Organizational Structure, Voluntary Disclosure, and Investment Efficiency

Hyun Hwang*

The University of Texas at Austin

Date: December, 2019

Abstract

An important role of corporate disclosure is to improve the efficiency of capital investment, and a key process in capital investment is firms’ internal allocation of capital across their multiple projects. This paper examines a multi-project firm’s disclosure behavior and its effect on investment efficiency. I identify conditions under which the multi-project firm withholds more information than a group of stand-alone firms. Despite less disclosure, I show that the multi-project firm enjoys higher investment efficiency than the stand-alone firms. The results suggest that organizational structure affects not only how capital is internally allocated but also firms’ disclosure behavior. In addition, corporate disclosure and internal capital allocation are substitute in improving capital investment efficiency.

JEL codes: D23; D82; D83; L22; L25; M41

Keywords: Organizational structure, internal capital allocation, corporate disclosure, investment efficiency

*Email address: hyun.hwang@mccombs.utexas.edu. This study is part of my dissertation at Carnegie Mellon University. I am greatly indebted to Carlos Corona (Co-Chair), Pierre Jinghong Liang (Co-Chair), Tim Baldenius, Jonathan Glover, Eunhee Kim, Austin Sudbury, and Erina Ytsma for their guidance and help. I also thank workshop participants at Carnegie Mellon University, the University of Texas at Austin, the University of Chicago, Hong Kong Polytechnic University, the 2019 Conference on the Convergence of Financial and Managerial Accounting, the 2019 AAA Annual Meeting, and the 2019 Junior Accounting Theory Conference.
1. Introduction

An important question in accounting is “whether and to what extent financial reporting facilitates the allocation of capital to the right investment projects (Roychowdhury, Shroff, and Verdi [2019]).” The allocation of capital takes two steps: i) capital providers supply a firm with capital, and ii) the firm allocates the raised capital to its investment projects. The disclosure literature has generally focused on the former. Specifically, the literature has investigated determinants that facilitate corporate disclosure, which helps the capital providers to make efficient investment decisions (see Beyer, Cohen, Lys, and Walther [2010], Stocken [2013] for a review). However, the effect of firms’ internal capital allocation on their disclosure behavior remains an open question, and this is important to understand how corporate disclosure can improve the efficiency of capital investment from a more holistic perspective.

Corporate finance research has studied organizational structure and its effects on internal capital allocation. The literature has investigated in particular whether a firm with multiple projects (i.e., the multi-divisional form or M-form) allocates capital better than an external-capital-markets benchmark, as depicted in Figure 1 (see Stein [2003]; Gertner and Scharfstein [2012] for a review). The research argues that the investment efficiency of multi-project firms relative to the benchmark depends on the degree of the information asymmetry between the firms and capital providers (e.g., Stein [1997]). Information asymmetry is also the main focus in the disclosure literature, and, in light of this intersection, Arrow [2015] calls for more research on the incentives for information sharing and its implications on organizational structure and internal capital allocation during his lecture “Future Directions of Research in the Coasean Tradition.”

![Figure 1: Two-project firms and stand-alone firms](image-url)
In this paper, I propose a disclosure model of a two-project firm in the context of internal capital allocation. In so doing, I address the following two questions: Does internal capital allocation induce the two-project firm to withhold more information than a group of two stand-alone firms? If it does, does the two-project firm perform worse than the group of two stand-alone firms in terms of investment efficiency? To answer the research questions, I make the following modeling assumptions. First, I follow the assumption of Verrecchia [1983] about corporate disclosure. That is, a firm’s manager is privately informed about project profitability and chooses to either disclose or withhold his private information about each project. The disclosure is credible, but it is costly and decreases project profitability. After the manager’s disclosure choice, capital providers make capital investment decisions. Second, as in Stein [1997], the firm’s manager can decide the allocation of capital across the two projects.

The analysis of the model delivers two main results. First, if disclosure cost is at an intermediate level, the two-project firm withholds more information than a group of two stand-alone firms. Thus, the results show that organizational structure affects not only how capital is allocated across the projects, but also firms’ disclosure behavior. Second, despite less disclosure, the two-project firm enjoys a higher investment efficiency than the group of the two stand-alone firms. The results suggest that disclosure and internal capital allocation are substitute in improving capital investment efficiency. Thus, in the presence of internal capital allocation, less disclosure may not be indicative of inefficient capital investment.

The intuition of the first result is as follows. The standard result of the disclosure literature suggests that the capital providers become concerned about their investment in a stand-alone upon no disclosure. This induces the capital providers not to supply the firm with capital. Thus, the manager has a strong incentive to disclose good news to avoid no capital investment. However, if a firm owns two independent projects and withholds information, the capital providers are less concerned about their investment, for the following reasons. First, the manager is more likely to be informed of good news about at least one project, due to the independence of the two projects. Second, the manager allocates capital to the best project to maximize the expected profit from the project. Lastly, no costly disclosure implies a higher project profitability. Thus, even without disclosure, the capital providers supply the two-project firm
with capital that is enough to implement one project. Thus, the two-project firm has a weaker incentive for disclosure and withholds more information than a group of two stand-alone firms. Both the two-project firm and the stand-alone firm exhibit the same disclosure behavior with either low or high disclosure cost. With a high level of disclosure cost, the two-project firm and stand-alone firm always withhold information because costly disclosure renders projects negative-NPV. In contrast, with a low level of disclosure cost, no disclosure is interpreted as the two-project firm hiding bad news. As a result, both the stand-alone firms and the two-project firms have a strong incentive to disclose their private information.

The second result shows that despite less disclosure, the investment efficiency of the two-project firm is higher than that of the two stand-alone firms with an intermediate level of disclosure cost. This is because the two-project firm faces, on average, less of an underinvestment problem than the stand-alone firm. Upon no disclosure from the stand-alone firm, the capital providers are unwilling to invest capital in the firm, although it might possess a positive-NPV project. The stand-alone firm is not able to release its private information, because the costly disclosure would render the project negative-NPV. However, the two-project firm can raise capital for a positive-NPV project without costly disclosure, because the capital providers become confident in their investment due to the internal capital allocation by the manager. There are chances that both projects might be negative-NPV, which leads to an over-investment problem. However, the expected cost of the over-investment problem is low, because of a low likelihood of having two negative-NPV projects simultaneously. Thus, the results suggest that internal capital allocation and costly disclosure are substitute in improving investment efficiency, so less disclosure does not necessarily imply inefficient investment.

I also show that if a firm owns sufficiently many independent projects, the firm withholds information about every project but enjoys the highest investment efficiency. That is, corporate disclosure becomes irrelevant in increasing investment efficiency. The intuition comes from the law of large numbers. That is, upon no disclosure, the capital providers may not necessarily know which projects are positive-NPV, but they know the number of positive-NPV projects owned by the firm. This information is sufficient for the capital providers to make capital investment decisions, because they know that the manager will allocate capital to profitable
Thus, the manager has no incentive to incur the cost to reveal any information about the projects. In equilibrium, the manager remains silent about project profitability; the capital providers supply the firm with capital that is enough to implement every positive-NPV project, and the manager implements the positive-NPV projects.

This paper investigates firms’ disclosure behavior by considering both external and internal capital allocation. Thus, the paper helps explain the role of corporate disclosure in improving investment efficiency from a more holistic perspective. The paper builds on several earlier works such as Coase [1990], Sunder [1997], and Zingales [2000]. Sunder [1997] emphasizes that “to understand accounting, the firm itself must be understood.” I follow this call by investigating accounting problems in the context of organizational structure. Specifically, my paper is related to the literature of voluntary disclosure and internal capital markets.

To begin, the paper contributes to the literature on voluntary disclosure by investigating voluntary disclosure of multiple signals in the presence of both external and internal capital markets. The literature has studied voluntary disclosure of multiple signals. For example, Kirschenheiter [1997] and Pae [2005] consider a setting in which a manager chooses to disclose two signals about the future cash flow of the firm. Einhorn and Ziv [2007] show that if different activities cannot be measured with the same level of precision, the two-project firm discloses less information. Other papers have examined firms’ choice of the aggregation of multiple signals (e.g., Arya, Frimor, and Mittendorf [2010]; Arya and Glover [2014]; Ebert, Simons, and Stecher [2017]). The literature has also investigated voluntary disclosure and capital investment. For example, Bertomeu, Beyer, and Dye [2011] jointly examine voluntary disclosure, capital structure, and the cost of capital. Cheynel [2013] considers the general equilibrium effect of voluntary disclosure on investment efficiency and the cost of capital.

My paper also contributes to the literature on internal capital markets by investigating the roles of corporate disclosure in explaining the relative benefits of organizing multiple projects under the same roof (see Gertner and Scharfstein [2012] for a review). Researchers have emphasized both the importance of and lack of research on information sharing in affecting firm boundaries (e.g., Holmstrom and Roberts [1998]; Arrow [2015]). My paper responds to their
suggestions by analyzing external disclosure in the context of Stein’s [1997] model of internal capital allocation. Consequently, I identify conditions under which internal capital markets perform better than an external capital-market benchmark with respect to a level of disclosure friction. Laux [2001] shows that organizing multiple projects under the same roof is beneficial because it becomes easier to incentivize the agent to exert effort. This effect comes from imperfect correlation among multiple projects, a diversification effect. My paper also depends on a diversification effect to investigate firms’ disclosure behavior.

This paper is organized as follows. In Section 2, I analyze the model of the stand-alone firm. I analyze the model of the two-project firm in section 3. In Section 4, I analyze the model of the firm with many projects. Section 5 concludes the paper.

2. Stand-alone Firm

2.1. Model

Players. Consider a variation of the model in Stein [1997]. A founder (she) owns an investment project, which requires both managerial labor and financial capital. Thus, she needs to hire a professional manager and raise capital from a group of financiers. The founder derives utility from the cash flows from the project, net of payments to the financiers, or wages to the manager, and the reservation wage of the manager is normalized at zero. Thus, the founder is concerned with the expected net cash flows from the investment project, that is, the efficiency of capital investment. The manager (he) always prefers more capital investment (i.e., an empire builder), but given capital investment, he maximizes the expected net cash flows from the project. The capital market is competitive, and the financiers break even. The market interest rate is normalized at zero. All the players are risk-neutral.

Technology and Information Structure. The investment project requires one unit of capital to generate cash flow $x \in \{R, 0\}$, where $R > 1$. $x = R$ occurs with probability $p$, and $x = 0$ occurs with $1 - p$. In this paper, I call $p$ the “success probability.” As a professional manager, he privately observes success probability $p$ about the investment project, and it is common
knowledge that \( p \) is uniformly distributed over \([0,1]\). The manager can credibly disclose his private information about success probability \( p \) in the sense of Verrecchia [1983]. Specifically, the manager makes disclosure choice \( d \in \{c, 0\} \). If \( d = c \) is chosen, \( p \) is disclosed at the expense of disclosure cost \( c \geq 0 \); if \( d = 0 \) is chosen, the manager remains silent about \( p \) and saves disclosure cost \( c \).

![Figure 2: A Stand-alone Firm](image)

**Capital Investment.** The financiers make capital investment decisions by choosing \( k \in \{1,0\} \). If \( k = 0 \) is chosen, no capital investment is made. If \( k = 1 \) is chosen, the financing contract is signed, and the project is implemented with one unit of capital. If the project generates cash flow \( x = R \), the financiers receive \( R_\ell \) and the founder receives \( R - R_\ell \). If the project generates \( x = 0 \), both receive zero cash flows. Repayment \( R_\ell \) is set such that the financiers break even:

\[
E[p|d]R_\ell - (1 + d) = 0. \tag{1}
\]

\( E[p|d] \) is the financiers’ expectation about success probability \( p \), given the manager’s disclosure choice \( d \in \{c, 0\} \), and \( 1 + d \) is the total investment cost. Thus, repayment \( R_\ell \) depends on the manager’s disclosure choice \( d \). By rearranging the break-even condition in (1), repayment \( R_\ell \) can be expressed as

\[
R_\ell(d) = \frac{1 + d}{E[p|d]}
\]

Notice that repayment \( R_\ell(d) \) to the financiers is paid from the project’s cash flow \( x = R \). This implies that \( R_\ell(d) \) needs to be lower than or equal to \( R \) so that the repayment upon \( x = R \) is credible. \( R_\ell(d) > R \) implies that the project cannot generate enough cash flow for the financiers to break even. Thus, the financiers choose \( k = 1 \) if \( R_\ell(d) \leq R \) and \( k = 0 \) otherwise.
notational convenience, I will omit arguments if there is no confusion. The following table summarizes the timeline of the model:

<table>
<thead>
<tr>
<th>Date 1</th>
<th>Date 2</th>
<th>Date 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manager privately observes $p \sim U[0,1]$.</td>
<td>Financiers make capital investment decision by choosing $k \in {1,0}$.</td>
<td>If $k = 1$, project generates cash flow $x \in {R, 0}$, which is distributed to the financiers and the founder.</td>
</tr>
<tr>
<td>Manager makes disclosure choice $d \in {c, 0}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 3: Timeline – Stand-alone Firm

**Date 1 – Disclosure.** The manager privately observes success probability $p$ and makes disclosure choice $d \in \{c, 0\}$. The empire-building manager’s objective is to induce capital investment from the financiers and, given capital investment, maximize the expected net cash flow from the project:

$$\max_{d \in \{c, 0\}} \{p(R - R_\ell(d)) + \beta\}k(d),$$

where $\beta > 0$ captures the manager’s level of private benefit from capital investment. I assume that $\beta$ is sufficiently large that the manager always prefers more capital investment.\(^1\) I assume that $d = 0$ is chosen, if the manager believes that the financiers will not invest their capital upon $d = c$; that is, if $k(c) = 0$.

**Date 2 – Investment.** Repayment $R_\ell$ is determined to satisfy the break-even condition in (1). If repayment $R_\ell$ satisfies $R_\ell \leq R$, the financiers invest capital in the project (i.e., $k = 1$) and the financing contract is signed. Otherwise, the financiers choose $k = 0$ not to invest capital in the project.

**Date 3 – Outcome.** If $k = 1$ is chosen, the project generates cash flow $x \in \{R, 0\}$. The project generates $x = R$ with probability $p$. In this case, $R_\ell$ is distributed to the financiers and

---

\(^1\) If $\beta$ is small, a first-best financing contract can be written so that the manager is induced not to implement a negative-NPV project; this renders the disclosure technology irrelevant. In the Appendix, I show the value of $\beta$ above which the first-best financing contract is not feasible.
$R - R_\ell$ is distributed to the founder. The project generates $x = 0$ with probability $1 - p$. In this case, both the founder and the financiers receive zero cash flows.

**The Equilibrium Concept.** The equilibrium solution concept is a Perfect Bayesian Equilibrium (PBE). A PBE is characterized by a set of decisions and repayment $R_\ell$ such that

1. at date 1, $d^* = \arg \max_{d \in \{c, 0\}} \{p(R - R_\ell(d)) + \beta\} \times k(d)$ maximizes the manager’s expected utility, given $p$, $k(d)$ and $R_\ell(d) = (1 + d)/E[p|d]$;
2. if $d = c$ leads to $k(c) = 0$, $d = 0$ is chosen;
3. at date 2, given $d^* \in \{c, 0\}$, if $R_\ell(d^*) \leq R$, $k = 1$ is chosen. Otherwise, $k = 0$ is chosen;
4. the players have rational expectations at each date. In particular, the players’ beliefs about each other’s strategies are consistent with the Bayes rule, if possible.

In the following section, I analyze the model of the stand-alone firm, and this becomes the benchmark for the analysis of the two-project firm.

### 2.2. Analysis

Consider a stand-alone firm with a single investment project. I use backward induction to evaluate the ex-ante net cash flows of the stand-alone firm and the efficiency of capital investment. This serves as the benchmark for the analysis of the two-project firm in Section 3.

**Date 3 – Outcome.** If capital investment has been made (i.e., $k = 1$), the project generates cash flow $x = R$ with probability $p$. Then, cash flow $R$ is split between the founder, $R - R_\ell$, and the financiers, $R_\ell$. The project generates $x = 0$ with probability $1 - p$. In this case, both the founder and the financiers receive zero cash flows.

**Date 2 – Investment.** Given the manager’s disclosure choice $d \in \{c, 0\}$, the financiers evaluate the expected success probability, $E[p|d]$. Let $p_{BE}(d)$ denote the probability that makes the project break-even, given $d \in \{c, 0\}$: $p_{BE}(d)R - 1 - d = 0$. Then, $p_{BE}(d)$ is be expressed as

$$p_{BE}(d) = \frac{1 + d}{R}.$$
The financiers compare $p_{BE}(d)$ with the expected success probability $E[p|d]$. If $E[p|d]$ is higher than or equal to $p_{BE}(d)$, the financiers believe that the project is not negative-NPV. This is because $E[p|d] \geq p_{BE}(d)$ implies that

$$E[p|d]R - (1 + d) \geq p_{BE}(d)R - (1 + d) = 0.$$ 

In this case, there exists $R_\ell \in [0, R]$ that satisfies the break-even condition in (1), and capital investment is made (i.e., $k = 1$). If $E[p|d] < p_{BE}(d)$, the project is negative-NPV; there does not exist $R_\ell \in [0, R]$ that satisfies the break-even condition. Thus, $k = 0$ is chosen. Then, the financiers’ investment rule can be summarized as follows:

$$k(d) = \begin{cases} 
1, & \text{if } E[p|d] \geq p_{BE}(d); \\
0, & \text{otherwise.}
\end{cases}$$

**Date 1 – Disclosure.** The manager chooses $d \in \{c, 0\}$ to induce capital investment from the financiers and maximize the expected net cash flow from the project:

$$\max_{d \in \{c, 0\}} \{p(R - R_\ell(d)) + \beta\}k(d).$$

Lemma 1 summarizes both the disclosure strategy by the manager and the investment strategy by the financiers. The proofs for lemmas and propositions are located in the Appendix.

**Lemma 1.** Let $c \geq 0$ and $R > 1$ be given. The equilibrium disclosure strategy of the manager and the investment strategy of the financiers are as follows.

(i) Suppose $c \geq 1$. Then, the manager chooses $d = 0$. Upon $d = 0$, the financiers believe $p \leq 1$ and choose $k = 1$ if $R \geq 2$ and $k = 0$ if $R < 2$.

(ii) Suppose that $c < 1$ and $R \geq 1 + c$. Then, the manager chooses $d = c$ if $p \geq p_{BE}(c) = \frac{1+c}{R}$ and $d = 0$ if $p < p_{BE}(c) = \frac{1+c}{R}$. Upon $d = 0$, the financiers believe $p < p_{BE}(c) = \frac{1+c}{R}$ and choose $k = 0$. Upon $d = c$, the financiers choose $k = 1$.

(iii) Suppose that $c < 1$ and $R < 1 + c$. Then, the manager chooses $d = 0$ for $p \leq 1$. Upon $d = 0$, the financiers believe $p \leq 1$ and choose $k = 0$. 
Part (i) of Lemma 1 shows that if disclosure cost \( c \) is sufficiently high, the manager finds it unprofitable to disclose even the highest success probability \( p = 1 \). Thus, the financiers do not interpret no disclosure as the manager hiding a low success probability, and they invest capital in the firm as long as the unconditional NPV of the project is nonnegative: \( E[p]R - 1 \geq 0 \Rightarrow R \geq 2 \). Part (ii) shows that with a lower disclosure cost \( c \), no disclosure is interpreted as the manager having observed a negative-NPV project, and the financiers do not invest capital upon no disclosure. Thus, the manager who has observed \( p \geq p_{BE}(c) \) chooses to reveal it to avoid no capital investment from the financiers. Part (iii) considers a condition in which disclosure cost \( c \) is low but the NPV of the project is negative upon disclosure. Thus, no information is released. In addition, the condition also implies that the unconditional NPV of the project is negative, so there is no capital investment. Based on Lemma 1, Proposition 1 examines the ex-ante net cash flow of the stand-alone firm.

**Proposition 1.** Let \( c \geq 0 \) and \( R > 1 \) be given. The ex-ante net cash flow of the stand-alone firm is as follows.

(i) Suppose \( c \geq 1 \). Then, the ex-ante net cash flow is \( \max\left\{ \frac{1}{2} R - 1, 0 \right\} \).

(ii) Suppose that \( c < 1 \) and \( R \geq 1 + c \). Then, the ex-ante net cash flow is \( \frac{(R-1-c)^2}{2R} \).

(iii) Suppose that \( c < 1 \) and \( R < 1 + c \). Then, the ex-ante net cash flow is zero.

Proposition 1 is concerned with the ex-ante net cash flow of the stand-alone firm given disclosure cost \( c \). Part (i) of Proposition 1 shows that with a high disclosure cost \( c \), the ex-ante net cash flow is the greater of zero or the unconditional NPV of the project. Intuitively, Lemma 1 shows that with a high disclosure cost, the financiers invest capital in the stand-alone firm as long as the unconditional NPV of the project is nonnegative. In this case, there is an *over-investment* problem, if success probability \( p \) is lower than \( p_{BE}(0) \). If the unconditional NPV of the project is negative, the financiers choose not to invest capital in the firm. In this case, there is an *under-investment* problem, if success probability \( p \) is higher than \( p_{BE}(0) \). Part (ii) shows that
with a lower disclosure cost, the ex-ante net cash flow increases with a lower disclosure $c$. Intuitively, with a lower disclosure cost $c$, the manager finds it profitable to reveal that the project is not negative-NPV (i.e., $p \geq p_{BE}(c)$), and a lower disclosure cost implies a higher net cash flow for the firm. In addition, the manager does not disclose $p < p_{BE}(c)$, which enables the financiers to avoid investing capital in a negative NPV project with $p < p_{BE}(0)$. This mitigates the over-investment problem and increasing the ex-ante net cash flow. However, there still is an under-investment problem upon no disclosure, because the firm might possess a project with $p \geq p_{BE}(0)$ and $p < p_{BE}(c)$. This project is positive-NPV without costly disclosure. However, the manager withholds the success probability, because disclosure would render it negative-NPV. Part (iii) shows that the ex-ante net cash flow is zero, because Lemma 1 shows that no capital investment is made under this condition. In the next section, I consider the firm with two projects and investigate the effect of internal capital allocation on the firm’s disclosure strategy and the ex-ante net cash flow of the two-project firm.

3. Two-project Firm

3.1. Model

Thus far, we have focused on the firm with a single project. In this section, I consider a firm with projects 1 and project 2. Project $i \in \{1, 2\}$ requires one unit of capital and generates cash flow $x_i \in \{R, 0\}$. $x_i = p_i$ occurs with probability $p_i$ and $x_i = 0$ with $1 - p_i$; $p_i$ follows a uniform distribution over $[0, 1]$; and $p_1$ and $p_2$ are independent.

The manager privately observes both $p_1$ and $p_2$ and makes disclosure choices $d_i \in \{c, 0\}$ about the two success probabilities. If the manager chooses $d_i = c$, $p_i$ is disclosed at the expense of disclosure cost $c$; if $d_i = 0$ is chosen, $p_i$ is withheld. Thus, there are four possible disclosure choices: both $p_1$ and $p_2$ are disclosed at disclosure cost $2c$, one of the success probabilities is disclosed at disclosure cost $c$, and neither of the success probabilities is disclosed. Given the manager’s disclosure choices $d_1$ and $d_2$, the financiers decide whether to invest two, one, or zero units of capital in the firm by choosing $k \in \{2, 1, 0\}$. 
Internal Capital Allocation. The manager has control rights with respect to internal capital allocation (e.g., Gertner, Scharfstein, and Stein [1994]; Stein [1997]). Specifically, if the financiers choose $k = 1$ to make one unit of capital investment in the firm, the manager decides which project to implement by choosing $I \in \{1, 0\}$: if $I = 1$ is chosen, project 1 is implemented; if $I = 0$ is chosen, project 2 is implemented. If the implemented project generates $R$, the financiers receive $R_{\ell} \in [0, R]$ and the founder receives $R - R_{\ell}$. Thus, given $k = 1$, the manager chooses $I \in \{1, 0\}$ to maximize the expected net cash flow from the project:

$$p_1(R - R_{\ell})I + p_2(R - R_{\ell})(1 - I).$$

If $k = 2$ is chosen, both project 1 and project 2 are implemented. Since both projects are implemented, internal capital allocation becomes irrelevant. In this case, the structure is the same as that of the stand-alone firm: upon cash flow $x_i = R$ from project $i \in \{1, 2\}$, the financiers receive $R_{\ell,i}$ and the founder receives $R - R_{\ell,i}$. If $k = 0$ is chosen, none of the projects is implemented, rendering internal capital allocation unviable. The game has four dates:

<table>
<thead>
<tr>
<th>Date 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manager privately observes $p_i \sim U[0,1]$ for $i \in {1, 2}$</td>
</tr>
<tr>
<td>Manager chooses $d_i \in {c, 0}$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Date 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financiers choose $k \in {2, 1, 0}$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Date 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>If $k = 1$, manager chooses $I \in {1, 0}$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Date 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash flow $x_i \in {R, 0}$ generated and distributed.</td>
</tr>
</tbody>
</table>

Figure 5: Timeline – Two-project Firm
**Date 1 – Disclosure.** The manager privately observes both $p_1$ and $p_2$; he then decides whether to disclose success probability $p_i$ by choosing $d_i \in \{c, 0\}$ for $i \in \{1,2\}$. The manager makes disclosure choices to induce capital investment from the financiers and maximize the expected net cash flow from the projects. Since the financiers choose either $k = 2$, $k = 1$, or $k = 0$, the manager considers three possibilities when making his disclosure choices:

$$
\max_{d_1, d_2 \in \{c, 0\}} \left\{ \sum_{i=1}^2 p_i \left( R - R_{\ell,i}(d_1, d_2) \right) + 2\beta, \begin{array}{l} p_1 (R - R_{\ell}(d_1, d_2)) I + p_2 (R - R_{\ell}(d_1, d_2))(1 - I) + \beta, \\ 0 \end{array} \right. \right\}.
$$

**Date 2 – Investment.** The financiers choose $k \in \{2,1,0\}$ to decide whether to invest two, one, or zero units of capital in the firm. If the financiers believe that both projects are not negative-NPV, they choose $k = 2$. In this case, repayment $R_{\ell,i}$ is determined for project $i \in \{1,2\}$ such that the break-even condition for each project, $E[p_i|d_1, d_2] \times R_{\ell,i} - (1 + d_i) = 0$, is satisfied:

$$
R_{\ell,i} = \frac{1 + d_i}{E[p_i|d_1, d_2]}, i \in \{1,2\}.
$$

In the case of $x_i = R$, the financiers receive $R_{\ell,i} \in [0, R]$ and the founder receives $R - R_{\ell,i}$. The financiers choose $k = 1$, if they believe that one unit of capital investment is not negative-NPV. In the case of project success, the financiers receive $R_{\ell} \in [0, R]$ and the founder receives $R - R_{\ell}$. If the financiers believe that capital investment is negative-NPV, they choose $k = 0$ and neither project is implemented.

**Date 3 – Internal Capital Allocation.** With $k = 1$, the manager chooses $I \in \{1,0\}$ to decide which project to implement to maximize the expected net cash flow from the project, as described in the manager’s objective function in (2). If $k = 2$, both project 1 and project 2 are implemented, rendering internal capital allocation irrelevant. If $k = 0$, no project is implemented, rendering internal capital allocation unviable.
**Date 4 – Outcome.** The implemented projects generate cash flows, which are distributed to the founder and the financiers.

### 3.2. Analysis

In this section, I investigate the disclosure strategy of the manager of the two-project firm and the investment strategy of the financiers. Recall the discussion of Part (ii) of Lemma 1 that with a low disclosure cost, the financiers do not invest capital in the stand-alone firm upon no disclosure. This leads to the under-investment problem, because the firm might possess a positive-NPV project. I derive the conditions under which the two-project firm can raise capital for one project without disclosure, which mitigates the under-investment problem. I use backward induction to solve the model.

**Date 4 – Outcome.** If both projects are implemented (i.e., $k = 2$), project $i \in \{1, 2\}$ generates cash flow $x_i = R$ with probability $p_i$ and $x_i = 0$ with probability $1 - p_i$. In the case of project $i$’s success, the financiers receive $R_{\ell_i}$ and the founder receives $R - R_{\ell_i}$. With $x_i = 0$, both receive zero cash flows. If one unit of capital investment is made (i.e., $k = 1$) and the implemented project generates $R$, the financiers receive $R_{\ell}$ and the founder receives $R - R_{\ell}$. If both projects are not implemented (i.e., $k = 0$), both the founder and the financiers receive no cash flows.

**Date 3 – Internal Capital Allocation.** If $k = 1$ has been chosen by the financiers, the manager decides which project to implement. The manager’s objective is to maximize the expected net cash flow from the project implementation, and he chooses $I \in \{1, 0\}$ to maximize

$$\max_{I \in \{1, 0\}} p_1(R - R_{\ell})I + p_2(R - R_{\ell})(1 - I).$$

To maximize the expected net cash flow from the project, the manager chooses $I = 1$ if $p_1 \geq p_2$ and $I = 0$ otherwise. This is an example of the “smarter-money” effect of internal capital allocation, as described in Stein [2003]. Williamson [1975] argues that “this assignment of cash flow to high yield uses is the most fundamental attribute of the M-form enterprise.” The manager’s capital allocation strategy is summarized in Lemma 2.
Lemma 2. Suppose $k = 1$. Then, the manager chooses $I = 1$ if $p_1 \geq p_2$ and $I = 0$ if $p_1 < p_2$.

Date 1 and Date 2 – Disclosure and Investment. At date 2, the financiers make capital investment decisions by choosing $k \in \{2, 1, 0\}$. At date 1, the manager makes a disclosure decision by choosing $d_1$ and $d_2$. Lemma 1 shows that if disclosure cost $c$ is low (i.e., $c < 1$), capital investment is not made without costly disclosure. In this section, I assume that there exist conditions under which one unit of capital investment is made in the two-project firm (i.e., $k = 1$) even if the manager chooses $d_1 = d_2 = 0$. I investigate how the capital investment upon no disclosure affects the disclosure behavior of the two-project firm. In Proposition 2, I derive the conditions under which such capital investment takes place upon no disclosure.

Suppose that $p_i < p_{BE}(c)$ and $p_j \geq p_{BE}(c)$, where $i \neq j$. Then, the manager-withholds success probability $p_i$ (i.e., $d_i = 0$), because the disclosure of $p_i < p_{BE}(c)$ would render project $i$ negative-NPV. When choosing to disclose success probability $p_j$, the manager needs to compare the expected payoff upon $d_j = c$ and $d_j = 0$. Let $V_{ND}$ denote the financiers’ evaluation about the expected NPV of one unit of capital investment upon $d_1 = d_2 = 0$, and let $A$ denote the corresponding expected success probability of the implemented project. Then, $V_{ND}$ is expressed as

$$V_{ND} = A \times R - 1.$$

If we have $V_{ND} \geq 0$ so that one unit of capital investment is not negative-NPV, the financiers choose $k = 1$. In this case, the break-even condition for the financiers is $A \times R - 1 = 0$, and $R = 1/A$ is set for repayment. Then, the manager discloses $p_j$ if the disclosure leads to a lower repayment than $1/A$. Let $p^* \geq p_{BE}(c)$ denote the probability such that the disclosure of $p_j = p^*$ leads to the same repayment as $1/A$:

$$\frac{1}{A} = \frac{1 + c}{p^*}.$$
Then, the manager chooses to disclose $p_j$ if it is equal to or greater than $p^*$. Lemma 3 investigates the effect of the existence of such $p^*$ on the manager’s disclosure and the financiers’ investment strategy. Proposition 2 derives the condition under which such $p^*$ exists.

**Lemma 3.** Suppose that $c < 1$ and $R \geq 1 + c$. Let $p^* \geq \frac{1+c}{R}$ be given such that $\frac{1}{A} = \frac{1+c}{p^*}$ is satisfied for $A \in [0,1]$.

(i) $V_{ND} = A \times R - 1 \geq 0$.

(ii) The financiers choose $k = 2$ if $d_1 = d_2 = c$ and $k = 1$ otherwise.

(iii) The disclosure strategy of the manager of the two-project firm is as follows:

(a) The manager chooses $d_i = c$ and $d_j = 0$ if

$$p_i \geq p^* \text{ and } p_j < p_{BE}(c) = \frac{1+c}{R}.$$ 

(b) The manager chooses $d_1 = d_2 = 0$ if

$$p_i < p^* \text{ and } p_j < p_{BE}(c) = \frac{1+c}{R}, i \neq j.$$ 

(c) The manager chooses $d_1 = d_2 = c$ if $p_i \geq p_{BE}(c) = \frac{1+c}{R}$ for $i \in \{1,2\}$.

Lemma 3 shows that if the firm can induce one unit of capital from the financiers, the two-project firm withholds more information than a group of the two stand-alone firms. Suppose that there are two stand-alone firms, firm 1 and firm 2, with independent success probabilities. Recall from Lemma 1 that with $c < 1$ and $R \geq c + 1$, both firm 1 and firm 2 fail to attract capital investment from the financiers upon the withholding of their success probabilities $p_i < p_{BE}(c)$. This induces the managers of both firm 1 and firm 2 to disclose any success probability $p_i \geq p_{BE}(c)$ to avoid no capital investment. This is described in Figure 6A. Lemma 3 shows that if, upon no disclosure, the financiers believe that one unit of capital investment is not negative-NPV (i.e., $V_{ND} \geq 0$), the two-project firm can withhold both $p_1, p_2 < p_{BE}(c)$ and still manage to
raise capital for one project. As a result, the two-project firm has a weaker incentive to disclose its success probabilities, leading to more withholding of success probabilities. This is described in Figure 6B. Note that if \( p_1, p_2 \geq p_{BE}(c) \), the empire-building manager reveals both probabilities so that the two projects can be implemented.

\[
p_1, p_2 \geq p_{BE}(c),
\]

This is described in Figure 6B. Note that if \( p_1, p_2 \geq p_{BE}(c) \), the empire-building manager reveals both probabilities so that the two projects can be implemented.

\[
\begin{align*}
  p_1, p_2 \geq p_{BE}(c) \\
  d_1 = 0; d_2 = c \\
  d_1 = d_2 = 0
\end{align*}
\]

Figure 6A: Disclosure Choices of Firm 1 and Firm 2

\[
\begin{align*}
  p_1, p_2 \geq p_{BE}(c) \\
  d_1 = 0; d_2 = c \\
  d_1 = d_2 = 0
\end{align*}
\]

Figure 6B: Disclosure Choices of Two-project Firm

I now turn to the derivation of \( V_{ND} \), that is, the financiers’ evaluation about the expected NPV of one unit of capital investment upon no disclosure (i.e., \( d_1 = d_2 = 0 \)). The financiers evaluate \( V_{ND} \) based on their belief about success probability \( p_1 \) and success probability \( p_2 \) upon no disclosure, which is specified in Lemma 3; they consider three possibilities about \( p_1 \) and \( p_2 \). First, both \( p_1 \) and \( p_2 \) are less than the break-even probability with disclosure (i.e., \( p_{BE}(c) \)). Second, \( p_1 \) is greater than \( p_{BE}(c) \) but less than \( p^* \); \( p_2 \) is less than \( p_{BE}(c) \). Third, \( p_2 \) is greater than \( p_{BE}(c) \) but less than \( p^* \); \( p_1 \) is less than \( p_{BE}(c) \). Thus, the financiers’ evaluation about \( V_{ND} \) depends on \( p^* \). The following proposition derives the conditions under which the two-project firm can afford to raise capital for one project upon no disclosure.
**Proposition 2.** Let $0 \leq c < 1$ and $R \geq 1 + c$ be given. Let $A(p^*)$ denote the expected success probability of the project upon $d_1 = d_2 = 0$, based on the financiers’ belief about $p_1$ and $p_2$ upon the manager’s disclosure choice in Lemma 3.

(i) If $c < 1/2$, there does not exist $p^* \geq \frac{1+c}{R}$ such that $\frac{1}{A(p^*)} = \frac{1+c}{p^*}$. Then, the manager chooses $d_i = 0$ if $p_i < \frac{1+c}{R}$ and $d_i = c$ otherwise. The financiers choose $k = 2$ upon $d_1 = d_2 = c$, $k = 1$ upon $d_i \neq d_j$, and $k = 0$ upon $d_1 = d_2 = 0$.

(ii) Suppose $\frac{1}{2} \leq c < 1$. Then, there exists the unique $p^* \geq \frac{1+c}{R}$ such that $\frac{1}{A(p^*)} = \frac{1+c}{p^*}$. In addition, $\frac{\partial}{\partial c} p^*(c) > 0$ and $\frac{\partial}{\partial R} p^*(R) < 0$. Then, the disclosure and investment strategy are characterized in Lemma 3.

Part (i) of Proposition 2 shows that if disclosure cost $c$ is low (i.e., $c < 1/2$), the two-project firm exhibits the same disclosure behavior as the two stand-alone firms. Intuitively, a low disclosure cost $c$ implies that the two-project firm can easily reveal both $p_1$ and $p_2$ at a relatively low cost. Thus, the financiers perceive $d_1 = d_2 = 0$ as the manager hiding low $p_1$ and $p_2$. As a result, upon no disclosure, no capital investment is made, making internal capital allocation upon no disclosure unviable. With unviable internal capital allocation across the projects, the two-project firm is the same as the two stand-alone firms in terms of disclosure behavior.

Part (ii) of Proposition 2 shows that if disclosure cost $c$ is at an intermediate level, the two-project firm can implement one project without costly disclosure. This is because of both the diversification effect of independent success probabilities and internal capital allocation across the two projects. The diversification effect implies that upon no disclosure, the financiers believe that the two-project firm is more likely to possess at least one profitable project than the stand-alone firm. Moreover, from Lemma 2, the financiers know that if $k = 1$, the manager allocates capital to the most profitable project. Thus, the combination of the effects of diversification and internal capital allocation induces the financiers to make one unit of capital investment in the two-project firm. Part (ii) also shows that the manager of the two-project firm withholds more
information if disclosure cost $c$ is higher. This is intuitive, because the financiers interpret nondisclosure as the two-project firm withholding information due to higher disclosure cost. With this financiers’ belief, the two-project firm can withhold more information. On the other hand, the two-project firm withholds less information if cash flow $R$ upon success is higher. With a higher $R$, the project becomes more profitable. Thus, firms with a lower success probability can also afford to incur the cost to disclose its private information to avoid no capital investment from the financiers. Thus, a higher cash flow $R$ upon success makes it hard for the two-project firm to withhold low success probabilities. The next lemma summarizes the equilibrium capital investment and disclosure decisions of the two-project firm under conditions of a high disclosure cost.

**Lemma 4.** Suppose that either $c \geq 1$ or $R < 1 + c$ such that $c < 1$. Then, the manager always chooses $d_1 = d_2 = 0$. If $R < \frac{3}{2}$, the financiers choose $k = 0$. If $\frac{3}{2} \leq R < 2$, the financiers choose $k = 1$. If $R \geq 2$, the financiers choose $k = 2$.

Lemma 4 shows that if disclosure is not economically viable, both the two-project firm and the stand-alone firm withhold information. Thus, the two-project firm exhibits the same disclosure behavior as the two stand-alone firms. However, there are conditions under which the two-project firm can still implement one project without disclosure. This is also driven by the effects of both the diversification effect of independent projects and internal capital allocation, which induce the financiers to believe that one unit of capital investment in the two-project firm is not negative-NPV. This result is consistent with that of Stein [1997], which shows the benefit of internal capital allocation across independent projects without credible communication between firms and financiers. In the next analysis, I calculate the ex-ante net cash flow of the two-project firm and show that it is at least as much as the ex-ante net cash flows of a group of the two stand-alone firms.
Proposition 3. Let \( c \geq 0 \) and \( R > 1 \) be given.

(i) If \( \frac{1}{2} \leq c < 1 \) and \( R \geq 1 + c \), the ex-ante net cash flow of the two-project firm is greater than the ex-ante net cash flow of the two stand-alone firms.

(ii) If \( \frac{1}{2} < c \) and \( R \geq 1 + c \), the ex-ante net cash flow of the two-project firm is the same as the ex-ante net cash flow of the two stand-alone firms.

(iii) If either \( c \geq 1 \) or \( R < 1 + c \) for \( c < 1 \), the ex-ante net cash flow of the two-project firm is at least as much as the ex-ante net cash flows of the two stand-alone firms.

Proposition 3 shows that at an intermediate level of disclosure cost, the two-project firm withholds more information but exhibits higher ex-ante net cash flows than a group of the two stand-alone firms. The result appears counterintuitive, because higher ex-ante net cash flow (i.e., higher investment efficiency) is associated with less disclosure to the financiers. However, less disclosure is not to be interpreted as indicative of inefficient capital investment, because under these conditions, the two-project firm faces, on average, less of an under-investment problem than the stand-alone firm. The stand-alone firm faces an under-investment problem if it possesses a positive-NPV project but cannot implement the project. This problem arises if the costly disclosure makes the project negative-NPV. The two-project firm can overcome this under-investment problem without costly disclosure; the financiers are induced to make capital investment in the two-project firm, because they are confident with their capital investment due to the internal capital allocation over the independent projects. The two-project firm might also face an over-investment problem, if it possesses two negative-NPV projects. However, the financiers believe that such an event is not likely to occur, due to the diversification effect of the independent projects and the allocation of capital to the best project. Thus, the expected cost of the over-investment problem is lower than the expected benefit of mitigating the under-investment problem. The result indicates that both corporate disclosure and organizational structure play an important role in improving capital allocation efficiency, and they need to be considered simultaneously. In addition, they are substitute in increasing investment efficiency:
information is withheld if investment efficiency is higher with internal capital allocation upon no disclosure than external capital allocation with costly disclosure.

Part (ii) shows that if the disclosure cost is low, the two-project firms share the same ex-ante value as the two stand-alone firms. With a low disclosure cost, the financiers interpret no disclosure as bad news. Thus, the internal capital allocation does not help the two-project firm to attract investment from the financiers, and the internal capital allocation becomes unviable. In this case, the two-project firm and a group of two stand-alone firms are the same in terms of disclosure behavior and capital investment. Part (iii) considers the ex-ante net cash flows of the two-project firm under the condition in which information is always withheld, as in Lemma 4. In this case, the ex-ante net cash flow of the two-project firm is at least as much as the ex-ante net cash flows of the two stand-alone firms. This is because with the internal capital allocation, the two-project firm faces, on average, a lower under-investment problem than the stand-alone firm.

4. Many-project Firm

In this section, I investigate the impact of organizing many projects on the manager’s disclosure strategy and the ex-ante net cash flow of the firm. Consider a firm that owns a continuum of project \( i \in [0,1] \), and each project requires one unit of capital to generate \( x_i = R \) with success probability \( p_i \) and \( x_i = 0 \) with \( 1 - p_i \). It is common knowledge that the manager observes each \( p_i \), which is independently and uniformly distributed over \([0,1]\). Proposition 4 summarizes the firm’s disclosure behavior and ex-ante net cash flow.

**Proposition 4.** Suppose that a founder owns a continuum of project \( i \in [0,1] \) with \( p_i \) independently and uniformly distributed over \([0,1]\), and the manager observes \( p_i \) for \( i \in [0,1] \).

(i) \( d_i = 0 \) is chosen for \( i \in [0,1] \), and the financiers provide \( 1 \times \left( 1 - \frac{1}{R} \right) \) units of capital.

(ii) The ex-ante net cash flow from the projects is \( \frac{(R-1)^2}{2R} \).
Proposition 4 shows that with a continuum of projects, the manager withholds information about the projects, but the ex-ante net cash flow of the firm is the same as the ex-ante net cash flow under full information (i.e., $c = 0$). Intuitively, by the law of large numbers, the financiers know that the manager has observed a mass $1 - \frac{1}{R}$ of positive-NPV projects with $p_t \geq \frac{1}{R'}$, although they do not know which projects are profitable. The financiers do not need to know which projects are profitable, because they know that the manager will implement the most profitable projects. Thus, there is no information asymmetry between the manager and the financiers about the profitability of the implemented projects; the financiers provide capital for the implementation of a mass $1 - \frac{1}{R}$ of profitable projects, and the manager implements every positive-NPV project. As a result, the manager has no incentive to incur cost $c$ to disclose any success probabilities; disclosure would simply lower the net cash flow for the founder.

In summary, if the manager can observe a continuum of success probabilities $p_t$ for $t \in [0,1]$, which is independently distributed, internal capital allocation by the manager replaces external capital allocation with disclosure. As a result, disclosure becomes irrelevant in mitigating the information asymmetry between the firm and the financiers.

5. Conclusion

In this paper, I jointly consider both firms’ disclosure and their internal capital allocation decisions. I show that internal capital allocation can affect firms’ disclosure behavior. Specifically, I compare the disclosure behavior of the two-project firms with that of the two stand-alone firms, and I show that the two-project firm has a weaker incentive to disclose information than a group of the two stand-alone firms. In addition, I show that despite less disclosure, the investment efficiency of the two-project firm is higher than that of a group of the two stand-alone firms with an intermediate level of disclosure cost. The result implies that both disclosure and internal capital allocation play an important role in affecting the efficiency of capital investment. They are substitute in improving capital investment efficiency, so less disclosure does not necessarily imply capital investment inefficiency. I further demonstrate the
substitutability between disclosure and internal capital allocation by investigating the disclosure behavior of a firm with sufficiently many projects; the firm does not disclose any information but achieves the full-information investment efficiency. Overall, the analysis suggests that disclosure behavior and organizational structure interact with each other in the context of internal capital allocation. As Coase (1990) argues, “the theory of the accounting system is part of the theory of the firm.”

6. Appendix

Proof of Lemma 1

(i) Let $c \geq 1$ be given. First, consider $2 \geq R$, which implies $\frac{1}{2} \geq p_{BE}(0) = \frac{1}{R}$. Suppose that the financier’s belief about $p$ upon $d = 0$ is $p \in [0,1]$. Then, $E[p|d = 0] = \frac{1}{2} \geq \frac{1}{R} = p_{BE}(0)$, so $k = 1$ is chosen and $R_{\ell}(d = 0) = 2$. Given the financiers’ belief, the manager chooses $d = 0$ for $p \in [0,1]$: if $c$ is large, the project with $d = c$ becomes negative-NPV: $p < \frac{1+c}{R} = p_{BE}(c)$; if $c$ is not large so that $p \geq \frac{1+c}{R} = p_{BE}(c)$, $c \geq 1$ implies that $R_{\ell}(d = c) = \frac{1+c}{p} \geq 2 = R_{\ell}(d = 0)$. That is, the payoff from $d = 0$ is always at least as much as the payoff from $d = c$:

$$\text{Payoff with } d = 0 \geq p \left( R - \frac{1+c}{p} \right) + \beta.$$ 

Thus, $d = 0$ is chosen for $p \in [0,1]$, consistent with the financiers’ belief about $p$ upon $d = 0$.

Consider $c \geq 1$ and $R < 2$, which implies $\frac{1}{2} < \frac{1}{R}$. Suppose that the financiers’ belief about $p$ upon $d = 0$ is $p \in [0,1]$: then, $E[p|d = 0] = \frac{1}{2} < \frac{1}{R} = p_{BE}(0)$ and $k = 0$ is chosen. Given the financiers’ belief, the manager chooses $d = 0$ for any $p \in [0,1]$. This is because the project is negative-NPV upon $d = c$ for $p \in [0,1]$: $p \leq 1 < \frac{1+c}{R} = p_{BE}(c)$ due to $c \geq 1$ and $R < 2$. Thus, $k = 0$ is chosen upon $d = c$. Since $d = c$ leads to $k = 0$, the manager chooses $d = 0$ for any $p \in [0,1]$, consistent with the financiers’ belief $p$ upon $d = 0$. 

(ii) Consider $c < 1$ and $R \geq 1 + c$. Suppose that the financiers’ belief upon $d = 0$ is $p \in \left[0, \frac{1+c}{R}\right)$. Then, $E[p|d = 0] = \frac{1+c}{2} \times \frac{1}{R} < \frac{1}{R} = p_{BE}(0)$. Thus, the financiers choose $k = 0$ upon $d = 0$. The financiers also choose $k = 0$ if $p < \frac{1+c}{R} = p_{BE}(c)$ is disclosed. If $p \geq \frac{1+c}{R} = p_{BE}(c)$ is disclosed, the financiers choose $k = 1$. Given the financiers’ belief, the manager chooses $d = c$ if $p \geq \frac{1+c}{R}$, because $d = 0$ leads to $k = 0$. $d = 0$ is chosen for $p \in \left[0, \frac{1+c}{R}\right)$, because $d = c$ also leads to $k = 0$. Thus, the manager’s disclosure strategy is consistent with the financiers’ belief about $p$ upon $d = 0$.

(iii) Consider $c < 1$ and $R < 1 + c$. Then, the project is negative-NPV upon disclosure for any $p \in [0,1]$, because $p \times R \leq R < 1 + c$. Thus, upon $d = 0$, the financiers believe that $p \in [0,1]$. Then, $E[p|d = 0]R - 1 = \frac{1}{2}R - 1 < 0$, because $R < 1 + c$ implies that $\frac{1}{2}R < \frac{1+c}{2} < 1$ for $c < 1$. Thus, the financiers choose $k = 0$ upon $d = 0$. Given the financiers’ belief, the manager chooses $d = 0$ for $p \in [0,1]$; the disclosure of any $p \in [0,1]$ renders the project negative-NPV. Thus, the manager’s disclosure strategy is consistent with the financiers’ belief about $p$ upon $d = 0$.

Q.E.D.

Proof of Proposition 1

(i) Part (i) of Lemma 1 shows that if $\frac{1}{2}R - 1 \geq 0$, $k = 1$ is always chosen. Thus, the ex-ante cash flow from the project is the unconditional NPV of the project, which is $\frac{1}{2}R - 1$. Part (i) of Lemma 1 also shows that if $\frac{1}{2}R - 1 < 0$, $k = 0$ is always chosen. Thus, the ex-ante cash flow from the project becomes zero.

(ii) Part (ii) of Lemma 1 shows that $k = 1$ and $d = c$ are chosen if $p \geq \frac{1+c}{R}$, and $k = 0$ and $d = 0$ are chosen otherwise. Then, the ex-ante cash flow from the project becomes

$$\Pr\left(p \geq \frac{1+c}{R}\right) \times \left( E[p|p \geq \frac{1+c}{R}]R - 1 - c\right) + \left(1 - \Pr\left(p \geq \frac{1+c}{R}\right)\right) \times 0 = \frac{(R - 1 - c)^2}{2R}.$$
Part (iii) of Lemma 1 shows that \( k = 0 \) is always chosen. Thus, the ex-ante cash flow from the project is zero.

\[
Q.E.D.
\]

**Proof of Lemma 3**

(i) Notice that \( p^* \geq \frac{1+c}{R} \) implies \( \frac{1}{p^*} \leq \frac{R}{1+c} \). Then, by multiplying \((1 + c)\) on both sides, we have \( \frac{1+c}{p^*} \leq R \). Since \( \frac{1}{A} = \frac{1+c}{p^*} \), we have \( \frac{1}{A} \leq R \implies A \geq \frac{1}{R} \), which implies \( V_{ND} = A \times R - 1 \geq 0 \).

(ii) I show that \( k = 2 \) is chosen upon \( d_1 = d_2 = c \) and \( k = 1 \) is chosen otherwise, given the financiers’ belief about \( p_1 \) and \( p_2 \) upon the manager’s disclosure choice, as in (a), (b), and (c) of Part (iii) of Lemma 3.

Consider Part (a) in Part (iii): \( p_i \geq p^* \) and \( p_j < \frac{1+c}{R} \) upon \( d_i = c \) and \( d_j = 0 \), where \( i \neq j \).

Then, \( p_i \geq p^* \geq p_{BE}(c) = \frac{1+c}{R} \), which implies that project \( i \) is not negative-NPV. Also, based on the financiers’ belief, \( E[p_j|p_i, d_j = 0] = E[p_j|p_j < p_{BE}(c)] = \frac{1+c}{2R} \). Notice that \( c < 1 \) implies \( \frac{1+c}{2R} < \frac{1}{R} = p_{BE}(0) \). That is, the financiers believe that project \( j \) is negative-NPV. Then, Lemma 2 implies that the financiers believe that if \( k = 1 \), project \( i \) will be implemented. Thus, \( R_\ell = \frac{1+c}{p_i} \leq R \), and they choose \( k = 1 \).

Consider Part (b) in Part (iii): \( p_i < p^* \) and \( p_j < \frac{1+c}{R} \) upon \( d_1 = d_2 = 0 \), where \( i \neq j \). By assumption, \( V_{ND} \geq 0 \implies A \geq \frac{1}{R} = p_{BE}(0) \). Thus, \( R_\ell = \frac{1}{A} \leq R \), and \( k = 1 \) is chosen. The financiers do not choose \( k = 2 \), because they believe that at least one of the two projects is negative-NPV.

Consider Part (c) in Part (iii): \( p_1, p_2 \geq \frac{1+c}{R} = p_{BE}(c) \) upon \( d_1 = d_2 = c \). Then, since \( p_1 \) and \( p_2 \) are greater than or equal to \( p_{BE}(c) \), we have \( R_{\ell,i} = \frac{1+c}{p_i} \leq R \) and \( k = 2 \) is chosen.
(iii) Given the financiers’ investment strategy, I show that the manager’s optimal disclosure strategy is consistent with the financiers’ investment strategy. Consider Part (a): \( p_i \geq p^* \geq p_{BE}(c) = \frac{1+c}{R} \) and \( p_j < \frac{1+c}{R} \), where \( i \neq j \). \( p_j \) is withheld, because the disclosure of \( p_j < p_{BE}(c) \) reveals to the financiers that project \( j \) is negative-NPV. \( p_i \) is disclosed, because repayment with \( d_j = 0 \) and \( d_i = c, \frac{1+c}{p_i} \), is lower than repayment with \( d_j = d_i = 0, \frac{1}{A} \cdot \frac{1+c}{p_i} > \frac{1+c}{p^*} = \frac{1}{A} \).

Consider Part (b): \( p_i < p^* \) and \( p_j < \frac{1+c}{R} \), where \( i \neq j \). \( p_j \) is withheld, because the disclosure of \( p_j < p_{BE}(c) \) reveals that project \( j \) is negative-NPV upon disclosure. \( p_i \) is withheld, because repayment with \( d_i = c, \frac{1+c}{p_i} \), is higher than repayment with \( d_j = d_i = 0, \frac{1}{A} \cdot \frac{1+c}{p_i} > \frac{1+c}{p^*} = \frac{1}{A} \).

Consider Part (c): \( p_1, p_2 \geq \frac{1+c}{R} \). If \( d_1 = d_2 = c \), the financiers learn that both projects are positive-NPV upon disclosure and choose \( k = 2 \). If either \( p_1 \) or \( p_2 \) is withheld, the financiers choose \( k = 1 \). Since the manager derives private benefit from more capital investment, \( d_1 = c \) and \( d_2 = c \) are chosen as a result.

Q.E.D.

**Proof of Proposition 2**

(i) Let \( B(p^*) \) denote the probability of the manager choosing \( d_1 = d_2 = 0 \), based on the disclosure strategy in Lemma 3, and \( p^* \geq \frac{1+c}{R} \). Then, with probability \( \left( \frac{1+c}{R} \right)^2 \), we have \( p_1, p_2 \leq p_{BE}(c) = \frac{1+c}{R} \), and \( d_1 = d_2 = 0 \) is chosen. With probability \( 2 \times \frac{1+c}{R} \left( p^* - \frac{1+c}{R} \right) \), we have \( p_i \leq \frac{1+c}{R} \) and \( p_j \in \left[ \frac{1+c}{R}, p^* \right] \), where \( i \neq j \), \( d_1 = d_2 = 0 \) are chosen. Then, we have

\[
B(p^*) = \left( \frac{1+c}{R} \right)^2 + 2 \times \frac{1+c}{R} \left( p^* - \frac{1+c}{R} \right) = \frac{(1+c)(2p^*R-1-c)}{R^2}.
\]

I now derive \( A(p) \). There are two cases to consider.
1) Conditional upon \( d_1 = d_2 = 0 \) and \( k = 1 \), we have \( p_1, p_2 \leq \frac{1+c}{R} \) with probability
\[
\frac{1}{B(p^*)} \times \left( \frac{1+c}{R} \right)^2.
\]
Then, Lemma 2 implies that the manager implements the project with
\( p_i \geq p_j \), where \( i \neq j \). Then, the expected value of the maximum of two independent
variables that follow a uniform distribution over \( \left[ 0, \frac{1+c}{R} \right] \) is
\[
E \left[ \max \{ p_1, p_2 \} \right] = \frac{2}{3} \left( 1 + \frac{c}{R} \right).
\]

2) Conditional upon \( d_1 = d_2 = 0 \) and \( k = 1 \), we have \( p_i \leq \frac{1+c}{R} \) and \( p_j \in \left( \frac{1+c}{R}, p^* \right) \), where
\( i \neq j \), with probability \( \frac{1}{B(p^*)} \times 2 \left( \frac{1+c}{R} \right) \left( p^* - \frac{1+c}{R} \right) \). Then, Lemma 2 implies that project \( j \) is
implemented, and the expected success probability of the implemented project is
\[
E \left[ p_j \right| p_j \in \left( \frac{1+c}{R}, p^* \right) \right] = \frac{1+c}{R} + p^*.
\]

Then, the expected success probability \( A(p^*) \) upon \( d_1 = d_2 = 0 \) and \( k = 1 \) is
\[
A(p^*) = \frac{1}{B(p^*)} \left[ \left( \frac{1+c}{R} \right)^2 \times \frac{2}{3} \left( 1 + \frac{c}{R} \right) + 2 \left( \frac{1+c}{R} \right) \left( p^* - \frac{1+c}{R} \right) \times \frac{1+c}{R} + p^* \right]
\]
\[
= \frac{3(p^*)^2 R^2 - (1+c)^2}{3R(2p^*R - 1 - c)}.
\]

The equilibrium threshold \( p^* \) satisfies the condition \( \frac{1}{A(p^*)} = \frac{1+c}{p^*} \). By rearranging, we have \( \frac{A(p^*)}{p^*} = \frac{1}{1+c} \). Notice that \( \frac{A(p)}{p} \) is decreasing in \( p \) for \( p \geq \frac{1+c}{R} \).
\[
\frac{\partial}{\partial p} \left( \frac{A(p)}{p} \right) = -\frac{(1+c)(3pR - 1 - c)(pR - 1 - c)}{3p^2 R(1 + c - 2pR)^2} < 0 \text{ for } p \geq \frac{1+c}{R}.
\]

Also, at \( p = \frac{1+c}{R} \), \( \frac{A(p)}{p} = \frac{2}{3} \). Thus, for \( p^* \geq \frac{1+c}{R} \) to exist such that \( \frac{A(p^*)}{p^*} = \frac{1}{1+c} \), we need \( c \geq \frac{1}{2} \) so
that \( \frac{2}{3} \geq \frac{1}{1+c} \).
If \( c < \frac{1}{2} \), we have \( \frac{A(p)}{p} = \frac{2}{3} < \frac{1}{1+c} \) at \( p = \frac{1+c}{R} \); then, there does not exist \( p^* \) satisfying
\[
\frac{A(p^*)}{p^*} = \frac{1}{1+c} \quad \text{for } p^* > \frac{1+c}{R}.
\]
This implies that \( \frac{1}{A(p^*)} > \frac{1+c}{p^*} \) for any \( p^* \geq \frac{1+c}{R} \), and the disclosure belief in Lemma 3 cannot be applied. Thus, upon \( d_1 = d_2 = 0 \), the financiers believe \( p_1, p_2 < \frac{1+c}{R} \) and choose \( k = 0 \); upon \( d_i = c \) and \( d_j = 0 \), they believe \( p_i < \frac{1+c}{R} \) and choose \( k = 1 \) as long as \( p_j \geq \frac{1+c}{R} \); upon \( d_1 = d_2 = c \), they choose \( k = 2 \) as long as \( p_i, p_j \geq \frac{1+c}{R} \). Given the financiers’ belief about \( p_1 \) and \( p_2 \), the manager chooses \( d_1 = d_2 = c \) if \( p_1, p_2 \geq \frac{1+c}{R} \), because \( d_i = 0 \) or \( d_j = 0 \) leads to \( k \in \{0,1\} \). The manager chooses \( d_i = c \) and \( d_j = 0 \) if \( p_i \geq \frac{1+c}{R} \) and \( p_j < \frac{1+c}{R} \). This is because \( d_i = 0 \) leads to \( k = 0 \) and \( d_j = c \) renders the project negative-NPV. The manager chooses \( d_1 = d_2 = 0 \) if \( p_1, p_2 < \frac{1+c}{R} \), because \( d_i = c \) renders the project negative-NPV. Thus, the manager’s disclosure strategy is consistent with the financiers’ belief.

(ii) Based on the proof in Part (i), with \( \frac{1}{2} \leq c < 1 \), we have
\[
p^* = \frac{(1+c)\sqrt{3\{\sqrt{3} + \sqrt{4c^2 - 1}\}}}{6R(1-c)}.
\]
Notice that \( \frac{(1+c)\sqrt{3\{\sqrt{3} - \sqrt{4c^2 - 1}\}}}{6R(1-c)} \leq \frac{1+c}{R} \) for \( \frac{1}{2} \leq c < 1 \). This can be shown as follows. The inequality implies \( \frac{(1+c)\sqrt{3\{\sqrt{3} - \sqrt{4c^2 - 1}\}}}{6R(1-c)} \leq 1 \) if \( c = 1/2 \), \( \frac{(1+c)\sqrt{3\{\sqrt{3} - \sqrt{4c^2 - 1}\}}}{6(1-c)} = 1 \). In addition, \( \frac{\partial}{\partial c} \left( \frac{\sqrt{3\{\sqrt{3} - \sqrt{4c^2 - 1}\}}}{6(1-c)} \right) \leq 0 \) for \( \frac{1}{2} \leq c < 1 \) due to the nonpositive numerator; this is because i) \( \frac{\partial}{\partial c} \left( 1 - 4c + \sqrt{3(4c^2 - 1)} \right) = 4\frac{3c^2}{\sqrt{4c^2 - 1}} - 4 > 0 \) for \( \frac{1}{2} \leq c < 1 \) due to \( 4c^2 - 1 < 3c^2 \Rightarrow c^2 < 1 \) for \( \frac{1}{2} \leq c < 1 \), and ii) \( 1 - 4c + \sqrt{3(4c^2 - 1)} = 0 \) at \( c = 1 \). Thus, the unique solution of \( p^* \geq \frac{1+c}{R} \) is
\[
p^* = \frac{(1+c)\sqrt{3\{\sqrt{3} + \sqrt{4c^2 - 1}\}}}{6R(1-c)}.
\]

29
In addition, we have, for $\frac{1}{2} \leq c < 1$,
\[
p^{**}(c) = \frac{2c - 1 + 2c^2(2 - c) + \sqrt{3(4c^2 - 1)}}{R(1 - c)^2 \sqrt{3(4c^2 - 1)}} > 0; \quad p^{**}(R) = -\frac{(1 + c)(\sqrt{3} + 4c^2 - 1)}{2 \sqrt{3}(1 - c)R^2} < 0.
\]

Proof of Lemma 4

Consider $R < \frac{3}{2}$. Upon $d_1 = d_2 = 0$, the financiers believe $p_i \in [0,1]$ for $i \in \{1,2\}$. They do not choose $k = 2$, because the break-even condition for each project is not satisfied: $E[p_i]R - 1 = \frac{1}{2}R - 1 < 0$. The financiers do not choose $k = 1$ either. Based on Lemma 2, the financiers evaluate the expected success probability upon $k = 1$, that is, $E[\max\{p_1, p_2\}] = 2/3$; the break-even condition is not satisfied with $k = 1$ because the capital investment is negative-NPV: $E[\max\{p_1, p_2\}]R - 1 = \frac{2}{3}R - 1 < 0$. Thus, the financiers choose $k = 0$. Given the financiers’ belief, the manager chooses $d_1 = d_2 = 0$ for any $p_i \in [0,1]$, $i \in \{1,2\}$, which is consistent with the financiers’ belief. The manager does not have an incentive to choose $d_i = c$ for $i \in \{1,2\}$. This is because given either $c \geq 1$ and $R < \frac{3}{2}$ or $R < 1 + c$, the costly disclosure would render the project negative-NPV; that is, $p_i R < 1 + c$ for any $p_i \in [0,1]$.

Consider $\frac{3}{2} \leq R < 2$. Upon $d_1 = d_2 = 0$, the financiers believe $p_i \in [0,1]$ for $i \in \{1,2\}$. They choose $k = 1$, because the break-even condition upon $k = 1$ is satisfied: $E[\max\{p_1, p_2\}] \times R_\ell = 1 \Rightarrow R_\ell = \frac{1}{E[\max\{p_1, p_2\}]} = \frac{3}{2} \leq R$. Then, the manager chooses $d_1 = d_2 = 0$ for $p_i \in [0,1]$, $i \in \{1,2\}$, which is consistent with the financiers’ belief. The manager has no incentive to choose $d_i = c$, because given either $c \geq 1$ and $\frac{3}{2} \leq R < 2$ or $R < 1 + c$, the costly disclosure would render the project negative-NPV: $p_i R < 1 + c$ for $p_i \in [0,1]$.

Consider $R \geq 2$ and $c \geq 1$. Upon $d_1 = d_2 = 0$, the financiers believe $p_i \in [0,1]$ for $i \in \{1,2\}$. They choose $k = 2$, because the break-even condition is satisfied for each project: $E[p_i] \times R_{\ell,i} = 1 \Rightarrow R_{\ell,i} = 2 \leq R$. Given the financiers’ belief, the manager chooses $d_1 = d_2 =$
0 for any \( p_i \in [0,1] \), which is consistent with the financiers’ belief. If \( c \) is large, project \( i \) with \( d_i = c \) becomes negative-NPV: \( p \times R < 1 + c \). If \( c \) is not so large that \( p \geq \frac{1+c}{R} \), \( c \geq 1 \) implies that repayment upon \( d_i = c, R_{\ell,i} = \frac{1+c}{p_i} \), is higher than repayment upon \( d_i = 0 \), which is \( R_{\ell,i} = 2 \). With \( R \geq 2 \), the case of \( R < 1 + c \) such that \( c < 1 \) does not apply, because \( R \geq 2 \) and \( R < 1 + c < 2 \) contradict each other.

Q.E.D.

Proof of Proposition 3

(i) Suppose \( \frac{1}{2} \leq c < 1 \) and \( R \geq 1 + c \). Part (ii) of Proposition 2 implies that the manager chooses \( d_1 = d_2 = c \) if \( p_1, p_2 \geq \frac{1+c}{R} \), and this is realized with probability \( \left(1 - \frac{1+c}{R}\right)^2 \). In this case, the two projects are implemented, and the expected value of each project conditional on \( p_1, p_2 \geq \frac{1+c}{R} \) becomes

\[
E[p_i | p_i \geq \frac{1+c}{R}] = \frac{1}{2}(R - 1 - c).
\]

With probability \( 2 \times (1 - p^*) \frac{1+c}{R}, p_i \in [p^*, 1] \) and \( p_j < \frac{1+c}{R} = p_{BE}(c) \), where \( i \neq j \) and \( p^* \geq \frac{1+c}{R} \). Then, \( p_i \) is disclosed and project \( i \) is implemented. The expected value of project \( i \) becomes

\[
E[p_i | p_i \geq p^*, p_j < p_{BE}(c)] = \frac{1 + p^*}{2} R - 1 - c.
\]

With probability \((p^*)^2 - \left(p^* - \frac{1+c}{R}\right)^2 \), the manager chooses \( d_1 = d_2 = 0 \), and project \( i \) with \( p_i \geq p_j \) is implemented, where \( i \neq j \). In this case, the expected value of the implemented project is \( V_{NB}(p^*) = A(p^*)R - 1 \). Then, the ex-ante cash flow of the two-project firm is as follows:

\[
2 \left(1 - \frac{1+c}{R}\right)^2 \frac{1}{2}(R - 1 - c) + 2 \frac{1+c}{R} (1 - p^*) \left(\frac{1 + p^*}{2} R - 1 - c\right) + \left((p^*)^2 - \left(p^* - \frac{1+c}{R}\right)^2 \right) (A(p^*)R - 1).
\]
Recall the equilibrium condition for $p^*$: \( \frac{1}{A(p^*)} = \frac{1 + c}{p^*} \Rightarrow A(p^*) = \frac{p^*}{1 + c} \). Then, by substituting $A(p^*) = \frac{p^*}{1 + c}$ and rearranging, the ex-ante cash flow of the two-project firm is reduced to

\[
\left( \frac{R - 1 - c}{R} \right)^2 + \left( p^* - \left( p^* - \frac{1 + c}{R} \right) c \right) \left( p^* - \frac{1 + c}{R} \right).
\]

This is greater than the ex-ante value of the two stand-alone firms, $2 \times \frac{(R - 1 - c)^2}{2R} = \frac{(R - 1 - c)^2}{R}$, because $p^* \geq \frac{1 + c}{R}$ and $p^* > \left( p^* - \frac{1 + c}{R} \right) c$.

(ii) If $c < \frac{1}{2}$, the financiers choose $k = 0$ upon $d_1 = d_2 = 0$ due to Part (i) of Proposition 2. Then, the manager chooses $d_i = c$ when $p_i \geq \frac{1 + c}{R}$ to avoid no capital investment for $i \in \{1, 2\}$. Thus, the ex-ante net cash flow of the two-project firm is

\[
2 \times \left\{ \Pr \left( p_i \geq \frac{1 + c}{R} \right) \times \left( E \left[ p_i \mid p_i \geq \frac{1 + c}{R} \right] R - 1 - c \right) \right\} = \frac{(R - 1 - c)^2}{R},
\]

which is the same as the ex-ante net cash flows of the two stand-alone firms.

(iii) I show that the ex-ante net cash flow of the two-project firm is at least as much as the ex-ante net cash flows of the two stand-alone firms if either $c \geq 1$ or $R < 1 + c$ such that $c < 1$.

Suppose $c \geq 1$. Consider $R \geq 2$. Then, Lemma 1 shows that the stand-alone firm’s project is implemented, and Lemma 4 shows that the two-project firm implements the two projects. Thus, the ex-ante net cash flows of the two stand-alone firms and the two-project firm are the same. Second, consider $R < 2$. Then, Lemma 1 shows that the stand-alone firm’s project is not implemented. However, Lemma 4 shows that the two-project firm implements one project if $\frac{3}{2} \leq R < 2$ and zero projects if $R < \frac{3}{2}$. Thus, if $\frac{3}{2} \leq R < 2$, the ex-ante net cash flow of the two-project firm is $E[\max\{p_1, p_2\}] \times R - 1 = \frac{2}{3} R - 1 \geq 0$. Thus, the ex-ante net cash flow of the two-project firm is at least as much as the ex-ante net cash flows of the two stand-alone firms.

Suppose that $R < 1 + c$ such that $c < 1$. Then, $R \geq 2$ is ruled out because $R < 1 + c$ and $c < 1$ cannot be satisfied at the same time. First, consider $\frac{3}{2} \leq R < 2$. Then, Part (iii) of
Proposition 1 shows that the ex-ante net cash flow of the stand-alone firm is zero. However, Lemma 4 shows that if \( \frac{3}{2} \leq R < 2 \), the financiers choose \( k = 1 \) for the two-project firm and \( k = 0 \) with \( R < \frac{3}{2} \). Then, with \( \frac{3}{2} \leq R < 2 \), the ex-ante net cash flow of the two-project firm becomes 
\[
E[\max(p_1, p_2)]R - 1 = \frac{2}{3}R - 1 \geq 0.
\]
Thus, the ex-ante net cash flow of the two-project firm is at least as much as the ex-ante net cash flows of the two stand-alone firms.

Q.E.D.

Proof of Proposition 4

(i) Upon no disclosure, the financiers believe that the manager has observed a mass \( 1 - \frac{1}{R} \) of project \( p_i \geq p_{BE}(0) = 1/R \). Thus, the break-even condition for the financiers who provide capital for a mass \( 1 - \frac{1}{R} \) of the projects becomes 
\[
\left(1 - \frac{1}{R}\right) \left\{ E[p|p \geq \frac{1}{R}]R_{\ell} - 1 \right\} = 0.
\]
Then, we have 
\[
R_{\ell} = \frac{2}{\frac{1}{1+1/R}} < R,
\]
because 
\[
R - R_{\ell} = \frac{R-1}{1+R}R > 0.
\]
Thus, the financiers provide \( 1 \times \left(1 - \frac{1}{R}\right) \) units of capital to the firm. With \( 1 - \frac{1}{R} \) unit of capital, the manager implements every project with \( p_i \geq \frac{1}{R} \). Given the financiers’ belief upon no disclosure, the manager has no incentive to disclose any success probabilities. The disclosure of \( p_i < \frac{1}{R} \) leads to no capital investment by the financiers, because the project is negative-NPV upon disclosure. The disclosure of \( p_i \geq \frac{1}{R} \) does not induce more capital investment than a \( 1 \times \left(1 - \frac{1}{R}\right) \), and the cash flows for the founder are decreased due to disclosure cost \( c \).

(ii) The ex-ante net cash flow can be calculated as follows:
\[
Pr(p_i \geq \frac{1}{R}) \times \int_{1/R}^{1} p_i \left(R - \frac{1}{E[p|p \geq \frac{1}{R}]}\right) \, di = \left(1 - \frac{1}{R}\right) E[p_i|p_i \geq \frac{1}{R}] \frac{R - 1}{1 + R} = \frac{(R - 1)^2}{2R}
\]
Q.E.D.
First-Best Financing Contract

Suppose that the following financing contract is signed between the founder and the financiers. If the project is implemented and \( x = R \), the financiers receive \( R \ell \) and the founder receives \( R - R \ell \). If the project is implemented and \( x = 0 \), both receive zero cash flows. If the project is not implemented, the financiers pay \( R_0 \) to the founder. \( R_0 \) and \( R \ell \) are expressed as follows.

\[
R_\ell = \frac{2(1 - p_{BE} + Rp_{BE}^2 + \beta p_{BE})}{1 + p_{BE}^2};
R_0 = \frac{p_{BE}(1 - p_{BE})(R(1 + p_{BE}) - 2) + \beta(1 - p_{BE}^2)}{1 + p_{BE}^2},
\]

where \( p_{BE} = 1/R \). \( R_\ell \) and \( R_0 \) are related such that \( p_{BE}(R - R_\ell) + \beta = R_0 \).

Under the financing contract, only a positive-NPV project (i.e., a project with \( p \geq 1/R = p_{BE} \)) is implemented, because the manager’s payoff with capital investment is at least as much as the payoff without capital investment (i.e., \( R_0 \)):

\[
p(R - R_\ell) + \beta \geq p_{BE}(R - R_\ell) + \beta = R_0.
\]

If \( p < 1/R = p_{BE} \), the project is not implemented because the manager’s payoff with \( R_0 \) is higher than the payoff with capital investment: \( R_0 = p_{BE}(R - R_\ell) + \beta \geq p(R - R_\ell) + \beta \). Then, the ex-ante net cash flow from the project is the same as the ex-ante net cash flow from the project under full information:

\[
\Pr(p \geq p_{BE}) E[p|p \geq p_{BE}](R - R_\ell) + \Pr(p < p_{BE}) R_0 = (1 - p_{BE}) \left(\frac{1 + p_{BE}}{2} R - 1\right).
\]

The financiers’ break-even condition, \( (1 - p_{BE}) \left(\frac{1 + p_{BE}}{2} R_\ell - 1\right) = p_{BE}R_0 \), is satisfied with \( R_\ell \) and \( R_0 \) specified above. Then, \( R_\ell = \frac{2(1-p_{BE}+Rp_{BE}^2+\beta p_{BE})}{1+p_{BE}^2} \leq R \) must be satisfied so that the financiers can obtain \( R_\ell \) upon \( x = R \). Thus, if \( R_\ell > R \) or \( \beta > \frac{1-p_{BE}}{p_{BE}} \left(\frac{1+p_{BE}}{2} R - 1\right) \),

the break-even condition cannot be satisfied, and the financing contract is not signed.

Q.E.D.
References

https://www.coase.org/conferences/2015washingtonconferencearrow.pdf


