Internet Appendix for
"Lying to Speak the Truth: Selective Manipulation and Improved Information Transmission"

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ABSTRACT

This Internet Appendix discusses a variation of our model and provides information about numerical work used in the paper.

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IA.A. Report as a Signal of Effort

This section of the Internet Appendix analyzes a variation of our model in which the report is a signal of the manager's effort choice rather than the firm's cash flow. Section IV.D of the manuscript includes a description of this alternative model and a summary of the results.

IA.A.1. The Model

We study an agency model with two risk-neutral parties, a board of directors and a manager, that takes place over times 0, 1, 2, and 3. At time 0, the board (the principal) chooses the firm's governance system (explained below) and hires a manager (the agent) to run the firm. The board represents the interests of shareholders and offers the manager a contract that maximizes the value of the firm, net of the cost of managerial compensation. At time 1, the manager exerts an unobservable effort to enhance the value of the firm. At time 2, the firm's accounting system produces a public report concerning the manager's performance. A key feature of our model is that this report can be manipulated by the manager. At time 3, the firm's terminal cash flow v is realized and paid out to shareholders.

The firm's cash flow is either high ($v = v_h$) or low ($v = v_\ell < v_h$). The distribution of v depends on the manager's effort choice $e \in \{0, 1\}$. If the manager exerts high effort ($e = 1$), v is equal to v_h with probability one; if she exerts low effort ($e = 0$), v is equal to v_h with probability $\lambda < 1$ and equal to v_ℓ with probability $1 - \lambda$. The manager's private utility cost of exerting high effort, denoted by c, is drawn from a uniform distribution over the interval $[0, \bar{c}]$; the cost of low effort is normalized to zero. The manager's effort choice e and effort cost c are her private information and hence cannot be used for contracting purposes. To make the problem interesting, we assume that $\bar{c} > (1 - \lambda)(v_h - v_\ell)$, which ensures that inducing high effort is suboptimal when a high cost of effort c is realized.

Prior to the realization of the cash flow v, the firm's accounting system generates a report r, providing noisy information to the market about the manager's effort choice (and thus the value of the firm). This report can take on one of two values, r_h or r_ℓ. Absent any managerial intervention, the report is correlated with the manager's effort choice as follows:

$$\text{prob}\left[r = r_h \,|\, e = 1\right] = \text{prob}\left[r = r_\ell \,|\, e = 0\right] = \delta, \tag{IA.1}$$

where $\delta \in \left(\frac{1}{2}, 1\right)$. For simplicity, we assume that the report r is independent of the firm's cash flow v, conditional on the manager's effort choice e. Note that r is nevertheless an informative signal about v: A favorable report r_h increases the likelihood of a high effort choice and hence of a high cash flow, whereas an unfavorable report r_ℓ decreases it. The parameter δ captures the quality of the firm's accounting system. It represents various accounting standards and conventions in the economy as well as firm- and auditor-specific factors such as the transparency of the firm's operations and the auditor's experience in the industry.

Although the report is produced by the firm's accounting system, the manager can influence its outcome—for example, by exploiting any leeway in accounting rules or by hiding information from the auditor. Specifically, we assume that, by incurring a utility cost g, the manager can turn an unfavorable report r_ℓ into a favorable report r_h with probability ϕ. We allow for the possibility of mixed-strategy equilibria and denote by $m \in [0, 1]$ the probability with which the manager takes such an action.

The manipulation cost g may reflect the time spent coming up with creative ways to manage the firm's earnings or the effort involved in convincing an auditor to sign off on a biased report. This cost is influenced by the legal system in which the firm operates, but firm-specific factors are also relevant, such as the rigor of the firm's accounting system and internal controls, the skills and independence of the firm's accounting and internal audit teams, the independence and experience of the board's audit committee, the choice of external auditors, etc. The firm commits to its broader governance system before the manager signs the contract, and the cost g captures the ease or difficulty of manipulating the report r. Note that the cost g accrues to the manager, not the firm. However, the firm bears an indirect cost of manipulation: When the equilibrium contract induces selective manipulation, the manager anticipates that she may incur the disutility g after exerting high effort, which makes it more costly to incentivize effort.

We assume that the board of directors can improve the firm's governance—and hence increase the manager's manipulation cost—at no cost to the firm. That is, at time 0 the board can choose any $g \geq 0$, without having to spend any resources. The board chooses the firm's governance system and the manager's contract to maximize the value of the firm, net of the cost of managerial compensation. A contract specifies the manager's compensation as a function of the report r and the terminal cash flow v. The manager is risk neutral, has no wealth, and is protected by limited liability so that all payments must be nonnegative. Her reservation level of utility is normalized to zero.

IA.A.2. Equilibrium Analysis

In this section, we solve for the manager's optimal compensation contract. Our specification of the set of available contracts is without loss of generality in the sense that it is fully consistent with the revelation principle. Thus, we can restrict attention to truthful direct revelation mechanisms.

In the ensuing analysis, let $w(r, v|c)$ denote the compensation scheme under the direct mechanism. The fact that the manager has no wealth means that all compensation payments must be nonnegative. This implies that the manager's participation constraint is trivially satisfied: By choosing to exert zero effort and to not manipulate the report, the manager can always achieve a nonnegative payoff.

IA.A.3. Preliminary Results

We first show that, under the optimal contract, the manager's effort choice is characterized by a cost threshold $\hat{c}$ such that the manager exerts high effort if and only if $c < \hat{c}$. This follows immediately from incentive-compatibility considerations. Suppose a manager with a cost of effort c finds it optimal to choose the high effort level. A manager with a strictly smaller cost $c' < c$ faces exactly the same feasible actions and continuation payoffs as the manager with a cost c: If she also chooses the high effort level, then the continuation payoffs for each feasible action are identical for c and c', but the payoff of the manager with the lower cost c' is larger because her cost of effort is smaller. The continuation payoffs after choosing the low effort level are identical for the two managers, because the cost of exerting low effort is zero. It must therefore be optimal for a manager with a cost $c' < c$ to also choose the high effort level. Conversely, if a manager with a cost of effort c finds it optimal to choose the low effort level, then a manager with a strictly higher cost $c'' > c$ must also find it optimal to choose the low effort level.

LEMMA 1: *There exists a threshold* $\hat{c} \in [0, \bar{c}]$ *such that the optimal contract induces high managerial effort (i.e.,* $e = 1$*) for all* $c < \hat{c}$ *and low managerial effort (i.e.,* $e = 0$*) for all* $c > \hat{c}$*.*

Note that the manager is never indifferent between the high and the low effort level, except when her cost of effort is exactly at the threshold, $c = \hat{c}$. In equilibrium, under an optimal contract shareholders are also indifferent between inducing high and low managerial effort when $c = \hat{c}$, but not for any other realizations of c.[2] Since both shareholders and the manager are indifferent if and only if the zero-probability event $c = \hat{c}$ occurs, we can ignore mixed strategies concerning the manager's effort choice e.

Our next result concerns the manipulation decision that the optimal contract induces the manager to take. We demonstrate that this decision depends on the manager's cost of effort only through its effect on the manager's effort choice e. This is not surprising, because the cost c has no *direct* effect (besides its effect on effort choice) on the manipulation decision that the firm wants to induce: For a given effort choice e, the cost c does not affect the firm's cash flow v or the report r and, hence, has no impact on the shareholders' expected payoff.

LEMMA 2: *For any two effort costs* c *and* c'*, for which the optimal contract induces the same effort choice* e*, the optimal contract also induces the same manipulation decision* m*.*

Lemma 1 shows that, under the optimal contract, the manager's effort choice is identical for all realizations of the cost parameter c below the threshold $\hat{c}$ and for all realizations above the threshold $\hat{c}$. Together with the result in Lemma 2, this implies that any allocation resulting from an optimal direct mechanism can be implemented through a menu of contracts that pools all managers of type $c < \hat{c}$ and of type $c > \hat{c}$.

LEMMA 3: *The optimal mechanism can be implemented by offering the manager a menu of contracts that pools all types* $c \in [0, \hat{c})$ *and all types* $c \in (\hat{c}, \bar{c}]$*.*

Without loss of generality, we can thus set $w(r, v|c) = w_1(r, v)$ for all $c \in [0, \hat{c})$ and $w(r, v|c) = w_0(r, v)$ for all $c \in (\hat{c}, \bar{c}]$, where the subscript 1 (respectively, 0) indicates the region of parameter values c for which the optimal contract induces high (respectively, low) managerial effort. The optimal compensation scheme can hence be characterized by the menu $\mathcal{W} = \{\boldsymbol{w}_0, \boldsymbol{w}_1\}$, where

[2] We analyze the optimal choice of the cost threshold $\hat{c}$ in Proposition 5 below.

$\boldsymbol{w}_e = (w_e(r_h, v_h), w_e(r_\ell, v_h), w_e(r_h, v_\ell), w_e(r_\ell, v_\ell))$. For notational convenience, we also define the *manipulation schedule* $\mathcal{M} = (m_0, m_1) \in [0,1]^2$ as the manipulation choices that the board wants to induce, where m_e is the desired manipulation choice for a given effort choice e.

IA.A.4. The Principal's Problem

The optimal contract that the board offers the manager maximizes the shareholders' expected payoff, that is, the firm's expected cash flow net of the manager's expected compensation. We solve for the optimal contract in three steps. First, for a given cost threshold $\hat{c}$ and manipulation schedule $\mathcal{M}$, we characterize the compensation scheme $\mathcal{W}$ and manipulation cost g that induce the manager to exert high effort if and only if $c \le \hat{c}$ and to make the desired manipulation decisions at minimum cost to the firm. Second, for a given cost threshold $\hat{c}$, we compare the firm's profit across different manipulation schedules $\mathcal{M}$. We show that it is never optimal to incentivize the manager to manipulate a low report when she exerted low effort, which allows us to restrict our attention to contracts that may or may not induce manipulation when the manager exerted high effort. Third, for each potentially optimal manipulation schedule, we solve for the cost threshold $\hat{c}$ that maximizes the firm's expected profit. We then compare the expected profits generated by these contracts and determine which contract is optimal for a given set of parameter values.

To simplify the notation, let $\pi_{e,m_e}(r, v)$ denote the probability that a report $r \in \{r_h, r_\ell\}$ and a cash flow $v \in \{v_h, v_\ell\}$ is produced when the manager chooses effort level $e \in \{0, 1\}$ and makes the manipulation decision $m_e \in [0, 1]$. For example, if the manager exerts high effort ($e = 1$), the firm generates a high cash flow with certainty; it generates a high report with probability δ in case the manager chooses not to manipulate ($m_1 = 0$) and with probability $\delta + (1-\delta)\,\phi$ in case the manager chooses to manipulate ($m_1 = 1$). Thus, we have $\pi_{1,m_1}(r_h, v_h) = \delta + (1 - \delta)\,\phi\, m_1$. The probabilities of the other possible outcomes are defined analogously (see the proof of Proposition 1). Based on the results stated in Lemmas 2 to 3, we can then express the manager's expected compensation as

$$\left(\frac{\hat{c}}{\bar{c}}\right) \sum_{r,v} \pi_{1,m_1}(r, v)\, w_1(r, v) + \left(1 - \frac{\hat{c}}{\bar{c}}\right) \sum_{r,v} \pi_{0,m_0}(r, v)\, w_0(r, v). \tag{IA.2}$$

For a given cost threshold $\hat{c}$ and manipulation schedule $\mathcal{M}$, the optimal contract $\mathcal{C} = (\mathcal{W}, g)$

minimizes the expected payment to the manager subject to the nonnegativity constraints

$$g \geq 0, \quad w_0(r,v) \geq 0, \quad w_1(r,v) \geq 0, \qquad \forall r \in \{r_h, r_\ell\}, v \in \{v_h, v_\ell\}, \tag{IA.3}$$

and the following incentive compatibility (IC) constraints that ensure that the manager takes the desired actions: First, to induce the manager to follow the manipulation schedule $\mathcal{M}$, the compensation scheme has to satisfy the constraints

$$\phi\left[w_1(r_h, v_h) - w_1(r_\ell, v_h)\right] - g \begin{cases} \leq 0 & \text{if } m_1 = 0, \\ = 0 & \text{if } m_1 \in (0,1), \\ \geq 0 & \text{if } m_1 = 1, \end{cases} \tag{IA.4}$$

$$\phi\left[\lambda\big(w_0(r_h, v_h) - w_0(r_\ell, v_h)\big) + (1-\lambda)\big(w_0(r_h, v_\ell) - w_0(r_\ell, v_\ell)\big)\right] - g \begin{cases} \leq 0 & \text{if } m_0 = 0, \\ = 0 & \text{if } m_0 \in (0,1), \\ \geq 0 & \text{if } m_0 = 1. \end{cases} \tag{IA.5}$$

These constraints ensure that the manager's expected benefit from manipulation, which turns an unfavorable report r_ℓ into a favorable report r_h with probability ϕ, outweighs (respectively, does not outweigh) her manipulation cost g when $m_e = 1$ (respectively, when $m_e = 0$).

Second, for the manager to exert high effort when $c < \hat{c}$ and to exert low effort when $c > \hat{c}$, we must have

$$\sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) - \hat{c} - (1-\delta) g m_1 \geq \max_{m \in [0,1]} \sum_{r,v} \pi_{0,m}(r,v)\, w_1(r,v) - \delta g m, \tag{IA.6}$$

$$\sum_{r,v} \pi_{0,m_0}(r,v)\, w_0(r,v) - \delta g m_0 \geq \max_{m \in [0,1]} \sum_{r,v} \pi_{1,m}(r,v)\, w_0(r,v) - \hat{c} - (1-\delta) g m. \tag{IA.7}$$

The above IC constraints take into account the fact that the manager's effort choice affects the distribution of the firm's report r and hence the likelihood that the manager will incur the manipulation cost g. The probability of an unmanipulated low report is $1-\delta$ when the manager exerts high effort and δ when the manager exerts low effort.

Finally, to ensure that the manager truthfully reports her effort cost c, it must be that

$$\sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) - c - (1-\delta)gm_1 \geq$$

$$\max_{e\in\{0,1\},m\in[0,1]} \sum_{r,v} \pi_{e,m}(r,v)\, w_0(r,v) - e\big(c + (1-\delta)gm\big) - (1-e)\delta gm, \quad \forall c \in [0,\hat{c}), \quad \text{(IA.8)}$$

$$\sum_{r,v} \pi_{0,m_0}(r,v)\, w_0(r,v) - \delta gm_0 \geq$$

$$\max_{e\in\{0,1\},m\in[0,1]} \sum_{r,v} \pi_{e,m}(r,v)\, w_1(r,v) - e\big(c + (1-\delta)gm\big) - (1-e)\delta gm, \quad \forall c \in (\hat{c},\bar{c}]. \quad \text{(IA.9)}$$

For a given cost threshold $\hat{c}$ and manipulation schedule $\mathcal{M}$, the principal's optimization problem is thus to minimize the manager's expected compensation in (IA.2), subject to the constraints in (IA.3)–(IA.9).

IA.A.5. No Manipulation vs. Selective Manipulation

In this section, we derive the optimal contract for various manipulation schedules $\mathcal{M} \in [0,1]^2$, taking the cost threshold $\hat{c}$ as given. We demonstrate that it is never optimal to incentivize the manager to manipulate a low report when she exerted low effort.

We begin our analysis by characterizing the optimal no-manipulation contract, that is, the optimal contract that induces the manager to never manipulate the report, irrespective of her chosen effort level. The following proposition shows that the optimal no-manipulation contract rewards the manager only when both the firm's cash flow and its report are high (i.e., when $v = v_h$ and $r = r_h$), which allows for the strongest inference that the manager exerted high effort. The cost of manipulation, g^n, is set such that manipulation is never optimal for the manager.

PROPOSITION 1: *For any cost threshold $\hat{c} \in [0,\bar{c}]$, the optimal no-manipulation contract $\mathcal{C}^n$ consists of a compensation scheme*

$$w_0^n(r,v) = w_1^n(r,v) = \begin{cases} \frac{\hat{c}}{\delta-\lambda(1-\delta)} & \text{if } r = r_h \text{ and } v = v_h, \\ 0 & \text{otherwise,} \end{cases} \quad \text{(IA.10)}$$

and a manipulation cost

$$g^n \geq \frac{\phi\,\hat{c}}{\delta - \lambda(1-\delta)}. \tag{IA.11}$$

This contract induces the manager to exert high effort if $c \leq \hat{c}$ and low effort if $c > \hat{c}$, and to follow the manipulation schedule $m_0 = m_1 = 0$, at minimum cost.

Despite the fact that the manager has private information about her cost of effort c, shareholders cannot benefit from offering the manager a menu of type-specific contracts with different compensation schemes depending on the (truthfully reported) cost of effort. The reason is that both the principal and the agent are risk neutral in our setting: Both parties care only about the expected value of payments, contingent on the manager's actions e and m. Thus, any compensation scheme that leads to the same expected contingent payments as the one in (IA.10) is optimal, as long as it satisfies the incentive compatibility constraints. For example, the compensation scheme could include lotteries after r and v have been realized or it could offer a fixed payment if the manager announces a cost $c > \hat{c}$ (instead of a payment that is contingent on r and v). It also means that setting the compensation scheme $\boldsymbol{w}_0$ equal to $\boldsymbol{w}_1$ is optimal: This choice of $\boldsymbol{w}_0$ (i) incentivizes a manager with a cost $c > \hat{c}$ to exert low effort, and (ii) ensures that the expected compensation of a low-effort manager is equal to the minimum amount required by the truth-telling constraint in (IA.9). Intuitively, there are no real effects if a manager with a cost $c > \hat{c}$ falsely reports a cost below $\hat{c}$, as long as she thereafter chooses the desired effort level $e = 0$ and does not manipulate.

We next turn to the optimal contract that prompts the manager to implement the manipulation schedule $m_0 = 0$ and $m_1 = 1$, that is, that induces the manager to manipulate a low report if she exerted high effort, but not if she exerted low effort. We refer to such a contract as a selective-manipulation contract.

PROPOSITION 2: *For any cost threshold $\hat{c} \in [0, \bar{c}]$, the optimal selective-manipulation contract $\mathcal{C}^s$ consists of a compensation scheme*

$$w_0^s(r,v) = w_1^s(r,v) = \begin{cases} \frac{\hat{c}}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} & \textit{if } r = r_h \textit{ and } v = v_h, \\ 0 & \textit{otherwise.} \end{cases} \tag{IA.12}$$

and a manipulation cost

$$g^s = \frac{\lambda\phi\,\hat{c}}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}. \tag{IA.13}$$

This contract induces the manager to exert high effort if $c \leq \hat{c}$ and low effort if $c > \hat{c}$, and to follow the manipulation schedule $m_0 = 0$ and $m_1 = 1$, at minimum cost.

As in the no-manipulation case, the manager only receives a compensation when the outcome $v = v_h$ and $r = r_h$ is observed, which allows for the strongest inference that the manager exerted high effort. The cost of manipulation, g^s, is chosen such that only a manager who exerted high effort has an incentive to manipulate a low report r_ℓ. It should not be chosen larger than necessary, because the manager anticipates that she may have to incur the cost g^s if she exerts high effort, and an increase in g^s therefore requires an increase in the promised compensation. Thus, the principal optimally sets g^s equal to $\lambda\,\phi\,w^s(r_h, v_h)$, the minimum amount required to prevent the manager from manipulating a low report when she exerted low effort.

Our next result shows that it is never optimal for shareholders to incentivize the manager to manipulate a low report when she exerted low effort or to play a mixed manipulation strategy when she exerted high effort.

PROPOSITION 3: *An optimal contract (i) does not induce manipulation after low effort and (ii) does not induce a randomized manipulation decision after high effort.*

A contract that incentivizes a manager who exerted high effort to use mixed strategies when making her manipulation decision cannot be optimal for two reasons. First, compared to the selective-manipulation contract $\mathcal{C}^s$, which always induces manipulation of a low report r_ℓ after high effort, inducing such behavior with a probability of less than one makes the public report r less informative about the manager's effort choice. Second, inducing $m_1 \in (0,1)$ requires a higher manipulation cost $g > g^s$ because the cost must make a manager who chose the high effort level (and hence correctly anticipates a high cash flow v_h) indifferent between manipulating and not manipulating. In contrast, under the selective-manipulation contract $\mathcal{C}^s$ a manager who exerted low effort is kept indifferent, whereas a manager who exerted high effort strictly prefers manipulation. Inducing mixed strategies over the choice of m_1 thus causes two inefficiencies for shareholders: The link between effort and compensation is weakened, and the required increase in the manipulation cost g makes it more costly for shareholders to incentivize high managerial effort. Similarly, incentivizing manipulation by a manager who exerted low effort has a negative effect: It reduces the informativeness of the report about the manager's effort choice and hence increases the compensation payment required to induce high managerial effort.

Proposition 3 implies that, for any desired cost threshold $\hat{c}$, the optimal contract is either the no-manipulation contract $\mathcal{C}^n$ defined in Proposition 1 (which prevents manipulation entirely) or the selective-manipulation contract $\mathcal{C}^s$ defined in Proposition 2 (which permits manipulation only after high effort). The following proposition compares the manager's expected compensation under these two contracts (taking the threshold $\hat{c}$ as given).

PROPOSITION 4: *Let* $\kappa = \frac{(1-\delta)(1-\lambda)}{\delta-\lambda(1-\delta)} \in (0,1)$. *Then,*

1. *for any cost threshold* $\hat{c} \in (0, \kappa\bar{c})$, *the expected compensation under the no-manipulation contract* $\mathcal{C}^n$ *defined in Proposition 1 is strictly higher than the expected compensation under the selective-manipulation contract* $\mathcal{C}^s$ *defined in Proposition 2;*

2. *for any cost threshold* $\hat{c} \in (\kappa\bar{c}, \bar{c}]$, *the expected compensation under the no-manipulation contract* $\mathcal{C}^n$ *defined in Proposition 1 is strictly lower than the expected compensation under the selective-manipulation contract* $\mathcal{C}^s$ *defined in Proposition 2.*

Proposition 4 shows that the board offers a selective-manipulation contract if it wants to implement a low cost threshold $\hat{c}$, and a no-manipulation contract if it prefers to induce a high cost threshold $\hat{c}$. To understand this result, we need to analyze the expected payments to the manager under the two contracts. Selective manipulation has two effects. First, it makes the report r more informative about the manager's effort choice: It increases the likelihood that a high-effort manager generates a favorable report r_h from δ to $\delta + (1-\delta)\,\phi$, while leaving the likelihood that a low-effort manager produces such an outcome unchanged. This improved informativeness allows for a more efficient compensation contract: It reduces the payment required to induce a high level of effort. Second, the selective-manipulation contract must prevent a manager who exerted low effort from manipulating. This is achieved by setting a sufficiently high cost of manipulation g^s. However, this cost must be borne by a manager who exerted high effort and, due to bad luck, generated an unfavorable report r_ℓ. Anticipating this possibility, the manager hence becomes more hesitant to exert high effort in the first place: Manipulating a low report selectively when $e = 1$ effectively increases the manager's cost of exerting high effort by the amount of her expected manipulation cost, $(1-\delta)g^s$. This makes it more costly for shareholders to incentivize effort provision. The increase in the payment $w^s(r_h, v_h)$ necessary to induce high effort partly undoes the reduction in $w^s(r_h, v_h)$ made possible by the improved informativeness of the report r.

For a manager with a cost $c > \hat{c}$, the net effect of switching to a selective-manipulation contract is easy to determine. The probability of receiving a compensation payment is the same under both contracts, $\lambda(1-\delta)$, but the payment is lower under the selective-manipulation contract. Thus, a low-effort manager earns a lower expected compensation under the selective-manipulation contract. For a manager with a cost $c < \hat{c}$, in contrast, the expected compensation is increased. The promised payment is lower, but selective manipulation increases the probability of receiving a payment. The increased probability more than offsets the reduction in the payment and, as a result, a high-effort manager's expected compensation under the selective-manipulation contract exceeds that under the no-manipulation contract:[3]

$$(\delta + (1-\delta)\phi)\left(\frac{\hat{c}}{\delta - \lambda(1-\delta) + (1-\delta)(1-\lambda)\phi}\right) > \delta\left(\frac{\hat{c}}{\delta - \lambda(1-\delta)}\right). \tag{IA.14}$$

This does not mean, however, that a high-effort manager receives a higher expected utility under the selective-manipulation contract. On the contrary, the increase in the manager's expected compensation is more than offset by the cost of manipulation that she expects to incur. This is intuitive. The effort IC constraint in (IA.6) is binding under both contracts, so if switching from a no-manipulation contract to a selective-manipulation contract reduces the low-effort manager's expected payoff (it does, because of the improved informativeness of r), it must also reduce the high-effort manager's expected payoff (net of the expected cost of manipulation).

When considering whether to offer a selective-manipulation contract, the firm trades off the reduction in expected compensation due to the improved informativeness of the report against the increased cost of inducing effort caused by the expected cost of manipulation. Which of these two effects dominates depends on how likely the firm is to face either a low-cost or a high-cost manager,

[3]The expressions on the left- and right-hand side of inequality (IA.14) are identical for $\phi = 0$, and the expression on the left-hand side is increasing in ϕ.

which in turn depends on the choice of the cost threshold $\hat{c}$. For low values of $\hat{c}$, the manager is unlikely to exert high effort and hence to manipulate the report. In this case, shareholders prefer the selective-manipulation contract, because the deadweight loss due to the manager's manipulation cost is small compared to the reduction in the expected compensation due to improved information transmission. For high values of $\hat{c}$, the opposite is the case. The manager is likely to exert high effort and hence to incur the manipulation cost. Shareholders thus prefer the no-manipulation contract, because the deadweight loss due to the manager's manipulation cost is large compared to the reduction in the expected compensation due to improved information transmission.

An inspection of $\kappa\bar{c}$, the maximum value of the cost threshold $\hat{c}$ for which shareholders prefer the selective-manipulation contract to the no-manipulation contract, shows that it decreases in both δ and λ. This is consistent with the intuition described above. A more informative unmanipulated report (higher δ) reduces the benefit of selective manipulation, thereby making the selective-manipulation contract relatively less attractive. A higher λ improves the chances of a low-effort manager generating a high cash flow v_h, which makes it more beneficial for her to manipulate a low report r_ℓ. Since manipulation by a low-effort manager is never optimal, this means that the manipulation cost g^s must be increased, which in turn requires an increase in the compensation payment $w^s(r_h, v_h)$ (to incentivize a low-cost manager to exert high effort). Thus, an increase in λ makes selective manipulation less attractive for the firm.

IA.A.6. Optimal Contract

Our analysis in Section IA.A.5 shows that the optimal contract to implement a given cost threshold $\hat{c}$ is either a no-manipulation contract or a selective-manipulation contract. We now endogenize the board's choice of the threshold $\hat{c}$ and analyze which of these two contracts is optimal in different situations. We show that the board's decision depends on the size of the ratio $\frac{v_h - v_\ell}{\bar{c}}$. The numerator, $v_h - v_\ell$, is the increase in cash flow that effort can generate; it is divided by $\bar{c}$, which captures the average cost of effort (since c is uniformly distributed over the interval $[0, \bar{c}]$). We interpret this ratio as a measure of the productivity of effort. This is intuitive if the ratio is multiplied by $(1 - \lambda)$, since $(1 - \lambda)(v_h - v_\ell)$ is the expected value of the incremental cash flow when high effort is exerted instead of low effort. We show that when effort is only moderately productive, the board chooses a low threshold $\hat{c}$ and implements it using a selective-manipulation contract. In contrast, when effort is highly productive, the board implements a higher threshold $\hat{c}$ using a no-manipulation contract. As a first step, we derive the optimal value of the threshold $\hat{c}$ for each type of contract.

PROPOSITION 5: *Under the no-manipulation contract $\mathcal{C}^n$ defined in Proposition 1, firm value is maximized at a cost threshold of*

$$\hat{c}_n = \max\left\{\frac{1}{2}\left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta - \lambda(1-\delta)}\right), 0\right\}. \tag{IA.15}$$

In contrast, under the selective-manipulation contract $\mathcal{C}^s$ defined in Proposition 2, firm value is maximized at a cost threshold of

$$\hat{c}_s = \max\left\{\frac{1}{2}\left(\frac{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}{\delta - (1-\delta)(\lambda - \phi)}\right)\left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}\right), 0\right\}. \tag{IA.16}$$

Under both types of contract, the board may optimally choose not to incentivize effort provision: If the expected value-added of high effort, $(1-\lambda)(v_h - v_\ell)$, is small, the optimal cost threshold $\hat{c}$ is equal to zero (which is implemented by setting all compensation payments equal to zero). The board implements a positive cost threshold $\hat{c} > 0$ (and hence induces high effort provision by a manager with a cost $c < \hat{c}$) only if effort is sufficiently productive. An inspection of (IA.15) and (IA.16) reveals that $\hat{c}_n = 0$ if $\hat{c}_s = 0$, but not vice versa. This means that, for some parameter values, the board incentivizes effort provision by the manager only under the selective-manipulation contract. The expression for $\hat{c}_s$ in (IA.16) immediately implies the following result.

COROLLARY 1: *The optimal contract implements a cost threshold $\hat{c} > 0$ (i.e., incentivizes the manager to exert high effort with a strictly positive probability) if and only if*

$$\frac{v_h - v_\ell}{\bar{c}} > \frac{\lambda(1-\delta)}{(1-\lambda)\left[\delta - (1-\delta)(\lambda - \phi + \lambda\phi)\right]}. \tag{IA.17}$$

Having determined the optimal cost threshold $\hat{c}$ under the no-manipulation and the selective-manipulation contract, we can now solve for the optimal contract by analyzing which of these two contracts generates a higher firm value when the cost threshold is chosen optimally (i.e., when $\hat{c}$ is set to $\hat{c}_n$ under the no-manipulation contract and to $\hat{c}_s$ under the selective-manipulation contract).

PROPOSITION 6: *If the condition*

$$\frac{v_h - v_\ell}{\bar{c}} \leq \frac{1-\delta}{\delta - \lambda(1-\delta)} \left(\frac{1}{1-\lambda} + \sqrt{\frac{\delta - (1-\delta)(\lambda - \phi)}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}} \right) \tag{IA.18}$$

is satisfied, then the optimal contract is the selective-manipulation contract $\mathcal{C}^s$ defined in Proposition 2. If the above condition is not satisfied, then the optimal contract is the no-manipulation contract $\mathcal{C}^n$ defined in Proposition 1.

As discussed above, the ratio $\frac{v_h - v_\ell}{\bar{c}}$ determines which contract is optimal. There are three distinct regions. If $\frac{v_h - v_\ell}{\bar{c}}$ is so small that condition (IA.17) is violated, the board optimally offers a contract that induces low effort for any cost of effort $c > 0$ (by setting $\hat{c} = 0$). In this case, both the selective-manipulation contract (with $w^s(r_h, v_h) = 0$ and $g^s = 0$) and the no-manipulation contract (with $w^n(r_h, v_h) = 0$ and $g^n = 0$) generate the same firm value. For intermediate values of $\frac{v_h - v_\ell}{\bar{c}}$ such that both conditions (IA.17) and (IA.18) are satisfied, the board optimally offers a selective-manipulation contract that induces high effort if $c < \hat{c}_s$ and incentivizes manipulation of a report r_ℓ if the manager exerted high effort. For high values of $\frac{v_h - v_\ell}{\bar{c}}$ such that condition (IA.17) is satisfied but condition (IA.18) is violated, the board optimally offers a no-manipulation contract that induces high effort if $c < \hat{c}_n$ and prevents all manipulation.

These results about the optimal contract are consistent with the intuition we provided in Section IA.A.5, comparing the costs of implementing a *given* threshold $\hat{c}$ using either a no-manipulation contract or a selective-manipulation contract. As discussed above, a selective-manipulation contract is preferred by the board when $\hat{c}$ is low, since the improved informativeness under such a contract decreases the expected compensation of a low-effort manager (with $c > \hat{c}$), whom the board is more likely to face when $\hat{c}$ is low. In contrast, a no-manipulation contract is preferred by the board when $\hat{c}$ is high, since selective manipulation makes inducing high effort more costly and, as a result, increases the expected compensation of a high-effort manager (with $c < \hat{c}$), whom the board is more likely to face when $\hat{c}$ is high. An inspection of (IA.15) and (IA.16) shows that both $\hat{c}_n$ and $\hat{c}_s$ are increasing in $\frac{v_h - v_\ell}{\bar{c}}$.

IA.A.7. Proofs

Proof of Lemma 1. We prove this result by contradiction. Suppose the result does not hold. Then, there must exist a cost $c_0 > 0$ that induces effort choice $e = 0$ and a cost $c_1 > c_0$ that induces effort choice $e = 1$. Thus, letting $U(e, m, c)$ denote the manager's expected utility if she chooses effort e and manipulation strategy m when facing a cost of effort c (that she reports truthfully), we must have

$$U(0, m_0, c_0) \geq U(1, m_1, c_0), \tag{IA.19}$$

$$U(1, m_1, c_1) \geq U(0, m_0, c_1), \tag{IA.20}$$

where m_e denotes the manager's optimal manipulation choice for a given effort choice e. Furthermore, let $\hat{U}(e, m, c, c')$ denote a type-c manager's expected utility from choosing e and m when she mimics the behavior of a type-c' manager (i.e., claims to be of type c' and chooses e and m accordingly). Since a type-c_0 manager prefers not to mimic the behavior of a type-c_1 manager, we have

$$U(0, m_0, c_0) \geq \hat{U}(1, m_1, c_0, c_1) > U(1, m_1, c_1), \tag{IA.21}$$

where the last inequality follows from the fact that $c_1 > c_0$. Similarly, since a type-c_1 manager prefers not to mimic the behavior of a type-c_0 manager, we have

$$U(1, m_1, c_1) \geq \hat{U}(0, m_0, c_1, c_0) = U(0, m_0, c_0), \tag{IA.22}$$

where the equality follows from the fact that the effort cost does not directly affect the manager's expected utility if she chooses low effort $e = 0$. Clearly, the two inequalities in (IA.21) and (IA.22) are inconsistent with each other, proving that such a case cannot exist. The result must therefore be true. □

Proof of Lemma 2. For a given effort choice e, the manager's cost of effort does not affect the distribution of the firm's cash flow v or the report r. Thus, if the manager chooses the same effort level e when her effort cost is either c or c', her continuation payoffs and hence her incentives to engage in manipulation are the same in both cases. Furthermore, since the firm's cash flow

v depends on the manger's effort cost only through its effect on the manager's effort choice e, if shareholders find it optimal to induce the manager to manipulate an unfavorable report r_ℓ with probability m when her effort cost is c, doing so must also be optimal when the manager's effort cost is c', as long as the manager's optimal effort choice is the same for c and c'. □

Proof of Lemma 3. From Lemma 1, it follows that all manager types $c \in [0, \hat{c})$ choose the same effort $e = 1$ and hence make the same manipulation decision m_1 (Lemma 2). Thus, these types face the same probability of generating outcome (r, v), for all $r \in \{r_h, r_\ell\}$ and $v \in \{v_h, v_\ell\}$. This means that, under an incentive-compatible mechanism, these types must all receive the same expected compensation. Otherwise, they would all report to be of the type that generates the highest expected compensation. Without loss of generality, we can therefore set $w(r, v|c) = w_1(r, v)$, for all $c \in [0, \hat{c})$. An analogous argument holds for all manager types $c \in (\hat{c}, \bar{c}]$, so that, without loss of generality, we can set $w(r, v|c) = w_0(r, v)$, for all $c \in (\hat{c}, \bar{c}]$. □

Proof of Proposition 1. We derive the optimal no-manipulation contract by first considering a simplified optimization problem and then showing that the solution to this simplified problem is also a solution to the full optimization problem in (IA.2)–(IA.9).

To simplify the notation, let $\pi_{e,m_e}(r, v)$ denote the probability that a report $r \in \{r_h, r_\ell\}$ and a cash flow $v \in \{v_h, v_\ell\}$ is produced when the manager chooses effort level $e \in \{0, 1\}$ and follows the manipulation schedule $m_e \in [0, 1]$. That is,

$$\pi_{1,m_1}(r_h, v_h) = \delta + (1 - \delta)\,\phi\, m_1, \tag{IA.23}$$

$$\pi_{0,m_0}(r_h, v_h) = \lambda(1 - \delta + \delta\,\phi\, m_0), \tag{IA.24}$$

$$\pi_{1,m_1}(r_\ell, v_h) = (1 - \delta)(1 - \phi\, m_1), \tag{IA.25}$$

$$\pi_{0,m_0}(r_\ell, v_h) = \lambda\,\delta(1 - \phi\, m_0), \tag{IA.26}$$

$$\pi_{1,m_1}(r_h, v_\ell) = 0, \tag{IA.27}$$

$$\pi_{0,m_0}(r_h, v_\ell) = (1 - \lambda)(1 - \delta + \delta\,\phi\, m_0), \tag{IA.28}$$

$$\pi_{1,m_1}(r_\ell, v_\ell) = 0, \tag{IA.29}$$

$$\pi_{0,m_0}(r_\ell, v_\ell) = (1 - \lambda)\delta(1 - \phi\, m_0). \tag{IA.30}$$

Also, define $\Delta\pi_{m_0,m_1}(r,v) = \pi_{1,m_1}(r,v) - \pi_{0,m_0}(r,v)$.

We begin by rewriting the principal's objective function in (IA.2). Setting $e = 0$ and $m = m_0$ on the right-hand side of (IA.8) yields

$$\sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) \geq \sum_{r,v} \pi_{0,m_0}(r,v)\, w_0(r,v) + \hat{c} + G(m_0,m_1), \tag{IA.31}$$

where $G(m_0,m_1)$ denotes the difference in the manager's expected manipulation cost when she exerts high rather than low effort, that is, $G(m_0,m_1) = [(1-\delta)m_1 - \delta m_0]\, g$. Similarly, setting $e = 1$ and $m = m_1$ on the right-hand side of (IA.9), we have

$$\sum_{r,v} \pi_{0,m_0}(r,v)\, w_0(r,v) \geq \sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) - \hat{c} - G(m_0,m_1). \tag{IA.32}$$

An inspection of (IA.31) and (IA.32) shows that both constraints must be binding, and the principal's objective function can therefore be written as

$$\min_{\boldsymbol{w}_0,\boldsymbol{w}_1,g} \sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right)\left(\hat{c} + G(m_0,m_1)\right). \tag{IA.33}$$

We next consider a simplified optimization problem. In particular, we solve for the optimal compensation scheme $\boldsymbol{w}_1$ that implements an effort choice characterized by the threshold $\hat{c} \in (0,\bar{c}]$ for a given manipulation schedule $m_0 = m_1 = 0$ and (temporarily) ignore the contracting variables $\boldsymbol{w}_0$ and g, the effort-choice constraint in (IA.7) (for the case when $c > \hat{c}$), and the truth-telling constraints in (IA.8) and (IA.9). Since $G(m_0,m_1) = 0$ when $m_0 = m_1 = 0$, the simplified problem is thus given by

$$\min_{\boldsymbol{w}_1} \sum_{r,v} \pi_{1,0}(r,v)\, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right)\hat{c} \tag{IA.34}$$

$$\text{s.t.} \sum_{r,v} \Delta\pi_{0,0}(r,v)\, w_1(r,v) \geq \hat{c} \tag{IA.35}$$

$$w_1(r,v) \geq 0, \quad \forall r \in \{r_h, r_\ell\}, v \in \{v_h, v_\ell\} \tag{IA.36}$$

Denoting the Lagrangian multiplier of the constraint in (IA.35) by ν and the respective multipliers of the limited liability constraints in (IA.36) by $\xi_{r,v}$, we derive the first order condition of the above optimization problem with respect to $w_1(r,v)$ as

$$\pi_{1,0}(r,v) - \nu\,\Delta\pi_{0,0}(r,v) - \xi_{r,v} = 0, \tag{IA.37}$$

with the complementary slackness condition $\xi_{r,v} w_1(r,v) = 0$. We first show that the IC constraint in (IA.35) must be binding. For the constraint to be satisfied for any $\hat{c} > 0$, the payment $w_1(r_h, v_h)$ or $w_1(r_\ell, v_h)$ must be strictly positive because $\Delta\pi_{0,0}(r_h, v_\ell) < 0$ and $\Delta\pi_{0,0}(r_\ell, v_\ell) < 0$. (Note that $\Delta\pi_{0,0}(r_\ell, v_h)$ may be positive or negative, whereas $\Delta\pi_{0,0}(r_h, v_h)$ is always positive.) If the constraint in (IA.35) were not binding for any $\hat{c} > 0$, the expected compensation in (IA.34) could therefore be reduced by lowering one of these positive payments without violating any constraints. Optimality thus requires that the IC constraint in (IA.35) be binding and that $\nu > 0$. Since $\pi_{1,0}(r,v) = 0$ and $\Delta\pi_{0,0}(r,v) < 0$ for the two outcomes (r_h, v_ℓ) and (r_ℓ, v_ℓ) and since $\nu > 0$, the first order condition in (IA.37) implies that $\xi_{r_h,v_\ell} > 0$ and $\xi_{r_\ell,v_\ell} > 0$. Thus, complementary slackness requires that $w_1(r_h, v_\ell) = w_1(r_\ell, v_\ell) = 0$. Furthermore, for the IC constraint in (IA.35) to hold for $\hat{c} > 0$, at least one of the two remaining payments, $w_1(r_h, v_h)$ and $w_1(r_\ell, v_h)$, must be positive. However, they cannot both be positive: If $\xi_{r_h,v_h} = \xi_{r_\ell,v_h} = 0$, the first order condition in (IA.37) would require that

$$\frac{\delta}{\delta - \lambda(1-\delta)} = \frac{\pi_{1,0}(r_h, v_h)}{\Delta\pi_{0,0}(r_h, v_h)} = \nu = \frac{\pi_{1,0}(r_\ell, v_h)}{\Delta\pi_{0,0}(r_\ell, v_h)} = \frac{1-\delta}{1-\delta-\lambda\delta}, \tag{IA.38}$$

which cannot hold since $\delta > \frac{1}{2}$ and $\lambda > 0$. Consequently, the IC constraint in (IA.35) implies that either

$$w_1(r_h, v_h) = \frac{\hat{c}}{\Delta\pi_{0,0}(r_h, v_h)} = \frac{\hat{c}}{\delta - \lambda(1-\delta)} \quad \text{and} \quad w_1(r_\ell, v_h) = 0 \tag{IA.39}$$

or

$$w_1(r_h, v_h) = 0 \quad \text{and} \quad w_1(r_\ell, v_h) = \frac{\hat{c}}{\Delta\pi_{0,0}(r_\ell, v_h)} = \frac{\hat{c}}{1-\delta-\lambda\delta}. \tag{IA.40}$$

The latter case is only feasible if $1 - \delta - \lambda\delta > 0$, since the payment $w_1(r_\ell, v_h)$ would otherwise be negative and hence violate the limited liability constraint in (IA.36). However, even if the payment scheme $w_1(r_h, v_h) = 0$ and $w_1(r_\ell, v_h) > 0$ is feasible, it is never optimal. To see this, consider an increase in $w_1(r_h, v_h)$ to $\varepsilon_1 > 0$ and a decrease in $w_1(r_\ell, v_h)$ by $\varepsilon_2 > 0$ such that the IC constraint in (IA.35) remains binding, that is,

$$\varepsilon_2 = \frac{\Delta\pi_{0,0}(r_h, v_h)}{\Delta\pi_{0,0}(r_\ell, v_h)}\,\varepsilon_1 = \frac{\delta - \lambda(1-\delta)}{1-\delta-\lambda\delta}\,\varepsilon_1. \tag{IA.41}$$

Such a change in payments would change the manager's expected compensation by

$$\pi_{1,0}(r_h, v_h)\,\varepsilon_1 - \pi_{1,0}(r_\ell, v_h)\,\varepsilon_2 = \delta\,\varepsilon_1 - (1-\delta)\,\frac{\delta-\lambda(1-\delta)}{1-\delta-\lambda\delta}\,\varepsilon_1 = -\frac{\lambda(2\delta-1)}{1-\delta-\lambda\delta}\,\varepsilon_1, \tag{IA.42}$$

which is negative since $\delta > \frac{1}{2}$ and $1-\delta-\lambda\delta > 0$. A positive payment $w_1(r_\ell, v_h)$ can therefore not be optimal. The optimal compensation scheme is hence given by $w_1(r_h, v_h) = \frac{\hat{c}}{\delta-\lambda(1-\delta)}$ and $w_1(r_\ell, v_h) = w_1(r_h, v_\ell) = w_1(r_\ell, v_\ell) = 0$. This is intuitive: The expected compensation in (IA.34) is minimized if the manager receives a positive payment only in the state of nature with the highest likelihood ratio $\frac{\pi_{1,0}(r,v)}{\pi_{0,0}(r,v)}$, which is state (r_h, v_h) in which both the report and the terminal cash flow signal high managerial effort.

Now consider the "no-manipulation" contract $\mathcal{C}^n = (\boldsymbol{w}_0^n, \boldsymbol{w}_1^n, g^n)$ with $w_1^n(r_h, v_h) = \frac{\hat{c}}{\delta-\lambda(1-\delta)}$ and $w_1^n(r_\ell, v_h) = w_1^n(r_h, v_\ell) = w_1^n(r_\ell, v_\ell) = 0$ as above, $w_0^n(r, v) = w_1^n(r, v)$ for all $r \in \{r_h, r_\ell\}$ and $v \in \{v_h, v_\ell\}$, and $g^n \geq \phi\, w_1^n(r_h, v_h)$. Since $\boldsymbol{w}_0$ and g are not part of the simplified problem, this contract is clearly a solution to the simplified problem in (IA.34)–(IA.36). Furthermore, since the objective functions in (IA.33) and (IA.34) are identical when $m_0 = m_1 = 0$ and since the constraints in (IA.35) and (IA.36) are implied by the constraints in (IA.6) and (IA.3), respectively, the contract $\mathcal{C}^n$ is also a solution to the full optimization problem characterized in Section IA.A.4 if it satisfies the additional constraints in (IA.3)–(IA.9).

The contract $\mathcal{C}^n$ clearly satisfies the nonnegativity constraints in (IA.3). Furthermore, any $g^n \geq \phi\, w_1^n(r_h, v_h)$ satisfies the manipulation incentive constraints in (IA.4) and (IA.5) when $m_0 = m_1 = 0$.

Since $g^n \geq \phi\, w_1^n(r_h, v_h)$, the right-hand side of (IA.6) is maximized by setting $m = 0$: The expected gain from manipulating, $\lambda\delta\phi\, w_1^n(r_h, v_h)$, is lower than the expected cost, δg^n. The constraint in (IA.6) then becomes identical to the constraint in (IA.35) and is binding. The right-hand side of (IA.7) is also maximized by setting $m = 0$: the expected gain from manipulating, $(1-\delta)\phi\, w_0^n(r_h, v_h)$, cannot exceed the expected cost, $(1-\delta)g^n$, when $g^n \geq \phi\, w_1^n(r_h, v_h)$. Since $\boldsymbol{w}_0^n = \boldsymbol{w}_1^n$, this means that the expression on the right-hand side of (IA.7) is identical to the expression on the left-hand side of (IA.6) when $m_1 = 0$. Furthermore, the expression on the left-hand side of (IA.7) is identical to the expression on the right-hand side of (IA.6) when $m_0 = 0$ because the right-hand side of (IA.6) is maximized by setting $m = 0$, as demonstrated above. Thus, the result that (IA.6) is binding implies that (IA.7) is also binding.

The truth-telling constraint in (IA.8) is implied by the constraint in (IA.6) when $e = 0$ on the right-hand side of (IA.8). To see this, note that, for $c = \hat{c}$, (IA.6) is identical to (IA.8) when $e = 0$ because $\boldsymbol{w}_0^n = \boldsymbol{w}_1^n$. Thus, (IA.8) must be satisfied for all $c \leq \hat{c}$ when $e = 0$. When $e = 1$, the constraint in (IA.8) is (weakly) more restrictive when $m = 0$ on the right-hand side: the expected gain from manipulating is $(1-\delta)\phi\, w_0^n(r_h, v_h)$ and hence cannot exceed the expected cost of $(1-\delta)g^n$ since $g^n \geq \phi\, w_1^n(r_h, v_h)$. This means that the constraint is trivially satisfied when $e = 1$ because, for $m = 0$ (and $m_1 = 0$), the expression on the left-hand side equals the expression on the right-hand side. Similarly, the truth-telling constraint in (IA.9) is implied by the constraint in (IA.7) when $e = 1$ on the right-hand side of (IA.9). To see this, note that, for $c = \hat{c}$, (IA.7) is identical to (IA.9) when $e = 1$ because $\boldsymbol{w}_0^n = \boldsymbol{w}_1^n$. Thus, (IA.9) must be satisfied for all $c \geq \hat{c}$ when $e = 1$. When $e = 0$, the constraint in (IA.9) is (weakly) more restrictive when $m = 0$ on the right-hand side: the expected gain from manipulating is $\lambda\delta\phi\, w_1^n(r_h, v_h)$ and hence is lower than the expected cost of δg^n since $g^n \geq \phi\, w_1^n(r_h, v_h)$. This means that the constraint is trivially satisfied when $e = 0$ because, for $m = 0$ (and $m_0 = 0$), the expression on the left-hand side equals the expression on the right-hand side. □

Proof of Proposition 2. The derivation of the optimal contract that induces manipulation by the manager when she exerted high effort but not when she exerted low effort (i.e., when $c < \hat{c}$) is similar to that of the optimal no-manipulation contract. We again first consider a simplified optimization problem that minimizes the cost of implementing an effort choice characterized by the threshold $\hat{c}$ for a given manipulation schedule $m_0 = 0$ and $m_1 = 1$ and then show that its solution is also a solution to the full optimization problem in (IA.2)–(IA.9). The simplified problem consists of the objective function in (IA.33) (ignoring the contracting variable $\boldsymbol{w}_0$), which is equivalent to the objective function in (IA.2) as demonstrated in the proof of Proposition 1, the effort-choice constraint in (IA.6) for the case when $c < \hat{c}$ (both for $m = 0$ and $m = 1$ on the right-hand side), and the nonnegativity constraint for $\boldsymbol{w}_1$ in (IA.3). Since $G(m_0, m_1) = (1-\delta)g \geq 0$ when $m_0 = 0$ and $m_1 = 1$, the simplified problem is thus given by

$$\min_{\boldsymbol{w_1},g} \sum_{r,v} \pi_{1,1}(r,v)\, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right)(\hat{c} + (1-\delta)g) \tag{IA.43}$$

$$\text{s.t.} \sum_{r,v} \Delta\pi_{0,1}(r,v)\, w_1(r,v) \geq \hat{c} + (1-\delta)g \tag{IA.44}$$

$$\sum_{r,v} \Delta\pi_{1,1}(r,v)\, w_1(r,v) \geq \hat{c} + (1-2\delta)g \tag{IA.45}$$

$$w_1(r,v) \geq 0, \quad \forall r \in \{r_h, r_\ell\}, v \in \{v_h, v_\ell\} \tag{IA.46}$$

Denoting the Lagrangian multiplier of the constraint in (IA.44) by ν, the multiplier of the constraint in (IA.45) by μ, and the respective multipliers of the limited liability constraints in (IA.46) by $\xi_{r,v}$, we derive the first order condition of the above optimization problem with respect to $w_1(r,v)$ as

$$\pi_{1,1}(r,v) - \nu\, \Delta\pi_{0,1}(r,v) - \mu\, \Delta\pi_{1,1}(r,v) - \xi_{r,v} = 0, \tag{IA.47}$$

with the complementary slackness condition $\xi_{r,v} w_1(r,v) = 0$, and the first order condition with respect to g as

$$-\left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta) + \nu(1-\delta) + \mu(1-2\delta) = 0. \tag{IA.48}$$

We first show that it is optimal to set $w_1(r_h, v_\ell) = w_1(r_\ell, v_\ell) = 0$. Suppose this is not the case (i.e., $w_1(r, v_\ell) > 0$ for $r = r_h$ or $r = r_\ell$). If $w_1(r, v_\ell) > 0$, complementary slackness requires that $\xi_{r,v_\ell} = 0$. But since $\pi_{1,1}(r,v) = 0$, $\Delta\pi_{0,1}(r,v) < 0$, and $\Delta\pi_{1,1}(r,v) < 0$ for the two outcomes (r_h, v_ℓ) and (r_ℓ, v_ℓ), this implies that the first order condition in (IA.47) can only be satisfied if $\nu = \mu = 0$ (the multipliers have to be nonnegative), which means that the IC constraints in (IA.44) and (IA.45) are not binding. This, in turn, implies that it is uniquely optimal to set $w_1(r_h, v_h) = w_1(r_\ell, v_h) = 0$ because $\pi_{1,1}(r_h, v_h) > 0$ and $\pi_{1,1}(r_\ell, v_h) > 0$. But this makes it impossible to elicit high effort for any nonzero $\hat{c}$: since $\Delta\pi_{0,1}(r_h, v_\ell) < 0$ and $\Delta\pi_{0,1}(r_\ell, v_\ell) < 0$, (IA.44) is violated if $w_1(r_h, v_h) = w_1(r_\ell, v_h) = 0$. Thus, we must have that $w_1(r_h, v_\ell) = w_1(r_\ell, v_\ell) = 0$.

We next argue that the IC constraints in (IA.44) and (IA.45) must both be binding. Suppose this is not the case. If the constraint in (IA.44) is slack, we must have $\nu = 0$. The first order condition in (IA.48) then implies that $\mu < 0$ (since $\delta > \frac{1}{2}$). But this violates the condition that the multiplier μ has to be nonnegative at the optimum. Thus, the constraint in (IA.44) must be binding. Similarly, if the constraint in (IA.45) is slack, we must have $\mu = 0$. Since a payment $w_1(r, v)$ can only be strictly positive if $\xi_{r,v} = 0$, the first order condition in (IA.47) then implies that $\nu = \frac{\pi_{1,1}(r,v)}{\Delta\pi_{0,1}(r,v)} = \frac{\pi_{1,1}(r,v)}{\pi_{1,1}(r,v)-\pi_{0,0}(r,v)}$. However, this expression either exceeds one (if $\pi_{1,1}(r, v) > \pi_{0,0}(r, v) > 0$) or it is nonpositive (if $\pi_{1,1}(r, v) < \pi_{0,0}(r, v)$). In both cases, it violates the first order condition in (IA.48) when $\mu = 0$, which requires that $\nu = 1 - \frac{\hat{c}}{\bar{c}} \in (0, 1]$ for any nonzero $\hat{c}$. Thus, the constraint in (IA.45) must be binding.

Since both IC constraints in (IA.44) and (IA.45) must be binding at the optimum, we obtain the following expression for g by subtracting (IA.45) from (IA.44) (and using the fact that $w_1(r_h, v_\ell) = w_1(r_\ell, v_\ell) = 0$):

$$g = \frac{1}{\delta} \sum_{r,v} \big(\Delta\pi_{0,1}(r, v) - \Delta\pi_{1,1}(r, v)\big)\ w_1(r, v) \tag{IA.49}$$

$$= \frac{1}{\delta} \sum_{r,v} \big(\pi_{0,1}(r, v) - \pi_{0,0}(r, v)\big)\ w_1(r, v) \tag{IA.50}$$

$$= \lambda\phi \left(w_1(r_h, v_h) - w_1(r_\ell, v_h)\right). \tag{IA.51}$$

Note that, with this choice of g, the two IC constraints in (IA.44) and (IA.45) become identical. We can therefore drop one of the constraints. Substituting g into the objective function in (IA.43) and the constraint in (IA.44), we can rewrite the optimization problem as

$$\min_{\boldsymbol{w}_1} \sum_{r,v} \pi_{1,1}(r, v)\, w_1(r, v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \Big[\hat{c} + (1-\delta)\lambda\phi \left(w_1(r_h, v_h) - w_1(r_\ell, v_h)\right)\Big] \tag{IA.52}$$

$$\text{s.t.} \sum_{r,v} \Delta\pi_{0,1}(r, v)\, w_1(r, v) = \hat{c} + (1-\delta)\lambda\phi \left(w_1(r_h, v_h) - w_1(r_\ell, v_h)\right) \tag{IA.53}$$

$$w_1(r_h, v_h) \geq 0, w_1(r_\ell, v_h) \geq 0, w_1(r_h, v_\ell) = 0, w_1(r_\ell, v_\ell) = 0 \tag{IA.54}$$

As before, denote the Lagrangian multiplier of the constraint in (IA.53) by ν and the multipliers of the limited liability constraints by ξ_{r_h,v_h} and ξ_{r_ℓ,v_h}. The first order conditions with respect to

$w_1(r_h, v_h)$ and $w_1(r_\ell, v_h)$ are then

$$\delta + (1-\delta)\phi - \left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta)\lambda\phi - \nu\left[\delta - (1-\delta)(\lambda - \phi + \lambda\phi)\right] - \xi_{r_h,v_h} = 0, \tag{IA.55}$$

$$(1-\delta)(1-\phi) + \left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta)\lambda\phi - \nu\left[(1-\delta)(1-\phi+\lambda\phi) - \lambda\delta\right] - \xi_{r_\ell,v_h} = 0, \tag{IA.56}$$

where we have substituted in the expressions for $\pi_{0,0}(r,v)$ and $\pi_{1,1}(r,v)$ from (IA.23)–(IA.26). For the IC constraint in (IA.53) to hold for $\hat{c} > 0$, at least one of the payments $w_1(r_h, v_h)$ and $w_1(r_\ell, v_h)$ must be positive. However, they cannot both be positive. If they were, complementary slackness would require that $\xi_{r_h,v_h} = \xi_{r_\ell,v_h} = 0$. But then the first order conditions in (IA.55) and (IA.56) would imply that

$$\frac{\delta + (1-\delta)\phi - \left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta)\lambda\phi}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} = \frac{(1-\delta)(1-\phi) + \left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta)\lambda\phi}{(1-\delta)(1-\phi+\lambda\phi) - \lambda\delta}, \tag{IA.57}$$

or, equivalently, that

$$\frac{\hat{c}}{\bar{c}} = 1 + \frac{2\delta - 1}{(1-\delta)(1-\lambda)\phi}, \tag{IA.58}$$

which cannot be the case because $\delta > \frac{1}{2}$ and $\hat{c} \leq \bar{c}$. Consequently, the IC constraint in (IA.53) implies that either

$$w_1(r_h, v_h) = \frac{\hat{c}}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} \quad \text{and} \quad w_1(r_\ell, v_h) = 0 \tag{IA.59}$$

or

$$w_1(r_h, v_h) = 0 \quad \text{and} \quad w_1(r_\ell, v_h) = \frac{\hat{c}}{(1-\delta)(1-\phi+\lambda\phi) - \lambda\delta}. \tag{IA.60}$$

In the former case, the payment $w_1(r_h, v_h)$ is positive because $\delta > \frac{1}{2}$. In the latter case, the payment $w_1(r_\ell, v_h)$ is positive only if $(1-\delta)(1-\phi+\lambda\phi) - \lambda\delta > 0$. Thus, the latter payment scheme may not be feasible because it may violate the limited liability constraint in (IA.54). However, even if it is feasible, this payment scheme is never optimal. To see this, consider an increase in $w_1(r_h, v_h)$ to $\varepsilon_1 > 0$ and a decrease in $w_1(r_\ell, v_h)$ by $\varepsilon_2 > 0$ such that the IC constraint in (IA.53) remains binding, that is,

$$\varepsilon_2 = \frac{\Delta\pi_{0,1}(r_h, v_h) - (1-\delta)\lambda\phi}{\Delta\pi_{0,1}(r_\ell, v_h) + (1-\delta)\lambda\phi}\,\varepsilon_1 = \frac{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}{(1-\delta)(1-\phi+\lambda\phi) - \lambda\delta}\,\varepsilon_1. \tag{IA.61}$$

Such a change in payments would change the manager's expected compensation by

$$\left[\pi_{1,1}(r_h, v_h) - \left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta)\lambda\phi\right]\varepsilon_1 - \left[\pi_{1,1}(r_\ell, v_h) + \left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta)\lambda\phi\right]\varepsilon_2$$

$$= \left[\delta + (1-\delta)\left(\phi - \left(1 - \frac{\hat{c}}{\bar{c}}\right)\lambda\phi\right)\right]\varepsilon_1 - \left[(1-\delta)\left(1 - \phi + \left(1 - \frac{\hat{c}}{\bar{c}}\right)\lambda\phi\right)\right]\varepsilon_2 \tag{IA.62}$$

$$= -\frac{\lambda\left[2\delta - 1 + \left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta)(1-\lambda)\phi\right]}{(1-\delta)(1-\phi+\lambda\phi) - \lambda\delta}\,\varepsilon_1, \tag{IA.63}$$

which is negative since $\delta > \frac{1}{2}$ and $(1-\delta)(1-\phi+\lambda\phi) - \lambda\delta > 0$. A positive payment $w_1(r_\ell, v_h)$ can therefore not be optimal. The optimal solution to the problem in (IA.43)–(IA.46) is thus given by the compensation scheme $w_1(r_h, v_h) = \frac{\hat{c}}{\delta - (1-\delta)(\lambda-\phi+\lambda\phi)}$, $w_1(r_\ell, v_h) = w_1(r_h, v_\ell) = w_1(r_\ell, v_\ell) = 0$, and the manipulation cost $g = \lambda\phi\, w_1(r_h, v_h)$.

Now consider the selective-manipulation contract $\mathcal{C}^s = (\boldsymbol{w}_0^s, \boldsymbol{w}_1^s, g^s)$ with $w_1^s(r_h, v_h) = \frac{\hat{c}}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}$, $w_1^s(r_\ell, v_h) = w_1^s(r_h, v_\ell) = w_1^s(r_\ell, v_\ell) = 0$, $g^s = \lambda\phi\, w_1^s(r_h, v_h)$ as above, and $w_0^s(r, v) = w_1^s(r, v)$ for all $r \in \{r_h, r_\ell\}$ and $v \in \{v_h, v_\ell\}$. Since $\boldsymbol{w}_0$ is not part of the simplified problem, this contract is clearly a solution to the simplified problem in (IA.43)–(IA.46). Furthermore, since the objective functions in (IA.33) and (IA.43) are identical when $m_0 = 0$ and $m_1 = 1$ and since the constraints in (IA.44), (IA.45), and (IA.46) are implied by the constraints in (IA.6) and (IA.3), the contract $\mathcal{C}^s$ is also a solution to the full optimization problem characterized in Section IA.A.4 if it satisfies the additional constraints in (IA.3)–(IA.9).

The contract $\mathcal{C}^s$ clearly satisfies the nonnegativity constraints in (IA.3). Furthermore, $g^s = \lambda\phi\, w_1^s(r_h, v_h)$ satisfies the manipulation incentive constraints in (IA.4) and (IA.5) when $m_0 = 0$ and $m_1 = 1$ ((IA.4) is slack and (IA.5) is binding).

Since $g^s = \lambda\phi\, w_1^s(r_h, v_h)$, the right-hand side of (IA.6) is the same for $m = 0$ and $m = 1$: the expected gain from manipulating, $\lambda\delta\phi\, w_1^s(r_h, v_h)$, is equal to the expected cost, δg^s. The constraint in (IA.6) then becomes identical to the constraint in (IA.44) and is binding. The right-hand side of (IA.7) is maximized by setting $m = 1$: the expected gain from manipulating, $(1-\delta)\phi\, w_0^s(r_h, v_h)$, exceeds the expected cost, $(1-\delta)g^s = (1-\delta)\lambda\phi\, w_1^s(r_h, v_h)$. Since $\boldsymbol{w}_0^s = \boldsymbol{w}_1^s$, this means that the expression on the right-hand side of (IA.7) is identical to the expression on the left-hand side of (IA.6) when $m_1 = 1$. Furthermore, the expression on the left-hand side of (IA.7) is identical to the expression on the right-hand side of (IA.6) when $m_0 = 0$ because the right-hand side of (IA.6) is maximized by setting $m = 0$, as demonstrated above. Thus, the result that (IA.6) is binding implies that (IA.7) is also binding.

The truth-telling constraint in (IA.8) is implied by the constraint in (IA.6) when $e = 0$ on the right-hand side of (IA.8). To see this, note that, for $c = \hat{c}$, (IA.6) is identical to (IA.8) when $e = 0$ because $\boldsymbol{w}_0^s = \boldsymbol{w}_1^s$. Thus, (IA.8) must be satisfied for all $c \leq \hat{c}$ when $e = 0$. When $e = 1$, the constraint in (IA.8) is more restrictive when $m = 1$ on the right-hand side: the expected gain from manipulating is $(1-\delta)\phi\, w_0^s(r_h, v_h)$ and hence exceeds the expected cost of $(1-\delta)g^s$ since $g^s = \lambda\phi\, w_1^s(r_h, v_h)$. This means that the constraint is trivially satisfied when $e = 1$ because, for $m = 1$ (and $m_1 = 1$), the expression on the left-hand side equals the expression on the right-hand side. Similarly, the truth-telling constraint in (IA.9) is implied by the constraint in (IA.7) when $e = 1$ on the right-hand side of (IA.9). To see this, note that, for $c = \hat{c}$, (IA.7) is identical to (IA.9) when $e = 1$ because $\boldsymbol{w}_0^s = \boldsymbol{w}_1^s$. Thus, (IA.9) must be satisfied for all $c \geq \hat{c}$ when $e = 1$. When $e = 0$, the constraint in (IA.9) is (weakly) more restrictive when $m = 0$ on the right-hand side: the expected gain from manipulating is $\lambda\delta\phi\, w_1^s(r_h, v_h)$ and hence equals the expected cost of δg^s since $g^s = \lambda\phi\, w_1^s(r_h, v_h)$. This means that the constraint is trivially satisfied when $e = 0$ because, for $m = 0$ (and $m_0 = 0$), the expression on the left-hand side equals the expression on the right-hand side. □

Proof of Proposition 3. We prove this result by showing (i) that any contract that induces manipulation decisions $m_0 > 0$ and $m_1 = 0$ is dominated by the no-manipulation contract $\mathcal{C}^n$ derived in Proposition 1, (ii) that any contract that induces manipulation decisions $m_0 > 0$ and $m_1 = 1$ is dominated by the selective-manipulation contract $\mathcal{C}^s$ derived in Proposition 2, and (iii) that any contract that induces manipulation decisions $m_0 \geq 0$ and $m_1 \in (0, 1)$ is dominated by the no-manipulation contract $\mathcal{C}^n$ as well.

As shown in the proof of Proposition 1, the manager's expected compensation can be written as

$$\sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right)\left(\hat{c} + (1-\delta) g m_1 - \delta g m_0\right). \tag{IA.64}$$

Since $g \geq 0$, the manager's expected compensation if $m_0 > 0$ can therefore not be lower than

$$\sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right)\left(\hat{c} + (1-\delta) g m_1\right), \tag{IA.65}$$

the expected compensation if $m_0 = 0$.

First, consider the case where $m_0 > 0$ and $m_1 = 0$. The IC constraint in (IA.6) then requires that

$$\sum_{r,v} \Delta\pi_{0,0}(r,v)\, w_1(r,v) \geq \hat{c}. \tag{IA.66}$$

This constraint is identical to the IC constraint in (IA.35) of the simplified problem analyzed in the proof of Proposition 1. Furthermore, the objective function of that problem in (IA.34) is identical to (IA.65) if $m_1 = 0$. The optimal no-manipulation contract $\mathcal{C}^n$ thus minimizes (the lower bound of) the manager's expected compensation in (IA.65) (with $m_1 = 0$) subject to the IC constraint in (IA.66) and the limited liability constraints $w_1(r,v) \geq 0$. But these constraints also have to be satisfied by any contract that implements the manipulation decisions $m_0 > 0$ and $m_1 = 0$. Furthermore, the additional constraints in (IA.3)–(IA.9) cannot reduce the manager's expected compensation. Hence, any contract that implements the cost threshold $\hat{c}$ and the manipulation decisions $m_0 > 0$ and $m_1 = 0$ is dominated by the no-manipulation contract $\mathcal{C}^n$.

Next, consider the case where $m_0 > 0$ and $m_1 = 1$. The IC constraint in (IA.6) then requires that

$$\sum_{r,v} \Delta\pi_{0,1}(r,v)\, w_1(r,v) \geq \hat{c} + (1-\delta) g, \tag{IA.67}$$

and that

$$\sum_{r,v} \Delta\pi_{1,1}(r,v)\, w_1(r,v) \geq \hat{c} + (1-2\delta) g. \tag{IA.68}$$

These constraints are identical to the IC constraints in (IA.44) and (IA.45) of the simplified problem analyzed in the proof of Proposition 2. Furthermore, the objective function of that problem in (IA.43) is identical to (IA.65) if $m_1 = 1$. The optimal selective-manipulation contract $\mathcal{C}^s$ thus minimizes (the lower bound of) the manager's expected compensation in (IA.65) (with $m_1 = 1$) subject to the IC constraints in (IA.67) and (IA.68) and the limited liability constraints $w_1(r,v) \geq 0$. But these constraints also have to be satisfied by any contract that implements the manipulation decisions $m_0 > 0$ and $m_1 = 1$. Furthermore, the additional constraints in (IA.3)–(IA.9) cannot reduce the manager's expected compensation. Hence, any contract that implements the cost threshold $\hat{c}$ and the manipulation decisions $m_0 > 0$ and $m_1 = 1$ is dominated by the selective-manipulation contract $\mathcal{C}^s$.

Finally, consider the case where $m_0 \geq 0$ and $m_1 \in (0,1)$. In this case, a manager who chose the high effort level must be indifferent between choosing $m_1 = 0$ and $m_1 = 1$. Thus, the IC constraints in (IA.4) and (IA.6) require that

$$g = \phi\left(w_1(r_h, v_h) - w_1(r_\ell, v_h)\right) \tag{IA.69}$$

and that

$$\sum_{r,v} \Delta\pi_{0,m_1}(r,v)\, w_1(r,v) \geq \hat{c} + (1-\delta) g m_1. \tag{IA.70}$$

Since $\Delta\pi_{0,m_1}(r_h, v_h) = \delta + (1-\delta)\phi\, m_1 - \lambda(1-\delta)$ and $\Delta\pi_{0,m_1}(r_\ell, v_h) = (1-\delta)(1-\phi\, m_1) - \lambda\delta$, substituting (IA.69) into (IA.70) yields

$$\sum_{r,v} \Delta\pi_{0,0}(r,v)\, w_1(r,v) \geq \hat{c}, \tag{IA.71}$$

which is identical to the effort IC constraint in (IA.35) of the simplified problem considered in the proof of Proposition 1. Furthermore, using (IA.69) we can write the lower bound of the manager's expected compensation in (IA.65) as

$$\sum_{r,v} \pi_{1,m_1}(r,v)\, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right)\left[\hat{c} + (1-\delta)\,\phi\left(w_1(r_h,v_h) - w_1(r_\ell,v_h)\right) m_1\right], \tag{IA.72}$$

which is equivalent to

$$\sum_{r,v} \pi_{1,0}(r,v)\, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right)\hat{c} + \frac{\hat{c}}{\bar{c}}(1-\delta)\,\phi\left(w_1(r_h,v_h) - w_1(r_\ell,v_h)\right) m_1, \tag{IA.73}$$

because $\pi_{1,m_1}(r_h, v_h) = \delta + (1-\delta)\phi\, m_1$ and $\pi_{1,m_1}(r_\ell, v_h) = (1-\delta)(1-\phi\, m_1)$. Since $g \geq 0$ and hence

$w_1(r_h, v_h) \geq w_1(r_\ell, v_h)$, the manager's expected compensation can therefore not be lower than

$$\sum_{r,v} \pi_{1,0}(r,v)\, w_1(r,v) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \hat{c}, \tag{IA.74}$$

the expected compensation in the no-manipulation case given by (IA.34). The optimal no-manipulation contract $\mathcal{C}^n$ thus minimizes (the lower bound of) the manager's expected compensation subject to the IC constraint in (IA.71) and the limited liability constraints $w_1(r,v) \geq 0$. But these constraints also have to be satisfied by any contract that implements the manipulation decisions $m_0 \geq 0$ and $m_1 \in (0,1)$. Furthermore, the additional constraints in (IA.3)–(IA.9) cannot reduce the manager's expected compensation. Hence, any contract that implements the cost threshold $\hat{c}$ and the manipulation decisions $m_0 \geq 0$ and $m_1 \in (0,1)$ is dominated by the no-manipulation contract $\mathcal{C}^n$. □

Proof of Proposition 4. From the objective function in (IA.33) and the compensation scheme in Proposition 1, it follows that, for any cost threshold $\hat{c} \in [0, \bar{c}]$, the expected compensation required to induce the manager to exert high effort if and only if $c \leq \hat{c}$ and to follow the no-manipulation schedule $m_0 = m_1 = 0$ is given by

$$\mathbb{E}w^n(\hat{c}) = \pi_{1,0}(r_h, v_h)\, w_1^n(r_h, v_h) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \hat{c} = \left(\frac{\lambda(1-\delta)}{\delta - \lambda(1-\delta)} + \frac{\hat{c}}{\bar{c}}\right) \hat{c}. \tag{IA.75}$$

Similarly, from (IA.33) and Proposition 2, it follows that the expected compensation necessary to induce the manager to exert high effort if and only if $c \leq \hat{c}$ and to follow the selective-manipulation schedule $m_0 = 0$ and $m_1 = 1$ is given by

$$\begin{aligned}\mathbb{E}w^s(\hat{c}) &= \pi_{1,1}(r_h, v_h)\, w_1^s(r_h, v_h) - \left(1 - \frac{\hat{c}}{\bar{c}}\right) \left(\hat{c} + (1-\delta) g^s\right) \\ &= \left(\frac{\delta + (1-\delta)\phi - \left(1 - \frac{\hat{c}}{\bar{c}}\right)(1-\delta)\lambda\phi}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} - 1 + \frac{\hat{c}}{\bar{c}}\right) \hat{c} \qquad \text{(IA.76)} \\ &= \left(\frac{\left[1 + \left(\frac{\hat{c}}{\bar{c}}\right)\phi\right](1-\delta)\lambda}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} + \frac{\hat{c}}{\bar{c}}\right) \hat{c}. \qquad \text{(IA.77)}\end{aligned}$$

For any cost threshold $\hat{c} > 0$, the expressions in (IA.75) and (IA.77) imply that $\mathbb{E}w^s(\hat{c}) \underset{(>)}{<} \mathbb{E}w^n(\hat{c})$ if and only if

$$\frac{\left[1 + \left(\frac{\hat{c}}{\bar{c}}\right)\phi\right](1-\delta)\lambda}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} \underset{(>)}{<} \frac{\lambda(1-\delta)}{\delta - \lambda(1-\delta)}, \tag{IA.78}$$

or, equivalently, if and only if

$$\frac{\hat{c}}{\bar{c}} \underset{(>)}{<} \frac{(1-\delta)(1-\lambda)}{\delta - \lambda(1-\delta)}. \tag{IA.79}$$

□

Proof of Proposition 5. For a given cost threshold $\hat{c} \in [0, \bar{c}]$, the value of the firm (net of the cost of managerial compensation) under the optimal no-manipulation contract $\mathcal{C}^n$ specified in Proposition 1 is given by

$$V_n(\hat{c}) = \left(\frac{\hat{c}}{\bar{c}}\right) v_h + \left(1 - \frac{\hat{c}}{\bar{c}}\right)\left(\lambda\, v_h + (1-\lambda)\, v_\ell\right) - \mathbb{E}w^n(\hat{c}), \tag{IA.80}$$

where the expected compensation $\mathbb{E}w^n(\hat{c})$ is given by (IA.75) in the proof of Proposition 4. Substituting the expression in (IA.75) into the above equation yields

$$V_n(\hat{c}) = V_0 + (1-\lambda)(v_h - v_\ell)\left(\frac{\hat{c}}{\bar{c}}\right) - \left(\frac{\lambda(1-\delta)}{\delta - \lambda(1-\delta)} + \frac{\hat{c}}{\bar{c}}\right)\hat{c}, \tag{IA.81}$$

where $V_0 = \lambda v_h + (1-\lambda) v_\ell$. Note that V_n is a strictly concave function of $\hat{c}$ with $V_n'(\bar{c}) < (1-\lambda)(v_h - v_\ell)/\bar{c} - 2 < 0$ because, by assumption, $(1-\lambda)(v_h - v_\ell) < \bar{c}$. Thus, if $V_n'(0) \geq 0$, the optimal cost threshold that maximizes V_n is uniquely determined by the first order condition

$$\hat{c}_n = \frac{1}{2}\left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta - \lambda(1-\delta)}\right). \tag{IA.82}$$

If $V_n'(0) < 0$, the above expression is negative and the optimal cost threshold is zero.

Similarly, the value of the firm under the optimal selective-manipulation contract $\mathcal{C}^s$ specified in Proposition 2 is given by

$$V_s(\hat{c}) = \left(\frac{\hat{c}}{\bar{c}}\right) v_h + \left(1 - \frac{\hat{c}}{\bar{c}}\right)\left(\lambda\, v_h + (1-\lambda)\, v_\ell\right) - \mathbb{E}w^s(\hat{c}), \tag{IA.83}$$

where the expected compensation $\mathbb{E}w^s(\hat{c})$ is given by (IA.77). Substituting the expression in (IA.77) into the above equation yields

$$V_s(\hat{c}) = V_0 + (1-\lambda)(v_h - v_\ell)\left(\frac{\hat{c}}{\bar{c}}\right) - \left(\frac{\left[1 + \left(\frac{\hat{c}}{\bar{c}}\right)\phi\right]\lambda(1-\delta)}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} + \frac{\hat{c}}{\bar{c}}\right)\hat{c}, \tag{IA.84}$$

where, as before, $V_0 = \lambda v_h + (1-\lambda)v_\ell$. Similarly to V_n, V_s is a strictly concave function of $\hat{c}$ with $V_s'(\bar{c}) < (1-\lambda)(v_h - v_\ell)/\bar{c} - 2 < 0$. Thus, if $V_s'(0) \geq 0$, the optimal cost threshold that maximizes V_s is uniquely determined by the first order condition

$$\hat{c}_s = \frac{1}{2}\left(\frac{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}{\delta - (1-\delta)(\lambda - \phi)}\right)\left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}\right). \tag{IA.85}$$

If $V_s'(0) < 0$, the above expression is negative and the optimal cost threshold is zero. □

Proof of Corollary 1. From Proposition 3, we know that, for any cost threshold $\hat{c}$, the optimal contract is either the no-manipulation contract $\mathcal{C}^n$ defined in Proposition 1 or the selective-manipulation contract $\mathcal{C}^s$ defined in Proposition 2. Proposition 5 shows that the optimal cost threshold under the no-manipulation contract, $\hat{c}_n$, is zero whenever the optimal cost threshold under the selective-manipulation contract, $\hat{c}_s$, is zero. Thus, a necessary and sufficient condition for the optimal contract to induce high effort is that $\hat{c}_s > 0$, which is equivalent to the condition in (IA.17). □

Proof of Proposition 6. From Proposition 3, we know that, for any cost threshold $\hat{c}$, the optimal contract is either the no-manipulation contract $\mathcal{C}^n$ defined in Proposition 1 or the selective-manipulation contract $\mathcal{C}^s$ defined in Proposition 2. Furthermore, Proposition 5 shows that firm value under the no-manipulation contract (respectively, the selective-manipulation contract) is maximized at a cost threshold of $\hat{c}_n$ (respectively, $\hat{c}_s$). Thus, to prove the result it suffices to show that $V_s(\hat{c}_s) \geq V_n(\hat{c}_n)$ if and only if (IA.18) is satisfied, where, as in the proof of Proposition 5, $V_n(\hat{c})$ denotes the firm value under the no-manipulation contract and $V_s(\hat{c})$ the firm value under the selective-manipulation contract.

The result that $V_s(\hat{c}_s) \geq V_n(\hat{c}_n)$ trivially holds if $\hat{c}_n = 0$ because $\max\{V_s(\hat{c}_s), V_s(0)\} \geq V_s(0) = V_n(0)$. Furthermore, since the right-hand side of (IA.18) exceeds the right-hand side of (IA.17), it follows that $\hat{c}_s > 0$ if (IA.18) is not satisfied. But if $\hat{c}_s > 0$, the fact that $V_s(\hat{c}_s) < V_n(\hat{c}_n)$ implies that $\hat{c}_n > 0$ as well. Thus, we are left to show that $V_s(\hat{c}_s) \geq V_n(\hat{c}_n)$ if and only if (IA.18) is satisfied in case $\hat{c}_s > 0$ and $\hat{c}_n > 0$.

If $\hat{c}_n > 0$, it follows from (IA.15) and (IA.81) that

$$V_n(\hat{c}_n) = V_0 + \frac{1}{\bar{c}}\left[\left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta - \lambda(1-\delta)}\right)\hat{c}_n - \hat{c}_n^2\right] = V_0 + \frac{\hat{c}_n^2}{\bar{c}}. \tag{IA.86}$$

Similarly, if $\hat{c}_s > 0$, from (IA.16) and (IA.84) we have

$$V_s(\hat{c}_s) = V_0 + \frac{1}{\bar{c}}\left[\left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}\right)\hat{c}_s \right.$$

$$\left. - \left(\frac{\delta - (1-\delta)(\lambda - \phi)}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}\right)\hat{c}_s^2\right] \tag{IA.87}$$

$$= V_0 + \left(\frac{\delta - (1-\delta)(\lambda - \phi)}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}\right)\frac{\hat{c}_s^2}{\bar{c}}. \tag{IA.88}$$

Thus, $V_s(\hat{c}_s) \geq V_n(\hat{c}_n)$ if and only if

$$\frac{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}{\delta - (1-\delta)(\lambda - \phi)}\left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}\right)^2 \geq$$

$$\left((1-\lambda)(v_h - v_\ell) - \frac{\lambda(1-\delta)\,\bar{c}}{\delta - \lambda(1-\delta)}\right)^2. \tag{IA.89}$$

Since $\hat{c}_n$ and $\hat{c}_s$ are positive, we can rewrite this condition as

$$\left(1 - \sqrt{\frac{\delta - (1-\delta)(\lambda - \phi)}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}}\right)\frac{(1-\lambda)(v_h - v_\ell)}{\bar{c}} \geq$$

$$\frac{\lambda(1-\delta)}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)} - \frac{\lambda(1-\delta)}{\delta - \lambda(1-\delta)}\sqrt{\frac{\delta - (1-\delta)(\lambda - \phi)}{\delta - (1-\delta)(\lambda - \phi + \lambda\phi)}}, \tag{IA.90}$$

or, since the term under the square root sign is greater than one, as

$$\frac{(1-\lambda)(v_h - v_\ell)}{\bar{c}} \leq \frac{\frac{\lambda(1-\delta)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} - \frac{\lambda(1-\delta)}{\delta-\lambda(1-\delta)}\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}{1 - \sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}. \tag{IA.91}$$

The term on the right-hand side of (IA.91) can be rearranged as follows:

$$\frac{\frac{\lambda(1-\delta)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} - \frac{\lambda(1-\delta)}{\delta-\lambda(1-\delta)}\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}{1-\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}$$

$$= \frac{1-\delta}{\delta-\lambda(1-\delta)} + \frac{\frac{\lambda(1-\delta)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} - \frac{1-\delta}{\delta-\lambda(1-\delta)} + \left(\frac{1-\delta}{\delta-\lambda(1-\delta)} - \frac{\lambda(1-\delta)}{\delta-\lambda(1-\delta)}\right)\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}{1-\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}} \tag{IA.92}$$

$$= \frac{1-\delta}{\delta-\lambda(1-\delta)} + \frac{\frac{\lambda(1-\delta)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} - \frac{1-\delta}{\delta-\lambda(1-\delta)} + \frac{(1-\delta)(1-\lambda)}{\delta-\lambda(1-\delta)}\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}{1-\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}} \tag{IA.93}$$

$$= \frac{1-\delta}{\delta-\lambda(1-\delta)} + \frac{(1-\delta)(1-\lambda)}{\delta-\lambda(1-\delta)}\left(\frac{\frac{\frac{\lambda(1-\delta)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} - \frac{1-\delta}{\delta-\lambda(1-\delta)}}{\frac{(1-\delta)(1-\lambda)}{\delta-\lambda(1-\delta)}} + \sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}{1-\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}\right) \tag{IA.94}$$

$$= \frac{1-\delta}{\delta-\lambda(1-\delta)} + \frac{(1-\delta)(1-\lambda)}{\delta-\lambda(1-\delta)}\left(\frac{\frac{\lambda}{1-\lambda}\left(\frac{\delta-\lambda(1-\delta)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} - \frac{1}{\lambda}\right) + \sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}{1-\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}\right) \tag{IA.95}$$

$$= \frac{1-\delta}{\delta-\lambda(1-\delta)} + \frac{(1-\delta)(1-\lambda)}{\delta-\lambda(1-\delta)}\left(\frac{-\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)} + \sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}{1-\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}}\right) \tag{IA.96}$$

$$= \frac{1-\delta}{\delta-\lambda(1-\delta)} + \frac{(1-\delta)(1-\lambda)}{\delta-\lambda(1-\delta)}\sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}. \tag{IA.97}$$

Thus, $V_s(\hat{c}_s) \geq V_n(\hat{c}_n)$ if and only if

$$\frac{v_h - v_\ell}{\bar{c}} \leq \frac{1-\delta}{\delta-\lambda(1-\delta)}\left(\frac{1}{1-\lambda} + \sqrt{\frac{\delta-(1-\delta)(\lambda-\phi)}{\delta-(1-\delta)(\lambda-\phi+\lambda\phi)}}\right). \tag{IA.98}$$

□

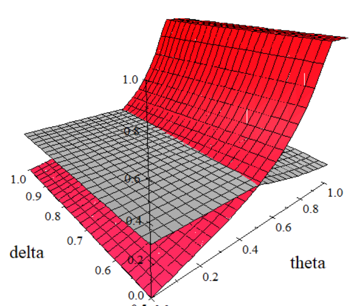
IA.B. Numerical Work

This section of the Internet Appendix provides details about the numerical analysis of how Γ changes in response to changes in λ_1 and λ_0. As we state in Section III.C of the manuscript, *"numerical calculations show that* Γ *increases in* λ_0 *and decreases in* λ_1*."* This conclusion is based on a number of numerical examples that we considered and that we illustrated in a series of animated 2D and 3D plots. We now briefly describe how we created these plots.

As a first step, we take the parameter values from Figure 1 and vary either λ_0 or λ_1. We do so using *Scientific WorkPlace 5.5*, which allows us to create animated GIF files. The two corresponding animated two-dimensional plots (one for changes in λ_0 and one for changes in λ_1) can be found in the folder `IA-C__Animated_GIFs` of the *Replication Code* ZIP file available on the *Journal of Finance* website (see the files `2D_Figure_1_increase_lam0.gif` and `2D_Figure_1_increase_lam1.gif`).

To make the GIFs more easily accessible, the HTML file `view_animated_GIFs.htm` displays all animated GIF files simultaneously (there are 17 animated GIF files in total). The first two animated plots show that Γ increases in λ_0 and decreases in λ_1.

The remaining animated GIFs show the same effects for a broader set of parameter values. They are three-dimensional plots with the variables δ and θ on two axes (with all feasible values shown: $\delta\left(\frac{1}{2},1\right]$, $\theta \in [0,1]$) and with either λ_0 or λ_1 increasing (within their respective feasible ranges). Specifically, in the first seven 3D animated pictures we continuously increase (in the animation) $\lambda_0 \in \left[0,\frac{1}{2}\lambda_1\right)$, with λ_1 fixed at different levels: $\lambda_1 \in \left\{\frac{1}{8},\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{7}{8},\frac{19}{20},\frac{999}{1000}\right\}$. In the remaining 3D animated pictures we continuously increase (in the animation) $\lambda_1 \in (2\lambda_0, 1]$, with λ_0 fixed at different levels: $\lambda_0 \in \left\{\frac{1}{100},\frac{1}{16},\frac{1}{8},\frac{1}{4},\frac{3}{8},\frac{7}{16},\frac{49}{100}\right\}$. These animated pictures suggest that Γ increases in λ_0 and decreases in λ_1. A static example of such a 3D plot is shown here:



Red: $\min\{\Gamma, 1\}$; Gray: $\min\{\Lambda_n, \Lambda_s\}$.

The code for these plots can be found in the LaTeX file `plots_2D_3D.tex` of the *Replication Code* ZIP file available on the *Journal of Finance* website. Note that this code requires the use of *Scientific WorkPlace 5.5.* After opening the file in *Scientific WorkPlace 5.5*, click on a picture, then on the red dot symbol at the bottom right of the picture; this opens the "VCam Plot" window, which allows the export of animated plots as GIF files.