The Role of Asymmetric Disclosure When Price Efficiency Affects Real Efficiency*

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Keywords: Timely loss recognition; Price informativeness; Feedback effect

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Abstract

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1 Introduction

Disclosure policies play an important role in shaping the information environment in financial markets. Not surprisingly, disclosure regulations have been an integral part in governments' efforts to regulate financial markets (Easterbrook and Fischel (1984), Greenstone, Oyer and Vissing-Jorgensen (2006)). Prior literature has evaluated disclosure policies by whether they help investors monitor self-interested managers and how they affect stock market liquidity.[1] Recent research demonstrates that traders' private information in the financial sector can affect the efficiency of the real sector. One such channel is via the feedback effect, under which traders' private information is revealed by the stock prices via the trading process and is utilized by firm managers in their investment decisions.[2] Since disclosure policies affect traders' incentives to produce and utilize private information, Goldstein and Yang (2017a) suggest that an alternative, efficiency-based benchmark is to evaluate disclosure policies by their impact on price informativeness.[3] We extend this line of inquiry by examining how asymmetry in disclosing good vs. bad news by firms affects informed traders' incentives to *trade* on their private information.

A key characteristic of accrual-based accounting reports, a main source of public disclosure by firms, is that bad news is more likely to be recognized and disclosed than good news, also known as the conservative principle in accounting (Basu (1997), Ball and Shivakumar (2005, 2006)). This feature manifests itself in many accounting standards such as lower-of-cost-or-market accounting for inventories, the impairment for long-lived assets and other-than-temporary impairments for held-to-maturity financial instruments. Different views exist on whether and why such asymme-

[1] See Diamond (1985) and Admati and Pfleiderer (2000) for examples of early studies. Goldstein and Yang (2017a) provide a recent review.

[2] The idea that prices provide useful information for guiding resource allocation decisions in the economy can be traced to Hayek (1945). Prices can contain decision-useful information because they aggregate the diverse *private* information possessed by traders which can be difficult to communicate directly to managers. See Dow and Gorton (1997) and Subrahmanyam and Titman (1999) for examples of theoretical studies of the feedback effects. See Luo (2005), Chen et al. (2006), and Foucault and Frésard (2012) for examples of empirical evidence consistent with the hypothesis that managers learn from prices. Bond et al. (2012) provide an excellent motivation and review for this literature.

[3] Indeed, Gao and Liang (2013) show that less disclosure may be desirable if it can increase investors' incentives to *acquire* information that increase investment efficiency under the feedback channel. Consistent with the crowding out effect predicted in Gao and Liang (2013), Jayaraman and Wu (2018) find that the investment to price sensitivity is lowered after mandatory segment disclosure.

try is a desirable feature in accounting standards. Some believe that forcing firms to report more negative news can overcome self-interested managers' incentives to withhold bad news and therefore help improve contracting efficiency and mitigate agency problems between firm managers and outside investors.[4] Others point out that it is not clear whether imposing asymmetry in accounting standards is the best solution to address firm-specific agency conflicts and call for alternative explanations (Lambert (2010)). We contribute to this debate by examining the role of asymmetric disclosure when managers' incentives are aligned with maximizing firm values. Specifically, we ask whether and how asymmetric disclosure can address frictions in the financial markets that affect both price efficiency and real efficiency.

Our analysis starts with the insight identified in Edmans, Goldstein, and Jiang (2015) that the presence of the feedback channel endogenously distorts informed traders' incentives to trade on their private information. Specifically, Edmans et al. (2015) show that the feedback channel increases an informed trader's incentives to trade (buy) on good news and decreases her incentives to trade (sell) on bad news.[5] This is because when the informed trader trades on her private information, the information is partially revealed through the trading process. With the feedback channel, such information allows the firm to adjust its investment scale accordingly and increases the firm's terminal value (relative to that without the feedback). However, the firm' adjustment has an asymmetric impact on the informed trader's trading profit, depending on whether the information is good or bad. When the informed trader observes that the state is good and therefore purchases the firm's stock, the purchase prompts the firm to adjust its investment upward which increases the firm's terminal value. This in turn increases the informed trader's profit from her long position. On the contrary, when the informed trader observes that the state is bad and sells on the bad news (i.e., taking a short position), the selling would prompt the firm to take a corrective action to mitigate the negative impact of the bad state. Consequently, the firm's value is higher (than without the feedback), which in turn decreases the informed trader's profit and reduces her incentives to trade

[4]See Ball (2001), Watts (2003), Bushman et al. (2004), and Kothari, Ramana and Skinner (2010) for summaries of this view. Kothari, Shu, and Wysocki (2009) document evidence consistent with the interpretation that managers withhold bad news.

[5]In this paper we use informed trader and speculator interchangeably.

on the bad news.

Edmans et al. (2015) assume that the firm does not observe any private information on its own. This assumption implies that there is no role for public disclosure because the market has strictly more information than the firm regarding the unknown state. We relax this assumption in our analysis by allowing both the firm and the speculator to privately and independently observe the true state with some probability. We model the firm's disclosure policy as a reporting system that can reveal the firm's private information with some probability. We refer to the disclosure probability as the reporting timeliness and differentiate two types of timeliness: the overall timeliness, which refers to the average probability with which the reporting system reveals the firm's private information, and the asymmetric timeliness, which refers to the differences in the probability that the system reveals bad news relative to good news. We refer to a system as a timely loss recognition (TLR) system when it features a higher probability of reporting bad news than good news. The opposite is true for a system with timely gain recognition (TGR).

Our main finding is that a TLR system reduces the asymmetry in the speculator's trading incentives and can improve the firm's investment efficiency. The intuition builds on the recognition that the speculator's trading profit, and therefore her incentives to trade, depend on the degree of information advantage she has vis-a-vis the market maker when the reporting system does not make a disclosure. Specifically, in our model, the informed speculator can have two information advantages about the underlying state: one is the advantage over the market maker and the other is the advantage over the firm. We show that holding the average timeliness constant, both advantages are higher under a TLR system than the alternative systems. This is because under a TLR system, the absence of an informative disclosure makes the market maker more optimistic about the good state and induces him to set a higher trading price accordingly. This increases the speculator's profit from selling when she observes the bad news and decreases her profit from buying when she observes the good news. Thus, she is more likely to trade on bad news with TLR than without. In addition, a non-disclosure under TLR would also enable a speculator who observes the negative information to assess higher likelihood that the firm has not been privately informed and therefore will not take the corrective investment action. This in turn increases the speculator's expected profit from selling on negative news (because she expects to pay a lower price to buy back the firm's share to cover her short position).

Thus, timely loss recognition has two effects on the firm's ex ante value: on the positive side, it incentivizes the speculator to trade on negative news. As a result, prices provide more useful information to guide the firm's decisions when the firm does not observe such information on its own. This effect increases the firm's terminal value. On the other hand, TLR also increases the speculator's information advantage over the market maker and the uninformed liquidity traders. Because the liquidity traders lose to the speculator on average, they would demand a discount when purchasing shares issued by the firm in the first place, which would lower the firm's initial selling price. Therefore, if the firm puts sufficiently large weight on the welfare of its liquidity traders, it may not choose a TLR reporting system. However, the speculator's trading gain and the liquidity traders' losses are transfers between them and do not directly add to the total surplus available from the firm's real decision. The total surplus would be higher when the price is more informative and the firm's investment is more efficient. We show that it is always (weakly) optimal to implement TLR from the social perspective, but not necessarily so from the firm's perspective.

We also explore the relation between the overall timeliness and TLR. We find them to be complements in that an increase in the overall timeliness necessitates an increase in timely loss recognition to facilitate the firm's learning from prices. Intuitively, in a regime with high overall timeliness, conditional on non-disclosure, the market marker expects the firm's value to be lower because he is more confident that the firm does not observe its own information and has to rely on the noisier information from price to make investment decision. Consequently, the market maker sets a lower price, squeezing the speculator's profit of trading on bad news and reducing her incentive to trade on negative news. To restore the incentive, a system with more TLR needs to be in place.

Our analysis offers a different theoretical justification for TLR in accounting standards. The extant literature mostly emphasizes TLR's stewardship role to overcome opportunistic managers' penchant to withhold bad news (Ball (2001), Watts (2003), Bushman et al. (2004), Kothari, Ramana and Skinner (2010)). However, Lambert (2010) points out that the argument for the stewardship value of TLR is mostly ad hoc and lacks a theoretical foundation. In addition, even if TLR had stewardship value, it is not clear why shareholders would not implement TLR on their own. In contrast, the friction in our analysis that justifies TLR arises not because managers have any nefarious motives but precisely because they want to utilize useful information in prices to maximize firm values. This in turn leads to an endogenous distortion in the speculators' incentives to trade on their information. Because it is unlikely, if possible at all, to directly contract with the market makers and speculators to mitigate the distortion, a plausible alternative is to use public disclosure policy to shape the information environments in the trading market. Further, we show that although TLR promotes investment efficiency, firms do not always have incentives to voluntarily adopt TLR. This tension therefore creates a demand for mandating TLR in disclosure regulations (such as accounting standards).

Our results also provide alternative explanations to existing empirical findings. Kothari, Shu, and Wysocki (2009) document evidence suggesting that managers withhold bad news and attribute it to self-interested managers hiding bad news. Our analysis suggests a more benevolent view in that managers withhold bad news to minimize market illiquidity created by informed trading. Prior studies have also documented positive associations between TLR in periodic financial reports and firm performance (e.g., Francis and Martin (2010), Bushman et al. (2011), Garcia Lara et al. (2016)). These results are often interpreted as evidence supporting the stewardship view of TLR. Our analysis suggests that similar correlations can be observed even when firm managers' incentives are perfectly aligned with maximizing investment efficiency. In our framework, TLR improves firms' investment efficiency via the feedback channel by motivating self-interested speculators to trade on their private information.

Our paper belongs to the large literature on the effects of disclosure policies. It is closely related to the studies on the relation between public information disclosure and private information in prices in the presence of the feedback channel (Dye and Sridhar (2002), Gao and Liang (2013), Goldstein and Yang (2017b), Arya and Mittendorf (2016)). We contribute to this literature by focusing specifically on the role of timely loss recognition on the speculators' incentives to trade on their private information. Moreover, our analysis highlights the effects of disclosure policies on the information environment in the absence of public disclosure, whereas most prior studies examine the effects of disclosed signals. This difference also distinguishes our study from the accounting literature on the effects of biased information disclosure (e.g., Gigler and Hemmer (2001), Chen et al. (2007), Gigler et al. (2009), Goex and Wagenhofer (2009), Gao (2013), Nan and Wen (2014), Armstrong et al. (2015)). In this literature, the reporting systems produce differentially informative signals for good news vs. bad news. In our model, both good and bad news disclosures are equally informative, but the informativeness of inferences by the market participants when there is no disclosure is different and depends on the disclosure timeliness, which in turn affects the market equilibrium.

2 Model

2.1 Setup

The model has four dates, $T \in \{0,1,2,3\}$, and four players: a firm, a speculator, a noise/liquidity trader, and a market maker. We assume that all players are risk-neutral with zero discount rate across dates.

Investment decision and firm value We denote the firm's terminal value as $v(\theta,d)$, which will be realized at $T=3$. The terminal value depends on both the underlying state of the world (θ) and the firm's investment decision (d). The underlying state can be either good ($\theta=H$) or bad ($\theta=L$) with equal likelihood, i.e., $\Pr(\theta=H)=\Pr(\theta=L)=\frac{1}{2}$.[6] The firm's investment is denoted by $d \in \{-1,0,1\}$, where $d=1$ stands for increasing investment, $d=-1$ for decreasing investment, and $d=0$ for maintaining the current level of investment. Changing the investment level (i.e.,

[6]As in Edmans et al. (2015), we impose symmetry for all parameters other than biases in disclosure policies to ensure that our results are not driven by asymmetry of other parameters.

$d = -1$ or 1) incurs an adjustment cost $c > 0$ for the firm, while maintaining the status quo (i.e., $d = 0$) does not.

When the firm maintains the status quo, its value $v(\theta, 0)$ is entirely determined by its asset in place, which is higher when the state is H than when the state is L, i.e., $v(\theta, 0) = R_\theta$ with $R_H > R_L$. When the firm adjusts its investment level, the firm value depends on both the asset in place R_θ and the outcome of the investment decision. When the state is H, the correct decision is to increase investment, which boosts the gross firm value by $x > 0$ and thus the net firm value becomes $v(H,1) = R_H + x - c$. Decreasing investment ($d = -1$) is the wrong decision and reduces the firm value by x: $v(H,-1) = R_H - x - c$. When the state is L, reducing investment ($d = -1$) is the right decision with $v(L,-1) = R_L + x - c$, while increasing investment is the wrong decision with $v(L,1) = R_L - x - c$. We assume $c < x$ so that only the correct adjustments are profitable.

Asymmetric timeliness in disclosure At $T = 0$, the firm privately observes a signal $\delta \in \{\theta, \phi\}$ that reveals the true state θ with probability f and is an uninformative null signal ϕ with probability $1 - f$. One can think of f as determined by the quality of the firm's internal information system, which we take as exogenously determined by factors outside of our model. Upon the realization of δ, the firm's external reporting system generates a signal $r \in \{\theta, \phi\}$ to be disclosed to the external market. To reflect the fact that the firm's internal information is often complex and soft in nature and therefore difficult to be credibly communicated to the external audience, we assume that the reported signal can only partially reveal the firm's internal information.[7] Specifically, we assume that the reporting system can only disclose a null signal ($r = \phi$) when the firm receives a null signal ($\delta = \phi$). When the firm privately observes θ, the reported signal can fully reveal θ with probability β_θ, and is a null signal uninformative about θ (i.e., $r = \phi$) with probability $1 - \beta_\theta$. We allow β_θ to depend on $\theta \in \{H, L\}$ to capture disclosure asymmetry, with β_H (β_L) denoting the probability that a good (bad) news is disclosed conditional on the firm has privately observed the good (bad) state.

[7] In the case of mandatory periodic performance reports such as the quarterly or annual reports filed by the SEC registrants in the U.S., this assumption also reflects the fact that the disclosed information needs to follow accounting rules and conventions, and therefore, may not fully convey the firm's internal information.

Clearly, reporting a null signal is equivalent to making no disclosure at all.[8] Thus, under a system with $\beta_L > \beta_H$, the market is more likely to receive a disclosure when the firm has observed the bad news than when the firm has observed the good news. This corresponds to the notion of timeliness loss recognition measured in the empirical literature (Basu (1997), Watts (2003)). For notational ease, we define $\beta = \frac{1}{2}(\beta_L + \beta_H)$ as the *overall timeliness* in disclosure and $\xi = \frac{1}{2}(\beta_L - \beta_H)$ as the degree of *asymmetric timeliness* in disclosing bad versus good news. Holding β constant, a system with a positive (negative) ξ exhibits *timely loss (gain) recognition* in that good news is less (more) likely to be disclosed than bad news. In Sections 2 and 3, we treat ξ as exogenously given and study how ξ affects the equilibria. We analyze the optimal level of ξ in Section 4. For the time being, we assume $\beta \in (0,1)$ is exogenously determined and mainly focus on the effects of *asymmetric timeliness.*[9]

Trading Subsequent to the firm's disclosure as described above, trading occurs at $T = 1$. A speculator (she) receives a private signal $\eta \in \{\theta, \phi\}$ which reveals θ with probability λ and is a null signal ϕ with probability $1-\lambda$. Thus, λ reflects the speculator's information endowment. Whether the speculator is privately informed of θ is not directly observable to other players in the model. As in Edmans et al. (2015), we assume the speculator can choose among three levels of trading, denoted by $s \in \{-1, 0, 1\}$. $s = 1$ (-1) means a buy (sell) order for 1 share of the firm's stock, while $s = 0$ stands for not trading. Trading (i.e., $s = 1$ or -1) imposes a cost $\kappa > 0$ on the speculator. The trading cost κ is commonly known and should be interpreted broadly: while direct trading costs from commissions are typically small, other indirect costs can be large, which may include borrowing costs paid to unmodeled third party financiers to finance her trades.

As in models of imperfect competition, the speculator does not trade (i.e., $s = 0$) when she is not privately informed of θ. A positive trading cost also implies that when θ is revealed by the firm's disclosure, the speculator will not trade. To rule out any equilibrium where the speculator trades against her private information (i.e., $s = 1$ when $\eta = L$ or $s = -1$ when $\eta = H$), we follow

[8] We will use no disclosure, null disclosure, and non-disclosure interchangeably.

[9] In Section 5, we relax this assumption and allow the firm to choose both β and ξ. Our results do not change qualitatively.

Edmans et al. (2015) and require that $R_H - x > R_L + x$. This implies that if the firm makes a wrong investment decision when $\theta = H$, its terminal value is still larger than that when the firm makes a correct investment decision when $\theta = L$.

In addition to the speculator, two other players participate in the trading stage of the game: a noise/liquidity trader and a market maker (he). The noise trader trades for liquidity reasons unrelated to the state of the world (θ) by submitting an order $z \in \{-1, 0, 1\}$, each with probability $\frac{1}{3}$. $z = 1$ (-1) means a buy (sell) order for 1 share of the firm's stock and $z = 0$ stands for not participating in trading. The speculator and the noise trader submit their orders simultaneously and anonymously to the market maker who absorbs orders using his own inventory. The market maker can only observe the total order flow $X = (s + z) \in \{-2, -1, 0, 1, 2\}$, but not its individual components s and z. The market maker sets the price equal to the expected firm's terminal value, conditional on the observed total order flow and the firm's disclosure (r), i.e., $P(X, r) \equiv E[v(\theta, d)|X, r]$.

When the firm does not observe θ but the speculator does, the firm can potentially use the information contained in stock prices to infer θ and choose its investment accordingly, a phenomenon commonly known as the feedback effect in the literature. Following prior literature (e.g., Dow, Goldstein, and Guembel (2016), Edmans et al. (2015)), we assume that the firm can observe the total order flow.[10] Since the firm makes its investment decision *after* trades are executed, it can observe the order flow X, update its posterior belief regarding θ, and make investment decisions accordingly. To highlight the feedback effect, we assume the likelihood that the speculator is privately informed of θ is sufficiently high such that $\lambda > \frac{2c}{x+c}$, which, as will be shown below, induces the firm to adjust its investment decisions in response to observed total order flow. In addition, we assume δ, η and z are all independently distributed.

Figure 1 summarizes the timeline of the model.

[10]Under the alternative assumption that the firm observes only P but not X, an alternative equilibrium can arise, in which the firm's investment decision is sub-optimal given the information in X. The assumption of observing X is reasonable, since in practice order flow information is provided by many security exchanges with only a short lag.

Figure 1: Timeline

T=0	T=1	T=2	T=3
• A disclosure policy $\{\beta_\theta\}$ (or equivalently $\{\beta, \xi\}$) is given. • The firm observes its private signal $\delta \in \{\theta, \phi\}$ where $\Pr(\delta = \theta) = f$ and $\Pr(\delta = \phi) = 1 - f$. • The firm discloses $r \in \{\theta, \phi\}$ according to the disclosure policy.	• The speculator privately observes $\eta \in \{\theta, \phi\}$ with $\Pr(\eta = \theta) = \lambda$ and submits her order $s \in \{-1,0,1\}$. • The noise trader submits a market order $z \in \{-1,0,1\}$ with equal probability. • The market maker observes the firm's disclosure r and the total order flow $X = s + z$ and sets price equal to the expected firm value: $P(X,r) = E(v\|X,r)$.	• The firm makes its investment decision $d \in \{-1,0,1\}$ to maximize the total expected firm value $E(v(\theta,d)\|\delta,X)$.	• The state of nature H (or L), the firm value θ, and each player's payoff are realized.

Objectives The speculator determines her trading strategy to maximize her expected gross trading profit net of any trading cost conditional on her information set. The firm makes investment decisions to maximize the firm's expected terminal value conditional on its information set. Finally, as mentioned earlier, the market maker sets the share price to the expected firm's terminal value conditional on his information set.

Equilibrium definition We employ the *Perfect Bayesian Equilibrium* as the solution concept for the model.[11]

Definition. A Perfect Bayesian Equilibrium consists of the speculator's trading strategy $s(\eta, r)$, the market maker's pricing strategy $P(X, r)$ and the firm's investment strategy $d(\delta, X)$ such that:

1. Given $P(X,r)$ and $d(\delta,X)$, the speculator's trading strategy $s(\eta,r)$ maximizes her expected trading profit net of trading cost: $s(\eta,r) \in \arg\max_s E[\{s(v-p) - |s| \times \kappa\} \mid \eta, r]$.

2. Given $s(\eta,r)$ and $d(\delta,X)$, the market maker sets price to the expected firm value $P(X,r) = E[v(\theta,d)|X,r]$.

3. Given $s(\eta,r)$ and $P(X,r)$, the firm makes the investment decision $d \in \{-1,0,1\}$ to maximize the expected firm value: $d(\delta,X) \in \arg\max_d E[v(\theta,d)|\delta,X]$.

[11] As in Edmans et al. (2015), we focus on pure strategy equilibria for expositional ease.

4. All players use the Bayes' rule to update their beliefs. Beliefs on outcomes not observed on the equilibrium path have to satisfy the Cho and Kreps' (1987) Intuitive criterion.

3 Solution

We solve the model with backward induction. We start with the firm's investment decision at $T=2$ and then derive the speculator's optimal trading strategy at $T=1$.

3.1 Firm's investment decision at $T=2$

Let μ_F^{Φ} be the firm's posterior belief that the state is H based on its information set $\Phi=\{\delta,X\}$.[12] Intuitively, the firm will invest if its posterior belief suggests that the state is more likely to be high than not. Let $\bar{\mu}$ denote the posterior belief under which the firm is indifferent between increasing investment and maintaining the status quo:

$$\bar{\mu}\,(R_H+x)+(1-\bar{\mu})\,(R_L-x)-c=\bar{\mu}R_H+(1-\bar{\mu})\,R_L \Rightarrow$$
$$\bar{\mu}=\frac{1}{2}+\frac{c}{2x}.$$

Similarly, let $\underline{\mu}$ denote the firm's posterior belief under which it is indifferent between decreasing investment and maintaining the status quo, then

$$\underline{\mu}\,(R_H-x)+(1-\underline{\mu})\,(R_L+x)-c=\underline{\mu}\,(R_H)+(1-\underline{\mu})\,R_L \Rightarrow$$
$$\underline{\mu}=\frac{1}{2}-\frac{c}{2x}.$$

Without loss of generality, we assume that when the firm is indifferent between maintaining the status quo or changing the investment level, it chooses the former ($d=0$). The following lemma summarizes the firm's optimal investment decisions.

[12] Throughout the paper, we use $\mu_i^{\Phi_i}$ to denote the player i's belief that the state is H based on the player's information set Φ_i. The subscript indicates the player identity and the subscript indicates the information set.

Lemma 1. *The firm optimally increases investment if $\mu_F^\Phi > \bar{\mu}$, decreases investment if $\mu_F^\Phi < \underline{\mu}$, and maintains the status quo if $\underline{\mu} \leq \mu_F^\Phi \leq \bar{\mu}$.*

Lemma 1 implies that the firm will change its investment level if and only if its posterior belief about state θ is sufficiently strong, that is, μ_F^Φ is either close to 1 or 0. Intuitively, since it is costly to change the status quo, the firm will do so only if its belief is significantly revised by the new information.

3.2 Equilibria of the trading game at $T = 1$

As mentioned earlier, a necessary condition for the speculator to trade is when the firm's disclosure is uninformative about the underlying state (i.e., $r = \phi$). This is because when the firm's disclosure reveals the true θ, the speculator has no information advantage over the market maker and hence expects no trading gain. As a result, she will not trade because trading incurs a positive cost κ. Furthermore, when the speculator does trade, the assumption that $R_H - x > R_L + x$ ensures that she will never buy when she observes that the state is bad (i.e., $\eta = L$) and will never sell when she learns that the state is good (i.e., $\eta = H$).

When the firm discloses a null signal (i.e., $r = \phi$), there are four possible pure strategy equilibria: (i) a no-trading (NT) equilibrium where the speculator never trades, (ii) a trading (T) equilibrium where the speculator always trades, (iii) a buy-not-sell (BNS) equilibrium where the speculator trades only when she observes that the state is good, and (iv) a sell-not-buy (SNB) equilibrium where the speculator trades only when she observes that the state is bad. As we show in the following analyses, how much of the speculator's information can be gleaned from the order flows differ across these equilibria, which in turn can affect the firm's investment efficiency when the feedback channel is present. To proceed, we first characterize and detail the conditions that can support each equilibrium. We then turn to our main focus, which is on how changes in the firm's disclosure policy can change the nature of equilibrium.

We start by defining t as the probability of state $\theta = H$ conditional on the firm disclosing a null signal ($r = \phi$) alone. Intuitively, t is the market maker's belief that the underlying state is

H after observing an uninformative disclosure by the firm but prior to observing the order flows. Straightforward algebra shows that

$$t \equiv Pr(\theta = H | r = \phi) = \frac{1}{2} + \frac{1}{2}\frac{f}{1-f\beta}\xi \tag{1}$$

$$\text{where } \frac{dt}{d\xi} = \frac{1}{2}\frac{f}{1-f\beta} > 0. \tag{2}$$

$\frac{dt}{d\xi} > 0$ implies that under the TLR disclosure policy (i.e.,$\xi > 0$), the market maker is more optimistic about the state being good ($t_{\xi>0} > \frac{1}{2}$) when there is no disclosure by the firm, i.e., no news means good news. Likewise, no news would make the market maker more pessimistic about state H under a TGR policy (i.e., $t_{\xi<0} < \frac{1}{2}$). Under both policies, the information content of no-disclosure is larger when the firm is more likely to be informed (i.e., f is large).

No-trading (NT) equilibrium In this equilibrium, the speculator does not trade and thus the order flow contains no information about the state on the equilibrium path ($X \in \{-1, 0, 1\}$). Consequently, the firm changes its investment level only based on its own private signal δ: increasing (decreasing) investment when $\delta = H$ (L) and maintaining the status quo when $\delta = \phi$.

Similarly, let $\mu_M^{r,X}$ denote the market maker's posterior belief for $\theta = H$ conditional on the firm's disclosure r and the order flow X (i.e., $\mu_M^{r,X} \equiv \Pr(\theta = H \mid r, X)$). It is easy to verify that when the firm's disclosure is uninformative, $\mu_M^{\phi,X} = t$ for all X where t is given in Eqn. (1) because there is no information from the order flows in the no-trading equilibrium. When the firm discloses the true state, $\mu_M^{H,X} = 1$ and $\mu_M^{L,X} = 0$ regardless of X.

To set price, the market maker considers not only the probability of the state but also the magnitude of the firm value in each state. The firm value depends on the firm's investment decision, which depends on the firm's internal information endowment. The market maker does not observe the firm's information endowment. Conditional on a null disclosure ($r = \phi$), his updated probability that the firm has observed the true state and therefore will make the correct investment decision, denoted by τ_θ, is given by

$$\tau_{\theta\in\{H,L\}} \equiv Pr(\delta=\theta|\theta, r=\phi) = \frac{f(1-\beta_\theta)}{1-f\beta_\theta}. \tag{3}$$

Subsequently, the market maker sets the price based on his beliefs as follows:

$$P_{NT}^{\phi} = tR'_H + (1-t)R'_L \tag{4}$$

$$\text{where } R'_\theta = R_\theta + \tau_\theta(x-c) \text{ for } \theta \in \{\text{H,L}\}. \tag{5}$$

For the no-trading equilibrium to sustain, it must be that when the firm and the market maker's beliefs/strategies are as described above, the speculator finds in her best interest not to deviate and start trading on her private information. Denote π_{sell}^{NT} (π_{buy}^{NT}) as the speculator's gross trading profit from selling (buying) when she deviates. Then the no-trading equilibrium will sustain if $\kappa > max\left\{\pi_{buy}^{NT}, \pi_{sell}^{NT}\right\}$. Proposition 1 completely characterizes the no-trading equilibrium.

Proposition 1. *No-trading equilibrium exists if and only if* $\kappa > \kappa^{NT} \equiv \max\left\{\pi_{buy}^{NT}, \pi_{sell}^{NT}\right\}$, *where* $\pi_{sell}^{NT} = \frac{2}{3}t\left(R'_H - R'_L\right)$ *and* $\pi_{buy}^{NT} = \frac{2}{3}(1-t)\left(R'_H - R'_L\right)$. *In this equilibrium, (i) the informed trader does not trade, (ii) the firm invests (divests) when it observes the state is H (L) and maintains status quo otherwise, and (iii) the market maker sets price to be* $R_\theta + x - c$ *when he observes an informative firm disclosure and sets the price to be* P_{NT}^{ϕ} *as in Eqn. (4) upon an uninformative disclosure.*

Proof. (All proofs are shown in the Appendix.) □

Proposition 1 relies on the off-equilibrium belief by the market maker and the firm that an order flow of 2 reveals that the speculator observes the state as H. As the proof in the Appendix shows, this belief is the only belief that survives the Intuitive Criterion. To understand the expression for π_{buy}^{NT} and π_{sell}^{NT}, note that when the speculator observes the true state, she expects the firm value as given by Eqn. (5), which is different from the trading price. If she were to buy when observing $\eta = H$, her expected profit (before trading cost) would be given by $\pi_{buy}^{NT} = Pr(X \neq 2|s=1)(R'_H -$

$P^{\phi}_{NT}) = \frac{2}{3}(R'_H - P^{\phi}_{NT})$. Similarly, her trading profit from selling upon observing $\eta = L$ would be $\pi^{NT}_{sell} = Pr(X \neq -2|s = -1)(P^{\phi}_{NT} - R'_L) = \frac{2}{3}(P^{\phi}_{NT} - R'_L)$.

Corollary 1. *Under the neutral disclosure policy of* $\xi = 0$, $\pi^{NT}_{sell}|_{\xi=0} = \pi^{NT}_{buy}|_{\xi=0}$.

Corollary 1 follows directly from the fact that $t = \frac{1}{2}$ when $\xi = 0$, as shown in Eqn. (1). The intuition is straightforward. Under a neutral policy, the firm discloses good news and bad news with the same probability. As a result, the market maker cannot glean information from the firm's null disclosure in setting prices. Consequently, the speculator's expected gross trading profit is the same regardless of whether she privately observes $\eta = H$ or $\eta = L$.

Trading (T) equilibrium In this equilibrium, the speculator buys on good news ($\eta = H$) and sells on bad news ($\eta = L$) when the firm discloses a null signal (i.e., $r = \phi$). Consequently, the order flow can be informative about the underlying state to the market maker and an uninformed firm. Take the order flows of $X = \pm 1$ for example. An uninformed firm (i.e., $\delta = \phi$) will update its posterior belief that the state is good as follows:[13]

$$\mu_F^{\phi,1} = \Pr(\theta = H|X = 1, \delta = \phi) = \frac{1}{2-\lambda};$$
$$\mu_F^{\phi,-1} = \Pr(\theta = H|X = -1, \delta = \phi) = \frac{1-\lambda}{2-\lambda}.$$

Since $\lambda > \frac{2c}{x+c}$ (as assumed), $\mu_F^{\phi,1} > \bar{\mu} = \frac{1}{2} + \frac{c}{2x}$ and $\mu_F^{\phi,-1} < \underline{\mu} = \frac{1}{2} - \frac{c}{2x}$where $\bar{\mu}$ and $\underline{\mu}$ are defined in Lemma 1. As a result, an uninformed firm will increase (decrease) investment when observing an order flow of $X = 1$ (-1).

Similarly, the market maker updates his belief about the state as:

$$\mu_M^{\phi,1} = \Pr(\theta = H|X = 1, r = \phi) = \frac{t}{t + (1-t)(1-\lambda)} > \frac{1}{2}; \tag{6}$$
$$\mu_M^{\phi,-1} = \Pr(\theta = H|X = -1, r = \phi) = \frac{t(1-\lambda)}{t(1-\lambda) + (1-t)} < \frac{1}{2}. \tag{7}$$

[13]The Appendix shows the detailed derivations for all conditional probabilities used throughout.

Thus, an order flow of $X = 1$ tilts the market maker's belief in favor of state H and leads to a higher price set by the market maker, whereas an order flow of -1 has the opposite effect.

Denote the speculator's expected gross trading profit from buying and selling as π^T_{buy} and π^T_{sell}, respectively. For the trading equilibrium to sustain, it must be that the speculator's trading cost is lower than the smaller of these profits: $\kappa < \min\left\{\pi^T_{buy}, \pi^T_{sell}\right\}$, i.e., when the trading cost is sufficiently low, the speculator always takes advantage of her private information. Proposition 2 characterizes the trading equilibrium.

Proposition 2. *Trading equilibrium exists if and only if* $\kappa < \kappa^T \equiv min\left\{\pi^T_{buy}, \pi^T_{sell}\right\}$, *where*

$$\pi^T_{buy} = \frac{1}{3}(1-t)\left[R'_H - R'_L\right] + \frac{1}{3}\left(1-\mu_M^{\phi,1}\right)\left[R_H - R_L + 2(1-\tau_L)x\right]; \tag{8}$$

$$\pi^T_{sell} = \frac{1}{3}t\left[R'_H - R'_L\right] + \frac{1}{3}\mu_M^{\phi,-1}\left[R_H - R_L - 2(1-\tau_H)x\right]. \tag{9}$$

In this equilibrium, (i) when the firm discloses a null signal, the speculator buys (sells) one unit when she observes the H (L) state. The speculator does not trade otherwise. (ii) The firm invests when it observes $\delta = H$ *or* $X > 0$*, divests when it observes* $\delta = L$ *or* $X < 0$*, and maintains status quo otherwise. (iii) The market maker sets the price as* $P(X,\theta) = R_\theta + x - c$ *when the firm discloses the state* ($r = \theta$)*; otherwise, he sets the price according to the order flow as below:*

$$\begin{aligned}
P(2,\phi) &= R_H + x - c \\
P(1,\phi) &= \mu_M^{\phi,1}(R_H + x - c) + \left(1-\mu_M^{\phi,1}\right)\left[\tau_L(R_L + x - c) + (1-\tau_L)(R_L - x - c)\right] \\
P(0,\phi) &= tR'_H + (1-t)R'_L \\
P(-1,\phi) &= \mu_M^{\phi,-1}\left[\tau_H(R_H + x - c) + (1-\tau_H)(R_H - x - c)\right] + \left(1-\mu_M^{\phi,-1}\right)(R_L + x - c) \\
P(-2,\phi) &= R_L + x - c
\end{aligned}$$

where $\mu_M^{\phi,1}$, $\mu_M^{\phi,-1}$ *and* $\tau_{\theta\in(H,L)}$ *are given in Eqns.* (6), (7), *and* (3), *respectively.*

The expressions for $P(-2,\phi)$,$P(0,\phi)$, and $P(2,\phi)$ are self-explanatory. To understand $P(1,\phi)$, note that it contains two components. The first component considers the possibility that the state is H which takes place with probability of $\mu_M^{\phi,1}$. In this case, the firm will correctly invest for sure, because both its private information (if it is informative) and the order flow suggests that the state is more likely to be H. This will result in a firm value $R_H + x - c$. The second component considers the possibility that the state is L (which takes place with probability $1-\mu_M^{\phi,1}$). In this case, with probability of τ_L, the firm is informed and takes the correct decision (of divesting), which generate a terminal value of $R_L + x - c$. However, with probability $1-\tau_L$, then the firm is uninformed, its decision to invest will turn out to be incorrect, which would lower the firm value to $R_L - x - c$. Similar argument applies to $P(-1,\phi)$.

Corollary 2. *Under the neutral disclosure policy of* $\xi = 0$*, (i)* $\pi^T_{sell}|_{\xi=0} < \pi^T_{buy}|_{\xi=0}$*, and (ii)* $\pi^T_{sell}|_{\xi=0} < \pi^T_{buy}|_{\xi=0} < \pi^{NT}_{buy}|_{\xi=0} = \pi^{NT}_{sell}|_{\xi=0}$.

Corollary 2(i) indicates that under a neutral disclosure policy, selling generates less profit than buying in the trading equilibrium. The intuition is from the fact that the firm learns from the observed order flow and adjusts its investment accordingly. Specifically, the firm increases investment when observing a positive order flow. This will increase the firm's terminal value when $\theta = H$, increasing the speculator's expected profit from a long position (buying). However, the opposite is true when the speculator observes $\theta = L$ and the firm decreases investment when observing a negative order flow. This is because if the speculator trades, it is rational for her to take a short position (sell) on bad news. Yet, the resulting negative order flow will prompt the firm to decrease investment, which increases the firm's expected terminal value, hence decreasing the speculator's expected profit from her short position. Consequently, under the feedback effect, the speculator has less incentive to trade on her negative news. As in Edmans et al. (2015), decision-useful negative news is less likely to be reflected in prices precisely because such information is used by firms for their investment decisions.

Corollary 2(ii) indicates that under a neutral disclosure policy, the trading profits from the trading equilibrium are strictly smaller than the profits that the speculator would obtain in the no-trading equilibrium if she deviated and started trading. The intuition follows the basic insight from the strategic trading models (Kyle (1985)), that is, the speculator's trading profit is higher when the market maker has less information than the speculator. In the trading equilibrium, the observed order flow partially reveals the speculator's private information to the market maker who in turn adjusts prices accordingly. This lowers the speculator's trading profit, resulting in both $\pi^T_{sell}\mid_{\xi=0}<\pi^{NT}_{sell}\mid_{\xi=0}$ and $\pi^T_{buy}\mid_{\xi=0}<\pi^{NT}_{buy}\mid_{\xi=0}$.

Buy-not-sell (BNS) equilibrium and Sell-not-buy (SNB) equilibrium Corollary 2(ii) imply that the intervals for the trading and no-trading equilibria ($\kappa<\kappa^T\equiv min\left\{\pi^T_{buy},\pi^T_{sell}\right\}$ and $\kappa>\kappa^{NT}\equiv\max\left\{\pi^{NT}_{buy},\pi^{NT}_{sell}\right\}$) do not span the entire range of the trading cost. When the trading cost falls outside of these intervals, different equilibria may emerge in which the speculator's strategy is one-sided. In these equilibria, the speculator either only buys on good news ($\eta=H$) and does not trade otherwise or only sells on bad news ($\eta=L$) and does not trade otherwise. These equilibria can be derived and characterized following similar logic as shown above. For space constraint, we relegate the detailed characterization of these equilibria to the Appendix and summarize the main insight in Proposition 3 below.

Proposition 3. *(i) If $\kappa\in[\pi^T_{sell},\pi^{NT}_{buy}]$, then there exists a Buy-not-sell (BNS) equilibrium where the speculator buys when the firm discloses a null signal and she observes $\eta=H$, and she does not trade otherwise. (ii) If $\kappa\in[\pi^T_{buy},\pi^{NT}_{sell}]$, then there exists a Sell-not-buy (SNB) equilibrium where the speculator sells when the firm discloses a null signal and she observes $\eta=L$, and she does not trade otherwise.*

It is easy to see that Corollary 2(ii) implies that under the neutral disclosure policy, the BNS equilibrium region strictly contains the SNB equilibrium region (i.e., $\left[\pi^T_{sell},\pi^{NT}_{buy}\right]\supset\left[\pi^T_{buy},\pi^{NT}_{sell}\right]$). This is the main insight from Edmans et al. (2015) that the feedback effect reduces the likelihood that negative private news will be traded and reflected in stock prices. However, Edmans et al. (2015) do not consider the effects of the firm's public disclosure policy, which is our focus in this paper. As we will show next, the fact that firms may not always need to learn from the stock price introduce additional uncertainty to the market maker and the speculator. This in turn enables the firm to design its disclosure policy to affect the nature of the equilibria in the market.

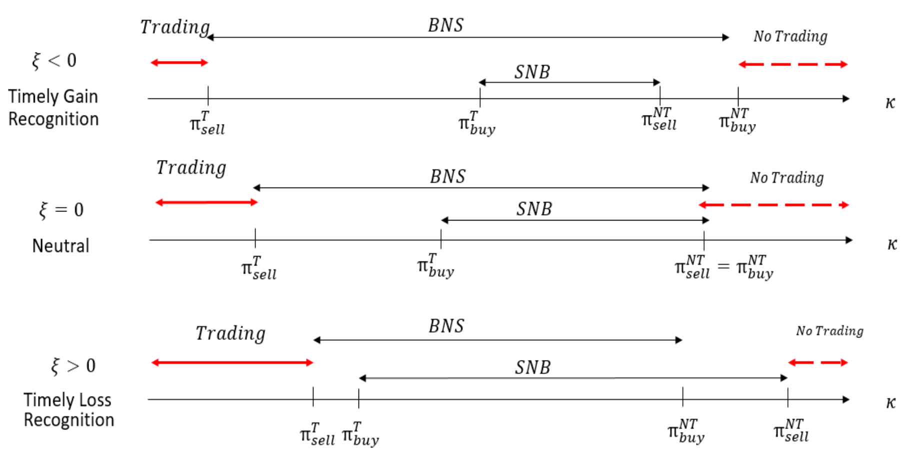
4 Effects of asymmetric timeliness in disclosure

Proposition 4 demonstrates how the trading and no-trading equilibrium interval would behave if ξ marginally moves away from 0.

Proposition 4. *(Effects of asymmetric timeliness in disclosure) (i)* $\frac{d\pi^T_{sell}}{d\xi} > 0$ *and* $\frac{d\pi^{NT}_{sell}}{d\xi} > 0$. *(ii)* $\frac{dmin\left\{\pi^T_{buy},\pi^T_{sell}\right\}}{d\xi}\ |_{\xi=0} > 0$ *in the neighborhood around* $\xi = 0$*. (iii) For any* $\hat{\xi} > 0$*,* $max\left\{\pi^{NT}_{buy},\pi^{NT}_{sell}\right\}|_{\xi=\hat{\xi}} > max\left\{\pi^{NT}_{buy},\pi^{NT}_{sell}\right\}|_{\xi=0}$*, and* $max\left\{\pi^{NT}_{buy},\pi^{NT}_{sell}\right\}|_{\xi=\hat{\xi}} > max\left\{\pi^{NT}_{buy},\pi^{NT}_{sell}\right\}|_{\xi=-\hat{\xi}}$*.*

Proposition 4 offers a key insight of our paper: timely loss recognition provides more incentive for the speculator to trade on her private information, which increases the information role of market prices to help the firm make more efficient investment decisions. Specifically, for the neutral system or any given TGR system, there exists a TLR system that generates a larger trading equilibrium region and a smaller no-trading equilibrium region. This insight is graphically represented in Figure 2, which illustrates the equilibrium intervals under a TGR, neutral, and TLR system, respectively. Specifically, consistent with Prop. 4(ii), the left side of Figure 2 shows that

Figure 2: Trading and no-trading equilibrium under different disclosure systems



in the neighborhood around $\xi = 0$, the trading equilibrium region is the largest under a TLR system, is smaller under a neutral policy (i.e., $\xi = 0$), and is the smallest under a TGR policy (with $\xi > 0$). Likewise, as shown in Proposition 4(iii), the right side of Figure 2 shows that a TLR system generates a smaller no-trading equilibrium region than both the neutral policy and a TGR system.

The intuition for these results comes from two effects of an asymmetric disclosure policy. The first is what we refer to as the State Uncertainty Effect. The State Uncertainty Effect affects the market maker's posterior belief of the state conditional on the disclosure of a null signal $r = \phi$. Under a TLR system ($\xi > 0$), bad news is disclosed more timely than good news, thus seeing $r = \phi$ induces the market maker to assess higher likelihood to $\theta = H$ than to $\theta = L$. This is reflected in Eq. (2) that $t \equiv Pr(\theta = H \mid r = \phi)$ is increasing in ξ. Ceteris paribus, this effect induces the market maker to set a higher price. This also means that a speculator who receives a private signal of $\eta = L$ can sell the share at a higher price and earn a higher profit ($\frac{d\pi_{sell}}{d\xi} > 0$).

The second effect of the disclosure asymmetry is what we refer to as the Information Endowment Uncertainty Effect. Under a TLR disclosure policy, conditional on the speculator observing $\eta = L$, she is more likely to believe that the firm is uninformed of the state (i.e., $\delta = \phi$) than when the firm discloses a null signal (i.e.,$r = \phi$). This is shown by noticing that $\tau_L \equiv Pr(\delta = L \mid L, r = \phi)$ (given in (3)) decreases in ξ as:

$$\frac{d\tau_L}{d\xi} = -\frac{f(1-f)}{(1-f\beta_L)^2} < 0.$$

When the speculator believes that the firm has not observed the true state, she is also more confident that the firm will maintain the status quo in the bad state, a decision that would lower the firm's terminal value and result in a lower price when the speculator buys back the share at $T = 3$ to cover the short position she takes at $T = 1$. This in turn increases her trading profit from selling ($\frac{d\pi_{sell}}{d\xi} > 0$).

Together, under a TLR policy, both the State Uncertainty Effect and the Information Endowment Uncertainty Effect improve the expected trading profit for a speculator observing negative news. This is not the case under a TGR policy ($\xi < 0$), under which the State Uncertainty Effect increases the speculator's trading profit from buying. Because a TGR policy is more likely to disclose good news than bad news, observing no disclosure ($r = \phi$) induces more pessimism in the market maker's belief. As a result, the market maker sets a lower price, ceteris paribus, which in turn enables a speculator who receives a private signal $\eta = H$ to buy the share at a lower price and increase her trading profit from buying ($\frac{d\pi_{buy}}{d\xi} > 0$). However, the Information Endowment Uncertainty effect reduces the speculator's trading profit from buying under a TGR policy. This is because when $\xi < 0$, observing $r = \phi$ makes the speculator (who has privately observed $\eta = H$) believe that the firm is more likely to be uninformed of the state (i.e., $\delta = \phi$) and therefore less likely to increase investment. Consequently, the speculator expects a lower trading profit from buying as the firm's terminal value is more likely to be low.

Corollary 1 shows that at $\xi = 0$, $\pi^{NT}_{buy} = \pi^{NT}_{sell}$. Both the State Uncertainty Effect and the Information Endowment Uncertainty Effect enhance π^{NT}_{sell} when ξ increases from zero, while the two effects counteract with each other when ξ decreases from zero. As such, the no-trading region shrinks under a conservative system with $\hat{\xi} > 0$ compared to either a symmetric system or a TGR system with $-\hat{\xi}$.

Corollary 2 shows that when $\xi = 0$, $\pi^{T}_{buy} > \pi^{T}_{sell}$ due to the feedback effect. This implies that the trading region is entirely determined by π^{T}_{sell} in the small neighborhood around $\xi = 0$. Consequently, because both the State Uncertainty Effect and the Information Endowment Uncertainty Effect increase π^{T}_{sell} when ξ increases from zero, the trading equilibrium region expands under the TLR and shrinks under TGR, compared to the neutral case.

5 Disclosure policy choices at $T = 0$

5.1 Optimal asymmetric timeliness

We have so far treated the disclosure policy parameter ξ as given and focused on characterizing the resulting equilibria. In this section, we examine the optimal choice of ξ at $T = 0$ from two perspectives. The first perspective is that of a social planner whose objective is assumed to be maximizing the total social welfare. In our case, this is equivalent to maximizing the firm's terminal value, because the speculator's trading gain and cost are simply wealth transfers from the noise trader to the speculator. Under this perspective, the social planner's problem can be characterized as

$$\max_{\xi} E\left\{v\left[\theta, d\left(\delta, X\right)\right]\right\} \; s.t. \; 0 \leq \beta \pm \xi \leq 1$$

A second, alternative perspective is that of a firm which cares not only about its terminal value, but also about the liquidity discount in its stock price. In this case, the firm's optimal disclosure problem can be characterized as

$$\max_{\xi} E\left\{v\left[\theta, d\left(\delta, X\right)\right]\right\} - \alpha E\left(\pi\right) \; s.t. \; 0 \leq \beta \pm \xi \leq 1$$

where π is the speculator's trading profit and $\alpha \in [0,1]$ is a commonly known parameter. An intuitive interpretation for α is that it represents the probability that the firm needs to subsequently issue shares to the liquidity trader to raise capital, and the issuing price is determined by the liquidity trader who will price protect himself against future losses to the speculator.[14]

Proposition 5. *For any level of trading cost κ, there exists a TLR system that is weakly preferred by the social planner.*

The key in proving Proposition 5 is to establish the ranking of the firm's expected terminal value under the four equilibria. Holding the firm's own information endowment constant, the firm's expected terminal value is maximized when the speculator is more likely to trade such that the firm can learn more from prices to make the correct investment decisions. Intuitively, the trading equilibrium brings both good news and bad news into prices via the speculator's trading. In contrast, the BNS/SNB equilibria bring only one-sided news into prices, and the no-trading equilibrium does not reflect any of the speculator's information in prices. Consequently, the firm's expected

[14] To elaborate, the firm's objective function can be generated by slightly revising the events at $T = 0$ as follows. At $T = 0.1$, the firm decides on ξ. At $T = 0.2$, with probability $1 - \alpha$, the firm has enough internal cash and does not need any external funding, and the game proceeds as in the rest of Figure 1. With probability α, the firm needs to raise a small amount of capital K by issuing shares to the liquidity trader at a selling price. If the liquidity trader declines, the game ends, the firm liquidates with terminal value equal to zero, and the liquidity trader gets zero. If the liquidity trader accepts the offer, the game proceeds as in the rest of Figure 1. Clearly, when the firm wishes to obtain the external funding, it will offer a price to cover the liquidity trader's expected trading loss, which generates the firm's objective function above.

value is the highest in the trading equilibrium, the second highest in the BNS/SNB equilibria,[15] and the lowest in the no-trading equilibrium. Since Proposition 4 (and Figure 2) shows that a TLR reporting system with $\hat{\xi} > 0$ expands the trading region and shrinks no trading equilibrium region, compared to a TGR system with $-\hat{\xi} < 0$, Proposition 5 follows immediately.

In contrast, Proposition 6 suggests that from the firm's perspective, the firm may prefer a TGR policy.

Proposition 6. *Fix any $\hat{\xi} > 0$. Consider a neutral policy ($\xi = 0$), a timely loss recognition policy with $\beta_H = \beta - \hat{\xi}$ and $\beta_L = \beta + \hat{\xi}$, and a timely gain recognition policy with $\beta_H = \beta + \hat{\xi}$ and $\beta_L = \beta - \hat{\xi}$. Suppose the trading equilibrium is sustained under all three policies. Then, the firm strictly prefers the timely gain recognition policy.*

When the trading cost is sufficiently low, the trading equilibrium prevails under all disclosure regimes. In this case, the speculator always trades and the firm always learns from prices, and the expected firm value is not affected by any asymmetry in timeliness, i.e. $E[v(\theta, d)]$ is the same for all three regimes. Compared to the TGR regime, a TLR regime gives the speculator a higher expected trading profit, thus lowers the firm's objective by increasing the liquidity discount due to adverse selection. The next proposition identifies a set of sufficient conditions under which the social planner and the firm's preferences coincide or diverge.

Proposition 7. *Suppose $\kappa > \kappa^T\ |_{\xi=0}$ and $\kappa - \kappa^T\ |_{\xi=0}$ is sufficiently small. (i) The social planner strictly prefers a TLR system. (ii) The firm prefers a TLR system if any of the following holds: (a) α (the weight the firm puts on liquidity discount) is sufficiently small; (b) f is sufficiently small, and x is sufficiently large relative to $R_H - R_L$; (c) f is sufficiently small and λ is sufficiently large. (iii) The firm prefers a TGR system if α is sufficiently large and any of the following holds: (a) f is sufficiently large, and x is sufficiently small relative to $R_H - R_L$; (b) f is sufficiently large and λ sufficiently small.*

[15] The proof in the Appendix shows that given that state $\theta = H$ and $\theta = L$ are equally likely ex ante, these two equilibria generate the same expected firm value.

When the trading cost κ lies just slightly above $\kappa^T_{sell}\mid_{\xi=0}$, the BNS equilibrium prevails with a symmetric disclosure policy or with a TGR policy for which ξ is not too negative, as implied by Proposition 5. When the disclosure policy turns to TLR (i.e., ξ increases from zero), there will be a discrete jump in the firm's expected terminal value because it can learn more information from prices when the trading equilibrium prevails. In contrast, as far as the firm's objective is concerned, a switch to the trading equilibrium also entails a discontinuous jump in the speculator's trading profit and thus a larger price discount. Whether the firm prefers a TLR policy depends on the trade-off between benefits of learning from prices and the cost of adverse selection discount in share offerings.

Proposition 7 shows that TLR is strictly preferred by the firm when it does not place too much weight on the liquidity discount (i.e., when α is small). The TLR policy also dominates when the firm does not have much private information and the impact of its investment decision is large relative to its asset in place. As f becomes smaller, the firm relies more on information gleaned from prices to make investment decision, which has a higher economic impact relative to the asset in place when x is large compared to $R_H - R_L$. Finally, a disclosure policy with TLR also prevails when the market is more likely to be better informed about the true state than the firm (i.e.,f is small and λ is large), as this is when the benefits for the firm to learn from prices are large. In contrast, when the firm already has a lot of private information (f is large), or when the speculator is not that well informed (λ is small), or when the firm's investment decision doesn't have much of an economic impact relative to its asset in place (x is small relative to $R_H - R_L$), the TGR policy dominates as it reduces the liquidity discount. The conditions under which the social planner's preferences diverge from that of the firm can help financial regulators evaluate when to introduce disclosure policies featuring timely loss regonition.

5.2 Endogenizing the overall and asymmetric timeliness

Up to this point, we have treated the overall timeliness (β) as exogenous and only focused on the impact of asymmetric timeliness (ξ). In this subsection, we allow both β and ξ to be free variables and look at the best combination of the two from both the social planner and the firm's perspective. Proposition 8 below shows that a TLR reporting system can still be optimal even if a perfect (and thus symmetric) system with $\beta = 1$ is feasible.

Proposition 8. *Suppose β and ξ are choice variables with $\beta \in [0,1]$ and $\beta \pm \xi \in [0,1]$. Let $\underline{\kappa} \equiv \left(\frac{1}{6} + \frac{1}{3}\left(\frac{1-\lambda}{2-\lambda}\right)\right)(R_H - R_L) - \frac{2}{3}\left(\frac{1-\lambda}{2-\lambda}\right)x$ and $\Delta \equiv \kappa - \underline{\kappa}$. When Δ is positive and sufficiently small, (i) from the social planner's perspective, any optimal (β,ξ) combination has β strictly less than 1. Furthermore, denote $\Phi_\beta \subseteq [\beta-1, 1-\beta]$ as the set of ξ such that the disclosure policy (β,ξ) maximizes the social planner's payoff, and define $\underline{\xi} \equiv \min_{t \in \Phi_\beta} t$. When β is sufficiently close to 1 and $\underline{\xi} \in (\beta-1, 1-\beta)$, $\underline{\xi}$ strictly increases with β. (ii) From the firm's perspective, if $\alpha > 0$ is sufficiently small, any optimal (β,ξ) combination has β strictly less than 1 and ξ strictly positive.*

Proposition 8 lays out the conditions under which both the social planner and the firm would prefer the optimal overall timeliness to be strictly less than 1 with TLR present. To see the intuition, note that when we fix ξ and increase β, the speculator becomes increasingly reluctant to trade on bad news ($\eta = L$) due to two forces. First, under a system with a higher β, a null disclosure suggests that the firm less likely to be informed internally and therefore is more likely to rely on the information from the order flow to make investment decision. Because the order flow is a noisier signal of the state than the firm's internal information, the firm's investment decision based on the order flow is more likely to be incorrect than a similar decision based on the firm's internal information. This would make the market maker set a lower price upon no disclosure. In contrast, when the speculator privately observes that the state is L, she will assess a higher firm value than the market maker because she is more confident (than the market maker) that the order flow will guide the firm to make the correct investment. Both forces lower the speculator's profit of selling on bad news. Thus, in order to restore the trading equilibrium, a TLR system needs to be in place. Setting $\beta = 1$ automatically imposes a symmetric system and thus leaves no room for adjusting ξ to restore the trading equilibrium.

Proposition 8 also implies that β and ξ are complements as an increase in the overall timeliness may necessitate an increase in timely loss recognition to facilitate learning from prices. To the extent that β is higher in countries with more developed stock markets, this result is consistent with empirical findings that firm disclosures in these markets also tend to feature TLR (Ball, Robin, and Wu (2001)). Interestingly, Edmans et al. (2015) express the concern that more developed markets may not necessarily be better at dealing with the endogenous friction generated by the feedback channel. Our analysis suggests that this concern can be mitigated by disclosure policies.

6 Conclusion

This paper analyzes how timely loss recognition affects firm performance via the feedback channel of financial markets. We show that by preempting more bad news, TLR changes the market dynamics when public disclosure is absent and mitigates the distortion of the feedback channel on price informativeness identified in Edmans, Goldstein, and Jiang (2015). Our results imply that in the absence of managerial incentive misalignment, the connection between stock price efficiency and real efficiency at the firm level via the feedback channel can affect the desirability of TLR. In the presence of the feedback channel, timely loss recognition can increase price informativeness and improve investment efficiency. In the absence of the feedback channel, however, timely loss recognition leads to higher information asymmetry among traders and thus a larger liquidity discount. Our analysis and results are consistent with the call by Goldstein and Yang (2017a) to evaluate disclosure policies by its effects on the efficiency of the real sector.

Our analysis highlights that disclosure policies affect the information environment of financial markets both directly and indirectly. The direct effect is by providing public information to the market. The indirect effect is by shaping the beliefs of market participants in the absence of public disclosure. Since public disclosure is a relatively infrequent event for most firms, understanding the indirect effects can help us better understand the relation between market behaviors and firm disclosure. Furthermore, to the extent that financial markets are unique in aggregating and revealing the diverse private information that is otherwise not available to the economy, it is also important to evaluate disclosure policies by their impact on traders' incentives to acquire and to trade their private information, especially when such private information can in turn affect decisions in the real sector.

Our analysis follows a recent trend in accounting research that highlights the role of accounting disclosure in broader market settings (e.g., Gao and Liang (2013), Chen, Lewis, Schipper and Zhang (2017), Plantin and Tirole (2018)). The general theme from these studies is to understand the role of accounting information in addressing a variety of frictions in financial markets other than the standard agency conflict (Jensen and Meckling (1976)). These frictions often rest on the part of the investors. For example, the investors may be short-term oriented (Gigler et al. (2014)), may not coordinate efficiently (Morris and Shin (2002), Angeletos and Pavan (2004)), or may face capacity constraint in processing information (Sims (2006)).[16] To the extent that policy makers and standard setters are concerned about a broad range of frictions that impede the stock market efficiency, future research can gain important insights by examining how to design disclosure system in those contexts.

References

[1] Admati, A. R., and Pfleiderer, P. (2000). Forcing firms to talk: Financial disclosure regulation and externalities. *The Review of Financial Studies* 13(3), 479-519.

[2] Agarwal, V., R. Vashishtha, and M. Venkatachalam (2017). Mutual fund transparency and corporate myopia. *The Review of Financial Studies* 31(5), 1966-2003.

[3] Angeletos, G. M., and A. Pavan (2004). Transparency of information and coordination in economies with investment complementarities. *American Economic Review* 94(2), 91-98.

[4] Armstrong, C. S., D.J. Taylor, and R.E. Verrecchia (2015). Asymmetric reporting. *Journal of Financial Reporting* 1(1), 15-32.

[16] These considerations have motivated empirical studies to examine the costs of more frequent disclosures (Agarwal, Vashishtha, and Venkatachalam (2017), Kraft, Vashishtha, and Venkatachalam (2018)) and of segment disclosure (Jayaraman and Wu (2018)).

[5] Arya, A., and B. Mittendorf (2016). On the synergy between disclosure and investment beauty contests. *Journal of Accounting and Economics* 61(2-3), 255-273.

[6] Ball, R. (2001). Infrastructure requirements for an economically efficient system of public financial reporting and disclosure. *Brookings-Wharton Papers on Financial Services* 2001(1), 127-169.

[7] Ball, R. and L. Shivakumar (2005). Earnings quality in UK private firms: comparative loss recognition timeliness. *Journal of Accounting and Economics* 39(1), 83–128.

[8] Ball, R. and L. Shivakumar (2006). The role of accruals in asymmetrically timely gain and loss recognition. *Journal of Accounting Research* 44(2), 207–242.

[9] Basu, S. (1997). The conservatism principle and the asymmetric timeliness of earnings. *Journal of Accounting and Economics* 24(1), 3-37.

[10] Bond, P., A. Edmans, and I. Goldstein (2012). The real effects of financial markets. *Annual Review Financial Economics* 4(1), 339-360.

[11] Bushman, R. M., J.D. Piotroski, and A.J. Smith (2004). What determines corporate transparency?. *Journal of Accounting Research* 42(2), 207-252.

[12] Bushman, R. M., and J.D. Piotroski (2006). Financial reporting incentives for conservative accounting: The influence of legal and political institutions. *Journal of Accounting and Economics* 42(1-2), 107-148.

[13] Bushman, R. M., J. D. Piotroski and A.J. Smith (2011). Capital allocation and timely accounting recognition of economic losses. *Journal of Business Finance & Accounting* 38(1-2), 1-33.

[14] Chen, Q., I. Goldstein, and W. Jiang (2006). Price informativeness and investment sensitivity to stock price. *The Review of Financial Studies* 20(3), 619–650.

[15] Chen, Q., T. Hemmer, and Zhang, Y. (2007). On the relation between conservatism in accounting standards and incentives for earnings management. *Journal of Accounting Research* 45(3), 541-565.

[16] Chen, Q., T. R. Lewis, K. Schipper, and Y. Zhang (2017). Uniform versus discretionary regimes in reporting information with unverifiable precision and a coordination role. *Journal of Accounting Research* 55(1), 153-196.

[17] Cho, I. K., and D.M. Kreps (1987). Signaling games and stable equilibria. *The Quarterly Journal of Economics* 102(2), 179-221.

[18] Diamond, D. W. (1985). Optimal release of information by firms. *The Journal of Finance* 40(4), 1071-1094.

[19] Dow, J., I. Goldstein, and A. Guembel (2017). Incentives for information production in markets where prices affect real investment. *Journal of the European Economic Association 15(4), 877-909.*

[20] Dow, J. and G. Gorton (1997). Stock market efficiency and economic efficiency: Is there a connection? *The Journal of Finance* 52(3), 1087–1129.

[21] Dye, R. A., and S. Sridhar (2002). Resource allocation effects of price reactions to disclosures. *Contemporary Accounting Research* 19(3), 385-410.

[22] Easterbrook, F. H., & Fischel, D. R. (1984). Mandatory disclosure and the protection of investors. *Virginia Law Review,* 669-715.

[23] Edmans, A., I. Goldstein, and W. Jiang (2015). Feedback effects, asymmetric trading, and the limits to arbitrage. *American Economic Review* 105(12), 3766–97.

[24] Edmans, A., S. Jayaraman, and J. Schneemeier (2017). The source of information in prices and investment-price sensitivity. *Journal of Financial Economics* 126(1), 74-96.

[25] Foucault, T. and L. Frésard (2012). Cross-listing, investment sensitivity to stock price, and the learning hypothesis. *The Review of Financial Studies* 25(11), 3305–3350.

[26] Francis, J. R. and X. Martin (2010). Acquisition profitability and timely loss recognition. *Journal of Accounting and Economics* 49(1), 161–178.

[27] Gao, P. (2013). A measurement approach to conservatism and earnings management. *Journal of Accounting and Economics* 55(2-3), 251–268.

[28] Gao, P. and P. Liang (2013). Informational feedback, adverse selection, and optimal disclosure policy. *Journal of Accounting Research* 51(5), 1133–1158.

[29] Gigler, F., C. Kanodia, H. Sapra, and R. Venugopalan (2009). Accounting conservatism and the efficiency of debt contracts. *Journal of Accounting Research* 47(3), 767-797.

[30] Goex, R. F., and A. Wagenhofer (2009). Optimal impairment rules. *Journal of Accounting and Economics* 48(1), 2-16.

[31] Goldstein, I. and L. Yang (2017a). Information disclosure in financial markets. *Annual Review of Financial Economics* (9), 101-125.

[32] Goldstein, I. and L. Yang (2017b). Good disclosure, bad disclosure. *Journal of Financial Economics forthcoming.*

[33] Greenstone M, Oyer P, Vissing-Jorgensen A. 2006. Mandated disclosure, stock returns, and the 1964 Securities Acts Amendments. *Quarterly Journal of Economics* 121:399–460.

[34] Hayek, F. A. (1945). The use of knowledge in society. *The American Economic Review* 35(4), 519–530.

[35] Jayaraman, S. and J. Wu (2018). Is silence golden? Real effects of mandatory disclosure. *Review of Financial Studies forthcoming.*

[36] Jensen, M. C., and W.H. Meckling (1976). Theory of the firm: Managerial behavior, agency costs and ownership structure. *Journal of Financial Economics* 3(4), 305-360.

[37] Kothari, S. P., K. Ramanna and D.J. Skinner (2010). Implications for GAAP from an analysis of positive research in accounting. *Journal of Accounting and Economics* 50(2-3), 246-286.

[38] Kothari, S. P., S. Shu, and P. D. Wysocki (2009). Do managers withhold bad news?. *Journal of Accounting Research* 47(1): 241-276.

[39] Kraft, A. G., R. Vashishtha and M. Venkatachalam (2018). Frequent financial reporting and managerial myopia. *The Accounting Review 93 (2), 249-275.*

[40] Kyle, A. S. (1985). Continuous auctions and insider trading. *Econometrica* 1315-1335.

[41] Lambert, R. (2010). Discussion of "Implications for GAAP from an analysis of positive research in accounting". *Journal of Accounting and Economics* 50(2-3), 287-295.

[42] Langberg, N., and K. Sivaramakrishnan (2010). Voluntary disclosures and analyst feedback. *Journal of Accounting Research* 48(3), 603-646.

[43] Lara, J. M. G., B. G. Osma, and F. Penalva (2016). Accounting conservatism and firm investment efficiency. *Journal of Accounting and Economics* 61(1), 221-238.

[44] Luo, Y. (2005). Do insiders learn from outsiders? evidence from mergers and acquisitions. *The Journal of Finance* 60(4), 1951–1982.

[45] Morris, S., and H.S. Shin (2002). Social value of public information. *American Economic Review* 92(5), 1521-1534.

[46] Nan, L. and X. Wen (2014). Financing and investment efficiency, information quality, and accounting biases. *Management Science* 60(9), 2308–2323.

[47] Plantin, G., and J. Tirole (2018). Marking to market versus taking to market. *American Economic Review* 108(8), 2246-76.

[48] Sims, C. A. (2006). Rational inattention: Beyond the linear-quadratic case. *American Economic Review* 96(2), 158-163.

[49] Subrahmanyam, A. and S. Titman (1999). The going-public decision and the development of financial markets. *The Journal of Finance* 54(3), 1045–1082.

[50] Watts, R.L., (2003). Conservatism in accounting part I: Explanations and implications. *Accounting Horizons* 17(3), 207-221.

Appendix: Proofs

Proof of Proposition 1: No-Trading equilibrium

For a given order flow X, the market maker's belief $\mu_M^{r=\phi,X}$, the uninformed firm's belief $\mu_F^{\phi,X}$, and the market price are presented in the following table:

Table A.1: No-trading equilibrium

Order Flow	Market Maker's belief	Firm's belief	Investment decision	Price
X	$\mu_M^{r=\delta,X}$	$\mu_F^{\delta=\phi,X}$	d	$p(X,\phi)$
-2	0	0	-1	$R_L+(x-c)$
-1	t	$\frac{1}{2}$	0	$tR'_H+(1-t)R'_L$
0	t	$\frac{1}{2}$	0	$tR'_H+(1-t)R'_L$
1	t	$\frac{1}{2}$	0	$tR'_H+(1-t)R'_L$
2	1	1	1	$R_H+(x-c)$.

Where $t=\Pr(\theta=H|r=\phi)=\frac{\frac{1}{2}(1-f\beta_H)}{\frac{1}{2}(1-f\beta_H)+\frac{1}{2}(1-f\beta_L)}=\frac{1}{2}+\frac{1}{2}\frac{f\xi}{(1-f\beta)}$. We assume that, when $X=2$ or $X=-2$ is observed off the equilibrium path, the market maker and the firm believe that the speculator knows that the state is H (or L). This is the only belief that survives the intuitive criterion, since the speculator would lose money if he trades against his private information.

If the speculator observes $\eta = H$ and deviates by buying, with equal probability of $\frac{1}{3}$, the order flow would be 2, 1, or 0. His expected payoff would be:

$$\begin{aligned}\pi_{buy}^{NT} &= \frac{1}{3}\left[R_H' - P(0,\phi)\right] + \frac{1}{3}\left[R_H' - P(1,\phi)\right] + \frac{1}{3}\left[R_H' - P(2,\phi)\right] \\ &= \frac{2}{3}(1-t)\left(R_H' - R_L'\right).\end{aligned}$$

In contrast, if the speculator observes $\eta = L$ and deviates by selling, his expected payoff would be:

$$\begin{aligned}\pi_{sell}^{NT} &= \frac{1}{3}\left[P(-1,\phi) - R_L'\right] + \frac{1}{3}\left[P(0,\phi) - R_L'\right] + \frac{1}{3}\left[R_L' - P(-2,\phi)\right] \\ &= \frac{2}{3}t\left(R_H' - R_L'\right).\end{aligned}$$

No-trading equilibrium sustains if and only if trading cost is higher than profits from deviating, i.e., $\kappa > \max\left\{\pi_{buy}^{NT}, \pi_{sell}^{NT}\right\} = \frac{2}{3}(R_H - R_L)\max\{t,(1-t)\}$. Q.E.D.

Proof of Proposition 2: Trading equilibrium

In this equilibrium, the speculator buys on good news and sells on bad news when the firm does not disclose ($r = \phi$). For a given order flow X, the market maker's belief $\mu_M^{r=\phi,X}$, the uninformed firm's belief $\mu_F^{\phi,X}$, and the market price are presented in the following table:

Table A.2: Trading Equilibrium

X	$\mu_M^{r=\delta,X}$	$\mu_F^{\delta=\phi,X}$	d	$p(X,\phi)$
-2	0	0	-1	$R_L + (x-c)$
-1	$\mu_M^{\phi,-1} = \frac{t(1-\lambda)}{t(1-\lambda)+(1-t)}$	$\frac{1-\lambda}{2-\lambda}$	-1	$\mu_M^{\phi,-1}(R_H + \tau_H x - (1-\tau_H)x - c) + \left(1-\mu_M^{\phi,-1}\right)(R_L + x - c)$
0	t	$\frac{1}{2}$	0	$t(R_H + \tau_H(x-c)) + (1-t)(R_L + \tau_L(x-c))$
1	$\mu_M^{\phi,1} = \frac{t}{t+(1-\lambda)(1-t)}$	$\frac{1-\lambda}{2-\lambda}$	1	$\mu_M^{\phi,1}(R_H + x - c) + \left(1-\mu_M^{\phi,1}\right)(R_L + \tau_L x - (1-\tau_L)x - c)$
2	1	1	1	$R_H + (x-c)$

The detailed derivations of $\mu_F^{\phi,1}$ and $\mu_F^{\phi,-1}$ are shown below:

$$
\begin{aligned}
\mu_F^{\phi,1} &= \frac{\Pr(\theta = H, X = 1|\delta = \phi)}{\Pr(\theta = H, X = 1|\delta = \phi) + \Pr(\theta = L, X = 1|\delta = \phi)} \\
&= \frac{\frac{1}{2}\frac{1}{3}\lambda + \frac{1}{2}\frac{1}{3}(1-\lambda)}{\frac{1}{2}\frac{1}{3}\lambda + \frac{1}{2}\frac{1}{3}(1-\lambda) + \frac{1}{2}\frac{1}{3}(1-\lambda)} = \frac{1}{2-\lambda} \\
\mu_F^{\phi,-1} &= \frac{\Pr(\theta = H, X = -1|\delta = \phi)}{\Pr(\theta = H, X = -1|\delta = \phi) + \Pr(\theta = L, X = -1|\delta = \phi)} \\
&= \frac{\frac{1}{2}\frac{1}{3}(1-\lambda)}{\frac{1}{2}\frac{1}{3}(1-\lambda) + \frac{1}{2}\frac{1}{3}\lambda + \frac{1}{2}\frac{1}{3}(1-\lambda)} = \frac{1-\lambda}{2-\lambda};
\end{aligned}
$$

The detailed derivations of $\mu_M^{\phi,1}$ and$\mu_M^{\phi,-1}$ are shown below:

$$
\begin{aligned}
\mu_M^{\phi,1} &= \frac{\Pr(\theta = H, X = 1|r = \phi)}{\Pr(\theta = H, X = 1|r = \phi) + \Pr(\theta = L, X = 1|r = \phi)} \\
&= \frac{\Pr(\theta = H|r = \phi)\Pr(X = 1|\theta = H, r = \phi)}{\Pr(\theta = H|r = \phi)\Pr(X = 1|\theta = H, r = \phi) + \Pr(\theta = L|r = \phi)\Pr(X = 1|\theta = L, r = \phi)} \\
&= \frac{t\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right]}{t\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right] + \frac{1}{3}(1-t)(1-\lambda)} = \frac{t}{t + (1-t)(1-\lambda)} > \frac{1}{2};
\end{aligned}
$$

$$
\begin{aligned}
\mu_M^{\phi,-1} &= \frac{\Pr(\theta = H, X = -1|r = \phi)}{\Pr(\theta = H, X = -1|r = \phi) + \Pr(\theta = L, X = -1|r = \phi)} \\
&= \frac{\Pr(\theta = H|r = \phi)\Pr(X = -1|\theta = H, r = \phi)}{\Pr(\theta = H|r = \phi)\Pr(X = -1|\theta = H, r = \phi) + \Pr(\theta = L|r = \phi)\Pr(X = -1|\theta = L, r = \phi)} \\
&= \frac{t\frac{1}{3}(1-\lambda)}{t\frac{1}{3}(1-\lambda) + (1-t)\left[\frac{1}{3}\lambda + \frac{1}{3}(1-\lambda)\right]} = \frac{t(1-\lambda)}{t(1-\lambda) + (1-t)} < \frac{1}{2}.
\end{aligned}
$$

If the speculator observes $\eta = H$ and buys one share, his expected profit is:

$$
\begin{aligned}
\pi_{buy}^T &= \frac{1}{3}\left[R'_H - P(0,\phi)\right] + \frac{1}{3}\left[R_H + x - c - P(1,\phi)\right] + \frac{1}{3}\left[R_H + x - c - P(2,\phi)\right] \\
&= \frac{1}{3}(1-t)\left(R'_H - R'_L\right) + \frac{1}{3}\left(1 - \mu_M^{\phi,1}\right)\left[R_H - R_L + 2(1-\tau_L)x\right].
\end{aligned}
$$

If the speculator observes $\eta = L$ and sells one share, his expected profit is:

$$\begin{aligned}\pi_{sell}^{T} &= \frac{1}{3}\left[P(0,\phi) - R_L'\right] + \frac{1}{3}\left[P(-1,\phi) - (R_L + x - c)\right] + \frac{1}{3}\left[P(-2,\phi) - (R_L + x - c)\right] \\ &= \frac{1}{3}t\left(R_H' - R_L'\right) + \frac{1}{3}\mu_M^{\phi,-1}\left[R_H - R_L - 2(1-\tau_H)x\right]\end{aligned}$$

Trading equilibrium sustains if and only if the cost of trading is lower than the gross trading profits from either buying or selling, i.e., $\kappa < min\left\{\pi_{buy}^{T}, \pi_{sell}^{T}\right\}$. Q.E.D.

Proof of Proposition 3

(i) Buy-no-sell (BNS) equilibrium In this equilibrium, when the firm does not disclose ($r = \phi$), the speculator buys on good news, but does not trade on bad news. For a given order flow X, the market maker's belief $\mu_M^{r=\phi,X}$, the uninformed firm's belief $\mu_F^{\phi,X}$, and the market price are presented in the following table:

Table A.3: Buy-no-sell Equilibrium

X		$\mu_M^{r=\delta,X}$	$\mu_F^{\delta=\phi,X}$	d	$p(X,\phi)$
-2		0	0	-1	$R_L + x - c$
-1		$\frac{t(1-\lambda)}{t(1-\lambda)+(1-t)}$	$\frac{1-\lambda}{2-\lambda}$	-1	$\mu_M^{\phi,-1}(R_H + \tau_H x - (1-\tau_H)x - c) + \left(1 - \mu_M^{\phi,-1}\right)(R_L + x - c)$
0		t	$\frac{1}{2}$	0	$tR_H' + (1-t)R_L'$
1		t	$\frac{1}{2}$	0	$tR_H' + (1-t)R_L'$
2		1	1	1	$R_H + x - c$

If the speculator privately observes $\eta = H$ and buys on this good news, his expected profit is:

$$\begin{aligned}\pi_{buy}^{BNS} &= \frac{1}{3}\left[R_H' - P(0,\phi)\right] + \frac{1}{3}\left[R_H' - P(1,\phi)\right] + \frac{1}{3}\left[R_H + x - c - P(2,\phi)\right] \\ &= \frac{2}{3}(1-t)\left(R_H' - R_L'\right) = \pi_{buy}^{NT}\end{aligned}$$

If the speculator privately observes $\eta = L$ and deviates to sell one share, his expected profit is:

$$\begin{aligned}\pi_{sell}^{BNS} &= \frac{1}{3}\left[P(0,\phi) - R_L'\right] + \frac{1}{3}\left[P(-1,\phi) - (R_L + x - c)\right] + \frac{1}{3}\left[P(-2,\phi) - (R_L + x - c)\right] \\ &= \frac{1}{3}t\left(R_H' - R_L'\right) + \frac{1}{3}\mu_M^{\phi,-1}\left[R_H - R_L - 2(1-\tau_H)x\right] = \pi_{sell}^T\end{aligned}$$

Buy-no-sell (BNS) equilibrium sustains if the profit from buying on good news (the profit of deviating to selling on bad news) is higher (lower) than the trading cost, i.e., when $\pi_{buy}^{NT} > \kappa > \pi_{sell}^T$.

(ii) Sell-not-buy (SNB) equilibrium In this equilibrium, when the firm does not disclose ($r = \phi$), the speculator does not trade on good news, but sells on bad news. For a given order flow X, the market maker's belief $\mu_M^{r=\phi,X}$, the uninformed firm's belief $\mu_F^{\phi,X}$, and the market price are presented in the following table:

Table A.4: Sell-no-buy Equilibrium

X	$\mu_M^{r=\delta,X}$	$\mu_F^{\delta=\phi,X}$	d	$p(X,\phi)$
-2	0	0	-1	$R_L + x - c$
-1	t	$\frac{1}{2}$	0	$tR_H' + (1-t)R_L'$
0	t	$\frac{1}{2}$	0	$tR_H' + (1-t)R_L'$
1	$\frac{t}{t+(1-\lambda)(1-t)}$	$\frac{1}{2-\lambda}$	1	$\mu_M^{\phi,1}(R_H + x - c) + \left(1 - \mu_M^{\phi,1}\right)(R_L + \tau_L x - (1-\tau_L)x - c)$
2	1	1	1	$R_H + x - c$

If the speculator privately observes $\eta = H$ and deviates to buy one share on this good news, his expected profit is:

$$\begin{aligned}\pi_{buy}^{SNB} &= \frac{1}{3}\left[R_H' - P(0,\phi)\right] + \frac{1}{3}\left[R_H + x - c - P(1,\phi)\right] + \frac{1}{3}\left[R_H + x - c - P(2,\phi)\right] \\ &= \frac{1}{3}(1-t)\left(R_H' - R_L'\right) + \frac{1}{3}\left(1 - \mu_M^{\phi,1}\right)\left[R_H - R_L + 2(1-\tau_L)x\right] = \pi_{buy}^T\end{aligned}$$

If the speculator privately observes $\eta = H$ and sells on this bad news, his expected profit is:

$$\begin{aligned}\pi_{sell}^{SNB} &= \frac{1}{3}\left[P(-1,\phi) - R'_L\right] + \frac{1}{3}\left[P(0,\phi) - R'_L\right] + \frac{1}{3}\left[P(-2,\phi) - (R_L + x - c)\right] \\ &= \frac{2}{3}t\left(R'_H - R'_L\right) = \pi_{sell}^{NT}\end{aligned}$$

Sell-no-buy (SNB) equilibrium sustains if the profit from selling on bad news (the profit of deviating to buying on good news) is higher (lower) than the trading cost, i.e., when $\pi_{sell}^{NT} > \kappa > \pi_{buy}^{T}$. Q.E.D.

Proof of Proposition 4

1. $\frac{d\pi_{sell}^{T}}{d\xi} > 0$ and $\frac{d\pi_{sell}^{NT}}{d\xi} > 0$ can be straightforwardly established by taking a derivative on π_{sell}^{T} and π_{sell}^{NT} with respective to ξ. Note that $\pi_{sell}^{NT} = \frac{2}{3}t\left(R'_H - R'_L\right)$ and $\pi_{sell}^{T} = \frac{1}{3}t\left(R'_H - R'_L\right) + \frac{1}{3}\mu_M^{\phi,-1}\left[R_H - R_L - 2\left(1-\tau_H\right)x\right]$, where $\frac{\partial t}{\partial \xi} > 0$, $\frac{\partial \tau_L}{\partial \xi} = -\frac{f(1-f)}{(1-f\beta_L)^2} < 0$ and $\frac{\partial \tau_H}{\partial \xi} = \frac{f(1-f)}{(1-f\beta_H)^2} > 0$. Thus $\frac{\partial(R'_H - R'_L)}{\partial \xi} > 0$, $\frac{\partial \mu_M^{\phi,-1}}{\partial \xi} > 0$.

2. With a neutral disclosure policy $\xi = 0$, we have $\pi_{sell}^{T} < \pi_{buy}^{T}$. Thus in the neighborhood of $\xi = 0$, $\kappa^T \equiv \min\left\{\pi_{buy}^{T}, \pi_{sell}^{T}\right\} = \pi_{sell}^{T}$, and $\frac{d\kappa^T}{d\xi}\mid_{\xi=0} = \frac{d\pi_{sell}^{T}}{d\xi}\mid_{\xi=0} > 0$.
   When $\xi < 0$, $\pi_{sell}^{T} < \pi_{buy}^{T}$, thus $\min\left\{\pi_{buy}^{T}, \pi_{sell}^{T}\right\}\mid_{\xi=-\hat{\xi}} = \pi_{sell}^{T}\mid_{\xi=-\hat{\xi}}$. Note that $\pi_{sell}^{T}\mid_{\xi=-\hat{\xi}} < \pi_{sell}^{T}\mid_{\xi=\hat{\xi}}$ since $\frac{d\pi_{sell}^{T}}{d\xi} > 0$. Furthermore, we have $\pi_{sell}^{T}\mid_{\xi=-\hat{\xi}} < \pi_{buy}^{T}\mid_{\xi=\hat{\xi}}$. To see this, examine Eq.(8) and (9), and note that $(R'_H - R'_L)\mid_{\xi=\hat{\xi}} > (R'_H - R'_L)\mid_{\xi=-\hat{\xi}}$, $(1-t)\mid_{\xi=\hat{\xi}} = t\mid_{\xi=-\hat{\xi}}$, and $\left(1-\mu_M^{\phi,1}\right)\mid_{\xi=\hat{\xi}} = \mu_M^{\phi,-1}\mid_{\xi=-\hat{\xi}}$. Thus $\min\left\{\pi_{buy}^{T}, \pi_{sell}^{T}\right\}\mid_{\xi=-\hat{\xi}} < \min\left\{\pi_{buy}^{T}, \pi_{sell}^{T}\right\}\mid_{\xi=\hat{\xi}}$, i.e., $\kappa_T\mid_{\xi=\hat{\xi}} > \kappa_T\mid_{\xi=-\hat{\xi}}$.

3. The no-trading equilibrium region is determined by $\kappa > \kappa_{NT} \equiv \max\left\{\pi_{buy}^{NT}, \pi_{sell}^{NT}\right\}$. For any

$\hat{\xi} > 0$, we have $\kappa_{NT} \mid_{\xi=\hat{\xi}} > \kappa_{NT} \mid_{\xi=0}$ and $\kappa_{NT} \mid_{\xi=\hat{\xi}} > \kappa_{NT} \mid_{\xi=-\hat{\xi}}$:

$$\begin{aligned}
\kappa_{NT} \mid_{\xi=\hat{\xi}} &= \max\left\{\pi_{buy}^{NT}, \pi_{sell}^{NT}\right\} \mid_{\xi=\hat{\xi}} = \pi_{sell}^{NT} \mid_{\xi=\hat{\xi}} \\
&= \frac{2}{3} t \mid_{\xi=\hat{\xi}} \left(R_H^{'} - R_L^{'}\right) \mid_{\xi=\hat{\xi}} \\
&> \frac{1}{3}\left(R_H - R_L\right) = \kappa_{NT} \mid_{\xi=0}; \\
\kappa_{NT} \mid_{\xi=-\hat{\xi}} &= \max\left\{\pi_{buy}^{NT}, \pi_{sell}^{NT}\right\} \mid_{\xi=-\hat{\xi}} = \pi_{buy}^{NT} \mid_{\xi=-\hat{\xi}} \\
&= \frac{2}{3}\left(1-t\right) \mid_{\xi=-\hat{\xi}} \left(R_H^{'} - R_L^{'}\right) \mid_{\xi=-\hat{\xi}} \\
&< \kappa_{NT} \mid_{\xi=\hat{\xi}}
\end{aligned}$$

Proof of Proposition 5

The social planner chooses ξ to maximize expected firm value $E\left\{v\left[\theta, d\left(\delta, X\right)\right]\right\}$, which depends on the equilibrium played. We first derive the expected firm value under all equilibria. Under the no-trading equilibrium, the firm does not learn from the market and changes investment level only when it is privately informed of the state. In this case, the expected firm value is:

$$E^{NT}\left[v\left(\theta, d\right)\right] = \frac{1}{2}\left(R_H + R_L\right) + f\left(x - c\right).$$

Under the buy-no-sell equilibrium, the firm can learn from the market when it does not have private information. When the speculator is present, (with probability λ), the firm learns from the market: when the state is H, the order flow can be 2, 1 or 0, and the firm increases investment when $X = 2$. When the state is L, the order flow can be 1, 0, -1, and the firm decreases investment when $X = -1$. Both happen with probability $\frac{1}{3}$ and improve firm value by $x - c$. When the speculator is not present (with probability $1 - \lambda$), the order flow can be 1, 0 or -1. The firm wrongly decreases investment when $X = -1$, which reduces firm value by c. Thus, the ex-ante expected firm value is:

$$E^{BNS}\left[v\left(\theta, d\right)\right] = \frac{1}{2}\left(R_H + R_L\right) + f\left(x - c\right) + \left(1 - f\right)\frac{1}{3}\left(\lambda x - c\right).$$

Similarly, under the sell-no-buy equilibrium, the ex-ante expected firm value is: $E^{SNB}\left[v\left(\theta,d\right)\right] = \frac{1}{2}\left(R_H+R_L\right)+f\left(x-c\right)+\left(1-f\right)\frac{1}{3}\left(\lambda x-c\right).$

Under the trading equilibrium, the firm can learn more from the market since the speculator trades more. The firm value is:

$$E^{T}\left[v\left(\theta,d\right)\right]=\frac{1}{2}\left(R_H+R_L\right)+f\left(x-c\right)+\left(1-f\right)\frac{2}{3}\left(\lambda x-c\right).$$

Given our assumption that $\lambda>\frac{2c}{x+c}$, we have $\lambda x-c>0$ and

$$E^{T}\left[v\left(\theta,d\right)\right]>E^{BNS}\left[v\left(\theta,d\right)\right]=E^{SNB}\left[v\left(\theta,d\right)\right]>E^{NT}\left[v\left(\theta,d\right)\right].$$

Note that ξ influences the expected firm value only through affecting the equilibrium played. Proposition 4 shows that, $\kappa_{NT}\mid_{\xi=\hat{\xi}}>\kappa_{NT}\mid_{\xi=-\hat{\xi}}$ and $\kappa_{T}\mid_{\xi=\hat{\xi}}>\kappa_{T}\mid_{\xi=-\hat{\xi}}$, which implies that for any given TGR system with $\xi=-\hat{\xi}<0$, the system with $\xi=\hat{\xi}$ gives a larger trading region and a smaller no-trading region. Furthermore, compared to the neutral system, there exists a TLR system with a sufficiently small $\hat{\xi}>0$ that gives a larger trading region and a smaller no-trading region. . Thus a TLR system that is weakly preferred by the social planner. Q.E.D.

Proof of Proposition 6

When the trading equilibrium prevails, the proof of Proposition 6 shows that firm value is maximized and does not depend on ξ. Thus, the firm's preference is entirely determined by $E\left(\pi\right)$, which can be written as follows:

$$\begin{aligned}
E\left(\pi\right)&=\frac{1}{2}\left(1-f\beta_H\right)\lambda\pi_{buy}^{T}+\frac{1}{2}\left(1-f\beta_L\right)\lambda\pi_{sell}^{T}\\
&=\lambda\left(1-f\beta\right)\left[t\pi_{buy}^{T}+\left(1-t\right)\pi_{sell}^{T}\right]\\
&=\lambda\left(1-f\beta\right)\{\frac{2}{3}t\left(1-t\right)\left(R_H'-R_L'\right)+\frac{1}{3}\left[t\left(1-\mu_M^{\phi,1}\right)+\left(1-t\right)\mu_M^{\phi,-1}\right]\left(R_H-R_L\right)\\
&+2(x-c)\left[\frac{1}{3}\left[t\left(1-\mu_M^{\phi,1}\right)\left(1-\tau_L\right)-\left(1-t\right)\mu_M^{\phi,-1}\left(1-\tau_H\right)\right]\right]\}
\end{aligned}$$

For any TLR policy $\hat{\xi} > 0$, we now show that there exists a TGR system with $-\hat{\xi}$ that can generate a lower $E(\pi)$. As $\tau_H > \tau_L$ if and only if $\xi > 0$, we have

$$R'_H - R'_L = R_H - R_L + (\tau_H - \tau_L)(x - c) \Longrightarrow$$

$$\left(R'_H - R'_L\right)|_{\xi=\hat{\xi}} > \left(R'_H - R'_L\right)|_{\xi=0} > \left(R'_H - R'_L\right)|_{\xi=-\hat{\xi}} .$$

Note that $t\,|_{\xi=-\hat{\xi}} = 1 - t\,|_{\xi=\hat{\xi}}$, and $\left(1-\mu_M^{\phi,1}\right)|_{\xi=-\hat{\xi}} = \mu_M^{\phi,-1}\,|_{\xi=\hat{\xi}}$. Thus, the first term in the curly bracket for $E(\pi)$ is larger under a TLR system than under an TGR system, the second are the same under both, while the third term is proportional to $\tau_H - \tau_L$ and is positive when the system is TLR, 0 when neutral, and negative when TGR. Thus, $E(\pi)$ is lower under $\xi = -\hat{\xi}$ than under $\xi = \hat{\xi}$. Finally, because both $t(1-t)$ and $t\left(1-\mu_M^{\phi,1}\right) + (1-t)\mu_M^{\phi,-1}$ are maximized at $t = \frac{1}{2}$, the trading profit is lower under an TGR system than under a neutral system. Q.E.D.

Proof of Proposition 7

When $\kappa > \kappa^T\,|_{\xi=0}$ and $\kappa - \kappa^T\,|_{\xi=0}$ is sufficiently small, the BNS equilibrium sustains under a neutral disclosure policy, while with a small $\hat{\xi} > 0$, $\pi^T_{sell} \geq \kappa$ and the trading equilibrium sustains.

1. From the proof of Propostion 5, $E^T[v(\theta,d)] > E^{BNS}[v(\theta,d)]$ and thus the social planner prefers a TLR system.

2. We study firm's preference by comparing a TLR system first to the neutral system and then to a TGR system. First we compare a TLR system and the neutral system. The speculator's profits under BNS equilibrium and under trading equilibrium with a sufficiently small $\hat{\xi} > 0$ are as follows, respectively:

$$E^{BNS}(\pi) = \frac{1}{6}\lambda(1-f\beta)(R_H - R_L)$$
$$\lim_{\xi\to 0} E^T(\pi) = \frac{1}{3}\lambda(1-f\beta)\left(\frac{1}{2} + \frac{1-\lambda}{2-\lambda}\right)(R_H - R_L)$$

Note that at $\xi = 0$, change in ξ has no first order effect on $E_c(\pi)$. Thus the firm prefers TLR to TGR when $E^T[v(\theta,d)] - E^{BNS}[v(\theta,d)] \geq \alpha \left[\lim_{\xi\to 0}\left(E^T(\pi) - E^{BNS}(\pi)\right)\right]$, i.e., $\frac{1}{3}(1-f)(\lambda x - c) \geq \frac{1}{3}\alpha\lambda(1-f\beta)\frac{1-\lambda}{2-\lambda}(R_H - R_L)$. When f is sufficiently small, the firm prefers a TLR system to the neutral system when Eq. (10) holds:

$$\lambda x - c \geq \alpha\lambda\frac{1-\lambda}{2-\lambda}(R_H - R_L) \tag{10}$$

Next we compare the firm's preference between a TGR system and a TLR system. Under a TGR system, the equilibrium is BNS and the speculator's profit is as follows:

$$E^{BNS}(\pi) = \frac{2}{3}\lambda t(1-t)(1-f\beta)\underbrace{[R_H - R_L + (\tau_H - \tau_L)(x-c)]}_{\downarrow as\, TGR \uparrow}.$$

The firm prefers TLR to TGR when $E^T[v(\theta,d)] - E^{BNS}[v(\theta,d)] \geq \alpha\left(E_c(\pi) - E^{BNS}(\pi)\right)$. Note that $\frac{\partial E^{BNS}(\pi)}{\partial \xi} > 0$ when $\xi < 0$, that is, $E^{BNS}(\pi)$ decreases as TGR increases. Thus, among all TGR systems, the speculator's profit is lowest when the firm chooses $\xi = \beta - 1$, which has $\beta_H = 1$, $\beta_L = 2\beta - 1$, and $t = \frac{1-f}{2(1-f\beta)}$. The firm prefers a TLR system to any TGR system if Eq. (11) holds:

$$\begin{aligned} &\frac{1-f}{1-f\beta}(\lambda x - c) \geq \qquad (11)\\ &\alpha\lambda\left\{\left(\frac{1}{2} + \frac{1-\lambda}{2-\lambda}\right)(R_H - R_L) - \frac{1-f}{1-f\beta}\left[1 - \frac{1-f}{2(1-f\beta)}\right]\left(R_H - R_L - \frac{2f(1-\beta)}{1-f(2\beta-1)}(x-c)\right)\right\}. \end{aligned}$$

When $f \to 0$ or $\beta \to 1$, the above becomes:

$$\lambda x - c \geq \alpha\lambda\frac{1-\lambda}{2-\lambda}(R_H - R_L) \tag{12}$$

It is straightforward that each of Part 2a - 2c is sufficient for equations (10) and (12).

3. Part 3 can be derived similarly to Part 2, hence omitted. Q.E.D.

Proof of Proposition 8

1. The social planner aims to maximize investment efficiency and thus perfers the trading equilibrium. Thus his optimal β must be strictly smaller than 1, because when $\beta = 1$ (which restricts $\xi = 0$), $\pi^T_{sell}(\beta = 1, \xi = 0) = \underline{\kappa} < \kappa$ implies that the trading equilibrium is not sustained. The social planner can restore the trading equilibrium by selecting $\beta_c < 1$ and $\xi = \xi_c$ slightly above 0 such that $\pi^T_{sell}(\beta = \beta_c, \xi = \xi_c) = \kappa$. This is feasible because $\frac{\partial \pi^T_{sell}}{\partial \beta} |_{\xi=0} < 0$, $\frac{\partial \pi^T_{sell}}{\partial \xi} > 0$, and κ is just slightly above $\underline{\kappa}$. Since β_c is close to 1 and ξ_c close to 0, we have $\kappa = \pi^T_{sell}(\beta = \beta_c, \xi = \xi_c) < \pi^T_{buy}(\beta = \beta_c, \xi = \xi_c)$, implying that trading equilibrium sustains. Clearly, fixing β_c, trading equilibrium does not sustain under any $\xi < \xi_c$, since $\frac{\partial \pi^T_{sell}}{\partial \xi} > 0$. As such, $\underline{\xi}(\beta_c) = \xi_c$ and $\pi^T_{sell}(\beta_c, \underline{\xi}) = \kappa$, which in turn implies $\frac{d\underline{\xi}}{d\beta_c} = -\frac{\partial \pi^T_{sell}(\beta_c, \underline{\xi})/\partial \beta_c}{\partial \pi^T_{sell}(\beta_c, \underline{\xi})/\partial \underline{\xi}}$ when $\underline{\xi}$ is not a corner solution. When β_c is close to 1 and $\underline{\xi}$ is close to 0, by continuity $\frac{\partial \pi^T_{sell}(\beta_c, \underline{\xi})}{\partial \beta_c} < 0$, and $\frac{\partial \pi^T_{sell}(\beta_c, \underline{\xi})}{\partial \underline{\xi}} = \frac{\partial \pi^T_{sell}(\beta_c, \underline{\xi})}{\partial t} \frac{\partial t}{\partial \underline{\xi}} > 0$. Thus $\frac{d\underline{\xi}}{d\beta_c} > 0$.

2. The firm aims to maximize $E(v(\theta, d)) - \alpha E(\pi)$. When α is sufficiently small, the firm optimally chooses its reporting system to minimize $E(\pi)$ while ensuring the trading equilibrium sustains. Thus, similar to the argument in part 1, the firm's optimal β must have $\beta < 1$. We prove that the firm prefers a TLR system in two steps.

- Step 1. The firm's optimal system cannot be a TGR system. We show that, for any TGR system, the firm can find a neutral system that gives a higher expected payoff. Suppose the optimal system has $\beta = \beta^*$ and $\xi = \xi^* < 0$, under which trading equilibrium sustains. Then $\pi^T_{sell}(\beta = \beta^*, \xi = \xi^*) \geq \kappa$. Then we can find a neutral system with $\beta = \beta' \in (\beta^*, 1)$ and $\xi = 0$ such that $\pi^T_{sell}\left(\beta = \beta', \xi = 0\right) = \kappa$. Such β' exists because $\pi^T_{sell}(\beta = \beta^*, \xi = 0) > \pi^T_{sell}(\beta = \beta^*, \xi = \xi^*) > \underline{\kappa}$, and $\pi^T_{sell}(\beta = 1, \xi = 0) = \underline{\kappa} < \kappa$. Since $\pi^T_{buy} > \pi^T_{sell}$ under a neutral system, the trading equilibrium sustains under System $(\beta', 0)$. To compare $E_{\beta,\xi}(\pi)$

under different systems, rewrite $E_{\beta,\xi}(\pi)$ as a function of β, π_{sell}^{T} and t:

$$\begin{aligned}
E_{\beta,\xi}(\pi) &= \frac{1}{3}(1-f\beta)\lambda t(1-t)* \\
&\quad \left(2\left(R_H'-R_L'\right)+\frac{(1-\lambda)}{1-\lambda(1-t)}\left(R_H-R_L+2\frac{1-f}{1-f\beta_L}x\right)+\frac{(1-\lambda)}{1-\lambda t}\left(R_H-R_L-2\frac{1-f}{1-f\beta_H}x\right)\right) \\
&= \underbrace{(1-f\beta)\lambda(1-t)2\pi_{sell}^{T}}_{H_1}+\underbrace{\frac{1}{3}\lambda(1-\lambda)(1-f)\left(\frac{1-t}{1-\lambda t}+\frac{t}{1-\lambda(1-t)}\right)x}_{H_2} \\
&\quad \underbrace{+\frac{1}{3}(1-f\beta)\lambda t(1-t)(1-\lambda)(R_H-R_L)\left[\frac{1}{1-\lambda(1-t)}-\frac{1}{1-\lambda t}\right]}_{H_3}
\end{aligned} \tag{13}$$

where we use $R_H'-R_L'=\left(\frac{3\pi_{sell}^{T}}{t}-\frac{(1-\lambda)}{1-\lambda t}(R_H-R_L)+\frac{(1-\lambda)}{1-\lambda t}\frac{1-f}{1-f\beta_H}2x\right)$. Since there is a one-to-one mapping between (β,ξ) and (β,t), the firm's problem is equivalent to pick (β,t), under the constraint $\pi_{sell}^{T}\geq\kappa$. Look at each componenet in Eq. (13), H_3 is smaller under a neutral system, since it is 0 (positive) under a neutral (TGR) system. For H_1+H_2, we have the following when $t\leq 1/2$:

$$\begin{aligned}
\frac{\partial(H_1+H_2)}{\partial t} &= -(1-f\beta)\lambda 2\kappa - f\frac{\partial\beta}{\partial t}\lambda(1-t)2\kappa+\frac{1}{3}\lambda(1-\lambda)(1-f)\frac{(2t-1)\lambda(-2+\lambda)(1-\lambda)}{(1-\lambda t)^2(1-\lambda(1-t))^2}x \\
&< -\frac{1-f\beta}{t}2\lambda\kappa+\frac{1}{3}\lambda(1-\lambda)(1-f)\frac{(2t-1)\lambda(-2+\lambda)(1-\lambda)}{(1-\lambda t)^2(1-\lambda(1-t))^2}x \\
&= -\frac{1-f\beta}{t}2\lambda\left(\frac{1}{6}(R_H-R_L)+\frac{1}{3}\frac{1-\lambda}{2-\lambda}\left(R_H-R_L-\frac{1-f}{(1-f\beta')}2x\right)\right) \\
&\quad +\frac{1}{3}\lambda(1-\lambda)(1-f)\frac{(2t-1)\lambda}{(1-\lambda t)^2(1-\lambda(1-t))^2}(-2+\lambda)(1-\lambda)x \\
&< -\frac{1-f}{t}2\lambda\frac{1}{3}x+\frac{1}{3}\lambda(1-f)\frac{(2t-1)\lambda}{(1-\lambda t)^2}(-2+\lambda)x \\
&< \frac{1}{3t}\lambda(1-f)x(-2+\lambda)<0
\end{aligned}$$

The first inequality uses $\frac{\partial\beta}{\partial t}>\frac{1-f\beta}{ft}$ when $t<\frac{1}{2}$. We can view β as an implicit function of t

since $\kappa = \pi_{sell}^{T} = \frac{1}{3}t\left(R_H' - R_L'\right) + \frac{1}{3}\frac{(1-\lambda)t}{1-\lambda t}\left(R_H - R_L - \frac{1-f}{t(1-f\beta)}x\right)$:

$$\begin{aligned}
\frac{\partial \beta}{\partial t} &= -\frac{\frac{\partial \pi_{sell}^{T}}{\partial t}}{\frac{\partial \pi_{sell}^{T}}{\partial \beta}} \\
&= -\frac{\frac{1}{3}\frac{t(1-\lambda)}{1-t\lambda}\left(\frac{1}{(1-f\beta)}\frac{1-f}{t^2}x\right) - \frac{1}{3}t\left(\frac{f}{2(1-f\beta)}\left(\frac{(2t-1)(1-f)}{t^2(1-t)f}\right)(x-c)\right) + (other\, positive\, terms)}{-\frac{1}{3}\frac{t(1-\lambda)}{1-t\lambda}\frac{f}{(1-f\beta)^2}\frac{1-f}{t}x + \frac{1}{3}t\left(\frac{f^2}{2(1-f\beta)^2}\left(\frac{(2t-1)(1-f)}{t(1-t)f}\right)(x-c)\right)} \\
&> \frac{1-f\beta}{ft} \quad \left(when\, t \le \frac{1}{2}\right)
\end{aligned}$$

The second inequality uses $R_H - R_L > 2x$, and the third inequality uses $1-2t \ge 0$ when $t \le \frac{1}{2}$. As a result, the liquidity discount at $\left(\beta = \beta', \xi = 0\right)$ is strictly lower than that at $(\beta = \beta^*, \xi = \xi^*)$, implying the firm strictly prefers a neutral system to a TGR system.

- Step 2. The firm strictly prefers a TLR system to a neutral system. Suppose otherwise. That is, the optimal system has $(\beta = \beta^*, \xi = 0)$. Then, we can construct a conservative system with $(\beta_c, \xi_c > 0)$ that gives the firm a smaller discount. To do so, take (β_c, ξ_c) such that $\xi_c > 0$ is sufficiently small and $\pi_{sell}^{T}(\beta_c, \xi_c) = \kappa$. Since ξ_c is small and $\frac{\partial \pi_{sell}^{T}}{\partial \beta}\mid_{\xi=0} < 0$, by continuity $\beta_c > \beta^*$, and $\pi_{buy}^{T}(\beta_c, \xi_c) > \pi_{sell}^{T}(\beta_c, \xi_c) = \kappa$, thus the trading equilibrium sustains. Clearly, when we move from $(\beta = \beta^*, \xi = 0)$ to (β_c, ξ_c), H_1 becomes smaller as both t and β increase, and H_3 is negative under TLR ($t > \frac{1}{2}$) and 0 under a neutral system ($t = \frac{1}{2}$). As to H_2, note that

$$\begin{aligned}
\frac{\partial H_2}{\partial t} &\propto \frac{(2t-1)\lambda}{(1-\lambda t)^2(1-\lambda(1-t))^2}(-2+\lambda)(1-\lambda) \\
&\le 0\, when\, t \ge \frac{1}{2}.
\end{aligned}$$

Hence, $\frac{\partial H_2}{\partial t} \le 0$ when $t \ge \frac{1}{2}$. Thus the firm strictly prefers a TLR system. Q.E.D.