Voluntary Disclosure and the Duty to Disclose

Ronald A. Dye

February 15, 2013

Abstract

This paper evaluates firms' disclosure decisions when they have a duty to disclose the material information in their possession. We posit that that, even when mandatory disclosure requirements exist, and firms incur penalties when they are caught violating the requirements, firms engage in cost-benefit calculations in deciding whether to comply with the requirements.

In this framework, the paper makes predictions regarding the dependence of a firm's equilibrium disclosure policy on various parameters of the model, including: the frequency the firm receives information, the threshold defining material information, the size of damage payments, the expected fraction of the firm's shares purchased while information is withheld, and the probability the firm is eventually detected having withheld material information.

The paper concludes by establishing a robust relationship between a firm's equilibrium disclosure policy and its equilibrium investment level.

1 Introduction

Many SEC regulations impose affirmative duties on firms to make disclosures of their material private information. For example, SEC Rule 10b-5 states that:

> "It shall be unlawful for any person, directly or indirectly, by the use of any means or instrumentality of interstate commerce, or of the mails or of any facility of any national securities exchange, to....make any untrue statement of a material fact or to omit to state a material fact necessary in order to make the statements made, in the light of the circumstances under which they were made, not misleading..in connection with the purchase or sale of any security."[1]

If all publicly traded firms dutifully adhered to these and related mandatory disclosure requirements, there would be little allocational role or informational role for voluntary disclosures to play. Voluntary disclosures would become an addendum, limited to instances in which firms decide whether to disclose the private *im*material information in their possession.

Our main thesis is that, in practice, these mandates do not eliminate all firms' discretion in choosing what material information ("MI") they possess to disclose.[2] Just as posted maximum speed limits don't eliminate some motorists propensity to speed, mandated disclosure requirements don't result in all firms

[1] As another example, consider SEC Rule 12b-20. It stipulates that:

> "In addition to the information expressly required to be included in a statement or report, there shall be added such further material information, if any, as may be necessary to make the required statements, in the light of the circumstances under which they are made not misleading."

[2] Sunder [1997] emphasizes the point that the distinction between "mandatory" and "voluntary" actions is often artificial (see pages nine and ten of Sunder [1997]).

obeying those requirements. The plethora of SEC enforcement actions and private civil litigation involving firms accused of withholding MI is testament to firms exercising their discretion in deciding whether to obey such requirements. Imperfect monitoring and enforcement of disclosure requirements create scope for discretion in adhering to these requirements, as do limited and uncertain penalties for being caught violating the requirements. While regulatory actions making the disclosure of certain pieces of information mandatory may alter the cost-benefit calculus that firms go through in deciding whether to disclose or withhold the private information they possess, it does not eliminate the importance or usefulness of cost-benefit calculations to firms in making their disclosure decisions.

This paper develops a model that evaluates expected value-maximizing firms' disclosure decisions when the firms have a duty to disclose MI in their possession. When firms are caught violating that duty by withholding MI, they are liable for damages. The damage payments we examine reflect several features of damage payments in 10b-5 and related litigation: (1) the damage payments are due shareholders who purchased, but not shareholders who sold, shares in a firm during the time the firm withheld MI; (2) the damage payments are the product of: (2a) the difference between the "artificially high" market price of the firm that prevailed during the time the firm withheld MI and the ("as if") market price that would have prevailed during this time had the firm disclosed the MI; (2b) the number of shares investors purchased at the "artificially high" price; and (2c) a damages "multiplier" determining what fraction of investors' overpayments for the firm's shares are reimbursed; (3) the damage payments are paid by the firm itself, which effectively means these payments are paid by the firm's shareholders of record after the withholding of MI is detected; and (4) the firm bears additional legal costs, beyond the damages payments, when it is caught withholding MI.

The primary goal of the paper is to develop predictions about how firms' disclosure decisions are affected by mandatory requirements to disclose MI. To make such predictions, we must confront the thorny problem of determining how potential purchasers of a firm's shares value the shares in light of the potential damage payments the firm may have to pay them if it is subsequently caught withholding MI. This problem arises in part because investors who purchase a firm's shares while the firm withheld MI are both potential recipients of the damage payments (as parties "wronged" by the firm's withholding of MI) and also potential payers of the damage payments (as shareholders of the firm responsible for making damage payments to "wronged" parties), and also because the damage payments both depend on the actual price of the firm while the MI was withheld (as the damage payments are a function of the difference between this actual price and the "as if" price of the firm had the firm disclosed the MI) and also help to determine that actual price (as the anticipation of receiving damage payments affects investors' assessments of the value of purchasing the firm's shares).

These interdependencies are further exacerbated by the circular nature of the notion of materiality in 10b-5 matters. To describe the source of this circularity, we recall that withheld information is considered to be material only if a reasonable investor would view knowledge of that information to be important in assessing the firm's financial condition.[3] Operationally, this means that withheld information is material to

[3] While there are many variations in the way materiality is described verbally (e.g., as Oesterle [2011] notes, in one of its early attempts at explicitly defining materiality, the SEC in 1947 in Rule 405 defined material information as "those matters

investors only if the difference between the actual price of the firm while the information is withheld and the "as if" price of the firm were the withheld information disclosed is sufficiently large. But, since (as we noted above) the price of the firm while it withheld information depends on investors' prospect of receiving damage payments, and the prospect of receiving damage payments depends on whether the withheld information is considered material, it follows that whether the withheld information is considered material is defined in terms of itself.

So, in the course of developing a model that allows us to make empirically verifiable predictions about firms' disclosure decisions in the presence of "duties to disclose," we are forced to resolve these circularities that arise in each of: the definition of materiality, the determination of damage payments, and the specification of firms' equilibrium prices in the event they fail to disclose MI.

The analysis includes comparative statics results that yield predictions about how a firm's propensity to disclose MI it receives changes with changes in: the fraction of the firm's shares expected to be purchased by investors during the "damages period" (when the MI was withheld), the size of the damages multiplier, the probability some fact finder will determine that the firm withheld MI, the size of the other litigation costs (besides damage payments) the firm incurs, and the definition of what is considered to be MI. While some of our findings are intuitive (e.g., firms withhold MI less often as the fraction of shares they expect to be traded during the damages period rises), some of our findings are less so (e.g., sometimes, firms withhold MI more often as either the size of the damages multiplier or the probability that some fact finder detects ex post that the firms withheld MI increases). These counterintuitive findings arise because of the previously mentioned circularities that arise in 10b-5 litigation.

Another goal of the analysis is to connect these empirical predictions regarding a firm's disclosure policy to the efficiency of the firm's investment decisions. Establishing a relationship between a firm's disclosure policy and its investment policy is valuable for its own sake, but is also desirable because it connects one endogenous real variable (the efficiency of a firm's investment policy) that is difficult to assess with another easily observed, and hence empirically measurable, endogenous variable (the firm's disclosure policy). Our main finding here is that any change in an exogenous parameter of the model that leads the firm to disclose information less (resp., more) often in equilibrium serves to worsen (resp., enhance) the efficiency of the firm's investment decisions and hence leads to lower (resp., greater) expected firm value. We discuss the economics underlying this result later in the Introduction.

As far as we are aware, these results are new to the disclosure literature.

In the remainder of this introduction, we preview a sample of our results - those related to changes in the materiality threshold ("MT") that determines whether the information a firm withholds is material and hence whether the subsequent discovery of the firm's withholding of this information subjects the firm to

to which there is a substantial likelihood that a reasonable investor would attach importance"), and the notion of materiality has changed over time (as described in the next footnote), the substance of "materiality" in all these variations, as far as we are aware, always involves what impact investors' knowledge of the information would have on investors' perceptions of a firm's financial condition.

legal liability.[4] [5] Our main finding here is to show that the disclosure "cutoff" that determines whether the firm seeks to disclose or withhold the private information it receives is not monotonic in the MT. Specifically, we show that when the MT is sufficiently low, the cutoff rises, so an expected value-maximizing firm is less likely to disclose its private information if the MT is raised somewhat, but if the MT is sufficiently high, the cutoff falls, so an expected value-maximizing firm is more likely to disclose its private information if the MT is further increased.

What explains these results? When the MT is initially very low and then is raised somewhat, the situation firms face changes from one in which virtually all the information they receive is considered material - and hence subjects them to potential liability for withholding that information - to a situation where they have to opportunity to withhold some information without being penalized for the nondisclosure. Firms naturally exploit this expanded opportunity to withhold some information without being subject to penalty that, in a world where virtually all the information they receive is considered material, they would disclose.

But, when the MT is already high, a firm has considerable discretion in deciding what information to disclose or withhold without fear of facing legal reprisals for withholding information. Investors consequently become concerned that, in such an environment, a firm's lack of disclosure is more likely to be attributable to the firm having received but withheld unfavorable information rather than being attributable to the firm not having received information. If the MT increases from this already high level, investors become progressively more skeptical about the motivation underlying a firm's nondisclosure, and so they discount the price of nondisclosing firms more the higher the MT rises. The result that firms disclose more when the MT rises from an already high level is a reflection of the fact that firms respond rationally to investors' increased discounting of nondisclosing firms' shares by disclosing more of the private information they receive.

In addition to generating empirical predictions about the relationship between the MT and a firm's disclosure decisions, this finding also leads to predictions about the effects of changing the MT on a firm's equilibrium investment level and its expected market value. We show that whenever the MT goes up (resp., down), the firm's equilibrium investment level and its ex ante market value goes down (resp., up) provided the MT starts at a sufficiently low (resp., high) level. Alternatively put, if a change in the MT threshold results in the firm's equilibrium "no disclosure" price going up, that same change in the MT results in the firm's equilibrium investment level and (ex ante) equilibrium market value going down.

We show that this latter phenomenon is general and robust: for any parameter of the model, not just for the parameter describing the MT, a (common knowledge) change in the parameter that results in the firm in equilibrium disclosing its information less (resp., more) often results in the firm's equilibrium investment and its ex ante market value decreasing (resp., increasing). This outcome occurs because when a parameter change results in a change in the firm's equilibrium "no disclosure" price, this price change is a result of a change in investors' *inferences* about its value, rather than being a result of a change in objective *evidence*

[4] Since the interpretation of materiality has changed over time, developing comparative statics involving materiality is not just an academic exercise. For example, Arthur Levitt's 1998 speech at NYU, "The Numbers' Game," was considered to have changed the notion of what constitutes material information. (See, e.g., Zabel and Benjamin [2002]). Related, questions involving the desirability of changing materiality threshholds frequently have arisen in policy discussions (see, e.g., AICPA [1997] CPA Letter Supplements (Finance and Accounting section) at http://aicpa.com/pubs/cpaltr/nov97/suppl/fin.htm or the Environmental Law Institute's News and Analysis at http://elr.radcampaign.com/sites/default/files/articles/35.10666.pdf).

[5] This is just one illustration of our results. We leave the discussion of the economic effects of changing the other parameters of the model to the body of the text.

disclosed about its value. Since a value-maximizing firm will disclose the information it receives only when disclosure results in a higher market price for the firm than does nondisclosure, an increase in the firm's "no disclosure" price creates a higher hurdle the realized value of the firm's private information must exceed to make disclosure of that information worthwhile for the firm. Such a change thereby expands the set of the circumstances under which the firm will make no disclosure - and hence expands the set of circumstances under which the firm is priced based on investors' inferences about the firm's value (as opposed to being based on concrete information the firm discloses about its value). At the time a firm makes its investment decision, it anticipates this effect - that its future market value will be determined more often by investors' inferences about the firm's value than based on the firm's disclosures about its value - and so it responds to a change in a parameter that increases its "no disclosure" price by investing less.

This linkage between the effect of changing a parameter on a firm's equilibrium propensity to disclose information and the effect of changing that same parameter on the firm's equilibrium investment choices might seem obvious, or at least intuitive. But, this linkage stands in striking contrast to the conclusion of many extant models in the literature on endogenous disclosure policies that increased (that is, either more precise, or more frequent) disclosures lead to *reduced* equilibrium firm market values and (weakly) *reduced* equilibrium investment levels.[6]

In summary, the paper makes the following contributions. First, it determines firms' equilibrium disclosure policies in the presence of a "duty to disclose." Second, it derives a variety of empirical predictions related to firms' equilibrium disclosure policies. Third, it determines firms' equilibrium prices when firms are obligated to pay damages when they are caught withholding MI, where the damage payments are determined in a "10b-5 fashion," i.e., by the extent to which firms' prices during the period the MI was withheld are overstated. Fourth, it shows how various recursions that result from 10b-5 damage payments can be resolved. And fifth, it shows how firms' equilibrium disclosure policies are linked to their equilibrium investment decisions.

In terms of related literature: Heitzman, Wasley and Zimmerman [2010] is one of the few contemporary papers that studies the empirical implications of firms' disclosure decisions in the presence of requirements to disclose material information. Apart from the fact that HWZ is an empirical study and the present paper is a theoretical study (albeit one whose goal is to make empirical predictions), HWZ and the present paper adopt very different conceptual approaches to addressing firms' disclosure decisions when mandatory disclosure requirements are present. Specifically, HWZ proceed by assuming that all firms obey all mandated requirements to disclose material information, whereas our approach, as noted above, is premised on firms' undertaking cost-benefit calculations in deciding what information to disclose, regardless of whether the information is stipulated to be disclosed under some mandatory disclosure requirement. Other papers concerning materiality and disclosures include Patillo [1976] and Ward [1976] and Messier, Martinov-Bennie,

[6] For example, Pae [1999] notes that firm value in his model is *highest* when the precision of the private information the firm's manager can acquire and disclose is *lowest* (which is akin to disclosing the least amount of information possible in the present model), because that reduces the incentives of the manager to waste resources on acquiring and disclosing the information. In a different model, Pae [2004] observes that the manager's equilibrium effort choice in his model, which is akin to the manager's equilibrium effort level in the present model, is first-best, *independent* of the manager's propensity to make dislosures. As a third example, expected firm value in Verrecchia [1990] decreases in the frequency with which a firm in his model makes a disclosure, as Pae and others have observed previously (see, e.g., remarks by Pae [1999] at p. 474).

Aasmund Eilifsen [2005]. Pae [2005] identified some of the issues that would have to addressed were the existing literature on voluntary disclosures extended so as to include legal penalties for failure to disclose,[7] but Pae [2005] did not himself conduct such a study.

Finally, our paper is premised on the idea that firms at least occasionally receive private information pertinent to their accurate pricing which, sometimes, they do not disclose in equilibrium. For this premise to be valid, a setting must be considered where the "unravelling" results of Grossman [1981] and Milgrom [1981] do not apply. Dye [1985] and Jung and Kwon [1988] identified a natural setting in which firms in equilibrium selectively disclose the information they receive, which subsequently has been adopted by other researchers in their examination of firms' voluntary disclosure policies (see, e.g., King and Wallin [1991], Pae [2005], Shavell [1994]), based on the simple notion that investors are often uncertain what private value-relevant information firms have in their possession. We make use of this natural setting here.

The paper proceeds as follows. The next section, section 2, introduces the base model. In the base model, investment is taken as exogenous, and damages related to the withholding of MI are assessed against the firm when a fact finder subsequently discovers the firm withheld MI from investors. In section 3, the equilibrium of the base model is defined. Section 4 analyzes the equilibrium of the base model. Section 5 extends the base model to include endogenous investment. Section 6 contains conclusions. The Appendix contains the proofs of most results not proven in the text or accompanying footnotes.

2 Introduction to the Base Model[8]

At time 0, a firm knows that, by some later time 1, with probability p it will learn privately the realization x of some value-relevant random variable $\tilde{x}$ with cumulative distribution function ("cdf") $L(x)$, density $l(x)$, and support $[\underline{x}, \bar{x}]$. At time 0, the firm also recognizes that with probability $1-p$, it will receive no information by time 1. If the firm learns the realization of $\tilde{x}$, it may disclose that information or disclose nothing at time 2 (a time greater than time 1). Disclosing or withholding x are the firm's only options. It cannot, for example, "partially" disclose the information it receives, and it cannot distort the information it receives before disclosing it.[9] If the firm does not learn information, it necessarily makes no disclosure. If the firm receives information but does not disclose it by time 2, it never discloses it (at least within the time horizon considered in the model).

The goal of the firm in choosing its disclosure policy is taken to be to maximize the (per share) price of the firm at time 2. There is some strictly increasing function $v(x)$ representing the discounted expected present value of the firm's cash flows per share (gross of any damage payments) as of time 2. If the firm

[7] See Pae [2005], pp. 402-403.

[8] We introduce the base model in the text in as streamlined a manner as possible, with a minimum of asides, leaving to the accompanying footnotes commentary on the realism of the assumptions, alternative ways of formulating the model, etc.

[9] The presumption here that the firm would not partially disclose its information need not be assumed: it can be derived as a feature of any equilibrium (because any incomplete disclosure must be interpreted by investors as skeptically as possible in equilibrium, so a firm that disclosed that it received information by time 1 would be obliged to disclose the exact value of its information to avoid unfavorable pricing by skeptical investors; this is a manifestation of the "unravelling" result of Grossman [1981] and Milgrom [1981]).

Also, the presumption that the firm cannot distort the information it receives is based on the implicit premise that antifraud statutes are sufficiently great and sufficiently well enforced so as to deter distorted disclosures. For a model that studies biased voluntary disclosures, see, e.g., Einhorn and Ziv [2012].

discloses x, each share is priced at $v(x)$.[10] If the firm discloses nothing at time 2, the price of each share, denoted by P^{nd}, is the expected value of $v(x)$ net of any expected damage payments, as described below, the firm may be liable for. P^{nd} is endogenously determined.

All trading in the firm's shares takes place at time 2. Short sales are prohibited.[11] Neither the firm nor the firm's managers trade in the firm's shares.[12] All traders are risk-neutral, homogeneously informed, and learn information about the firm only through the firm's disclosures. Traders are of two types: "liquidity" traders whose trading decisions are exogenous and random (as of time 0), and many "rational" traders who buy or sell shares at time 2 depending on whether the firm's time 2 price is below or above their perceptions of the expected value of $v(\tilde{x})$ adjusted for any expected damage payments.[13] The total fraction of the firm's shares purchased by liquidity (resp., rational) traders at time 2 is described by the realization of the random (resp., deterministic) variable $\tilde{f}_l$ (resp., f_r). $\tilde{f}_l$ (resp., f_r) is exogenous (resp., endogenous) and in particular independent (resp., dependent) on the price P^{nd} of the firm in the event the firm makes no disclosure. Given realization f_l of $\tilde{f}_l$ and f_r, the total fraction of the firm's shares purchased at time 2 is denoted by $f \equiv f_l + f_r$.

If the firm withholds information at time 2, there is a fact finder - e.g., auditor, regulator, reporter, etc. - who, by time 3 (a time later than time 2) learns with probability q that the firm withheld information at time 2. When the fact finder learns the firm withheld information, the fact finder also learns, and discloses to the investing public, the value of the withheld information. The fact finder is assumed to make only one type of error: the fact finder may fail to detect the firm possessed undisclosed information at time 2, but he does not err by claiming the firm withheld information at time 2 when it did not.

The firm is assumed to be liable for damages if and only if the fact finder detects that the firm withheld MI at time 2.[14] Withheld information x is deemed material if the difference between the actual time 2 price of a firm's share when the firm makes no disclosure, P^{nd}, and what the time 2 price of a share would have have been had x been disclosed, $v(x)$, exceeds materiality threshold ("MT") Δ_m. That is, withheld information is material if and only if the following inequality holds:

$$P^{nd} - v(x) \geq \Delta_m.$$

In the following, we take the MT Δ_m to be fixed[15] and public information.

[10] Consistent with the preceding description, unless something explicitly to the contrary is stated, all firm values should be taken to be amounts *per share*.

[11] Either simply as an exogenous restriction on trades or because there is no follow-up market after time 2 on which time 2 short positions can be covered.

[12] As we are not interested in fomulating or studying a model of insider trading. In any event, sufficient large penalties for inside trading would prohibit insiders from engaging in such trading, absent disclosure of their private information.

[13] More precisely, there are presumed to be enough rational traders so that the equilibrium price is determined by their perceptions of the expected value of buying a share in the firm. (This assumption might seem to render the introduction of liquidity traders into the model superfluous. But, it will be clear from the discussion below that the presence of liquidity traders does in fact affect the equilibrium pricing of the firm's shares because the expected damage payments the firm will owe investors who purchase shares while material information is withheld will vary with the combined number of shares purchased by both rational and liquidity traders.)

[14] There are interpretations of SEC rule 10b-5 under which firms are not required to disclose all the new material information in their possession. Rather, firms are required to disclose the new material information they receive only if their previous disclosures of related information would result in investors having a misleading interpretation of a firm's financial condition absent "corrective" disclosures based on the new information. If one subscribes to this interpretation, the model in the text still applies without change, except that the distribution of the random variable $\tilde{x}$ of the firm's private information must now be understood to be that of the distribution of $\tilde{x}$ *conditional* on whatever information related to $\tilde{x}$ the firm previously disclosed.

We would like to thank Katherine Schipper for this observation.

[15] In a working paper we allow Δ_m to be described by a random variable, and we obtain results similar to those we report

When the fact finder discovers that the firm withheld MI, the firm is obliged to pay all of the investors who purchased the firm's shares at the price P^{nd} the damage payment $d(P^{nd} - v(x))$ per share, where $d > 0$ is a "damages multiplier" determined by a judge, some dispute resolution mechanism, etc.[16] Since there is substantial documentation that "wronged" shareholders are reimbursed for only a fraction of their actual losses,[17] we take $d < 1$ throughout the following. Given f and the firm is subsequently found to have withheld MI x, it follows that the the damage payments reduce the per share value of the firm by $d \times f \times (P^{nd} - v(x))$.

When the firm is liable for damages, it is posited to incur legal costs above and beyond the damage payments themselves. If there are in total n shares of the firm outstanding, the firm's total legal costs are taken to be the sum of costs that vary with the number of shares purchased at time 2, $k \times n \times f$, and costs that vary with the total dollar value of the damage payments, $t \times n \times d \times f \times (P^{nd} - v(x))$, for some constants k, $t \geq 0$.[18] [19] [20]

3 Definition of Equilibrium in the Base Model

A firm's disclosure policy is described by a set ND of the realizations of the firm's private information $\tilde{x}$ that, when received, the firm elects not to disclose. Since the firm's goal is posited to be to maximize its

below (except for those comparative statics below concerning the effects of a change in the (here deterministic) materiality threshhold).

[16] Note that even if liability rules were changed so that damage payments in principle could be awarded sellers, as well as purchasers, of the firm's shares, no sellers of the firm's shares would be awarded damage payments in this model, because sellers who sell shares while the firm withholds information receive more, not less, for those shares than they would had the firm disclosed the withheld information (because $P^{nd} > v(x)$ for all $x \in ND$).

[17] See, e.g., Ryan and Simmons [2009].

[18] Obviously, these total legal costs correspond to per share costs of kf and $tdf(P^{nd} - v(x))$ respectively.

[19] In this footnote, we make several comments concerning the correspondence between damage payments as portrayed in the model and actual damage payments in 10b-5 and related securities litigation. First, consistent with the "fraud on the market" theory of damages (Basic, Inc. v. Levinson, 485 U.S. 224 (1988)), investors purchasing shares at time 2 need not demonstrate reliance on the firm's disclosures (or, if the model were embellished to include them, the firm's financial reports) to be awarded damage payments. All that is necessary for an investor to be entitled to receive damage payments is that the investor demonstrate he purchased shares during the period in which the firm withheld MI from the capital market.

Second, that the magnitude of the assessed damages is proportional to the difference between the "artificially high" price of the firm arising from the firm having withheld MI and the "as if" price of the firm were the firm to have disclosed the MI, seems consistent with practice (see, e.g., Booth [2009]). Also, the model defines damages in terms of the losses of investors who *purchased* shares at time 2. The profits that shareholders who *sold* shares while the firm's value was overstated are disregarded, even though arguments have been made that, on efficiency grounds, such shareholders should be required to disgouge their profits (see, e.g, Posner [2007], at p. 483).

Third, the representation of materiality is "output-based" in the sense that whether the withheld information is deemed material is determined by the amount by which the price of the firm would have changed had information withheld by the firm been disclosed. This representation fits with the U.S. Supreme Court's notion of materiality as reported, for example, in the case of Basic Inc. v. Levinson, where the Court found that "materiality depends on the significance the reasonable investor would place on the withheld or misrepresented information." That is, omitted information is considered to be material if its disclosure would have had a sufficiently significant influence on the firm's price. (In contrast, an "input-based" notion of materiality would be described by criteria such as: earnings were overstated by x% or total assets were understated by y%, etc., without regard to how much the market value of the firm changes with those percentage changes in the firm's earnings, assets, etc.)

Fourth, when a firm is found to have withheld MI, the damages paid to the shareholders who purchased shares at the "artificially high" price P^{nd} established by the capital markets are compensated after the fact finder has discovered the material withholding. That is, in effect the shareholders who pay for these damages are not the shareholders of record at the time the tort (of withholding information) was committed, but rather are the shareholders of record as of (or shortly after) the time the tort is discovered. While it may be argued that better incentives to induce the firm to disclose its information would be achieved were the former set of shareholders made liable, the impracticality of identifying those former shareholders and making them pay is generally thought to rule out that alternative as infeasible.

Fifth, while we do not bother to adjust the notation to reflect this, we note that many of the parameters in the model could be replaced by random variables the realizations of which are not known at the time the firm must make its disclosure decision without affecting the results below.

[20] A numerical example of the effects of the damange payments and legal costs is presented in footnote 23 below.

expected market value as of time 2, and since $v(x)$ is taken to be monotone increasing in x, it is clear that if a firm prefers to disclose information x it receives at time 2, it also prefers to disclose any information $x' > x$ too. Thus, the "no disclosure" set ND is of the conventional "cutoff" form:

$$ND = \{x|x < x^c\} \tag{1}$$

for some $x^c \in [\underline{x}, \bar{x}]$.[21]

We simplify notation somewhat in the following by introducing the transformed random variable $\tilde{v} = v(\tilde{x})$ with cdf $G(v) \equiv \Pr(\tilde{v} \leq v)$, density $g(v)$, and support $[\underline{v}, \bar{v}] \equiv [v(\underline{x}), v(\bar{x})]$.[22] We assume that the density $g(\cdot)$ of $\tilde{v}$ is continuous on its support. In this new notation, the "no disclosure" set ND is given by $ND = \{v|v < v^c\}$ for some $v^c \in [\underline{v}, \bar{v}]$, where $v^c = P^{nd}$.

Investors who purchase shares of the firm at time 2 when the firm makes no disclosure take into account both their perceptions of $\tilde{v}$ and the expected damage payments they might receive (in the event the fact finder subsequently determines that the reason the firm did not disclose information was that it was withholding MI) when deciding how much to pay for the firm's shares. In the event the fact finder subsequently determines that MI x with associated value $v = v(x)$ was withheld, buying a share at price P^{nd} at time 2 entitles an investor to the sum of (i) the damage payment per share of $d(P^{nd} - v)$ and (ii) a share of the firm which - after paying damages to the fraction f of traders who bought shares at time 2 and also after paying the other legal costs - is worth (per share): $v - df(P^{nd} - v) - k - dtf(P^{nd} - v)$. It follows that when the fact finder discovers that the firm withheld MI, the "ex post" value of one share of the firm to an investor who purchases the share at time 2 is:[23]

$$d(P^{nd} - v) + v - df(P^{nd} - v) - k - dtf(P^{nd} - v) = v + d(1 - f(1 + t))(P^{nd} - v) - k. \tag{2}$$

To identify the equilibrium relationship between ND and P^{nd}, we first provide some additional detail regarding traders' assumed trading behavior. Rational traders are assumed to know the distribution, but not the realization, of the fraction $\tilde{f}_l$ of shares that will be purchased by liquidity traders at time 2 before they submit their own demands for the firm's shares then. Rational traders make a conjecture about the cutoff v^c the firm used in deciding whether to disclose its information, and they also make a conjecture about the aggregate fraction f_r of shares purchased by rational traders at time 2. In equilibrium, these conjectures are correct. The capital market is assumed to be competitive in that rational traders perceive their individual trading decisions to have no impact on either P^{nd} or f_r.

[21] Without loss of generality, we assume that if the firm is indifferent between disclosing its information and not disclosing it, the firm discloses.

[22] Recalling that $L(x)$ denotes the cdf of $\tilde{x}$, it follows that $G(v) = L(v^{-1}(v))$. Incidentally, the results that follow also hold for a random variable $\tilde{v}$ with infinite support provided that its mean $E[\tilde{v}]$ is finite and its density $g(\cdot)$ is continuous and bounded on its support.

[23] The following numerical example may help to clarify (2). Suppose $k = t = 0$ and there are in total 100 shares of the firm. Suppose also that the price per share when $\tilde{x} = x$, would have been $v(x) = \$30$ were x disclosed. Also suppose that if the firm does not disclose x, then the no disclosure price is $P^{nd} = 50$, that $d = .6$, and that 25 shares are purchased at time 2. If the fact finder discovers the firm withheld x, then the damages paid per share are $.6 \times (50 - 30) = \$12$, for a total of $\$300 = 12 \times 25$. Accordingly, the total value of the firm at time 3, after payment of damages, is $100 \times 30 - 300$ =\$2700, or $\$27/share$. Alternatively, one can calculate the ex post value of the firm per share directly as in the text as: $v(x) - df(P^{nd} - v(x)) =$ $\$30 - .6 \times .25 \times (50 - 30) = \$27$ If an investor who intended to purchase a share of the firm at time 2 anticpated that the firm was withholding information which, if disclosed, would result in its value per share being \$30, and so could collect damages of \$12 per share, the investor would be willing to pay \$39=27+12 per share (or, alternatively computed via (2) as $\$30 + .6 \times .75 \times (50 - 30) = \$39$).

$EVF(v^c, f_r)$ denotes a rational trader's perceptions of the expected value of *purchasing* a share of the firm at time 2 given his conjectures about f_r and v^c, and his knowledge of the distribution of $\tilde{f}_l$. $EVF(v^c, f_r)$ is computed by employing Bayes' rule; the details are in Appendix A. There, we show that:

$$EVF(v^c, f_r) = \frac{pG(v^c - \Delta_m)q}{1 - p + pG(v^c)} \times E[\tilde{v} + d \times (1 - (f_r + \tilde{f}_l)(1 + t)) \times (P^{ND} - \tilde{v}) - k|\tilde{v} < v^c - \Delta_m] + \tag{3}$$

$$\frac{p(G(v^c) - G(v^c - \Delta_m))q}{1 - p + pG(v^c)} \times E[\tilde{v}|v^c - \Delta_m \leq \tilde{v} < v^c] +$$

$$(1 - \frac{pq(v^c)G(v^c)}{1 - p + pG(v^c)}) \times \left(\frac{G(v^c)p(1 - q)}{(1 - p + pG(v^c))(1 - q)} \times E[\tilde{v}|\tilde{v} < v^c] + (1 - \frac{G(v^c)p(1 - q)}{1 - p + pG(v^c)(1 - q)}) \times E[\tilde{v}]\right).$$

Holding v^c and f_r fixed, the first line of this expression is the probability at time 2 the firm will subsequently be found by the fact finder to have withheld MI, given that the firm makes no disclosure at time 2, times the conditional expected value of a share in the firm given the firm is found having withheld MI. The second line is the probability the firm will be found by the fact finder to have withheld *im*material information at time 2, given that the firm makes no disclosure at time 2, times the conditional expected value of a share in the firm given that the firm is found having withheld *im*material information. The final line is the probability the fact finder will not find that the firm withheld any information at time 2, given that the firm makes no disclosure at time 2, times the conditional expected value of a share in the firm given that the fact finder fails to detect the firm withheld any information at time 2.

This specification of $EVF(v^c, f_r)$ is central to the definition of an equilibrium of the model, presented below.

Definition 1 *A competitive equilibrium of the base model where some rational traders choose to own shares of the firm at time 2 consists of: (a) a cutoff* v^c; *(b) a no disclosure set* $ND = \{v|v < v^c\}$; *(c) the price* P^{nd} in the event the firm makes no *disclosure*; (d) *the sum* f_r *of the fractions of shares purchased at time 2 by all rational traders who purchase shares at time 2; and (e) price* $P(x)$ *if the firm discloses* x, *such that: (i)* $P(x) = v(x)$ for all x; *(ii)* $P^{nd} = EVF(v^c, f_r)$; and *(iii)* $v^c = P^{nd}$.

The definition requires that: (i) $P(x) = v(x)$ whenever the firm discloses x; (ii) the price of a share when the firm makes no disclosure, P^{nd}, equals the value of the cash flows a trader expects to receive from purchasing a share of the firm at time 2 net of all potential damage payments and related legal costs; (iii) the cutoff v^c the firm uses in deciding whether to disclose information it receives at time 2 equals P^{nd}.[24]

4 Analysis of the Base Model

In the introduction to the paper, we noted that to determine both the equilibrium price of the firm and the size of the damage payments in 10b-5 and related litigation settings one has to take into consideration several circularities and recursions that naturally arise in such settings. Three specific circularities that must be

[24] If a firm makes no disclosure, the expected value a current shareholder gets from *selling* a share of the firm at time 2 is less than $EVF(v^c, f_r)$, because of the feature of 10b-5 and related litigation that sellers of a firm's shares, unlike purchasers of the firm's shares, do not collect damage payments in the event the fact finder subsequently reveals that the firm withheld material information at time 2.

Notwithstanding this difference, the equilibrium price of a share in the event the firm makes no disclosure, P^{nd}, must equal $EVF(v^c, f_r)$ if any rational traders own any shares in the firm in equilibrium, since any lower price would result in an arbitrage opportunity for rational traders (they would buy the shares as long as the expected value of what they purchased exceeded the purchase price) and any higher price would deter all rational traders from possessing any shares in the firm.

taken into account are: 1. the dependence of the size of damage payments on the firm's equilibrium price, and the dependence of the firm's equilibrium price on the size of damage payments; 2. the dependence of investors' perceptions of the value of a share of the firm on their view of the cutoff the firm uses to define its disclosure policy at the same time the cutoff the firm uses to define its disclosure policy depends on the firm's view of investors' perceptions of the value of a share of the firm; and 3. the circular definition of materiality[25]. All of these circularities and recursions are embedded in the single equation

$$v^c = EVF(v^c, f_r), \tag{4}$$

which follows directly from the definition of an equilibrium in the base model.

By substituting (3) into (4) and simplifying, in the next lemma we rewrite equation (4) in a way that facilitates analysis of the equilibrium of the base model.

Lemma 1 *Fix f_r and set $\bar{f} \equiv (\bar{f}_l + f_r)(1+t)$. Then, the equation*

$$E[\tilde{v}] - v^c = \frac{p}{1-p}\int_{\underline{v}}^{v^c} G(v)dv - \frac{pq}{1-p} \times \left[((1-\bar{f})d\Delta_m - k)G(v^c - \Delta_m) + (1-\bar{f})d\int_{\underline{v}}^{v^c-\Delta_m} G(v)dv\right] \tag{5}$$

is a rearrangement of the equation $EVF(v^c, f_r) = v^c$. The proof is in Appendix B.

To see how (5) helps to identify features of an equilibrium for a particular example before we proceed to obtain general comparative statics, we consider the solution to equation (5) when $\tilde{v}$ is uniformly distributed on $[0, \bar{v}]$ (so $G(v) = \frac{v}{\bar{v}}$ for $v \in [0, \bar{v}]$). When both v^c and $v^c - \Delta_m$ are in $[0, \bar{v}]$, then equation (5) is the same as:

$$\bar{v}/2 - v^c = \frac{p}{1-p}\int_{\underline{v}}^{v^c} \frac{v}{\bar{v}}dv - \frac{pq}{1-p} \times \left[((1-\bar{f})d\Delta_m - k)\frac{v^c - \Delta_m}{\bar{v}} + (1-\bar{f})d\int_{\underline{v}}^{v^c-\Delta_m} \frac{v}{\bar{v}}dv\right]. \tag{6}$$

After performing some algebra, one can show that, when $1 > \bar{f}$ and $\frac{\bar{v}\sqrt{1-p}}{1+\sqrt{1-p}} > \Delta_m$,[26] the unique positive solution of (6) is given by:[27]

$$v^c = \frac{-(\frac{1-p}{p}\bar{v} + k) + \sqrt{(\frac{1-p}{p}\bar{v} + k)^2 + (1 - qd(1-\bar{f}))(\frac{1-p}{p}\bar{v}^2 - qd(1-\bar{f})\Delta_m^2 + 2k\Delta_m)}}{1 - qd(1-\bar{f})}. \tag{7}$$

While various comparative statics can be derived from this expression for the cutoff v^c, the only one we mention presently is one involving Δ_m. Viewed as a function of Δ_m, the quadratic $-qd(1-\bar{f})\Delta_m^2 + 2k\Delta_m$ is obviously not monotonic in Δ_m, and so it follows from (7) that v^c itself is not monotonic in Δ_m. More precisely, (7) shows that v^c increases in Δ_m as long as $\frac{k}{qd(1-\bar{f})} > \Delta_m$, and v^c decreases in Δ_m for $\Delta_m > \frac{k}{qd(1-\bar{f})}$. The explanation for this nonmonotoncity was discussed in the Introduction. We show below that this nonmonotonicity is a feature of the equilibrium cutoff for any distribution for $\tilde{v}$, not just for $\tilde{v}$ that is uniformly distributed.

In the next section, we derive general predictions about the equilibrium of the base model.[28]

[25] See the fifth paragraph of the Introduction.

[26] The latter condion ensures that $v^c - \Delta_m > 0$.

[27] Deriving this expression simply entails solving a quadratic equation, and picking the positive root. Accordingly, the demonstration of the calculations leading to this expression are omitted.

[28] Because technical issues related to the existence and uniqueness of an equilibrium need not be broached in order to either make or understand predictions derived from the model, we forego a detailed discussion of these technical issues here. (These issues are discussed in a working paper available from the authors.) We note here only that, as is often the case in pricing models where risk-neutral traders are present, that the equilibrium total fraction f_r of shares purchased by rational traders is not uniquely determined. Accordingly, in some of the results that follow, we condition the results on the size of f_r.

5 Analysis of the base model

The following general predictions emerge from the base model. (In the statement of the theorem, we fix f_r and set $\bar{f} \equiv (\bar{f}_l + f_r)(1+t)$.)

Theorem 1 (*The Comparative Statics of the Equilibrium Cutoff*) *If either (a) $\bar{f} \geq 1$ or (b) $\bar{f} < 1$ and $\Delta_m < \frac{1-p}{pb}$, then the equilibrium no disclosure price/equilibrium cutoff is:*

(a) strictly decreasing in the probability p that information arrives at the firm;

(b) strictly decreasing in each of $\bar{f}$, t, and k.

(c) strictly decreasing in the materiality threshold Δ_m when $(1-\bar{f})d\Delta_m - k > 0$ and strictly increasing in the materiality threshold Δ_m when $(1-\bar{f})d\Delta_m - k < 0$;

(d) strictly increasing in the damages multiplier d when $\bar{f} < 1$ and strictly decreasing in the damages multiplier when $\bar{f} > 1$;

(e) strictly increasing in the probability q the fact finder discovers that the firm withheld MI when $(1-\bar{f})d\Delta_m > k$ and strictly decreasing in this probability when $\bar{f} > 1$.[29]

The proof of Theorem 1 is in Appendix C.

Theorem 1(a) confirms and extends a result known from the original Dye[1985] and Jung and Kwon[1988] models: if investors know that information arrives at a firm more often (i.e., p increases), they will become more skeptical of a firm that does not disclose information, and they reduce the firm's equilibrium "no disclosure" price accordingly. Theorem 1(a) documents that this result, which was known previously to be true only when all disclosures are voluntary, extends to cases where firms are subject to penalties for failing to disclose MI.

Theorem 1(b) is also clear: as $\bar{f}$ increases, and so more investors are anticipated to buy the firm's shares and hence become eligible to collect damages from a firm that withholds MI, a value-maximizing firm becomes less inclined to withhold the information it receives. The same forces also apply with respect to increases in either of the litigation cost parameters t or k.

Theorem 1(c) shows that the nonmontonicity of the equilibrium no disclosure price/cutoff in MT exhibited by the uniform distribution discussed above around (7) is a general phenomenon. What is perhaps surprising is that the MT level $\Delta_m = \frac{k}{(1-\bar{f})d}$ at which the cutoff v^c changes from increasing in Δ_m to decreasing in Δ_m is exactly the same in the general case - for an arbitrary distribution- as it was for the uniform distribution discussed above.

Theorem 1(d) reports that changes in the damages multiplier also has non-monotonic effects on the incentives of a firm to disclose its private information. If $\bar{f} < 1$, increases in the damages multiplier causes firms to withhold information more often. On its face, this is a surprising result: increases in the damages multiplier increase the penalty/liability the firm must pay in the event it is caught withholding information, and so it might seem that firms would respond by withholding information less often. To explain the result, note that increasing the damages multiplier d increases the "ex post" value of purchasing a share of the firm

[29]Note that there is a "hole" in this last characterization of comparative statics, that is the set $\bar{f} > 1$ and the set $(1-\bar{f})\bar{\beta}\Delta_m > \bar{k}$, while mutually exclusive, are not exhaustive. As inspection of the proof of this result shows, in the omitted region of parameter space, comparative statics appear to be ambiguous.

(2) in the event the firm withholds MI when $\bar{f} < 1$.[30] So, increasing the damages multiplier when $\bar{f} < 1$: (1) increases the value of a share of the firm in the event the firm makes no disclosure, and (2) has no impact on the value of a share of the firm when the firm discloses its information. Combined, these last two effects must induce the firm to disclose MI less often as d increases. When $\bar{f} > 1$,[31] this effect reverses itself: then, the combination of payouts to other shareholders and payouts in other legal costs are so large that increases in the damages multiplier under these conditions lead to a decrease in the value investors at time 2 are willing to pay for the firm's shares in the event the firm makes no disclosure. In turn, this last effect induces the firm to disclose its MI more often.

As Theorem 1(e) reports, an increase in the probability the fact finder detects the firm withheld MI may result in either an increase or a decrease in the firm's propensity to disclose information it receives, depending on the specific values of various the parameters of the model. This ambiguity is also, at first blush, surprising, as it might seem that improvements in the fact finder's ability to detect the firm withheld MI would always result in the firm withholding information less often. The ambiguous effect arises because investors who buy the firm's shares at time 2 are both payers of damage payments (as shareholders of record at time 3) and recipients of damage payments (as members of the class of shareholders entitled to receive damage payments in the event the fact finder discovers the firm withheld MI). If the payouts to others become too large (specifically, if $\bar{f} > 1$) when the firm is caught withholding MI, the value of a share in the firm to a trader purchasing the share drops as the fact finder's effectiveness increases, whereas if the payouts to others are sufficiently small (specifically i.e, $\bar{f}$ and k are sufficiently small so that $(1 - \bar{f})d\Delta_m > k$), the amount traders are willing to pay for a share of the firm when the firm makes no disclosure is increasing in the fact finder's effectiveness. The dependence of the firm's disclosure policy on a change in q is now explained in the same way it was in the discussion of Theorem 1(d): the firm becomes more (resp., less) inclined to disclose its information when q increases when parameter values change so as to decrease (resp., increase) the value investors attach to the firm's shares when the firm makes no disclosure.

Theorem 1 also implications for both the direction and size of the firm's price reaction to the *non*disclosure of information at time 2. In particular, it implies that there will be a negative price reaction attending the *lack* of a disclosure by the firm at time 2, assuming the parameters determining the litigation costs (k and t) are sufficiently small. To see this, first note that when $k = t = 0$, then even though there are damage payments in the model, those damage payments are *between* groups of shareholders in the firm, and hence the damage payments are, on net, zero sum. Therefore, the price of the firm at time zero is just the firm's unconditional expected value $E[\tilde{v}]$. When k and t are sufficiently small, it follows that the price of the firm at time 0 will be nearly $E[\tilde{v}]$ too. Next notice that $k \geq 0$ and $1 > q(1 - \bar{f})d$ (clearly, the latter inequality

[30] To see this, just substitute $\bar{f}$ for $f(1 + t)$ in (2) and recall that $P^{nd} > v$ for all v for which the firm prefers not to disclose its information, to conclude that the "ex post" value $v + d(1 - \bar{f})(P^{nd} - v) - k$ increases in d when $\bar{f} < 1$.

[31] Recall $\bar{f} \equiv (\bar{f}_l + f_r)(1 + t)$, so it is entirely plausible for $\bar{f} > 1$.

always holds since $q < 1, d < 1$ and $\bar{f} \geq 0$) imply:

$$\begin{aligned}
& RHS(5) \\
\geq \quad & \frac{p}{1-p}\int_{\underline{v}}^{v^c} G(v)dv - \frac{pq(1-\bar{f})d}{1-p} \times \left[\Delta_m G(v^c - \Delta_m) + \int_{\underline{v}}^{v^c - \Delta_m} G(v)dv\right] \\
\geq \quad & \frac{p}{1-p}\left(\int_{\underline{v}}^{v^c} G(v)dv - \Delta_m G(v^c - \Delta_m) - \int_{\underline{v}}^{v^c - \Delta_m} G(v)dv\right) \\
= \quad & \frac{p}{1-p}\left(\int_{v^c - \Delta_m}^{v^c} G(v)dv - \Delta_m G(v^c - \Delta_m)\right) \\
= \quad & \frac{p\Delta_m}{1-p}(G(\psi) - G(v^c - \Delta_m)) \\
> \quad & 0, \qquad (8)
\end{aligned}$$

(here, ψ is a number (by the mean value theorem for integrals) in the interval $(v^c - \Delta_m, v^c)$).[32] Since $LHS(5) = RHS(5)$ in equilibrium, and since we just demonstrated that $RHS(5) > 0$, it follows that $LHS(5) = E[\tilde{v}] - v^c > 0$ too. Therefore, since $E[\tilde{v}]$ is the time 0 price of the firm, and v^c is the price of the firm at time 2 when the firm makes no disclosure, we conclude there is a negative price reaction to no disclosure at time 2. Furthermore, since $E[\tilde{v}]$ does not change with changes in the parameters of the model, whereas - as Theorem 1 reports - v^c does change with changes in the parameters of the model, Theorem 1's conclusions immediately translate into comparative statics about the size of the negative price reaction to the firm's nondisclosure at time 2. For example, Theorem 1(c) implies that the negative price reaction to nondisclosure will be bigger as the MT Δ_m increases if $(1-\bar{f})d\Delta_m - k > 0$ and the negative price reaction will be smaller as the MT Δ_m increases when $(1-\bar{f})d\Delta_m - k < 0$. The comparative statics reported in Theorem 1(a), (d), and (e) similarly can be translated into statements about the size of the price reaction to the firm's nondisclosure at time 2.

6 Expanding the model to include endogenous investment

In this section, we extend the analysis of the preceding section by considering some of the real effects of voluntary disclosures when firms have a duty to disclose the material information they receive.[33] We do this by modifying the model of the previous section by making the distribution of $\tilde{v}$ depend on the firm's time 0 endogenous investment choice. Our primary goal in developing this extension is to connect the (not easily observed) efficiency of firms' equilibrium investment choices with observable, and empirically testable, predictions developed in preceding sections regarding firms' equilibrium disclosure policies. Our main finding is that changes in exogenous parameters that lead firms to engage in more (resp., less) disclosure lead to increased (resp., reduced) investment efficiency.

Formally, we now suppose that at time 0, the firm under investigation *privately* chooses an investment

[32] We know that $\psi \in (v^c - \Delta_m, v^c)$, as opposed to simply $\psi \in [v^c - \Delta_m, v^c]$, since $G(v)$, as a cdf, is strictly increasing over its support.

[33] The idea that financial reporting and disclosure decisions should be evaluated in terms of their real effects, and in particular, in terms of their effects on a firm's equilibrium investment choices, was initiated by Kanodia: see, e.g., Kanodia [1980], Kanodia and Lee [1998], and Kanodia and Mukherji [1996].

level $I \geq 0$.[34] The cost of investment I is \$$I$. The cdf of $\tilde{v}$ is affected by this investment choice, which we indicate by writing $G(v) = G(v|I)$; the associated density is written $g(v|I)$, with support $[\underline{v}, \bar{v}(I)]$.[35] All other aspects of the model of this section coincide with the model developed in prior sections.

We assume that the family of cdfs $\{G(\cdot|I)|I \geq 0\}$ indexing the distributions of the random variable $\tilde{v}$ generated by investment in a firm is continuously strictly ordered by stochastic dominance, as defined below. (In this definition, $G_I(v|I)$ and $g_I(v|I)$ respectively refer to the partial derivative of $G(v|I)$ and $g(v|I)$ with respect to I.)

Definition 2 *The family of cdfs $\{G(\cdot|I)|I \geq 0\}$ is said to be continuously strictly ordered by stochastic dominance provided, for each $I > 0$ and each $v > \underline{v}$:*

(i) $G_I(v|I) \leq 0$;

(ii) $\int_{\underline{v}}^{v} G_I(v'|I)dv' < 0$;

(iii) $G_I(v|I)$ and $g_I(v|I)$ both exist and are (jointly) continuous on $(\underline{v}, \bar{v}(I))$.

The crucial requirement in this definition is part (i), as it ensures that increasing the investment parameter I strictly "shifts the distribution of $\tilde{v}$ to the right" in the sense of first-order stochastic dominance (i.e., if $I_1 > I_2$, then $G(v|I_1) \leq G(v|I_2)$ for all v). Condition (ii) ensures that this "right-shifting" is strict in the sense that it occurs to some extent over any nondegenerate interval $(\underline{v}, v)$; and condition (iii) ensures that this right-shifting of the distributions occurs "continuously." One example (among many) of a family of cdfs that is continuously strictly ordered by stochastic dominance is the family of cumulative distribution functions corresponding to the set of uniform random variables $\{\tilde{v}_I|I > 0\}$ where $\tilde{v}_I$ is uniformly distributed on the interval $[0, I]$.[36]

The first natural question to ask when the distribution of a firm's value $\tilde{v}$ is affected by investment is how changes in investment[37] change the firm's equilibrium cutoff and hence change the firm's "no disclosure" price. Operationally, this question involves replacing the cdf $G(v)$ that appears in (5) with a cdf $G(v|I)$ drawn from the family $\{G(\cdot|I)|I \geq 0\}$ and determining how the cutoff $v^c = v^c(I)$ that solves that equation varies with I. The following corollary answers this question.

Corollary 1 *(How the Equilibrium Cutoff Varies with Investment) Assume: the family of cdfs $\{G(\cdot|I)|I \geq 0\}$ is continuously strictly ordered by stochastic dominance and there exist positive constants w and B such that $\frac{\partial}{\partial I}E_I[\tilde{v}] \geq w$ and $|G_I(\cdot|I)| \leq B$ for all $I > 0$. Then, the equilibrium cutoff v^c is strictly increasing in I.*

The proof of Corollary 1 is in Appendix C.

Corollary 1 demonstrates that, under mild technical conditions, when a firm increases the level of its investment I, then the firm's equilibrium no disclosure price v^c increases. Intuitively this outcome occurs

[34] Even when firms are obliged to produce balance sheets recording their investments and statements of cash flows that include an investing section that captures cash expenditures on investments, investors cannot be sure whether the investments recorded in these accounts constitute well-directed expenditures that increase the value-generating potential of the firms, rather than being frivolous and/or unproductive investments. Accordingly, even with the publication of balance sheets and statements of cash flows, it is appropriate to consider the actual productive investments firms engage in as private information to firms.

[35] As this notation indicates, we take the lower endpoint $\underline{v}$ of the supports to be common across all I, and we allow the upper endpoints of the supports to vary with I.

[36] To see this, just note that $G(v|I) = \frac{v}{I}$ for $v \in [0, I]$ and $G(v|I) = 1$ for $v > I$, so for all $v' > 0$ and all $I > 0$, $\int_0^{v'} G_I(v|I)dv = -\int_0^{\min\{v',I\}} \frac{v}{I^2}dv = -\frac{(\min\{v',I\})^2}{2I^2} < 0$.

[37] This is the only part of this section where we take I, or changes in I, to be exogenous. Thus, this corollary is a comparative static involving a fixed distribution for $\tilde{v}$ just as the results of Theorem 1 are also comparative statics for a fixed distribution of $\tilde{v}$.

because investors know that, as I increases, the firm's nondisclosure is not as indicative of bad news (as compared to when I is smaller), because high values of $\tilde{v}$ are more likely to occur whether or not the firm makes a disclosure.

To proceed with identifying the firm's equilibrium choice of investment (which, recall, we take to be private information to the firm[38]), we must distinguish between the investment choice the firm actually undertakes from the investment choice investors conjecture the firm undertakes. We (continue to) write I for the firm's actual investment choice, and we now introduce the notation $\hat{I}$ to represent investors' conjectures about the firm's investment choice. (In equilibrium, these will be one and the same.) The cutoff investors think the firm will use will be determined by the cutoff $v^c = v^c(\hat{I})$ that solves equation (5) with $G(v|\hat{I})$ replacing $G(v)$. When there is no ambiguity about what investment level $\hat{I}$ investors think the firm has made, we sometimes write $v^c(\hat{I})$ as $\hat{v}^c$.

It follows that, given $\hat{I}$ (and hence $\hat{v}^c = v^c(\hat{I})$) and I, the expected value of the firm gross of the cost of the actual investment, as perceived by the firm (i.e., the firm's managers) is given by:

$$\begin{aligned} V(\hat{v}^c, I) &\equiv (1-p+pG(\hat{v}^c|I))\hat{v}^c + p(1-G(\hat{v}^c|I))E[\tilde{v}|\tilde{v} > \hat{v}^c, I] \\ &= (1-p+pG(\hat{v}^c|I))\hat{v}^c + p\int_{\hat{v}^c}^{\bar{v}} vg(v|I)dv. \end{aligned} \tag{9}$$

That is, when investors think the firm uses cutoff $\hat{v}^c$, the expected value of the firm gross of the cost of investment is the probability the firm makes no disclosure $(1-p+pG(\hat{v}^c|I))$ times the price investors assign to the firm given no disclosure $(\hat{v}^c)$, plus the probability the firm makes a disclosure $(p\int_{\hat{v}^c}^{\bar{v}} g(v|I)dv)$ times its expected value given that the firm makes a disclosure $(\frac{\int_{\hat{v}^c}^{\bar{v}} vg(v|I)dv}{\int_{\hat{v}^c}^{\bar{v}} g(v|I)dv})$. So, the firm's expected value net of the cost of the cost of the actual investment I is given by[39]:

$$W(\hat{v}^c, I) \equiv V(\hat{v}^c, I) - I. \tag{10}$$

It follows that the firm's preferred investment choice is determined by the first-order condition $\frac{\partial}{\partial I}W(\hat{v}^c, I) = 0$, i.e., by:

$$pG_I(v^c(\hat{I})|I)v^c(\hat{I}) + p\int_{v^c(\hat{I})}^{\bar{v}} vg_I(v|I)dv - 1 = 0. \tag{11}$$

The following definition formalizes the notion of an equilibrium in this setting.

Definition 3 *An equilibrium consists of an investment level I^* and a no disclosure price $\hat{v}^c$ such that,*
(i) given the price $\hat{v}^c$:

$$I^* \in \arg\ max_{I\geq 0}\ W(\hat{v}^c, I); \tag{12}$$

(ii) the no disclosure price $\hat{v}^c$ is given by:

$$\hat{v}^c = v^c(I^*).$$

In words, an equilibrium must satisfy two conditions: first, taking investors' conjecture $\hat{I} = I^*$ about the firm's investment - as well as the "no disclosure" price $\hat{v}^c = v^c(I^*)$ implied by that conjecture I^* - as

[38] Notwithstanding that, in practice, reporting of investment levels occurs on a firm's balance sheet, and reporting of CAPX occurs on a firm's statement of cash flows, we believe it is appropriate for modeling purposes to view I as private, since the publicly disclosed expenditures on investment do not indicate whether those expenditures were efficient or effective.

[39] We ignore discounting in the following (or what amounts to the same thing, we assume that v is discounted to time zero).

given, the firm's preferred investment level I^* maximizes the expected value of the firm net of the cost of making the investment. That is, the investment level the firm actually adopts is the same investment level that investors conjectured the firm adopted. Second, the firm's "no disclosure" price $\hat{v}^c$ based on investors' conjecture I^* about the firm's investment is the "correct" no disclosure price in that it is the "no disclosure" price that corresponds to the investment level the firm actually adopts.[40]

We now show that all of the predictions of the model in Theorem 1 and Corollary 1 above where the distribution of $\tilde{v}$ was exogenous and fixed translate immediately into predictions about the firm's equilibrium level of investment and the firm's equilibrium expected market value in the model of this section where the distribution of $\tilde{v}$ is endogenous and varies with the firm's investment decision. Specifically, we show that a change in any exogenous parameter that results in the firm's equilibrium "no disclosure" price increasing (resp., decreasing) results in a lower (resp., higher) equilibrium level of investment and a lower (resp., higher) equilibrium expected value for the firm. Theorem 2 below states this result formally.

In preparation for the statement of Theorem 2, we let γ generically denote any one of the exogenous parameters of the model, i.e., $\gamma \in \{\Delta_m, \bar{f}, d, q, p, t, k\}$. We adopt the convention that when γ is assigned to be one of these parameters, all other parameters are held fixed. We let $v^c(\gamma)$ denote the equilibrium cutoff that solves (5) in the model of the preceding section, where the distribution $G(v)$ of $\tilde{v}$ was taken as exogenously given. We let $I(\gamma)$ denote the firm's equilibrium investment choice as a function of exogenous parameter γ. And finally, we let $P(\gamma)$ denote the equilibrium expected value of the firm as of the start of the model, i.e., $P(\gamma) \equiv E[\tilde{v}|I(\gamma)] - I(\gamma)$.

Theorem 2 *(The Comparative Statics of Equilibrium Investment) Assume the assumptions of Corollary 1, and either of conditions (a) or (b) of Theorem 1, hold. In the model of endogenous investment, for all the exogenous parameters* $\gamma \in \{\Delta_m, \bar{f}, d, q, p, t, k\}$, *we have:*

(i) $sgn\frac{dI(\gamma)}{d\gamma} = -sgn\frac{dv^c(\gamma)}{d\gamma}$ *and*

(ii) $sgn\frac{dP(\gamma)}{d\gamma} = -sgn\frac{dv^c(\gamma)}{d\gamma}$.

The proof of Theorem 2 is in Appendix D.

Theorem 2(i) states that a change in any parameter that increases the firm's equilibrium no disclosure price reduces the firm's equilibrium investment level, and vice versa. The intuition for this result was described in the Introduction. Theorem 2(ii) adds that a change in any parameter that increases the firm's equilibrium no disclosure price also reduces the firm's equilibrium ex ante expected market value. The latter result obtains because if investors do not get to see a firm's level actual of investment directly, the firm invariably invests too little relative to first-best in equilibrium. Consequently, a change in any parameter that leads the firm to invest more invariably leads to increases in the firm's expected market value, both because the firm's original (pre-parameter change) investment level was too low (relative to first-best) and because, even with the change in the parameter value, there is no possibility that the firm will overinvest (relative to first-best). Symmetric comments apply with respect to a change in any parameter that leads a firm to invest less: such a change invariably reduces the firm's equilibrium expected market value.

[40] Just as was the case in the preceding section, we are much more interested in characterizing features on an equilibrium in what follows rather than addressing technical issues related to the existence of equilibrium. Discussion of sufficient conditions for the existence of an equilibrium are available in a working paper available from the authors.

7 Conclusions

This paper extends the literature on disclosures by studying a firm's disclosure decisions in the presence of a regulatory-imposed duty to disclose material information. Specifically, it examines an expected value-maximizing firm's disclosure decisions in a model where, if the firm gets caught withholding material information, the firm itself is obliged to pay damages to investors who purchased the firm's shares while the material information was withheld. Consistent with 10b-5 litigation, the damage payments are taken to be a (typically fractional) multiple of the amount the investors overpaid for the firm's shares relative to what they would have paid for the shares had the information been disclosed.

An extensive set of comparative statics involving the firm's equilibrium disclosure policy is obtained. Among our results, we show that the probability a firm discloses its private information is increasing in each of the following parameters: the ex ante probability the firm received information; the fraction of shares of the firm expected to be purchased while the material information is withheld; the size of the firm's other litigation costs, in addition to the damage payments, the firm incurs associated with determining and settling damage payments; the size of the materiality threshold if that threshhold is initially high, and also - when a sufficiently large fraction of the firm's shares are expected to be purchased while the firm withheld material information - both the damages multiplier and the probability some fact finder will subsequently discover the firm withheld material information.

In the course of developing these predictions, we show how various circularities that arise naturally when a firm may be liable to make 10b-5 damage payments (such as: the potential damage payments depend on the firm's equilibrium "no disclosure" price at the same time the firm's equilibrium "no disclosure" price depends on these potential damage payments) can be resolved.

The paper concludes by establishing how a firm's equilibrium disclosure policy and its equilibrium investment level are linked. The analysis shows that, generally, a change in any exogenous parameter that reduces (resp., increases) a firm's propensity to withhold its private information also results in the firm increasing (resp., reducing) its equilibrium investment level.

8 References

Alexander, J., "Rethinking Damages in Securities Class Actions," *Stanford Law Review* 48, No. 6 (July 1996), pp. 1487-1537.

Booth, Richard, "Direct and Derivative Claims in Securities Fraud Litigation," *Virginia Law and Business Review* 4, 2 Fall 2009, pp. 277-294.

Dye, Ronald A., "Disclosure of Nonproprietary Information," *Journal of Accounting Research Vol. 23, No. 1,* Spring 1985, pp. 123-147.

Einhorn, E. and A. Ziv, "Biased Voluntary Disclosures," *Review of Accounting Studies* Volume 17 June 2012 No. 2 pp. 420-442.

Grossman, Sanford, "The Informational Role of Warranties and Private Disclosure About Product Quality," *Journal of Law and Economics,* 1981, 461–483.

Heitzman, Shane, Charles Wasley and Jerold Zimmerman, "The joint effects of materiality thresholds and voluntary disclosure incentives on firms' disclosure decisions," *Journal of Accounting and Economics* 49 (2010), pp. 109–132.

Jung, Woon-Oh and Young K. Kwon, "Disclosure When the Market is Unsure of Information Endowment of Managers," *Journal of Accounting Research* Vol 26, Spring 1988.

Kanodia, Chandra, "Effects of Shareholder Information on Corporate Decisions and Capital Market Equilibrium," *Econometrica* 48 No. 4 (May 1980), pp.923-953..

Kanodia, Chandra and D. Lee, "Investment and Disclosure: The Disciplinary Role of Periodic Performance Reports," *Journal of Accounting Research* 36 No. 1 (Spring 1998), pp.33-55.

Kanodia, Chandra and A. Mukherji,"Real Effects of Separating Investment and Operating Cash Flows," *Review of Accounting Studies* 1 No. 1 (March 1996), pp.51-71.

King, Ronald R. and David E. Wallin, "Voluntary Disclosures When Seller's Level of Information Is Unknown," *Journal of Accounting Research* Vol. 29, No. 1, Spring, 1991, pp. 96-108.

Messier, William F., Nonna Martinov-Bennie, Aasmund Eilifsen, "A Review and Integration of Empirical Research on Materiality: Two Decades Later," *Auditing: A Journal of Practice & Theory* (November 2005)

Milgrom, Paul, "Good News and Bad News: Representation Theorems and Applications," *Bell Journal of Economics,*Vol. 12, No. 2. (Autumn, 1981), pp. 380-391.

Oesterle, Dale, "The Overused and Under-defined Notion of "Material" in Securities Law," *University of Pennsylvania Journal of Business Law* Volume 14:1 (Fall 2011), pp. 167-187,

Pae, S. "Selective Disclosures in the Presence of Uncertainty about Information Endowment." *Journal of Accounting and Economics*, Vol. 39, No. 3 (September 2005): 383-409.

Pae, Suil, "Acquisition and Discretionary Disclosure of Private Information and Its Implications for Firms' Productive Activities," *Journal of Accounting Research* Vol. 37 No. 2 (Autumn, 1999), pp. 465-474.

Pae, Suil, "Corrigendum to: Acquisition and Discretionary Disclosure of Private Information and Its Implications for Firms' Productive Activities," *Journal of Accounting Research,* Vol. 42, No. 1 (Mar., 2004), p. 121.

Patillo, James, *The Concept of Materiality in Financial Reporting,* New York Financial Executives Research Foundation 1976.

Posner, R., *Economic Analysis of Law* 7th ed. Wolters Kluwer, New York, 2007.

Ryan, Ellen and Laura Simmons, "Securities Class Action Settlements 2009," *Cornerstone Research.*

Shavell, Steven, "Acquisition and Disclosure of Information Prior to Sale," *RAND Journal of Economics*, Volume 25, Number 1, Spring 1994, pages 20-36.

Spindler, J. C., "Vicarious Liability for Bad Corporate Goverance: Are We Wrong About 10b-5?" University of Southern California Working Paper, October 2008.

Sunder, Shyam, *Theory of Accounting and Control*, SouthWestern College Publishing, New York, 1997

Verrecchia, R. "Information Quality and Discretionary Disclosure." *Journal of Accounting and Economics* 12 (March 1990): 365-80.

Ward, Bart H., "An Investigation of the Materiality Construct in Auditing," *Journal of Accounting Research* Vol. 14, No. 1 (Spring, 1976), pp. 138-152.

Zabel, Richard and J. Benjamin Jr., "Reviewing Materiality in Accounting Fraud," *New York Law Journal* Jan. 15, 2002.

9 Appendix A Deriving the expression for $EVF(v^c, f_r)$

Fix f_r and set $\tilde{f} \equiv (f_r + \tilde{f}_l)(1+t)$. Then, the expected value of the firm given the firm made no disclosure, $EVF(v^c, f_r)$, is the sum of the expressions in the following bullet points:

- the conditional probability the firm is subject to damages times the firm's expected value given that it is subject to damages. That is:[41]

$$\frac{pG(v^c - \Delta_m)q}{1 - p + pG(v^c)} \times E[\tilde{v} + d \times (1 - \tilde{f}) \times (P^{ND} - \tilde{v}) - k | \tilde{v} < v^c - \Delta_m]. \tag{13}$$

- the probability the fact finder discovers that the firm withheld *immaterial* information times the firm's expected value given that the firm withheld immaterial information, i.e.,

$$\frac{p(G(v^c) - G(v^c - \Delta_m))q}{1 - p + pG(v^c))} \times E[\tilde{v} | v^c - \Delta_m \leq \tilde{v} < v^c].$$

- the probability the fact finder does not subsequently discover that the firm withheld any information,

$$1 - \frac{pqG(v^c)}{1 - p + pG(v^c)},$$

  times the conditional expected value of the firm given that the firm made no disclosure and the fact finder does not discover that the firm withheld any information:

$$\frac{G(v^c)p(1-q)}{1 - p + pG(v^c)(1-q)} \times E[\tilde{v} | \tilde{v} < v^c] + \left(1 - \frac{G(v^c)p(1-q)}{1 - p + pG(v^c)(1-q)}\right) \times E[\tilde{v}].$$

10 Appendix B Proof of Lemma 1

Fix f_r and set $\tilde{f} \equiv (f_r + \tilde{f}_l)(1+t)$ and $\bar{f} \equiv (f_r + \bar{f}_l)(1+t)$. Then, using the expression for $EFV(v^c, f_r)$ appearing in (3), multiply both sides of the equation $v^c = EFV(v^c, f_r)$ by $1 - p + pG(v^c)$ to get:

$$\begin{aligned}
& v^c(1 - p + pG(v^c)) \qquad (14) \\
= \; & pqG(v^c - \Delta_m) \times E[\tilde{v} - \tilde{k} + (1 - \tilde{f}) \times d \times (P^{ND} - \tilde{v}) - \tilde{k} | \tilde{v} < v^c - \Delta_m] + \\
& pq(G(v^c) - G(v^c - \Delta_m)) \times E[\tilde{v} | v^c - \Delta_m < \tilde{v} < v^c] + \\
& (1 - p + pG(v^c) - pqG(v^c)) \times \\
& \{\frac{G(v^c)p(1-q)E[\tilde{v} | \tilde{v} < v^c]}{1 - p + pG(v^c)(1-q)} + (1 - \frac{G(v^c)p(1-q)}{1 - p + pG(v^c)(1-q)}) \times E[\tilde{v}]\}.
\end{aligned}$$

[41] The denominator of the first term in (13) is the probability the firm makes no disclosure. It is the sum of: the probability the firm received no information $1-p$ and the probabily the firm received information and decided not to disclose the information $pG(v^c)$. The numerator of the first term in (13) is the probability the fact finder discovers the firm withheld material information, and so the quotient of the first term in (13) is the probability, given that the firm makes no disclosure, that the fact finder will subsequently discover that the firm withheld material information and hence is subject to damages. (Similar explanations for the other probability ratios appearing in the bullet points below are omitted.)

Rearranging terms and recognizing that, in equilibrium, $P^{ND} = v^c$, we get for the RHS(14):

$$pqG(v^c - \Delta_m) \times \{(1 - (1 - \bar{f})d)E[\tilde{v}|\tilde{v} < v^c - \Delta_m] + (1 - \bar{f})dv^c - k\} +$$

$$pq(G(v^c) - G(v^c - \Delta_m)) \times E[\tilde{v}|v^c - \Delta_m < \tilde{v} < v^c] +$$

$$(1 - p + p(1 - q)G(v^c) \times$$
$$\{\frac{G(v^c)p(1-q)E[\tilde{v}|\tilde{v} < v^c]}{1 - p + pG(v^c)(1-q)} + (1 - \frac{G(v^c)p(1-q)}{1 - p + pG(v^c)(1-q)}) \times E[\tilde{v}]\}.$$

This last expression can be rewritten as:

$$pqG(v^c - \Delta_m) \times \{(1 - (1 - \bar{f})d)E[\tilde{v}|\tilde{v} < v^c - \Delta_m] + (1 - \bar{f})dv^c - k\} +$$

$$pq(G(v^c) - G(v^c - \Delta_m)) \times E[\tilde{v}|v^c - \Delta_m < \tilde{v} < v^c] +$$

$$\{G(v^c)p(1-q)E[\tilde{v}|\tilde{v} < v^c] + (1 - p + p(1 - q)G(v^c) - G(v^c)p(1-q)) \times E[\tilde{v}]\}$$

or as

$$pqG(v^c - \Delta_m) \times \{(1 - (1 - \bar{f})d)E[\tilde{v}|\tilde{v} < v^c - \Delta_m] + (1 - \bar{f})dv^c - k\} +$$

$$pq(G(v^c) - G(v^c - \Delta_m)) \times E[\tilde{v}|v^c - \Delta_m < \tilde{v} < v^c] +$$

$$G(v^c)p(1-q)E[\tilde{v}|\tilde{v} < v^c] + (1 - p) \times E[\tilde{v}]$$

or as

$$pq \times \{(1 - (1 - \bar{f})d) \int_{\underline{v}}^{v^c - \Delta_m} vg(v)dv + ((1 - \bar{f})dv^c - k)G(v^c - \Delta_m)\} +$$

$$pq \times \int_{v^c - \Delta_m}^{v^c} vg(v)dv +$$

$$p(1-q) \int_{\underline{v}}^{v^c} vg(v)dv + (1 - p) \times E[\tilde{v}]$$

or as (using integration by parts to write the integral $\int_{\underline{v}}^{v^c - \Delta_m} vg(v)dv$ as $(v^c - \Delta_m)G(v^c - \Delta_m) - \int_{\underline{v}}^{v^c - \Delta_m} G(v)dv$, and performing similar integration by parts for the integrals $\int_{v^c - \Delta_m}^{v^c} vg(v)dv$ and $\int_{\underline{v}}^{v^c} vg(v)dv$)

$$pq \times \{(1 - (1 - \bar{f})d)((v^c - \Delta_m)G(v^c - \Delta_m) - \int_{\underline{v}}^{v^c - \Delta_m} G(v)dv)$$
$$+((1 - \bar{f})dv^c - k)G(v^c - \Delta_m)\} +$$

$$pq \times \{v^c G(v^c) - (v^c - \Delta_m)G(v^c - \Delta_m) - \int_{v^c - \Delta_m}^{v^c} G(v)dv\} +$$

$$p(1-q) \times \{v^c G(v^c) - \int_{\underline{v}}^{v^c} G(v)dv\} + (1 - p) \times E[\tilde{v}]$$

or as

$$pq \times \{(v^c - \Delta_m + (1 - \bar{f})d\Delta_m - k)G(v^c - \Delta_m) - (1 - (1 - \bar{f})d) \int_{\underline{v}}^{v^c - \Delta_m} G(v)dv\} +$$

$$pq \times \{v^c G(v^c) - (v^c - \Delta_m)G(v^c - \Delta_m) - \int_{v^c - \Delta_m}^{v^c} G(v)dv\} +$$

$$p(1-q)\times\{v^cG(v^c)-\int_{\underline{v}}^{v^c}G(v)dv\}+(1-p)\times E[\tilde{v}]$$

or as

$$pq\times\{((1-\bar{f})d\Delta_m-k)G(v^c-\Delta_m)-(1-(1-\bar{f})d)\int_{\underline{v}}^{v^c-\Delta_m}G(v)dv\}+$$

$$-pq\int_{v^c-\Delta_m}^{v^c}G(v)dv+$$

$$pv^cG(v^c)-p(1-q)\int_{\underline{v}}^{v^c}G(v)dv+(1-p)\times E[\tilde{v}]$$

or as

$$pq((1-\bar{f})d\Delta_m-k)G(v^c-\Delta_m)+pq(1-\bar{f})d\int_{\underline{v}}^{v^c-\Delta_m}G(v)dv+$$

$$-p\int_{\underline{v}}^{v^c}G(v)dv+pv^cG(v^c)+(1-p)\times E[\tilde{v}].$$

Therefore, after deleting $pv^cG(v^c)$ from both sides of the equation obtained by equating this last expression with LHS(14), and then dividing the resulting equation by $1-p$, we conclude that equation (14) is equivalent to the equation (5) in the text.■

11 Appendix C Proof of Theorem 1 and Corollary 1

When the equilibrium value for the cutoff v^c is considered to be a function of the model's parameters, i.e., $v^c=v^c(\Delta_m,\bar{f},k,t,d,q,p)$, then equation (5) is an identity in those parameters. The identity is preserved upon total differentiation with respect to any of these parameters. An easy way to differentiate (5) totally with respect to any parameter $\gamma\in\{\Delta_m,\ \bar{f},\ k,\ d,\ q,\ p,\ t\}$ is to first differentiate it with respect to v^c, i.e., to calculate:

$$\frac{\partial LHS(5)}{\partial v^c}\times\frac{dv^c}{d\gamma}+\frac{\partial LHS(5)}{\partial\gamma}=\frac{\partial RHS(5)}{\partial v^c}\times\frac{dv^c}{d\gamma}+\frac{\partial RHS(5)}{\partial\gamma}.$$

Since $\frac{\partial LHS(5)}{\partial v^c}=-1$, $\frac{\partial LHS(5)}{\partial\gamma}=0$, and

$$\frac{\partial RHS(5)}{\partial v^c}\equiv D(v^c)\equiv\frac{p}{1-p}\left[G(v^c)-q\left\{((1-\bar{f})d\Delta_m-k)g(v^c-\Delta_m)+(1-\bar{f})dG(v^c-\Delta_m)\right\}\right],$$

it follows that

$$\frac{dv^c}{d\gamma}=-\frac{\frac{\partial RHS(5)}{\partial\gamma}}{1+D(v^c)}.\tag{15}$$

To proceed, we first must sign the denominator $1+D(v^c)$ of $\frac{dv^c}{d\gamma}$ in (15).

Lemma 2 *If either (a) $\bar{f}\geq 1$ or (b) $\bar{f}<1$ and $\Delta_m<\frac{1-p}{pb}$ then $1+D(v^c)>0$.*

Proof. First suppose $\bar{f}\geq 1$. In this case, the lemma is obvious, since $D(v^c)$ is weakly bigger than

$$\frac{p}{1-p}G(v^c)-q(1-\bar{f})d(\Delta_mg(v^c-\Delta_m)+G(v^c-\Delta_m)),$$

and the latter is obviously positive if $\bar{f}\geq 1$. This proves case (a).

Now, suppose $\bar{f} < 1$ and $\Delta_m < \frac{1-p}{pb}$. Then, since $q(1-\bar{f})d < 1$ always holds, we conclude:

$$\begin{aligned} & G(v^c) - q\left\{((1-\bar{f})d\Delta_m - k)g(v^c - \Delta_m) + (1-\bar{f})dG(v^c - \Delta_m)\right\} \\ \geq \ & G(v^c) - q(1-\bar{f})d\{\Delta_m g(v^c - \Delta_m) + G(v^c - \Delta_m)\} \\ \geq \ & (G(v^c) - G(v^c - \Delta_m) - \Delta_m g(v^c - \Delta_m)) \\ = \ & \Delta_m(g(\xi) - g(v^c - \Delta_m)) \\ \geq \ & -\Delta_m b, \end{aligned}$$

(where the equality follows for some $\xi \in [v^c - \Delta_m, v^c]$ (by the mean value theorem) and the last inequality follows from $g(\cdot)$ being bounded above by b. It follows that

$$G(v^c) - q\left\{((1-\bar{f})d\Delta_m - k)g(v^c - \Delta_m) + (1-\bar{f})dG(v^c - \Delta_m)\right\} \geq -\Delta_m b.$$

Therefore, $1 + D(v^c) > 0$ surely holds when $1 - \frac{p\Delta_m b}{1-p} > 0$. This proves case (b). ■

When $1 + D(v^c) > 0$, we conclude by (15) that for any $\gamma \in \{\Delta_m, \bar{f}, d, q, p, k, t\}$:

$$sgn\frac{dv^c}{d\gamma} = -sgn\left(\frac{\partial RHS(5)}{\partial \gamma}\right). \tag{16}$$

This last observation is the basis for the proofs of most parts of Theorem 1.

Proof of Theorem 1(a)

Applying (16) when $\gamma = p$, we conclude

$$sgn\frac{dv^c}{dp} = -sgn\left(\int_{\underline{v}}^{v^c} G(v)dv - q \times \left[((1-\bar{f})d\Delta_m - k)G(v^c - \Delta_m) + (1-\bar{f})d\int_{\underline{v}}^{v^c - \Delta_m} G(v)dv\right]\right)$$

It follows that $sgn\frac{dv^c}{dp} < 0$, since

$$\begin{aligned} & \int_{\underline{v}}^{v^c} G(v)dv - q \times \left[((1-\bar{f})d\Delta_m - k)G(v^c - \Delta_m) + (1-\bar{f})d\int_{\underline{v}}^{v^c - \Delta_m} G(v)dv\right] \\ \geq \ & \int_{\underline{v}}^{v^c} G(v)dv - q \times \left[(1-\bar{f})d\Delta_m G(v^c - \Delta_m) + (1-\bar{f})d\int_{\underline{v}}^{v^c - \Delta_m} G(v)dv\right] \\ > \ & \int_{\underline{v}}^{v^c} G(v)dv - \left[\Delta_m G(v^c - \Delta_m) + \int_{\underline{v}}^{v^c - \Delta_m} G(v)dv\right] \\ \geq \ & \int_{v^c - \Delta_m}^{v^c} G(v)dv - \Delta_m G(v^c - \Delta_m) \\ = \ & \Delta_m\left[G(\xi) - G(v^c - \Delta_m)\right] \qquad (17) \\ > \ & 0. \end{aligned}$$

(The second to last line follows for some $\xi \in (v^c - \Delta_m, v^c)$ by the intermediate value theorem,[42] and the last inequality follows from $G(\cdot)$, as a cdf of a random variable with support $[\underline{v}, \bar{v}]$, is strictly increasing on $[\underline{v}, \bar{v}]$.)

Proof of Theorem 1(b)

Applying (16) when $\gamma = \bar{f}$, we conclude:

[42] It must be that $\xi \in (v^c - \Delta_m, v^c)$ and not $\xi = v^c - \Delta_m$ since $G(\cdot)$ is strictly increasing.

$$sgn\frac{dv^c}{d\bar{f}} = -sgn\left(d\Delta_m G(v^c - \Delta_m) + d\int_{\underline{v}}^{v^c - \Delta_m} G(v)dv\right) < 0.$$

Recalling the relationship between t and $\bar{f}$: $\bar{f} \equiv (\bar{f}_l + f_r)(1+t)$, from this last inequality we also immediately deduce $sgn\frac{dv^c}{dt} < 0$.

Similarly, applying (16) when $\gamma = k$, we conclude:

$$sgn\frac{dv^c}{dk} = -sgn\left(\frac{pq}{1-p} \times G(v^c - \Delta_m)\right) < 0.$$

Proof of Theorem 1(c)

Applying (16) to $\gamma = \Delta_m$, we conclude:

$$\begin{aligned} sgn\frac{dv^c}{d\Delta_m} &= -sgn\left(\frac{\partial RHS(5)}{\partial \Delta_m}\right) \\ &\quad -sgn\frac{pq \times ((1-\bar{f})d\Delta_m - k) \times g(v^c - \Delta_m)}{1-p} \qquad (18) \\ &= -sgn((1-\bar{f})d\Delta_m - k). \qquad (19) \end{aligned}$$

Thus, $sgn\frac{dv^c}{d\Delta_m}$ is negative or positive, depending on whether $(1-\bar{f})d\Delta_m - k > 0$ or $(1-\bar{f})d\Delta_m - k < 0$.

Proof of Theorem 1(d)

Applying (16) when $\gamma = d$, we conclude:

$$\begin{aligned} sgn\frac{dv^c}{dd} &= sgn\left((1-\bar{f})\left[\Delta_m G(v^c - \Delta_m) + \int_{\underline{v}}^{v^c - \Delta_m} G(v)dv\right]\right) \\ &= sgn(1-\bar{f}). \end{aligned}$$

Proof of Theorem 1(e)

Applying (16) when $\gamma = q$, we conclude:

$$sgn\frac{dv^c}{dq} = sgn\left[((1-\bar{f})d\Delta_m - k)G(v^c - \Delta_m) + (1-\bar{f})d\int_{\underline{v}}^{v^c - \Delta_m} G(v)dv\right],$$

and the latter is positive when $(1-\bar{f})d\Delta_m > k$ and is negative when $1 - \bar{f} < 0$.

Proof of Corollary 1

First, rewrite (5) with $G(\cdot)$ parameterized by I, i.e.:

$$\begin{aligned} & E_I[\tilde{v}] - v^c \\ = \; & \frac{p}{1-p}\int_{\underline{v}}^{v^c} G(v|I)dv - \frac{pq}{1-p} \times \left[((1-\bar{f})d\Delta_m - k)G(v^c - \Delta_m|I) + (1-\bar{f})d\int_{\underline{v}}^{v^c - \Delta_m} G(v|I)dv\right]. \end{aligned} \qquad (20)$$

Then, totally differentiate (20) with respect to I to get:

$$\begin{aligned} \frac{\partial}{\partial I}E_I[\tilde{v}] - \frac{dv^c}{dI} &= D(v^c)\frac{dv^c}{dI} + \frac{p}{1-p}\int_{\underline{v}}^{v^c} G_I(v|I)dv \\ &\quad -\frac{pq}{1-p} \times \left[((1-\bar{f})d\Delta_m - k)G_I(v^c - \Delta_m|I) + (1-\bar{f})d\int_{\underline{v}}^{v^c - \Delta_m} G_I(v|I)dv\right], \end{aligned}$$

or equivalently,

$$\begin{aligned}
&\frac{\partial}{\partial I}E_I[\tilde{v}] - \frac{p}{1-p}\int_{\underline{v}}^{v^c} G_I(v|I)dv + \frac{pq}{1-p} \times \left[((1-\bar{f})d\Delta_m - k)G_I(v^c-\Delta_m|I) + (1-\bar{f})d\int_{\underline{v}}^{v^c-\Delta_m} G_I(v|I)dv\right] \\
= \quad & (1+D(v^c))\frac{dv^c}{dI}.
\end{aligned}$$

Since we have previously shown that $1 + D(v^c)$ is positive, it follows that $\frac{dv^c}{dI}$ is positive provided

$$\frac{\partial}{\partial I}E_I[\tilde{v}] - \frac{p}{1-p}\int_{\underline{v}}^{v^c} G_I(v|I)dv + \frac{pq}{1-p} \times \left[((1-\bar{f})d\Delta_m - k)G_I(v^c-\Delta_m|I) + (1-\bar{f})d\int_{\underline{v}}^{v^c-\Delta_m} G_I(v|I)dv\right] > 0. \tag{21}$$

Since $k \geq 0$ and $q(1-\bar{f})d < 1$, LHS(21) is weakly bigger than

$$\begin{aligned}
&\frac{\partial}{\partial I}E_I[\tilde{v}] - \frac{p}{1-p}\int_{\underline{v}}^{v^c} G_I(v|I)dv + \frac{pq(1-\bar{f})d}{1-p} \times \left[(G_I(v^c-\Delta_m|I) + \int_{\underline{v}}^{v^c-\Delta_m} G_I(v|I)dv\right] \\
\geq \quad & \frac{\partial}{\partial I}E_I[\tilde{v}] - \frac{p}{1-p}\left(\int_{\underline{v}}^{v^c} G_I(v|I)dv - (\Delta_m G_I(v^c-\Delta_m|I) - \int_{\underline{v}}^{v^c-\Delta_m} G_I(v|I)dv\right) \\
= \quad & \frac{\partial}{\partial I}E_I[\tilde{v}] - \frac{p}{1-p}\left(\int_{v^c-\Delta_m}^{v^c} G_I(v|I)dv - \Delta_m G_I(v^c-\Delta_m|I)\right) \\
= \quad & \frac{\partial}{\partial I}E_I[\tilde{v}] - \frac{p\Delta_m}{1-p}(G_I(\xi|I) - G_I(v^c-\Delta_m|I)), \qquad (22)
\end{aligned}$$

where in this last expression, ξ is some constant $\xi \in [v^c - \Delta_m, v^c]$ (which exist by the intermediate value theorem). By assumption, there exist constants $w > 0$ and $B > 0$ such that for all $I > 0$, $\frac{\partial}{\partial I}E_I[\tilde{v}] \geq w$ and for all $I > 0$ and for all $v \in [\underline{v}, \bar{v}]$, $|G_I(v|I)| \leq B$. Therefore, (22) weakly exceeds

$$w + \frac{p\Delta_m}{1-p}G_I(v^c - \Delta_m|I) \geq w - \frac{p\Delta_m}{1-p}B.$$

Consequently, if $\Delta_m < \frac{w(1-p)}{Bp}$, it follows that $\frac{dv^c}{dI} > 0$.■

12 Appendix D Proof of Theorem 2

We illustrate the proof by considering Theorem 2i and the effects of changing the MI parameter Δ_m, holding all other parameters fixed. (The proof of Theorem 2ii, and the proof with other parameters, follow similarly.) To emphasize the dependence of the equilibrium investment and cutoff on Δ_m, we write:

$$I = I(\Delta_m) \text{ and } v^c = v^c(I(\Delta_m), \Delta_m). \tag{23}$$

Also, let the dependence of the first-order condition for I (in (11)), now denoted by N, also be made explicit:

$$N \equiv pG_I(v^c(I(\Delta_m), \Delta_m)|I(\Delta_m))v^c(I(\Delta_m), \Delta_m) + p\int_{v^c(I(\Delta_m),\Delta_m)}^{\bar{v}} vg_I(v|I(\Delta_m))dv - 1 = 0. \tag{24}$$

Differentiate N totally with respect to Δ_m to obtain:

$$\begin{aligned}
\frac{dN}{d\Delta_m} \quad &= \quad \frac{\partial N}{\partial v^c} \times \{\frac{\partial v^c(I(\Delta_m), \Delta_m)}{\partial I}\frac{dI(\Delta_m)}{d\Delta_m} + \frac{\partial v^c(I(\Delta_m), \Delta_m)}{\partial \Delta_m}\} + \frac{\partial N}{\partial I}\frac{dI(\Delta_m)}{d\Delta_m} + \frac{\partial N}{\partial \Delta_m} \\
&= \quad \left(\frac{\partial N}{\partial v^c} \times \frac{\partial v^c(I(\Delta_m), \Delta_m)}{\partial I} + \frac{\partial N}{\partial I}\right) \times \frac{dI(\Delta_m)}{d\Delta_m} + \frac{\partial N}{\partial v^c} \times \frac{\partial v^c(I(\Delta_m), \Delta_m)}{\partial \Delta_m} + \frac{\partial N}{\partial \Delta_m} = 0
\end{aligned} \tag{25}$$

Observe:

$$\begin{aligned}
\frac{\partial N}{\partial v_c} &= pG_I(v^c|I) < 0 \text{ (by first order stochastic dominance)} \qquad (26) \\
\frac{\partial v^c}{\partial I} &> 0 \text{ (by Corollary 1)} \\
Q &\equiv \frac{\partial N}{\partial I} = pG_{II}(v^c|I)v^c + p\int_{v^c}^{\bar{v}} v g_{II}(v|I)dv < 0 \\
&\quad \text{(as the second order condition corresponding to the first-order condition (24))} \\
\frac{\partial N}{\partial \Delta_m} &= 0 \text{ (as } N \text{ is not directly a function of } \Delta_m\text{)}.
\end{aligned}$$

Set $H \equiv \frac{\partial N}{\partial v_c} \times \frac{\partial v^c}{\partial I}$. The above establishes $H < 0$. Combined with $Q < 0$, we conclude $H + Q < 0$. Since (25) can be written as (leaving out arguments to improve readability):

$$(H + Q) \times \frac{dI(\Delta_m)}{d\Delta_m} + pG_I(v^c|I) \times \frac{\partial v^c}{\partial \Delta_m} = 0, \qquad (27)$$

it follows that $sgn\frac{dI(\Delta_m)}{d\Delta_m} = -sgn\frac{\partial v^c}{\partial \Delta_m}$. In similar fashion $sgn\frac{dI(\gamma)}{d\gamma} = -sgn\frac{v^c(\gamma)}{d\gamma}$ for all parameters $\gamma \in \{\bar{f}, k, q, p, t, k\}$ follows.■