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The Effects of Auditor Tenure on Fraud and Its Detection

Abstract

We examine the strategic effects of auditor tenure on the auditor’s testing strategy and the manager’s inclination to commit fraud. Most empirical studies find that longer audit tenure improves audit quality. Proponents of restricting auditor tenure express concern that longer tenures impair auditor independence and a ‘fresh look’ from a new auditor results in higher audit quality. Validating this concern requires testing whether the observed difference in audit quality between a continuing auditor and a change in auditor is less than the expected difference in audit quality. The findings of our model provide the theoretical guidance necessary for developing such tests. Our results show that audit risk (probability that fraud goes undetected) is lower in both periods for the continuing auditor than with a change in auditor. More importantly, we show that across both periods, expected undetected fraud is lower for the continuing auditor than with a change in auditor.
I. INTRODUCTION

Security market regulators have long debated the potential effects of firms hiring the same auditor over multiple periods. On one hand, some believe that as the length of time an auditor stays with a firm increases, the auditor becomes ‘too cozy’ with the company. The concern is that this familiarity may reduce the auditor’s skepticism about potential misstatements and, in turn, reduce the auditor’s effectiveness in detecting misstatements. On the other hand, some argue that auditing a firm over multiple periods increases the auditor’s knowledge of the company’s operations and system processes, thereby increasing his effectiveness in detecting misstatements. Notable frauds, such as Equity Funding, Comptronix, and Crazy Eddie that have occurred over several years, seem to support the first conjecture.

Interpreting these observable outcomes might be misleading without first considering the underlying economics of how multi-period audits affect the auditor’s choice of effort and its related impact on deterring and detecting fraud. The observation of an audit failure provides limited evidence regarding the (often unobservable) action choices of the auditor and manager that together determine the risk of undetected misstatements. The purpose of this paper, therefore, is to examine the strategic effects of multi-period audits, where the accumulation of audit evidence over time affects the auditor’s testing strategy and the manager’s inclination to commit fraud.

Proponents of restricting auditor tenure suggest that a ‘fresh look’ at a company by an auditor would result in higher quality audits. The concern expressed in PCAOB CONCEPT RELEASE (PCR) No. 2011-006: Auditor Independence and Audit Firm Rotation is that over time, the auditor can lose his ability to exercise professional skepticism despite adherence to the rules of independence. For regulators, practitioners, or accounting researchers to assess whether externalities exist that could potentially be attributable to an impairment in independence, the first step would be to assess the equilibrium strategies of a continuing auditor versus a change in
auditor strictly from the players maximizing their expected payoffs, where none of the payoffs violate the independence rules. Our model provides such a baseline.

Defond and Zhang (2014) point out that most studies find that longer audit tenure improves audit quality. Examples of such evidence include findings that longer audit tenure is associated with lower levels of discretionary accruals (Johnson, Khurana, and Reynolds 2002; Myers, Myers, and Omer 2003), higher earnings response coefficients (Ghosh and Moon 2005), and more issuances of going concern opinions (Louwers 1998; Knechel and Vanstraelen 2007). More directly, Cameran, Francis, Marra, and Pettinicchio (2015) document that earnings quality improves with longer audit tenure in Italy, where auditor rotation has been mandated since 1975. Further, Reid and Carcello (2017) document that the market perceives audit quality to be higher with longer audit tenure.

The challenge in interpreting these findings as evidence that longer audit tenure does not impair independence is that these studies are not able to calibrate the extent that audit quality would be expected to be higher with a continuing auditor, had there not been any impairment of independence. If there is indeed an impairment of independence, as is the concern expressed by the PCAOB and others, empirical researchers would be able to document such impairments only by testing whether the observed difference in audit quality between a continuing auditor and a change in auditor is less than the expected difference in audit quality. The equilibrium findings of our model provide the theoretical guidance necessary for developing such tests.

Our study investigates a two-period audit across two settings. In the first setting, we have a change in auditors from the first period to the second. In the second setting, one auditor continues auditing from one period to the next. In both settings, the auditor chooses audit effort while the manager chooses an amount of fraud, given he is the ‘dishonest’ manager type, where the players in the two settings possess the same payoff parameters. The prior probability that the manager is
dishonest is common knowledge, and equal to the risk of fraud that the manager is dishonest in the period 1. Furthermore, the game continues to period 2 only if fraud is not discovered at the end of period 1.

In both settings, the auditor in period 2 updates the probability that the manager is dishonest, based on period 1 audit effort and the public knowledge that no fraud was discovered in period 1. For the continuing auditor, he has direct knowledge of the audit testing performed in period 1. For the ‘new’ auditor in the change setting, his knowledge of period 1 audit effort is perfectly inferred, because of our simplifying assumption that the period 1 ‘old’ auditor’s costs and the manager’s payoffs are common knowledge.¹

The key difference between the two settings is that the continuing auditor not only has information acquired in period 1 to use in the period 2 audit, but he chooses the amount of period 1 audit effort to control expected audit costs across both periods. At the beginning of period 1, the continuing auditor plans how he will audit in each period in order to minimize his total expected costs, taking into consideration how he anticipates the manager will react to each period's effort choice. In contrast to when there is a change in auditors, the old auditor has no incentive to consider the effect of his period 1 effort choice on period 2 outcomes and costs.²

Overall, our results show that audit risk, which is the probability that fraud goes undetected, is lower in both periods for the continuing auditor than with a change in auditor. More importantly, our results also show that across both periods, expected undetected fraud is lower for the continuing auditor than with a change in auditor. However, we also find that the continuing auditor chooses less audit effort in the later period, relative to the effort choice of the new auditor

¹ In practice, the new auditor may not be able to perfectly infer period 1 audit effort, because he has only limited information of the manager’s and old auditor’s payoffs in period 1. However, this simplifying assumption biases against our results and highlights the differences in equilibria between the two settings.

² He also has no ability to do so, because at the beginning of period 1 the old auditor only knows that his engagement with the firm ends and has no knowledge of who the new auditor would be.
in the change setting. This result derives from the continuing auditor allocating more effort in period 1 than would be allocated with a change in auditors. While the appearance that a continuing auditor choosing less audit effort in the later period than does the new auditor might suggest an impairment of independence, this results strictly from the players maximizing their expected payoffs and not due the continuing auditor becoming ‘too cozy’ with the client.

This paper significantly adds to our knowledge of strategic auditing. Most strategic auditing models primarily involve just one period and do not take into account how the auditor might strategically use information from previous audit periods in the current period. Some models, such as Patterson and Smith (2007, 2016) and Smith, Tiras, Vichitlekarn (2000) do consider two-stage audits that allow the auditor to obtain information by testing internal controls in a first stage and then use that information for substantive testing in the second stage. Our two-period model differs from these two-stage models in that the effort in each period of our two-period model is designed to detect fraud. Because we have two distinct periods, we can compare the two settings: a continuing auditor that audits both periods and a setting in which the audit in each period is performed by a different auditor.

Corona and Randhawa (CR) (2010) also examine a two-period strategic audit setting, in which the manager in each period chooses fraud or no fraud and the auditor chooses an audit report. They demonstrate a circumstance in which an auditor that does not identify fraud in one period might not report fraud that is detected in a later period because it would highlight their failure in the previous period. By construction, the auditor in their model does indeed lack independence. But because their model does not provide any penalties or other disincentives for

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3 Other related studies include Newman, Patterson, and Smith (2001), Patterson and Smith (2003), Patterson and Noel (2003), and more recently, Laux and Newman (2008) and Patterson, Smith, and Tiras (2017).
the auditor to hide the fraud, their model cannot address the potential benefit to multi-period auditing.

The remainder of the paper is structured as follows. In section II, we lay out the two settings of our two-period model. Section III provides an equilibrium analysis while section IV provides a comparative analysis of both settings. Section V compares expected undetected fraud and audit risk across the two settings and Section VI relates our results to empirical insights. Section VII concludes the paper.

II. A TWO-PERIOD MODEL OF AUDITING

We consider a two-period model of auditing that includes an auditor and a possibly dishonest manager in two settings. The first setting depicts a situation where the ‘old’ auditor of period 1 is replaced with a ‘new’ auditor in period 2. In the second setting, one ‘continuing’ auditor audits both periods and chooses audit effort in the first period to minimize his expected audit costs across both periods.

Public information is revealed at the end of period 1 in both settings. At the end of period 1 the players know whether or not the auditor discovered fraud in period 1. If fraud is not detected, the game continues to period 2; otherwise, it ends at the end of period 1. The updated probability that the manager is dishonest depends upon the auditor’s effort in period 1. In the second (continuing) setting, that updated probability is affected by that same auditor’s effort choice in the first period.

In both settings, the auditor(s) and manager have the same basic payoffs while the expected payoffs for the auditor in periods 1 and 2 depend on whether or not there has been a change in auditors. The manager is dishonest with prior probability \( \theta \). The dishonest manager chooses an amount of fraud equal to \( a_1 \) and \( a_2 \) in periods 1 and 2, respectively. He receives net benefits of
and \( a_2 R_2 \) if no fraud is detected by the auditor, and his penalty for fraud detection is \(-\frac{a_i^2}{2} R_i \) and \(-\frac{a_i^2}{2} p_i \). The payoffs \( a_1 R_1 \) and \( a_2 R_2 \) represent the benefits of fraud less personal costs for committing fraud such as the manager's designs to circumvent the audit strategy. The rewards and penalties for periods 1 and 2 are independent across both periods.

The auditor in period 1 chooses audit effort equal to \( x_1 \) and the auditor in period 2 chooses audit effort equal to \( x_2 \). Given the manager is dishonest, the auditor detects fraud in periods 1 and 2 with probabilities \( 1 - \exp(-x_1) \) and \( 1 - \exp(-x_2) \), respectively. The auditor incurs a liability cost in each period for undetected fraud of \( a_1 L_1 \) and \( a_2 L_2 \), while \( k_1 \) and \( k_2 \) are cost multipliers for audit effort that yield a cost of audit effort in each period of \( x_1 k_1 \) and \( x_2 k_2 \).

**The Manager's Expected Payoff**

We work backwards to derive the manager's expected payoff by starting with period 2 and then incorporating it into period 1. The form of the manager's expected payoff does not change between settings. It remains the same whether or not a change in auditors occurs at the end of period 1. His fraud strategy can differ based upon which setting we analyze but the basic construct of his expected payoff remains the same.

The manager's period 2 expected payoff is as follows.

\[
M_2 = a_1 R_1 + a_2 R_2 \exp(-x_2) - \frac{a_i^2}{2} p_2 \left(1 - \exp(-x_2)\right) \tag{1}
\]

\( M_2 \) is the dishonest manager’s expected payoff in period 2, when a period 1 fraud goes undetected. Note that the reward from period 1 \( a_i R_i \) is retained in period 2. If fraud in period 2 is detected, we assume that the manager has already consumed the benefits of any undetected fraud committed in
period 1. We assume that the penalty for detected fraud in period 2 is independent of any undetected fraud in period 1.

The dishonest manager's expected payoff at the beginning of period 1 is as follows.

\[ M_1 = M_2 \exp(-x_i) - \frac{a^2}{2} p_1 (1 - \exp(-x_i)) \]  

\[ (2) \]

\( M_1 \) reflects the fact that any reward from fraud over the two periods occurs only if the period 1 fraud goes undetected. Thus, if the period 1 fraud goes undetected, which occurs with probability \( \exp(-x_i) \), the manager expects a payoff associated with \( M_2 \). On the other hand, if the auditor detects the period 1 fraud, the manager receives zero benefit from fraud and is penalized \( \frac{a^2}{2} p_1 \).

Next we consider the auditor's expected payoffs where its characterization depends on whether or not a change in auditors has occurred at the end of period 1.

The Auditor's Expected Payoff

Setting 1: A Change in Auditors

Assume there is a change in auditors at the end of period 1 that everyone knows will occur at the start of the game. In this case, each auditor maximizes his own expected payoff, the new auditor for period 2 and the old auditor for period 1. Except for the undiscovered fraud in the first period, no public information from the first period is accrued by the new auditor in period 2. However, the second period new auditor can infer the effort choice of the old auditor because we assume that the new auditor knows the payoff parameters of the manager and the old auditor.\(^4\)

Thus, if no fraud is detected in the first period, the new auditor updates the probability that the manager is dishonest given the inferred choice of period 1 audit effort.

\[^4\] Details of the equilibrium that excludes knowledge regarding the old auditor payoffs are available from the authors upon request.
In order to compare the two settings, we must assume that the expected liability costs resulting from undetected fraud in period 1 are independent from the expected liability costs resulting from undetected fraud in period 2. In other words, we ignore the impact of detection probability in period 2 on the costs facing the period 1 auditor (in either setting). We assume that the likelihood that undetected fraud in period 1 is subsequently brought to light in the future is exogenously captured by the parameter $L_1$.

Below we express the expected payoffs in periods 1 and 2 for the new ($A_{2\text{new}}$) and old ($A_{1\text{old}}$) auditors.

**New auditor – second period**

$$A_{2\text{new}} = -\Pr(D \mid ND) a_2 L_2 \exp(-x_2) - x_2 k_2$$  \hspace{1cm} (3)

where $\Pr(D \mid ND) = \frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1 - \theta)}$ is the probability of the dishonest type manager ($D$), given that fraud was not discovered ($ND$) in period 1.

The expected payoff for the predecessor auditor follows.

**Old auditor – first period**

$$A_{1\text{old}} = -\theta a_1 L_1 \exp(-x_1) - x_1 k_1$$  \hspace{1cm} (4)

The sequence of events for setting 1 is given in Figure 1.

**Setting 2: A Continuing Auditor**

Next we assume that the same auditor audits both period 1 and period 2. We start at the end of the game in formulating the auditor's total expected payoff. The continuing auditor incurs all the liability and effort costs for periods 1 and 2. The auditor's second period payoff is

$$A_{2\text{cont}} = -\Pr(D \mid ND) \left( L_1 a_1 + L_2 a_2 \exp(-x_2) \right) - x_2 k_2$$  \hspace{1cm} (5)
where \( \Pr(D \mid ND) = \frac{\theta \exp(-x_i)}{\theta \exp(-x_i) + (1 - \theta)} \) is the probability of the dishonest type, given no fraud is detected in period 1.\(^5\) The continuing auditor maintains his expected loss \( L_i a_i \) for an undiscovered first period fraud whether or not fraud is discovered in the second period. Moreover, because the auditor audits both periods, he can choose \( x_1 \) and \( x_2 \) to minimize his costs across both periods and he can also use the amount of period 1 audit effort to infer the likelihood that the manager is the dishonest type if no fraud is discovered in period 1. A key advantage of auditing both periods is the ability to choose \( x_1 \) that provides the best benefits in choosing \( x_2 \), including an \( x_1 \) that will provide optimal updating of information for period 2.

Then we have the auditor's first period expected payoff of

\[
A_{\text{cont}} = (\theta \exp(-x_1) + (1 - \theta)) A_{\text{cont}} - x_i k_1
\]

(6)

where \( \theta \exp(-x_1) + (1 - \theta) \) is the unconditional probability that no fraud is detected in period 1, given the choice of audit effort \( x_1 \).

In order to compare the two settings, it is important that the payoffs are the same across the two settings except that the auditor in the continuing setting is able to optimize his choices of \( x_1 \) and \( x_2 \) across time because he audits both periods. It is important, then, that the expected payoff to the continuing auditor in (6) (the total expected payoff over the two periods) be equal to the expected payoffs of the old and new auditors in the change setting as long as \( x_1, x_2, a_1, \) and \( a_2 \) are the same. To see that this is the case, we can add the expected payoff in (3) to the expected payoff in (4) multiplied by the probability that the game proceeds to the second period and we

\(^5\) In this expression and all those that follow, ‘cont’ in the subscript is short for ‘continuing.’
obtain expression (6). The only differences between the two settings, then, derive from differences in the auditor and manager strategies.

The sequence of events for setting 2 is given in Figure 2. Our next section will describe our equilibria for the two settings.

III. EQUILIBRIUM ANALYSIS

Setting 1: A Change in Auditors

With a change in auditors, we use expressions (1) – (4) to solve for our equilibrium. First note that the manager's expected payoff at the beginning of the game is

\[ M_i = \left( a_i R_i + a_2 R_2 \exp(-x_2) - \frac{a_i^2}{2} p_2 \left(1 - \exp(-x_2)\right) \right) \exp(-x_i) - \frac{a_i^2}{2} p_i \left(1 - \exp(-x_i)\right) \] (7)

which yields first order conditions

\[ \frac{dM_i}{da_i} = R_i \exp(-x_i) - a_i p_i \left(1 - \exp(-x_i)\right) = 0 \] (8)

and

\[ \frac{dM_i}{da_2} = \left( R_2 \exp(-x_2) - a_2 p_2 \left(1 - \exp(-x_2)\right) \right) \exp(-x_i) = 0. \] (9)

Conditions (8) and (9) yield fraud choices of \( a_i \) and \( a_2 \), respectively.

\[ a_i = \frac{R_i \exp(-x_i)}{p_i \left(1 - \exp(-x_i)\right)} \quad \text{and} \quad a_2 = \frac{R_2 \exp(-x_2)}{p_2 \left(1 - \exp(-x_2)\right)} \] (10)

Expression (10) says that the manager selects the amount of fraud equal to the ratio of its expected marginal benefit to its expected marginal penalty. Thus, as audit effort increases the relative expected benefit of fraud decreases.

The auditor's first order conditions follow from expressions (3) and (4).
\begin{align}
\frac{dA_{\text{old}}}{dx_1} &= \theta a_1 L_1 \exp(-x_1) - k_1 = 0 \\
\frac{dA_{\text{new}}}{dx_2} &= \Pr(D1ND)a_2 L_2 \exp(-x_2) - k_2 = 0
\end{align}

Conditions (11) and (12) yield effort choices of $x_1$ and $x_2$, respectively.

\begin{align}
x_1 &= \log \left[ \frac{a_1 L_1 \theta}{k_1} \right] \text{ and } x_2 = \log \left[ \frac{a_2 L_2}{k_2} \frac{\theta \exp(-x_1)}{\theta \exp(-x_1)+(1-\theta)} \right]
\end{align}

We use expressions (10) and (13) to obtain our equilibrium, which is stated in Proposition 1.

**Proposition 1:** Given a change in auditors, the unique equilibrium strategies for the manager, the old auditor and the new auditor are as follows.

**Manager:**

\[ a_1 = \frac{R_1}{p_1 \left( -1 + \frac{1}{2} \left( 1 + \sqrt{1 + \frac{4 L_1 R_1}{k_1 p_1 \theta}} \right) \right)} \text{ and } \]

\[ a_2 = \frac{R_2}{p_2 \left( -1 + \frac{1}{2} \left( 1 + \sqrt{1 + \frac{4 L_2 R_2}{k_2 p_2 \theta} \theta \exp(-x_1)} \frac{\theta \exp(-x_1)+(1-\theta)}{\theta \exp(-x_1)+(1-\theta)} \right) \right)} \]

**Auditor$_{old}$**

\[ x_1 = \log \left( \frac{1}{2} \left( 1 + \sqrt{1 + \frac{4 \theta R_1 L_1}{p_1 k_1}} \right) \right) \]

**Auditor$_{new}$**

\[ x_2 = \log \left( \frac{1}{2} \left( 1 + \sqrt{1 + \frac{4 R_2 L_2}{p_2 k_2} \frac{\theta \exp(-x_1)}{\theta \exp(-x_1)+(1-\theta)}} \right) \right) \]

*(All proofs are in the Appendix)*

This provides a benchmark to compare the equilibrium with a continuous auditor.
Setting 2: A Continuing Auditor

Next we derive the equilibrium when the auditor continues as the auditor of period 2 after auditing period 1. The characterization of the fraud amounts in expression (10) remains the same in this setting where

\[ a_1 = \frac{R_1 \exp(-x_1)}{p_1(1-\exp(-x_1))} \quad \text{and} \quad a_2 = \frac{R_2 \exp(-x_2)}{p_2(1-\exp(-x_2))}. \]

In this setting, we first solve for period 2 audit effort and then period 1 audit effort because information has been revealed between periods. When no fraud is detected in period 1, the auditor uses his period 1 information to reassess the probability that the manager is dishonest and knowing this is the case chooses his effort in period 1, accordingly, to minimize his total expected costs across both periods. Based on his period 2 expected payoff in expression (5), the auditor's period 2 first order condition is equal to the following.

\[ \frac{dA_{2\text{cont}}}{dx_2} = \Pr(D \mid ND)(L_2a_2\exp[-x_2]) - k_2 \Rightarrow \]

\[ x_2 = \log \left[ \frac{a_2L_2}{k_2} \frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1-\theta)} \right] = \log \left[ \frac{L_2a_2}{k_2} \frac{\theta}{\theta \exp[-x_1] + (1-\theta)} \right] - x_1 \]

Then we substitute for \( x_2 \) into \( A_1 \) and with some rearranging we obtain the following.

\[ A_{1\text{cont}} = \begin{pmatrix} -L_1a_1\theta \exp(-x_1) \\ -(\theta \exp(-x_1) + (1-\theta))k_2 \left(1-x_1 + \log \left[ \frac{L_2a_2}{k_2} \frac{\theta}{\exp(-x_1) + (1-\theta)} \right] \right) \end{pmatrix} - x_1k_1 \]

which we use to obtain the first order condition for \( x_1 \).

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\( ^6 \) The equilibrium, given a continuing auditor, is technically defined implicitly based on four equilibrium conditions. Due to the nature of our payoffs a fully explicit solution is not possible. However for convenience and to enable us to more intuitively compare the equilibrium to one with a change in auditors, we define each of the players strategies as functions of the auditor's period 1 effort choice \( x_1 \).
Next we substitute for $a_1$ into (16) and solve for $x_2$.

\[
x_2 = \log \left[ \frac{1}{2} \left( 1 + \sqrt{1 + \frac{4L_2R_2}{k_2p_2} \frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1-\theta)}} \right) \right],
\]

which we need to obtain $a_2$.

\[
a_2 = \frac{R_2}{p_2 \left( -1 + \frac{1}{2} \left( 1 + \sqrt{1 + \frac{4L_2R_2}{k_2p_2} \frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1-\theta)}} \right) \right)}
\]

The form of the equilibrium strategies for $a_2$ and $x_2$ in the continuous case is the same as those for the new auditor, given a change in auditors. However, the values are different due to the differing audit effort strategies in period 1 for the two settings.

Unlike the old auditor who has no ability and no incentive to control or minimize period 2 audit costs, the continuing auditor chooses his first period audit effort knowing that he will face a possibly dishonest manager in period 2 if no fraud is detected in period 1. Based on expression (16), we have the equilibrium condition for $x_1$, labeled $H[x_1]$.

\[
H[x_1] = \theta \exp(-x_1) \left( L_1a_1[x_1] + k_2 + k_2x_2[x_1] \right) + (1-\theta)k_2 - k_1 = 0
\]

where $x_2[x_1]$ is the value of $x_2$ derived in expression (17) and $a_1[x_1] = \frac{R_1 \exp(-x_1)}{p_1 \left( 1 - \exp(-x_1) \right)}$.

Expression (19) implicitly defines the equilibrium value of $x_1$ because we cannot explicitly solve for $x_1$. Furthermore, each of our equilibrium strategies are defined in terms of $x_1$. 

\[\text{13}\]
Proposition 2: When the auditor continues from period 1 to period 2 the unique equilibrium strategies for the manager and auditor are as follows.

Manager

\[ a_1 = \frac{R_1 \exp(-x_1)}{p_1(1 - \exp(-x_1))} \quad \text{and} \]
\[ a_2 = \frac{R_2}{p_2 \left[ -1 + \frac{1}{2} \left( 1 + \sqrt{1 + \frac{4L_2R_2}{k_2p_2} \theta \exp(-x_1) + (1 - \theta)} \right) \right]} \]

Auditor

\[ x_1 = \text{Log} \left[ \frac{\theta \left( L_2 a_1 [x_1] + k_2 (1 + x_2 [x_1]) \right)}{k_1 - (1 - \theta)k_2} \right] \quad \text{and} \]
\[ x_2 = \text{Log} \left[ \frac{1}{2} \left( 1 + \sqrt{1 + \frac{4L_2R_2}{k_2p_2} \theta \exp(-x_1) + (1 - \theta)} \right) \right] \]

where \( x_1 \) satisfies \( H\left[ x_1 \right] = 0 \).

The equilibrium defined in Proposition 2 has several interesting characteristics. First, we see that \( x_1 \) and \( x_2 \) are inversely related, all else held constant. In Proposition 2, \( x_1 \) decreases in \( x_1 \) without regard to a change in payoff parameters. If no fraud is discovered in period 1 for a high \( x_1 \) then the probability that the manager is the dishonest type is smaller and the auditor may find a reduction in \( x_2 \) to be optimal. Of course, the equilibrium relation between \( x_1 \) and \( x_2 \) must be considered with regard to a change in a payoff parameter. We explore this more thoroughly in the next section that provides a comparative analysis.

Second, expression (19) that implicitly defines the equilibrium value of \( x_1 \) shows that the first period audit effort \( x_1 \) is greater than what would occur in period 1 when we have a change in auditors. Note that expression (19) can be written as
where $H_{1_{\text{period}}}[x_1] = \theta \exp(-x_1) L_i a_i[x_1] - k_i$ is the equilibrium condition for a single period game or alternatively, for the old auditor in period 1. As part of our proof to Proposition 2, we show that expression (19) decreases in $x_1$ and expression (20) shows that point-wise it has larger values than that for a single period game. Thus, the continuous auditor chooses greater period 1 audit effort than the auditor who knows he is leaving at the end of period 1, ceteris paribus. As a result, the amount of fraud committed in the first year is less for a continuing auditor relative to an auditor who only audits period 1.

Finding fraud sooner rather than later is more valuable to the continuing auditor, which translates into a smaller amount of period 2 effort relative to the new auditor. (See Corollary 1). Despite the new auditor's ability to update based on the inferred period 1 audit effort, the continuing auditor chooses lower audit effort in period 2 because period 1 audit effort is higher and he estimates a lower probability of fraud going into period 2. Due to the lower assessment of the probability of fraud for period 2, one might infer that there has been a reduction in skepticism. Furthermore, the loss of skepticism appears to be more for the continuing auditor versus the new auditor. However, the lower probability assessment of fraud for the continuing auditor at the beginning of the second period is solely due to his choice of higher audit effort in the first period and the fact that fraud was not detected.

Third, we see that allowing the auditor to continue auditing in period 2 effectively reduces the marginal cost of his period 1 audit effort by an amount equal to $(1 - \theta)k_2$. As the prior probability that the manager is the honest type increases, the continuing auditor's first period
marginal cost of audit effort decreases. And, in the limit as $k_2 \to \frac{k_1}{1-\theta}$, audit effort approaches ‘perfect auditing’ in period 1.

These results, which relate to comparisons within each period are formally presented in Corollary 1.

**Corollary 1:** For the continuing auditor relative to a change in auditors:

1. The amount of period 1 fraud is smaller and the amount of period 1 audit effort is larger.

2. The amount of period 2 fraud is larger and the amount of period 2 audit effort is smaller.

Next we perform a comparative analysis on our equilibrium results, given various changes in payoff parameters.

**IV. COMPARATIVE ANALYSIS OF CHANGES IN THE AUDITOR’S AND MANAGER’S PAYOFFS**

In this section, we consider how the players' strategies change when the payoff parameters change. For setting 1 (the change setting), the auditor’s and manager’s period 1 strategies ($x_1$ and $a_1$ respectively) depend only upon the period 1 payoff parameters. But, because the auditor can update the likelihood of the manager’s dishonesty based upon the first period outcome, the second period strategies for the auditor and manager depend upon period 1 parameters. For setting 2 (the continuing setting), all four strategies are jointly determined because the manager’s first period fraud choice is determined by the auditor’s first period effort choice. And, the auditor’s first period effort choice is jointly determined with the auditor’s second period effort choice. This is demonstrated by the equilibrium condition for $x_1$ provided in expression (19).
Proposition 3: The following table demonstrates how changes in the parameters, $L_i$, $R_i$, $p_i$, and $k_i$ for $i \in \{1, 2\}$ affect the auditor’s effort choice in periods 1 and 2 and the manager’s fraud choice in periods 1 and 2 for the continuing and change settings. With the exception of the effects of changes in period 2 payoff parameters on period 1 strategies (the shaded cells), the comparative results are the same for both settings. The period 2 payoff parameters do not affect the auditor’s or manager’s period 1 strategies in the change setting.

<table>
<thead>
<tr>
<th>Increase in Period 1 payoff parameters</th>
<th>Effect on strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_1$ for continuing and change settings</td>
</tr>
<tr>
<td>$L_1$</td>
<td>+</td>
</tr>
<tr>
<td>$R_1$</td>
<td>+</td>
</tr>
<tr>
<td>$p_1$</td>
<td>−</td>
</tr>
<tr>
<td>$k_1$</td>
<td>−</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Increase in Period 2 payoff parameters</th>
<th>$x_1$ for continuing setting</th>
<th>$x_1$ for change setting</th>
<th>$\alpha_1$ for continuing setting</th>
<th>$\alpha_1$ for change setting</th>
<th>$x_2$ for continuing setting</th>
<th>$\alpha_2$ for continuing setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_2$</td>
<td>+</td>
<td>0</td>
<td>−</td>
<td>0</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>$R_2$</td>
<td>+</td>
<td>0</td>
<td>−</td>
<td>0</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$p_2$</td>
<td>−</td>
<td>0</td>
<td>+</td>
<td>0</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>$k_2$</td>
<td>+</td>
<td>0</td>
<td>−</td>
<td>0</td>
<td>−</td>
<td>+</td>
</tr>
</tbody>
</table>

(proof in the Appendix)

The above table is constructed as four quadrants. The top-left quadrant relates to the effects that changes in period 1 payoff parameters have on the period 1 auditor and manager strategies. Similarly, the bottom-right quadrant relates to how changes in period 2 payoff parameters affect period 2 auditor and manager strategies. Note that these two quadrants are identical. That is because changes in the payoff parameters within a given period induce the intuitive changes in the auditor’s and manager’s strategies. Increases in the auditor’s litigation exposure in a given period ($L_i$) increases audit effort and decreases fraud in that period whereas
increases in the auditor’s cost parameter for auditing in a given period \( (k_i) \) decreases audit effort and increases fraud in that period. On the other hand, the increases in the manager’s benefit parameter in a given period \( (R_i) \) increase fraud and audit effort in that period and increases in the manager’s penalty parameter in that period \( (p_i) \) decrease fraud and audit effort in that period. These results are true across both settings.  

Next consider the top-right quadrant of the table. Notice that every effect is the exact opposite of the effect in the top-left quadrant. That is because any increase in a period 1 strategy will be traded off for a decrease in a period 2 strategy and vice versa. This is true for both settings because in both settings, the period 2 auditor updates the probability that the manager is dishonest based on the fact that the period 1 auditor did not detect fraud. Of course, since the continuing auditor exerts more audit effort in period 1, his updating places a higher probability that the manager is honest than the new auditor in the change setting. 

The bottom left quadrant of the table illustrates the differential impact of the continuing auditor on auditing and fraud. In the continuing auditor setting, the auditor’s payoffs and strategies are tied across the two periods. The auditor is allocating effort optimally across the two periods in order to minimize the overall costs of undetected fraud. The old auditor in the change setting is incapable of this strategic choice. As a result, the period 2 payoffs, which affect the period 2 strategies do not affect the period 1 strategies for the auditor or the manager in the change setting.

The period 1 strategies in the change setting (see Proposition 1) do not involve the period 2 parameters. So changes in these parameters do not affect period 1 strategies of the auditor or manager in the change setting. For the continuing auditor setting, the period 1 strategy for the auditor is tied to the period 2 strategies for all parameters, except \( k_2 \). With the exception of \( k_2 \),

increases in the incentives for audit effort in period 2 also drive increases in the incentives for audit effort in period 1.

For $L_2$, $R_2$, and $p_2$, we assess changes in $x_i$ by using the equilibrium condition,

$$H[x_i] = 0$$

in expression (19).

$$H[x_i] = \theta \exp(-x_i)(L_i a_i[x_i] + k_2 + k_2 x_i[x_i]) + (1 - \theta) k - k_i = 0$$

Because $\frac{dH[x_i]}{dx_i} < 0$, we know that the sign of $\frac{dx_i}{dg} = \frac{dH[x_i]/dx_i}{-dH[x_i]/dx_i}$ depends only on the sign of $dH[x_i]/dg$ where $g$ stands for a general payoff parameter. Consequently, as an example,

$$\frac{dx_i}{dR_2} > 0 \text{ because } \frac{dH[x_i]}{dR_2} > 0 \text{ where } \frac{dx_2[x_i]}{dR_2} > 0.$$  Moreover,

$$\frac{da_1}{dR_2} = \frac{\partial a_1}{\partial R_2} + \frac{\partial a_1}{\partial x_1} \frac{dx_1}{dR_2} < 0 \text{ where } \frac{\partial a_1}{\partial R_2} < 0, \frac{\partial a_1}{\partial x_1} < 0 \text{ and } \frac{dx_1}{dR_2} > 0.$$  The results for $\frac{dx_1}{dL_2}$ and $\frac{dx_1}{dp_2}$ follow the same logic.

For changes in $k_2$,

$$\frac{dx_i}{dk_2} = \frac{dH[x_i]/dk_2}{-dH[x_i]/dx_i} > 0 \text{ where } \frac{dH[x_i]}{dk_2} > 0 \text{ and }$$

$$\frac{da_1}{dk_2} = \frac{\partial a_1}{\partial k_2} + \frac{\partial a_1}{\partial x_1} \frac{dx_1}{dk_2} < 0 \text{ where } \frac{\partial a_1}{\partial k_2} < 0 \text{ and } \frac{dx_1}{dk_2} > 0.$$  Effectively, an increase in $k_2$ reduces the marginal cost of period 1 audit effort.

Figure 3 depicts a graphical example of audit effort and fraud in each of the two periods, for a change in auditors versus a continuing auditor and illustrates the results of Corollary 1. Audit effort is higher in period 1 and lower in period 2 while fraud is lower in period 1 and higher in period 2 for the continuing auditor relative to a change in auditors. The figure also illustrates the results in Proposition 3 that relate to changes in period 2’s audit cost $k_2$. Period 2 audit effort
decreases in \( k_2 \) and period 2 fraud increases in \( k_2 \) for both settings. On the other hand, period 1 audit effort increases in \( k_2 \) for the continuing auditor but it is unaffected by changes in \( k_2 \) for the change auditor. The continuing auditor, knowing that he goes on to audit period 2, audits more aggressively in period 1 and therefore chooses a higher period 1 effort than the change auditor. The dishonest manager responds to these audit effort effects by decreasing period 1 fraud in the continuing auditor setting, but changes in \( k_2 \) have no effect on fraud in period 1 in the change setting.

The next section shows that audit quality is always higher for continuing auditor, where we measure audit quality either expected undetected fraud or audit risk.

V. EXPECTED UNDETECTED FRAUD AND AUDIT RISK

We begin our analysis of audit quality by first comparing the two settings in terms of expected undetected fraud.

**Expected Undetected Fraud**

Expected undetected fraud is measured as the probability that fraud occurs, multiplied by the probability that fraud is not detected (given that a fraud has occurred), multiplied by the amount of fraud. As this measure decreases, audit quality increases.

**Setting 1: A Change in Auditors**

For setting 1 where we have a change in auditors, expected undetected fraud (\( EUF \)) over two periods simplifies to

\[
\theta a_1 \exp(-x_1) + \Pr(D|ND) a_2 \exp(-x_2) =
\]

\[
EUF_{\text{change}} = \frac{k_1}{L_1} + \frac{k_2}{L_2}.
\]

(21)
**Setting 2: A Continuing Auditor**

Expected undetected fraud in setting 2 is equal to

\[ \theta a_i \exp(-x_1) + \Pr(D \mid ND) a_2 \exp(-x_2) = \]

\[ EUF_{\text{cont}} = \frac{k_1 - (1 - \theta)k_2}{L_1 + \frac{k_2}{a_i \theta} (1 + x_2)} + \frac{k_2}{L_2} \]  \hspace{1cm} (22)

We see that \( EUF_{\text{cont}} < EUF_{\text{change}} \), which we formalize as Proposition 4, because clearly

\[ \frac{k_1 - (1 - \theta)k_2}{L_1 + \frac{k_2}{a_i \theta} (1 + x_2)} < \frac{k_1}{L_1}. \]

**Proposition 4:** Expected undetected fraud over the two periods, given a continuing auditor is strictly less than that given a change in auditors.

Despite the ‘fresh look’ adopted by the new auditor in period 2, the strategic aspects of a continuing auditor provide benefits that are not achieved by a change in auditor. The continuing auditor chooses period 1 effort that anticipates the manager’s strategy over the two periods and thus has the ability to minimize his total expected costs. Moreover, when a change in auditors is known to occur, the old auditor is only interested in how his strategic choice affects his own period 1 expected costs.

Figure 4 depicts a graphical example illustrating Proposition 4. While the comparison is valid for all parameters, we demonstrate the relationship between \( EUF_{\text{cont}} \) and \( EUF_{\text{change}} \) in terms of the auditor’s liability parameter in period 2, \( L_2 \). Note that \( EUF_{\text{cont}} < EUF_{\text{change}} \) for all values of \( L_2 \) and a change in \( L_2 \) affects \( EUF_{\text{cont}} \) to a relatively greater extent.
Audit Risk

Audit risk is the probability that the financial statements contain undetected fraud. Furthermore, when we consider audit risk, we compare audit risk for each individual audit period across the two settings.

Setting 1: A change in Auditors

When there is a change in auditors, audit risk in period 1 \( AR_{\text{old}} \) is

\[
AR_{\text{old}} = \theta \exp(-x_1)
\]

and in period 2 audit risk \( AR_{\text{new}} \) equals

\[
AR_{\text{new}} = \frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1 - \theta)} \exp(-x_2)
\]

where \( x_1 \) and \( x_2 \) are from Proposition 1.

Setting 2: A Continuing Auditor

When there is a continuing auditor, audit risk in period 1 \( AR_{\text{cont}} \) is equal to

\[
AR_{\text{cont}} = \theta \exp(-x_1)
\]

and in period 2 audit risk \( AR_{\text{cont}} \) equals

\[
AR_{\text{cont}} = \frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1 - \theta)} \exp(-x_2)
\]

where \( x_1 \) and \( x_2 \) are from Proposition 2.

Proposition 5: Audit risk in each period given a continuing auditor is strictly less than that given a change in auditors.

Based on the equilibrium condition \( H[x_i] = 0 \), we know that first period audit effort in setting two is higher than that in setting one and we have the following result,

\[
AR_{\text{old}} > AR_{\text{cont}}
\]
which implies that audit risk is higher for the change auditor in period 1. Figure 5 provides a graphical representation of a comparison of period 1 audit risk across settings 1 and 2 as they relate to the auditor’s period 1 liability parameter \( L_1 \). Again any parameter could be used to illustrate these relationships. Despite a higher audit effort in period 2 for a change in auditors, compared to a continuing auditor, audit risk remains lower in period 2 for the continuing auditor. This occurs because period 2 audit risk in setting 2 includes the updated probability

\[
\frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1 - \theta)}
\]

that the manager is the fraudulent type, where \( x_1 \) is greater for the continuing auditor. Together our findings for expected undetected fraud and audit risk in each period suggest that audit quality is higher in the continuing setting than in the change setting.

VI. EMPRICAL INSIGHTS

Regulators, practitioners, and academic researchers continue to debate whether the efficiencies that result from an extended audit tenure outweigh the potential impairment of auditor independence from the auditor becoming ‘too cozy’ with the client. As with other studies, we find audit quality is higher for the continuing auditor, in that both audit risk and expected undetected fraud are lower for a continuing auditor than with a new auditor. Our study demonstrates, that without empirical and experimental researchers developing an expectations model that reflects the extent audit quality would differ in the absence of an impairment of independence, the evidence to date is inconclusive on whether an impairment does or does not exist.

The concern about impairment of independence continues to be fueled by the observed phenomenon in later periods that audit effort is smaller and the amount of fraud is larger for the continuing auditors than for a new auditor. A key finding of our study, however, is that the continuing auditor would typically exert more effort in the earlier period to discover fraud, which
in turn results in less audit effort and more fraud in the later period. This, of course, has the appearance that audit tenure impairs auditor independence. While an impairment of independence may indeed exist, to conclude from observing lower adult effort a higher fraud in later periods of an auditor's tenure requires first calibrating the expected differences between a continuing auditor and a change in auditor, again in the absence of an impairment of independence.

With the collapse of Arthur Andersen, new concerns arose about auditor tenure eroding auditor independence. Rather than creating mandatory audit firm rotations to alleviate these concerns, the Sarbanes-Oxley Act of 2002 (SOX) enhanced the provision for mandatory partner rotation by reducing the rotation period to a maximum of five years. In terms of our model, mandating partner rotation potentially increases the cost of auditing in the later periods for a new continuing auditor firm partner, represented by $k_2$.

The desired effects of SOX reducing the rotation period was to increase audit quality by reducing the possibility that audit partner tenure impairs auditor independence. Gipper, Hail, and Leuz (2017) analyze a large scale sample of partner rotations in the U.S. to test for this possibility, but find no evidence that audit quality increased with the SOX requirement. The results from our study provide an explanation as to why the above study did not find an improvement in their indicators of audit quality. Our comparative statics indicate that as $k_2$ increases, audit effort by the continuing auditor (in this case, a continuing audit firm with a different lead audit partner) decreases even more in the later audit period relative to the new auditor. The amount of fraud in the later period also increases as $k_2$ increases.

While mandatory audit partner rotation might be effective in reducing the possible impairment of independence from auditor tenure, the untended consequences of mandatory audit partner rotation are potentially opposite of SOX’s goal to increase audit quality. By applying the results of our comparative statics to future experimental and empirical studies, future tests will be
able to identify whether the positive effects of mandatory audit partner rotation on auditor
independence outweighs the unintended negative effects of mandatory audit partner rotation
increasing $k_2$, the cost of auditing in the later period,

\textbf{VII. CONCLUSION}

We consider a game-theoretic model of auditing where a manager possibly commits fraud
over two periods and an auditor attempts to detect the fraud, if it exists. We compare two settings
of this model. In the first, a different auditor audits each period. And while the auditor in the
second period can update the probability that the manager is a dishonest, the incentives of the
auditors in each period are independent. In the second setting a single auditor audits both periods.
In this case, the continuing auditor can allocate audit effort efficiently across the two periods to
minimize the combined costs of auditing and litigation exposure across the two periods. As a
result, a continuing auditor chooses a higher amount of period 1 audit effort and the manager
chooses a lesser amount of period 1 fraud. If there is no fraud detected at the end of period 1, the
continuing auditor chooses a lower amount of period 2 audit effort relative to a new period 2
auditor, because the period 1 evidence indicates that there is a lower risk of a dishonest-type
manager.

Our results also show that two measures of audit quality are always higher for a continuing
auditor. This occurs despite a lower level of period 2 audit effort for the continuing auditor when
compared to period 2 audit effort given a change in auditors. We find audit risk (probability that
fraud goes undetected) is lower in \textit{each} period for the continuing auditor. More importantly, total
expected undetected fraud across the two periods is lower for the continuing auditor.

In addition, we show how the period 1 and period 2 strategies for the auditor and manager
change for changes in the various payoff parameters. Not surprisingly, we find an increase in the
period 1 auditor liability costs increases the period 1 audit effort and decreases the period 1 amount of fraud. These results are also true for period 2. By analyzing a multi-period audit setting, we are also able to show that changes in period 1 parameters have the same intuitive effects on period 2 strategies in both settings. Any period 1 parameter that would induce higher audit effort in period 1, would cause audit effort in period 2 to decrease. Any period 1 parameter that would induce more fraud in period 1, would induce less fraud in period 2.

Finally, our results provide guidance for future research. The concern expressed by regulators and others is that longer audit tenures could result in an impairment of independence. While most empirical studies find that longer audit tenure improves audit quality, these studies cannot address whether longer audit tenure impairs independence without first calibrating the extent that audit quality would be expected to be higher with a continuing auditor, had there not been any impairment of independence. The equilibrium findings of our model provide the theoretical guidance necessary for developing these expectations. Further, we show the conclusions drawn by regulators and others that a continuing auditor’s independence is likely impaired when observing lower audit effort and higher amounts of fraud in the later audit periods might be a direct result of efficient allocation of audit effort by a continuing auditor, rather than an impairment of independence. Our study provides the necessary theoretical guidance for experimental and empirical researchers to disentangle whether such observations are evidence of an impairment, or evidence of the continuing auditor effectively allocating effort.
FIGURE 1
Timeline for a change in auditors

<table>
<thead>
<tr>
<th>Period 1</th>
<th>Period 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dishonest manager chooses the amount of fraud $a_1$.</td>
<td>If fraud is detected the game ends; otherwise we go on to period 2.</td>
</tr>
<tr>
<td>The old auditor chooses audit effort $x_1$, optimizing only over period 1.</td>
<td>A new auditor is hired who infers effort $x_2$.</td>
</tr>
<tr>
<td>Audit risk is measured, given no detection.</td>
<td>Audit risk is measured, given no detection.</td>
</tr>
</tbody>
</table>

Dishonest manager chooses fraud $a_2$, knowing that the new auditor infers period 1 effort but only optimizes over period 2.

The new auditor updates the probability of the dishonest manager and chooses effort $x_3$. 
FIGURE 2
Timeline for the continuing auditor

<table>
<thead>
<tr>
<th>Period 1</th>
<th>Period 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dishonest manager chooses the amount of fraud $a_1$, knowing the auditor optimizes over both periods.</td>
<td>If fraud is detected the game ends; otherwise we go on to period 2.</td>
</tr>
<tr>
<td>The continuing auditor chooses audit effort $x_1$, knowing he uses $x_1$ to update and plans his choice of effort $x_2$ to optimize his expected payoffs over both periods.</td>
<td>The auditor continues in period 2.</td>
</tr>
<tr>
<td>Audit risk is measured, given no detection</td>
<td>Audit risk is measured, given no detection</td>
</tr>
<tr>
<td>Dishonest manager chooses fraud $a_2$, knowing that the continuing auditor has planned for effort $x_2$, taking into account his $x_1$ choice.</td>
<td>The continuing auditor implements $x_2$.</td>
</tr>
</tbody>
</table>
FIGURE 3
Audit Effort and the Amount of Fraud in Each of the Two Periods
(Note that in calculating audit effort and fraud in the example below, $k_i = 2$)

Panel A: Audit Effort in Each of the Two Periods

Panel B: The Amount of fraud in Each of the Two Periods
**FIGURE 4**
Expected Undetected Fraud for a Change in Auditors Versus a Continuing Auditor

Expected Undetected Fraud ($EUF$)

$EUF_{\text{change}}$

$EUF_{\text{cont}}$

$L_2$
FIGURE 5
A Comparison of Audit Risk in Periods 1 and 2
for a Change in Auditors Versus a Continuing Auditor

Panel A: Period One Audit Risk

Panel B: Period Two Audit Risk
Appendix

Proof of Proposition 1:

Period 1 equilibrium strategies

From expressions (10) and (13) we have

\[ a_i = \frac{R_i \exp(-x_i)}{p_i(1 - \exp(-x_i))} = \frac{R_i}{p_i(-1 + \exp(x_i))} \quad \text{and} \quad x_i = \log \left[ \frac{\theta a_i L_i}{k_i} \right] \Rightarrow \exp(x_i) = \frac{\theta L_i a_i}{k_i}. \]

Thus, \( a_i = \frac{R_i k_i}{p_i(\theta L_i a_i - k_i)} \)

\[ \Rightarrow \theta L_i p_i a_i^2 - k_i p_i a_i - R_i k_i = 0, \text{ which is a quadratic equation}. \]

The solution to a general quadratic of \( ay^2 + by + c = 0 \) is \( y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \).

Therefore, \( a_i = \frac{k_i p_i + \sqrt{k_i^2 p_i^2 + 4(\theta L_i p_i R_i) k_i}}{2\theta L_i p_i} = \frac{k_i + k_i \sqrt{1 + \frac{4(\theta L_i R_i)}{k_i p_i}}}{2\theta L_i} \Rightarrow \)

\[ a_i = \frac{k_i}{2\theta L_i} \left( 1 + \sqrt{1 + \frac{4(\theta L_i R_i)}{k_i p_i}} \right) \]

Substituting into \( x_i = \log \left[ \frac{\theta a_i L_i}{k_i} \right] \) we get \( x_i = \log \left[ \frac{1}{2} \left( 1 + \sqrt{1 + \frac{4(\theta L_i R_i)}{k_i p_i}} \right) \right] \)

Substituting \( x_i \) back into \( a_i \) we have \( a_i = \frac{R_i}{p_i \left( -1 + \frac{1}{2} \left( 1 + \sqrt{1 + \frac{4(\theta L_i R_i)}{k_i p_i}} \right) \theta \right)} \)

We also have \( x_2 = \log \left[ \frac{a_2 L_2}{k_2} \frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1 - \theta)} \right] \) and \( a_2 = \frac{R_2 \exp(-x_2)}{p_2 (1 - \exp(-x_2))} \) where
$a_2$ and $x_2$ are solved for in a similar fashion and $\theta$ is replaced with the updated probability of
\[
\frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1-\theta)}.
\]

**Proof of Proposition 2:**

First, we derive the auditor’s first period expected payoff, given fixed $a_1$ and $a_2$

Recall that $x_2$ must satisfy $\frac{dA_{2\text{cont}}}{dx_2} = \Pr(D|ND)(L_2 a_2 \exp([-x_2]) - k_2 = 0$ (expression (14) in the paper). This results in

$$
\exp(-x_2) = \frac{k_2}{L_2 a_2} \frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1-\theta)} \Rightarrow
$$

$$
x_2 = \log \left[ \frac{L_2 a_2}{k_2} \frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1-\theta)} \right] = \log \left[ \frac{L_2 a_2}{k_2} \frac{1}{\theta \exp(-x_1) + (1-\theta)} \right] - x_1
$$

We also have

$$
A_{1\text{cont}} = \left( \theta \exp(-x_1) + (1-\theta) \right) A_2 - x_1 k_1 \quad \text{where} \quad A_{2\text{cont}} = -\Pr(D|ND)(L_2 a_1 + L_2 a_2 \exp(-x_2)) - x_2 k_2.
$$

Then substitute for $x_2$ with $a_2$ fixed and we obtain the following for $A_{2\text{cont}}$.

$$
A_{2\text{cont}} = \Pr(D|ND) \left( -L_2 a_1 - k_2 \frac{\theta \exp(-x_1) + (1-\theta)}{\theta \exp(-x_1)} \right) - k_2 \left( -x_1 + \log \left[ \frac{L_2 a_2}{k_2} \frac{\theta}{\theta \exp(-x_1) + (1-\theta)} \right] \right)
$$

Substitute back into $A_{1\text{cont}}$.

$$
A_{1\text{cont}} = \left( \theta \exp(-x_1) + (1-\theta) \right) \cdot 
\begin{vmatrix}
\theta \exp(-x_1) & -L_2 a_1 - k_2 \frac{\theta \exp(-x_1) + (1-\theta)}{\theta \exp(-x_1)} \\
\theta \exp(-x_1) + (1-\theta) & -k_2 
\end{vmatrix}^{-x_1 k_1}
$$

$$
\begin{vmatrix}
-L_2 a_1 - k_2 \frac{\theta \exp(-x_1) + (1-\theta)}{\theta \exp(-x_1)} \\
-x_1 + \log \left[ \frac{L_2 a_2}{k_2} \frac{\theta}{\theta \exp(-x_1) + (1-\theta)} \right]
\end{vmatrix}
$$

where $\left( \theta \exp(-x_1) + (1-\theta) \right)$ is the probability that no fraud is detected in period 1.
Then,

\[
A_{\text{cont}} = \left\{ \begin{array}{cl} 
-L_1 a_1 \theta \exp(-x_i) - k_2 \left( \theta \exp(-x_i) + (1 - \theta) \right) \\
-(\theta \exp(-x_i) + (1 - \theta)) k_2 \left( -x_i + \log \left[ \frac{L_2 a_2}{k_2} \frac{\theta}{\theta \exp(-x_i) + (1 - \theta)} \right] \right) \end{array} \right\} - x_i k_i
\]

\[
= \left\{ \begin{array}{cl} 
-L_1 a_1 \theta \exp(-x_i) \\
-(\theta \exp(-x_i) + (1 - \theta)) k_2 \left( -x_i + \log \left[ \frac{L_2 a_2}{k_2} \frac{\theta}{\theta \exp(-x_i) + (1 - \theta)} \right] \right) \end{array} \right\} - x_i k_i
\]

The above is expression (15) in the paper.

Next take the first derivative of \( A_{\text{cont}} \) with respect to \( x_i \).

\[
\frac{dA_{\text{cont}}}{dx_i} = L_1 a_1 \theta \exp(-x_i) + \theta \exp(-x_i) k_2 \left( -x_i + \log \left[ \frac{L_2 a_2}{k_2} \frac{\theta}{\theta \exp(-x_i) + (1 - \theta)} \right] \right) 
- (\theta \exp(-x_i) + (1 - \theta)) k_2 \left( -1 + \log \left[ \frac{L_2 a_2}{k_2} \frac{\theta}{\theta \exp(-x_i) + (1 - \theta)} \right] \right) / dx_i \right) - k_i
\]

\[
= L_1 a_1 \theta \exp(-x_i) + \theta \exp(-x_i) k_2 \left( -x_i + \log \left[ \frac{L_2 a_2}{k_2} \frac{\theta}{\theta \exp(-x_i) + (1 - \theta)} \right] \right) 
- (\theta \exp(-x_i) + (1 - \theta)) k_2 \left( -1 + \frac{\theta \exp(-x_i)}{\theta \exp(-x_i) + (1 - \theta)} \right) - k_i
\]

Thus, we have,

\[
\frac{dA_{\text{cont}}}{dx_i} = \theta \exp(-x_i) \left( L_1 a_1 + k_2 \left( -x_i + \log \left[ \frac{L_2 a_2}{k_2} \frac{\theta}{\theta \exp(-x_i) + (1 - \theta)} \right] \right) \right) \right) + (1 - \theta) k_2 - k_i \tag{A1}
\]

Take the second derivative and we get
\[ \frac{d^2 A_{\text{cont}}}{dx_1^2} = -\theta \exp(-x_1) \left( L_i a_i + k_2 \left( 1 - x_1 + \log \left[ \frac{L_i a_i}{k_2} \frac{\theta}{\theta \exp(-x_1) + (1 - \theta)} \right] \right) \right) \]

\[ -\theta \exp(-x_1) k_2 \left( 1 - \frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1 - \theta)} \right) < 0 \]

where note that \( x_2 = -x_1 + \log \left[ \frac{L_i a_i}{k_2} \frac{\theta}{\theta \exp(-x_1) + (1 - \theta)} \right] > 0 \)

Thus, \( A_{\text{cont}} \) is concave in \( x_1 \).

Expression (A1) is our basis for implicitly defining \( x_1 \). We substitute for \( a_i \)
and \( a_2 \) where \( a_i, a_2 \) and \( x_2 \) are functions of \( x_1 \).

We know that \( a_2 = \frac{R_i \exp(-x_2)}{p_2 (1 - \exp(-x_2))} \). Thus we substitute for \( a_2 \) in

\[ x_2 = \log \left[ \frac{L_i a_i}{k_2} \frac{\theta}{\theta \exp(-x_1) + (1 - \theta)} \right] - x_1 \]

and solve for \( x_2 \).

\[ x_2 = \log \left[ \frac{L_i \frac{R_i \exp(-x_2)}{p_2 (1 - \exp(-x_2))}}{k_2} \frac{\theta}{\theta \exp(-x_1) + (1 - \theta)} \right] - x_1 \Rightarrow \]

\[ \exp(x_2) = \frac{L_i R_i}{p_2 k_2 \theta \exp(-x_1) + (1 - \theta)} \quad \text{and} \]

\[ (\exp(x_2))^2 - \exp(x_2) - \frac{L_i R_i}{p_2 k_2 \theta \exp(-x_1) + (1 - \theta)} = 0 \]

In applying the quadratic formula \( ay^2 + by + c = 0 \) is \( y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \), we get
\[
\exp(x_2) = \frac{1}{2} \left( 1 + \sqrt{1 + \frac{4L_2R_2}{p_2k_2} \frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1 - \theta)} } \right) \quad \text{and thus,} \\
\]

\[
x_2 = \log \left[ \frac{1}{2} \left( 1 + \sqrt{1 + \frac{4L_2R_2}{p_2k_2} \frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1 - \theta)} } \right) \right] \\
\]

Next substitute for \( x_2 \) in \( a_2 \). We start with

\[
a_2 = \frac{R_2 \exp(-x_2)}{p_2 (1 - \exp(-x_2))} = \frac{R_2}{p_2 (\exp(x_2) - 1)} .
\]

Then we have

\[
a_2 = \frac{R_2}{p_2 \left( -1 + \frac{1}{2} \left( 1 + \sqrt{1 + \frac{4L_2R_2}{p_2k_2} \frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1 - \theta)} } \right) \right)} > 0
\]

So now we can rewrite expression (A1) in terms of \( \theta_1 \) that defines the equilibrium condition for \( x_1 \).

\[
H[\theta_1] = \theta \exp(-x_1) \left( L_1a_1[x_1] + k_2 + k_2x_2[x_1] \right) + (1 - \theta)k_2 - k_1 = 0 
\]

(A2)

where

\[
a_1[x_1] = \frac{R_1 \exp(-x_1)}{p_1 (1 - \exp(-x_1))} \quad \text{and}
\]

\[
x_2[x_1] = \log \left[ \frac{1}{2} \left( 1 + \sqrt{1 + \frac{4L_2R_2}{p_2k_2} \frac{\theta \exp(-x_1)}{\theta \exp(-x_1) + (1 - \theta)} } \right) \right] \\
\]

Finally we know that expression (A2) yields a unique \( x_1 \) and hence unique values of \( a_1, a_2 \) and \( x_2 \) because we show below that based on our assumption of \( (k_1 - (1 - \theta)k_2) > 0 \), \( \frac{dH[x_1]}{dx_1} < 0 \).

\[
\frac{dH[x_1]}{dx_1} = -\theta \exp(-x_1) \left( L_1a_1[x_1] + k_2 + k_2x_2[x_1] \right) + \theta \exp(-x_1) \left( L_1 \frac{da_1[x_1]}{dx_1} + k_2 \frac{dx_2[x_1]}{dx_1} \right)
\]

(A3)

and after substituting in condition (A2), we have
\[ \frac{dH[x_i]}{dx_i} = -(k_i - (1-\theta)k_2) + \theta \exp(-x_i) \left( L_i \frac{da_1[x_i]}{dx_i} + k_2 \frac{dx_2[x_i]}{dx_i} \right) < 0 \]  

(A4)

where \( \frac{da_1[x_i]}{dx_i} = -\frac{R_i \exp(x_i)}{p_i \left( \exp(x_i) - 1 \right)^2} < 0 \) and

\[ \frac{dx_2}{dx_i} = -\frac{2 \exp(x_i)(1-\theta)\theta L_i R_2}{k_2 p_2 \left( \exp(x_i) - 1 \right)^2 \left[ 1 + \frac{4 L_2 R_2}{p_2 k_2} \theta \exp(-x_i) \right]} < 0 \]  

(A5)

This also indicates that to determine the sign of how \( x_i \) changes with respect to a parameter

we would calculate

\[ \frac{dx_i}{dg} = \frac{-dH[x_i]}{dg} \]  

where \( g \) stands for a parameter (e.g. \( L_i \)). We know that \( \frac{-dH[x_i]}{dx_i} > 0 \) so that

the sign of \( \frac{dx_i}{dg} \) is the same as the sign of \( \frac{dH[x_i]}{dg} \) where \( x_i \) is fixed.

**Proof of Corollary 1:** First note that the characterization of fraud in both settings is from expression (10) where

\[ a_1 = -\frac{R_i \exp(-x_i)}{p_i \left( 1 - \exp(-x_i) \right)} \quad \text{and} \quad a_2 = -\frac{R_2 \exp(-x_2)}{p_2 \left( 1 - \exp(-x_2) \right)} . \]

Furthermore, \( \frac{da_1}{dx_i} = -\frac{R_i \exp(x_i)}{p_i \left( \exp(x_i) - 1 \right)^2} < 0 \) and \( \frac{da_2}{dx_2} = -\frac{R_1 \exp(x_2)}{p_i \left( \exp(x_2) - 1 \right)^2} < 0 \). (A6)

We know from Proposition 1 that for the ‘old’ and ‘new’ auditors of periods 1 and 2 we have
Moreover Proposition 2 shows that $x_1^{old}$ is smaller compared to audit effort in period 1 for the continuing auditor and thus we know from (A6) that the amount of period 1 fraud is greater for the change auditor.

Proposition 2 shows that for the continuing versus the change auditor in period 2

$$x_2^{con} = \log \left[ \frac{1}{2} \left( 1 + \sqrt{\frac{4L_1 R_1}{k_1 p_1}} \right) \theta \exp \left( -x_1^{con} \right) \right] < \log \left[ \frac{1}{2} \left( 1 + \sqrt{\frac{4L_2 R_2}{p_2 k_2}} \theta \exp \left( -x_1^{new} \right) \right) \theta \exp \left( -x_1^{new} \right) \right] = x_2^{new}$$

because $\frac{\theta \exp (-x_i)}{\theta \exp (-x_i) + (1-\theta)}$ decreases in $x_i$. Thus, the amount of fraud in period 2 is more for a continuing auditor than for the ‘new’ auditor and the amount of audit effort is less.

**Proof of Proposition 3:**

We begin by demonstrating the results in the top-left quadrant and the bottom right quadrant of the table. We begin with the results in the top-left quadrant

$x_1$ increases in $L_i$ and $R_i$, while decreasing in $p_i$ and $k_i$.

**Setting 1- old auditor:**

$x_1$ increases in $L_i$

$$\frac{dx_1}{dL_i} = \frac{2R_i \theta}{k_1 p_1 + 4L_1 R_1 \theta + \sqrt{k_1^2 p_1^2 + 4L_1 R_1 \theta k_1 p_1}} > 0$$

$x_1$ increases in $R_i$
\[
\frac{dx_1}{dR_1} = \frac{2L_1\theta}{k_1p_1 + 4L_1R_1\theta + \sqrt{k_1^2p_1^2 + 4L_1R_1\theta k_1p_1}} > 0
\]
x_1 decreases in \( p_1 \).

\[
\frac{dx_1}{dp_1} = -\frac{2R_1L_1\theta}{p_1\left(k_1p_1 + 4L_1R_1\theta + \sqrt{k_1^2p_1^2 + 4L_1R_1\theta k_1p_1}\right)} < 0
\]
x_1 decreases in \( k_1 \).

\[
\frac{dx_1}{dk_1} = -\frac{2R_1L_1\theta}{k_1\left(k_1p_1 + 4L_1R_1\theta + \sqrt{k_1^2p_1^2 + 4L_1R_1\theta k_1p_1}\right)} < 0
\]

**Setting 2 -- the continuous auditor**

Recall (A2) or \( H[x_1] = \theta \exp(-x_1)\left(L_1a_1[x_1] + k_2 + k_2x_2[x_1] + (1 - \theta)k_2 - k_1 \right) \)

As discussed in the proof to Proposition 2, \( \frac{dx_1}{dg} = -\frac{dH[x_1]}{dx_1} \) where \( g \) stands for a general parameter (e.g. \( L_1 \) ). We know that \( -\frac{dH[x_1]}{dx_1} > 0 \) so that the sign of \( \frac{dx_1}{dg} \) is the same as the sign of \( \frac{dH[x_1]}{dx_1} \) where \( x_1 \) is fixed.

\( x_1 \) increases in \( L_1 \).

\[
\frac{dH[x_1]}{dL_1} = \theta \exp(-x_1)a_1[x_1] > 0
\]

\( x_1 \) increases in \( R_1 \).

\[
\frac{dH[x_1]}{dR_1} = \theta \exp(-x_1) L_1 \frac{1}{p_1(\exp(x_1) - 1)} > 0
\]

\( x_1 \) decreases in \( p_1 \).
\[
\frac{dH}{dp_i} = -\theta \exp(-x_i) L_i \frac{R_i}{p_i^2(\exp(x_i)-1)} < 0
\]

\(x_i\) decreases in \(k_i\).

\[
\frac{dH}{dk_i} = -1 < 0
\]

\(a_i\) increases in \(R_i\), and \(k_i\), while decreasing in \(L_i\) and \(p_i\).

Setting 1 -- the old auditor:

\(a_i\) increases in \(R_i\)

\[
\frac{da_i}{dR_i} = \frac{k_i}{\sqrt{k_i^2 p_i^2 + 4 L_i R_i \theta k_i p_i}} > 0
\]

\(a_i\) increases in \(k_i\),

\[
\frac{da_i}{dk_i} = \frac{1}{2 L_i \theta} \left( 1 + \frac{p_i k_i + 2 L_i R_i \theta}{\sqrt{k_i^2 p_i^2 + 4 L_i R_i \theta k_i p_i}} \right) > 0
\]

\(a_i\) decreases in \(L_i\)

\[
\frac{da_i}{dL_i} = -\frac{k_i}{2 L_i \theta^2} \left( 1 + \frac{p_i k_i + 2 L_i R_i \theta}{\sqrt{k_i^2 p_i^2 + 4 L_i R_i \theta k_i p_i}} \right) < 0
\]

\(a_i\) decreases in \(p_i\).

\[
\frac{da_i}{dp_i} = -\frac{k_i R_i}{p_i \sqrt{k_i^2 p_i^2 + 4 L_i R_i \theta k_i p_i}} < 0
\]

\(a_i\) decreases in \(\theta\).

\[
\frac{da_i}{d\theta} = -\frac{k_i}{2 L_i \theta^2} \left( 1 + \frac{k_i p_i + 2 L_i R_i \theta}{\sqrt{k_i^2 p_i^2 + 4 L_i R_i \theta k_i p_i}} \right) < 0
\]

Setting 2 -- a continuous auditor.
increases in $R_i$.  

$$\frac{\partial a_i}{\partial R_i} = \frac{\partial a_i}{\partial x_i} \frac{\partial x_i}{\partial R_i} \left( k_i - (1 - \theta) k_2 \right) - \theta \exp(-x_i) \left( L_i \frac{\partial a_i}{\partial x_i} + k_2 \frac{\partial x_2}{\partial x_i} \right)$$

$$= \frac{\partial a_i}{\partial R_i} \left\{ 1 - \frac{-\theta \exp(-x_i) \left( L_i \frac{\partial a_i}{\partial x_i} \right)}{(k_i - (1 - \theta) k_2) - \theta \exp(-x_i) \left( L_i \frac{\partial a_i}{\partial x_i} + k_2 \frac{\partial x_2}{\partial x_i} \right)} \right\} > 0$$

where $\frac{\partial a_i}{\partial R_i} > 0$.

$\quad$

$a_i$ increases in $k_i$. 

$$\frac{\partial a_i}{\partial k_i} = \frac{\partial a_i}{\partial x_i} \frac{\partial x_i}{\partial k_i} > 0$$

where $\frac{\partial a_i}{\partial k_i} = 0$, $\frac{\partial a_i}{\partial x_i} < 0$ and $\frac{\partial x_i}{\partial k_i} < 0$.

$\quad$

$a_i$ decreases in $L_i$.  

Similarly, $\frac{\partial a_i}{\partial L_i} = \frac{\partial a_i}{\partial x_i} \frac{\partial x_i}{\partial L_i} < 0$

$\quad$

$a_i$ decreases in $p_i$. 

$$\frac{\partial a_i}{\partial p_i} = \frac{\partial a_i}{\partial x_i} \frac{\partial x_i}{\partial p_i} \left( k_i - (1 - \theta) k_2 \right) - \theta \exp(-x_i) \left( L_i \frac{\partial a_i}{\partial x_i} + k_2 \frac{\partial x_2}{\partial x_i} \right) < 0$$

Now we prove the results for the bottom-right quadrant of the table.

$x_2$ increases in $L_2$ and $R_2$, while decreasing in $p_2$ and $k_2$.

The proof is essentially the same as in part 1 above for the new auditor.

For the continuous auditor we have the following.
\[ \frac{\partial x_2}{\partial x_1} = -\frac{2 \exp(x_1) L_2 R_2 (1-\theta) \theta}{k_2 p_2 \left( \exp(x_1)(1-\theta)e^{\theta} \right)^2 SQ_2 (1 + SQ_2)} < 0 \text{ where} \]

\[ SQ_2 = \sqrt{1 + \frac{4 L_2 R_2}{k_2 p_2} \theta \exp(-x_1) \left( \exp(-x_1) + (1-\theta) \right) } > 1 \]

And we use the expression
\[
\frac{dH}{dx_1} = -(k_1 - (1-\theta) k_2) + \theta \exp(-x_1) \left( L_1 \frac{da_1}{dx_1} + k_2 \frac{dx_2}{dx_1} \right) \text{ from (A4).} 
\]

\[ x_2 \text{ increases in } L_2. \]

\[
\frac{dx_2}{dL_2} = \frac{\partial x_2}{\partial L_2} + \frac{\partial x_2}{\partial x_1} \frac{dx_1}{dL_2} = \frac{\partial x_2}{\partial L_2} + \frac{\partial x_2}{\partial x_1} \frac{dH}{dx_1} - \theta \exp(-x_1) k_2 \frac{\partial x_2}{\partial L_2} - \theta \exp(-x_1) \left( L_1 \frac{da_1}{dx_1} + k_2 \frac{dx_2}{dx_1} \right) 
\]

\[
= \frac{\partial x_2}{\partial L_2} \left( 1 - \frac{-\theta \exp(-x_1) k_2 \frac{\partial x_2}{\partial x_1}}{(k_1 - (1-\theta) k_2) - \theta \exp(-x_1) \left( L_1 \frac{da_1}{dx_1} + k_2 \frac{dx_2}{dx_1} \right)} \right) > 0 
\]

where the second term inside the parentheses is less than one, resulting in a positive sum within the parentheses. Finally, \[ \frac{\partial x_2}{\partial L_2} = \frac{2 \exp(x_1) \theta}{k_2 p_2 \left( (1-\theta) + \exp(x_1) \theta \right) SQ_2 (1 + SQ_2)} > 0 . \]

\[ x_2 \text{ increases in } R_2. \]

\[ \frac{dx_2}{dR_2} = \frac{\partial x_2}{\partial R_2} + \frac{\partial x_2}{\partial x_1} \frac{dx_1}{dR_2} \text{ and similar to the derivation above} \]

\[
= \frac{\partial x_2}{\partial R_2} + \frac{\partial x_2}{\partial x_1} \left( k_1 - (1-\theta) k_2 \right) - \theta \exp(-x_1) \left( L_1 \frac{da_1}{dx_1} + k_2 \frac{dx_2}{dx_1} \right) 
\]

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\[ \frac{\partial x_2}{\partial R_2} \left( 1 - \frac{-\theta \exp(-x_1) k_2 \frac{\partial x_2}{\partial x_1}}{k_1 - (1-\theta)k_2 - \theta \exp(-x_1) \left( L_1 \frac{\partial a_1}{\partial x_1} + k_2 \frac{\partial x_2}{\partial x_1} \right)} \right) > 0 \text{ where } \frac{\partial x_2}{\partial R_2} > 0. \]

\( x_2 \) decreases in \( p_2 \).

Similarly, \( \frac{dx_2}{dp_2} = \frac{\partial x_2}{\partial p_2} \left( 1 - \frac{-\theta \exp(-x_1) k_2 \frac{\partial x_2}{\partial x_1}}{k_1 - (1-\theta)k_2 - \theta \exp(-x_1) \left( L_1 \frac{\partial a_1}{\partial x_1} + k_2 \frac{\partial x_2}{\partial x_1} \right)} \right) < 0 \)

where \( \frac{\partial x_2}{\partial p_2} < 0. \)

\( x_2 \) decreases in \( k_2 \).

\[ \frac{dx_2}{dk_2} = \frac{\partial x_2}{\partial k_2} + \frac{\partial x_2}{\partial x_1} \frac{(1-\theta) + \theta \exp(-x_1) \left( k_2 \frac{\partial x_2}{\partial k_2} + 1 + x_2 \right)}{k_1 - (1-\theta)k_2 - \theta \exp(-x_1) \left( L_1 \frac{\partial a_1}{\partial x_1} + k_2 \frac{\partial x_2}{\partial x_1} \right)} \]

\[ \frac{\partial x_2}{\partial k_2} + \frac{\partial x_2}{\partial x_1} \frac{\theta \exp(-x_1) \left( k_2 \frac{\partial x_2}{\partial k_2} \right)}{k_1 - (1-\theta)k_2 - \theta \exp(-x_1) \left( L_1 \frac{\partial a_1}{\partial x_1} + k_2 \frac{\partial x_2}{\partial x_1} \right)} \]

\[ + \frac{\partial x_2}{\partial x_1} \frac{(1-\theta) + \theta \exp(-x_1) (1 + x_2)}{k_1 - (1-\theta)k_2 - \theta \exp(-x_1) \left( L_1 \frac{\partial a_1}{\partial x_1} + k_2 \frac{\partial x_2}{\partial x_1} \right)} \]

\[ = \frac{\partial x_2}{\partial k_2} \left( 1 - \frac{-\theta \exp(-x_1) k_2 \frac{\partial x_2}{\partial x_1}}{k_1 - (1-\theta)k_2 - \theta \exp(-x_1) \left( L_1 \frac{\partial a_1}{\partial x_1} + k_2 \frac{\partial x_2}{\partial x_1} \right)} \right) \]

\[ + \frac{\partial x_2}{\partial x_1} \frac{(1-\theta) + \theta \exp(-x_1) (1 + x_2)}{k_1 - (1-\theta)k_2 - \theta \exp(-x_1) \left( L_1 \frac{\partial a_1}{\partial x_1} + k_2 \frac{\partial x_2}{\partial x_1} \right)} < 0 \]
where \[ \frac{\partial x_2}{\partial k_2} = -\frac{2\exp(-x_1)L_2R_2\theta}{k_2^2p_2((1-\theta) + \theta\exp(-x_1))SQ_2(1+SQ_2)} < 0. \]

\( x_2 \) increases in \( \theta \).

\[ \frac{dx_2}{d\theta} = \frac{\partial x_2}{\partial \theta} + \frac{\partial x_2}{\partial x_1} \frac{dx_1}{d\theta} > 0 \] can only be shown to be true algebraically when \( k_2 \) is sufficiently large.

However numerically \( x_2 \) increases in \( \theta \) for all values of \( k_2 \).

\( a_2 \) increases in \( R_2 \), and \( k_2 \), while decreasing in \( L_2 \) and \( p_2 \).

For Setting 1, the proof is essentially the same as in part 3 above.

**Setting 2 -- the continuous auditor**

Recall that one way to write \( a_2 \) is \[ a_2 = \frac{R_2\exp(-x_1[x_1])}{p_2(1-\exp(-x_1[x_1]))} \]

\( a_2 \) increases in \( R_2 \).

\[ \frac{da_2}{dR_2} = \frac{\partial a_2}{\partial R_2} + \frac{\partial a_2}{\partial x_1} \frac{dx_1}{dR_2} > 0 \] because

\[ \frac{dH[x_1]}{dR_2} = \theta\exp(-x_1)k_2\frac{\partial x_2}{\partial R_2} > 0 \Rightarrow \frac{dx_1}{dR_2} > 0 \]

\[ \frac{\partial a_2}{\partial x_1} = \frac{\partial a_2}{\partial x_2} \frac{dx_2}{dx_1} > 0 \] where \( \frac{\partial a_2}{\partial x_2} < 0 \) and \( \frac{dx_2}{dx_1} < 0 \). Finally, \( \frac{\partial a_2}{\partial R_2} > 0 \).

\( a_2 \) increases in \( k_2 \).

For this proof we use the form of \( a_2 = \frac{R_2\exp(-x_2)}{p_2(1-\exp(-x_2))} \).

We have \[ \frac{da_2}{dk_2} = \frac{\partial a_2}{\partial k_2} + \frac{\partial a_2}{\partial x_2} \frac{dx_2}{dk_2} > 0 \]

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where $\frac{\partial a_2}{\partial k_2} = 0$, $\frac{\partial a_2}{\partial x_2} < 0$ and $\frac{dx_2}{dk_2} < 0$, which is shown to be true above in part 2.

$a_2$ decreases in $L_2$.

For this proof we use the form of $a_2 = \frac{R_1 \exp(-x_2)}{p_2 \left(1 - \exp(-x_2)\right)}$.

And $\frac{da_2}{dL_2} = \frac{\partial a_2}{\partial L_2} + \frac{\partial a_2}{\partial x_2} \frac{dx_2}{dL_2} < 0$ where $\frac{\partial a_2}{\partial L_2} = 0$, $\frac{\partial a_2}{\partial x_2} < 0$ and $\frac{dx_2}{dL_2} > 0$

which is shown to be true above in part 2.

$a_2$ decreases in $p_2$

$\frac{da_2}{dp_2} = \frac{\partial a_2}{\partial p_2} + \frac{\partial a_2}{\partial x_1} \frac{dx_1}{dp_2} < 0$, which is proved similarly to $\frac{da_2}{dR_2}$ above.

Now we consider the top-right quadrant of the table:

$x_2$ decreases in $R_i$, while $a_2$ increases in $R_i$.

The proof that follows applies to both Setting 1 and Setting 2.

$x_2$ decreases in $R_i$.

$\frac{dx_2}{dR_i} = \frac{\partial x_2}{\partial R_i} + \frac{\partial x_2}{\partial x_1} \frac{dx_1}{dR_i} < 0$ where $\frac{\partial x_2}{\partial R_i} = 0$, $\frac{\partial x_2}{\partial x_1} < 0$ and $\frac{dx_1}{dR_i} > 0$ and where $\frac{dx_1}{dR_i} > 0$ from the proofs above for the top-right quadrant.

$a_2$ increases in $R_i$.

$\frac{da_2}{dR_i} = \frac{\partial a_2}{\partial R_i} + \frac{\partial a_2}{\partial x_2} \frac{dx_2}{dR_i} > 0$ where $\frac{\partial a_2}{\partial R_i} = 0$, $\frac{\partial a_2}{\partial x_2} = \frac{\partial a_2}{\partial x_1} \frac{dx_2}{dR_i} > 0$ and $\frac{dx_1}{dR_i} > 0$

$x_2$ increases in $p_1$

$\frac{dx_2}{dp_1} = \frac{\partial x_2}{\partial p_1} + \frac{\partial x_2}{\partial x_1} \frac{dx_1}{dp_1} > 0$ where $\frac{\partial x_2}{\partial p_1} = 0$, $\frac{\partial x_2}{\partial x_1} < 0$ and $\frac{dx_1}{dp_1} < 0$
$a_2$ decreases in $p_1$.
\[
\frac{da_2}{dp_1} = \frac{\partial a_2}{\partial p_1} + \frac{\partial a_2}{\partial x_1} \frac{dx_1}{dp_1} < 0 \quad \text{where} \quad \frac{\partial a_2}{\partial p_1} = 0, \quad \frac{\partial a_2}{\partial x_1} = \frac{\partial a_2}{\partial x_2} \frac{\partial x_2}{\partial x_1} > 0 \text{ and } \frac{dx_1}{dp_1} < 0
\]

$x_2$ decreases in $L_1$.
\[
\frac{dx_2}{dL_1} = \frac{\partial x_2}{\partial L_1} + \frac{\partial x_2}{\partial x_1} \frac{dx_1}{dL_1} < 0 \quad \text{where} \quad \frac{\partial x_2}{\partial L_1} = 0, \quad \frac{\partial x_2}{\partial x_1} < 0 \text{ and } \frac{dx_1}{dL_1} > 0
\]

$a_2$ increases in $L_1$.
\[
\frac{da_2}{dL_1} = \frac{\partial a_2}{\partial L_1} + \frac{\partial a_2}{\partial x_1} \frac{dx_1}{dL_1} > 0 \quad \text{where} \quad \frac{\partial a_2}{\partial L_1} = 0, \quad \frac{\partial a_2}{\partial x_1} = \frac{\partial a_2}{\partial x_2} \frac{\partial x_2}{\partial x_1} > 0 \text{ and } \frac{dx_1}{dL_1} > 0
\]

$x_2$ increases in $k_1$ -- for both Settings 1 and 2, the following is true.
\[
\frac{dx_2}{dk_1} = \frac{\partial x_2}{\partial k_1} + \frac{\partial x_2}{\partial x_1} \frac{dx_1}{dk_1} > 0
\]

where \( \frac{\partial x_2}{\partial k_1} = 0, \frac{\partial x_2}{\partial x_1} < 0 \) and \( \frac{dx_1}{dk_1} < 0 \)

Finally, we consider the results in the bottom-left quadrant of the table. This proof only applies to Setting 2 because the strategies in setting 1 do not depend on any of the period 2 payoff parameters.

$x_1$ increases in $R_2$.
\[
\frac{dx_1}{dR_2} > 0 \text{ because } \frac{dH[x_1]}{dR_2} = \theta \exp[-x_1] k_2 \frac{dx_2}{dR_2} > 0
\]

$a_1$ decreases in $R_2$.
\[
\frac{da_1}{dR_2} = \frac{\partial a_1}{\partial R_2} + \frac{\partial a_1}{\partial x_1} \frac{dx_1}{dR_2} < 0 \quad \text{where} \quad \frac{\partial a_1}{\partial R_2} = 0, \quad \frac{\partial a_1}{\partial x_1} < 0 \text{ and } \frac{dx_1}{dR_2} > 0
\]

$x_1$ decreases in $p_2$.
\[
\frac{dx_1}{dp_2} < 0 \text{ because } \frac{dH[x_1]}{dp_2} = \theta \exp[-x_1] k_2 \frac{dx_2}{dp_2} < 0
\]

$a_1$ increases in $p_2$.
\[
\frac{da_1}{dp_2} = \frac{\partial a_1}{\partial p_2} + \frac{\partial a_1}{\partial x_1} \frac{dx_1}{dp_2} > 0 \quad \text{where} \quad \frac{\partial a_1}{\partial p_2} = 0, \quad \frac{\partial a_1}{\partial x_1} < 0 \quad \text{and} \quad \frac{dx_1}{dp_2} < 0
\]

\[x_1 \text{ increases in } L_2\]
\[
\frac{dx_1}{dL_2} > 0 \quad \text{because} \quad \frac{dH[x_1]}{dL_2} = \theta \exp[-x_1] \frac{\partial x_1}{\partial L_2} > 0
\]

\[a_1 \text{ decreases in } L_2 .\]
\[
\frac{da_1}{dL_2} = \frac{\partial a_1}{\partial L_2} + \frac{\partial a_1}{\partial x_1} \frac{dx_1}{dL_2} < 0 \quad \text{where} \quad \frac{\partial a_1}{\partial L_2} = 0, \quad \frac{\partial a_1}{\partial x_1} < 0 \quad \text{and} \quad \frac{dx_1}{dL_2} > 0
\]

Only for Setting 2 is the following true because \(\frac{\partial x_1}{\partial x_2} = 0\) for the old auditor in Setting 1 and thus

\[x_1 \text{ is constant with respect to changes in } k_2.\]

\[x_1 \text{ increases in } k_2\]
\[
\frac{dx_1}{dk_2} > 0 \quad \text{because} \quad \frac{dH[x_1]}{dk_2} = \theta \exp[-x_1] \left( k_2 \frac{\partial x_1}{\partial k_2} + 1 + x_2 \right) + (1 - \theta)
\]
\[
= 1 - \theta + \theta \exp[-x_1] \left( - \frac{2 \exp[-x_1] L_2 R_2 \theta}{k_2 p_2 \left( (1 - \theta) + \theta \exp[-x_1] \right) SQ_2 (1 + SQ_2)} + 1 + x_2 \right) > 0
\]

where \(SQ_2 = \sqrt{1 + \frac{4 L_2 R_2}{\frac{\theta \exp[-x_1]}{k_2 p_2 \theta \exp[-x_1] + (1 - \theta)}}} > 1\) and

\[
\frac{2 \exp[-x_1] L_2 R_2 \theta}{k_2 p_2 \left( (1 - \theta) + \theta \exp[-x_1] \right) SQ_2 (1 + SQ_2)}
\]
\[
= \frac{2 \exp[-x_1] L_2 R_2 \theta}{k_2 p_2 \left( (1 - \theta) + \theta \exp[-x_1] \right) \left( SQ_2 + 1 + \frac{4 L_2 R_2}{k_2 p_2 \theta \exp[-x_1] + (1 - \theta)} \right)}
\]
\[
= \frac{2 \exp[-x_1] L_2 R_2 \theta}{\left( k_2 p_2 \left( (1 - \theta) + \theta \exp[-x_1] \right) (SQ_2 + 1) + 4 \theta \exp[-x_1] k_2 R_2 \right)} < 1
\]
\(a_2\) decreases in \(k_i\)

\[
\frac{da_2}{dk_i} = \frac{\partial a_2}{\partial k_i} + \frac{\partial a_2}{\partial x_1} \frac{dx_1}{dk_i} < 0 \text{ where } \frac{\partial a_2}{\partial k_i} = 0, \frac{\partial a_2}{\partial x_1} > 0 \text{ and } \frac{dx_1}{dk_i} < 0
\]

\(a_i\) decreases in \(k_2\)

\[
\frac{da_i}{dk_2} = \frac{\partial a_i}{\partial k_2} + \frac{\partial a_i}{\partial x_1} \frac{dx_1}{dk_2} < 0 \text{ where } \frac{\partial a_i}{\partial k_2} = 0, \frac{\partial a_i}{\partial x_1} < 0 \text{ and } \frac{dx_1}{dk_2} > 0
\]

**Proof of Proposition 4:**

Expected undetected fraud, given a continuing auditor is strictly less than that given a change in auditors.

When there is a change in auditors, expected undetected fraud for the first period is

\[
\theta \exp(-x_1)a_i = \theta \exp\left(-\log\left(\frac{1}{2} \left(1 + \sqrt{1 + \frac{4\theta R L_1}{p_i k_i}}\right)\right)\right) \frac{k_i}{2\theta L_1} \left(1 + \sqrt{1 + \frac{4\theta R L_1}{p_i k_i}}\right)
\]

\[
= \theta \frac{1}{\left(\frac{1}{2} \left(1 + \sqrt{1 + \frac{4\theta R L_1}{p_i k_i}}\right)\right) 2\theta L_1} \frac{k_i}{1 + \sqrt{1 + \frac{4\theta R L_1}{p_i k_i}}} = \frac{k_i}{L_i} \frac{k_i}{L_i} \text{. Other undetected fraud amounts are derived in a similar fashion}
\]

\(EUF_{\text{change}} = \frac{k_1}{L_1} + \frac{k_2}{L_2}\) and \(EUF_{\text{cont}} = \frac{k_1(1-\theta)k_2}{L_1 + \frac{k_2}{L_2}(1+x_2)} + \frac{k_2}{L_2} < \frac{k_1}{L_1} + \frac{k_2}{L_2}\)

Thus, \(EUF_{\text{cont}} < EUF_{\text{change}}\).

**Proof of Proposition 5:**

Audit risk in period 1 for both settings is \(\theta \exp(-x_1)\) and as we have shown in Proposition 2, audit effort for the continuing auditor is higher in period 1. Thus, audit risk is lower for the continuing auditor.

Audit risk in period 2 for the ‘new’ auditor is equal to
\[ AR_{2\,\text{new}} = \frac{\theta \exp(-x_i)}{\theta \exp(-x_i) + (1-\theta)} \exp(-x_2) = \frac{\exp(-x_i)}{\theta \exp(-x_i) + (1-\theta)} \frac{2\theta}{1 + \sqrt{1 + \frac{4R_2 L_2}{p_2 k_2} \theta \exp(-x_i)} (\theta \exp(-x_i) + (1-\theta))} \]

and audit risk for the continuing auditor in period 2 is

\[ AR_{2\,\text{cont}} = \frac{\theta \exp(-x_i)}{\theta \exp(-x_i) + (1-\theta)} \exp(-x_2) = \frac{\exp(-x_i)}{\theta \exp(-x_i) + (1-\theta)} \frac{2\theta}{1 + \sqrt{1 + \frac{4L_2 R_2}{k_2 p_2} \theta \exp(-x_i)} (\theta \exp(-x_i) + (1-\theta))} \]

Because

\[
d\left( \frac{\exp(-x_i)}{\theta \exp(-x_i) + (1-\theta)} \frac{2\theta}{1 + \sqrt{1 + \frac{4L_2 R_2}{k_2 p_2} \theta \exp(-x_i)} (\theta \exp(-x_i) + (1-\theta))} \right) = \frac{\exp(x_i)(1-\theta)\theta}{(\exp(x_i)(1-\theta)+\theta)^2 \sqrt{\frac{\exp(x_i) k_2 p_2 (1-\theta) + (k_2 p_2 + 4L_2 R_2)\theta}{k_2 p_2 (\exp(x_i)(1-\theta)+\theta)}}} < 0 \text{ and } x_{i\,\text{old}} < x_{i\,\text{cont}} ,
\]

\[ AR_{2\,\text{new}} < AR_{2\,\text{cont}} . \]
References


