MULTIPLE BANKING AS A COMMITMENT NOT TO RESCUE

Paul Povel

ABSTRACT

We show why investors may prefer not to be a firm's unique lender, even if they are in a strong bargaining position. Some firms need additional funds after a first investment: providing additional funds is rational after the first investment is sunk, but together the two investments are unprofitable. A unique lender will always provide additional funds and make losses. Two creditors can commit not always to provide funds: inefficient negotiations over debt forgiveness may end with a project's liquidation, which is harmful ex post, but helpful ex ante, if it keeps entrepreneurs with nonpromising projects from initially requesting funds.

1. INTRODUCTION

This paper analyzes a bank's incentives to forgive debt and refinance a distressed firm. We compare the decision of a unique lender with that of two banks, which have jointly provided a loan to the firm. We show that banks may prefer such co-financing, even if they enjoy a strong bargaining position relative to the firm. The main difference between single and multiple banking lies in the negotiations that are necessary, if the firm cannot repay its debt but it could profitably be refinanced.

Suppose that refinancing is profitable, once an initial investment is sunk, but that ex ante it is not. Some firms will need refinancing, others not, and the

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creditors would like to finance the latter, only. The entrepreneurs of the respective firms, however, who are informed about their prospective financial needs, are only interested in receiving a loan, irrespective of whether it will be performing well or badly. If the creditors could commit not to refinance a firm, the entrepreneurs with ex ante unprofitable firms would prefer to be inactive, instead of being forced to liquidate their firm prematurely. A single lender cannot credibly commit to being tough, as it is always sequentially rational to refinance a distressed firm, once the initial loan is sunk. We argue that introducing multiplicity on the side of the lenders can make such a commitment possible. Even if they agree on the need to rescue the firm, two lenders will have to bargain about the distribution of the overall loss. Asymmetric information between the banks is the cause of inefficiencies in the rescue decision: with positive probability the firm is not refinanced, and it is liquidated, instead.

There is a large literature now, which analyzes the effects of single or multiple lending on the decisions of a firm. One strand of the literature analyzes the effects that the structure of the creditors' claims has on the possibilities to reorganize a distressed firm. Gertner and Scharfstein (1991) and Detragiache (1994) for instance assume that bonds are held by atomistic investors and therefore cannot be renegotiated. They analyze the effects of different bankruptcy regimes on the possibilities to reorganize a distressed firm.

These effects can be used strategically by a firm, i.e. different financial structures can be used to achieve different goals. Several papers have asked the question why a firm may prefer to have one or many creditors. The difference between the market-based financial system in the U.S. and the bank-based system in Germany and Japan are striking, and an analysis of the relative advantages of the two systems is an important research program.

A frequently stated advantage of the "main bank" financial system in Germany and Japan is that distressed firms are rescued more frequently (see e.g. Hoshi et al., 1990, for the case of Japan, and Edwards & Fischer, 1994, for the case of Germany). Some theoretical papers have analyzed the conditions under which "main bank" finance is more efficient than a system with multiple lenders (see e.g. Dewatripont & Maskin, 1995; Fischer, 1990; von Thadden, 1995). As Edwards and Fischer (1990) conclude, however, these models are not compatible with the empirical evidence for the German case. While in the models at most one "main bank" can emerge, in reality a German firm has more than one "Hausbank." The question to analyze is thus why we may observe more than one nonatomistic lender. Several answers are possible.

First, one could argue that banks are risk averse and want to spread out their risk exposure by sharing risks with their competitors. This is certainly true, but not a very satisfying explanation from a theoretical point of view. Banks are usually thought of as "large," compared with the size of the average firm. They should therefore be able to diversify away most of their risks, as was modeled in Diamond (1984). This makes them de facto risk neutral, and they should not suffer from risk exposure. After all, it is the banks' business to deal with risks and to allocate them optimally, and not to avoid risks. Additionally, it would be interesting to know whether there is more behind multiple banking than mere risk-sharing.

Second, a bank may lack the funds to finance a project. Dewatripont and Maskin (1995) suggested that such smallness could be a solution to the soft budget constraint problem in centralized economies. Inability to finance a project exclusively may be a real problem when firms are very large. However, even in cases when the firms are very small, compared with their banks, we find multiplicity. As before, there is a need for additional explanations.

Third, firms may want to have many banks because this protects them from being exploited by too strong a partner, as was suggested in von Thadden (1992). This third rationale for multiple banking implies that neither the banks nor the firms enjoy exceptionally strong bargaining positions in their relationship. This contrasts with the general perception that in bank-dominated financial systems, banks are in a stronger position. Many situations can occur in which a firm has to rely on its bank or banks and in which the bank can cheaply "punish" earlier unfriendly behaviour.

Finally, some authors analyze the use of multiple claimants, holding different types of securities, in solving agency problems: the investors may have poor incentives either to really monitor their debtor, or to make proper use of their information (e.g. to liquidate a firm). See e.g. Diamond (1993), Berglöf and von Thadden (1995), Dewatripont and Tirole (1994), Rajan and Winton (1995), and Repullo and Suarez (1995).

The present paper offers a rationale for multiplicity, which complements the explanations above. We argue that multiplicity is requested by the banks, who use it as a commitment device for eventual renegotiations of the lending contracts. The inefficiencies that arise in rescue negotiations (the banks have to determine their respective degrees of debt forgiveness) are a threat for entrepreneurs with bad projects. If the inefficiencies are sufficiently strong, this allows the banks to deter nonprofitable projects, and to finance high quality ones, only.

The idea that multiplicity can serve as a commitment device was first stated in Hellwig (1991). Dewatripont and Maskin (1995) analyze the role of "multiple lending" in hardening the "soft budget constraint" of a firm. In their model, however, multiplicity is a credible commitment not to rescue only because of the assumption that lenders are "small," and cannot provide both an initial and a refinancing loan. Bolton and Scharfstein (1996) analyze a renegotiation problem that is similar in spirit to ours. In their model, too, multiplicity is used as a commitment to be inefficient in renegotiations, with the result that high quality firms borrow from two creditors, while low quality firms prefer to borrow from a single creditor. Our model differs from theirs in several aspects. First, we work in a complete contracting environment. There is no variable in this model, which is “observable but not verifiable.” In Bolton and Scharfstein (1996), the entrepreneur can hide the returns of the project, and claim that the returns had been low. An optimal contract “punishes” him by threatening to liquidate the assets that are still valuable to him. In our model, the banks want to keep away nonprofitable projects, i.e. projects with a low probability of being successful. Second, we model the renegotiation process explicitly, and base it on observations from a financial system with “main banks.” Bolton and Scharfstein (1996) use the Nash Bargaining Solution and the Shapley Value, instead, to model bargaining outcomes.

Other related work includes Yosha (1995) and Bhattacharya and Chiesa (1995), who analyze the strategic use of single or multiple lending as a commitment device with respect to nonfinancial decisions. More precisely, they study the relative advantages of public or bank lending, if the two regimes have different effects on how sensitive information can leak to a firm’s competitors. They thus provide more and richer explanations for multilateral lending, which add new aspects to the purely financial models.

A second contribution of this paper is the development of a new model of inefficient bargaining, which has realistic features. We model the negotiations between the banks as a *war of attrition*. As soon as the banks have been informed that the firm must be refinanced, negotiations start. In these negotiations, each of the two banks tries to convince its opponent to write down the larger fraction of its claims. A rescue is only possible if one of the banks gives in: it frees the way to a rescue of the firm by accepting its opponent’s rescue plan. The reason why the banks eventually give in is that a rescue may become impossible, and the firm has to be liquidated. Each bank has a privately known valuation for the business relationship with the firm, which it loses if the latter is liquidated. The impossibility to rescue can arise at any time, as soon as the parties have started to bargain, and the longer the rescue is delayed, the more likely it becomes that the banks are forced to liquidate the firm. If a bank has a high valuation at risk, it has strong incentives to accept its opponent’s plan, only to ensure that the firm is rescued. As the opponent could have an even higher valuation, however, it also has an incentive to hold out for a while. This tradeoff determines the banks’ strategies in the war of attrition.

Admati and Perry (1991), Fernandez and Glazer (1991), and Abreu and Gul (2000) are other papers, in which two parties must come to an agreement in time consuming negotiations. We could have used variants of these models, instead of the war of attrition, to capture the inefficiencies of the renegotiation process.

The models in the three papers, however, are somewhat technical, too, and do not generate more elegant results than our model. We believe, therefore, that the war of attrition is a good compromise between the requirements for the analysis and the tractability of the results.

The rest of the paper is structured as follows: In Section 2, the projects and the entrepreneurs are introduced, and the difficulties of a single bank are discussed. The model is extended in Section 3, where two banks finance a firm, and renegotiate if it must be refinanced. These renegotiations are modeled as a war of attrition. Section 4 solves this model to find the equilibrium of the renegotiation stage, as well as that of the whole game. Section 5 presents some empirical evidence, and discusses implications and extensions of the model. Section 6 concludes. Proofs are in the Appendix A.

2. THE MODEL WITH ONE BANK

There is a large number of entrepreneurs who can start one project each. Each entrepreneur privately knows the type of project that he can start, either "good" or "bad." The proportion of entrepreneurs with "good" projects, γ, is common knowledge.

The timing of a project is the following. In the first period, an investment I must be sunk. In period 2 the project types become publicly observable. Payoffs are earned in the third (the last) period. A "good" project earns $R > I$, while a "bad" project earns zero. Both project types can be liquidated, which earns r, where $0 \leq r < R$. A "bad" project can be "rescued" in period 2: if an additional amount J is invested, a payoff $\bar{R}$ is earned, instead of zero.

Assumption 1. It is profitable to rescue a "bad" project in period 2, as $\bar{R} - J > r$. However, it is not profitable to finance a "bad" project ex ante: $\bar{R} - J - I < 0$. Neither should a random sample of projects be financed: $\gamma(R - I) + (1 - \gamma)(\bar{R} - J - I) < 0$.

The entrepreneurs' payoffs depend on whether their projects were started and completed. If a project was not started, the entrepreneur earns zero utility. If the project was started, and either completed successfully (if "good") or rescued (if "bad"), his utility is $M > 0$. If a project was started and then liquidated, this causes harm to the entrepreneur, and his payoff is $-m$ (where $m > 0$).

The entrepreneurs have no wealth of their own, and need outside finance to start their projects. We assume that a project cannot be separated from its entrepreneur. "Good" projects cannot be continued without him, and "bad" projects cannot be rescued – both types would have to be liquidated. The entrepreneurs are protected by limited liability. No punishment can be used legally to influence the entrepreneurs' decisions, except for the liquidation of the project, which gives them negative utility.

As we assume that it is not profitable to finance a cross section of projects, an investor must find a way to separate the "good" from the "bad" projects. Ideally, only the former would be financed. A bank could propose a contract which specifies that "bad" projects are liquidated in period 2. It would like to commit never to refinance, as this would prevent the entrepreneurs with "bad" projects from applying for initial loans I. Unfortunately, as one can easily verify, such a threat is not credible. Entrepreneurs with both "good" and "bad" projects will apply for I, as those with "good" projects have nothing to fear, and those with "bad" projects know that there will be a rescue. As a result, the single bank faces a random sample of projects, and it has to reject all loan requests. Due to a lack of commitment no project is undertaken, even though there would be valuable investment opportunities.

3. THE MODEL WITH TWO BANKS

The lack of a commitment possibility in the case of a single bank can be overcome (at least partially) by having more than one creditor for each project. If each of two banks provides, say, half of the initial loan, both have some rights over the returns of the firm at $t = 3$. If the entrepreneur asks for the additional loan J, a part of the total investment will have to be written off. The banks will bargain over how much each should forgive. If this bargaining is sufficiently inefficient, and the consequences of this inefficiency cause harm to the entrepreneur, the underinvestment problem can be solved.

It will be shown below, that two banks can commit to rescue with a probability which is strictly smaller than one. There is a critical value for this probability, which we denote by $\overline{q}$. It is determined by the entrepreneurs' utility functions:

$$\overline{q}M - (1 - \overline{q})m = 0. \tag{1}$$

If an entrepreneur's "bad" project is rescued with probability $\overline{q}$ and liquidated with probability $(1 - \overline{q})$, his expected payoff is exactly zero. He is thus indifferent between applying for a loan, and being inactive (which earns a sure payoff zero). If the rescue probability is strictly below $\overline{q}$, he prefers not to apply for the loan. In this case, only the entrepreneurs with "good" projects apply for funding. Therefore, if the banks can credibly commit not to refinance with a probability larger than $(1 - \overline{q})$, multiple banking strictly dominates bilateral lending relationships.

The model with two banks incorporates some observations about private workouts and bankruptcy negotiations that are reported in the business press, in empirical and descriptive papers (e.g. Edwards & Fischer, 1994; Fischer, 1990), in studies on the banking system and insolvency procedures in Germany, and in the large literature on the reform of the bankruptcy laws in Germany. These observations, or "stylized facts," are:

(1) Banks seem to have a strong bargaining position.
(2) The parties involved try to keep the negotiations secret.
(3) The banks want to terminate the negotiations quickly.
(4) It is likely that customers and suppliers are lost if they hear that there are rescue negotiations.
(5) Whether to rescue or not is rarely subject to dispute.
(6) The parties rather bargain about who is to sacrifice how much.

We have used these observations to construct a model of debt renegotiations, such that it captures important elements of an existing financial system, and it generates results which can again be compared with reality. To do so, we must expand the model with a single bank, by adding some assumptions. Two comments will be helpful before this is done. First, all additional assumptions could have been added to the model with a single bank, without changing any of the results. This has not been done, as it would have complicated the exposition unnecessarily. Second, we will make assumptions that are much more restrictive than is necessary to generate the results. Again, this is done to simplify the notation. Where assumptions are "extreme," we mention this fact, and discuss weaker alternatives.

We model the renegotiation process between the two banks of a firm as a *war of attrition*. Each of the two banks tries to convince its opponent to carry the burden of refinancing. An outside observer of the negotiations will find that no progress is being made for a while: the banks fail to come to an agreement on how to split the overall loss $\overline{R} - I - J$, if there should be a rescue. The negotiations can end in two different ways. Either one of the banks gives in, i.e. it accepts the rescue plan of its opponent. Or fate turns against the firm: a rescue becomes impossible for exogenous reasons, and it must be liquidated. In the latter case, each bank incurs a loss (additional to the financial loss). The size of this loss is privately known by the respective bank. In equilibrium, the higher it is, the more a bank fears liquidation, and the less it is willing to reject its opponent's rescue plan.

We now introduce the extensions of the single banking model, incorporating the observations listed above. The equilibrium of the war of attrition will be analyzed in Section 4.

The first observation above states that banks are the main players in rescue negotiations. This is captured by assuming that they are the only bargaining parties, and by assuming that the courts strictly enforce *Absolute Priority Rules*. These rules specify that no party may receive any of the returns of the firm, if the banks have neither been repaid in full, nor have agreed to such a payment.

Observation 2 describes how the banks want to keep the negotiations secret. It is helpful in achieving this goal to conclude an agreement as quickly as possible (see Observation 3). The reason for this wish for secrecy lies in the bankruptcy laws, which in most countries favour the banks (France is a notable exception). The assets of the firm usually are used as collateral for the loans from the banks, and absolute priority rules enforce the need to repay these claims first. The customers and suppliers are the parties who typically do badly in bankruptcy. Similar to a bank run, they have every incentive to request what they are owed, as soon as they discover the firm's problems, and not to engage in any new trades (except possibly on a cash-only basis). We model this sensitivity of a rescue to the cooperation of these parties as a heavily reduced form of Observation 4.

Assumption 2. At any time during the rescue negotiations, the public can discover that there are such negotiations going on. This happens by the time t with probability $F(t)$. If the negotiations have been discovered, a rescue becomes immediately impossible, and the project must be liquidated.

Assumption 2 is stronger than is necessary for the results. Nevertheless, it is not unrealistic. Firms whose assets consist almost exclusively of human capital are an example. If the competitors of an advertising company find out that it is in difficulties, they will try to hire its best employees on the spot. Robbed of its most valuable "assets," the distressed company is not worth rescuing anymore, and must be liquidated. For this reason, a formal insolvency in this industry can end after a couple of hours. Furthermore, there is anecdotal evidence from the U.K., which indicates that secrecy may be a crucial requirement for a successful rescue. The Bank of England assists in the rescue of distressed large companies, by coordinating the parties' efforts as soon as possible. It is not uncommon that in the negotiation meetings the parties have to use coded names to identify the distressed firms, even if everybody is informed about the real ones. Secrecy may also be relevant if without it potential customers are lost; for example, airlines may not able to get any more advance bookings if their customers fear being stranded abroad in case of the airline's bankruptcy filing.

We have to make some technical assumptions, in order to make the model tractable:

Assumption 3. The "discovery technology" F of the public has a mass point with measure $\pi > 0$ at $t = 2$, and a density f with support $(2, \tau]$, where $\tau < \infty$.

The mass point at $t = 2$ is necessary for the uniqueness of the equilibrium strategies.[1] These are determined by two differential equations, the solution of which is not unique without a socalled boundary condition. The mass point leads to a static lottery over rescue and liquidation at $t = 2$, which gives us such a boundary condition. This lottery is a logic extension of the dynamic war of attrition game to a discrete pre-stage, and is therefore used in the model: as will be shown below, the dynamic war of attrition is the limiting case of a discrete time game, if the length of a time unit becomes infinitesimal.

Assumption 3 further restricts the support of f to a finite interval. The reason for this is that the results would be difficult to interpret if $\tau = \infty$ (it would be possible that the banks bargain endlessly). It is by no means a necessary assumption. Furthermore, one can easily imagine why the firm's distress should be discovered in finite time. For example, there may be legal obligations to make the distress publicly known if certain contingencies arise.

Observation 5 states that the negotiating parties normally agree that the firm should be rescued (if they start to negotiate). This is captured by the complete information about the costs and returns of a rescue, and by the assumption that a rescue is profitable (Assumption 1). Not everything is common knowledge between the negotiating parties, however.

Assumption 4. After signing the initial loan contract, each bank B_i develops a privately known valuation ℓ_i for the business relations with the firm. The bank loses ℓ_i if the firm is liquidated. The valuations are independently and identically distributed, with a common probability density function g (g is strictly positive on its support $\mathbb{R}_+$, continuous and differentiable; denote the cumulative distribution function by G).

There are many possible interpretations for the loss of ℓ_i if the firm is liquidated. For instance, it may be an estimate of future profits from dealing with the firm. Alternatively, the bank may incur costs or lose profits because the liquidation of its debtor damages its public image or leads to tighter supervision by the banking regulator. Finally, ℓ_i may parametrize agency problems within the bank. A bank manager's career prospects may be worsened, if "his" firm must be liquidated. Similarly, the bank manager and the entrepreneur may have become good friends. In both cases, the decision making unit in the bank would lose something if the firm is liquidated, and would prefer to rescue it.

The banks' willingness to assist a distressed debtor is frequently underlined in studies of the German financial system (see e.g. Schneider-Lenné, 1992). It is questioned in Fischer (1990). His evidence, however, is based on interviews with insolvency practitioners, and can therefore be assumed to be biased to the banks' disadvantage. In their analysis of private workouts in the U.S., Gilson et al. (1990)

conclude that restructuring is the more likely, the more debt is owed to banks. This may be caused by the banks' superior skills and capabilities in attempting to rescue a firm, but it may also signal that banks are more willing to rescue a firm than other creditors. In the model this willingness to rescue is captured by the valuation ℓ_i.

Assumption 4 and the next assumption jointly capture Observation 6, that the banks bargain about who has to bear how much of the loss. The set of outcomes that the banks can achieve is restricted to simplify the analysis, that is how the net surplus s (the returns $\bar{R}$ minus the cost J and the opportunity cost r) from rescuing can be split (it is positive because of Assumption 1).

Assumption 5. The banks fight for the whole surplus $s := (\bar{R} - r - J)$. No offer to share the surplus is made or accepted. If one bank gives in it receives its share r_i in the liquidation value r of the firm from the other bank, where r_1 and r_2 are specified in the initial contract. The winning bank is committed to rescue the firm immediately, but may keep the returns for itself.

As before (in Assumption 2), the formulation of Assumption 5 is much stronger than necessary. A sharing rule saying that the gross surplus $\bar{R} - J$ can only be shared in proportions α and $(1 - \alpha)$, where $\alpha \neq (1/2)$, would be sufficient. This would lead to significant complications of the analysis, however, which are not rewarded by the additional insight that one gets.

This completes the introduction of the model with two banks. As one can easily see, the assumptions that have been added in this section could also have been introduced in the single bank model, without changing anything. The loss of a valuation ℓ_i if the firm is liquidated would make a single lender even more willing to refinance a "bad" project. This rescue happens already without the valuation, however.

In the model with one bank, a strategy for the bank consisted of a financing and a refinancing decision. In the case with two banks it is slightly more complicated. We first consider the part of the strategy which is used in the rescue negotiations. If a firm needs refinancing, the sequence of events is the following. First, the banks decide whether they want to give in immediately. If none of the banks has given in, the negotiations are discovered with probability π, and the firm must be liquidated. With probability $(1 - \pi)$ the continuous time war of attrition starts. We assume that if both banks give in simultaneously, each "wins" with probability 1/2.

A strategy is a function $T_i : \mathbb{R}_+ \to [2, \tau]$, which determines for each moment of time whether a bank B_i with valuation ℓ_i should give in or not. It will be shown in Section 4, that if the equilibrium strategy tells this bank to stop at time $T_i(\ell_i)$, it will stop at every later time, as well. Thus, we will define T_i as determining the first time at which a bank plans to stop. This includes the static lottery which is played because of the mass point in F at $t = 2$.

One may wonder why the banks cannot renegotiate the lending contract, after it has been signed. Both are fully aware of the inefficiency that will arise, if the contract is renegotiated using the war of attrition. Why cannot one bank (or a third bank) take over all debt for a flat price? Suppose B_1 would make such an offer to B_2. B_2 would claim to have a valuation $\ell_2 = 0$ and not to fear the war of attrition, in order to increase the takeover price. B_1 would claim to have the same valuation, to decrease the price. None of the two has any incentive to admit having a positive valuation, until a rescue is really needed. In this case, however, the war of attrition will start. The time that passes by is the only credible information about one's valuation, as talk is "cheap," and neither before nor during the war of attrition the parties can renegotiate more efficiently. Even a bank with valuation $\ell_i = \infty$ would wait until a rescue is necessary, as it might be that the opponent gives in. Nothing is lost by waiting until $t = 2$, at which time both banks can prevent a liquidation with probability one by giving in.

4. EQUILIBRIUM STRATEGIES

The first step in solving the renegotiation game is to determine which types would want to start the war of attrition, and which types would prefer to give in immediately, in order to secure the rescue of the firm. If no bank gives in immediately, the negotiations are discovered with probability π (the mass point in F), and the firm is liquidated. With probability $(1 - \pi)$ the continuous time war of attrition starts.

A bank with a very high valuation at stake will not want to gamble for the surplus s, and stop immediately. We must determine which is the lowest valuation, for which this is still true. Denote this cut-off value of bank B_i with λ_i. If its valuation is $\ell_i > \lambda_i$, it should strictly prefer to give in immediately, while if it is $\ell_i < \lambda_i$, it should want to start the war of attrition, and plan to stop later than $t = 2$.

We define λ_i as the valuation with which a bank B_i is indifferent between giving in immediately, and starting the war of attrition, if it is sure that the opponent will either give in immediately (with probability $1 - G(\lambda_2)$), or will start the war of attrition without giving in (with probability $G(\lambda_2)$).

Consider the bank with valuation $\ell_i = \lambda_i - \varepsilon$, where $\varepsilon > 0$. Given the definition of λ_i, there must be a $\delta > 0$, such that it will strictly prefer to start the negotiations, if the probability that the opponent gives in immediately, as soon as the negotiations have started, is δ. Thus, a bank with a valuation below λ_i has an incentive to hold out for a strictly positive amount of time. A bank with a valuation higher than λ_i, however, strictly prefers to give in immediately.

Lemma 1. The cut-off values λ_1 and λ_2 are defined implicitly by

$$\lambda_1 = \frac{s}{2\pi}\left(\frac{1-G(\lambda_2)}{G(\lambda_2)}\right) \quad \text{and} \quad \lambda_2 = \frac{s}{2\pi}\left(\frac{1-G(\lambda_1)}{G(\lambda_1)}\right). \tag{2}$$

Since λ_i is continuous and monotonic in λ_j, a symmetric solution exists. It can happen that there are multiple solutions, since the two equations in Lemma 1 must be solved simultaneously. We assume that the banks play the symmetric solution in this case, and denote the common cut-off value with λ.[2]

As was mentioned before, the war of attrition is only one of many possible ways to model negotiations with inefficient delays. The model could have been slightly simplified by assuming that the support of G is bounded (see Assumption 4). Suppose it was common knowledge that the highest value ℓ_i that a bank can attribute to its business relationship with a firm is $A < \infty$, because a bank's line manager cannot "bet the ranch." The results would be qualitatively the same, except that we would have $\lambda = A$. Our formulation allows for banks that give in immediately with a certain probability, depending on the parameters of the model.

If both banks decided to stay in the game, the war of attrition starts. A strategy T_i in this war of attrition specifies the earliest instant at which a bank wishes to stop, given the realisation of its potential loss, ℓ_i. Lemma 2 derives some characteristics that equilibrium strategies must have. In Proposition 1 we will show that these necessary conditions are also sufficient conditions for the existence of a unique equilibrium, together with the boundary conditions that are determined in Lemma 1.

Lemma 2. Let T_1 and T_2 be equilibrium strategies of the game defined above. Then the strategy T_i is (i) strictly decreasing in the liquidation loss ℓ_i, (ii) continuous, (iii) differentiable, and (iv) bank B_i stops at τ if and only if $\ell_i = 0$.

In equilibrium it will never be the case that the bank with the higher loss level will decide to stay in longer than its opponent. The threat of the public's discovery must have strictly more weight in a bank's reasoning the higher ℓ_i is, while the gain from winning, the surplus s, is constant. Only a bank with zero liquidation loss will wait until τ, and it will not want to stop earlier than τ. A bank with strictly positive loss level will either stop immediately at $t = 2$ (if it has costs $\ell_i \geq \lambda$) or at some moment after $t = 2$ but earlier than τ.

At every moment, both players update their beliefs about the opponent's valuation. Since T_i is continuous and strictly decreasing, each ℓ_i is mapped one-to-one with a stopping time $T_i(\ell_i)$. T_i can be inverted to yield a function $L_i : [2, \tau] \to \mathbb{R}_+$. At each instant t_i there is a valuation $L_i(t_i)$ with which a bank would plan to stop. As time passes by, a player's expectation about the maximal valuation that his opponent could possibly have decreases. Bygones are not "bygones" in this game: every second that passes by signals information about a bank's valuation, and is relevant for the present and future decisions of the opponent.

As was mentioned above, the finiteness of τ is not a necessary condition for the tractability of the model. If the function f had an infinite support, then Lemma 2.(iv) would state that banks with zero liquidation loss never stop, and banks with strictly positive loss levels plan to stop at some finite time.

L_i, the inverse of the strategy function T_i, is the lowest cost level that would make bank B_i want to stop at time t. It will be helpful for characterising the equilibrium strategies in the following. These are determined by finding for each moment t_1 a valuation $L_1(t_1)$, such that B_1 is exactly indifferent between stopping at t_1, and waiting for a small amount of time Δ, and giving in then (the derivation is similar to that of the cut-off values λ_i).

If the bank gives in at time t_1, its payoff is r_1 for sure. We require this payoff to be equal to the expected payoff, if it decides to wait until $t_1 + \Delta$:

$$r_1 = \frac{G(L_2(t_1+\Delta))}{G(L_2(t_1))}\left[\frac{(F(t_1+\Delta)-F(t_1))}{1-F(t_1)}(r_1 - L_1(t_1)) + \frac{(1-F(t_1+\Delta))}{1-F(t_1)}r_1\right]$$
$$+\frac{G(L_2(t_1))-G(L_2(t_1+\Delta))}{G(L_2(t_1))}\left[\frac{F(t_1+\Delta)-F(t_1)}{1-F(t_1)}(r_1 - L_1(t_1))\right.$$
$$\left.+\frac{1-F(t_1+\Delta)}{1-F(t_1)}(\bar{R}-r_2-J)\right]. \tag{3}$$

The second expected payoff (on the right-hand side of Eq. (3)) has four components. The opponent may have a low valuation, and plan to give in later than $t_1 + \Delta$. By this time, the negotiations may have been discovered, and the firm must be liquidated. The bank receives its share r_1 of the liquidation value r, but loses $L_1(t_1)$. If the negotiations are not discovered, it will give in at time $t_1 + \Delta$, which earns r_1. On the other hand, the opponent may plan to give in between t_1 and $t_1 + \Delta$. As before, the negotiations may be discovered, or they may not. In the latter case, the firm is rescued. The bank pockets the surplus $\bar{R} - J$, and pays r_2 to the opponent. We abstract from the possibility that both may give in at $t_1 + \Delta$ simultaneously, as the probability that this happens is negligible.

Equation (3) can be simplified by rearranging, subtracting r_1 on both sides, and by substituting s for $(\bar{R} - r_1 - r_2 - J)$. A division of both sides by Δ leads to

$$\frac{G(L_2(t_1))-G(L_2(t_1+\Delta))}{G(L_2(t_1))\Delta}\left(\frac{1-F(t_1+\Delta)}{1-F(t_1)}\right)s$$
$$= -\left(\frac{F(t_1+\Delta)-F(t_1)}{(1-F(t_1))\Delta}\right)L_1(t_1). \tag{4}$$

Since the strategies are differentiable everywhere it is possible to take the limit as Δ goes to zero. The same procedure can be repeated for the second bank, and we get a system of two differential equations:

$$L_2'(t_1) = -\left(\frac{G(L_2(t_1))}{g(L_2(t_1))}\right)\left(\frac{f(t_1)}{1-F(t_1)}\right)\frac{L_1(t_1)}{s}, \tag{5}$$

$$L_1'(t_1) = -\left(\frac{G(L_1(t_2))}{g(L_1(t_2))}\right)\left(\frac{f(t_2)}{1-F(t_2)}\right)\frac{L_2(t_2)}{s}. \tag{6}$$

Given the strategy of the opponent, Eq. (5) determines the optimal response of bank B_1, if it has loss level $L_1(t_1) = \ell_1$ (the two are equivalent, if the equilibrium strategy tells bank B_i with cost level ℓ_i to stop at time t_i) and bank B_2 plays strategy $L_2(\cdot)$. If Eq. (5) were an inequality, B_1 would either want to wait longer than t_1 (if $<$), or it would want to have stopped earlier (if $>$).

Since by Assumption 2 the probability density function g is strictly positive on $\mathbb{R}_+$, G has an inverse function $G^{-1} : [0, 1] \to \mathbb{R}_+$. Equations (5) and (6) can be integrated, and this leads to the following reaction function for bank B_i:

$$L_j(t_i) = G^{-1}\left(G(\lambda)\exp\left\{-\int_2^{t_i}\left(\frac{f(t)}{1-F(t)}\right)\frac{L_i(t)}{s}\,\mathrm{d}t\right\}\right). \tag{7}$$

Equation (7) implicitly describes the strategy of bank B_j that makes bank B_i exactly indifferent between stopping at t_i and stopping at $t_i + \Delta$ (where Δ is a small amount of time), given its cost level $L_i(t_i)$. The analogous can be done to derive the strategy of the other bank. The solution to these two equations will give us the equilibrium strategies for the banks. We will continue with the differential equations (5) and (6), and show that there is a unique equilibrium. The reaction functions will be helpful in Section 5, where we present some comparative statics.

With the help of the differential equations and the boundary conditions it is now possible to describe the equilibrium strategies of the players for the whole renegotiation game.

Proposition 1. The renegotiation game has a unique symmetric Bayesian equilibrium, which is implicitly described by the system of differential equations (5) and (6), and the boundary conditions $T_1(\lambda) = T_2(\lambda) = 0$. The equilibrium strategy for bank B_i is to stop at time t if and only if $\ell_i \geq L_i(t)$, where $L_i(t)$ is determined in Eq. (7).

We can now find the equilibrium strategies for the whole game with two banks, including the financing decision. Whether an entrepreneur with a bad project applies for a loan in the first period depends on the probability with which his project is rescued in the second period. In Eq. (1) we determined an upper bound $\overline{q}$ to this probability, such that "bad" projects are not financed.

Proposition 2. If the probability of non-rescue due to bargaining delays is high enough,

$$\int_0^{\lambda} 2\,F(T_1(\ell_1))G(\ell_1)g(\ell_1)\,\mathrm{d}\ell_1 \geq \frac{M}{M+m}, \tag{8}$$

the entrepreneurs will apply for the initial loan if and only if the project is of the "good" type.

Proposition 2 is the main result of the paper. There are cases in which a financial system with multiple banking performs strictly better than one with single bank lending. If the condition in Eq. (8) is met, the banks prefer to require co-financing by a second bank to being a single lender.

5. EMPIRICAL IMPLICATIONS

The main result of the paper is that banks might want to syndicate a loan to a firm, if they fear to find themselves in a harmfully weak bargaining position if the firm has to be refinanced. The loan is shared for strategic reasons, and the banks propose to share even if they have all bargaining power. There can be other reasons for why loans are syndicated, however, like (see Section 1) risk aversion, the sheer size of the loan, or because the strong competition on the lenders' side. These reasons complement each other, and it is not clear which one was the most important if a loan has been shared.

There is some empirical work on this question for the U.S. and for Germany. For the U.S., Gilson et al. (1990) have analyzed the performance of private workouts. One of their results is that debt restructurings are more likely if the number of lenders is small, which could support the result above. For the case of Germany, Fischer (1990) and Edwards and Fischer (1994) report that all but the very small firms have several "main banks," which could be interpreted as supporting the conclusions in this paper.

Interesting evidence is reported in Armendariz (1999). She analyzes the performance of several development banks, i.e. the default rates of their loans. Some of these banks require that projects are co-financed by commercial banks, while others usually are the unique providers of capital. The former enjoy considerably less arrears in the repayment of their loans. Her interpretation of these facts is that the requirement of co-financing hardens the Soft Budget Constraint of development projects, exactly what the results above suggest.

A similar observation can be made if firms grow: suppose that for a small firm $\bar{R}_s - J_s > I_s$, while for a larger firm $\bar{R}_l - J_l < I_l$. Then a "main bank" could require that a growing firm finds a second main lender, for instance by committing to finance only a fraction of a major investment. Similarly, a bank could require co-financing if fixed costs of rescuing a firm are higher than the net surplus s for small firms, but lower for larger firms.

We now analyze other implications of the model. The equilibrium strategies of all parties are unique, and therefore we can analyze the effects of varying some of the parameters of the model.

Proposition 3. A higher expected value of the firm $\bar{R}$, a lower liquidation value r and a lower additional loan J lead to later concessions. This in turn implies that the liquidation of a "bad" firm becomes more likely.

The intuition behind Proposition 3 is clear: if the prize is increased, and the expected costs of fighting remain unchanged, the banks have an incentive to fight longer. The implications for rescue negotiations are surprising, however. Of two otherwise identical candidates for a rescue, the one with a higher post-rescue return $\bar{R}$, i.e. the more profitable, is more likely to be liquidated. Similarly, the one with a lower liquidation value is more likely to be liquidated. This seems to be counterintuitive, as usually we would expect a valuable rescue to be undertaken. The result follows from two modeling assumptions. First, the negotiations are inefficient, as the "cake" that is to be split can disappear at any time. Second, the banks' valuations for the surplus from a rescue and for the rescue itself are independent. Suppose that s depends on the number of employees of the firm, and that the banks' public relations suffer if they cause unemployment by not assisting a distressed debtor (they lose ℓ_i). In this case we would expect a bank to be more willing to rescue if the firm is larger.

A "valuable" firm could therefore be rescued for different reasons, either because a rescue is profitable (large s), or because failing to rescue would cause indirect costs (large ℓ_i). The second reason is an incentive problem that is similar to the one underlying our assumption: once a project has been financed, its investors have too strong incentives to refinance it (see Mitchell, 1993, or Aghion et al., 1999, on the problems that this can cause for banking regulation).

The result should hold, however, in situations in which the valuations ℓ_i are small, compared with the surplus from a rescue, s. One could analyze the refinancing decisions of foreign banks, that care less about their public image outside their home country. Similarly, one could analyze these decisions in sectors, regions, or during time periods, in which unemployment and bankruptcies are not considered as being major problems.

A further implication of Proposition 3 concerns the allocation of the assets of a distressed firm. Many bankruptcy procedures are court-led, and contain rules that are meant to protect the interests of all parties. This may make it difficult to use the assets in the most efficient way, as for instance their quick sale to the highest bidder. The liquidation value of a firm is therefore lower than necessary if a formal procedure is started, with the consequence that a rescue becomes less likely.

The variables R and I (the return of a "good" project and the initial investment) have no effect on the strategies, because of the simplified structure of the model. As was suggested above, we could allow a bank's valuation ℓ_i to depend on the size of its stake in the firm. The larger the loan, the more the bank is exposed to public scrutiny, and the more it will therefore be willing to cover up "mistakes" by rescuing the firm.

Similarly, the relative shares r_i in the liquidation value r play no role. The reason for this is that the bank receives a payment of at least r_i whatever the outcome of the negotiations. We could easily change the sharing rule such that r_i plays a role in the banks' renegotiation strategies. For instance, a sharing rule could require that the bank that gives in receives a share $\alpha < 1/2$ of the surplus.

Next, consider a variation in the public discovery technology, the density function f. Suppose that π remains constant, and that f is changed to f_1 such that the hazard rate is higher (the term $f/(1-F)$ on the RHS of Eq. (7)). Assume that this makes the second discovery technology is superior, i.e. it becomes more skewed to the left. The RHS of Eq. (7) becomes more negative, and in order to restore the equilibrium L_2 must become steeper and L_1 must decrease.

Proposition 4. Assume that early discovery becomes more likely, such that the hazard rate of the discovery technology $f/(1-F)$ increases. Then the banks tend to give in earlier.

Rescue negotiations can become more difficult to hide, if the disclosure requirements for banks or firms are tightened. The introduction of a new business paper in a region can have a similar effect. The effect of a change in the discovery technology by varying π is similar: an increase in π leads to a reduced stopping time for all types (see Lemma 1). Unfortunately the effect on the likelihood of liquidation is not easy to specify for the general case, as two effects are opposed: the banks stop earlier but discovery becomes more likely. This would be interesting, as one could derive implications for disclosure rules of stock markets, or for the benefits of having a more transparent economy. Consider the following change, however:

Proposition 5. Suppose that the support of f is rescheduled such that $f_1(t) = f(\alpha \cdot t)$, where $\alpha < 1$. Then the banks tend to stop earlier, but the probability of liquidation is unchanged.

Suppose that the speed of all information channels is increased symmetrically. In this case the moment of sure discovery τ has an effect on the stopping time of a bank with cost $\ell_i = 0$, but not on the *relative* stopping times of the other types (as it does not appear in the derivations). In this case, the improvement of the discovery technology had no material effect. Thus, stricter disclosure requirements can be neutral, and therefore (depending on the parameters) welfare reducing or improving.

Similarly, we can analyze changes in the distribution of types. Here the "hazard rate" is somewhat complicated, as the types are revealed "backwards," i.e. the first types that reveal themselves by stopping are those with high costs ℓ_i. The "reversed" hazard rate is thus $g(\ell)/G(\ell)$. We encounter the same difficulties as in Proposition 4, as we can determine (using the equilibrium conditions (5) and (6)) the effect on the banks' strategies, but not the effect on the probability of liquidation.

Proposition 6. Assume that the probability of ℓ being low is higher, such that the "reverse" hazard rate of the type distribution g/G increases. Then the banks tend to give in earlier.

This seems to be a surprising result, as one would expect "tougher" banks to hold out longer. However, the result states that a bank with type ℓ will stop earlier. This is intuitive, as it must be more pessimistic about its strength relative to other types. The overall effect cannot be determined without making assumptions on the functional forms of f and g.

Negotiation costs can easily be introduced to the model. They have been omitted for simplicity, but can be expected to have an effect on rescue negotiations. Examples for such costs are the need to set up a management team which analyzes the firm's state and the rescue plans (i.e. the opportunity costs of sending bank managers to attend negotiations), legal costs (the costs of hiring lawyers), or the material costs of planning and negotiating (expenses for business consultants and industry experts, travel expenses).

Proposition 7. Assume that each bank incurs a continuous cost c per unit of time dt, while the negotiations take place. Then the banks tend to stop earlier than in the case of no costs, and rescues are more likely.

Even though this type of bargaining costs reduces the net surplus from a rescue, this material loss has no effect on the banks' decisions. At each instant, the past costs are sunk, and "bygones are bygones." However, c has an effect on the decision whether to wait another infinitesimal amount of time. It decreases the expected payoff from waiting, and therefore the banks stop earlier with higher costs. Thus, while the already incurred costs have no effect, the costs that have to be incurred if the negotiations continue are relevant for the decision to stop.

Finally, the entrepreneurs' utility functions are relevant. As m, the utility loss that an entrepreneur incurs if his project is liquidated, increases, funds become available for more parameter settings. Thus, there is a use in this model for the stigma that is attached to a business failure. While we do not want to suggest that this is a good way of solving incentive problems, we can conclude from the model that the financing patterns of two regions or industries should be different if bankruptcy is "not a big deal" in one of them, while it has strong negative connotations in the other.

6. CONCLUSIONS

This paper studies the difference between single and multiple banking. It concentrates on renegotiation problems, which are shown to be solved better in the case of multiple banking. We assume that entrepreneurs ask banks for loans, such that they can start projects. These may be of a "good" or "bad" type, where the type of a project can be observed by the respective entrepreneur, only. "Bad" projects need refinancing at an intermediate stage, which makes them nonprofitable from an ex ante perspective. However, once the initial loan is lost, refinancing is better than the only alternative, liquidation.

A single bank cannot commit not to refinance a bad project, which would keep entrepreneurs with "bad" projects from applying for a loan. Two banks, however, can commit not to refinance with some probability. The reason for this are inefficiencies in the negotiations between the banks, when they have to agree on their respective degree of debt forgiveness. If the probability of liquidation is sufficiently high, entrepreneurs with "bad" projects do not ask for a loan at all.

We model the negotiations as a war of attrition. Each of the two banks incurs a privately known loss, if the firm is liquidated, and therefore would like to have it refinanced. Additionally, refinancing is profitable, once the initial loan is sunk. The banks have to agree on how to split the costs and revenues, if they refinance the firm. These negotiations take time, and the longer they last, the more likely it becomes that a rescue becomes impossible (for exogenous reasons). In order to prevent this, the banks plan to "give in" after a while, i.e. to let the opponent pocket the gain from rescuing, only to make sure that the firm is refinanced. There is a unique equilibrium in this game: the higher the potential loss, the earlier a bank decides to give in. The negotiations can last for a while, if both banks' potential losses are low, and therefore the firm is liquidated with positive probability.

The model is designed to isolate the advantage of multiplicity for the lenders. We thus abstract from many aspects which are relevant for the choice between bilateral and multilateral finance, as well as for reorganisation procedures. One of these is the tradeoff between single and multiple banking. Bolton and Scharfstein (1996) analyze a case where either single or multiple lending may be optimal, and also derive results for voting rules, as well as for the optimal use of assets as collateral. Similarly, the effects of different bankruptcy laws need further analysis. In the model the two banks decide to share the highest priority rank. It would be interesting to analyze a model in which their claims have different ranks. A further topic for future analysis is whether and how a distressed firm is rescued, if the banks do not enjoy the highest priority rank.

NOTES

1. A simple alternative to the mass point assumption will be discussed below, see Lemma 1.
2. A sufficient condition for uniqueness can be found by inverting $\lambda_2(\lambda_1)$, and requiring that the slope of this inverse is never equal to the slope of $\lambda_1(\lambda_2)$. It is, however, difficult to interpret:

$$\frac{g(\ell)}{[G(\ell)]^2}\,\frac{g(G^{-1}[s/(2\pi\ell+s)])}{[G(G^{-1}[s/(2\pi\ell+s)])]^2} \neq \left(\frac{2\pi}{s}\right)^2 \quad \forall \ell \in \mathbb{R}_+.$$

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APPENDIX A: PROOFS

Proof of Lemma 1

(A.1) compares the respective payoffs for bank B_1 with valuation λ_1, given λ_2:

$$G(\lambda_2)r_1 + (1 - G(\lambda_2))\left(\frac{\overline{R} - J - r}{2} + r_1\right)$$
$$= (1 - G(\lambda_2))(\overline{R} - r_2 - J) + G(\lambda_2)\left[(1 - \pi)(\overline{R} - r_2 - J) + \pi(r_1 - \lambda_1)\right]. \tag{A.1}$$

The left-hand side of Eq. (A.1) is the expected payoff if bank B_1 gives in immediately. With probability $G(\lambda_2)$ the opponent has a low valuation and does not give in. The firm is rescued, and the bank receives r_1. With probability $1 - G(\lambda_2)$ the opponent gives in, as well, and the net surplus is shared (in expected terms).

The right-hand side of (A.1) is the payoff if the bank gives in as soon as the war of attrition has started. With probability $1 - G(\lambda_2)$ the opponent has a high valuation and will give in immediately. The bank rescues, pockets the surplus $\overline{R} - J$, and pays r_2 to the opponent. With probability $G(\lambda_2)$ the war of attrition starts. It is discovered with probability π, and the firm is liquidated. With probability $(1 - \pi)$, the game could continue, but by definition the bank plans to stop, which earns r_1.

Some simplifications of (A.1) and of an analogous equation for bank B_2 lead to the two equations in Lemma 1. There is always an interior solution for the cut-off levels: If λ_i goes to zero, then $\lambda_j(\lambda_i)$ goes to infinity, while if λ_i goes to infinity it goes to zero.

Proof of Lemma 2

(i) We first show that T_i is nonincreasing, and then that it is strictly decreasing. By utility-maximisation it must be the case that

$$\mathcal{V}_1(t_1, T_2(\cdot), \ell_1) \geq \mathcal{V}_1(t_1', T_2(\cdot), \ell_1) \quad \forall t_1', \quad \forall t_1 = T_1(\ell_1) \tag{A.2}$$

and

$$\mathcal{V}_1(t_1', T_2(\cdot), \ell_1') \geq \mathcal{V}_1(t_1, T_2(\cdot), \ell_1') \quad \forall t_1, \quad \forall t_1' = T_1(\ell_1'), \tag{A.3}$$

where $\mathcal{V}_i(t_i, T_j(\cdot), \ell_i)$ is the expected payoff of bank B_i with cost level ℓ_i, if it stops at t_i, and bank B_j plays strategy $T_j(\cdot)$:

$$\begin{aligned}\mathcal{V}_i(t_i, T_j(\cdot), \ell_i) = {} & \Pr\{T_j(\ell_j) \geq t_i\}\,(F(t_i)(r_i - \ell_i) + (1 - F(t_i))r_i) \\ & + \int_{\{\ell_j | T_j(\ell_j) < t_i\}} \big[F(T_j(\ell_j))\,\big(r_i - \ell_i - (\bar{R} - r_j - J)\big) \\ & + (\bar{R} - r_j - J)\big]\, g(\ell_j)\,\mathrm{d}\ell_j. \qquad \text{(A.4)}\end{aligned}$$

The payoff of a bank depends on the chosen stopping time t_i, the opponent's strategy T_j and the (privately known) loss ℓ_i of losing the firm. With probability $\Pr\{T_j(\ell_j) \geq t_i\}$ the opponent plans to stop later than t_i. If the public discovered the negotiations (this happens with probability $F(t_i)$), the payoff is $(r_i - \ell_i)$. If the secret was kept well, the bank receives r_i from bank B_j who rescues the firm. The second term of Eq. (A.4)is the equivalent if the opponent plans to stop earlier. Here the bank receives $(\bar{R} - J)$ if the firm can be rescued and pays r_j to the opponent.

We can rewrite the two inequalities Eqs (A.2) and (A.3) using Eq. (A.4). Subtracting the RHS of Eq. (A.3) from the LHS of Eq. (A.2), and the LHS of Eq. (A.3) from the RHS of Eq. (A.2), we get

$$\begin{aligned}&\Pr\{T_2(\ell_2) \geq t_1\}F(t_1)(\ell_1' - \ell_1) + \int_{\{\ell_2 | T_2(\ell_2) < t_1\}} F(T_2(\ell_2))(\ell_1' - \ell_1)g(\ell_2)\,\mathrm{d}\ell_2 \\ &\quad \geq \Pr\{T_2(\ell_2) \geq t_1'\}F(t_1')(\ell_1' - \ell_1) + \int_{\{\ell_2 | T_2(\ell_2) < t_1'\}} F(T_2(\ell_2))(\ell_1' - \ell_1)g(\ell_2)\,\mathrm{d}\ell_2\end{aligned}$$

or, rearranging,

$$\begin{aligned}&[(1 - \Pr\{T_2(\ell_2) < t_1\})(F(t_1) - F(t_1'))](\ell_1' - \ell_1) \\ &\quad \geq \left[\int_{\{\ell_2 | t_1' < T_2(\ell_2) < t_1\}} F(t_1')g(\ell_2)\,\mathrm{d}\ell_2 - \int_{\{\ell_2 | t_1' < T_2(\ell_2) < t_1\}} F(T_2(\ell_2))g(\ell_2)\,\mathrm{d}\ell_2\right] \\ &\quad \times (\ell_1' - \ell_1).\end{aligned}$$

If $t_1 > t_1'$, the following holds:

$$\begin{aligned}&(1 - \Pr\{T_2(\ell_2) < t_1\})(F(t_1) - F(t_1')) \\ &\quad \geq 0 \geq \int_{\{\ell_2 | t_1' < T_2(\ell_2) < t_1\}} [F(t_1') - F(T_2(\ell_2))]g(\ell_2)\,\mathrm{d}\ell_2,\end{aligned}$$

and it must be the case that $\ell_1' \geq \ell_1$. On the other hand, if $t_1' > t_1$:

$$
\begin{aligned}
&(1 - \Pr\{T_2(\ell_2) < t_1\})(F(t_1') - F(t_1)) \\
&\quad \geq \Pr\{t_1 < T_2(\ell_2) < t_1'\}(F(t_1') - F(t_1)) \\
&\quad \geq \int_{\{\ell_2 | t_1 < T_2(\ell_2) < t_1'\}} [F(t_1') - F(T_2(\ell_2))] g(\ell_2)\, \mathrm{d}\ell_2,
\end{aligned}
$$

and it must be the case that $\ell_1 \geq \ell_1'$. Thus for all t_1, t_1', in equilibrium $(\ell_1' - \ell_1) \cdot (t_1' - t_1) \leq 0$, i.e. the strategies are nonincreasing in the liquidation loss.

Assume that T_1 is not strictly decreasing, i.e. there are $\ell_a, \ell_b > \ell_a$, such that for all $\ell \in [\ell_a, \ell_b]$, $T_1(\ell) = \theta$. Then there is an $\varepsilon > 0$ such that all types ℓ_2 with $T_2(\ell_2) \in (\theta - \varepsilon, \theta]$ prefer to wait until θ and stop then, if the opponent did not stop. Then the types $\ell \in [\ell_a, \ell_b]$ could gain by stopping at $\theta - \varepsilon$ instead of θ: The probability of winning is not affected, but the risk of losing ℓ is diminished.

(ii) Assume that T_1 is discontinuous at ℓ. Then there are $t_a, t_b > t_a$ such that a type ℓ never stops at any $t \in (t_1, t_2)$. A type ℓ_2 with $T_2(\ell_2) \in [t_a, t_b)$ would thus wait only until t_a, and stop if the opponent did not stop. This implies that no one stops at any $t \in (t_a, t_b)$. But then there are types $\hat{\ell}_1$ and an $\varepsilon > 0$ such that $T_1(\hat{\ell}_1) \in [t_b, t_b + \varepsilon]$, who prefer stopping at some $t \in (t_a, t_b)$.

(iii) Assume that T_i is not differentiable at ℓ.

(a) Let the left-hand derivative be higher than the right-hand derivative (T_i is flatter to the left of ℓ). Then there is an $\varepsilon > 0$ such that no type ℓ_j stops at any $t \in (T_i(\ell) - \varepsilon, T_i(\ell)]$. It pays to wait longer since after the point of discontinuity it becomes relatively likely that the opponent stops. This holds since both f and g are continuous and differentiable.
(b) Let the right-hand derivative be higher. Then there is an $\varepsilon > 0$ such that no type ℓ_j stops at $t \in [T_i(\ell), T_i(\ell) + \varepsilon)$. It pays to stop earlier since it becomes more likely that B_i stops immediately after $T_i(\ell)$.

In both cases T_j is not continuous, contradicting (ii) above.

(iv) If type $\ell_1 = 0$ stops at $\theta < \tau$, all types with higher loss level stop earlier because of Lemma 2(i). Then in equilibrium no type of the other bank should stop later than θ. There is an $\varepsilon > 0$ such that types $\ell_1 \in (0, \varepsilon]$ find it profitable to wait until θ and wait for the opponent to stop.

Proof of Proposition 1

The differential equations are Lipschitz-continuous on $[2, \tau]$ which implies that a solution exists and is unique (see e.g. Birckhoff & Rota, 1978, Chap. 6). At each

$t \in [2, \tau]$, the (expected) payoffs from stopping or non-stopping can be compared, as was done in deriving Eq. (5). Since the strategies are strictly decreasing, at $t < T_1(\ell_1)$, i.e. if $L_1(t) > \ell_1$, the payoff to bank B_i with loss level ℓ_1 will be higher if it waits. The opposite holds for $L_1(t) < \ell_1$. For all $t \geq t_1$, type $L_1(t_1)$ can only decrease his payoff by waiting, and will stop whenever possible.

The players constantly update their beliefs using Bayes' Rule. If a player stops at the wrong time (this is the only deviation that is possible) the opponent will have no difficulties in updating his beliefs: If a player stops too early, the game is over and beliefs are not relevant anymore. If a player waits too long, the strategy tells him to stop immediately: Type ℓ_i stops at any time t if $t > T_i(\ell_i)$. Again, the opponent can update his beliefs without problems.

Proof of Proposition 2

Follows directly from the Assumptions and Proposition 1.

Proof of Proposition 3

The reaction curves L_i (see Eq. (7)) are shifted outward, if s is increased. The indirect effect via the cut-off value λ goes in the same direction: $\lambda_i(\lambda_j)$ is shifted outward, as well (see Lemma 1).

Proof of Propositions 4, 5 and 6

As Proposition 3: analyze the equilibrium conditions Eqs (5) and (6), and the indirect effect via the cutoff value λ in Lemma 1.

Proof of Proposition 7

Equation (7) is changed to

$$L_2(t_1) = G^{-1}\left(G(\lambda)\exp\left\{-\int_2^{t_1}\left[\left(\frac{f(t)}{1-F(t)}\right)\frac{L_1(t)}{s} + \frac{c}{s}\right]\mathrm{d}t\right\}\right). \tag{A.5}$$

A comparison of Eqs (A.5) with (7) shows that all types will want to stop earlier, including zero-cost types.