The Effects of Auditor Tenure on Fraud and Its Detection

Evelyn R. Patterson
Kelley School of Business
Indiana University
evpatter@iupui.edu
Phone: (317)278-7843
Fax: (317) 274-3312

J. Reed Smith*
Kelley School of Business
Indiana University
jrsmith2@iupui.edu
Phone: (317)274-0867
Fax: (317) 274-3312

Samuel L. Tiras
Kelley School of Business
Indiana University
stiras@iu.edu
Phone: (317)274-3420
Fax: (317) 274-3312

February 2018

We would like to thank the participants at the AAA Auditing Section Mid-Year meeting and the AAA National Meeting along with Amir Michael, Paul Newman and workshop participants at the Rotman School of Management, University of Toronto for their helpful comments and discussions. We are also appreciative of the financial support given by the Kelley School of Business, Indiana University through the summer research grant program.

*Corresponding author

The Effects of Auditor Tenure on Fraud and Its Detection

Abstract

We examine the strategic effects of auditor tenure on the auditor's testing strategy and the manager's inclination to commit fraud. Most empirical studies find that longer audit tenure improves audit quality. Proponents of restricting auditor tenure express concern that longer tenures impair auditor independence and a 'fresh look' from a new auditor results in higher audit quality. Validating this concern requires testing whether the observed difference in audit quality between a continuing auditor and a change in auditor is less than the expected difference in audit quality. The findings of our model provide the theoretical guidance necessary for developing such tests. Our results show that audit risk (probability that fraud goes undetected) is lower in both periods for the continuing auditor than with a change in auditor. More importantly, we show that across both periods, expected undetected fraud is lower for the continuing auditor than with a change in auditor.

I. INTRODUCTION

Security market regulators have long debated the potential effects of firms hiring the same auditor over multiple periods. On one hand, some believe that as the length of time an auditor stays with a firm increases, the auditor becomes 'too cozy' with the company. The concern is that this familiarity may reduce the auditor's skepticism about potential misstatements and, in turn, reduce the auditor's effectiveness in detecting misstatements. On the other hand, some argue that auditing a firm over multiple periods increases the auditor's knowledge of the company's operations and system processes, thereby increasing his effectiveness in detecting misstatements. Notable frauds, such as *Equity Funding*, *Comptronix,* and *Crazy Eddie* that have occurred over several years, seem to support the first conjecture.

Interpreting these observable outcomes might be misleading without first considering the underlying economics of how multi-period audits affect the auditor's choice of effort and its related impact on deterring and detecting fraud. The *observation* of an audit failure provides limited evidence regarding the (often unobservable) action choices of the auditor and manager that *together* determine the risk of undetected misstatements. The purpose of this paper, therefore, is to examine the strategic effects of multi-period audits, where the accumulation of audit evidence over time affects the auditor's testing strategy and the manager's inclination to commit fraud.

Proponents of restricting auditor tenure suggest that a 'fresh look' at a company by an auditor would result in higher quality audits. The concern expressed in PCAOB CONCEPT RELEASE (PCR) No. 2011-006: *Auditor Independence and Audit Firm Rotation* is that over time, the auditor can lose his ability to exercise *professional skepticism* despite adherence to the rules of independence. For regulators, practitioners, or accounting researchers to assess whether externalities exist that could potentially be attributable to an impairment in independence, the first step would be to assess the equilibrium strategies of a continuing auditor versus a change in auditor strictly from the players maximizing their expected payoffs, where none of the payoffs violate the independence rules. Our model provides such a baseline.

Defond and Zhang (2014) point out that most studies find that longer audit tenure improves audit quality. Examples of such evidence include findings that longer audit tenure is associated with lower levels of discretionary accruals (Johnson, Khurana, and Reynolds 2002; Myers, Myers, and Omer 2003), higher earnings response coefficients (Ghosh and Moon 2005), and more issuances of going concern opinions (Louwers 1998; Knechel and Vanstraelen 2007). More directly, Cameran, Francis, Marra, and Pettinicchio (2015) document that earnings quality improves with longer audit tenure in Italy, where auditor rotation has been mandated since 1975. Further, Reid and Carcello (2017) document that the market perceives audit quality to be higher with longer audit tenure.

The challenge in interpreting these findings as evidence that longer audit tenure does not impair independence is that these studies are not able to calibrate the extent that audit quality would be expected to be higher with a continuing auditor, had there not been any impairment of independence. If there is indeed an impairment of independence, as is the concern expressed by the PCAOB and others, empirical researchers would be able to document such impairments only by testing whether the observed difference in audit quality between a continuing auditor and a change in auditor is less than the expected difference in audit quality. The equilibrium findings of our model provide the theoretical guidance necessary for developing such tests.

Our study investigates a two-period audit across two settings. In the first setting, we have a change in auditors from the first period to the second. In the second setting, one auditor continues auditing from one period to the next. In both settings, the auditor chooses audit effort while the manager chooses an amount of fraud, given he is the 'dishonest' manager type, where the players in the two settings possess the same payoff parameters. The prior probability that the manager is dishonest is common knowledge, and equal to the risk of fraud that the manager is dishonest in the period 1. Furthermore, the game continues to period 2 only if fraud is not discovered at the end of period 1.

In both settings, the auditor in period 2 updates the probability that the manager is dishonest, based on period 1 audit effort and the public knowledge that no fraud was discovered in period 1. For the continuing auditor, he has direct knowledge of the audit testing performed in period 1. For the 'new' auditor in the change setting, his knowledge of period 1 audit effort is perfectly inferred, because of our simplifying assumption that the period 1 'old' auditor's costs and the manager's payoffs are common knowledge.[1]

The key difference between the two settings is that the continuing auditor not only has information acquired in period 1 to use in the period 2 audit, but he chooses the amount of period 1 audit effort to control expected audit costs across *both* periods. At the beginning of period 1, the continuing auditor plans how he will audit in each period in order to minimize his total expected costs, taking into consideration how he anticipates the manager will react to each period's effort choice. In contrast to when there is a change in auditors, the old auditor has no incentive to consider the effect of his period 1 effort choice on period 2 outcomes and costs.[2]

Overall, our results show that audit risk, which is the probability that fraud goes undetected, is lower in both periods for the continuing auditor than with a change in auditor. More importantly, our results also show that across both periods, expected undetected fraud is lower for the continuing auditor than with a change in auditor. However, we also find that the continuing auditor chooses less audit effort in the later period, relative to the effort choice of the new auditor

[1] In practice, the new auditor may not be able to perfectly infer period 1 audit effort, because he has only limited information of the manager's and old auditor's payoffs in period 1. However, this simplifying assumption biases against our results and highlights the differences in equilibria between the two settings.

[2] He also has no ability to do so, because at the beginning of period 1 the old auditor only knows that his engagement with the firm ends and has no knowledge of who the new auditor would be.

in the change setting. This result derives from the continuing auditor allocating more effort in period 1 than would be allocated with a change in auditors. While the appearance that a continuing auditor choosing less audit effort in the later period than does the new auditor might suggest an impairment of independence, this results strictly from the players maximizing their expected payoffs and not due the continuing auditor becoming 'too cozy' with the client.

This paper significantly adds to our knowledge of strategic auditing. Most strategic auditing models primarily involve just one period and do not take into account how the auditor might strategically use information from previous audit periods in the current period. Some models, such as Patterson and Smith (2007, 2016) and Smith, Tiras, Vichitlekarn (2000) do consider two-stage audits that allow the auditor to obtain information by testing internal controls in a first stage and then use that information for substantive testing in the second stage.[3] Our *two-period* model differs from these two-stage models in that the effort in each period of our two-period model *is* designed to detect fraud. Because we have two distinct periods, we can compare the two settings: a continuing auditor that audits both periods and a setting in which the audit in each period is performed by a different auditor.

Corona and Randhawa (CR) (2010) also examine a two-period strategic audit setting, in which the manager in each period chooses fraud or no fraud and the auditor chooses an audit report. They demonstrate a circumstance in which an auditor that does not identify fraud in one period might not report fraud that is detected in a later period because it would highlight their failure in the previous period. By construction, the auditor in their model does indeed lack independence. But because their model does not provide any penalties or other disincentives for

---

[3] Other related studies include Newman, Patterson, and Smith (2001), Patterson and Smith (2003), Patterson and Noel (2003), and more recently, Laux and Newman (2008) and Patterson, Smith, and Tiras (2017).

the auditor to hide the fraud, their model cannot address the potential benefit to multi-period auditing.

The remainder of the paper is structured as follows. In section II, we lay out the two settings of our two-period model. Section III provides an equilibrium analysis while section IV provides a comparative analysis of both settings. Section V compares expected undetected fraud and audit risk across the two settings and Section VI relates our results to empirical insights. Section VII concludes the paper.

II. A TWO-PERIOD MODEL OF AUDITING

We consider a two-period model of auditing that includes an auditor and a possibly dishonest manager in two settings. The first setting depicts a situation where the 'old' auditor of period 1 is replaced with a 'new' auditor in period 2. In the second setting, one 'continuing' auditor audits both periods and chooses audit effort in the first period to minimize his expected audit costs across *both* periods.

Public information is revealed at the end of period 1 in both settings. At the end of period 1 the players know whether or not the auditor discovered fraud in period 1. If fraud is not detected, the game continues to period 2; otherwise, it ends at the end of period 1. The updated probability that the manager is dishonest depends upon the auditor's effort in period 1. In the second (continuing) setting, that updated probability is affected by that same auditor's effort choice in the first period.

In both settings, the auditor(s) and manager have the same basic payoffs while the *expected* payoffs for the auditor in periods 1 and 2 depend on whether or not there has been a change in auditors. The manager is dishonest with prior probability θ. The dishonest manager chooses an amount of fraud equal to a_1 and a_2 in periods 1 and 2, respectively. He receives net benefits of

a_1R_1 and a_2R_2 if no fraud is detected by the auditor, and his penalty for fraud detection is $-\frac{a_1^2}{2}p_1$ and $-\frac{a_2^2}{2}p_2$. The payoffs a_1R_1 and a_2R_2 represent the benefits of fraud less personal costs for committing fraud such as the manager's designs to circumvent the audit strategy. The rewards and penalties for periods 1 and 2 are independent across both periods.

The auditor in period 1 chooses audit effort equal to x_1 and the auditor in period 2 chooses audit effort equal to x_2. Given the manager is dishonest, the auditor detects fraud in periods 1 and 2 with probabilities $1-\exp(-x_1)$ and $1-\exp(-x_2)$, respectively. The auditor incurs a liability cost in each period for undetected fraud of a_1L_1 and a_2L_2, while k_1 and k_2 are cost multipliers for audit effort that yield a cost of audit effort in each period of x_1k_1 and x_2k_2.

The Manager's Expected Payoff

We work backwards to derive the manager's expected payoff by starting with period 2 and then incorporating it into period 1. The form of the manager's expected payoff does not change between settings. It remains the same whether or not a change in auditors occurs at the end of period 1. His fraud strategy can differ based upon which setting we analyze but the basic construct of his expected payoff remains the same.

The manager's period 2 expected payoff is as follows.

$$M_2 = a_1R_1 + a_2R_2\exp(-x_2) - \frac{a_2^2}{2}p_2\left(1-\exp(-x_2)\right) \tag{1}$$

M_2 is the dishonest manager's expected payoff in period 2, when a period 1 fraud goes undetected. Note that the reward from period 1 a_1R_1 is retained in period 2. If fraud in period 2 is detected, we assume that the manager has already consumed the benefits of any undetected fraud committed in period 1. We assume that the penalty for detected fraud in period 2 is independent of any undetected fraud in period 1.

The dishonest manager's expected payoff at the beginning of period 1 is as follows.

$$M_1 = M_2 \exp(-x_1) - \frac{a_1^2}{2} p_1 \left(1 - \exp(-x_1)\right) \tag{2}$$

M_1 reflects the fact that any reward from fraud over the two periods occurs only if the period 1 fraud goes undetected. Thus, if the period 1 fraud goes undetected, which occurs with probability $\exp(-x_1)$, the manager expects a payoff associated with M_2. On the other hand, if the auditor detects the period 1 fraud, the manager receives zero benefit from fraud and is penalized $\frac{a_1^2}{2} p_1$.

Next we consider the auditor's expected payoffs where its characterization depends on whether or not a change in auditors has occurred at the end of period 1.

The Auditor's Expected Payoff

Setting 1: A Change in Auditors

Assume there is a change in auditors at the end of period 1 that everyone knows will occur at the start of the game. In this case, each auditor maximizes his own expected payoff, the new auditor for period 2 and the old auditor for period 1. Except for the undiscovered fraud in the first period, no public information from the first period is accrued by the new auditor in period 2. However, the second period new auditor can infer the effort choice of the old auditor because we assume that the new auditor knows the payoff parameters of the manager and the old auditor.[4] Thus, if no fraud is detected in the first period, the new auditor updates the probability that the manager is dishonest given the inferred choice of period 1 audit effort.

[4] Details of the equilibrium that excludes knowledge regarding the old auditor payoffs are available from the authors upon request.

In order to compare the two settings, we must assume that the expected liability costs resulting from undetected fraud in period 1 are independent from the expected liability costs resulting from undetected fraud in period 2. In other words, we ignore the impact of detection probability in period 2 on the costs facing the period 1 auditor (in either setting). We assume that the likelihood that undetected fraud in period 1 is subsequently brought to light in the future is exogenously captured by the parameter L_1.

Below we express the expected payoffs in periods 1 and 2 for the new $\left(A_{2new}\right)$ and old $\left(A_{1old}\right)$ auditors.

New auditor – second period

$$A_{2new} = -\Pr\left(D \mid ND\right) a_2 L_2 \exp\left(-x_2\right) - x_2 k_2 \tag{3}$$

where $\Pr\left(D \mid ND\right) = \dfrac{\theta \exp\left(-x_1\right)}{\theta \exp\left(-x_1\right) + \left(1-\theta\right)}$ is the probability of the dishonest type manager (*D*), given that fraud was not discovered (*ND*) in period 1.

The expected payoff for the predecessor auditor follows.

Old auditor – first period

$$A_{1old} = -\theta a_1 L_1 \exp\left(-x_1\right) - x_1 k_1 \tag{4}$$

The sequence of events for setting 1 is given in Figure 1.

Setting 2: A Continuing Auditor

Next we assume that the same auditor audits both period 1 and period 2. We start at the end of the game in formulating the auditor's total expected payoff. The continuing auditor incurs all the liability and effort costs for periods 1 and 2. The auditor's second period payoff is

$$A_{2cont} = -\Pr\left(D \mid ND\right)\left(L_1 a_1 + L_2 a_2 \exp\left(-x_2\right)\right) - x_2 k_2 \tag{5}$$

where $\Pr(D \mid ND) = \dfrac{\theta\exp(-x_1)}{\theta\exp(-x_1)+(1-\theta)}$ is the probability of the dishonest type, given no fraud is detected in period 1.[5] The continuing auditor maintains his expected loss $L_1 a_1$ for an undiscovered first period fraud whether or not fraud is discovered in the second period. Moreover, because the auditor audits both periods, he can choose x_1 and x_2 to minimize his costs across both periods and he can also use the amount of period 1 audit effort to infer the likelihood that the manager is the dishonest type if no fraud is discovered in period 1. A key advantage of auditing both periods is the ability to choose x_1 that provides the best benefits in choosing x_2, including an x_1 that will provide optimal updating of information for period 2.

Then we have the auditor's first period expected payoff of

$$A_{1cont} = \left(\theta\exp(-x_1)+(1-\theta)\right)A_{2cont} - x_1 k_1 \tag{6}$$

where $\theta\exp(-x_1)+(1-\theta)$ is the unconditional probability that no fraud is detected in period 1, given the choice of audit effort x_1.

In order to compare the two settings, it is important that the payoffs are the same across the two settings except that the auditor in the continuing setting is able to optimize his choices of x_1 and x_2 across time because he audits both periods. It is important, then, that the expected payoff to the continuing auditor in (6) (the total expected payoff over the two periods) be equal to the expected payoffs of the *old* and *new* auditors in the change setting as long as x_1, x_2, a_1, and a_2 are the same. To see that this is the case, we can add the expected payoff in (3) to the expected payoff in (4) multiplied by the probability that the game proceeds to the second period and we

[5] In this expression and all those that follow, 'cont' in the subscript is short for 'continuing.'

obtain expression (6). The only differences between the two settings, then, derive from differences in the auditor and manager strategies.

The sequence of events for setting 2 is given in Figure 2. Our next section will describe our equilibria for the two settings.

III. EQUILIBRIUM ANALYSIS

Setting 1: A Change in Auditors

With a change in auditors, we use expressions (1) – (4) to solve for our equilibrium. First note that the manager's expected payoff at the beginning of the game is

$$M_1=\left(a_1R_1+a_2R_2\exp(-x_2)-\frac{a_2^2}{2}p_2\left(1-\exp(-x_2)\right)\right)\exp(-x_1)-\frac{a_1^2}{2}p_1\left(1-\exp(-x_1)\right) \tag{7}$$

which yields first order conditions

$$\frac{dM_1}{da_1}=R_1\exp(-x_1)-a_1p_1\left(1-\exp(-x_1)\right)=0 \tag{8}$$

and

$$\frac{dM_1}{da_2}=\left(R_2\exp(-x_2)-a_2p_2\left(1-\exp(-x_2)\right)\right)\exp(-x_1)=0\,. \tag{9}$$

Conditions (8) and (9) yield fraud choices of a_1 and a_2, respectively.

$$a_1=\frac{R_1\exp(-x_1)}{p_1\left(1-\exp(-x_1)\right)}\quad\text{and}\quad a_2=\frac{R_2\exp(-x_2)}{p_2\left(1-\exp(-x_2)\right)} \tag{10}$$

Expression (10) says that the manager selects the amount of fraud equal to the ratio of its expected marginal benefit to its expected marginal penalty. Thus, as audit effort increases the relative expected benefit of fraud decreases.

The auditor's first order conditions follow from expressions (3) and (4).

$$\frac{dA_{1old}}{dx_1} = \theta a_1 L_1 \exp(-x_1) - k_1 = 0 \tag{11}$$

$$\frac{dA_{2new}}{dx_2} = \Pr(D \mid ND) a_2 L_2 \exp(-x_2) - k_2 = 0 \tag{12}$$

Conditions (11) and (12) yield effort choices of x_1 and x_2, respectively.

$$x_1 = Log\left[\frac{a_1 L_1}{k_1}\theta\right] \text{ and } x_2 = Log\left[\frac{a_2 L_2}{k_2}\frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1-\theta)}\right] \tag{13}$$

We use expressions (10) and (13) to obtain our equilibrium, which is stated in Proposition 1.

Proposition 1: Given a change in auditors, the unique equilibrium strategies for the manager, the old auditor and the new auditor are as follows.

Manager:

$$a_1 = \frac{R_1}{p_1\left(-1 + \frac{1}{2}\left(1 + \sqrt{1 + \frac{4L_1R_1}{k_1p_1}\theta}\right)\right)} \quad \text{and}$$

$$a_2 = \frac{R_2}{p_2\left(-1 + \frac{1}{2}\left(1 + \sqrt{1 + \frac{4L_2R_2}{k_2p_2}\frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1-\theta)}}\right)\right)}$$

Auditor$_{old}$

$$x_1 = Log\left(\frac{1}{2}\left(1 + \sqrt{1 + \frac{4\theta R_1 L_1}{p_1 k_1}}\right)\right)$$

Auditor$_{new}$

$$x_2 = Log\left(\frac{1}{2}\left(1 + \sqrt{1 + \frac{4R_2L_2}{p_2k_2}\frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1-\theta)}}\right)\right)$$

(*All proofs are in the Appendix*)

This provides a benchmark to compare the equilibrium with a continuous auditor.

Setting 2: A Continuing Auditor

Next we derive the equilibrium when the auditor continues as the auditor of period 2 after auditing period 1. The characterization of the fraud amounts in expression (10) remains the same in this setting where

$$a_1 = \frac{R_1 \exp(-x_1)}{p_1(1-\exp(-x_1))} \quad \text{and} \quad a_2 = \frac{R_2 \exp(-x_2)}{p_2(1-\exp(-x_2))}.$$

In this setting, we first solve for period 2 audit effort and then period 1 audit effort because information has been revealed between periods.[6] When no fraud is detected in period 1, the auditor uses his period 1 information to reassess the probability that the manager is dishonest and knowing this is the case chooses his effort in period 1, accordingly, to minimize his total expected costs across both periods. Based on his period 2 expected payoff in expression (5), the auditor's period 2 first order condition is equal to the following.

$$\frac{dA_{2cont}}{dx_2} = \Pr(D \mid ND)\left(L_2 a_2 \exp[-x_2]\right) - k_2 \Rightarrow \tag{14}$$

$$x_2 = Log\left[\frac{a_2 L_2}{k_2}\frac{\theta \exp(-x_1)}{\theta \exp(-x_1)+(1-\theta)}\right] = Log\left[\frac{L_2 a_2}{k_2}\frac{\theta}{\theta \exp[-x_1]+(1-\theta)}\right] - x_1$$

Then we substitute for x_2 into A_1 and with some rearranging we obtain the following.

$$A_{1cont} = \begin{pmatrix} -L_1 a_1 \theta \exp(-x_1) \\ \quad -\left(\theta \exp(-x_1)+(1-\theta)\right)k_2\left(1 - x_1 + Log\left[\frac{L_2 a_2}{k_2}\frac{\theta}{\exp(-x_1)+(1-\theta)}\right]\right) \end{pmatrix} - x_1 k_1 \tag{15}$$

which we use to obtain the first order condition for x_1.

---

[6] The equilibrium, given a continuing auditor, is technically defined implicitly based on four equilibrium conditions. Due to the nature of our payoffs a fully explicit solution is not possible. However for convenience and to enable us to more intuitively compare the equilibrium to one with a change in auditors, we define each of the players strategies as functions of the auditor's period 1 effort choice x_1.

$$\frac{dA_{1cont}}{dx_1} =$$

$$\theta \exp(-x_1)\left(L_1 a_1 + k_2\left(1 - x_1 + Log\left[\frac{L_2 a_2}{k_2}\frac{\theta}{\theta \exp(-x_1) + (1-\theta)}\right]\right)\right) + (1-\theta)k_2 - k_1 = 0 \tag{16}$$

Next we substitute for a_2 into x_2 and solve for x_2.

$$x_2 = Log\left[\frac{1}{2}\left(1 + \sqrt{1 + \frac{4L_2R_2}{k_2p_2}\frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1-\theta)}}\right)\right], \tag{17}$$

which we need to obtain a_2.

$$a_2 = \frac{R_2}{p_2\left(-1 + \frac{1}{2}\left(1 + \sqrt{1 + \frac{4L_2R_2}{k_2p_2}\frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1-\theta)}}\right)\right)} \tag{18}$$

The *form* of the equilibrium strategies for a_2 and x_2 in the continuous case is the same as those for the new auditor, given a change in auditors. However, the *values* are different due to the differing audit effort strategies in period 1 for the two settings.

Unlike the old auditor who has no ability and no incentive to control or minimize period 2 audit costs, the continuing auditor chooses his first period audit effort knowing that he will face a possibly dishonest manager in period 2 if no fraud is detected in period 1. Based on expression (16), we have the equilibrium condition for x_1, labeled $H[x_1]$.

$$H[x_1] = \theta \exp(-x_1)\left(L_1 a_1[x_1] + k_2 + k_2 x_2[x_1]\right) + (1-\theta)k_2 - k_1 = 0 \tag{19}$$

where $x_2[x_1]$ is the value of x_2 derived in expression (17) and $a_1[x_1] = \dfrac{R_1 \exp(-x_1)}{p_1(1 - \exp(-x_1))}$.

Expression (19) implicitly defines the equilibrium value of x_1 because we cannot explicitly solve for x_1. Furthermore, each of our equilibrium strategies are defined in terms of x_1.

Proposition 2: When the auditor continues from period 1 to period 2 the unique equilibrium strategies for the manager and auditor are as follows.

Manager

$$a_1 = \frac{R_1 \exp(-x_1)}{p_1(1-\exp(-x_1))} \quad \text{and}$$

$$a_2 = \frac{R_2}{p_2\left(-1+\frac{1}{2}\left(1+\sqrt{1+\frac{4L_2R_2}{k_2p_2}\frac{\theta\exp(-x_1)}{\theta\exp(-x_1)+(1-\theta)}}\right)\right)}$$

Auditor

$$x_1 = Log\left[\frac{\theta\left(L_1a_1[x_1]+k_2(1+x_2[x_1])\right)}{k_1-(1-\theta)k_2}\right] \quad \text{and}$$

$$x_2 = Log\left[\frac{1}{2}\left(1+\sqrt{1+\frac{4L_2R_2}{k_2p_2}\frac{\theta\exp(-x_1)}{\theta\exp(-x_1)+(1-\theta)}}\right)\right]$$

where x_1 satisfies $H[x_1]=0$.

The equilibrium defined in Proposition 2 has several interesting characteristics. First, we see that x_1 and x_2 are inversely related, all else held constant. In Proposition 2, x_2 decreases in x_1 without regard to a change in payoff parameters. If no fraud is discovered in period 1 for a high x_1 then the probability that the manager is the dishonest type is smaller and the auditor may find a reduction in x_2 to be optimal. Of course, the equilibrium relation between x_1 and x_2 must be considered with regard to a change in a payoff parameter. We explore this more thoroughly in the next section that provides a comparative analysis.

Second, expression (19) that implicitly defines the equilibrium value of x_1 shows that the first period audit effort x_1 is greater than what would occur in period 1 when we have a change in auditors. Note that expression (19) can be written as

$$H[x_1] = H_{1period}[x_1] + \theta \exp(-x_1)(k_2 + k_2 x_2[x_1]) + (1-\theta)k_2 = 0 \tag{20}$$

where $H_{1period}[x_1] = \theta \exp(-x_1) L_1 a_1[x_1] - k_1$ is the equilibrium condition for a single period game or alternatively, for the old auditor in period 1. As part of our proof to Proposition 2, we show that expression (19) decreases in x_1 and expression (20) shows that point-wise it has larger values than that for a single period game. Thus, the continuous auditor chooses greater period 1 audit effort than the auditor who knows he is leaving at the end of period 1, ceteris paribus. As a result, the amount of fraud committed in the first year is less for a continuing auditor relative to an auditor who only audits period 1.

Finding fraud sooner rather than later is more valuable to the continuing auditor, which translates into a smaller amount of period 2 effort relative to the new auditor. (See Corollary 1). Despite the new auditor's ability to update based on the inferred period 1 audit effort, the continuing auditor chooses lower audit effort in period 2 because period 1 audit effort is higher and he estimates a lower probability of fraud going into period 2. Due to the lower assessment of the probability of fraud for period 2, one might infer that there has been a reduction in skepticism. Furthermore, the loss of skepticism *appears* to be more for the continuing auditor versus the new auditor. However, the lower probability assessment of fraud for the continuing auditor at the beginning of the second period is solely due to his choice of higher audit effort in the first period and the fact that fraud was not detected.

Third, we see that allowing the auditor to continue auditing in period 2 effectively reduces the marginal cost of his period 1 audit effort by an amount equal to $(1-\theta)k_2$. As the prior probability that the manager is the honest type increases, the continuing auditor's first period marginal cost of audit effort decreases. And, in the limit as $k_2 \rightarrow \frac{k_1}{1-\theta}$, audit effort approaches 'perfect auditing' in period 1.

These results, which relate to comparisons within each period are formally presented in Corollary 1.

Corollary 1: For the continuing auditor relative to a change in auditors:

1. The amount of period 1 fraud is smaller and the amount of period 1 audit effort is larger.

2. The amount of period 2 fraud is larger and the amount of period 2 audit effort is smaller.

Next we perform a comparative analysis on our equilibrium results, given various changes in payoff parameters.

IV. COMPARATIVE ANAYSIS OF CHANGES IN THE AUDITOR'S AND MANAGER'S PAYOFFS

In this section, we consider how the players' strategies change when the payoff parameters change. For setting 1 (the change setting), the auditor's and manager's period 1 strategies (x_1 and a_1 respectively) depend only upon the period 1 payoff parameters. But, because the auditor can update the likelihood of the manager's dishonesty based upon the first period outcome, the second period strategies for the auditor and manager depend upon period 1 parameters. For setting 2 (the continuing setting), all four strategies are jointly determined because the manager's first period fraud choice is determined by the auditor's first period effort choice. And, the auditor's first period effort choice is jointly determined with the auditor's *second* period effort choice. This is demonstrated by the equilibrium condition for x_1 provided in expression (19).

Proposition 3: The following table demonstrates how changes in the parameters, L_i, R_i, p_i, and k_i for $i \in \{1,2\}$ affect the auditor's effort choice in periods 1 and 2 and the manager's fraud choice in periods 1 and 2 for the continuing and change settings. With the exception of the effects of changes in period 2 payoff parameters on period 1 strategies (the shaded cells), the comparative results are the same for both settings. The period 2 payoff parameters do not affect the auditor's or manager's period 1 strategies in the change setting.

	Effect on strategy					
Increase in Period 1 payoff parameters	x_1 for continuing and change settings		α_1 for continuing and change settings		x_2 for continuing and change settings	α_2 for continuing and change settings
L_1	+		−		−	+
R_1	+		+		−	−
p_1	−		−		+	+
k_1	−		+		+	−
Increase in Period 2 payoff parameters	x_1 for continuing setting	x_1 for change setting	α_1 for continuing setting	α_1 for change setting	x_2 for continuing and change settings	α_2 for continuing and change settings
L_2	+	0	−	0	+	−
R_2	+	0	−	0	+	+
p_2	−	0	+	0	−	−
k_2	+	0	−	0	−	+

(proof in the Appendix)

The above table is constructed as four quadrants. The top-left quadrant relates to the effects that changes in period 1 payoff parameters have on the period 1 auditor and manager strategies. Similarly, the bottom-right quadrant relates to how changes in period 2 payoff parameters affect period 2 auditor and manager strategies. Note that these two quadrants are identical. That is because changes in the payoff parameters *within* a given period induce the intuitive changes in the auditor's and manager's strategies. Increases in the auditor's litigation exposure in a given period (L_i) increases audit effort and decreases fraud in that period whereas increases in the auditor's cost parameter for auditing in a given period (k_i) decreases audit effort and increases fraud in that period. On the other hand, the increases in the manager's benefit parameter in a given period (R_i) increase fraud and audit effort in that period and increases in the manager's penalty parameter in that period (p_i) decrease fraud and audit effort in that period. These results are true across both settings.

Next consider the top-right quadrant of the table. Notice that every effect is the exact *opposite* of the effect in the top-left quadrant. That is because any *increase* in a period 1 strategy will be traded off for a *decrease* in a period 2 strategy and vice versa. This is true for both settings because in both settings, the period 2 auditor updates the probability that the manager is dishonest based on the fact that the period 1 auditor did not detect fraud. Of course, since the continuing auditor exerts more audit effort in period 1, his updating places a higher probability that the manager is honest than the new auditor in the change setting.

The bottom left quadrant of the table illustrates the differential impact of the continuing auditor on auditing and fraud. In the continuing auditor setting, the auditor's payoffs and strategies are tied across the two periods. The auditor is allocating effort optimally across the two periods in order to minimize the overall costs of undetected fraud. The old auditor in the change setting is incapable of this strategic choice. As a result, the period 2 payoffs, which affect the period 2 strategies do not affect the period 1 strategies for the auditor or the manager in the change setting.

The period 1 strategies in the change setting (see Proposition 1) do not involve the period 2 parameters. So changes in these parameters do not affect period 1 strategies of the auditor or manager in the change setting. For the continuing auditor setting, the period 1 strategy for the auditor is tied to the period 2 strategies for all parameters, except k_2. With the exception of k_2, increases in the incentives for audit effort in period 2 also drive increases in the incentives for audit effort in period 1.

For L_2, R_2, and p_2, we assess changes in x_1 by using the equilibrium condition, $H[x_1]=0$ in expression (19).

$$H[x_1]=\theta\exp(-x_1)\left(L_1a_1[x_1]+k_2+k_2x_2[x_1]\right)+(1-\theta)k_2-k_1=0$$

Because $\frac{dH[x_1]}{dx_1}<0$, we know that the sign of $\frac{dx_1}{dg}=\frac{dH[x_1]/dg}{-dH[x_1]/dx_1}$ depends only on the sign of $dH[x_1]/dg$ where *g* stands for a general payoff parameter. Consequently, as an example, $\frac{dx_1}{dR_2}>0$ because $\frac{dH[x_1]}{dR_2}>0$ where $\frac{dx_2[x_1]}{dR_2}>0$. Moreover, $\frac{da_1}{dR_2}=\frac{\partial a_1}{\partial R_2}+\frac{\partial a_1}{\partial x_1}\frac{dx_1}{dR_2}<0$ where $\frac{\partial a_1}{\partial R_2}=0$, $\frac{\partial a_1}{\partial x_1}<0$ and $\frac{dx_1}{dR_2}>0$. The results for $\frac{dx_1}{dL_2}$ and $\frac{dx_1}{dp_2}$ follow the same logic.

For changes in k_2, $\frac{dx_1}{dk_2}=\frac{dH[x_1]/dk_2}{-dH[x_1]/dx_1}>0$ where $\frac{dH[x_1]}{dk_2}>0$. and $\frac{da_1}{dk_2}=\frac{\partial a_1}{\partial k_2}+\frac{\partial a_1}{\partial x_1}\frac{dx_1}{dk_2}<0$ where $\frac{\partial a_1}{\partial x_1}<0$ and $\frac{dx_1}{dk_2}>0$. Effectively, an increase in k_2 reduces the marginal cost of period 1 audit effort.

Figure 3 depicts a graphical example of audit effort and fraud in *each* of the two periods, for a change in auditors versus a continuing auditor and illustrates the results of Corollary 1. Audit effort is higher in period 1 and lower in period 2 while fraud is lower in period 1 and higher in period 2 for the continuing auditor relative to a change in auditors. The figure also illustrates the results in Proposition 3 that relate to changes in period 2's audit cost k_2. Period 2 audit effort decreases in k_2 and period 2 fraud increases in k_2 for *both* settings. On the other hand, period 1 audit effort *increases* in k_2 for the continuing auditor but it is unaffected by changes in k_2 for the change auditor. The continuing auditor, knowing that he goes on to audit period 2, audits more aggressively in period 1 and therefore chooses a higher period 1 effort than the change auditor. The dishonest manager responds to these audit effort effects by decreasing period 1 fraud in the continuing auditor setting, but changes in k_2 have no effect on fraud in period 1 in the change setting.

The next section shows that audit quality is always higher for continuing auditor, where we measure audit quality either expected undetected fraud or audit risk

V. EXPECTED UNDETECTED FRAUD AND AUDIT RISK

We begin our analysis of audit quality by first comparing the two settings in terms of expected undetected fraud.

Expected Undetected Fraud

Expected undetected fraud is measured as the probability that fraud occurs, multiplied by the probability that fraud is not detected (given that a fraud has occurred), multiplied by the amount of fraud. As this measure decreases, audit quality increases.

Setting 1: A Change in Auditors

For setting 1 where we have a change in auditors, expected undetected fraud (*EUF)* over two periods simplifies to

$$\theta a_1 \exp\left(-x_1\right) + \Pr\left(D \mid ND\right) a_2 \exp\left(-x_2\right) =$$

$$EUF_{change} = \frac{k_1}{L_1} + \frac{k_2}{L_2}\,. \tag{21}$$

Setting 2: A Continuing Auditor

Expected undetected fraud in setting 2 is equal to

$$\theta a_1 \exp(-x_1) + \Pr(D \mid ND) a_2 \exp(-x_2) =$$

$$EUF_{cont} = \frac{k_1 - (1-\theta)k_2}{L_1 + \frac{k_2}{a_1\theta}(1+x_2)} + \frac{k_2}{L_2} \tag{22}$$

We see that $EUF_{cont} < EUF_{change}$, which we formalize as Proposition 4, because clearly

$$\frac{k_1 - (1-\theta)k_2}{L_1 + \frac{k_2}{a_1\theta}(1+x_2)} < \frac{k_1}{L_1}.$$

> **Proposition 4:** Expected undetected fraud over the two periods, given a continuing auditor is strictly less than that given a change in auditors.

Despite the 'fresh look' adopted by the new auditor in period 2, the strategic aspects of a continuing auditor provide benefits that are not achieved by a change in auditor. The continuing auditor chooses period 1 effort that anticipates the manager's strategy over the two periods and thus has the ability to minimize his total expected costs. Moreover, when a change in auditors is known to occur, the old auditor is only interested in how his strategic choice affects his own period 1 expected costs.

Figure 4 depicts a graphical example illustrating Proposition 4. While the comparison is valid for *all* parameters, we demonstrate the relationship between EUF_{cont} and EUF_{change} in terms of the auditor's liability parameter in period 2, L_2. Note that $EUF_{cont} < EUF_{change}$ for all values of L_2 and a change in L_2 affects EUF_{cont} to a relatively greater extent.

Audit Risk

Audit risk is the probability that the financial statements contain undetected fraud. Furthermore, when we consider audit risk, we compare audit risk for each individual audit period across the two settings.

Setting 1: A change in Auditors

When there is a change in auditors, audit risk in period 1 $\left(AR_{1old}\right)$ is

$$AR_{1old} = \theta \exp\left(-x_1\right)$$

and in period 2 audit risk $\left(AR_{2new}\right)$ equals

$$AR_{2new} = \frac{\theta \exp\left(-x_1\right)}{\theta \exp\left(-x_1\right) + \left(1-\theta\right)} \exp\left(-x_2\right)$$

where x_1 and x_2 are from Proposition 1.

Setting 2: A Continuing Auditor

When there is a continuing auditor, audit risk in period 1 $\left(AR_{1cont}\right)$ is equal to

$$AR_{1cont} = \theta \exp\left(-x_1\right)$$

and in period 2 audit risk $\left(AR_{2cont}\right)$ equals

$$AR_{2cont} = \frac{\theta \exp\left(-x_1\right)}{\theta \exp\left(-x_1\right) + \left(1-\theta\right)} \exp\left(-x_2\right)$$

where x_1 and x_2 are from Proposition 2.

Proposition 5: Audit risk in each period given a continuing auditor is strictly less than that given a change in auditors.

Based on the equilibrium condition $H\left[x_1\right] = 0$, we know that first period audit effort in setting two is higher than that in setting one and we have the following result,

$$AR_{1old} > AR_{1cont} ,$$

which implies that audit risk is higher for the change auditor in period 1. Figure 5 provides a graphical representation of a comparison of period 1 audit risk across settings 1 and 2 as they relate to the auditor's period 1 liability parameter L_1. Again any parameter could be used to illustrate these relationships. Despite a higher audit effort in period 2 for a change in auditors, compared to a continuing auditor, audit risk remains lower in period 2 for the continuing auditor. This occurs because period 2 audit risk in setting 2 includes the updated probability $\frac{\theta \exp(-x_1)}{\theta \exp(-x_1)+(1-\theta)}$ that the manager is the fraudulent type, where x_1 is greater for the continuing auditor. Together our findings for expected undetected fraud and audit risk in each period suggest that audit quality is higher in the continuing setting than in the change setting.

VI. EMPRICAL INSIGHTS

Regulators, practitioners, and academic researchers continue to debate whether the efficiencies that result from an extended audit tenure outweigh the potential impairment of auditor independence from the auditor becoming 'too cozy' with the client. As with other studies, we find audit quality is higher for the continuing auditor, in that both audit risk and expected undetected fraud are lower for a continuing auditor than with a new auditor. Our study demonstrates, that without empirical and experimental researchers developing an expectations model that reflects the extent audit quality would differ in the absence of an impairment of independence, the evidence to date is inconclusive on whether an impairment does or does not exist.

The concern about impairment of independence continues to be fueled by the observed phenomenon in later periods that audit effort is smaller and the amount of fraud is larger for the continuing auditors than for a new auditor. A key finding of our study, however, is that the continuing auditor would typically exert more effort in the earlier period to discover fraud, which in turn results in less audit effort and more fraud in the later period. This, of course, has the appearance that audit tenure impairs auditor independence. While an impairment of independence may indeed exist, to conclude from observing lower adult effort a higher fraud in later periods of an auditor's tenure requires first calibrating the expected differences between a continuing auditor and a change in auditor, again in the absence of an impairment of independence.

With the collapse of Arthur Andersen, new concerns arose about auditor tenure eroding auditor independence. Rather than creating mandatory audit firm rotations to alleviate these concerns, the *Sarbanes-Oxley Act of 2002* (SOX) enhanced the provision for mandatory *partner* rotation by reducing the rotation period to a maximum of five years. In terms of our model, mandating partner rotation potentially increases the cost of auditing in the later periods for a new *continuing* auditor firm partner, represented by k_2.

The desired effects of SOX reducing the rotation period was to increase audit quality by reducing the possibility that audit partner tenure impairs auditor independence. Gipper, Hail, and Leuz (2017) analyze a large scale sample of partner rotations in the U.S. to test for this possibility, but find no evidence that audit quality increased with the SOX requirement. The results from our study provide an explanation as to why the above study did not find an improvement in their indicators of audit quality. Our comparative statics indicate that as k_2 increases, audit effort by the continuing auditor (in this case, a continuing audit firm with a different lead audit partner) decreases even more in the later audit period relative to the new auditor. The amount of fraud in the later period also increases as k_2 increases.

While mandatory audit partner rotation might be effective in reducing the possible impairment of independence from auditor tenure, the untended consequences of mandatory audit partner rotation are potentially opposite of SOX's goal to increase audit quality. By applying the results of our comparative statics to future experimental and empirical studies, future tests will be able to identify whether the positive effects of mandatory audit partner rotation on auditor independence outweighs the unintended negative effects of mandatory audit partner rotation increasing k_2, the cost of auditing in the later period,

VII. CONCLUSION

We consider a game-theoretic model of auditing where a manager possibly commits fraud over two periods and an auditor attempts to detect the fraud, if it exists. We compare two settings of this model. In the first, a different auditor audits each period. And while the auditor in the second period can update the probability that the manager is a dishonest, the incentives of the auditors in each period are independent. In the second setting a single auditor audits both periods. In this case, the continuing auditor can allocate audit effort efficiently across the two periods to minimize the combined costs of auditing and litigation exposure across the two periods. As a result, a continuing auditor chooses a higher amount of period 1 audit effort and the manager chooses a lesser amount of period 1 fraud. If there is no fraud detected at the end of period 1, the continuing auditor chooses a *lower* amount of period 2 audit effort relative to a new period 2 auditor, because the period 1 evidence indicates that there is a lower risk of a dishonest-type manager.

Our results also show that two measures of audit quality are always higher for a continuing auditor. This occurs despite a lower level of period 2 audit effort for the continuing auditor when compared to period 2 audit effort given a change in auditors. We find audit risk (probability that fraud goes undetected) is lower in *each* period for the continuing auditor. More importantly, total expected undetected fraud across the two periods is lower for the continuing auditor.

In addition, we show how the period 1 and period 2 strategies for the auditor and manager change for changes in the various payoff parameters. Not surprisingly, we find an increase in the period 1 auditor liability costs increases the period 1 audit effort and decreases the period 1 amount of fraud. These results are also true for period 2. By analyzing a multi-period audit setting, we are also able to show that changes in period 1 parameters have the same intuitive effects on period 2 strategies in both settings. Any period 1 parameter that would induce higher audit effort in period 1, would cause audit effort in period 2 to decrease. Any period 1 parameter that would induce more fraud in period 1, would induce less fraud in period 2.

Finally, our results provide guidance for future research. The concern expressed by regulators and others is that longer audit tenures could result in an impairment of independence. While most empirical studies find that longer audit tenure improves audit quality, these studies cannot address whether longer audit tenure impairs independence without first calibrating the extent that audit quality would be expected to be higher with a continuing auditor, had there not been any impairment of independence. The equilibrium findings of our model provide the theoretical guidance necessary for developing these expectations. Further, we show the conclusions drawn by regulators and others that a continuing auditor's independence is likely impaired when observing lower audit effort and higher amounts of fraud in the later audit periods might be a direct result of efficient allocation of audit effort by a continuing auditor, rather than an impairment of independence. Our study provides the necessary theoretical guidance for experimental and empirical researchers to disentangle whether such observations are evidence of an impairment, or evidence of the continuing auditor effectively allocating effort.

FIGURE 1

Timeline for a change in auditors

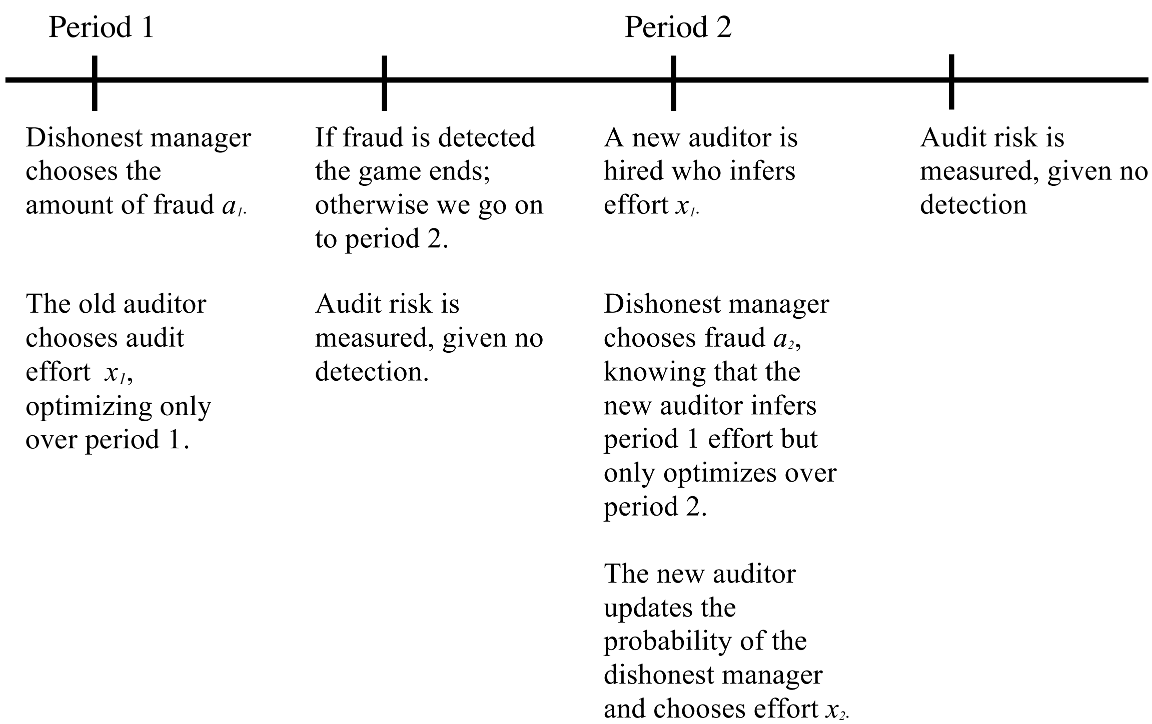


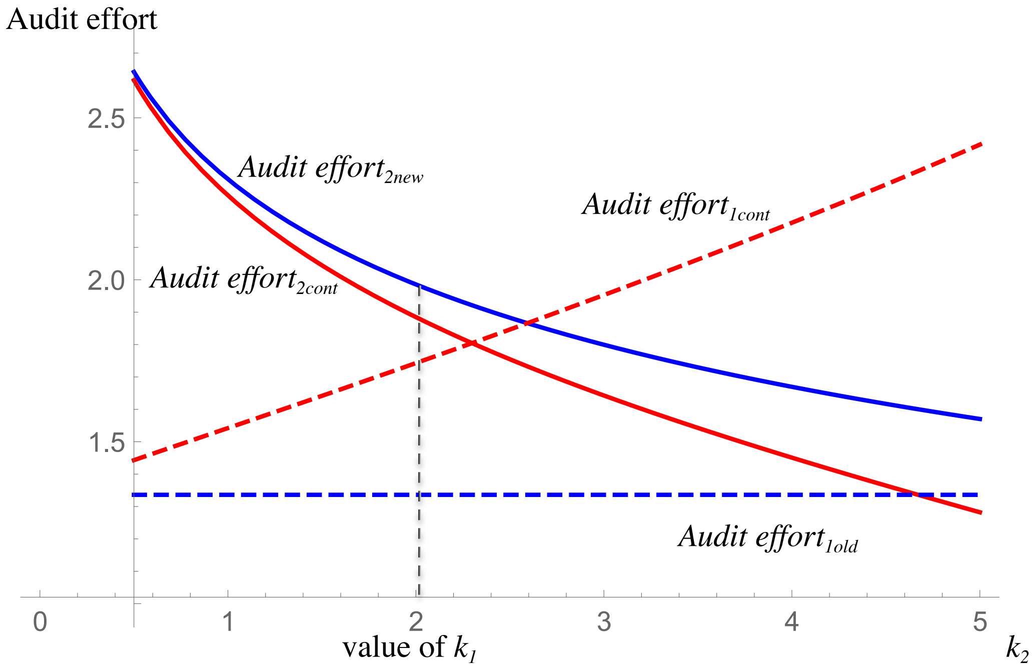
FIGURE 2
Timeline for the continuing auditor

Period 1		Period 2	
Dishonest manager chooses the amount of fraud a_1, knowing the auditor optimizes over both periods.	If fraud is detected the game ends; otherwise we go on to period 2.	The auditor continues in period 2.	Audit risk is measured, given no detection.
The continuing auditor chooses audit effort x_1, knowing he uses x_1 to update and plans his choice of effort x_2 to optimize his expected payoffs over both periods.	Audit risk is measured, given no detection	Dishonest manager chooses fraud a_2, knowing that the continuing auditor has planned for effort x_2, taking into account his x_1 choice. The continuing auditor implements x_2.	

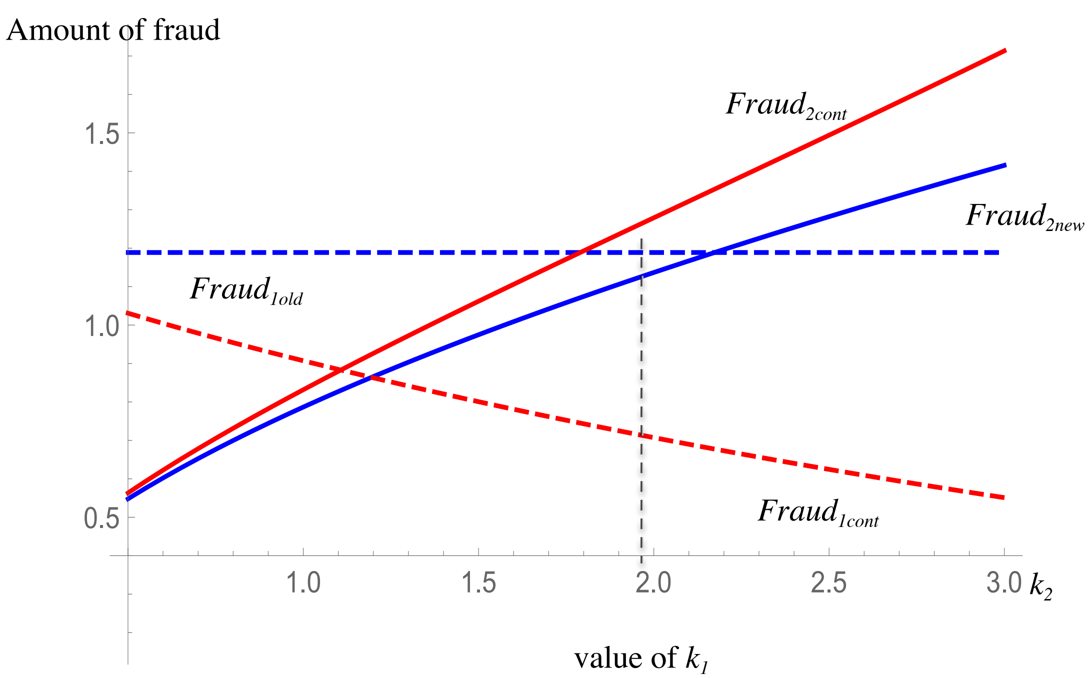
FIGURE 3

Audit Effort and the Amount of Fraud in Each of the Two Periods

(Note that in calculating audit effort and fraud in the example below, $k_1 = 2$)



Panel A: *Audit Effort in Each of the Two Periods*



Panel B: *The Amount of fraud in Each of the Two Periods*

FIGURE 4
Expected Undetected Fraud for a Change in Auditors
Versus a Continuing Auditor

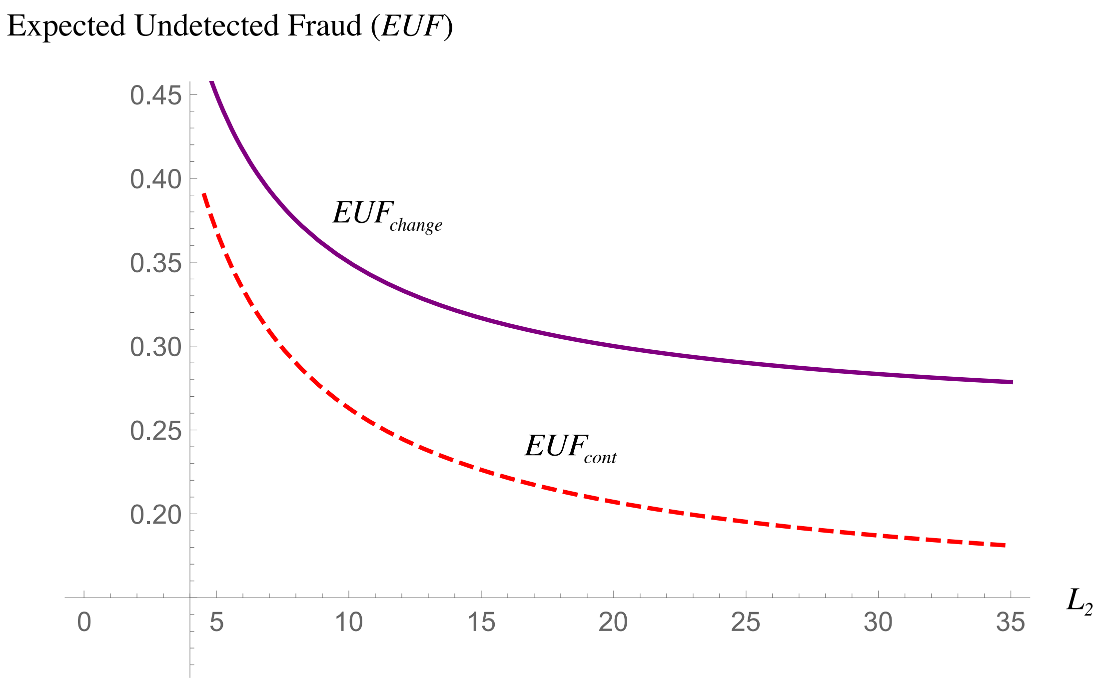
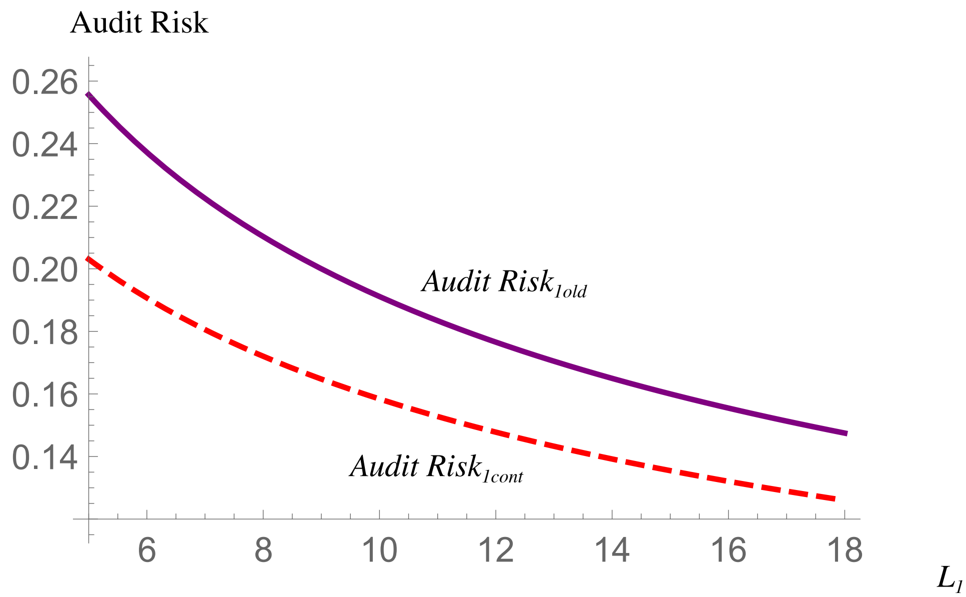
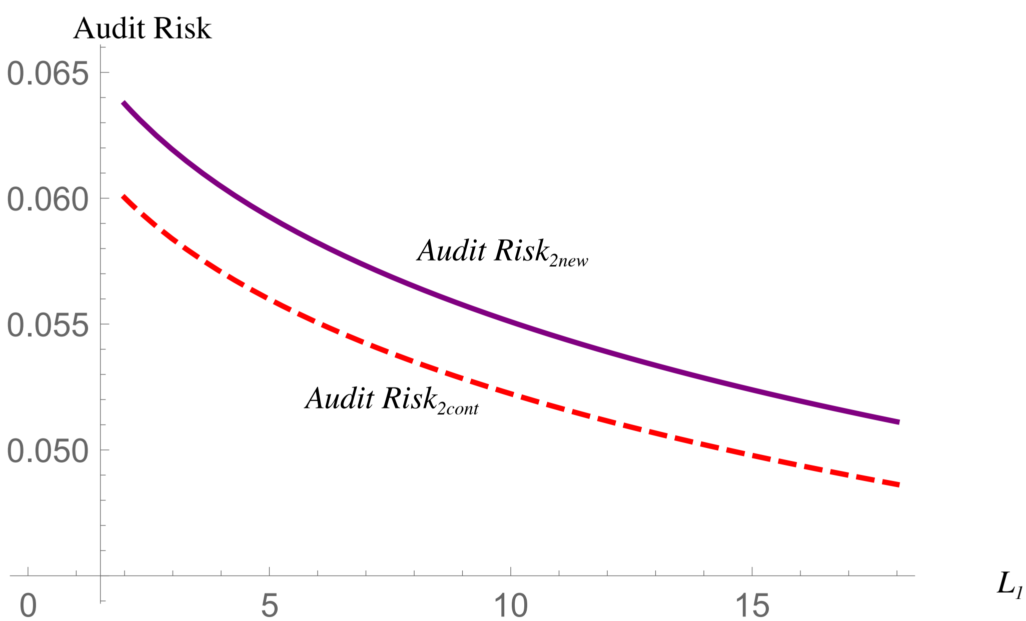


FIGURE 5
A Comparison of Audit Risk in Periods 1 and 2
for a Change in Auditors Versus a Continuing Auditor



Panel A: *Period One Audit Risk*



Panel B: *Period Two Audit Risk*

Appendix

Proof of Proposition 1:

Period 1 equilibrium strategies

From expressions (10) and (13) we have

$$a_1 = \frac{R_1 \exp(-x_1)}{p_1(1-\exp(-x_1))} = \frac{R_1}{p_1(-1+\exp(x_1))} \text{ and } x_1 = Log\left[\frac{\theta a_1 L_1}{k_1}\right] \Rightarrow \exp(x_1) = \frac{\theta L_1 a_1}{k_1} .$$

Thus, $a_1 = \dfrac{R_1 k_1}{p_1(\theta L_1 a_1 - k_1)}$

$\Rightarrow \theta L_1 p_1 a_1{}^2 - k_1 p_1 a_1 - R_1 k_1 = 0$, which is a quadratic equation.

The solution to a general quadratic of $ay^2 + by + c = 0$ is $y = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Therefore, $a_1 = \dfrac{k_1 p_1 + \sqrt{k_1{}^2 p_1{}^2 + 4\theta L_1 p_1 R_1 k_1}}{2\theta L_1 p_1} = \dfrac{k_1 + k_1\sqrt{1+\dfrac{4\theta L_1 R_1}{k_1 p_1}}}{2\theta L_1} \Rightarrow$

$$a_1 = \frac{k_1}{2\theta L_1}\left(1+\sqrt{1+\frac{4\theta L_1 R_1}{k_1 p_1}}\right)$$

Substituting into $x_1 = Log\left[\dfrac{\theta a_1 L_1}{k_1}\right]$ we get $x_1 = Log\left[\dfrac{1}{2}\left(1+\sqrt{1+\dfrac{4\theta L_1 R_1}{k_1 p_1}}\right)\right]$

Substituting x_1 back into a_1 we have $a_1 = \dfrac{R_1}{p_1\left(-1+\dfrac{1}{2}\left(1+\sqrt{1+\dfrac{4 L_1 R_1}{k_1 p_1}\theta}\right)\right)}$

We also have $x_2 = Log\left[\dfrac{a_2 L_2}{k_2}\dfrac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1-\theta)}\right]$ and $a_2 = \dfrac{R_2 \exp(-x_2)}{p_2(1-\exp(-x_2))}$ where

a_2 and x_2 are solved for in a similar fashion and θ is replaced with the updated probability of

$$\frac{\theta\exp(-x_1)}{\theta\exp(-x_1)+(1-\theta)}.$$

Proof of Proposition 2:

First, we derive the auditor's first period expected payoff, given fixed a_1 and a_2

Recall that x_2 must satisfy $\frac{dA_{2cont}}{dx_2}=\Pr(D\mid ND)\left(L_2 a_2\exp[-x_2]\right)-k_2=0$ (expression (14) in the paper). This results in

$$\exp(-x_2)=\frac{k_2}{L_2 a_2}\frac{\theta\exp(-x_1)+(1-\theta)}{\theta\exp(-x_1)}\Rightarrow$$

$$x_2=Log\left[\frac{L_2 a_2}{k_2}\frac{\theta\exp(-x_1)}{\theta\exp(-x_1)+(1-\theta)}\right]=Log\left[\frac{L_2 a_2\theta}{k_2}\frac{1}{\theta\exp(-x_1)+(1-\theta)}\right]-x_1$$

We also have

$A_{1cont}=\left(\theta\exp(-x_1)+(1-\theta)\right)A_2-x_1k_1$ where $A_{2cont}=-\Pr(D\mid ND)\left(L_1a_1+L_2a_2\exp(-x_2)\right)-x_2k_2$.

Then substitute for x_2 with a_2 fixed and we obtain the following for A_{2cont}.

$$A_{2cont}=\Pr(D\mid ND)\left(-L_1a_1-k_2\frac{\theta\exp(-x_1)+(1-\theta)}{\theta\exp(-x_1)}\right)-k_2\left(-x_1+Log\left[\frac{L_2a_2}{k_2}\frac{\theta}{\theta\exp(-x_1)+(1-\theta)}\right]\right)$$

Substitute back into A_{1cont}.

$$A_{1cont}=\left(\theta\exp(-x_1)+(1-\theta)\right)\bullet\left(\begin{array}{l}\frac{\theta\exp(-x_1)}{\theta\exp(-x_1)+(1-\theta)}\left(-L_1a_1-k_2\frac{\theta\exp(-x_1)+(1-\theta)}{\theta\exp(-x_1)}\right)\\ \qquad -k_2\left(-x_1+Log\left[\frac{L_2a_2}{k_2}\frac{\theta}{\theta\exp(-x_1)+(1-\theta)}\right]\right)\end{array}\right)-x_1k_1$$

where $\left(\theta\exp(-x_1)+(1-\theta)\right)$ is the probability that no fraud is detected in period 1.

Then,

$$A_{1cont} = \begin{pmatrix} -L_1 a_1 \theta \exp(-x_1) - k_2\left(\theta \exp(-x_1) + (1-\theta)\right) \\ \qquad -\left(\theta \exp(-x_1) + (1-\theta)\right) k_2 \left(-x_1 + Log\left[\dfrac{L_2 a_2}{k_2} \dfrac{\theta}{\theta \exp(-x_1) + (1-\theta)} \right] \right) \end{pmatrix} - x_1 k_1$$

$$= \begin{pmatrix} -L_1 a_1 \theta \exp(-x_1) \\ \qquad -\left(\theta \exp(-x_1) + (1-\theta)\right) k_2 \left(1 - x_1 + Log\left[\dfrac{L_2 a_2}{k_2} \dfrac{\theta}{\theta \exp(-x_1) + (1-\theta)} \right] \right) \end{pmatrix} - x_1 k_1$$

The above is expression (15) in the paper.

Next take the first derivative of A_{1cont} with respect to x_1.

$$\frac{dA_{1cont}}{dx_1} = L_1 a_1 \theta \exp(-x_1) + \theta \exp(-x_1) k_2 \left(1 - x_1 + Log\left[\frac{L_2 a_2}{k_2} \frac{\theta}{\theta \exp(-x_1) + (1-\theta)} \right] \right)$$
$$- \left(\theta \exp(-x_1) + (1-\theta)\right) k_2 \left(-1 + d\left(Log\left[\frac{L_2 a_2}{k_2} \frac{\theta}{\theta \exp(-x_1) + (1-\theta)} \right] \right) / dx_1 \right) - k_1$$

$$= L_1 a_1 \theta \exp(-x_1) + \theta \exp(-x_1) k_2 \left(1 - x_1 + Log\left[\frac{L_2 a_2}{k_2} \frac{\theta}{\theta \exp(-x_1) + (1-\theta)} \right] \right)$$
$$- \left(\theta \exp(-x_1) + (1-\theta)\right) k_2 \left(-1 + \frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1-\theta)} \right) - k_1$$

Thus, we have,

$$\frac{dA_{1cont}}{dx_1} = \theta \exp(-x_1) \left(L_1 a_1 + k_2 \left(1 - x_1 + Log\left[\frac{L_2 a_2}{k_2} \frac{\theta}{\theta \exp(-x_1) + (1-\theta)} \right] \right) \right) + (1-\theta) k_2 - k_1 \qquad \text{(A1)}$$

Take the second derivative and we get

$$\frac{d^2 A_{1cont}}{dx_1^{\ 2}} = -\theta \exp\left(-x_1\right)\left(L_1 a_1 + k_2\left(1 - x_1 + Log\left[\frac{L_2 a_2}{k_2} \frac{\theta}{\theta \exp\left(-x_1\right) + \left(1-\theta\right)} \right]\right)\right)$$

$$- \theta \exp\left(-x_1\right) k_2 \left(1 - \frac{\theta \exp\left(-x_1\right)}{\theta \exp\left(-x_1\right) + \left(1-\theta\right)} \right) < 0$$

where note that $x_2 = -x_1 + Log\left[\frac{L_2 a_2}{k_2} \frac{\theta}{\theta \exp\left(-x_1\right) + \left(1-\theta\right)} \right] > 0$

Thus, A_{1cont} is concave in x_1.

Expression (A1) is our basis for implicitly defining x_1. We substitute for a_1 and a_2 where a_1, a_2 and x_2 are functions of x_1.

We know that $a_2 = \frac{R_2 \exp\left(-x_2\right)}{p_2\left(1-\exp\left(-x_2\right)\right)}$. Thus we substitute for a_2 in

$$x_2 = Log\left[\frac{L_2 a_2}{k_2} \frac{\theta}{\theta \exp\left(-x_1\right) + \left(1-\theta\right)} \right] - x_1$$

and solve for x_2.

$$x_2 = Log\left[\frac{L_2 \frac{R_2 \exp\left(-x_2\right)}{p_2\left(1-\exp\left(-x_2\right)\right)}}{k_2} \frac{\theta}{\theta \exp\left(-x_1\right) + \left(1-\theta\right)} \right] - x_1 \Rightarrow$$

$$\exp\left(x_2\right) = \frac{L_2 R_2}{p_2 k_2 \left(\exp\left(x_2\right) - 1\right)} \frac{\theta \exp\left(-x_1\right)}{\theta \exp\left(-x_1\right) + \left(1-\theta\right)} \quad \text{and}$$

$$\left(\exp\left(x_2\right)\right)^2 - \exp\left(x_2\right) - \frac{L_2 R_2}{p_2 k_2} \frac{\theta \exp\left(-x_1\right)}{\theta \exp\left(-x_1\right) + \left(1-\theta\right)} = 0$$

In applying the quadratic formula $\left(\text{for } ay^2 + by + c = 0 \text{ is } y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right)$, we get

$$\exp(x_2)=\frac{1}{2}\left(1+\sqrt{1+\frac{4L_2R_2}{p_2k_2}\frac{\theta\exp(-x_1)}{\theta\exp(-x_1)+(1-\theta)}}\right)$$ and thus,

$$x_2=Log\left[\frac{1}{2}\left(1+\sqrt{1+\frac{4L_2R_2}{p_2k_2}\frac{\theta\exp(-x_1)}{\theta\exp(-x_1)+(1-\theta)}}\right)\right]$$

Next substitute for x_2 in a_2. We start with

$$a_2=\frac{R_2\exp(-x_2)}{p_2\left(1-\exp(-x_2)\right)}=\frac{R_2}{p_2\left(\exp(x_2)-1\right)}\ .$$

Then we have

$$a_2=\frac{R_2}{p_2\left(-1+\frac{1}{2}\left(1+\sqrt{1+\frac{4L_2R_2}{p_2k_2}\frac{\theta\exp(-x_1)}{\theta\exp(-x_1)+(1-\theta)}}\right)\right)}>0$$

So now we can rewrite expression (A1) in terms of x_1 that defines the equilibrium condition for x_1.

$$H[x_1]=\theta\exp(-x_1)\left(L_1a_1[x_1]+k_2+k_2x_2[x_1]\right)+(1-\theta)k_2-k_1=0 \qquad \text{(A2)}$$

where

$$a_1[x_1]=\frac{R_1\exp(-x_1)}{p_1\left(1-\exp(-x_1)\right)}$$ and

$$x_2[x_1]=Log\left[\frac{1}{2}\left(1+\sqrt{1+\frac{4L_2R_2}{p_2k_2}\frac{\theta\exp(-x_1)}{\theta\exp(-x_1)+(1-\theta)}}\right)\right]$$

Finally we know that expression (A2) yields a unique x_1 and hence unique values of a_1, a_2 and x_2 because we show below that based on our assumption of $\left(k_1-(1-\theta)k_2\right)>0$, $\frac{dH[x_1]}{dx_1}<0$.

$$\frac{dH[x_1]}{dx_1}=-\theta\exp(-x_1)\left(L_1a_1[x_1]+k_2+k_2x_2[x_1]\right)+\theta\exp(-x_1)\left(L_1\frac{da_1[x_1]}{dx_1}+k_2\frac{dx_2[x_1]}{dx_1}\right) \qquad \text{(A3)}$$

and after substituting in condition (A2), we have

$$\frac{dH[x_1]}{dx_1} = -\left(k_1 - (1-\theta)k_2\right) + \theta\exp(-x_1)\left(L_1\frac{da_1[x_1]}{dx_1} + k_2\frac{dx_2[x_1]}{dx_1}\right) < 0 \tag{A4}$$

where $\frac{da_1[x_1]}{dx_1} = -\frac{R_1}{p_1}\frac{\exp(x_1)}{\left(\exp(x_1)-1\right)^2} < 0$ and

$$\frac{dx_2}{dx_1} = -\frac{2\exp(x_1)(1-\theta)\theta L_2 R_2}{\left\{\begin{array}{c} k_2 p_2\left(\exp(x_1)(1-\theta)+\theta\right)^2\sqrt{1+\frac{4L_2R_2}{p_2k_2}\frac{\theta\exp(-x_1)}{\theta\exp(-x_1)+(1-\theta)}}\bullet \\ \left(1+\sqrt{1+\frac{4L_2R_2}{p_2k_2}\frac{\theta\exp(-x_1)}{\theta\exp(-x_1)+(1-\theta)}}\right)\end{array}\right\}} < 0 \tag{A5}$$

This also indicates that to determine the sign of how x_1 changes with respect to a parameter we would calculate

$\frac{dx_1}{dg} = \frac{\frac{dH[x_1]}{dg}}{-\frac{dH[x_1]}{dx_1}}$ where *g* stands for a parameter (e.g. L_1). We know that $-\frac{dH[x_1]}{dx_1} > 0$ so that

the sign of $\frac{dx_1}{dg}$ is the same as the sign of $\frac{dH[x_1]}{dg}$ where x_1 is fixed.

Proof of Corollary 1: First note that the characterization of fraud in both settings is from expression (10) where

$a_1 = \frac{R_1\exp(-x_1)}{p_1\left(1-\exp(-x_1)\right)}$ and $a_2 = \frac{R_2\exp(-x_2)}{p_2\left(1-\exp(-x_2)\right)}$.

Furthermore, $\frac{da_1}{dx_1} = -\frac{R_1}{p_1}\frac{\exp(x_1)}{\left(\exp(x_1)-1\right)^2} < 0$ and $\frac{da_2}{dx_2} = -\frac{R_1}{p_1}\frac{\exp(x_2)}{\left(\exp(x_2)-1\right)^2} < 0$. (A6)

We know from Proposition 1 that for the ‘old’ and ‘new’ auditors of periods 1 and 2 we have

$$x_1^{old} = Log\left[\frac{1}{2}\left(1+\sqrt{1+\frac{4\theta L_1 R_1}{k_1 p_1}}\right)\right] \text{ and } x_2^{new} = Log\left[\frac{1}{2}\left(1+\sqrt{1+\frac{4L_2R_2}{p_2k_2}\frac{\theta\exp\left(-x_1^{new}\right)}{\theta\exp\left(-x_1^{new}\right)+\left(1-\theta\right)}}\right)\right].$$

Moreover Proposition 2 shows that x_1^{old} is smaller compared to audit effort in period 1 for the continuing auditor and thus we know from (A6) that the amount of period 1 fraud is greater for the change auditor.

Proposition 2 shows that for the continuing versus the change auditor in period 2

$$x_2^{cont} = Log\left[\frac{1}{2}\left(1+\sqrt{1+\frac{4L_2R_2}{p_2k_2}\frac{\theta\exp\left(-x_1^{cont}\right)}{\theta\exp\left(-x_1^{cont}\right)+\left(1-\theta\right)}}\right)\right]<$$

$$Log\left[\frac{1}{2}\left(1+\sqrt{1+\frac{4L_2R_2}{p_2k_2}\frac{\theta\exp\left(-x_1^{new}\right)}{\theta\exp\left(-x_1^{new}\right)+\left(1-\theta\right)}}\right)\right]=x_2^{new}$$

because $\dfrac{\theta\exp\left(-x_1\right)}{\theta\exp\left(-x_1\right)+\left(1-\theta\right)}$ decreases in x_1. Thus, the amount of fraud in period 2 is more for a continuing auditor than for the 'new' auditor and the amount of audit effort is less .

Proof of Proposition 3:

We begin by demonstrating the results in the top-left quadrant and the bottom right quadrant of the table. We begin with the results in the top-left quadrant

x_1 increases in L_1 and R_1, while decreasing in p_1 and k_1.

Setting 1- old auditor:

x_1 increases in L_1

$$\frac{dx_1}{dL_1}=\frac{2R_1\theta}{k_1p_1+4L_1R_1\theta+\sqrt{k_1^{\ 2}p_1^{\ 2}+4L_1R_1\theta k_1p_1}}>0$$

x_1 increases in R_1

$$\frac{dx_1}{dR_1} = \frac{2L_1\theta}{k_1p_1 + 4L_1R_1\theta + \sqrt{k_1^2 p_1^2 + 4L_1R_1\theta k_1 p_1}} > 0$$

x_1 decreases in p_1

$$\frac{dx_1}{dp_1} = -\frac{2R_1L_1\theta}{p_1\left(k_1p_1 + 4L_1R_1\theta + \sqrt{k_1^2 p_1^2 + 4L_1R_1\theta k_1 p_1}\right)} < 0$$

x_1 decreases in k_1.

$$\frac{dx_1}{dk_1} = -\frac{2R_1L_1\theta}{k_1\left(k_1p_1 + 4L_1R_1\theta + \sqrt{k_1^2 p_1^2 + 4L_1R_1\theta k_1 p_1}\right)} < 0$$

Setting 2 -- the continuous auditor

Recall (A2) or $H[x_1] = \theta\exp(-x_1)\left(L_1a_1[x_1] + k_2 + k_2x_2[x_1]\right) + (1-\theta)k_2 - k_1$

As discussed in the proof to Proposition 2, $\frac{dx_1}{dg} = \frac{\frac{dH[x_1]}{dg}}{-\frac{dH[x_1]}{dx_1}}$ where *g* stands for a general parameter (e.g. L_1) . We know that $-\frac{dH[x_1]}{dx_1} > 0$ so that the sign of $\frac{dx_1}{dg}$ is the same as the sign of $\frac{dH[x_1]}{dg}$ where x_1 is fixed.

x_1 increases in L_1.

$$\frac{dH[x_1]}{dL_1} = \theta\exp(-x_1)a_1[x_1] > 0$$

x_1 increases in R_1.

$$\frac{dH[x_1]}{dR_1} = \theta\exp(-x_1)L_1\frac{1}{p_1\left(\exp(x_1)-1\right)} > 0$$

x_1 decreases in p_1.

$$\frac{dH[x_1]}{dp_1} = -\theta \exp(-x_1) L_1 \frac{R_1}{p_1{}^2(\exp(x_1)-1)} < 0$$

x_1 decreases in k_1.

$$\frac{dH[x_1]}{dk_1} = -1 < 0$$

a_1 increases in R_1, and k_1, while decreasing in L_1 and p_1.

Setting 1 -- the old auditor:

a_1 increases in R_1

$$\frac{da_1}{dR_1} = \frac{k_1}{\sqrt{k_1{}^2 p_1{}^2 + 4L_1R_1\theta k_1 p_1}} > 0$$

a_1 increases in k_1,

$$\frac{da_1}{dk_1} = \frac{1}{2L_1\theta}\left(1 + \frac{p_1k_1 + 2L_1R_1\theta}{\sqrt{k_1{}^2 p_1{}^2 + 4L_1R_1\theta k_1 p_1}}\right) > 0$$

a_1 decreases in L_1

$$\frac{da_1}{dL_1} = -\frac{k_1}{2L_1{}^2\theta}\left(1 + \frac{p_1k_1 + 2L_1R_1\theta}{\sqrt{k_1{}^2 p_1{}^2 + 4L_1R_1\theta k_1 p_1}}\right) < 0$$

a_1 decreases in p_1.

$$\frac{da_1}{dp_1} = -\frac{k_1R_1}{p_1\sqrt{k_1{}^2 p_1{}^2 + 4L_1R_1\theta k_1 p_1}} < 0$$

a_1 decreases in θ.

$$\frac{da_1}{d\theta} = -\frac{k_1}{2L_1\theta^2}\left(1 + \frac{k_1p_1 + 2L_1R_1\theta}{\sqrt{k_1{}^2 p_1{}^2 + 4L_1R_1\theta k_1 p_1}}\right) < 0$$

Setting 2 -- a continuous auditor.

a_1 increases in R_1.

$$\frac{da_1}{dR_1}=\frac{\partial a_1}{\partial R_1}+\frac{\partial a_1}{\partial x_1}\frac{\theta\exp(-x_1)\left(L_1\frac{\partial a_1}{\partial R_1}\right)}{\left(k_1-(1-\theta)k_2\right)-\theta\exp(-x_1)\left(L_1\frac{\partial a_1}{\partial x_1}+k_2\frac{\partial x_2}{\partial x_1}\right)}$$

$$=\frac{\partial a_1}{\partial R_1}\left(1-\frac{-\theta\exp(-x_1)\left(L_1\frac{\partial a_1}{\partial x_1}\right)}{\left(k_1-(1-\theta)k_2\right)-\theta\exp(-x_1)\left(L_1\frac{\partial a_1}{\partial x_1}+k_2\frac{\partial x_2}{\partial x_1}\right)}\right)>0$$

where $\frac{\partial a_1}{\partial R_1}>0$.

a_1 increases in k_1.

$\frac{da_1}{dk_1}=\frac{\partial a_1}{\partial k_1}+\frac{\partial a_1}{\partial x_1}\frac{dx_1}{dk_1}>0$ where $\frac{\partial a_1}{\partial k_1}=0$, $\frac{\partial a_1}{\partial x_1}<0$ and $\frac{dx_1}{dk_1}<0$.

a_1 decreases in L_1.

Similarly, $\frac{da_1}{dL_1}=\frac{\partial a_1}{\partial x_1}\frac{dx_1}{dL_1}<0$

a_1 decreases in p_1.

$$\frac{da_1}{dp_1}=\frac{\partial a_1}{\partial p_1}\left(1-\frac{-\theta\exp(-x_1)\left(L_1\frac{\partial a_1}{\partial x_1}\right)}{\left(k_1-(1-\theta)k_2\right)-\theta\exp(-x_1)\left(L_1\frac{\partial a_1}{\partial x_1}+k_2\frac{\partial x_2}{\partial x_1}\right)}\right)<0$$

Now we prove the results for the bottom-right quadrant of the table.

x_2 increases in L_2 and R_2, while decreasing in p_2 and k_2.

The proof is essentially the same as in part 1 above for the new auditor.

For the continuous auditor we have the following.

$$\frac{\partial x_2}{\partial x_1}=-\frac{2\exp(x_1)L_2R_2(1-\theta)\theta}{k_2p_2\left(\exp(x_1)(1-\theta)+\theta\right)^2SQ_2(1+SQ_2)}<0$$ where

$$SQ_2=\sqrt{1+\frac{4L_2R_2}{k_2p_2}\frac{\theta\exp(-x_1)}{\theta\exp(-x_1)+(1-\theta)}}>1$$

And we use the expression

$$\frac{dH[x_1]}{dx_1}=-\left(k_1-(1-\theta)k_2\right)+\theta\exp(-x_1)\left(L_1\frac{da_1[x_1]}{dx_1}+k_2\frac{dx_2[x_1]}{dx_1}\right)$$ from (A4) .

x_2 increases in L_2 .

$$\frac{dx_2}{dL_2}=\frac{\partial x_2}{\partial L_2}+\frac{\partial x_2}{\partial x_1}\frac{dx_1}{dL_2}$$

$$=\frac{\partial x_2}{\partial L_2}+\frac{\partial x_2}{\partial x_1}\frac{\dfrac{dH[x_1]}{dL_2}}{\dfrac{-dH[x_1]}{dx_1}}=\frac{\partial x_2}{\partial L_2}+\frac{\partial x_2}{\partial x_1}\frac{\theta\, exp(-x_1)k_2\dfrac{\partial x_2}{\partial L_2}}{\left(k_1-(1-\theta)k_2\right)-\theta\, exp(-x_1)\left(L_1\dfrac{da_1}{dx_1}+k_2\dfrac{dx_2}{dx_1}\right)}$$

$$=\frac{\partial x_2}{\partial L_2}\left(1-\frac{-\theta\exp(-x_1)k_2\dfrac{\partial x_2}{\partial x_1}}{\left(k_1-(1-\theta)k_2\right)-\theta\exp(-x_1)\left(L_1\dfrac{\partial a_1}{\partial x_1}+k_2\dfrac{\partial x_2}{\partial x_1}\right)}\right)>0$$

where the second term inside the parentheses is less than one, resulting in a positive sum within the parentheses. Finally, $\frac{\partial x_2}{\partial L_2}=\frac{2\exp\left(x_1^{-1}\right)R_2\theta}{k_2p_2\left((1-\theta)+\exp\left(x_1^{-1}\right)\theta\right)SQ_2(1+SQ_2)}>0$.

x_2 increases in R_2 .

$$\frac{dx_2}{dR_2}=\frac{\partial x_2}{\partial R_2}+\frac{\partial x_2}{\partial x_1}\frac{dx_1}{dR_2}$$ and similar to the derivation above

$$=\frac{\partial x_2}{\partial R_2}+\frac{\partial x_2}{\partial x_1}\frac{\theta\exp(-x_1)k_2\dfrac{\partial x_2}{\partial R_2}}{\left(k_1-(1-\theta)k_2\right)-\theta\exp(-x_1)\left(L_1\dfrac{\partial a_1}{\partial x_1}+k_2\dfrac{\partial x_2}{\partial x_1}\right)}$$

$$=\frac{\partial x_2}{\partial R_2}\left(1-\frac{-\theta\exp(-x_1)k_2\frac{\partial x_2}{\partial x_1}}{\left(k_1-(1-\theta)k_2\right)-\theta\exp(-x_1)\left(L_1\frac{\partial a_1}{\partial x_1}+k_2\frac{\partial x_2}{\partial x_1}\right)}\right)>0 \quad \text{where } \frac{\partial x_2}{\partial R_2}>0 .$$

x_2 decreases in p_2 .

Similarly, $\frac{dx_2}{dp_2}=\frac{\partial x_2}{\partial p_2}\left(1-\frac{-\theta\exp(-x_1)k_2\frac{\partial x_2}{\partial x_1}}{\left(k_1-(1-\theta)k_2\right)-\theta\exp(-x_1)\left(L_1\frac{\partial a_1}{\partial x_1}+k_2\frac{\partial x_2}{\partial x_1}\right)}\right)<0$

where $\frac{\partial x_2}{\partial p_2}<0$.

x_2 decreases in k_2 .

$$\frac{dx_2}{dk_2}=\frac{\partial x_2}{\partial k_2}+\frac{\partial x_2}{\partial x_1}\frac{(1-\theta)+\theta\exp(-x_1)\left(k_2\frac{\partial x_2}{\partial k_2}+1+x_2\right)}{\left(k_1-(1-\theta)k_2\right)-\theta\exp(-x_1)\left(L_1\frac{\partial a_1}{\partial x_1}+k_2\frac{\partial x_2}{\partial x_1}\right)}$$

$$=\frac{\partial x_2}{\partial k_2}+\frac{\partial x_2}{\partial x_1}\frac{\theta\exp(-x_1)\left(k_2\frac{\partial x_2}{\partial k_2}\right)}{\left(k_1-(1-\theta)k_2\right)-\theta\exp(-x_1)\left(L_1\frac{\partial a_1}{\partial x_1}+k_2\frac{\partial x_2}{\partial x_1}\right)}$$

$$+\frac{\partial x_2}{\partial x_1}\frac{(1-\theta)+\theta\exp(-x_1)(1+x_2)}{\left(k_1-(1-\theta)k_2\right)-\theta\exp(-x_1)\left(L_1\frac{\partial a_1}{\partial x_1}+k_2\frac{\partial x_2}{\partial x_1}\right)}$$

$$=\frac{\partial x_2}{\partial k_2}\left(1-\frac{-\theta\exp(-x_1)\left(k_2\frac{\partial x_2}{\partial x_1}\right)}{\left(k_1-(1-\theta)k_2\right)-\theta\exp(-x_1)\left(L_1\frac{\partial a_1}{\partial x_1}+k_2\frac{\partial x_2}{\partial x_1}\right)}\right)$$

$$+\frac{\partial x_2}{\partial x_1}\frac{(1-\theta)+\theta\exp(-x_1)(1+x_2)}{\left(k_1-(1-\theta)k_2\right)-\theta\exp(-x_1)\left(L_1\frac{\partial a_1}{\partial x_1}+k_2\frac{\partial x_2}{\partial x_1}\right)}<0$$

where $\frac{\partial x_2}{\partial k_2} = -\frac{2\exp(-x_1)L_2R_2\theta}{k_2{}^2 p_2\left((1-\theta)+\theta\exp(-x_1)\right)SQ_2(1+SQ_2)} < 0$.

x_2 increases in θ .

$\frac{dx_2}{d\theta} = \frac{\partial x_2}{\partial\theta} + \frac{\partial x_2}{\partial x_1}\frac{dx_1}{d\theta} > 0$ can only be shown to be true algebraically when k_2 is sufficiently large.

However numerically x_2 increases in θ for all values of k_2 ,

a_2 increases in R_2 , and k_2 , while decreasing in L_2 and p_2 .

For *Setting 1*, the proof is essentially the same as in part 3 above.

Setting 2 -- the continuous auditor

Recall that one way to write a_2 is $a_2 = \frac{R_2\exp(-x_2[x_1])}{p_2(1-\exp(-x_2[x_1]))}$

a_2 increases in R_2 .

$\frac{da_2}{dR_2} = \frac{\partial a_2}{\partial R_2} + \frac{\partial a_2}{\partial x_1}\frac{dx_1}{dR_2} > 0$ because

$\frac{dH[x_1]}{dR_2} = \theta\exp(-x_1)k_2\frac{\partial x_2}{\partial R_2} > 0 \Rightarrow \frac{dx_1}{dR_2} > 0$

$\frac{\partial a_2}{\partial x_1} = \frac{\partial a_2}{\partial x_2}\frac{\partial x_2}{\partial x_1} > 0$ where $\frac{\partial a_2}{\partial x_2} < 0$ and $\frac{\partial x_2}{\partial x_1} < 0$. Finally, $\frac{\partial a_2}{\partial R_2} > 0$.

a_2 increases in k_2 .

For this proof we use the form of $a_2 = \frac{R_2\exp(-x_2)}{p_2(1-\exp(-x_2))}$.

We have $\frac{da_2}{dk_2} = \frac{\partial a_2}{\partial k_2} + \frac{\partial a_2}{\partial x_2}\frac{dx_2}{dk_2} > 0$

where $\frac{\partial a_2}{\partial k_2}=0$, $\frac{\partial a_2}{\partial x_2}<0$ and $\frac{dx_2}{dk_2}<0$, which is shown to be true above in part 2.

a_2 decreases in L_2 .

For this proof we use the form of $a_2=\frac{R_2\exp(-x_2)}{p_2(1-\exp(-x_2))}$.

And $\frac{da_2}{dL_2}=\frac{\partial a_2}{\partial L_2}+\frac{\partial a_2}{\partial x_2}\frac{dx_2}{dL_2}<0$ where $\frac{\partial a_2}{\partial L_2}=0$, $\frac{\partial a_2}{\partial x_2}<0$ and $\frac{dx_2}{dL_2}>0$

which is shown to be true above in part 2.

a_2 decreases in p_2

$\frac{da_2}{dp_2}=\frac{\partial a_2}{\partial p_2}+\frac{\partial a_2}{\partial x_1}\frac{dx_1}{dp_2}<0$,which is proved similarly to $\frac{da_2}{dR_2}$ above.

Now we consider the top-right quadrant of the table:

x_2 decreases in R_1 , while a_2 increases in R_1 .

The proof that follows applies to both *Setting 1* and *Setting 2*.

x_2 decreases in R_1 .

$\frac{dx_2}{dR_1}=\frac{\partial x_2}{\partial R_1}+\frac{\partial x_2}{\partial x_1}\frac{dx_1}{dR_1}<0$ where $\frac{\partial x_2}{\partial R_1}=0,$ $\frac{\partial x_2}{\partial x_1}<0$ and $\frac{dx_1}{dR_1}>0$ and where

$\frac{dx_1}{dR_1}>0$ from the proofs above for the top-right quadrant.

a_2 increases in R_1 .

$\frac{da_2}{dR_1}=\frac{\partial a_2}{\partial R_1}+\frac{\partial a_2}{\partial x_1}\frac{dx_1}{dR_1}>0$ where $\frac{\partial a_2}{\partial R_1}=0,$ $\frac{\partial a_2}{\partial x_1}=\frac{\partial a_2}{\partial x_2}\frac{\partial x_2}{\partial x_1}>0$ and $\frac{dx_1}{dR_1}>0$

x_2 increases in p_1

$\frac{dx_2}{dp_1}=\frac{\partial x_2}{\partial p_1}+\frac{\partial x_2}{\partial x_1}\frac{dx_1}{dp_1}>0$ where $\frac{\partial x_2}{\partial p_1}=0,$ $\frac{\partial x_2}{\partial x_1}<0$ and $\frac{dx_1}{dp_1}<0$

a_2 decreases in p_1.

$$\frac{da_2}{dp_1} = \frac{\partial a_2}{\partial p_1} + \frac{\partial a_2}{\partial x_1}\frac{dx_1}{dp_1} < 0 \quad \text{where } \frac{\partial a_2}{\partial p_1} = 0,\ \frac{\partial a_2}{\partial x_1} = \frac{\partial a_2}{\partial x_2}\frac{\partial x_2}{\partial x_1} > 0 \text{ and } \frac{dx_1}{dp_1} < 0$$

x_2 decreases in L_1

$$\frac{dx_2}{dL_1} = \frac{\partial x_2}{\partial L_1} + \frac{\partial x_2}{\partial x_1}\frac{dx_1}{dL_1} < 0 \quad \text{where } \frac{\partial x_2}{\partial L_1} = 0,\ \frac{\partial x_2}{\partial x_1} < 0 \text{ and } \frac{dx_1}{dL_1} > 0$$

a_2 increases in L_1.

$$\frac{da_2}{dL_1} = \frac{\partial a_2}{\partial L_1} + \frac{\partial a_2}{\partial x_1}\frac{dx_1}{dL_1} > 0 \quad \text{where } \frac{\partial a_2}{\partial L_1} = 0,\ \frac{\partial a_2}{\partial x_1} = \frac{\partial a_2}{\partial x_2}\frac{\partial x_2}{\partial x_1} > 0 \text{ and } \frac{dx_1}{dL_1} > 0$$

x_2 increases in k_1 -- for both Settings 1 and 2, the following is true. $\frac{dx_2}{dk_1} = \frac{\partial x_2}{\partial k_1} + \frac{\partial x_2}{\partial x_1}\frac{dx_1}{dk_1} > 0$

where $\frac{\partial x_2}{\partial k_1} = 0$, $\frac{\partial x_2}{\partial x_1} < 0$ and $\frac{dx_1}{dk_1} < 0$

Finally, we consider the results in the bottom-left quadrant of the table. This proof only applies to *Setting 2* because the strategies in setting 1 do not depend on any of the period 2 payoff parameters.

x_1 increases in R_2

$$\frac{dx_1}{dR_2} > 0 \text{ because } \frac{dH[x_1]}{dR_2} = \theta \exp[-x_1] k_2 \frac{\partial x_2}{\partial R_2} > 0$$

a_1 decreases in R_2.

$$\frac{da_1}{dR_2} = \frac{\partial a_1}{\partial R_2} + \frac{\partial a_1}{\partial x_1}\frac{dx_1}{dR_2} < 0 \quad \text{where } \frac{\partial a_1}{\partial R_2} = 0,\ \frac{\partial a_1}{\partial x_1} < 0 \text{ and } \frac{dx_1}{dR_2} > 0$$

x_1 decreases in p_2

$$\frac{dx_1}{dp_2} < 0 \text{ because } \frac{dH[x_1]}{dp_2} = \theta \exp[-x_1] k_2 \frac{\partial x_2}{\partial p_2} < 0$$

a_1 increases in p_2.

$$\frac{da_1}{dp_2}=\frac{\partial a_1}{\partial p_2}+\frac{\partial a_1}{\partial x_1}\frac{dx_1}{dp_2}>0 \quad \text{where } \frac{\partial a_1}{\partial p_2}=0,\ \frac{\partial a_1}{\partial x_1}<0 \text{ and } \frac{dx_1}{dp_2}<0$$

x_1 increases in L_2

$$\frac{dx_1}{dL_2}>0 \text{ because } \frac{dH[x_1]}{dL_2}=\theta\exp[-x_1]k_2\frac{\partial x_2}{\partial L_2}>0$$

a_1 decreases in L_2 .

$$\frac{da_1}{dL_2}=\frac{\partial a_1}{\partial L_2}+\frac{\partial a_1}{\partial x_1}\frac{dx_1}{dL_2}<0 \quad \text{where } \frac{\partial a_1}{\partial L_2}=0,\ \frac{\partial a_1}{\partial x_1}<0 \text{ and } \frac{dx_1}{dL_2}>0$$

Only for Setting 2 is the following true because $\frac{\partial x_1}{\partial x_2}=0$ for the old auditor in *Setting 1* and thus x_1 is constant with respect to changes in k_2.

x_1 increases in k_2

$$\frac{dx_1}{dk_2}>0 \text{ because } \frac{dH[x_1]}{dk_2}=\theta\exp(-x_1)\left(k_2\frac{\partial x_2}{\partial k_2}+1+x_2\right)+(1-\theta)$$

$$=1-\theta+\theta\exp(-x_1)\left(-\frac{2\exp(-x_1)L_2R_2\theta}{k_2p_2\left((1-\theta)+\theta\exp(-x_1)\right)SQ_2\left(1+SQ_2\right)}+1+x_2\right)>0$$

where $SQ_2=\sqrt{1+\frac{4L_2R_2}{k_2p_2}\frac{\theta\exp(-x_1)}{\theta\exp(-x_1)+(1-\theta)}}>1$ and

$$\frac{2\exp(-x_1)L_2R_2\theta}{k_2p_2\left((1-\theta)+\theta\exp(-x_1)\right)SQ_2\left(1+SQ_2\right)}$$

$$=\frac{2\exp(-x_1)L_2R_2\theta}{k_2p_2\left((1-\theta)+\theta\exp(-x_1)\right)\left(SQ_2+1+\frac{4L_2R_2}{k_2p_2}\frac{\theta\exp(-x_1)}{\theta\exp(-x_1)+(1-\theta)}\right)}$$

$$=\frac{2\exp(-x_1)L_2R_2\theta}{\left(k_2p_2\left((1-\theta)+\theta\exp(-x_1)\right)\left(SQ_2+1\right)+4\theta\exp(-x_1)L_2R_2\right)}<1$$

a_2 decreases in k_1

$$\frac{da_2}{dk_1}=\frac{\partial a_2}{\partial k_1}+\frac{\partial a_2}{\partial x_1}\frac{dx_1}{dk_1}<0 \quad \text{where } \frac{\partial a_2}{\partial k_1}=0,\ \frac{\partial a_2}{\partial x_1}>0 \text{ and } \frac{dx_1}{dk_1}<0$$

a_1 decreases in k_2

$$\frac{da_1}{dk_2}=\frac{\partial a_1}{\partial k_2}+\frac{\partial a_1}{\partial x_1}\frac{dx_1}{dk_2}<0 \quad \text{where } \frac{\partial a_1}{\partial k_2}=0,\ \frac{\partial a_1}{\partial x_1}<0 \text{ and } \frac{dx_1}{dk_2}>0$$

Proof of Proposition 4:

Expected undetected fraud, given a continuing auditor is strictly less than that given a change in auditors.

When there is a change in auditors , expected undetected fraud for the first period is

$$\theta\exp\left(-x_1\right)a_1=\theta\exp\left(-Log\left(\frac{1}{2}\left(1+\sqrt{1+\frac{4\theta R_1L_1}{p_1k_1}}\right)\right)\right)\frac{k_1}{2\theta L_1}\left(1+\sqrt{1+\frac{4\theta R_1L_1}{p_1k_1}}\right)$$

$$=\theta\frac{1}{\left(\frac{1}{2}\left(1+\sqrt{1+\frac{4\theta R_1L_1}{p_1k_1}}\right)\right)}\frac{k_1}{2\theta L_1}\left(1+\sqrt{1+\frac{4\theta R_1L_1}{p_1k_1}}\right)=\frac{k_1}{L_1}.$$ Other undetected fraud amounts are derived in a similar fashion

$$EUF_{change}=\frac{k_1}{L_1}+\frac{k_2}{L_2} \text{ and } EUF_{cont}=\frac{k_1-(1-\theta)k_2}{L_1+\frac{k_2}{a_1\theta}(1+x_2)}+\frac{k_2}{L_2} < \frac{k_1}{L_1}+\frac{k_2}{L_2}$$

Thus, $EUF_{cont}<EUF_{change}$.

Proof of Proposition 5:

Audit risk in period 1 for both settings is $\theta\exp\left(-x_1\right)$ and as we have shown in Proposition 2, audit effort for the continuing auditor is higher in period 1. Thus, audit risk is lower for the continuing auditor.

Audit risk in period 2 for the 'new' auditor is equal to

$$AR_{2\,new} = \frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1-\theta)} \exp(-x_2) = \frac{\exp(-x_1)}{\theta \exp(-x_1) + (1-\theta)} \frac{2\theta}{\left(1 + \sqrt{1 + \frac{4R_2L_2}{p_2k_2} \frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1-\theta)}}\right)}$$

and audit risk for the continuing auditor in period 2 is

$$AR_{2\,cont} = \frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1-\theta)} \exp(-x_2) =$$

$$\left(\frac{\exp(-x_1)}{\theta \exp(-x_1) + (1-\theta)}\right) \frac{2\theta}{1 + \sqrt{1 + \frac{4L_2R_2}{k_2p_2} \frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1-\theta)}}} .$$

Because

$$\frac{d\left(\left(\frac{exp(-x_1)}{\theta\, exp(-x_1) + (1-\theta)}\right) \frac{2\theta}{1 + \sqrt{1 + \frac{4L_2R_2}{k_2p_2} \frac{\theta\, exp(-x_1)}{\theta\, exp(-x_1) + (1-\theta)}}}\right)}{dx_1} =$$

$$-\frac{exp(x_1)(1-\theta)\theta}{\left(exp(x_1)(1-\theta)+\theta\right)^2 \sqrt{\frac{exp(x_1)k_2p_2(1-\theta) + (k2p2 + 4L_2R_2)\theta}{k_2p_2\left(exp(x_1)(1-\theta)+\theta\right)}}} < 0 \text{ and } x_1^{old} < x_1^{cont} \ ,$$

$AR_{2\,new} < AR_{2\,cont}$.

References

Cameran, M., J.R. Francis, A. Marra, and A.Pettinicchio. 2015. Are there adverse consequences of mandatory auditor rotation? Evidence from the Italian experience. ***Auditing: A Journal of Practice and Theory*** 34(2): 1-24.

Corona, C. and R.S. Randhawa. 2010. The auditor's slippery slope: An analysis of reputational incentives. ***Management Science*** 56(6): 924-937.

Ghosh, A. and D. Moon. 2005. Auditor tenure and perceptions of audit quality. ***The Accounting Review*** 80(2): 585–612.

Gipper, B., L. Hail, and C. Leuz. 2017. On the economics of audit partner tenure and rotation: evidence from PCAOB data. Stanford University working paper.

Johnson, V.E., I.K. Khurana, and K.J. Reynolds. 2002. Audit-Firm tenure and the quality of financial reports. ***Contemporary Accounting Research*** 19(4): 637–660.

Knechel, R.W. and A. Vanstraelen. 2007. The relationship between auditor tenure and audit quality implied by going concern opinions. ***Auditing: A Journal of Practice & Theory*** 26(1): 113–131.

Laux, V., and D.P. Newman. 2010. Auditor liability and client acceptance decisions. ***The Accounting Review*** 85(1): 261-285.

Louwers, T.J. 1998. The relation between going-concern opinions and the auditor's loss function. ***Journal of Accounting Research*** 36(1): 143–156.

Myers, J.N., L.A. Myers, and T.C. Omer. 2003. Exploring the term of the auditor-client relationship and the quality of earnings: A case for mandatory auditor rotation. ***The Accounting Review*** 78(3): 779-799.

Newman, D.P., E.R. Patterson, and J.R. Smith. 2001. The influence of potentially fraudulent reports on audit risk assessment and planning. ***The Accounting Review*** 76(1): 59-80.

Patterson, E.R., and J. Noel. 2003. Audit strategies and multiple fraud opportunities of misreporting and defalcation. ***Contemporary Accounting Research*** 20(3): 519-549.

Patterson, E.R., and J.R. Smith. 2003. Materiality uncertainty and earnings misstatement. ***The Accounting Review*** 78(3): 819-846.

Patterson, E.R. and J.R. Smith. 2007. The effects of Sarbanes-Oxley on auditing and internal control strength. ***The Accounting Review*** 82(2): 427-455.

Patterson, E.R. and J.R. Smith. 2016. The strategic effects of Auditing Standard No. 5 in a multi-location setting. ***Auditing: A Journal of Practice & Theory*** 35(1): 119-138.

Patterson, E.R., J.R. Smith, and S.L. Tiras. 2017. The interrelation between audit quality and managerial reporting choices and its effects on financial reporting quality. Indiana University working paper.

Reid, L.C. and J.V. Carcello. 2017. Investor reaction to the prospect of mandatory audit firm rotation. ***The Accounting Review*** 92(1): 183-211.

Smith, J.R., S.L. Tiras, and S. Vichitlekarn. 2000. The Interaction between Internal Control Assessment and Substantive Testing in Audits for Fraud. ***Contemporary Accounting Research*** 17(2): 327-356.