

It is important to note that the investor takes the cutoff value $\theta_c$ as given when choosing the optimal $\alpha$. The reason is that while entrepreneurs can predict the amount of information produced in equilibrium, they do not observe the actual amount produced by the investor. The cutoff value $\theta_c(\hat{\alpha})$ therefore depends on the entrepreneur's beliefs about $\alpha$, which we denote by $\hat{\alpha}$, rather than the actual $\alpha$.

Assumption 5 ensures that the problem in (8) has an interior optimum. Substituting the expression for the sales price $P_e$ derived in Section 3.1 into the investor's objective function, we derive the first-order condition for a maximum with respect to $\alpha$ as:

$$\int_{\theta_c(\hat{\alpha})}^{\bar{\theta}} \frac{\theta (1 - \beta) (1 - \lambda X) R}{\bar{\theta}} \, d\theta = \phi'(\alpha). \tag{9}$$

It is easily verified that the amount of information production defined by equation (9) is the unique maximum, since the cost function $\phi(\cdot)$ is strictly convex in $\alpha$.

In equilibrium, the entrepreneur's beliefs about the investor's behavior must be correct. Thus, the amount of information production expected by the entrepreneur has to coincide with the actual amount of information production. The equilibrium amount of information production can therefore be obtained by setting $\hat{\alpha}$ equal to $\alpha$ in the above first order condition. The following proposition identifies a sufficient condition for its uniqueness.

**Proposition 2.** *Suppose that:*

$$\beta\left(1-\beta\right)\left(\bar{\theta}\left(1-\lambda X\right)R\right)^{2} < \phi''(\alpha), \tag{10}$$

*for all $\alpha \in [0,1]$. Then, the equilibrium amount of information produced by the investor in the stage game, $\alpha^*$, is uniquely determined by the following equation:*

$$\int_{\theta_c(\alpha^*)}^{\bar{\theta}} \frac{\theta\left(1-\beta\right)\left(1-\lambda X\right)R}{\bar{\theta}}\,d\theta = \phi'(\alpha^*), \tag{11}$$

*where the cutoff value $\theta_c$ is defined by equation (6).*

In the ensuing analysis, we assume that the investor's cost function $\phi$ is sufficiently convex, so that the above inequality holds for all $\alpha \in [0,1]$.

## 3.4 Value of Commitment

Our analysis in Section 3.2 shows that entrepreneurs benefit from more information production. Their expected profit is strictly increasing in $\alpha$. We now argue that increasing the equilibrium amount of information production above the level of $\alpha^*$ makes the investor better off as well. We use the term "equilibrium amount" to stress the fact that entrepreneurs are aware of this change.

An increase in the equilibrium amount of information production affects the investor's chances to identify an undervalued project in two ways: first, for a given investment strategy, it increases the probability that the investor receives an informative signal; second, it increases the likelihood of an investment by reducing the cutoff value $\theta_c$. Note that the second effect plays no role in the derivation of the optimal information choice $\alpha^*$, since the investor's decision is not observable to entrepreneurs. Thus, the investor does not internalize the positive effect that an increase in information production has on the entrepreneur's investment activity. Formally, this can be seen by evaluating the derivative of the investor's expected

profit with respect to $\alpha$ at the point $\alpha = \alpha^*$:

$$\frac{d\pi(\alpha, \theta_c(\alpha))}{d\alpha}\bigg|_{\alpha=\alpha^*} = -\frac{\alpha^* \theta_c(\alpha^*)(R - P_e)}{\bar{\theta}} \frac{d\theta_c(\alpha)}{d\alpha}\bigg|_{\alpha=\alpha^*} \tag{12}$$

$$= \frac{\alpha^* \beta (1 - \beta) \theta_c^3(\alpha^*)(1 - \lambda X)^2 R^2}{\bar{\theta}}, \tag{13}$$

where we have used the fact that, for a given cutoff value $\theta_c$, the derivative at $\alpha^*$ is zero according to the first order condition in (11).

The above expression is clearly positive, which implies that the investor would be better off if he could commit to an amount of information production that exceeds $\alpha^*$ before the entrepreneur makes her investment decision. In fact, since the entrepreneur benefits from an increase in information production as well, such a commitment would lead to a Pareto improvement. Of course, in a one-shot game, any announcement that the investor makes regarding his choice of $\alpha$ will not be credible. Entrepreneurs understand that the investor only announces a large amount of information production to encourage investment, and that he has no intention to follow through.

## 4  Liquidity and Investment in the Repeated Game

The analysis of the one-shot game in the previous section has shown that the investor would benefit from the ability to commit to an information production strategy. In fact, if the investor's choice of $\alpha$ were observable to entrepreneurs before they make their investment decision, the resulting equilibrium would be a strict Pareto improvement. We now examine whether the repeated interactions of the long-lived investor with entrepreneurs in an infinite-horizon game give rise to incentives that would allow him to effectively commit to more information production. In particular, we are interested in the conditions under which an implicit agreement between entrepreneurs and the investor is self-enforcing, according to which (i) the investor chooses a level of information production that exceeds $\alpha^*$, and (ii)

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entrepreneurs invest in the risky project even when their success probability falls short of $\theta_c(\alpha^*)$.

In a repeated setting, the agents' decisions depend on the history of the game. The history for the long-lived investor includes both the public history and the history of actions that he has taken. Short-lived entrepreneurs only observe the public history. In our economy, the public history in period $t$ includes, for each period $\tau = 0, \ldots, t-1$, (i) the entrepreneur's investment decision and (ii) the entrepreneur's and the investor's actions taken in the bargaining game (i.e., the price at which the entrepreneur offered her project for sale, the investor's decision of whether or not to accept the offer, the investor's counteroffer if he declined the entrepreneur's offer and the bargaining process continued, and the entrepreneur's response to the counteroffer).

The difficulty that entrepreneurs face is that they cannot tell for sure whether the investor complied with the implicit agreement described above, since his information production decision is not part of the public history. Thus, this is a game of *imperfect public monitoring*. Rather than observing the investor's actions directly, entrepreneurs can only infer information about them indirectly from the outcome of the bargaining game. If the project succeeds and the the investor agrees to buy it at a price of $P_e$, it is obvious that he is informed. Similarly, if he rejects the offer (and, consequently, specifies a price of zero in his counteroffer), entrepreneurs can conclude that the investor has no information. If the project fails, however, the outcome of the bargaining game does not reveal whether or not the investor has any information about it. In both cases, the investor would reject the entrepreneur's offer and make a counteroffer with a price of zero. Moreover, even if the investor turns out to be informed, it does not mean that he necessarily adhered to his equilibrium strategy. He could have chosen a lower $\alpha$ than entrepreneurs anticipated and just got lucky.

In perfect monitoring games, every history induces a continuation game that is strategically identical to the original repeated game. Thus, these games have a recursive structure, which considerably simplifies the characterization of equilibria. Unfortunately, such a recur-

sive structure does not hold in imperfect monitoring games. Because long-lived players have private information, there is no continuation game induced by the public history alone; and a player's private history is not known by the other players. A recursive structure can be recovered, however, if one restricts attention to *public strategies*, i.e., to strategies that, in every period, depend only on the public history. In our economy, the restriction to public strategies is without loss of generality, since short-lived entrepreneurs play public strategies by definition (they have no private histories), and the long-lived investor therefore always has a public strategy as a best reply.[10] Once we restrict attention to public strategies, a natural equilibrium concept to use is that of a Perfect Public Equilibrium (PPE).[11]

**Definition 1** (Fudenberg, Levine, and Maskin (1994)). *A perfect public equilibrium is a profile of public strategies that, beginning at any date t and given any public history, form a Nash equilibrium from that point on.*

The fact that in imperfect monitoring games, deviations cannot be unambiguously detected has important implications for the nature of the equilibrium. In order to create intertemporal incentives supporting behavior in which players do not myopically optimize, some realizations that are especially likely to arise in the event of a deviation must be followed by low continuation values ("punishments"). In contrast to the perfect-monitoring case where punishments are off the equilibrium path and thus are never carried out, imperfect monitoring ensures that these punishments will sometimes occur on the equilibrium path. This happens despite the fact that the players know when the punishment is triggered that they all have in fact followed their equilibrium strategies. The players therefore prefer the punishments to be as lenient as possible, consistent with creating the appropriate intertemporal incentives.

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[10]For a more general version of this argument, see Mailath and Samuelson (2006), chapter 7.

[11]Note that a PPE together with any beliefs consistent with Bayes' rule form a Perfect Bayesian Equilibrium. In contrast, a PBE is not necessarily recursive and thus does not have to be a PPE.

## 4.1 Trigger-Strategy Equilibria

There are many equilibria in the repeated game. This is not surprising given that, according to the Folk Theorem with Imperfect Public Monitoring (Fudenberg, Levine, and Maskin, 1994), any feasible payoff that Pareto dominates the minmax payoff of the stage game can arise in a PPE of the infinitely repeated game when players are sufficiently patient. In this section, we construct equilibria in "trigger strategies" along the lines of Green and Porter (1984) with the following features:

(i) The relationship between entrepreneurs and the investor alternates between normal phases and punishment phases. The game starts in the normal phase.

(ii) In *normal phases*, the investor acquires an amount $\alpha_n \geq \alpha^*$ of information and entrepreneurs with a success probability greater than $\theta_c(\alpha_n)$ invest in the risky project.

(iii) Play remains in the normal phase as long as the investor accepts an entrepreneur's offer to buy a successful project at a price of $P_e$. If the investor rejects the offer, makes a counteroffer of zero, and the subsequent payoff turns out to be $XR$, play switches to the punishment phase for $T$ periods.

(iv) In *punishment phases*, entrepreneurs and the investor play the equilibrium strategies of the stage game regardless of the realized outcomes: the investor produces an amount $\alpha^*$ of information and entrepreneurs choose a cutoff value of $\theta_c(\alpha^*)$. After the $T$ periods end, play returns to the normal phase.

This trigger-strategy profile is characterized by three parameters: the amount of information production in the normal phase $(\alpha_n)$, the length of the punishment phase $(T)$, and the set of outcomes of the bargaining game that trigger the punishment phase. It is easy to see that such equilibria exist. If we set $\alpha_n = \alpha^*$, the strategies prescribe that the static equilibrium of the stage game be played in every period, which is clearly an equilibrium.

The above strategy profile can be viewed as an incentive scheme. The choice of $\alpha_n > \alpha^*$ is optimal for the investor only if he is penalized for producing less information. Since entrepreneurs do not observe the investor's information acquisition decision, penalties must be based on variables that are correlated with the investor's unobserved behavior. This is the case for asset sales: the probability that a successful entrepreneur can sell her project at a nonzero price is linearly increasing in $\alpha$, the probability that the investor is informed. Entrepreneurs must therefore penalize the investor when he refuses to buy a project that later generates a positive cash flow. This is exactly what is achieved by the above strategy profile. By switching to a higher cutoff value of $\theta_c(\alpha^*)$, entrepreneurs reduce their investment activity, thereby lowering the investor's chances to purchase an undervalued project.

We now derive conditions under which the trigger-strategy profile constitutes a PPE. These conditions guarantee that no player has an incentive to deviate from her equilibrium strategy at any stage of the game. The *one-shot deviation principle* allows us to restrict attention to simple one-shot deviations, i.e., deviations with the property that there exists a unique history for which the deviation strategy differs from the equilibrium strategy. A public strategy profile is a PPE if and only if there are no profitable one-shot deviations.[12]

Clearly, short-lived entrepreneurs cannot benefit from a deviation, since the investment decision specified by their equilibrium strategy is a best response to the investor's information production decision in each period. Further, the investor's information acquisition decision in the punishment phase, $\alpha^*$, is a best response to the entrepreneur's investment policy defined by the cutoff value $\theta_c(\alpha^*)$. We are therefore left to show that there are no profitable deviations for the investor in the normal phase of the game.

We begin our analysis by calculating the investor's expected payoff from the equilibrium strategy profile. A key fact about perfect public equilibria that follows from their recursive structure is that the payoffs to such equilibria are stationary. When all players follow the strategy profile specified above, the investor's expected discounted payoff from period $t$ on

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[12]See, e.g., Mailath and Samuelson (2006).

can be written as follows, assuming that in period $t$ the game is in the normal phase:

$$\Pi_n = (1 - \delta)\,\pi(\alpha_n, \theta_c(\alpha_n))$$
$$+ \delta\left(\int_{\theta_c(\alpha_n)}^{\bar{\theta}} \frac{(1 - \alpha_n)\,\lambda\,\theta}{\bar{\theta}}\,d\theta\right)\Pi_p + \delta\left(1 - \int_{\theta_c(\alpha_n)}^{\bar{\theta}} \frac{(1 - \alpha_n)\,\lambda\,\theta}{\bar{\theta}}\,d\theta\right)\Pi_n, \qquad (14)$$

where $\Pi_p$ denotes the investor's expected discounted payoff at the beginning of the punishment phase, which is equal to:

$$\Pi_p = (1 - \delta^T)\,\pi(\alpha^*, \theta_c(\alpha^*)) + \delta^T\,\Pi_n. \qquad (15)$$

Equation (14) says that in the normal phase, the investor acquires an amount $\alpha_n$ of information and the entrepreneur invests in the risky project if $\theta \geq \theta_c(\alpha_n)$. The investor's expected profit is thus given by $\pi(\alpha_n, \theta_c(\alpha_n))$. Despite his efforts, there is a chance of $1 - \alpha_n$ that the investor does not receive any information about the value of the risky project in the current period. Entrepreneurs in subsequent periods will be able to detect the absence of an informative signal, if the entrepreneur in the current period invests in the risky project (which happens when $\theta \geq \theta_c(\alpha_n)$), the investor rejects the offer to buy the project, and the project yields a cash flow of $XR$. For a given success probability $\theta$, the probability that this event occurs is $(1 - \alpha_n)\,\lambda\,\theta$. In this case, the game switches to the punishment phase. With the complementary probability, the game remains in the normal phase, so that the investor's expected discounted payoff is $\Pi_n$ again next period. Equation (15) specifies the investor's expected discounted profit at the beginning of the punishment phase. For $T$ periods, the investor's expected profit is reduced to that of the one-shot game, $\pi(\alpha^*, \theta_c(\alpha^*))$, after which the game reverts back to the normal phase. By stationarity, $\Pi_n$ and $\Pi_p$ do not depend on time.

Using the above expressions, the investor's incentive constraint can be written as:

$$\alpha_n \in \arg\max_{\alpha} (1-\delta)\,\pi(\alpha, \theta_c(\alpha_n))$$

$$+ \delta\left(\int_{\theta_c(\alpha_n)}^{\bar{\theta}} \frac{(1-\alpha)\,\lambda\,\theta}{\bar{\theta}}\,d\theta\right)\Pi_p + \delta\left(1 - \int_{\theta_c(\alpha_n)}^{\bar{\theta}} \frac{(1-\alpha)\,\lambda\,\theta}{\bar{\theta}}\,d\theta\right)\Pi_n, \quad (16)$$

This equation expresses the tradeoff that the investor faces. If he acquires less information, his profit in the current period increases, since $\alpha_n$ is not a best response to the entrepreneur's investment strategy characterized by a cutoff value of $\theta_c(\alpha_n)$ for $\alpha_n > \alpha^*$. This follows from our analysis of the stage game in Section 3. However, choosing a lower $\alpha$ increases the investor's chances of triggering a punishment phase, which yields an expected discounted payoff of $\Pi_p$ instead of $\Pi_n$.

The unique optimum of the maximization problem in (16) is defined by the following first order condition:[13]

$$\left((1-\beta)(1-\lambda X)R + \frac{\delta}{1-\delta}\lambda(\Pi_n - \Pi_p)\right)\left(\int_{\theta_c(\alpha_n)}^{\bar{\theta}} \frac{\theta}{\bar{\theta}}\,d\theta\right) = \phi'(\alpha_n), \qquad (17)$$

where we have used the fact that, in equilibrium, the entrepreneur's beliefs must be correct. This, in fact, establishes the following result.

**Proposition 3.** *For given values of $\delta$, $\alpha_n$, and $T$, the strategy profile described above is a PPE if and only if equation (17) holds.*

Of course, this characterization does not prove the existence of trigger-strategy equilibria in which the amount of information production in the normal phase strictly exceeds $\alpha^*$. Comparing equation (17) to the equilibrium condition of the one-shot game in equation (11) reveals that for such equilibria to exist, the investor's payoff in the normal phase of the game, $\Pi_n$, has to be greater than his payoff at the beginning of the punishment phase, $\Pi_p$. We now

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[13]The second derivative of (16) with respect to $\alpha$ is $-\phi''(\alpha)$, which is clearly negative for all $\alpha \in [0,1]$, since the cost function is strictly convex.

show that this is the case for (some) $\alpha > \alpha^*$.

Rearranging equations (14) and (15) yields, after some computations:

$$\Pi_n - \Pi_p = \frac{(1-\delta)\left(1-\delta^T\right)\left(\pi(\alpha_n, \theta_c(\alpha_n)) - \pi(\alpha^*, \theta_c(\alpha^*))\right)}{1 - \delta + \delta\left(1-\delta^T\right)\left(\int_{\theta_c(\alpha_n)}^{\bar{\theta}} \frac{(1-\alpha_n)\lambda\theta}{\theta}\,d\theta\right)}. \tag{18}$$

Thus, $\Pi_n > \Pi_p$ if the investor's per-period profit in the normal phase exceeds that in the punishment phase.[14] From our analysis of the commitment case in Section 3.4, we know that this is the case for values of $\alpha_n$ that are slightly greater than $\alpha^*$. More precisely, we have shown that there exists an $\bar{\alpha} > \alpha^*$, such that, for all $\alpha_n \in (\alpha^*, \bar{\alpha})$, $\pi(\alpha_n, \theta_c(\alpha_n)) > \pi(\alpha^*, \theta_c(\alpha^*))$. This implies that for $\alpha_n \in (\alpha^*, \bar{\alpha})$, the investor's marginal benefit from producing more information in the normal phase of the repeated game is higher than that in the one-shot game. If the discount factor $\delta$ is sufficiently high, this increase in the marginal benefit outweighs the increase in the marginal cost at the point $\alpha^*$, which means that the investor's incentive constraint in (17) has to hold for some $\alpha_n > \alpha^*$.

**Proposition 4.** *If the investor is sufficiently patient, there exist trigger-strategy equilibria in which the amount of information acquired by the investor in the normal phase is strictly higher than that in the one-shot game, i.e., $\alpha_n > \alpha^*$.*

Since the game starts in the normal phase, the highest profit for the investor is obtained by maximizing $\Pi_n$ with respect to $\alpha_n$ and $T$, subject to the constraint in equation (17). As is easily verified from equations (14) and (15), $\Pi_n$ is increasing in $\alpha_n$, the amount of information acquired in the normal phase (at least over the interval $(\alpha^*, \bar{\alpha})$), and decreasing in $T$, the length of the punishment phase. The difference between $\Pi_n$ and $\Pi_p$, however, is an increasing function of $T$. This follows immediately from equation (18). Thus, the optimal trigger strategy trades off a larger amount of information production in the normal phase of the game against a longer punishment phase.

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[14]Of course, the length of the punishment phase has to be at least one period, i.e., $T \geq 1$.

## 4.2 Endogenous Liquidity Cycles

The trigger-strategy equilibrium constructed in the previous section is characterized by fluctuations in market liquidity in the form of stochastic cycles. Periods of high liquidity are followed by periods of relative illiquidity. The length of these cycles depends on the realization of the entrepreneurs' investment return and on the outcome of the investor's information collection efforts. The low-liquidity regime is triggered by the failure of an entrepreneur to sell a successful project. These liquidity dry-ups lead to a reduction in investment activity, which serves as a "punishment" to discipline the behavior of investors.

Liquidity in our model economy is procyclical. Periods of increased market liquidity are associated with high economic growth. It is important to note that causality runs in both directions. On the one hand, liquid asset markets attract more investment, since successful projects can be sold at a "reasonable" price, thereby reducing the agency cost associated with the asset substitution problem. On the other hand, larger scale investments make liquidity provision more profitable, since investors have a better chance to acquire assets at "fire-sale prices." This illustrates that liquidity is essentially a coordination phenomenon in our setting.

Unlike other explanations in the literature, our results do not rely on exogenous changes in economic productivity or the availability of liquid funds. Rather, fluctuations in liquidity arise endogenously from the need to provide intertemporal incentives to investors to collect information. Of course, changes in liquidity could also be generated through changes in the profitability of investment projects or in the cost of information production in our economy. The contribution of our paper is to show that market liquidity may fluctuate over time even in the absence of exogenous shocks to the economy.

## 5  Conclusion

This paper develops a general equilibrium model of liquidity based on adverse selection. Risky assets are illiquid due to adverse selection. The degree of adverse selection and hence

the liquidity of these assets depends on the endogenous information structure in the market. Liquidity provision is modeled as a repeated game with imperfect public monitoring. We construct trigger-strategy equilibria along the lines of Green and Porter (1984) that are characterized by stochastic fluctuations in market liquidity and economic activity. Periods of high liquidity are followed by periods of relative illiquidity. The former are associated with high economic growth. That is, liquidity is procyclical in our economy.

Unlike other explanations of variations in aggregate liquidity, our results are not driven by exogenous shocks to the economy. Rather, fluctuations in liquidity arise endogenously from the need to create incentives for investors to engage in costly information production.

## Appendix

**Proof of Proposition 2.** Our analysis in Section 3.3 shows that, for any $0 < \theta_c < \bar{\theta}$, the investor's optimization problem in (8) has a unique interior optimum. This follows from Assumption 5. Since the cutoff value $\theta_c$ is itself a function of the entrepreneur's beliefs about $\alpha$, the equilibrium amount of information production is a fixed point of the function that maps the amount expected by the entrepreneur into the amount chosen by the investor, which is implicitly defined by equation (9). In order to prove the existence of such a fixed point, we have to show that the equilibrium condition obtained by setting $\hat{\alpha}$ equal to $\alpha$ in equation (9) has a root in the interval $[0, 1]$. This equilibrium condition is given by:

$$g(\alpha) = 0, \qquad \text{where} \quad g(\alpha) = \int_{\theta_c(\alpha)}^{\bar{\theta}} \frac{\theta \, (1 - \beta) \, (1 - \lambda X) \, R}{\bar{\theta}} \, d\theta - \phi'(\alpha), \tag{19}$$

and $\theta_c(\alpha)$ is defined by equation (6). First, note that $g(\alpha)$ is a continuous function. Further, we have $g(0) > 0$, since $\theta_c(0) < \bar{\theta}$ (Assumption 3), $\lambda X < 1$ (Assumption 2), and $\phi'(0) = 0$ (Assumption 5). Assumption 5 also implies that $\lim_{\alpha \to 1} g(\alpha) = -\infty$. Thus, it follows from the *intermediate value theorem* that $g(\alpha)$ has a root in the interval $[0, 1]$. Moreover, there will be a unique root, if the above function is strictly decreasing in $\alpha$. The first derivative of $g(\alpha)$ is given by:

$$g'(\alpha) = \frac{\beta \, (1 - \beta) \, (\theta_c(\alpha))^3 \, (1 - \lambda X)^2 R^2}{\bar{\theta}} - \phi''(\alpha). \tag{20}$$

Since $\theta_c(\alpha) < \bar{\theta}$, a sufficient condition for this expression to be negative is given by the inequality in (10). This proves that the equilibrium amount of information production, $\alpha^*$, is uniquely determined by equation (11) as long as the constraint in (10) holds for all $\alpha \in [0, 1]$. ∎

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