The Term Structure of Recovery Rates*†

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Abstract

There is widespread agreement that corporate recovery rates are time-varying, but empirical work in this area is limited due to the econometric difficulty in isolating default probabilities from recovery rates. In this paper, we identify the dynamics of the term structure of recovery rates by simultaneously using the information from senior and subordinate credit default swaps. We estimate a reduced form model on forty-six firms across different industries and show that recovery rates change rapidly in response to economic events. We find that, on average, the term structure of expected recovery rates is downward sloping. However, an inversion takes place during bad economic times, during which it is upward sloping. Thus, during such periods, the market expects higher recoveries conditional on short-term survival. The inversion of the recovery term structure during economic downturns is more pronounced for firms in distressed industries. Overall, we provide strong empirical evidence for the cyclical nature of recovery.

JEL Classification: G01, G12
Keywords: credit default swap; no-arbitrage; stochastic recovery rate; seniority; term structure

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1 Introduction

Credit spreads consist of two main components: the probability of default and the loss given default.[1] While much is known on the modeling of default probabilities, the same cannot be said for risk-neutral recovery rates. The lack of empirical work on the modeling of risk-neutral recovery rates is primarily due to the fact that the two spread components are difficult to identify using a single financial instrument. Instead, the standard practice is to assume a constant recovery rate. Unfortunately, this assumption is not realistic given the mounting evidence in support of time-varying recovery rates in the credit risk literature. For instance, Altman, Brady, Resti and Sironi (2005) examine recovery rates for corporate bond defaults between 1982 and 2002 and find that they vary significantly over time and are negatively correlated with default rates.[2]

In this paper, we use information from both senior and subordinate credit default swaps (CDS) to isolate the recovery rate component. The senior CDS are insurance contracts for the senior unsecured bonds while the subordinate CDS are insurance contracts for the subordinate or lower tier-2 bonds. In the event of default, the expected loss given default is larger for the subordinate contract. Consequently, the spread of subordinate CDS contracts is larger than the spread of their corresponding senior contracts despite having the *same* default probability. This seniority-driven gap in the CDS spreads is due to the difference in their recovery rates. We exploit this seniority-driven gap in spreads to estimate the dynamics of the recovery term structure. The estimation results show that both the default intensity as well as the recovery components vary significantly over time. The average risk-neutral recovery rate across firms in our sample is between 32.7 and 36.8 percent for the senior contracts, while it is between 14.8 and 16.4 percent for the subordinate contracts. Our measures of estimated recovery rates are economically plausible and consistent with realized recovery rates reported in the literature. In addition, we show that recovery rates implied by the CDS contracts are highly responsive to important corporate events such as accounting news with significant impact on the lender's ability to recover the debt.

To our knowledge, this is the first paper that utilizes information from credit default swap contracts with multiple seniorities in order to estimate the stochastic dynamics of the risk-neutral recovery rates. There are only a few other studies that consider a stochastic recovery rate model. Bakshi, Madan and Zhang (2006) examine which modeling assumption for the recovery rate best supports the data using a sample of BBB-rated bonds. Our approach differs from theirs in a number of important ways. First, their model assumes that the dynamics of recovery rates and the default intensity are governed solely by the factor that drives the risk-free rate. In other words, they do not account for any firm-specific factors; where as in our approach, the dynamics of the recovery rate and the default intensity depend on latent

[1] The loss given default is defined as one minus the recovery rate.

[2] See also Acharya, Bharath and Srinivasan (2007).

firm-specific factors as well as the risk-free term structure. Second, Bakshi, Madan and Zhang (2006) impose a negative correlation between the default intensity and the recovery rate while in our approach, we let the data determine their relationship.

Christensen (2007) estimates a stochastic recovery model using CDS data for the Ford Motor corporation.[3] His estimates are based only on information from senior CDS contracts and hence the identification of the recovery rate dynamics is weak compared to a setting using both the senior and subordinate contracts.[4] In addition, the scale of our empirical exercise is much larger than that in Christensen (2007): in contrast to his single firm, we estimate the model using multiple seniority CDS contracts for forty-six firms.

Our empirical approach consists of jointly modeling the senior and subordinate CDS contracts in a four-factor reduced form framework. The model allows for stochastic interest rates, stochastic default intensity, and stochastic loss given default. We model the short rate dynamics, the default intensity as well as the loss given default using a quadratic specification. This approach ensures that the short rate and the default intensity is always positive and that the loss given default is bounded between zero and hundred percent. We estimate the model on forty-six firms which have CDS contracts trading for both seniorities. The data set spans the period from January 1, 2001 to March 7, 2008.

Using a novel estimation approach, we obtain a number of important findings about recovery rate dynamics. First, we document a sharp decline in the recovery rates during the financial crisis. More specifically, we show that the average recovery rate falls dramatically from mid-2007 onwards, which marks the onset of the financial crisis. Second, we find that the term structure of expected recovery implied by the CDS contracts differs significantly before and during the 2008 financial crisis. Preceding the crisis, the term structure of expected recovery is downward sloping while it inverts or becomes upward sloping during the crisis period. During good economic times, there is more uncertainty about the future state of the economy as the default horizon increases. Investors are risk averse about this economic uncertainty and command a recovery risk premium for investing in the longer term debt contracts, resulting in the downward sloping recovery rate term structure. On the other hand, an upward sloping term structure during the crisis period shows that the market expects the firms to recover less if they were to default in the midst of the recession than if they were to survive and default at future dates when the economy picks up. We find that the inversion in the term structure of expected recovery during the crisis is more prominent for firms within the financial sector (an industry in distress). The upward sloping term structure of recovery during bad economic times is in line with the finding of Zhang (2009) who shows that realized recovery rates are negatively correlated with lagged macroeconomic conditions.

Third, we find that the increase in CDS spreads during the financial crisis is mainly caused

[3] Karoui (2007) also estimates a stochastic recovery model using a discrete time framework.

[4] The issue about the recovery rate identification using a single seniority contract is discussed by Christensen (2005) in a simulation study.

by the increase in default probabilities. However, although the change in loss given default is small relative to the change in default probabilities, it is economically significant. Interestingly, we find that the increase in loss given default during the financial crisis is much larger at the short end of the term structure. The relatively larger increase in loss given default at short horizon explains why the term structure of recovery flattens or inverts during the financial crisis period.

Fourth, we find that industry characteristics are an important determinant of the recovery rate, which is consistent with the finding in Acharya, Bharath and Srinivasan (2007) who studies realized recovery rates on an extensive set of firms. More specifically, we show that recovery rates decrease most dramatically for financial firms during the crisis period. In addition, we examine the impact of firm-specific characteristics on the cross-sectional differences between the risk-neutral recovery rates across firms. The only firm-specific characteristics that are found to impact the risk-neutral recovery rates are profitability and leverage. This finding echoes Acharya, Bharath and Srinivasan (2007) who find similar results using realized recovery rates as the dependent variable.

Finally, our fifth empirical finding is that the risk-neutral default probabilities and recovery rates are negatively correlated.

The rest of the paper is organized as follows. Section 2 discusses the literature that is closely related to our paper. The readers are referred to Altman (2006) and Schuermann (2004) for a detailed literature review of the work on recovery rates. Section 3 introduces the model while Section 4 discusses the data and the estimation method. Section 5 presents the empirical results and Section 6 concludes.

2 Related literature

Our paper builds on the work by Madan, Guntay and Unal (2003) and Madan and Unal (1998) who use information from more than one type of security to infer risk-neutral recovery rates. Their model, however, does not allow for stochastic recovery rates. On the other hand, our approach allows the recovery rate to change depending on the state of the world and hence provides insights about the term structure of risk-neutral recovery rates. Jarrow (2001) also proposes a framework using both debt and equity prices in order to separately identify the recovery rates and the default probabilities. He shows that the use of both debt and equity prices can facilitate the identification of default probabilities and a constant recovery rate parameter.

Pan and Singleton (2008) show that use of the term structure of CDS spreads allows for separate identification of the default intensity and the recovery rates. They estimate a reduced form model using the entire term structure of sovereign CDS spreads over a five and half year sample. However, their setup only allows for the identification of a constant recovery parameter but not the recovery rate dynamics. In a related paper, Schneider, Sögner and

Veža (2009) estimate a jump model with a constant recovery rate parameter across a large cross-section of firms. The mean implied risk-neutral recovery rate reported in their study is around 79 percent which is rather large relative to the historical realized recovery that is found to be between 40 and 50 percent (see Altman and Kishore (1996) and Emery, Ou, and Tennant (2008)). In addition, this seemingly large magnitude of the risk-neutral recovery rate implies a negative recovery risk premium which is economically counterintuitive.

Le (2007) uses information from option prices to estimate the dynamics of the risk-neutral default intensity. The estimated default intensity is then used together with the five year maturity CDS spreads to compute the implied recovery rates. This method assumes that the CDS and equity options markets are fully integrated. If the equity options and CDS markets are segmented, the estimates of the default probabilities will not be representative of the true default probabilities. As a result, the implied recovery rates could be negative or higher than one. In fact, Le (2007) constrains his recovery rates to avoid negativity. In a related paper, Carr and Wu (2009) model credit default swaps and equity options jointly and estimate a constant recovery rate model. However, they note that their estimates should be treated cautiously because of the possible segmentation between the equity options and the credit default swaps market.

Das and Hanouna (2009) use the term structure of CDS spreads together with stock price and volatility to extract the term structure of default probabilities and recovery rates. The recovery rate and default probabilities in their model are, however, driven by only one state variable which is the stock price. In addition, their model requires calibration at each point in time (i.e., they estimate different parameters every month) while we estimate our model using the time series information for the term structure of CDS spreads with multiple seniorities. Song (2008), in a related paper, shows that the recovery rates can be separated from the default probabilities using a series of cross-sectional no-arbitrage relationships between the spot and forward credit default swaps. However, as with most other papers in the literature, he estimates a static recovery rate model for sovereign CDS.[5]

In addition to the above mentioned literature, there is a large body of literature that studies the relationship between realized recovery rates and default rates. The general conclusion of these papers is that the recovery rates are lower when the default rates are higher. Frye (2000a) proposes a model for bank loans where the probability of default and the recovery rate depend on the same systematic risk factor. He shows that in bad economic times, there is an increase in default rate and a fall in the value of the collateral which gives rise to a negative relationship between the default probabilities and the recovery rates.[6] Jokivuolle and Peura (2003) obtain a similar negative relationship between the default rates and the recovery rates in a modeling framework similar to that of Merton (1974). Carey and Gordy

[5] Berd (2005) also derives a no-arbitrage relationship between digital default swaps and conventional CDS contracts in order to extract risk-neutral recovery rates.

[6] Consistent with Frye (2000a), Düllmann and Trapp (2004) empirically show that systematic factors have a significant impact on the recovery rates of bonds and loans.

(2003) examine the empirical relationship between realized recovery rates and default rates using a large cross-section of data. They find that the correlation between default rates and recovery rates is on average close to zero. However, they find that there is a significant negative relationship between them during bad economic conditions. Altman, Brady, Resti and Sironi (2005) and Hu and Perraudin (2002) also find a negative relationship between the default and realized recovery rates using different empirical approaches. Finally, Chava, Stefanescu and Turnbull (2006) jointly model the physical default probabilities and the realized recovery rates using observable covariates. Consistent with the literature, they find a negative relationship between the physical default probabilities and the recovery rates. In addition, they find that the magnitude of the correlation varies with the credit cycle. Since there is widespread empirical support for the correlation between default probabilities and recovery rates, we allow for correlation between the two through their common dependence on the factors driving the risk-free term structure.

Our paper differs from the aforementioned studies in two important ways. First, we study the stochastic dynamics of risk-neutral recovery rates. Second, we use CDS contracts with multiple seniorities. Consequently, our estimation approach results in improved identification, which leads to important new findings about the term structure of expected recovery rates.

3 Model

3.1 Default-free model

We start with the model for default-free bonds. Let r_t denote the instantaneous default-free interest rate. Following Duffee (1999), we assume that the short rate dynamics are described by two latent factors. We assume that r_t has a quadratic specification that is given by

$$r_t = \left(\delta_0 + \delta_1 X_{1,t} + \delta_2 X_{2,t}\right)^2, \tag{3.1}$$

where $X_{1,t}$ and $X_{2,t}$ are the latent factors that drive the short rate dynamics.[7] The risk-neutral dynamics of these latent factors are given by

$$\begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{bmatrix} \begin{bmatrix} X_{1,t-1} \\ X_{2,t-1} \end{bmatrix} + \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}. \tag{3.2}$$

Equivalently, we can write the above dynamics in the following vector form

$$X_t^r = \mu^r + \rho^r X_{t-1}^r + \Sigma^r \varepsilon_t^r, \tag{3.3}$$

[7] See for example Ahn, Dittmar and Gallant (2002), Ang, Boivin and Dong (2008), Brandt and Chapman (2008), Constantinides (1992), Leippold and Wu (2002), Li and Zhao (2006), and Longstaff (1989) for quadratic term structure models. See Gourieroux and Monfort (2007) for an application of a discrete time quadratic factor model to mortality intensity modeling.

where X_t^r and μ^r are 2×1 vectors, ρ^r and Σ^r are 2×2 matrices and $\varepsilon_t^r \sim N(0, I)$. For parsimony, we impose zero correlation between the term structure factors. Note that we apply the superscript r to the variables in (3.3) to indicate that they are specific to the short rate dynamics.

The price of a zero-coupon bond at time t that matures in h periods is given by

$$B(t, t+h) = E_t^Q \left[\exp(-\sum_{j=0}^{h-1} r_{t+j}) \right], \tag{3.4}$$

where E_t^Q indicates the expectation under the risk-neutral measure. Given the dynamics in equations (3.1) and (3.3), the price of a default-free zero coupon bond can be written as

$$B(t, t+h) = \exp(A_h + B_h' X_t^r + X_t^{r\prime} C_h X_t^r), \tag{3.5}$$

where the coefficients A_h, B_h and C_h are given by recursive relations in appendix A.

3.2 Credit default swap valuation

We model default as a surprise event driven by a Poisson process.[8] The risk-neutral intensity for the Poisson process at time t is defined as λ_t. The probability of surviving at least h periods conditional on no default up until time t is

$$Q_t[\tau > t+h] = E_t^Q \left[\exp(-\sum_{j=0}^{h-1} \lambda_{t+j}) \right]. \tag{3.6}$$

The default intensity is assumed to depend on the same latent factors that drive the short rate dynamics and two additional latent factors which are credit-risk specific. Similar to the short rate dynamics, we assume a quadratic specification for the default intensity

$$\lambda_t = (\alpha_0 + \alpha_1 X_{1,t} + \alpha_2 X_{2,t} + \alpha_3 X_{3,t} + \alpha_4 X_{4,t})^2 . \tag{3.7}$$

Given the dynamics of the default intensity in (3.7) and the dynamics of the short rate in (3.1), we can write their joint dynamics in the following quadratic form

$$r_t + \lambda_t = \gamma_0 + \gamma_1' X_t + X_t' \Omega X_t, \tag{3.8}$$

where

$$X_t = \begin{bmatrix} X_t^r \\ X_t^c \end{bmatrix}$$

[8]This framework is based on the work by Duffie and Singleton (1997, 1999), Jarrow and Turnbull (1995), and Lando (1998).

denote a 4×1 vector, γ_0 is a scalar, γ_1 is a 4×1 vector and Ω is a 4×4 matrix.[9] The vector of latent state variables, X_t, can be decomposed into two components. The first component is specific to the short rate factors and is denoted by $X_t^r = [X_{1,t}\ X_{2,t}]'$. The second component is $X_t^c = [X_{3,t}\ X_{4,t}]'$, it consists of the credit-risk specific factors denoted by the superscript c. The state variables are assumed to follow the following dynamics

$$X_t = \mu + \rho X_{t-1} + \Sigma \varepsilon_t, \tag{3.9}$$

where X_t and μ are 4×1 vectors, ρ and Σ are 4×4 matrices and $\varepsilon_t \sim N(0, I)$.

For the valuation of a credit default swap, we first consider the payments by the protection buyer. Let S denote the annual CDS spread. The protection buyer promises to make payments $S\Delta$ on each coupon date, conditional on no default by the reference obligor, where Δ is the time between successive payment dates in years. For simplicity, we assume that the payments are equally spaced. If a credit event occurs, the protection buyer receives a payment from the protection seller and the contract terminates. The present value of the payment by the protection buyer on a payment date that is h periods ahead is

$$PB(t, t+h) = E_t^Q \left[S\Delta \cdot \exp(-\sum_{j=0}^{h-1} r_{t+j} + \lambda_{t+j}) \right]. \tag{3.10}$$

In appendix B, we show that

$$E_t^Q \left[\exp(-\sum_{j=0}^{h-1} r_{t+j} + \lambda_{t+j}) \right] = \exp(F_h + G_h' X_t + X_t' H_h X_t), \tag{3.11}$$

where the coefficients F_h, G_h and H_h are derived recursively.

The protection seller makes a payment of LGD_{t+h-1}, which is the loss given that the default occurs between time interval $t+h-1$ and $t+h$. We assume that if a default event occurs during the interval $(t+h-1, t+h]$, payment by the protection seller is made at the end of the interval. The present value of the promised payment by the protection seller if a default happens between $t+h-1$ and $t+h$ is

$$\begin{aligned} PS(t, t+h) &= E_t^Q \left[LGD_{t+h-1} \cdot \exp(-\sum_{j=0}^{h-1} r_{t+j}) \cdot \mathbf{1}_{t+h-1<\tau\leq t+h} \right] \qquad (3.12) \\ &= E_t^Q \left[LGD_{t+h-1} \cdot \exp(-\sum_{j=0}^{h-1} r_{t+j}) \cdot (Q_t(\tau > t+h-1) - Q_t(\tau > t+h)) \right]. \end{aligned}$$

[9] See appendix B for further details.

3.3 The loss given default

We assume that the loss given default is defined as

$$LGD_{t+h-1} = \exp\left(-\left(\beta_0 + \beta_1 X_{1,t+h-1} + \beta_2 X_{2,t+h-1} + \beta_3 X_{3,t+h-1} + \beta_4 X_{4,t+h-1}\right)^2\right) \quad (3.13)$$

Given the above dynamics, we can rewrite the expression for the present value of the payment by the protection seller if a default happens between $t+h-1$ and $t+h$ as

$$PS(t,t+h) = E_t^Q \left[\begin{array}{c} LGD_{t+h-1} \cdot \exp(-\sum_{j=0}^{h-1} r_{t+j}) \cdot \exp(-\sum_{j=0}^{h-2} \lambda_{t+j}) \\ -LGD_{t+h-1} \cdot \exp(-\sum_{j=0}^{h-1} r_{t+j}) \cdot \exp(-\sum_{j=0}^{h-1} \lambda_{t+j}) \end{array} \right] \quad (3.14)$$

Solving the conditional expectation, we show in appendix B that the payment by protection seller can be written in a more compact form

$$PS(t,t+h) = \exp(M_h + N_h' X_t + X_t' O_h X_t) - \exp(J_h + K_h' X_t + X_t' L_h X_t), \quad (3.15)$$

where the coefficients M_h, N_h, O_h, J_h, K_h, and L_h are derived recursively. The recursive relations are provided in appendix B.

The spread of a CDS contract is set such that the present value of the payments by protection buyer is equal to the present value of the payments by protection seller. That is,

$$\sum_{h=1}^{\frac{T}{\Delta}} PB(t,t+h\cdot\Delta\cdot N) = \sum_{k=1}^{N\cdot T} PS(t,t+k), \quad (3.16)$$

where N refers to the number of trading days in a year, Δ refers to the time period between two successive premium payments, and T refers to the maturity in years of the CDS contract. The time period for payments between two successive premium payments is 0.25 years. This corresponds to the payment frequency of the U.S. credit default swaps, which we use in our study.

3.4 Senior and subordinate contracts

The difference between the senior and subordinate CDS spreads is due to the difference in their expected recovery rates. While the loadings on the default intensity are the same for both senior and subordinate contracts, the loadings on their loss given default will be different (see equation (3.13)). To distinguish the spread and the loss given default for two contracts with different seniority, we apply a superscript SEN (i.e., β_i^{SEN}) to the loadings on the senior contracts, and a superscript SUB (i.e., β_i^{SUB}) to the loadings on the subordinate contracts.

For parsimony, the latent credit risk specific factor $X_{3,t}$ is allowed to impact only default intensity while the latent credit risk specific factor $X_{4,t}$ is allowed to impact only the loss given default (i.e., we assume $\beta_3^{SEN} = 0$, $\beta_3^{SUB} = 0$, and $\alpha_4 = 0$). We normalize the loading on the loss given default latent factor for the subordinate contract, i.e., $\beta_4^{SUB} = 1$. In addition, we assume that the off-diagonal elements in ρ and Σ are zero. In our model, the correlation between recovery rates and default intensity is governed by their common dependence on interest rate factors. Although this assumption is restrictive, it is imposed primarily to reduce the parameter space, which makes the large scale empirical estimation feasible. Even with this assumption, the estimation takes around four to five days for one firm. Since we have forty-six firms in our sample, the estimation is still numerically demanding.

3.5 Identification

The spread of a CDS contract is determined from equation (3.16). Using equation (3.10), equation (3.12) and equation (3.16), the spread of senior and subordinate CDS contracts is defined as follows.

$$S^{SEN} = \frac{\sum_{k=1}^{N\cdot T} E_t^Q \left[LGD_{t+k-1}^{SEN} \cdot \exp(-\sum_{j=0}^{k-1} r_{t+j}) \cdot (Q_t(\tau > t+k-1) - Q_t(\tau > t+k)) \right]}{\Delta \sum_{h=1}^{\frac{T}{\Delta}} E_t^Q \left[\exp(-\sum_{j=0}^{h\cdot\Delta\cdot N-1} r_{t+j} + \lambda_{t+j}) \right]}, \quad (3.17)$$

$$S^{SUB} = \frac{\sum_{k=1}^{N\cdot T} E_t^Q \left[LGD_{t+k-1}^{SUB} \cdot \exp(-\sum_{j=0}^{k-1} r_{t+j}) \cdot (Q_t(\tau > t+k-1) - Q_t(\tau > t+k)) \right]}{\Delta \sum_{h=1}^{\frac{T}{\Delta}} E_t^Q \left[\exp(-\sum_{j=0}^{h\cdot\Delta\cdot N-1} r_{t+j} + \lambda_{t+j}) \right]}, \quad (3.18)$$

where S^{SEN} is the CDS spread for the senior contract with maturity T, S^{SUB} is the CDS spread for the subordinate contract with maturity T, Δ is the time period between successive premium payment dates, LGD^{SEN} is the loss given default for the senior contract and LGD^{SUB} is the loss given default for the subordinate contract (note that the loss given default for both seniorities is governed by equation (3.13) with different factor loadings). The ratio of the CDS spread for senior contract to subordinate contract is defined as below.

$$\frac{S^{SEN}}{S^{SUB}} = \frac{\sum_{k=1}^{N\cdot T} E_t^Q \left[LGD_{t+k-1}^{SEN} \cdot \exp(-\sum_{j=0}^{k-1} r_{t+j}) \cdot (Q_t(\tau > t+k-1) - Q_t(\tau > t+k)) \right]}{\sum_{k=1}^{N\cdot T} E_t^Q \left[LGD_{t+k-1}^{SUB} \cdot \exp(-\sum_{j=0}^{k-1} r_{t+j}) \cdot (Q_t(\tau > t+k-1) - Q_t(\tau > t+k)) \right]}$$

Notice that the conditional default probability and the discount factor appear both in the numerator and the denominator. Therefore, it can be shown that the impact of the discount factor and the conditional default probability on the ratio is minimal. This suggests that the ratio of the senior to subordinate CDS spreads is driven primarily by the parameters governing the stochastic loss given default for the two seniorities. Thus, the use of the contracts with different seniorities facilitates the identification of the parameters governing the stochastic loss given default.

4 Data and Estimation

4.1 Data

We collect daily data for all the single name CDS contracts that are denominated in US dollars between January 1, 2001 and March 7, 2008. For both seniorities, we obtain CDS spreads for the three, five and seven year maturities. We concentrate on these maturities because they are the most liquid. We eliminate firms that have less than one year of data for both the senior and subordinate contracts. The final data set consists of forty-six firms. We obtain the risk-free term structure data from Bloomberg. We use the six-month Libor rates and the swap rates with maturities of one, two, three, four, five, seven and ten years.

In order to investigate the cross sectional determinants of the recovery rates, we obtain data for the firm-specific characteristics by mapping the CDS contracts to the corresponding identifiers in CRSP, Compustat, and Optionmetrics. The firm-specific variables that we use include operating income divided by sales, tangibility, leverage, q-ratio, size, trailing one year returns and option implied volatility. Operating income is the quarterly operating income before depreciation and taxes, and sales is the end-of-the quarter sales. We define tangibility as the ratio of the property, plant and equipment to the total book value of assets. Leverage is defined as the ratio of the long term debt to the sum of long term debt and market value of equity. The q-ratio is the ratio of the market value of assets to the book value of assets. Size is defined as the log of the book value of assets. The data on the market value of equity are obtained from CRSP. The data on debt, book value of assets, property, plant and equipment are obtained from Compustat. We obtain information about the implied volatility from Optionmetrics. The implied volatility that we use corresponds to the 30 day at-the-money put options.

Table 1a reports the summary statistics for the firms in our sample. The table includes the sample averages and the standard deviations of the CDS spreads for each maturity and seniority. The table also includes ratings, tickers, the start and the end date of data for each firm.[10] The firms are evenly distributed between A, BBB, and BB rating categories while we

[10]We assign an integer value to each rating class from 1 (AAA) to 6 (B). The ratings are then averaged for the entire sample and rounded off to the nearest integer. The table lists the average rating translated back to

have six firms that are rated AAA or AA and five firms that are rated B. Table 1a indicates a substantial variation in the CDS spreads across firms. The lowest average spread for the senior contracts in our sample is for Freddie Mac and Fannie Mae; they have the highest credit ratings among the firms in our sample. The average subordinate CDS spreads is always higher than the average senior CDS spreads across all three maturities for all firms. In most cases, the term structure of credit spreads is upward sloping for both the senior and subordinate contracts. The exceptions are Bear Stearns, Countrywide Financial and Lehman Brothers, for which we have data starting only around December 2006. These firms have exceptionally large spreads at the short maturities starting in mid-2007, which suggests that they are in distress. The length of the data set is dictated by the availability of CDS data for both seniorities. The median firm in our sample has around 3.6 years of data while the shortest sample is for Lehman Brothers and the longest sample is for Citigroup. Some of the firms are either privately held or acquired during our sample period.

Table 1b presents the average spreads across rating and industry categories. The table also reports the summary statistics for the firm-specific characteristics. The average spread increases as the rating worsens for both seniorities. The term structure of spreads is generally upward sloping across all rating and industry categories. There is no clear relationship between leverage and CDS spreads when looking across different rating categories. This is primarily a result of the large number of financial firms in our sample which are mostly rated under the AAA/AA and A category. There is, however, a monotonic relationship between the firm implied volatility and the CDS spreads across different rating categories. Firms with worse credit ratings therefore have higher implied volatility which suggests that the implied volatilities contain useful information about default probabilities.[11]

Figure 1 shows the average spread for the senior and subordinate contracts for all three maturities. The sample period is from January 1, 2001 to March 7, 2008. Because the data availability differs significantly across firms, the number of firms that we use to compute the average varies through time. There is a sharp rise in the average spreads in mid 2002 and at that time the average term structure is essentially flat, i.e., the three maturities have similar spread levels. This is the time period during which the S&P 500 index was at its lowest level and the VIX index was at its highest level, which suggests that the market conditions were not good. The spreads increase gradually from mid 2007 onwards which marks the onset of the financial crisis period. As in the case of mid 2002, the term structure of CDS spreads is also flatter during the financial crisis.

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character rating.

[11] This is consistent with the findings of Carr and Wu (2008, 2009), Zhang, Zhou and Zhu (2009), and Cao, Yu and Zhong (2010).

4.2 Estimation method

We use the unscented Kalman filter to filter out the latent state variables in all the estimations. Consider the following nonlinear state-space system

$$X_t = V\left(X_{t-1}, \varepsilon_t\right) \tag{4.1}$$

$$Y_t = Q(X_t) + u_t, \tag{4.2}$$

where Y_t is a D-dimensional vector of observables, ε_t is the state noise, and u_t is the observation noise. We assume the market price of risk and the Radon-Nikodym derivative such that the dynamics of the state variables under physical measure are defined as follows[12]

$$\begin{aligned} X_t &= \mu^P + \rho^P X_{t-1} + \Sigma\varepsilon_t & (4.3)\\ \mu^P &= \mu + \Sigma\lambda_0 & (4.4)\\ \rho^P &= \rho + \Sigma\lambda_1. & (4.5) \end{aligned}$$

The parameter λ_0 is an $N \times 1$ vector, and λ_1 is a diagonal $N \times N$ matrix, where $N = 4$ is the number of state variables. The equation (4.3) for the transition function is Gaussian but the measurement function Q is determined by (3.11), (3.14) and (3.16), which is highly nonlinear. We therefore use the unscented Kalman filter, which is suitable for the nonlinear filtration. We use the square-root unscented Kalman filter proposed by Van der Merwe and Wan (2001), which is found to be numerically stable and computationally feasible. Other studies that apply the unscented Kalman filter to estimate the risk-free term structure and credit risk models include Carr and Wu (2009), Chen, Cheng, Fabozzi and Liu (2008), and Christoffersen, Jacobs, Karoui and Mimouni (2009).

Following Duffee (1999), we first estimate the short rate parameters. We then assume that the estimated short rate dynamic is the true process. The dynamics of the risk-free term structure factors is estimated only once and assumed to be same across all firms. In the next step, we estimate the credit risk model using only term structure factors by fitting the model to both the senior and subordinate credit default swaps. That is, we estimate the model where the default intensity and the loss given default dynamics depend only on the term structure factors. This approach allows us to obtain good starting values for the loadings on the term structure factors. In the final step, we estimate the full model that includes the latent factors

[12] The Radon–Nikodym derivative takes the form

$$\frac{dP}{dQ} = \exp\frac{1}{2}\left(2\varepsilon_t'\Lambda(X_t) - \Lambda(X_t)'\Lambda(X_t)\right)$$

In addition, we assume time varying prices of risk depending on the state variables

$$\Lambda(X_t) = \lambda_0 + \lambda_1 X_t.$$

of the default intensity and the loss given default. We use nonlinear least squares in all our estimations.

5 Results

5.1 Estimates and model fit

Table 2 presents the estimation results for the risk-free process. The table presents the dynamics of the two latent term structure specific factors as well as the factor loadings. The first factor is closely related to the long term interest rates. The correlation between the bootstrapped zero rates for 10 year maturity and the first factor is around 91%, the correlation between the changes in the 10 year zero rates and the changes in the first factor is around 94%. The second factor is closely related to the difference between the long term zero rates and the short term zero rates. The correlation between the difference in 10 year zero rates and six month zero rates with the second factor is -93%. Overall, our estimates suggest that the first factor is closely related to the level of the yield curve while the second factor is closely related to the slope of the yield curve, which is consistent with Duffee (1999). Both term structure factors are highly persistent under the risk-neutral measure; the first factor (X_1) is found to be slightly more persistent than the factor (X_2). The root-mean-squared errors (RMSE) range from 4.5 basis points to 9 basis points which is fairly low compared to the estimates in the literature. For example, the RMSE in Chen, Cheng, Fabozzi and Liu (2008) range from 13 to 24 basis points while in Duffee (1999), it is between 4 to 34 basis points. The relatively smaller RMSEs in our study could partly be a result of the sample period. Nevertheless, consistent with both Duffee (1999) and Chen, Cheng, Fabozzi and Liu (2008), we find the largest RMSE at the short and long end of the yield curve.

Table 3 presents the distribution of the parameter estimates across all firms in our sample. Int-Latent indicates the latent factor that is specific to the default intensity and Rec-Latent indicates the latent factor that is specific to the loss given default. Recall that the loadings of the Rec-Latent factor for the subordinate contracts are normalized to one and hence are not reported in the table. Without loss of generality, the risk-neutral mean of the latent factors is also normalized to zero. The loss-given-default-specific latent factor has a lower persistence than the latent factor that drives the default intensity. That is, the loss given default on average mean reverts at a faster rate than the default intensity. This finding is true under both the physical and the risk-neutral measures. Looking at the estimates for the constant in panel A of Table 3, we find that they are larger for the senior contracts relative to the subordinate contracts. This is consistent with our expectation of lower loss given default for senior contracts. Finally, the factor loadings for the default intensity as well as the loss given default vary substantially across firms. This is partly due to the fact that we have a large cross-section of firms and it is the cross-sectional difference in spreads that drives the variation in their loadings.

Panel B in Table 3 presents the distribution of the estimates for the measurement error standard deviation. The measurement error standard deviation is generally low.

Table 4 presents the model's performance in terms of the relative root mean squared errors and the mean absolute percentage errors. The table presents the average model errors across all firms as well as the breakdown for different rating and industry categories. The average relative RMSE across all firms is around 10% for the three and seven year senior contracts while it is 9% for the five year senior contract. The mean absolute percentage error, on the other hand, is between 7% and 8.6%. The relative RMSE as well as the mean absolute percentage error for the subordinate contracts are essentially similar to that for the senior contracts for all three maturities. In comparison, Chen, Cheng, Fabozzi and Liu (2008) find the percentage RMSE for the five year senior contract between 4% and 6% depending on industry categories. However, they fit a one-factor model to a single maturity while our model involves fitting the term structure of CDS spreads for two seniorities. We can reduce the RMSE by including more factors. However, additional latent factors would substantially increase the parameter space. We find that the computational cost of including an additional latent factor is substantially larger than the improvement in the RMSE. Finally, both the relative RMSE as well as the mean absolute percentage error is lowest for the firms in the highest and lowest rating category. The breakdown based on the industry categories shows that the model performs very well in fitting the term structure of CDS spreads for financial firms. Overall, our model performs very well in fitting the term structure of CDS spreads for both seniorities across all firms.

5.2 Time-varying recovery

This section explores the time-varying dynamics of the recovery rates that are implied by the CDS contracts. We examine the recovery rate dynamics along two dimensions. We first look at the time series of the maturity specific implied recovery rates. Subsequently, we study the term structure of expected recovery rates across all firms.

5.2.1 Maturity specific implied recovery

The maturity specific implied recovery, $R^{IMP}(t,T)$, is a constant recovery rate that can be applied to price a CDS contract with maturity T at time t. It is computed using the following equation

$$S(t,T) = \left(1 - R^{IMP}(t,T)\right) \cdot S_{Unit}(t,T) \tag{5.1}$$

where $S(t,T)$ is the model implied spread at time t for a T-maturity CDS contract, and $S_{Unit}(t,T)$ is the model implied spread of a corresponding unit contract (i.e., digital credit default swap) that pays one dollar in case of a default and zero otherwise. We compute the value of a unit contract by shutting off the parameters that govern the stochastic loss given default. A unit contract is not subject to the recovery risk and hence its spread fully reflects the term structure of the underlying firm's default risk. Consequently, the implied recovery can also be thought of as the implied fraction of the notional value that will be recovered when the firm underlying a specific CDS contract defaults. We look at the level of implied recovery rates instead of the level of expected recovery for a particular default horizon because it facilitates comparison with the existing literature. Moreover, the time series properties of the level of expected recovery rate for a five year default horizon is similar to the time series properties of the implied recovery rates for a five year CDS contract. The maturity specific implied recovery rate also allows us to decompose the source of the spread changes into two components. The first component is the change in spreads that is due to change in the recovery rate while the second is the change in spreads that is due to change in the default probabilities. We discuss the decomposition of the spread changes during the financial crisis period in Section 5.3.

The top two panels in Figure 2 plot the time series of the five year firm-wide implied recovery rate. We compute this by averaging the five year implied recovery rates across all firms on a daily basis. We do not report the time series of the recovery rates for other maturities because their dynamics look very similar. The top left (right) panel plots the firm-wide averages of the five year implied recovery for the senior (subordinate) CDS contracts. As expected, the firm-wide implied recovery rates for the senior contracts are significantly higher than those of the subordinate contracts.[13]

We find that the implied recovery of the senior and subordinate contracts are highly correlated. Nevertheless, there is a subtle difference between their dynamics. The top right panel of Figure 2 shows a steady decline for the implied recovery rates of the subordinate contracts between 2004 and 2008. On the other hand, for the senior contracts, the top left panel of Figure 2 shows that the implied recovery rate decreases most rapidly in the beginning of 2007 when the U.S. subprime mortgage market crisis started to unfold. The significant decline in the recovery rates during the financial crisis period is not dominated by a single firm, as evident from the graphs of the average recovery rates across different rating groups as well as from the average recovery rates of financial firms (see Figures 3 to 5). Figure 2 also shows that the average risk-neutral recovery rates are stable between 2004 and 2007 for both seniorities. This is consistent with the realized recovery rates reported in Moody's report (see Emery, Ou, and Tennant (2008)). The reported realized recovery rates during the same time period suggests a stable recovery rate of around 55% for the senior unsecured bonds.

Table 5a reports the time-series averages of the implied recovery at three different maturities for each firm in our sample. The table also reports the average five year unconditional

[13] There is a sharp drop in the implied recovery in 2003 for both the senior and the subordinate contracts. This is driven by the drop in the implied recovery for Fannie Mae and Freddie Mac when the accounting practices of the two firms came under intense scrutiny. We discuss the dynamics of the recovery rates for Fannie Mae and Freddie Mac in Section 5.4.

default probabilities as well as the correlations between the five year implied recovery and the five year unconditional default probability. Table 5a shows that Fannie Mae and Freddie Mac, the two government-backed entities, have the highest implied recovery among the firms in our sample. Boyd Gaming is the firm with the lowest implied recovery in our sample. The correlations between the five year implied recovery and the unconditional default probability are mostly negative.

Table 5b reports the averages of the implied recovery and the implied default probabilities for different groups of CDS contracts. Overall, we find that the implied recovery for the senior contracts is between 32.7% and 36.8%. As for the subordinate CDS contracts, we find that their implied recovery is between 14.8% and 16.4%, approximately half of the estimates that are implied by the senior contracts. Altman and Kishore (1996) back out the realized recovery from a sample of 696 defaulted bonds between 1971 and 1995. They find an average realized recovery of 47.65% for the senior unsecured bonds and an average realized recovery of 31.34% for the subordinate bonds. In comparison to Altman and Kishore (1996), our results suggest that there is a *positive* recovery risk premium of 12 % to 15% for senior contracts and 14% to 16% for subordinate contracts. In contrast to our finding, Schneider, Sögner and Veža (2009) estimate a constant recovery model and find that the risk-neutral recovery rate for the firms in their sample is about 79%. This suggests that the recovery risk premium is negative and hence investors prefer to take on the recovery risk, which seems rather implausible.[14] Our estimates of the implied recovery rate are therefore economically more plausible than those in Schneider, Sögner and Veža (2009). Furthermore, our estimates of the implied recovery for the financial firms in comparison to those reported in Altman and Kishore (1996) also suggest that these firms have a positive recovery risk premium. The average realized recovery rate for senior and subordinate bonds categorized under the financial industry in Altman and Kishore (1996) is 38.68% and 24.81% respectively. The average implied recovery for the financial firms in our sample is between 33% and 37% for senior contracts while it is between 13% and 15% for subordinate contracts. On the contrary, the average recovery rate for financial firms reported in Schneider, Sögner and Veža (2009) is 72%.

Table 5b also shows that although the unconditional default probability is larger for firms with worse credit ratings, no such pattern is observed for recovery rates. We therefore find that firm rating do not drive cross-sectional differences in recovery rates. This is consistent with Altman and Kishore (1996) who show that the bond ratings do not have an impact on the recovery rates once the seniority of the bonds is taken into consideration. Note that credit ratings are solely assigned based on the firm's default probability, which explains why

[14] Schneider, Sögner and Veža (2009) provide the following numbers for the realized recovery rates obtained from Moody's and Fitch Ratings research report for their sample: recovery rates were 58.8% in 2005, 55% in 2006 and 51% in 2007. However, even these numbers are lower than the overall average of 79% estimated in their paper, implying a negative recovery risk premium. On the other hand, in comparison to Schneider, Sogner and Veza (2009) and our risk-neutral recovery rate estimates, Le (2007) find the average risk-neutral recovery rates for CDS contracts with five year maturity between 41% and 59% for the time period spanning from 2002 to 2005.

we observe a clear monotonic relationship between the risk-neutral default probabilities and the credit ratings.

Table 5b also reports the average correlations between the five year implied recovery and the five year unconditional default probability for different groups of CDS contracts. We find that their correlations are mostly negative. This is consistent with the vast literature that documents decline in recovery rates during the economic downturn, when default is more likely.[15] The exception for the strong negative correlations between implied recovery and default probability is for the BBB-rated firms, which mostly belong to the Retail Trade industry.

5.2.2 The term structure of expected recovery rates

In this subsection, we look at the evolution of the term structure of expected recovery rates over time. In order to explore the dynamics of the term structure of recovery rates, we turn to the expected recovery, which is defined as

$$R^{EXP}(t,h) = 1 - E_t^Q[LGD_{t+h-1}], \tag{5.2}$$

where $E_t^Q[LGD_{t+h-1}]$ follows from equation (3.13). The expected recovery, $R^{EXP}(t,h)$, represents the market's expectation of the recovery rate at time t if the firm were to default at time $t+h$ for $h \geq 0$. Therefore, unlike the implied recovery, the expected recovery provides insights on the term structure of the recovery rate at any given time point.

The middle panels in Figure 2 plot the time series of the slope of expected recovery. We plot the daily average values of the slopes across all firms. We define the slope of expected recovery as the difference between the seven year expected recovery ($R^{EXP}(t, 7 \text{ years})$) and the three month expected recovery ($R^{EXP}(t, 3 \text{ months})$). The middle left panel plots the slope of expected recovery for the senior CDS contracts while the middle right panel plots the slope of expected recovery for the subordinate CDS contracts. We find that the slopes of expected recovery are negative throughout our sample period. This shows that on average, the term structure of recovery rate is downward sloping. The market expects that a smaller fraction of the CDS's notional value can be recovered as the insured horizon increases. Our finding for the downward sloping term structure of expected recovery suggests that the recovery risk is priced in the CDS contracts. Longer maturity CDS contracts are exposed to more uncertainty about the firm's loss given default. Investors are risk averse about this uncertainty and hence command a recovery risk premium for investing in the longer term debt contracts.

The middle panels of Figure 2 show that the slope (i.e., the term structure) of expected recovery is time-varying. This further emphasizes that the dynamics of the firm's recovery is

[15] See for example Acharya, Bharath and Srinivasan (2007), Altman (2006), Altman, Brady, Resti and Sironi (2005), Bakshi, Madan and Zhang (2006), Carey and Gordy (2003), Chava, Stefanescu and Turnbull (2006), Frye (2000a and 2000b), Hu and Perraudin (2002), Jarrow (2001), and Jokivuolle and Peura (2003).

much more complex than those implied by the constant recovery models. We compare the time-varying dynamics of the recovery term structure to that of the CDS spreads. This helps us answer whether the term structure of recovery rate can be explained by the changes in the term structure of the CDS spreads. The bottom left panel of Figure 6 plots the time-series of the difference between the seven and three year CDS spreads. These values represent the daily averages across all firms. To save space, we only report the slope of the CDS spreads for the senior contracts. We do not find a robust relationship between the slopes of expected recovery and that of CDS spreads on an aggregate level.[16]

A closer look at the middle two panels in Figure 2 shows that the slope of expected recovery is approximately zero during the 2008 subprime crisis. During this period, the level of implied recovery is also lowest, which implies that the market prices the short-term and the long-term CDS contracts in such a way that they have an equally low recovery rate. The low level of short-term expected recovery is consistent with the prevailing view that the fire-sale effect as well as the illiquidity may depress the firm's ability to recover its fair value if it were to default in the midst of economic distress (see Altman (2006) and Schuermann (2004)). We discuss the term structure of expected recovery during the subprime crisis in Section (5.3).

Overall, our results in Figure 2 provide strong evidence in support of the time-varying recovery. We find that the recovery rate dynamics are time-varying in its level as well as its term structure. We also confirm that the level of implied recovery that we extracted from the CDS prices is consistent with other empirical studies. In addition, we find that the implied recovery rates are significantly lower during the financial crisis.

5.2.3 Investment and non-investment grade firms

We next look at the recovery rate dynamics for firms with different credit ratings. Figure 3 and 4 plot the implied and expected recovery along with the default probabilities for firms that fall under the investment grade and non-investment grade categories. We plot the daily average values for the firms within each category. We define investment grade firms as those that have a rating of BBB and above, while non-investment grade firms are those that have a rating below BBB. We do not have sufficient CDS data for the non investment grade firms before mid 2004 to compute the daily averages and hence we do not report their time-series before mid 2004.[17]

Figures 3 and 4 again show strong evidence for the time-varying recovery rate. The same patterns in the recovery rate dynamics that we document in Figure 2 are also found in both the investment grade and non-investment grade firms. In addition, the time series averages show that the recovery rate for the investment grade contracts is slightly higher than the re-

[16] Note that there is a strikingly large drop in the slope of the CDS spreads between 2002 and 2003; this is primarily driven by the CDS spreads of Cox Communications. The S&P 500 index was also at its all time low between 2002 and 2003, and the spreads were unusually large for the firms for which we have available data.

[17] We require at least five firms in the sample for the computation of the average across firms at any given point in time.

covery rate for the non investment grade contracts between 2005 and 2007 for both seniorities. However, from mid 2007 onwards, this positive difference in the recovery rates between the investment and non investment grade firms disappears. The average implied recovery rate for the senior and subordinate contracts of the non-investment grade firms is 34.3% and 16.1% respectively. As for the investment grade firms, the average implied recovery rate for the same sample period is 36.3% for the senior contracts and 18.7% for the subordinate contracts. Thus, as before, on average the recovery rates are not significantly affected by the entity ratings once the seniority of the contract is taken into consideration. On the other hand, the implied default probability of non-investment grade firms at the five year horizon is significantly higher than that of investment grade firms. This is not surprising since the entity ratings are driven entirely by their default probabilities.

5.2.4 Financial firms

Because financial firms played an important role in the 2008 crisis, it is interesting to study their recovery dynamics separately. Figure 5 plots the time-series of the implied and the expected recovery for the 17 financial firms in our sample. The values shown represent their daily averages.[18]

Most of the financial firms in our sample have an "A" rating or higher, and hence their five year unconditional default probabilities are relatively low up until the start of the subprime crisis in 2007 (see bottom right panel of Figure 5). The slope of the CDS spreads for the financial firms are fairly stable up until the beginning of the subprime crisis. The bottom left panel of Figure 5 shows that the slope of the CDS spreads become inverted half way through the year 2007. Therefore, it is more expensive to insure a financial firm through a 3-year CDS than through a 7-year CDS contract. Similarly, the middle panels of Figure 5 shows that the slope of expected recovery also becomes inverted half way through 2007 for both seniorities. This suggests that the market expects the financial firms to recover less of its value if it were to default during the financial crisis than if they were to survive and default at a later date. Overall, our findings confirm that the fear of illiquidity and of the fire-sale effect is significantly priced in the CDS spreads between 2007 and 2008. The market expects the recovery rate to be lower during the crisis which in turn raises the spread of the short-term CDS contracts relative to those at the longer horizons.

[18] Notice that in the bottom left panel, there is a sharp vertical line in the slope of the CDS spread near the end of the sample period. This vertical line represents a large drop followed by a reversal in the recovery term structure of Countrywide Financial. On January 8, 2008 the stock price for Countrywide Financial dropped by 28% relative to the previous day while the spreads for the three year maturity almost doubled for both seniorities. The spreads for the seven year maturity contract on the other hand increased but not by the same proportion; this caused a large negative CDS slope for Countrywide Financial. This was followed by an upward jump in slope on January 11, 2008 when Bank of America announced its plan to purchase Countrywide Financial for $4.1 billion in stock.

5.3 The impact of the financial crisis

5.3.1 Decomposing CDS spread changes

To study the impact of the financial crisis on the recovery rates and the default probabilities, we compare their dynamics in February 2008 (during the subprime crisis) to those in February 2007 (before the crisis) when the market conditions were rather normal. This approach allows us to study the separate role that the default probability and the recovery rate play in the pricing of CDS during the subprime crisis. For instance, how much of the CDS spread changes are due to the rise in default probabilities? How much of it is due to the fall in the implied recovery?

We perform our analysis as follows. Recall that our measure of the implied recovery rate (see section (5.2.1)) for maturity T can be thought of as the constant recovery rate that can be applied to price a CDS contract with maturity T. Following from equation (5.1), the log change in the CDS spreads between any two dates t and $t+\tau$ can be computed as follows

$$\log\left(\frac{S(t+\tau,T)}{S(t,T)}\right) = \log\left(\frac{1-R^{IMP}(t+\tau,T)}{1-R^{IMP}(t,T)}\right) + \log\left(\frac{S_{Unit}(t+\tau,T)}{S_{Unit}(t,T)}\right). \tag{5.3}$$

Using equation (5.3), we can isolate the CDS spread changes into two components. The first term on the right hand side of equation (5.3) is the log change in loss given default which is defined as one minus the implied recovery. It is easy to see that the change in loss given default depends on the change in implied recovery rate. Therefore, we can use the log ratio of the loss given default to represent the component in the CDS spread change that is due to the change in recovery rate dynamics. The second term in equation (5.3) is the change in the spread of a unit contract. We recall that a unit contract is a derivative that pays a dollar in case of default and zero otherwise. It therefore only depends on the term structure of default probabilities. Consequently, the log ratio of the spreads of a unit contract in equation (5.3) represents the component in the CDS spread changes that is due to the change in default probabilities.

Table 6 reports the log ratios of the model-implied loss given default between February 2008 and February 2007. We also report in this table the log ratios of the market spreads for the same period. The log ratios are computed for each day in February 2008 for all firms. We then compute the average log ratio over the month for each firm in our sample. We report the average ratios across all firms as well as the breakdown based on industry and rating categories. A few interesting results emerge from this exercise. The increase in market spreads is larger at the shorter horizon (three year maturity) than at the longer horizons (five and seven year maturity) for both seniorities. A substantial change in the CDS spread between 2007 and 2008 is due to the change in loss given default. Nevertheless, the increase in spreads is largely due to the increase in default probabilities. For instance, the overall increase in the market spreads for three-year senior CDS contracts is 204% while the increase in their loss given default is about 33%. This translates to a 171% increase that can be attributed to the change in default probabilities. The impact of the loss given default on the overall credit spread change is thus rather low relative to that of the default probability. In addition, we find that the impact of loss given default on the overall CDS spread changes is lower for the subordinate contracts than for the senior contracts.

The breakdown based on the rating categories shows that the increase in market spreads is significantly larger for the higher rating firms relative to the lower rating firms. This is not surprising because most of the financial firms in our sample have AAA/AA rating. These firms have the largest increase in spreads during the financial crisis period. As far as the relative change in loss given default is concerned, the increase in loss given default during the financial crisis is larger at the short horizon than at the long horizon across all rating categories. The largest increase in the loss given default for both seniorities and all three maturities is for the AAA/AA rating category, which contain the government sponsored entities (GSE) Fannie Mae and Freddie Mac. We do not observe a significant change in the loss given default for subordinate contracts in the lowest rating category in our sample (B rated firms). This is because these firms have a loss given default before the crisis which is close to the face value of the bond on average (i.e., close to zero recovery) and as a result, we do not observe a large change in their loss given default during the crisis period. On the other hand, we find that the senior contracts of B-rating firms experience a significant increase in the loss given default during the financial crisis period.

A closer look at the breakdown of the averages based on industry categories suggests that the increase in market spreads in February 2008 relative to the previous year is largest for the financial firms. The increase in loss given default is largest for the construction industry followed by the firms that fall under the financial category. This is not surprising since the two main industries that were in distress during the financial crisis are construction and financials. As before, the breakdown based on the industry category also shows a larger rise in loss given default for short maturity contracts relative to long maturity contracts for both seniorities.

5.3.2 Changes in the term structure of recovery

Figure 6 plots the average term structure of expected recovery for the firms for which we have the available data in February 2007 and in February 2008. We also plot, the average CDS spreads as well as the average default probabilities that are implied by the model. The x-axis in each panel represents the time horizon in months. The left y-axis and the dark solid line correspond to the results for the February 2007 CDS contracts. The dotted line and the right y-axis correspond to the results for the February 2008 CDS contracts.

The top two panels of Figure 6 show that the average expected recovery rate before 2008 monotonically decreases as the horizon increases. The expected recovery rate, on the other hand, exhibits a U-shape pattern during the subprime crisis. The results are robust for both the senior and subordinate contracts. The U-shape pattern in the expected recovery explains why the slope of expected recovery in Figure 2 is close to zero in 2008. Recall that we define the slope as the difference between the 7–year expected recovery and the 3–month expected recovery.

The U-shape pattern in the expected recovery is more pronounced for the senior contracts than for the subordinate contracts. Overall, the U-shape pattern in the expected recovery provides an important economic insight on the market expectation of the recovery level in 2008. During the crisis, investors believed that the firms would be able to recover more of its value if they were to default after the crisis was over.

We also observe a similar pattern in the credit spreads during the financial crisis period. While the credit spreads are upward sloping before the crisis period, the credit spreads increases from three to five year maturity and then slightly decreases or flattens out from five to seven year maturity during the financial crisis period. This pattern is similar to what we observe for the expected recovery rates. The comparison of the term structure of default probabilities show that the term structure is convex before the financial crisis while it is concave during the financial crisis period. This suggests that the relative increase in default probabilities during the financial crisis period is higher at the short horizon. Therefore, we observe that not only does the shape of the term structure of default probabilities change during the financial crisis period, the shape of the term structure of expected recovery also changes.

We now turn our focus to the term structure of expected recovery for the financial firms. Because the CDS spreads of these firms changed most significantly during the subprime crisis, it is interesting to see whether their recovery rate term structure also exhibits a similar U-shape pattern in 2008. Figure 7 shows the term structure of expected recovery for individual financial firms before and during the financial crisis. To conserve space, we graph the term structure for senior contracts only. The panels correspond to an individual firm denoted by their ticker. The general patterns that arise in Figure 7 is that the recovery rate term structure for the financial firms becomes upward sloping in 2008. This finding is robust for both the senior and the subordinate contracts. In addition, this result is consistent with our previous finding in Figure 5 which shows that the financial firms have significantly lower short-term recovery rates relative to the long-term recovery rates during the subprime crisis period.

In summary, we find that the term structure of expected recovery can exhibit many shapes. In the pre-crisis period, the term structure of expected recovery is downward sloping. However during the subprime crisis, the term structure of expected recovery can have a U-shaped pattern as well as a monotonically upward sloping shape. We find that the upward sloping shape is more prominent for the financial firms. The changes in the shape of expected recovery rates during the crisis period emphasizes the need to incorporate stochastic recovery when pricing credit sensitive instruments. The assumption of a constant recovery that is commonly used in the literature implies that the recovery rate is constant through time and that the risk-neutral recovery rate is same regardless of when the actual default happens. Our results demonstrate that these two assumptions are violated in the data. Zhang (2009) shows that the realized recovery rates are negatively associated with the macroeconomic conditions at the origination of the loans. Our results of upward sloping term structure of risk-neutral recovery rates during the financial crisis are in line with his findings.

5.4 Time-varying recovery: case studies

This section examines how important information events impact the time-series of the implied recovery at an individual firm level. More specifically, we look at four big financial companies: Fannie Mae, Freddie Mac, Lehman Brothers and Washington Mutual. We study these firms because they were significantly impacted by the credit risk crisis as well as drew large media coverage. There are a few reasons as to why it is important to study the firm's recovery rate dynamics in relation to its news events. First, we show that our estimates are economically meaningful in a sense that they are linked to the arrival of important news. Second, if the recovery rate is highly responsive to the important economic news, then it further emphasizes the importance of the stochastic recovery rate feature for the pricing of CDS contracts.

5.4.1 Fannie Mae and Freddie Mac

The top left panel of Figure 5 shows that the implied recovery rate among the financial firms fall from 70 to 40 percent in 2003. This is the time period around which the accounting malpractices of Fannie Mae and Freddie Mac were exposed. We now examine these two firms in greater detail. The top two panels of Figure 8 and 9 plot the implied recovery for a five year CDS contract (senior and subordinate) for Fannie Mae and Freddie Mac respectively. Figures 8 and 9 also plot their market five year CDS spreads, their model-implied five year CDS spreads, their implied five year unconditional default probabilities, their one year trailing stock return, and their leverage. A few noticeable patterns emerge from the graphs. The model performs very well in fitting the spreads for both seniorities. Figure 8 and 9 show that the implied recovery rates of their senior and subordinate contracts vary substantially over time. In addition, the average implied recovery for a five year senior contract is 71% for Fannie Mae and 66% for Freddie Mac. These values are relatively large compared to other firms in our sample. Moreover, we see that the implied recovery rates of Fannie Mae and Freddie Mac drop significantly in mid-2003 before climbing back to their conventional values. On the other hand, we do not find any significant changes in their unconditional default probabilities during this period. Therefore, changes in the CDS spreads of Fannie Mae and Freddie Mac in mid 2003 are mostly due to the fall in recovery rates.

Our estimates of the implied recovery are highly responsive to important news events. For instance, on January 16, 2003, Fannie Mae posted the results for the fiscal year of 2002. The reported earnings fell substantially in the fourth quarter of 2002 relative to earnings in the same quarter of previous year. The fall in earnings was primarily due to the adopted accounting standard used by Fannie Mae in mark-to-market practice of the derivative trading. Fannie Mae used derivatives to manage interest rate risk and its loss due to derivatives in the last quarter of 2002 was \$ 1.88 billion. As a result, Fannie Mae posted a net income of \$ 4.53 per share for 2002 compared to the net income of \$ 5.72 per share in the previous year. If we look at the implied recovery on the announcement date (January 16, 2003), we observe a one day drop of around 5% (from 80% to around 75 %) in the implied recovery even though the spreads actually dropped from 22.57 to 22.36 basis points. This large drop in the recovery rate also appears for the subordinate contract.

Another important news item that significantly impacted Freddie Mac occured on June 9, 2003. On that day, Freddie Mac fired three of its top executives because they refused to cooperate in the investigation of the firm's accounting practices. Freddie Mac was re-auditing the results for the past three years because its previous auditor, Arthur Andersen, misapplied accounting rules. As a result of the news, the stock plunged 16% on that day, and the option implied volatility jumped from 24 % to 50%. The implied recovery for the senior contract dropped from 79% to 75% while the spreads rose from 19 basis points to 22 basis points. The implied recovery for the subordinate contract dropped from 46% to 41%. This event continued to cause a gradual drop in the implied recovery between June 2003 and August 2003 for both Freddie Mac and Fannie Mae because lawmakers were pushing for more oversight and regulation on these firms. In fact, the implied recovery continually dropped to 50% over the next three months for the senior contract of both firms. Two other accounting related events (one in late 2004 and the other in March 2007) also caused a similarly large drop in recovery rates for the two firms.[19]

The fact that the news about accounting errors had such a large impact on recovery rates is not surprising since the financial statements are an important source of information for lenders. The quality of accounting information impacts the lenders' estimates of future cash flows from which the debt repayments are serviced. Bharath, Sunder and Sunder (2008) show that firms with poorer accounting quality are more likely to post collateral when borrowing from banks. This suggests that banks, being sophisticated investors, expect lower recovery in the event of default for the firms that have poor accounting quality and as a result require these firms to post additional collateral. This is exactly what we observe in the case of Fannie Mae and Freddie Mac. Our estimates of the risk-neutral recovery rates decline as the information about bad accounting practices from the two firms becomes public.

[19]In late 2004, Fannie Mae was under investigation for its accounting practices. The Office of Federal Housing Enterprise Oversight released a report on September 20, 2004, alleging widespread accounting errors. This event caused a rise in spreads from 18 to 23 basis points for senior contract and 36 to 48 basis points for subordinate contract and a decline in recovery by 9% (from 76% to 67%) for the senior and a decline in recovery by 9% (from 47% to 38%) for subordinate within next three days. Finally, in March 2007, the mortgage crisis started to unfold and at the same time United States house financial services committee put forward the bill intended to avoid the repeat of the financial scandals that affected both Fannie Mae and Freddie Mac. The executives of both firms argued that the bill intended for stricter regulations of the two firms would not only hurt them but affect the already weakened mortgage market. This event also caused similarly large drop in the recovery rates for both firms.

5.4.2 Lehman Brothers and Washington Mutual: recovery risk premia

Lehman Brothers and Washington Mutual defaulted in the second half of 2008. We can therefore compare our model-implied expected recovery to the after-default realized recovery for these two firms.[20] Figure 10 graphs the average term structure of expected recovery against the future default horizons for the first week of March 2008. The left scale on the y-axis corresponds to the senior contract and the right scale on the y-axis corresponds to the subordinate contract. The solid line indicates the term structure of expected recovery for the senior contract and the dotted line indicates the term structure of expected recovery for the subordinate contract. The dash-dot line indicates the time around which the CDS auctions for the firms were settled.[21] We see that the model implied expected recovery around the time when the CDS contracts for Lehman brothers were settled is about 5.1% for the senior contract and 0.73% for the subordinate contract. The results from CDS auctions suggest a recovery rate of 8.6% of Lehman Brothers, which is 1.5 times the magnitude of our model-implied expected recovery for the senior contracts. This suggests a recovery risk premium of around 3.5% for the senior contract. The proportion of risk premium is therefore large; about 41%. In the case of Washington Mutual, the model implied expected recovery around the time when the CDS contracts were settled is about 11% for the senior contract. The realized recovery rate for Washington Mutual from the CDS auction is 57%, which suggests a recovery risk premium of around 46%. The proportion of risk premium is about 80%, which is larger than that of Lehman brothers.[22] An important finding of this analysis is that the CDS contracts in March 2008 had already incorporated a very low expected recovery rate into their prices. Investors are therefore adequately compensated for taking on such recovery risk; the recovery risk premium is positive and large for both firms. In addition, it shows that in the case of Lehman Brothers our model is able to capture the rather low expected recovery that is consistent with the realized recovery.

5.5 Determinants of the recovery rate

In this section, we explore whether the variables that have been used to explain realized recovery rates in existing work can explain the cross-sectional differences in the recovery rates

[20] Both Lehman Brothers and Washington Mutual had a credit event in September 2008. The credit event for Washington Mutual, Inc was triggered because Washington Mutual Bank owned by Washington Mutual, Inc was placed into the receivership of the Federal Deposit Insurance Corporation (FDIC) and later sold to J P Morgan Chase. This caused Washington Mutual, Inc to file for voluntary Chapter 11 bankruptcy after losing its banking subsidiary. Lehman Brothers filed for Chapter 11 bankruptcy on Sep. 15, 2008 due to large losses and devaluation of its assets.

[21] The information about the CDS auction results is available on http://www.creditfixings.com/. Helwege, Maurer, Sarkar and Wang (2009) show that the results of the CDS auctions are in line with the after default traded price of the bonds.

[22] In case of Lehman Brothers, filing of Chapter 11 resulted in fire sale while in case of Washington Mutual, Inc, part of the assets were acquired by J P Morgan Chase. This explains the large difference in the realized recovery rate for the two firms. The model implied expected recovery for Washington Mutual suggests that the CDS contracts incorporate the possibility of liquidation of assets as opposed to acquisition.

that are implied by the CDS spreads. We use operating income divided by sales, tangibility, leverage, size, q-ratio, volatility, and one-year trailing return to explain the cross-sectional differences in the implied recovery rates. Since the balance sheet data that we use is available at a quarterly frequency, our dependent variable is the average implied recovery rate over the fiscal quarter for each firm. We run the following pooled regression:

$$R_i^{IMP}(t,5) = \alpha + \beta \cdot X_i(t) + \varepsilon_i(t)$$

where $R_i^{IMP}(t,5)$ is the average implied recovery of firm i over the fiscal quarter t for a senior contract with five year maturity and $X_i(t)$ is the vector of firm-specific characteristics for firm i at time t. We only report the results from the regression on the implied recovery rates of five year senior contracts because we obtain the same conclusions when running the regression on the implied recovery rates for other maturities as well as for the subordinate contracts.

The variable operating income divided by sales is used to capture the profitability of a firm. We expect that firms with higher income per unit of sales will have higher recovery rates. Altman and Kishore (1996) document that firms with more tangible assets have higher recovery rates. In addition, tangible assets are relatively easy to transfer to another firm and as a result may fetch higher value. Thus, firms that have a large proportion of tangible assets relative to the total assets should have higher recovery rates. The variable q-ratio measures the growth opportunities of a firm. Firms with higher market value of assets relative to book value of assets imply more growth opportunities. We therefore expect a positive relationship between the recovery rates and the q-ratio. As in Acharya, Bharath and Srinivasan (2007), we argue that firms with higher leverage should have lower recovery rates due to their difficulty in resolving the bankruptcy proceedings. We also include the one-year returns as well as the volatility for each firm. We expect firms with a significant drop in their stock price relative to the previous year to have lower recovery. Finally, an increase in the firm's volatility indicates an increase in the variation of the expected cash flows and hence results in a lower recovery rate for their bondholders.

Table 7 reports the results of the pooled regression where the dependent variable is the implied recovery rate for the senior contract with five year maturity. The regression includes only those firms that have firm specific characteristics data available. Consistent with our expectation, we find that the measure of firm profitability (income per unit sales) has a positive and significant impact on the risk-neutral recovery rates. Specifically, a 1 percent increase in operating income per unit sales results in an increase of 0.28 to 0.38 percent in the risk neutral recovery rates. These numbers are similar in magnitude to those found in Acharya, Bharath and Srinivasan (2007). In addition, we also find that the firm leverage has a negative and significant impact on the implied recovery rates. A 1 percent increase in the firm leverage results in a 0.22 to 0.31 percent drop in the recovery rates. The firm's implied volatility has marginal impact and a correct sign on the risk-neutral recovery rates.

Surprisingly, none of the other firm specific characteristics have any significant impact on the risk-neutral recovery rates. A possible explanation is that our dependent variable is the risk-neutral recovery rates which are embedded with risk premium. If the recovery risk premium is not affected by the firm specific characteristics that we consider, then the cross-sectional rankings between these firms based on their characteristics may not be the same under the physical and the risk neutral measures.

6 Conclusion

Existing empirical work on the dynamics of risk-neutral recovery rates is limited due to the econometric difficulty in isolating the default risk component from the recovery component. In this paper, we circumvent this identification issue by using the term structure of senior and subordinate credit default swaps (CDS) simultaneously to estimate a stochastic recovery credit risk model. Using a reduced form framework, we estimate the dynamics of default intensity and loss given default (1-recovery) on a large sample of daily senior and subordinate CDS data between 2001 and 2008.

Overall, our model performs very well in terms of fitting the term structure of CDS spreads for both seniorities. The estimated recovery rates are economically plausible in a sense that they imply a positive recovery risk premium when compared to studies that estimate the realized recovery rates. In addition, we show that recovery rates implied by the CDS contracts are highly responsive to important corporate events such as accounting news with significant impact on the lender's ability to recover the debt.

During the 2008 financial crisis period, we find that the increase in loss given default is significantly larger at the short horizon relative to the long horizon. This disproportionate change in the expected recovery rates between short and long horizon results in the inversion of the recovery rate term structure: from downward sloping before the financial crisis period to upward sloping during the financial crisis period. Furthermore, this inversion is more prominent for the firms which are in distress during the crisis period, i.e. financial firms.

Finally, we find that firm specific characteristics such as profitability and leverage have a significant impact in explaining the cross-sectional differences between risk-neutral recovery rates across firms. Firm ratings, on the other hand, do not have an impact on risk-neutral recovery rates.

Overall, the use of information from senior and subordinate credit default swaps jointly significantly improves the identification of the recovery rates. The estimated model is able to capture the stylized facts and provide new insights about the term structure of recovery rates. Our results provide strong evidence for the importance of time-varying recovery rate in credit risk models.

Appendix A: Default-free model

We want to price a default free zero coupon bond

$$B(t,t+h) = E_t^Q\left[\exp(-\sum_{j=0}^{h-1} r_{t+j})\right] \tag{A.1}$$

We can re-write equation (3.1) in the form

$$\begin{aligned} r_t &= \left(\delta_0 + \delta_1 X_{1,t}^r + \delta_2 X_{2,t}^r\right)^2 \\ &= \left(\delta_0 + \delta' X_t^r\right)^2 \\ &= \delta_0^2 + 2\delta_0 \delta' X_t^r + X_t^{r\prime} \delta\delta' X_t^r \end{aligned} \tag{A.2}$$

where δ and X_t^r are 2×1 vectors. Recall that from equation (3.3), the dynamics of the state variables is as below:

$$X_t^r = \mu^r + \rho^r X_{t-1}^r + \Sigma^r \varepsilon_t^r \tag{A.3}$$

ρ^r and Σ^r are 2×2 matrices.

We compute the expectation in equation (A.1) using law of iterative expectations. Recall that the price of a one period ahead zero coupon risk-free bond at time $t + h - 1$ is

$$B(t+h-1,t+h) = E_{t+h-1}^Q[\exp(-r_{t+h-1})] = \exp(-r_{t+h-1}) \tag{A.4}$$

Substituting expression (A.2) we have

$$\begin{aligned} B(t+h-1,t+h) &= \exp[-(\delta_0^2 + 2\delta_0\delta' X_{t+h-1}^r + X_{t+h-1}^{r\prime}\delta\delta' X_{t+h-1}^r)] \\ &= \exp(A_1 + B_1' X_{t+h-1}^r + X_{t+h-1}^{r\prime} C_1 X_{t+h-1}^r) \end{aligned} \tag{A.5}$$

where

$$\begin{array}{rcl} A_1 = & -\delta_0^2 & \text{scalar} \\ B_1 = & -2\delta_0\delta' & \text{a } 2\times 1 \text{ vector} \\ C_1 = & -\delta\delta' & \text{a } 2\times 2 \text{ matrix} \end{array} \tag{A.6}$$

The price of a two period ahead zero coupon bond at time $t + h - 2$ is

$$\begin{aligned} B(t+h-2,t+h) &= E_{t+h-2}^Q[\exp(-r_{t+h-1} - r_{t+h-2})] \\ &= \exp(-\delta_0^2 - 2\delta_0\delta' X_{t+h-2}^r - X_{t+h-2}^{r\prime}\delta\delta' X_{t+h-2}^r) \times \\ &\quad E_{t+h-2}^Q[\exp(A_1 + B_1' X_{t+h-1}^r + X_{t+h-1}^{r\prime} C_1 X_{t+h-1}^r)] \\ &= \exp(-\delta_0^2 - 2\delta_0\delta' X_{t+h-2}^r - X_{t+h-2}^{r\prime}\delta\delta' X_{t+h-2}^r) \times \\ &\quad \exp(A_1 + B_1'\left(\mu^r + \rho^r X_{t+h-2}^r\right) + \left(\mu^r + \rho^r X_{t+h-2}^r\right)' C_1 \left(\mu^r + \rho^r X_{t+h-2}^r\right)) \times \\ &\quad E_{t+h-2}^Q\left[B_1'\Sigma^r\varepsilon_{t+h-1}^r + 2\left(\mu^r + \rho^r X_{t+h-2}^r\right)' C_1 \Sigma^r \varepsilon_{t+h-1}^r + \varepsilon_{t+h-1}^{r\prime}\Sigma^{r\prime} C_1 \Sigma^r \varepsilon_{t+h-1}^r\right] \end{aligned} \tag{A.7}$$

Now, the expectation of an exponential of a quadratic gaussian random variable can be computed as follows. Let

$$Q = \epsilon' V \epsilon + a'\epsilon + d \tag{A.8}$$

Then the expectation of the exponential of Q is given by

$$E[\exp(tQ)] = \exp(-\frac{1}{2}\ln(\det(I - 2t\ \Gamma\ V)) + td + \frac{1}{2}ta'(\Gamma^{-1} - 2tV)^{-1}at) \qquad (A.9)$$

where ϵ is an $N \times 1$ vector described by a multi-variate normal distribution $\epsilon \sim N(0, \Gamma)$ and det indicates determinant.[23]

Comparing equation (A.9) and the expectation E^Q_{t+h-2} in the last line in equation (A.7) we have, $\epsilon = \varepsilon^r_{t+h-1}$, $V = \Sigma^{r\prime} C_1 \Sigma^r$, $a' = (B_1'\Sigma^r + 2\left(\mu^r + \rho^r X^r_{t+h-2}\right)' C_1 \Sigma^r)$, $\Gamma = I$, $t = 1$ and $d = 0$. Using this equivalence and organizing the common terms together, the expectation in equation (A.7) can be written as below.

$$B(t+h-2, t+h) = \exp(A_2 + B_2' X^r_{t+h-2} + X^{r\prime}_{t+h-2} C_2 X^r_{t+h-2}) \qquad (A.10)$$

where

$$A_2 = \begin{array}{c} -\delta_0^2 + (A_1 + B_1'\mu^r + \mu^{r\prime} C_1 \mu^r) \\ +\frac{1}{2}(\Sigma^{r\prime} B_1 + 2\Sigma^{r\prime} C_1 \mu^r)'(I - 2\Sigma^{r\prime} C_1 \Sigma^r)^{-1}(\Sigma^{r\prime} B_1 + 2\Sigma^{r\prime} C_1 \mu^r) \\ -\frac{1}{2}\ln[\det(I - 2\Sigma^{r\prime} C_1 \Sigma^r)] \end{array} \qquad (A.11)$$

$$B_2' = -2\delta_0\delta' + (B_1 + 2C_1\mu^r)'\rho^r + 2(\Sigma^{r\prime} B_1 + 2\Sigma^{r\prime} C_1 \mu^r)'(I - 2\Sigma^{r\prime} C_1 \Sigma^r)^{-1}\Sigma^r C_1 \rho^{r\prime} \qquad (A.12)$$

and

$$C_2 = -\delta\delta' + \rho^{r\prime} C_1 \rho^r + 2\left(\Sigma^{r\prime} C_1 \rho^r\right)'\left(I - 2\Sigma^{r\prime} C_1 \Sigma^r\right)^{-1}\Sigma^{r\prime} C_1 \rho^r \qquad (A.13)$$

This process is repeated and the expectation in equation (A.1) is

$$B(t, t+h) = \exp(A_h + B_h' X_t^r + X_t^{r\prime} C_h X_t^r) \qquad (A.14)$$

$$A_h = \begin{array}{c} -\delta_0^2 + (A_{h-1} + B_{h-1}'\mu^r + \mu^{r\prime} C_{h-1} \mu^r) \\ +\frac{1}{2}(\Sigma^{r\prime} B_{h-1} + 2\Sigma^{r\prime} C_{h-1} \mu^r)'(I - 2\Sigma^{r\prime} C_{h-1} \Sigma^r)^{-1}(\Sigma^{r\prime} B_{h-1} + 2\Sigma^{r\prime} C_{h-1} \mu^r) \\ -\frac{1}{2}\ln[\det(I - 2\Sigma^{r\prime} C_{h-1} \Sigma^r)] \end{array} \qquad (A.15)$$

$$B_h' = -2\delta_0\delta' + (B_{h-1} + 2C_{h-1}\mu^r)'\rho^r + 2(\Sigma^{r\prime} B_{h-1} + 2\Sigma^{r\prime} C_{h-1} \mu^r)'(I - 2\Sigma^{r\prime} C_{h-1} \Sigma^r)^{-1}\Sigma^r C_{h-1} \rho^{r\prime} \qquad (A.16)$$

and

$$C_h = -\delta\delta' + \rho^{r\prime} C_{h-1} \rho^r + 2\left(\Sigma^{r\prime} C_{h-1} \rho^r\right)'\left(I - 2\Sigma^{r\prime} C_{h-1} \Sigma^r\right)^{-1}\Sigma^{r\prime} C_{h-1} \rho^r \qquad (A.17)$$

Appendix B: Credit default swap valuation

The derivation of equation (3.11)

The sum of the default intensity and the short rate is given by

$$\begin{aligned} r_t + \lambda_t &= (\delta_0 + \delta_1 X_{1,t} + \delta_2 X_{2,t})^2 + \\ &\quad (\alpha_0 + \alpha_1 X_{1,t} + \alpha_2 X_{2,t} + \alpha_3 X_{3,t} + \alpha_4 X_{4,t})^2 \end{aligned} \qquad (B.1)$$

[23] The proof is given in Mathai and Provost (1992, p. 40).

which can be re-written as below:

$$\begin{aligned} r_t + \lambda_t &= (\delta_0 + \delta' X_t^r)^2 \\ &\quad + (\alpha_0 + \alpha^{r\prime} X_t^r + \alpha^{c\prime} X_t^c)^2 \\ r_t + \lambda_t &= \gamma_0 + \gamma_1' X_t + X_t' \Omega X_t \end{aligned} \tag{B.2}$$

where

$$\gamma_0 = \delta_0^2 + \alpha_0^2 \tag{B.3}$$

$$\gamma_1 = \begin{bmatrix} 2(\delta_0 \delta' + \alpha_0 \alpha^{r\prime}) \\ 2\alpha_0 \alpha^{c\prime} \end{bmatrix} \tag{B.4}$$

$$\Omega = \begin{bmatrix} \delta\delta' + \alpha^r \alpha^{r\prime} & \alpha^r \alpha^{c\prime} \\ \alpha^c \alpha^{r\prime} & \alpha^c \alpha^{c\prime} \end{bmatrix} \tag{B.5}$$

$$X_t = \begin{bmatrix} X_t^r \\ X_t^c \end{bmatrix} \tag{B.6}$$

$\delta = [\delta_1 \ \delta_2]'$, $\alpha^r = [\alpha_1 \ \alpha_2]'$, $\alpha^c = [\alpha_3 \ \alpha_3]'$ $X_t^r = [X_{1,t}, \ X_{2,t}]'$ and $X_t^c = [X_{3,t} \ X_{4,t}]'$. Recall from equation (3.9), the dynamics of the state variables is as below

$$X_t = \mu + \rho X_{t-1} + \Sigma \varepsilon_t, \tag{B.7}$$

where ρ and Σ are 4×4 matrices while μ and X_t are 4×1 vectors. Notice that the expectation in equation (A.1) is same as the equation (3.11) and the dynamics of the state variables in equation (A.3) are also similar to the dynamics in equation (B.7). Therefore, the derivation of the expectation in equation (3.11) follows as in appendix A and is given as below

$$E_t^Q \left[\exp(-\sum_{j=0}^{h-1} r_{t+j} + \lambda_{t+j}) \right] = \exp(F_h + G_h' X_t + X_t' H_h X_t), \tag{B.8}$$

where F_h, G_h, and H_h is computed using the recursions defined as follows with the initial conditions $F_1 = -\gamma_0$, $G_1 = -\gamma_1$ and $H_1 = -\Omega$.

$$F_h = \begin{array}{c} -\gamma_0 + (F_{h-1} + G_{h-1}' \mu + \mu' H_{h-1} \mu) \\ +\frac{1}{2}(\Sigma' G_{h-1} + 2\Sigma' H_{h-1} \mu)' (I - 2\Sigma' H_{h-1} \Sigma)^{-1} (\Sigma' G_{h-1} + 2\Sigma' H_{h-1} \mu) \\ -\frac{1}{2} \ln[\det(I - 2\Sigma' H_{h-1} \Sigma)] \end{array} \tag{B.9}$$

$$G_h' = -\gamma_1' + (G_{h-1} + 2H_{h-1}\mu)'\rho + +2(\Sigma' G_{h-1} + 2\Sigma' H_{h-1}\mu)'(I - 2\Sigma' H_{h-1}\Sigma)^{-1} \Sigma H_{h-1} \rho' \tag{B.10}$$

and

$$H_h = -\Omega + \rho' H_{h-1} \rho + 2\left(\Sigma' H_{h-1} \rho\right)' (I - 2\Sigma' H_{h-1} \Sigma)^{-1} \Sigma' H_{h-1} \rho \tag{B.11}$$

The derivation of equation (3.14)

This sub-section shows the derivation of equation (3.14). We derive each term in the expectation separately. We start with the first term in the expectation i.e.,

$$Y^1(t, t+h) = E_t^Q \left[LGD_{t+h-1} \times \exp(-\sum_{j=0}^{h-1} r_{t+j}) \times \exp(-\sum_{j=0}^{h-2} \lambda_{t+j}) \right] \tag{B.12}$$

where $Y^1(t, t+h)$ denotes the expectation of the first term in equation (3.14) at time t for horizon h, and LGD_{t+h-1} follows from equation (3.13) and re-written below.

$$\begin{aligned} LGD_{t+h-1} &= \exp\left(-\left(\beta_0 + \beta_1 X_{1,t+h-1} + \beta_2 X_{2,t+h-1} + \beta_3 X_{3,t+h-1} + \beta_4 X_{4,t+h-1}\right)^2\right) \\ &= \exp\left(-\left(\beta_0 + \beta^{r\prime} X^r_{t+h-1} + \beta^{c\prime} X^c_{t+h-1}\right)^2\right) \end{aligned} \quad \text{(B.13)}$$

$\beta^r = [\beta_1 \; \beta_2]'$ is the vector of coefficients associated with the term structure factors and $\beta^c = [\beta_3 \; \beta_4]'$ is the vector of coefficients associated with the credit-risk specific factors.

The expectation in equation (B.12) can be re-written as follows.

$$Y^1(t, t+h) = E^Q_t \left[LGD_{t+h-1} \times \exp(-r_{t+h-1}) \times \exp(-\sum_{j=0}^{h-2} r_{t+j} + \lambda_{t+j}) \right] \quad \text{(B.14)}$$

As before, we compute the expectation in the above expression using the law of iterative expectation. We start with the conditional expectation at time $t+h-1$ and work backwards i.e.,

$$\begin{aligned} Y^1(t+h-1, t+h) &= E^Q_{t+h-1}\left[LGD_{t+h-1} \times \exp(-r_{t+h-1})\right] \\ &= \exp(M_1 + N_1' X_{t+h-1} + X_{t+h-1}' O_1 X_{t+h-1}) \end{aligned} \quad \text{(B.15)}$$

where

$$M_1 = -\left(\delta_0^2 + \beta_0^2\right) \quad \text{(B.16)}$$

$$N_1 = -\begin{bmatrix} 2(\delta_0 \delta' + \beta_0 \beta^{r\prime}) \\ 2\beta_0 \beta^{c\prime} \end{bmatrix} \quad \text{(B.17)}$$

$$O_1 = -\begin{bmatrix} \delta\delta' + \beta^r \beta^{r\prime} & \beta^r \beta^{c\prime} \\ \beta^c \beta^{r\prime} & \beta^c \beta^{c\prime} \end{bmatrix} \quad \text{(B.18)}$$

From here onwards, the derivation of the expectation in equation (B.14) is similar to the derivation of the equation (B.8) and (A.1). More specifically, the conditional expectation at time t is

$$Y^1(t, t+h) = \exp(M_h + N_h' X_t + X_t' O_h X_t) \quad \text{(B.19)}$$

where

$$M_h = \begin{array}{c} -\gamma_0 + (M_{h-1} + N_{h-1}'\mu + \mu' O_{h-1}\mu) \\ +\frac{1}{2}(\Sigma' N_{h-1} + 2\Sigma' O_{h-1}\mu)'(I - 2\Sigma' O_{h-1}\Sigma)^{-1}(\Sigma' N_{h-1} + 2\Sigma' O_{h-1}\mu) \\ -\frac{1}{2}\ln[\det(I - 2\Sigma' O_{h-1}\Sigma)] \end{array} \quad \text{(B.20)}$$

$$N_h' = -\gamma_1' + (N_{h-1} + 2O_{h-1}\mu)'\rho + +2(\Sigma' N_{h-1} + 2\Sigma' O_{h-1}\mu)'(I - 2\Sigma' O_{h-1}\Sigma)^{-1}\Sigma O_{h-1}\rho' \quad \text{(B.21)}$$

and

$$O_h = -\Omega + \rho' O_{h-1}\rho + 2\left(\Sigma' O_{h-1}\rho\right)'\left(I - 2\Sigma' O_{h-1}\Sigma\right)^{-1}\Sigma' O_{h-1}\rho \quad \text{(B.22)}$$

with the initial conditions M_1, N_1 and O_1 from equation (B.16), equation (B.17) and equation (B.18) respectively.

In what follows, we show the derivation of the second term in the expectation of equation

(3.14).

$$Y^2(t,t+h) = E_t^Q\left[LGD_{t+h-1}\cdot \exp\left(-\sum_{j=0}^{h-1} r_{t+j}+\lambda_{t+j}\right)\right] \tag{B.23}$$

The superscript 2 in $Y^2(t,t+h)$ indicates the expectation of the second term. Again, we derive the expectation by working backwards. We start with the conditional expectation at time $t+h-1$

$$Y^2(t+h-1,t+h) = E_{t+h-1}^Q\left[LGD_{t+h-1}\times \exp\left(-r_{t+h-1}-\lambda_{t+h-1}\right)\right] \tag{B.24}$$

Using equation (B.2) and equation (B.13),

$$\begin{aligned} Y^2(t+h-1,t+h) &= \exp\left(-\left(\beta_0+\beta^{r\prime}X^r_{t+h-1}+\beta^{c\prime}X^c_{t+h-1}\right)^2\right)\times \\ &\quad \exp\left(-\left(\gamma_0+\gamma_1' X_{t+h-1}+X'_{t+h-1}\Omega X_{t+h-1}\right)\right) \\ &= \exp(J_1+K_1' X_{t+h-1}+X'_{t+h-1}L_1 X_{t+h-1}) \end{aligned} \tag{B.25}$$

where

$$J_1 = -(\delta_0^2+\beta_0^2+\alpha_0^2) \tag{B.26}$$

$$K_1 = -\begin{bmatrix} 2(\delta_0\delta'+\beta_0\beta^{r\prime}+\alpha_0\alpha^{r\prime}) \\ 2\left(\beta_0\beta^{c\prime}+\alpha_0\alpha^{c\prime}\right)\end{bmatrix} \tag{B.27}$$

$$L_1 = -\begin{bmatrix} \delta\delta'+\beta^r\beta^{r\prime}+\alpha^r\alpha^{r\prime} & \beta^r\beta^{c\prime}+\alpha^r\alpha^{c\prime} \\ \beta^c\beta^{r\prime}+\alpha^c\alpha^{r\prime} & \beta^c\beta^{c\prime}+\alpha^c\alpha^{c\prime}\end{bmatrix} \tag{B.28}$$

As before, from here onwards, the derivation of the expectation in equation (B.23) is similar to the derivation of the equation (B.8) and (A.1). More specifically, the conditional expectation at time t is

$$Y^2(t,t+h) = \exp(J_h+K_h' X_t+X_t' L_h X_t) \tag{B.29}$$

where

$$J_h = \begin{array}{c} -\gamma_0+(J_{h-1}+K'_{h-1}\mu+\mu' L_{h-1}\mu) \\ +\frac{1}{2}(\Sigma' K_{h-1}+2\Sigma' L_{h-1}\mu)'(I-2\Sigma' L_{h-1}\Sigma)^{-1}(\Sigma' K_{h-1}+2\Sigma' L_{h-1}\mu) \\ -\frac{1}{2}\ln[\det(I-2\Sigma' L_{h-1}\Sigma)] \end{array} \tag{B.30}$$

$$K_h' = -\gamma_1'+(K_{h-1}+2L_{h-1}\mu)'\rho++2(\Sigma' K_{h-1}+2\Sigma'_{h-1}L_{h-1}\mu)'(I-2\Sigma' L_{h-1}\Sigma)^{-1}\Sigma L_{h-1}\rho' \tag{B.31}$$

and

$$L_h = -\Omega+\rho' L_{h-1}\rho+2\left(\Sigma' L_{h-1}\rho\right)'(I-2\Sigma' L_{h-1}\Sigma)^{-1}\Sigma' L_{h-1}\rho \tag{B.32}$$

with the initial conditions J_1, K_1 and L_1 from equation (B.26), equation (B.27) and equation (B.28) respectively.

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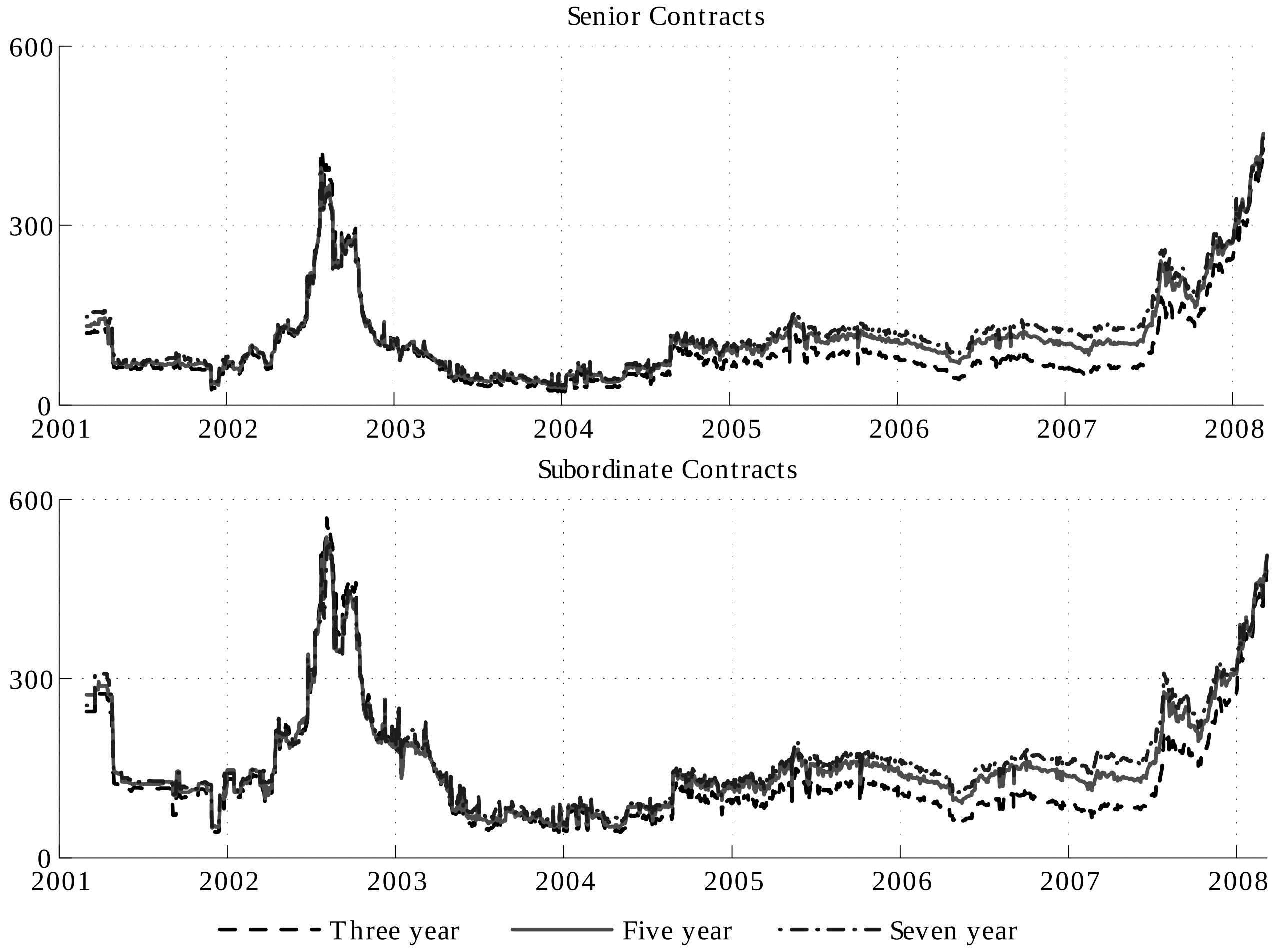
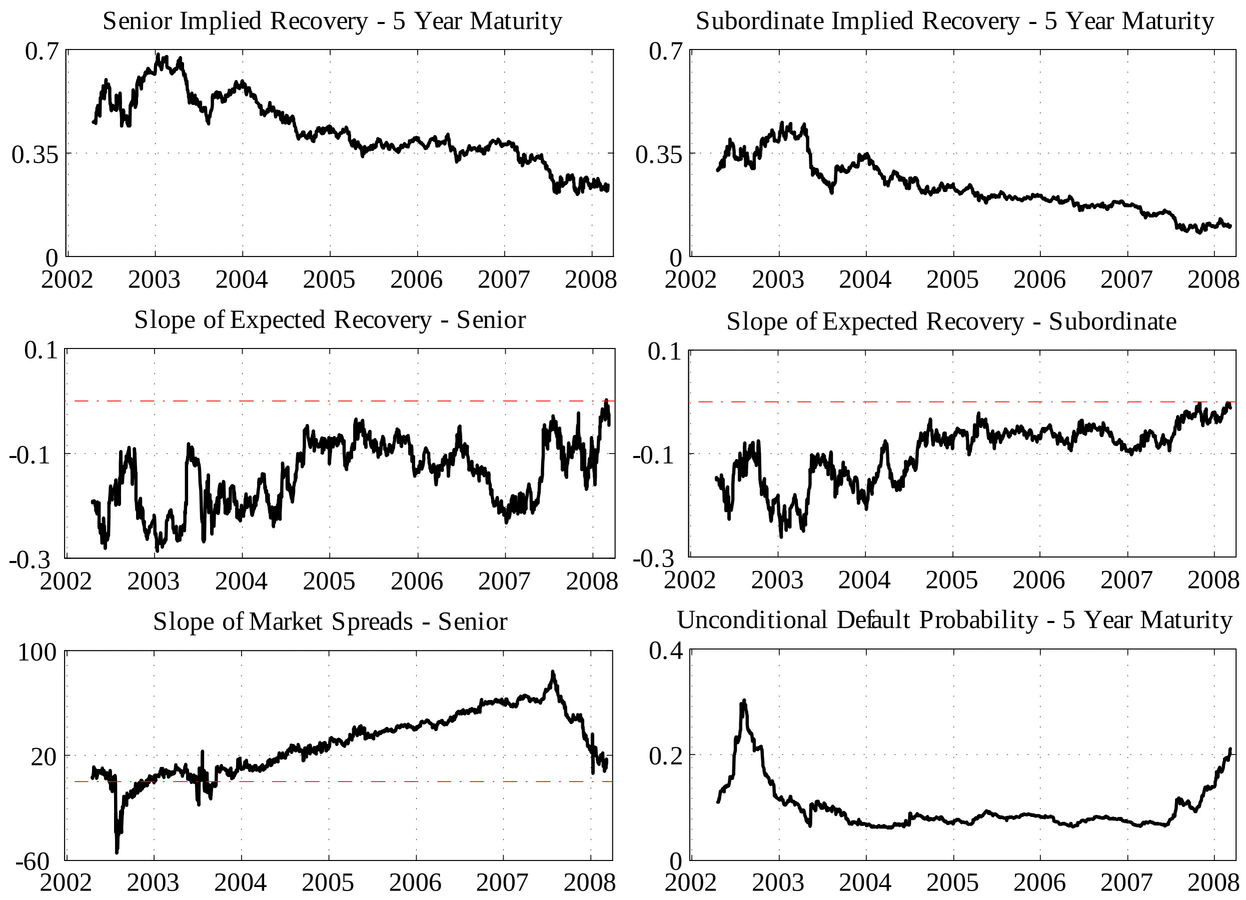
Figure 1: Summary statistics: average market spreads



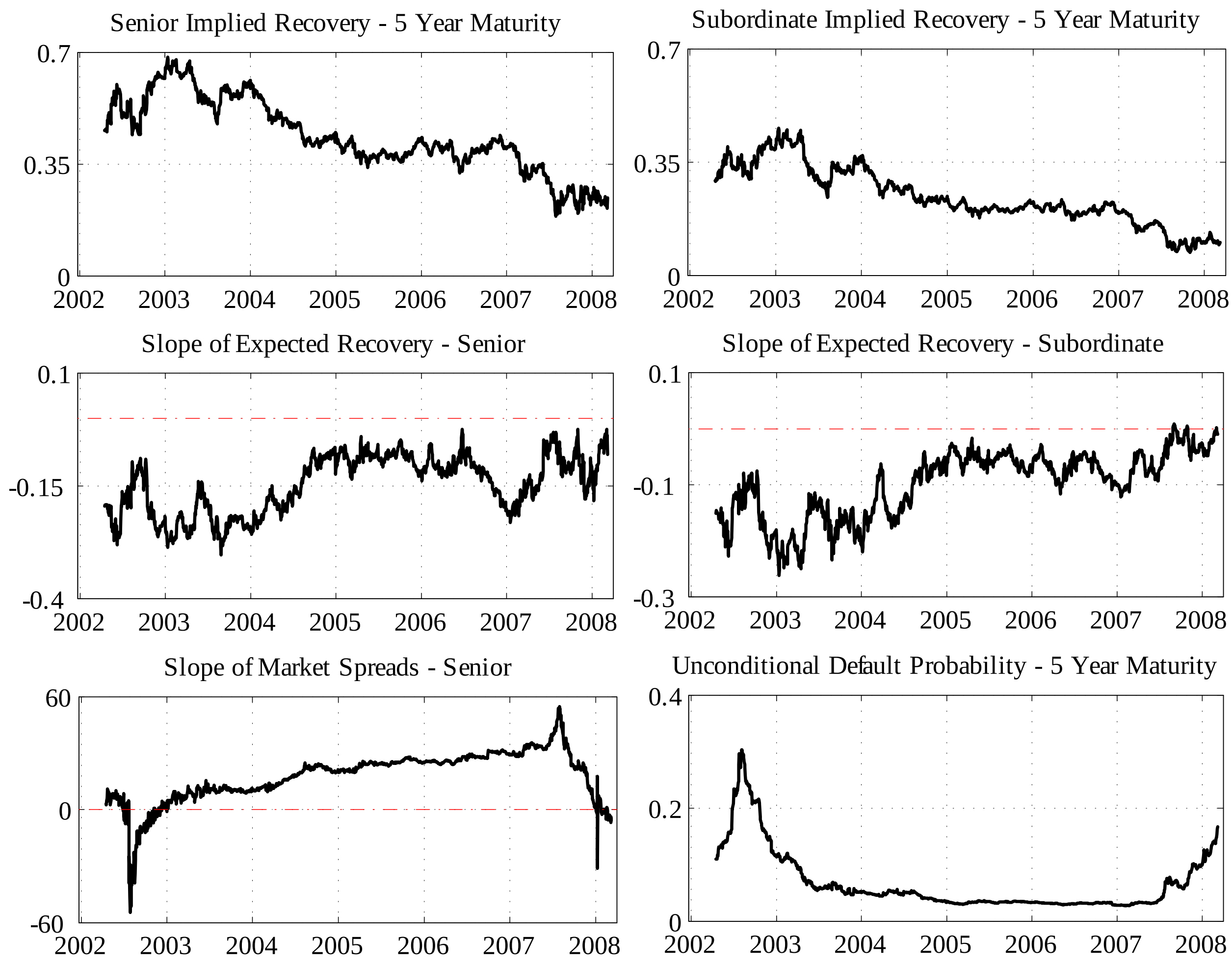
Notes to Figure: We show the time series of the average market spread (in basis points) for both senior and subordinate contracts for all three maturities. The number of firms used to compute the average varies over time, it increases as the time horizon increases since we have data for a lot more firms in the later part of the sample period.

Figure 2: Recovery dynamics: average across all firms



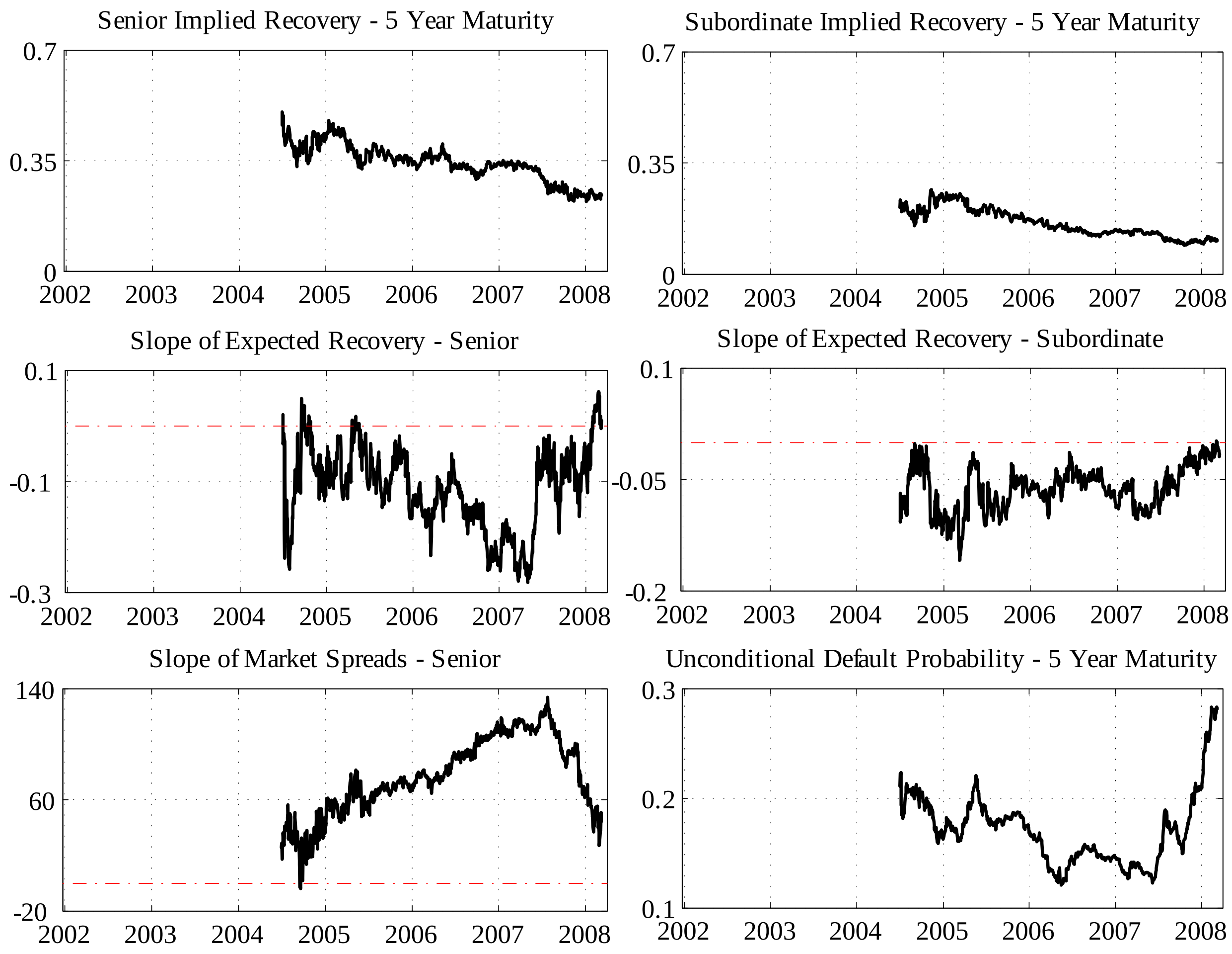
Notes to Figure: The top two panels show the average implied recovery for the five year maturity contract for both seniorities across all firms in our sample. The middle two panels present the average slope of the expected recovery defined as the expected recovery at the seven year horizon minus the expected recovery at the three month horizon for both seniorities. The bottom two panels present the average slope of the market spreads (defined as the difference between seven year credit spreads and the three year credit spreads) and the average five year unconditional default probability. All averages are computed using available data for all firms in our sample. The average is computed if there are at least five firms with available data on a given date.

Figure 3: Recovery dynamics: average across firms above investment grade



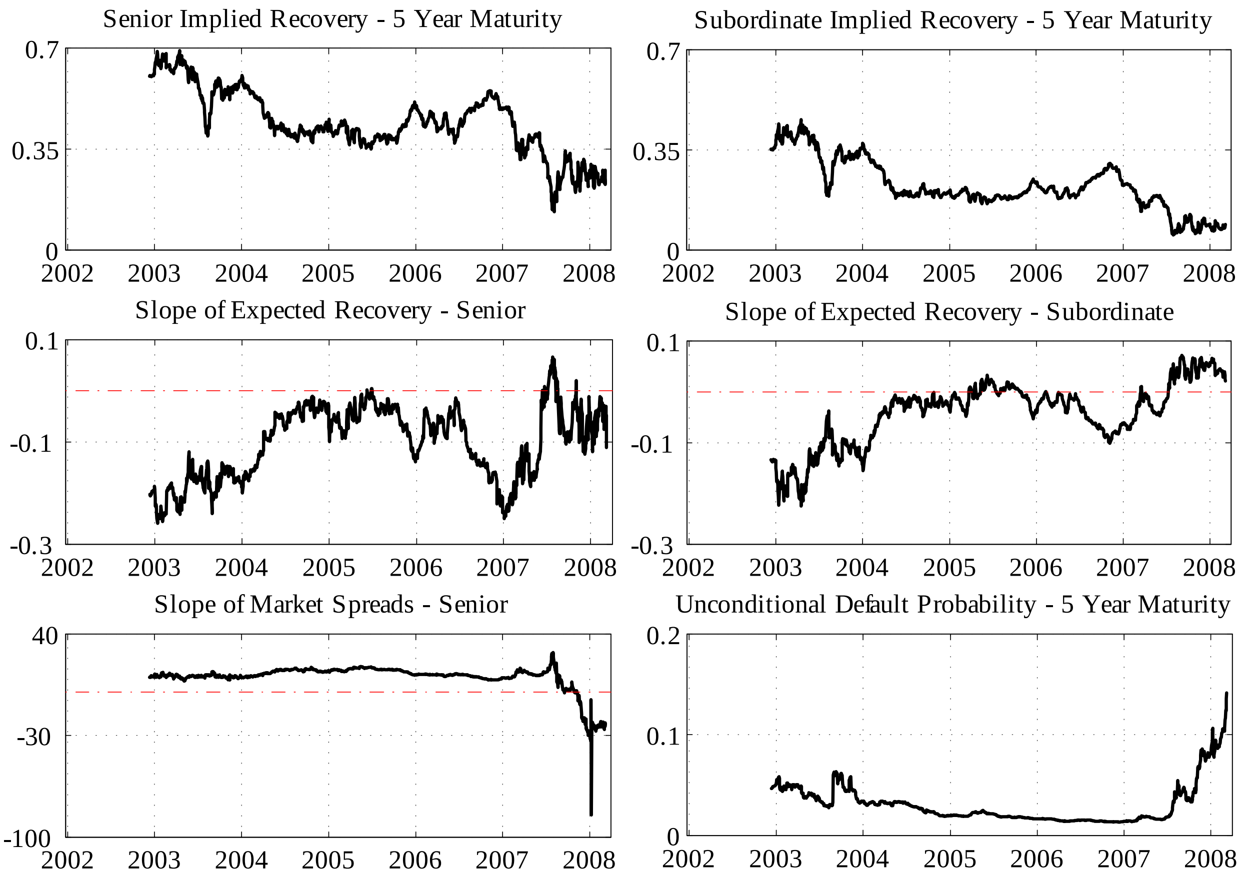
Notes to Figure: The top two panels show the average implied recovery for the five year maturity contract for both seniorities across all firms rated above investment grade in our sample. The middle two panels present the average slope of the expected recovery defined as the expected recovery at the seven year horizon minus the expected recovery at the three month horizon for both seniorities. The bottom two panels present the average slope of the market spreads (defined as the difference between seven year credit spreads and the three year credit spreads) and the average five year unconditional default probability. All averages are computed using available data for all firms rated above investment grade in our sample. The average is computed if there are at least five firms with available data on a given date.

Figure 4: Recovery dynamics: average across firms below investment grade



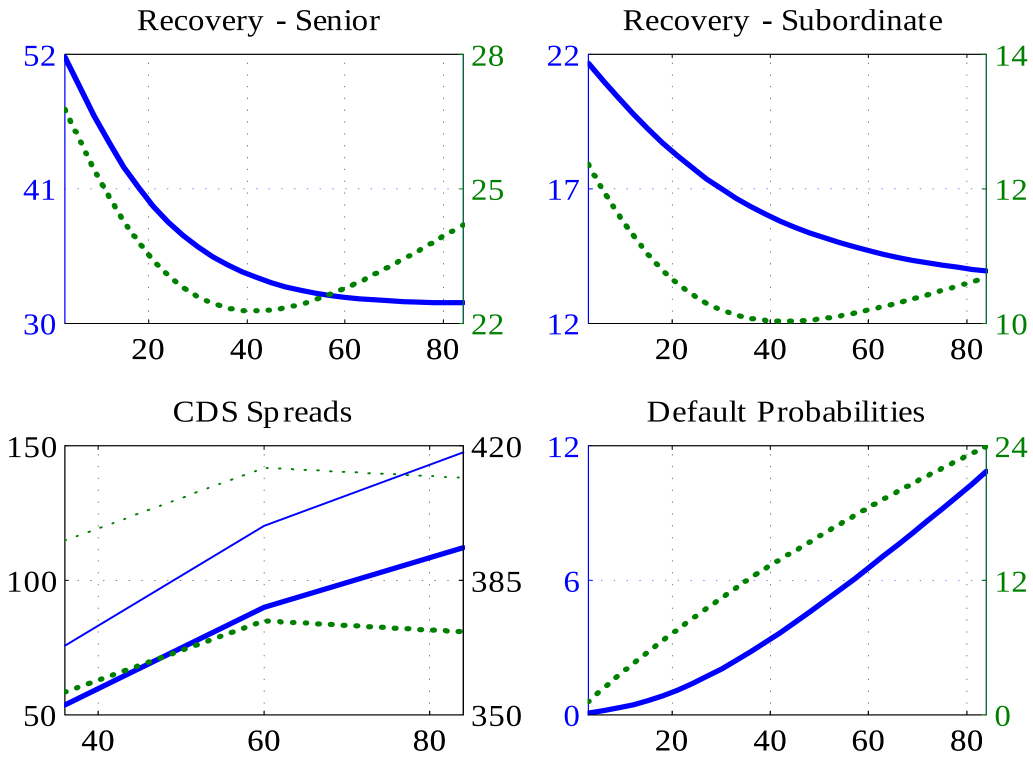
Notes to Figure: The top two panels show the average implied recovery for the five year maturity contract for both seniorities across all firms rated below investment grade in our sample. The middle two panels present the average slope of the expected recovery defined as the expected recovery at the seven year horizon minus the expected recovery at the three month horizon for both seniorities. The bottom two panels present the average slope of the market spreads (defined as the difference between seven year credit spreads and the three year credit spreads) and the average five year unconditional default probability. All averages are computed using available data for all firms that are rated below investment grade in our sample. The average is computed if there are at least five firms with available data on a given date.

Figure 5: Recovery dynamics: average across firms in financial industry



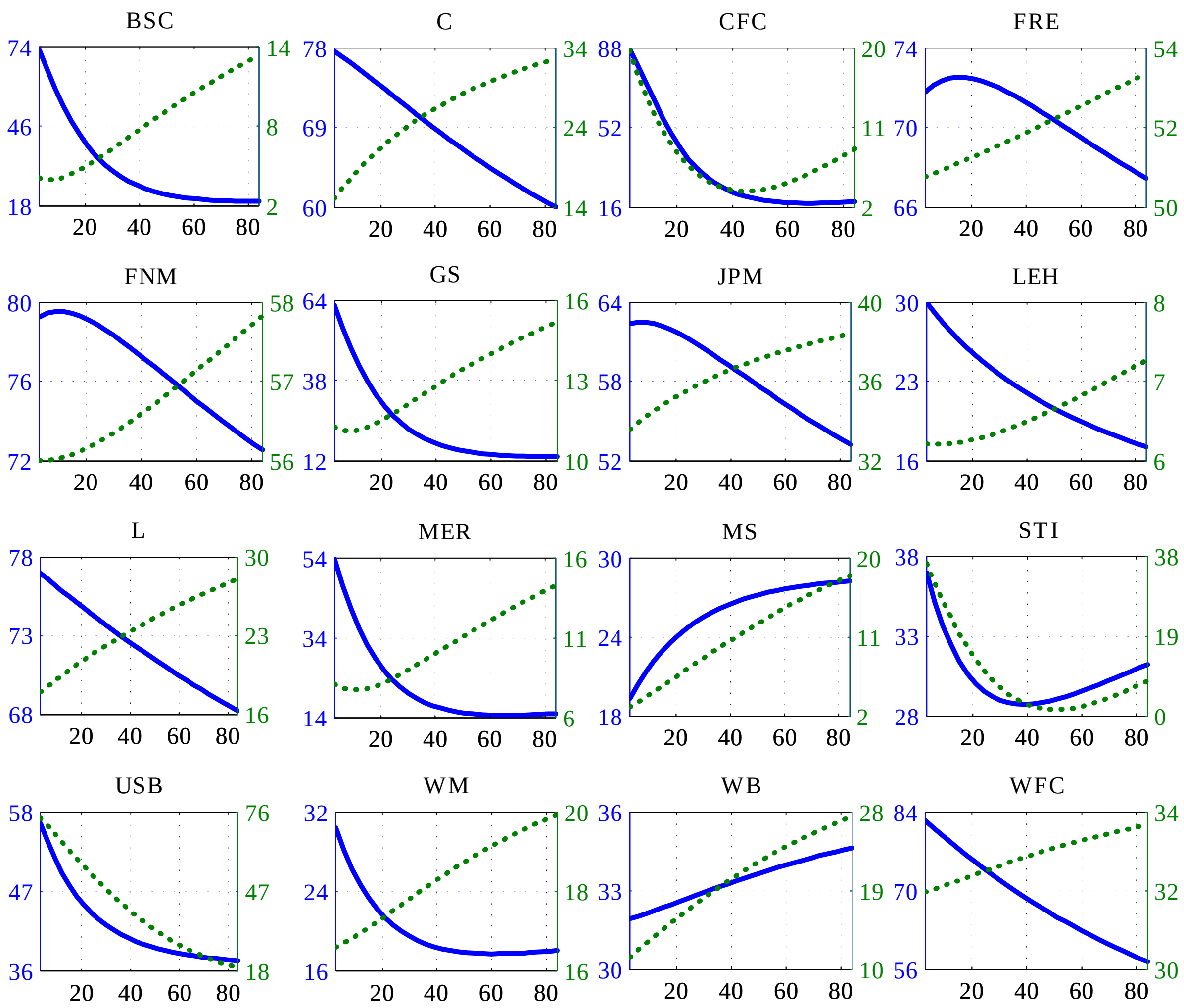
Notes to Figure: The top two panels show the average implied recovery for the five year maturity contract for both seniorities across all firms that can be classified as financials in our sample. The middle two panels present the average slope of the expected recovery defined as the expected recovery at the seven year horizon minus the expected recovery at the three month horizon for both seniorities. The bottom two panels present the average slope of the market spreads (defined as the difference between seven year credit spreads and the three year credit spreads) and the average five year unconditional default probability. All averages are computed using available data for all firms in financial industry in our sample. The average is computed if there are at least five firms with available data on a given date. As the year increases, the number of firms in the sample increases.

Figure 6: Average term structure of expected recovery before and during financial crisis



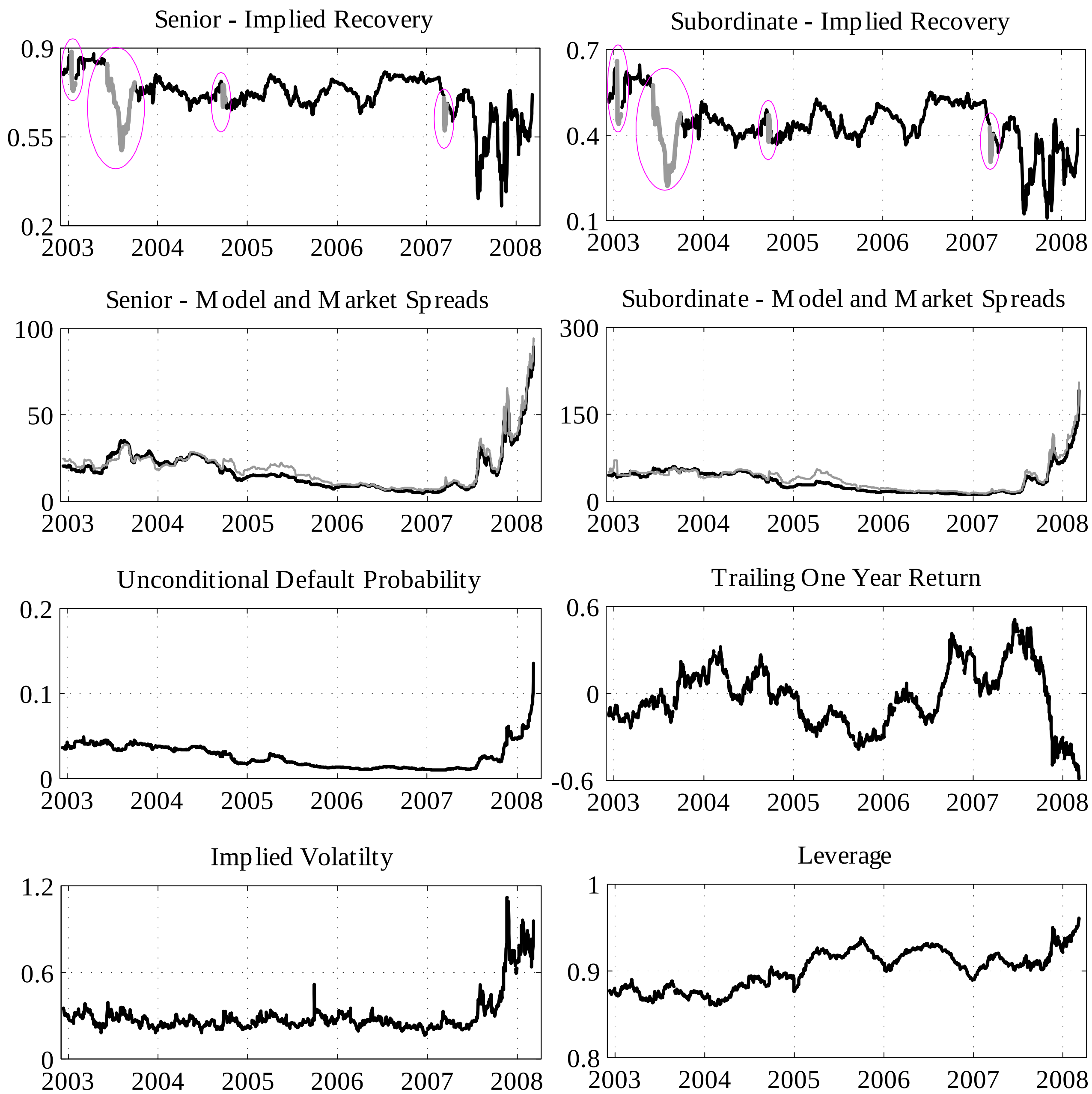
Notes to Figure: We graph the average term structure of expected recovery across all firms in our sample in February 2007 versus the average term structure of expected recovery in February 2008 for both senior and subordinate contract. We also graph the term structure of average credit spreads in February 2007 versus the term structure of credit spreads in February 2008 and the term structure of unconditional default probabilities in February 2007 versus the term structure of unconditional default probabilities in February 2008. The average is computed using only those firms that have available data for both time periods. The expected recovery and default probabilities are expressed in percentages while the credit spreads are expressed in basis points. The x-axis is expressed in months. The left y-axis corresponds to the results for February 2007 while the right y-axis corresponds to the results for February 2008. The solid line is the average for February 2007, while the dotted line is the average for February 2008. In the bottom left panel, the thick (thin) solid line is for the senior contract (subordinate contract) for February 2007 while the thick (thin) dotted line is for the senior contract (subordinate contract) for February 2008.

Figure 7: Term structure of expected recovery for individual financial firms before and during financial crisis



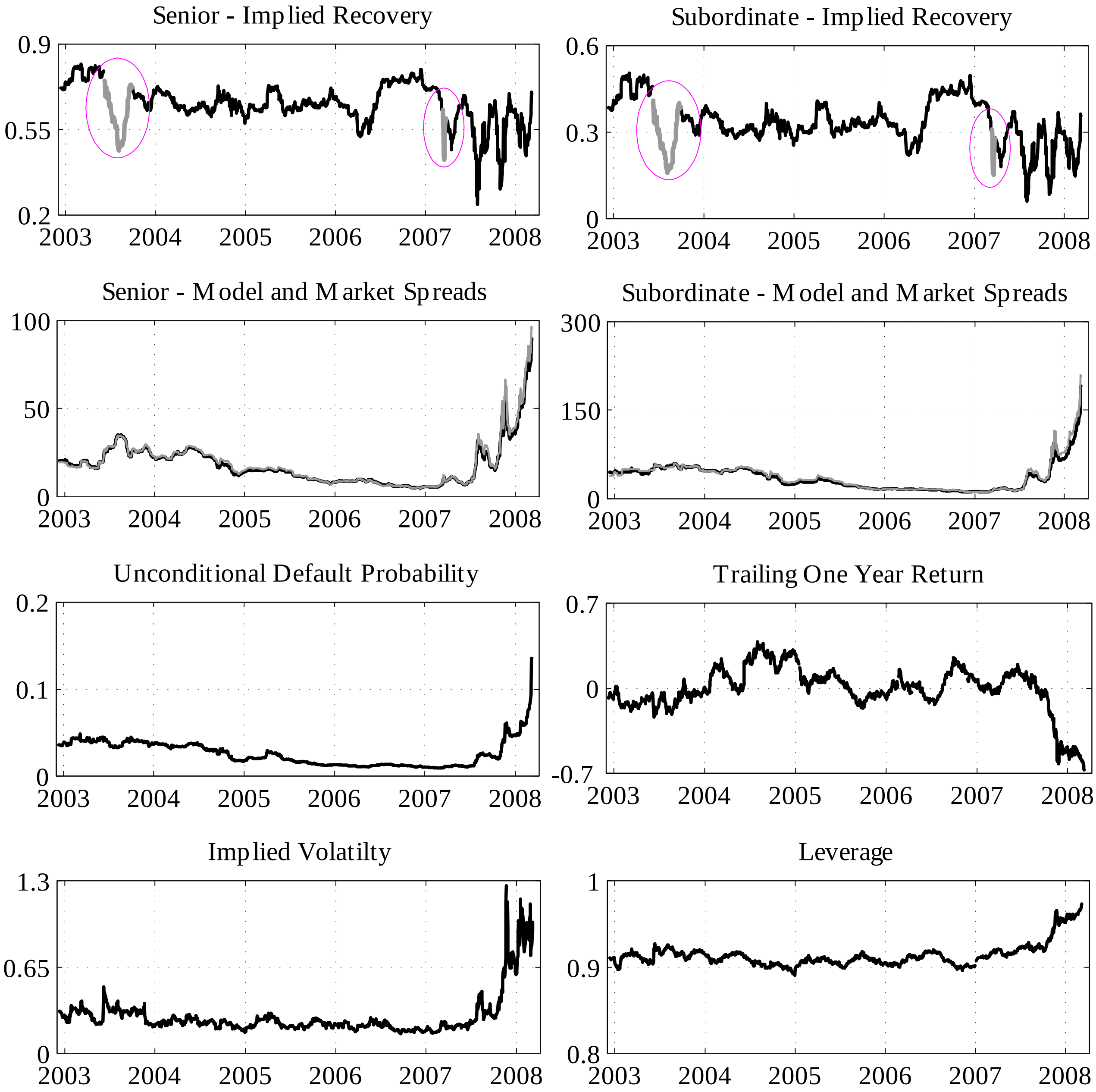
Notes to Figure: We graph the average term structure of expected recovery in February 2007 versus the average term structure of expected recovery in February 2008 for senior contracts of individual financial firms in our sample. The expected recovery rates are expressed in percentages. The x-axis is expressed in months. The left y-axis corresponds to the results for February 2007 while the right y-axis corresponds to the results for February 2008. The solid line is the average for February 2007, while the dotted line is the average for February 2008. The name of the firm is represented by the ticker. Firm tickers are listed in Table 1b.

Figure 8: Case study: Fannie Mae



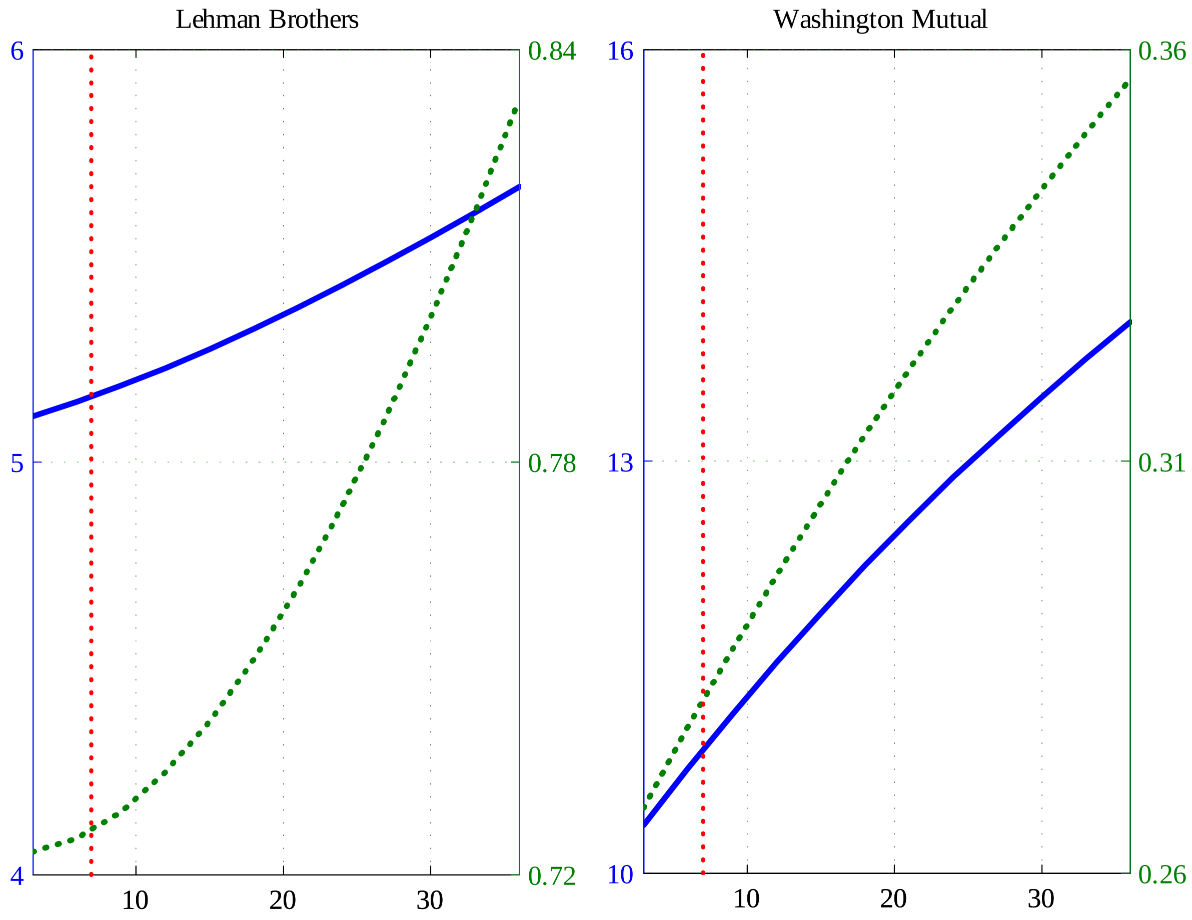
Notes to Figure: The top two panels present the time series of implied five year recovery rates for both senior and subordinate contracts. The panels in the second row present the market (in gray) and model implied spreads (in black) for senior and subordinate contracts with five year maturity. The panels in third row present the model implied five year unconditional default probability and the trailing one year return of the firm, and the panels in row four present the 30 day at-the-money put option implied volatility and leverage computed as the ratio of long term debt to the sum of long term debt and market value of equity. The data points in gray (with circles around them) in top two panels indicate the events discussed in the text.

Figure 9: Case study: Freddie Mac



Notes to Figure: The top two panels present the time series of implied five year recovery rates for both senior and subordinate contracts. The panels in the second row present the market (in gray) and model implied spreads (in black) for senior and subordinate contracts with five year maturity. The panels in third row present the model implied five year unconditional default probability and the trailing one year return of the firm, and the panels in row four present the 30 day at-the-money put option implied volatility and leverage computed as the ratio of long term debt to the sum of long term debt and market value of equity. The data points in gray (with circles around them) in top two panels indicate the events discussed in the text.

Figure 10: Model implied expected recovery around default: Lehman Brothers and Washington Mutual



Notes to Figure: We show the average term structure of expected recovery for Lehman Brothers and Washington Mutual in the first week of March 2008, which is the last week of data in our sample. The dash-dot line indicates the time around which CDS contracts for Lehman Brothers and Washington Mutual were settled after their default, the solid line indicates the term structure of recovery for the senior contract and the dotted line indicates the term structure of recovery for the subordinate contract. The x-axis is expressed in months. The left y-axis indicates the expected recovery for the senior contract in percentages, and the right y-axis indicates the expected recovery for the subordinate contract in percentages.

Table 1a: Summary statistics: individual firms

Firm Name	Ticker	Entity Ratings	Data Availability	Senior - Market Spread (bps)						Subordinate - Market Spread (bps)					
				3 Yr		5 yr		7 Yr		3 Yr		5 yr		7 Yr	
				Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
Amkor Tech	AMKR	B	05/03-03/08	499	222	572	215	589	210	729	300	811	298	904	299
Bear Stearns	BSC	A	12/06-03/08	104	100	104	85	103	78	118	109	117	94	117	87
Best Buy	BBY	BBB	04/03-03/08	36	25	51	28	58	26	45	29	76	48	74	24
Boyd Gaming	BYD	BB	12/05-03/08	185	131	272	139	305	134	188	132	279	135	318	133
Cap One Bank	COF	BBB	04/04-03/08	45	76	58	74	62	67	56	93	68	78	75	72
Citigroup	C	AA	02/01-03/08	21	20	28	21	33	21	28	24	38	26	43	26
Cox Communications*	COX	BBB	03/01-03/08	88	100	103	91	112	85	152	194	167	178	181	186
Countrywide Financial	CFC	A	12/06-03/08	354	413	296	318	274	280	373	429	313	330	293	289
D R Horton	DHI	BBB	03/06-01/08	140	144	173	126	189	110	209	175	229	145	265	129
Freddie Mac	FRE	AAA	12/02-03/08	13	11	19	13	22	13	27	21	37	24	41	23
Fannie Mae	FNM	AAA	09/02-03/08	14	10	20	12	23	12	30	21	41	24	46	23
Goldman Sachs	GS	AA	12/06-03/08	47	38	54	34	57	33	56	42	62	38	64	35
Harrahs Entmt *	HET	BBB	07/04-03/08	145	212	200	227	228	228	165	227	229	250	264	257
Health Mgmt. Assoc.	HMA	BBB	01/05-03/08	103	123	145	154	164	160	111	130	162	167	182	171
Iron Mountain	IRM	BB	05/05-03/08	179	68	256	75	283	77	182	70	261	81	295	83
JPMorgan Chase & Co	JPM	A	01/01-03/08	29	20	38	21	43	21	38	27	49	28	56	30
KB Home	KBH	BB	06/03-03/08	174	110	209	93	218	88	210	140	254	127	268	118
Kerr Mcgee Corp*		BB	03/06-03/08	18	9	30	16	39	18	23	11	35	16	45	18
Kohls Corp	KSS	A	01/04-03/08	26	22	40	27	49	26	29	25	41	29	50	32
L 3 Comms. Corp	LLL	BBB	08/05-03/08	79	28	123	37	145	42	91	33	133	38	151	42
Lehman Brothers	LEH	A	01/07-03/08	85	68	88	58	87	52	95	73	97	64	94	55
Loews Corp	L	A	08/03-03/08	28	25	38	27	45	26	46	52	54	46	64	46
MGM MIRAGE	MGM	BB	06/04-03/08	139	67	197	70	223	74	171	70	248	70	285	78
Merrill Lynch & Co Inc	MER	A	03/06-03/08	48	59	53	52	55	46	56	66	62	58	66	54
Morgan Stanley	MS	A	06/04-03/08	31	38	39	33	44	29	38	40	46	35	50	31
Navistar International	NAV	B	03/05-03/08	254	90	330	96	355	95	282	91	361	95	374	77
Neiman Marcus*		B	01/06-03/08	145	97	224	103	258	97	208	114	328	122	374	117
Office Depot	ODP	BBB	12/03-03/08	50	48	74	57	85	54	58	52	88	59	94	56
Omnicare	OCR	BBB	08/04-03/08	132	60	184	66	211	76	138	72	196	79	222	81
Sungard Systems*		B	08/06-03/08	239	101	344	101	385	96	316	127	443	114	492	107
Sanmina SCI Corp	SANM	BB	01/05-03/08	144	58	194	71	216	76	319	113	412	137	450	145
Sinclair Broadcast	SBGI	BB	10/04-03/08	202	61	259	52	287	45	192	62	273	51	300	41
Solectron Corp*	SLR	B	04/06-10/07	102	39	158	56	186	59	182	58	278	81	313	88
SunTrust Banks	STI	A	09/04-03/08	16	15	22	20	28	19	21	16	35	28	38	29
TJX Companies	TJX	A	08/03-07/07	17	6	27	9	35	12	30	12	43	15	55	18
TRW Automotive	TRW	BB	10/05-03/08	174	92	244	98	273	99	224	92	281	97	309	96
Tesoro Corp	TSO	BB	11/04-03/08	79	33	113	38	130	39	101	46	145	48	166	45
Time Warner	TWX	BBB	04/02-02/08	95	151	106	129	114	120	145	218	147	185	153	169
Toll Brothers	TOL	BBB	04/04-03/08	88	84	115	70	127	61	117	94	147	76	161	69
Triad Hospitals*	TRI	BB	09/04-03/08	112	46	161	61	185	61	170	72	228	81	255	80
Tribune Company*	TRB	BB	10/03-03/08	293	658	310	602	315	555	302	633	316	576	334	535
U S Bancorp	USB	AA	11/04-03/08	14	11	20	13	24	14	19	15	26	15	31	14
United Rents	URI	BB	07/04-03/08	249	144	327	161	355	166	364	171	466	172	494	173
Washinton Mutual	WM	A	05/04-03/08	71	134	75	106	77	93	81	146	90	127	94	114
Wachovia Corp	WB	A	03/02-03/08	22	27	28	29	32	27	27	31	36	33	40	34
Wells Fargo & Co	WFC	AA	10/03-03/08	14	15	21	17	25	16	19	18	27	21	32	20

Notes: We report summary statistics for the forty-six firms in our dataset. 3, 5 and 7 Yr indicate sample statistics for the three, five and seven year credit default swap spreads in basis points respectively. The table also includes information about the firm ticker and the corresponding overall credit rating. The column Data Availability presents the information about the sample start and end dates (in mm/yy format) for each firm. * indicates the firms that are either privately held or acquired during our sample period. The ticker of the firms that are held privately for the entire sample is left blank.

Table 1b: Summary statistics: average across firms

Category	Senior (bps)			Subordinate (bps)			Income/ Sales	Tangibility	LEV	Q Ratio	Size	FIV	# of Firms
	3 Yr	5 Yr	7 Yr	3 Yr	5 Yr	7 Yr							
Overall	112	142	155	143	180	197	0.30	0.20	0.45	1.31	10.55	0.33	46
AAA/AA	21	27	31	30	38	43	0.63	0.01	0.65	1.08	13.44	0.26	6
A	69	71	73	79	82	85	0.41	0.09	0.52	1.29	12.22	0.32	12
BBB	91	121	136	117	149	166	0.17	0.19	0.31	1.44	9.64	0.32	11
BB	162	214	236	204	266	293	0.22	0.38	0.39	1.34	8.84	0.34	12
B	248	326	354	343	444	491	0.12	0.21	0.45	1.30	8.87	0.52	5
Mining/Contr	105	132	143	140	166	185	0.17	0.18	0.32	1.33	9.08	0.37	4
Manufacturing	203	256	276	279	342	375	0.11	0.23	0.39	1.26	8.95	0.41	8
Communications	156	206	227	175	233	259	0.32	0.27	0.41	1.49	8.81	0.34	3
Retail Trade	68	100	116	85	129	145	0.09	0.27	0.12	2.13	8.90	0.32	6
F,I,R	56	59	61	66	70	73	0.52	0.01	0.62	1.05	13.00	0.30	17
Services	158	219	245	204	275	305	0.24	0.47	0.41	1.35	9.38	0.30	8

Notes: We report the average credit default swap spreads for both seniorities across all maturities as well as the average firm-characteristics. The table presents overall average as well as the averages by rating and industry categories. Income/Sales is defined as the quarterly operating income before depreciation and taxes divide by quarterly sales. Tangibility is defined as the property, plant and equipment divide by total book value of assets. LEV indicates leverage defined as the ratio of long term debt to the sum of long term debt and market value of equity. Q-Ratio is defined as the ratio of market value of assets to book value of assets. Size is defined as the log of book value of assets. FIV is the 30 day at-the-money put option implied volatility. F, I, R indicates Finance, Insurance, and Real Estate. Mining/Contr. includes firms in the Mining and Construction businesses. Some firms in our sample do not have available data for firm-specific characteristics, those firms are ignored when computing firm specific characteristic averages.

Table 2: Risk-free term structure parameter estimates

Panel A: Risk-free term structure factor loadings and dynamics		
	X_1	X_2
δ_0	0.03859	
δ_1	0.18497	
δ_2		0.23072
ρ	0.99941	0.99781
σ x 100	0.00712	0.00705
$\mu/(1-\rho)$	-0.06031	-0.04994
ρ^P	0.99280	0.99585
$\mu^P/(1-\rho^P)$	-0.07123	-0.05867

Panel B: Risk-free term structure model RMSE (bps) and measurement error standard deviation (bps)								
	6 Months	1 Year	2 Year	3 Year	4 Year	5 Year	7 Year	10 Year
RMSE	8.99	5.50	8.10	6.60	5.13	4.74	4.57	8.67
ME-Std	1.25	1.15	2.30	4.41	1.60	2.50	2.90	1.12

Notes: Panel A reports parameter estimates for the risk-free term structure factors. X_1 and X_2 indicate the two latent risk-free term structure factors estimated using Unscented Kalman Filter. The factor dynamics and short rate loadings for the risk-free term structure are estimated using 6 month Libor rate, 1, 2, 3, 4, 5, 7 and 10 year maturity swap rates. The parameters with superscript "P" indicate the parameters under the physical measure. Recall that the off-diagonal elements of the σ as well as the ρ matrix are assumed to be zero and hence, not reported. Panel B reports RMSE and measurement error standard deviations for the risk free term structure. ME-Std in Panel B indicates the Measurement Error Standard Deviation and RMSE indicates the Root Mean Squared Error.

Table 3: Estimated parameter distribution: CDS valuation model

Panel A: Parameter distribution															
	Intensity Loadings				Senior LGD Loadings				Subordinate LGD Loadings			Risk-Neutral Factor Dynamics			
Prctile	α_0 x 100	α x 100			β_0^{SEN}	β^{SEN}			β_0^{SUB}	β^{SUB}		Int-Latent		Rec-Latent	
	Constant	X_1	X_2	Int-Latent	Constant	X_1	X_2	Rec-Latent	Constant	X_1	X_2	ρ	σ x 100	ρ	σ x 100
Mean	0.65	0.03	-6.21	-16.63	1.95	5.14	20.85	4.89	0.27	0.28	-0.32	0.9993	56.56	0.9797	0.496
25%	0.10	-5.05	-17.44	-0.13	0.07	-0.36	-6.74	0.95	0.05	-0.26	-4.11	0.9991	0.45	0.9991	0.0040
50%	0.37	-0.73	-7.71	0.03	0.78	2.27	2.64	1.38	0.27	0.65	1.27	0.9995	4.64	0.9994	0.0486
75%	0.72	2.93	0.04	0.71	1.90	8.84	23.83	4.44	0.37	1.10	4.15	0.9996	25.60	0.9996	0.7115
Std Dev	0.92	8.10	18.79	107.77	3.03	12.51	49.43	7.69	0.27	4.07	7.47	0.0007	258.17	0.1399	0.949

Panel A: Continued				
	Factor Dynamics -Physical Measure			
Prctile	Int-Latent		Rec-Latent	
	ρ^P	μ^Px 100	ρ^P	μ^Px 100
Mean	0.9753	-1.4582	0.9579	6.928
25%	0.9612	-0.0783	0.9407	-0.0342
50%	0.9858	0.0000	0.9718	4.7044
75%	0.9996	0.0393	0.9960	14.0343
Std Dev	0.0409	9.3954	0.0542	33.897

Panel B: Measurement error standard deviation (bps)						
	Senior			Subordinate		
Prctile	3 Yr	5 Yr	7 Yr	3 Yr	5 Yr	7 Yr
Mean	0.99	0.80	0.85	1.20	1.11	1.14
25%	0.26	0.25	0.23	0.32	0.28	0.39
50%	0.59	0.59	0.64	0.85	0.75	0.75
75%	0.97	0.88	0.92	1.35	1.17	1.35
Std Dev	2.06	1.06	1.09	1.65	1.40	1.23

Notes: We report the estimated parameter distribution of the model across all firms. X_1 and X_2 indicate the risk-free term structure factors, Int-Latent indicates the intensity specific latent factor, and Rec-Latent indicates the latent factor specific to loss given default. LGD indicates loss given default. Panel A reports the distribution of the dynamics of the factors (under both physical and risk-neutral measures) as well as the factor loadings. The superscripts SEN and SUB indicate the loadings for the senior and the subordinate contracts respectively. Recall that the LGD loadings for the subordinate contract are normalized to 1, and the off diagonal elements of the ρ and σ matrix are assumed to be zero and hence, not reported. Panel B reports the distribution of the measurement error standard deviation in basis points.

Table 4: Model Fit

Category	Relative RMSE (%)						Mean Absolute Percentage Error (%)						# of Firms
	Senior			Subordinate			Senior			Subordinate			
	3 Yr	5 Yr	7 Yr	3 Yr	5 Yr	7 Yr	3 Yr	5 Yr	7 Yr	3 Yr	5 Yr	7 Yr	
Overall	10.7	8.7	10.2	10.5	9.1	10.7	8.3	7.0	8.3	8.0	7.1	8.6	46
AAA/AA	8.5	6.2	6.7	7.7	7.7	8.0	6.7	4.8	5.2	5.8	6.0	6.6	6
A	11.4	7.2	8.7	10.5	8.3	9.9	8.9	5.5	6.8	7.9	6.4	7.8	12
BBB	12.7	10.8	11.4	13.1	9.8	12.5	9.7	9.0	9.4	10.3	7.7	9.8	11
BB	10.8	9.8	13.3	10.6	10.1	12.4	8.5	8.1	11.2	8.3	7.9	10.2	12
B	7.2	8.3	7.9	7.8	8.6	8.0	5.4	6.6	5.9	5.6	6.9	6.4	5
Mining/Contr	11.5	10.5	12.4	9.8	7.4	12.4	9.5	8.7	10.6	7.9	6.0	10.4	4
Manufacturing	10.3	12.1	14.0	12.2	11.6	12.0	8.0	9.9	11.4	9.1	9.0	9.7	8
Communications	14.3	11.5	13.4	8.0	8.8	12.4	11.4	9.6	11.4	6.1	6.6	9.3	3
Retail Trade	9.7	8.8	10.6	9.6	10.8	12.1	7.2	7.0	8.4	7.2	8.4	9.8	6
F,I,R	10.1	6.6	7.5	9.9	7.8	8.9	8.0	5.1	5.8	7.4	6.0	7.0	17
Services	11.2	7.9	9.7	11.9	9.0	10.8	8.5	6.6	8.1	9.7	7.4	8.9	8

Notes: We report the average model errors (in terms of relative RMSE and mean absolute percentage error) across all firms as well as the average based on rating and industry categories. The measures of the model performance (relative RMSE and mean absolute percentage error) are computed individually for each firm and averaged across different groups. 3 Yr, 5 Yr and 7 Yr indicate the three, five and seven year maturity contracts. F, I, R indicates Finance, Insurance, And Real Estate. Mining/Contr. includes firms in the Mining and Construction businesses.

Table 5a: Average implied recovery, unconditional default probability and correlation for individual firms

Firm Names	Entity Ratings	Data Availability	Implied Recovery (%)						Correlation (%)		Default Prob.
			Senior			Subordinate			Sen.	Sub.	
			3 Yr	5 Yr	7 Yr	3 Yr	5 Yr	7 Yr	(5 Yr)	(5 Yr)	(5 Yr)
Amkor Tech	B	05/03-03/08	27.5	28.6	29.8	0.1	0.1	0.2	-12.6	-8.1	32.2
Bear Stearns	A	12/06-03/08	29.4	23.4	21.0	0.2	0.1	0.1	-43.4	-43.2	5.8
Best Buy	BBB	04/03-03/08	32.3	32.3	32.4	8.5	5.9	4.5	13.2	16.5	3.7
Boyd Gaming	BB	12/05-03/08	8.6	5.4	4.3	0.4	0.2	0.2	-24.6	-16.0	12.3
Cap One Bank	BBB	04/04-03/08	28.5	28.4	28.5	2.3	2.3	2.2	16.6	15.5	3.5
Citigroup	AA	02/01-03/08	55.2	52.5	50.8	37.6	36.3	35.5	-55.5	-62.6	2.7
Cox Communications*	BBB	03/01-03/08	53.3	52.7	52.4	31.5	30.5	30.1	18.8	18.8	10.9
Countrywide Financial	A	12/06-03/08	30.5	23.5	21.1	0.0	0.0	0.0	-26.1	-31.7	13.4
D R Horton	BBB	03/06-01/08	31.9	26.2	24.2	0.1	0.1	0.1	-46.6	-46.8	9.9
Freddie Mac	AAA	12/02-03/08	67.9	66.2	65.1	35.9	34.9	34.4	-17.0	-20.2	2.6
Fannie Mae	AAA	09/02-03/08	72.8	70.6	69.1	43.6	42.2	41.3	-7.6	-6.9	3.6
Goldman Sachs	AA	12/06-03/08	18.7	14.1	12.3	0.1	0.1	0.1	-45.5	-46.4	2.9
Harrahs Entmt *	BBB	07/04-03/08	46.1	46.1	46.0	40.8	40.8	40.8	-19.2	-21.4	14.3
Health Mgmt. Assoc.	BBB	01/05-03/08	21.7	14.9	12.5	0.1	0.0	0.0	18.9	17.4	7.5
Iron Mountain	BB	05/05-03/08	19.4	11.9	9.5	19.0	11.2	8.3	-57.8	-55.1	13.6
JPMorgan Chase & Co	A	01/01-03/08	46.4	44.6	43.6	31.2	30.7	30.5	-5.8	-10.3	3.3
KB Home	BB	06/03-03/08	46.9	43.0	41.1	30.9	28.2	26.9	-8.3	-9.5	16.0
Kerr Mcgee Corp*	BB	03/06-03/08	31.4	25.4	21.5	9.3	7.3	6.0	-54.3	-33.4	1.7
Kohls Corp	A	01/04-03/08	22.8	15.2	10.8	16.9	11.2	7.8	22.3	16.9	2.3
L 3 Comms. Corp	BBB	08/05-03/08	16.5	14.8	17.3	9.1	7.4	8.1	9.2	12.3	7.0
Lehman Brothers	A	01/07-03/08	13.2	12.3	11.7	3.6	3.3	3.1	-30.0	-22.2	4.7
Loews Corp	A	08/03-03/08	47.4	47.4	47.4	25.8	27.7	29.3	23.2	19.1	3.7
MGM MIRAGE	BB	06/04-03/08	23.1	24.3	25.1	5.6	5.8	5.9	-57.3	-39.7	12.0
Merrill Lynch & Co Inc	A	03/06-03/08	21.6	17.3	15.8	0.2	0.1	0.1	-28.8	-27.4	3.1
Morgan Stanley	A	06/04-03/08	27.2	27.2	27.3	13.3	13.1	13.0	-37.5	-31.7	2.5
Navistar International	B	03/05-03/08	40.1	39.0	38.7	33.4	32.4	32.2	-14.5	-14.5	22.8
Neiman Marcus*	B	01/06-03/08	34.8	32.9	33.0	1.7	1.7	1.7	-36.1	15.6	14.9
Office Depot	BBB	12/03-03/08	40.1	34.5	30.6	30.0	25.1	21.6	5.4	6.3	5.9
Omnicare	BBB	08/04-03/08	15.5	10.5	8.1	8.3	5.4	4.1	-19.8	-12.4	9.7
Sungard Systems*	B	08/06-03/08	25.4	22.5	21.6	0.4	0.4	0.4	-42.6	-39.6	20.0
Sanmina SCI Corp	BB	01/05-03/08	56.0	54.8	55.3	7.8	7.9	7.9	-29.9	-19.6	18.3
Sinclair Broadcast	BB	10/04-03/08	9.1	7.6	8.1	3.2	2.7	2.8	9.9	11.5	13.4
Solectron Corp*	B	04/06-10/07	48.8	49.2	49.6	8.7	9.0	9.2	-6.7	-5.8	13.7
SunTrust Banks	A	09/04-03/08	31.0	30.5	30.8	1.4	1.3	1.3	-47.8	-24.6	1.5
TJX Companies	A	08/03-07/07	54.5	54.0	53.7	31.4	30.6	30.2	7.8	5.6	2.8
TRW Automotive	BB	10/05-03/08	24.2	18.0	16.4	0.0	0.0	0.0	-64.6	-29.9	13.2
Tesoro Corp	BB	11/04-03/08	44.6	45.1	45.4	30.9	31.2	31.4	4.7	4.9	9.8
Time Warner	BBB	04/02-02/08	61.2	56.3	52.4	47.8	42.7	38.7	6.8	7.0	12.1
Toll Brothers	BBB	04/04-03/08	55.8	53.0	50.8	38.3	35.0	32.8	-7.8	-6.1	10.7
Triad Hospitals*	BB	09/04-03/08	52.9	51.9	51.1	31.3	31.8	32.1	20.2	20.0	15.5
Tribune Company*	BB	10/03-03/08	40.6	40.2	40.3	26.3	24.1	23.0	-41.1	-33.4	12.1
U S Bancorp	AA	11/04-03/08	36.3	32.1	29.6	2.0	1.8	1.6	-31.9	-31.6	1.3
United Rents	BB	07/04-03/08	54.1	53.0	52.1	38.5	36.7	35.4	-25.1	-34.8	29.0
Washinton Mutual	A	05/04-03/08	16.6	16.5	16.7	0.5	0.4	0.4	-17.0	-15.0	4.1
Wachovia Corp	A	03/02-03/08	31.4	32.0	32.6	11.2	11.2	11.2	-17.5	-7.3	2.0
Wells Fargo & Co	AA	10/03-03/08	52.7	50.4	48.5	35.7	33.9	32.6	-67.5	-64.1	1.9

Notes: We report the average implied recovery rate for each of the forty-six firms in our dataset for all three maturities and both seniorities. 3, 5 and 7 Yr indicate the three, five and seven year maturity contracts respectively. The column Data Availability presents the information about the sample start and end dates (in mm/yy format) for each firm. * indicates the firms that are either privately held or acquired during our sample period. The column correlation reports the time series correlation between the implied recovery rate for five year maturity contracts (both senior and subordinate) and the unconditional default probability for the five year horizon. The column default prob. reports the model implied five year unconditional default probability.

Table 5b: Average implied recovery, unconditional default probability and correlation for different categories

Category	Implied Recovery (%)						Default Probability (%)			Correlation		# of Firms
	Senior			Subordinate						Sen.	Sub.	
	3 Yr	5 Yr	7 Yr	3 Yr	5 Yr	7 Yr	3 Yr	5 Yr	7 Yr	(5 Yr)	(5 Yr)	
Overall	36.2	33.8	32.7	16.4	15.3	14.8	4.9	9.3	14.0	-18.9	-14.9	46
AAA/AA	50.6	47.7	45.9	25.8	24.9	24.2	1.2	2.5	4.0	-37.5	-38.6	6
A	31.0	28.7	27.7	11.3	10.8	10.6	2.5	4.1	5.8	-16.7	-14.3	12
BBB	36.6	33.6	32.3	19.7	17.7	16.6	4.8	8.7	13.0	-0.4	0.6	11
BB	34.2	31.7	30.9	16.9	15.6	15.0	7.1	13.9	20.9	-27.4	-19.6	12
B	35.3	34.4	34.5	8.9	8.7	8.7	10.4	20.7	31.3	-22.5	-10.5	5
Mining/Contr	41.5	36.9	34.4	19.7	17.6	16.4	5.1	9.6	14.3	-29.3	-24.0	4
Manufacturing	37.3	36.2	36.6	14.5	14.0	14.0	8.5	16.1	24.0	-19.4	-11.8	8
Communications	27.3	24.1	23.3	17.9	14.8	13.7	6.7	12.6	18.8	-9.7	-8.3	3
Retail Trade	33.3	29.9	28.1	16.1	13.3	11.7	3.1	6.6	10.7	-1.2	8.1	6
F,I,R	36.9	34.7	33.6	14.4	14.1	13.9	2.1	3.7	5.4	-25.8	-24.2	17
Services	36.6	34.3	33.1	20.6	19.8	19.2	7.9	15.3	22.9	-15.4	-13.4	8

Notes: We report the average implied recovery rates across all firms as well as the average based on rating and industry categories. The table also presents the average unconditional default probability for three, five and seven year maturities and the average correlation between the five year implied recovery rate and the five year unconditional default probability. F, I, R indicates Finance, Insurance, and Real Estate. Mining/Contr. includes firms in the Mining and Construction businesses. 3 Yr, 5 Yr, and 7 Yr indicate the three, five and seven year maturities respectively.

Table 6: Relative increase in loss given default during financial crisis

Category	Log Ratio of Implied Loss Given Default (%)						Log Ratio of Market Spreads (%)						# of Firms
	Senior			Subordinate			Senior			Subordinate			
	3 Yr	5 yr	7 Yr	3 Yr	5 yr	7 Yr	3 Yr	5 yr	7 Yr	3 Yr	5 yr	7 Yr	
Overall	32.5	25.4	21.7	15.6	12.7	10.7	204.2	160.2	134.6	188.6	148.1	124.0	43
AAA/AA	65.9	55.1	47.2	44.3	37.1	31.7	259.0	214.5	183.0	245.8	201.8	170.7	6
A	34.0	27.6	24.4	14.5	12.5	11.0	271.4	216.6	183.0	253.4	202.3	170.2	11
BBB	21.5	18.3	16.4	12.1	10.1	8.7	213.3	164.1	135.0	201.1	153.5	125.4	10
BB	21.9	14.2	11.1	9.4	6.2	4.6	130.6	99.7	85.2	119.4	92.6	79.5	12
B	37.2	25.7	20.7	3.1	2.2	1.8	135.2	95.2	75.6	101.0	71.6	56.6	4
Mining/Contr	53.3	29.5	18.2	35.4	21.2	14.1	165.8	113.1	86.1	147.7	104.7	82.0	3
Manufacturing	24.8	18.3	15.9	1.4	1.3	1.2	152.9	125.5	111.1	129.9	108.8	96.6	7
Communications	3.2	3.3	3.3	1.9	2.3	2.4	120.9	76.5	54.7	124.0	78.6	56.3	3
Retail Trade	13.4	10.0	8.7	1.7	1.3	1.2	217.3	165.2	135.1	197.6	150.2	122.7	5
F,I,R	44.9	37.6	33.1	24.6	21.0	18.3	274.9	223.7	190.2	258.6	209.5	176.9	17
Services	27.7	21.9	18.8	15.2	12.6	10.8	136.2	101.5	84.6	125.2	93.1	77.3	8

Notes: We compute the log ratio of implied loss given default on each date in February 2008 relative to the implied loss given default one year before. The table presents the average log ratio for both seniorities and across all three maturities. The table also presents the average percentage increase in the market spreads in February 2008 with respect to the spreads one year before. Mining/Contr. includes firms in the Mining and Construction businesses. 3 Yr, 5 Yr, and 7 Yr indicate the three, five and seven year maturities respectively.

Table 7: Cross-sectional determinants of implied recovery

	Spec. 1	Spec. 2	Spec. 3	Spec. 4	Spec. 5
Constant	**0.368**	**0.322**	**0.492**	**0.336**	**0.311**
	0.022	0.070	0.055	0.029	0.027
Income/Sales	**0.301**	**0.279**	**0.318**	**0.363**	**0.373**
	0.046	0.055	0.047	0.066	0.061
Tangibility	-0.059	-0.041	-0.016	-0.063	-0.088
	0.038	0.048	0.042	0.042	0.051
LEV	**-0.216**	**-0.219**	**-0.312**	**-0.266**	**-0.276**
	0.040	0.041	0.053	0.055	0.051
Size		0.005			
		0.007			
Q-ratio			-0.071		
			0.059		
FIV				-0.116	
				0.068	
Firm Ret.				0.014	0.003
				0.019	0.021
Hvol					-0.233
					0.169
Adj. R^2	12.1%	12.0%	13.9%	12.2%	13.8%
Nos	544	544	544	501	502

Notes: The dependent variable is the five year implied recovery rate for the senior contract computed as the average over the fiscal quarter for each firm. The table reports the estimated coefficient and the corresponding heteroscedasticity-consistent standard errors. The coefficients that are significant at 5% level are in bold. Income/Sales is defined as the quarterly operating income before depreciation and taxes divide by quarterly sales. Tangibility is defined as the property, plant and equipment divide by total book value of assets. LEV indicates leverage defined as the ratio of long term debt to the sum of long term debt and market value of equity. Q-ratio is defined as the ratio of market value of assets to book value of assets. Size is defined as the log of book value of assets. FIV is the 30 day put option implied volatility. Firm Ret. is the trailing one year firm specific return. Hvol is the annualized historical volatility computed as the standard deviation of returns over the previous one year.