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# Financial constraints and product market competition: ex ante vs. ex post incentives

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## Abstract

This paper analyzes the interaction of financing and output market decisions in a duopoly in which one firm is financially constrained and can borrow funds to finance production costs. Two ideas have been separately analyzed in previous work: Some authors argue that debt strategically affects a firm's output market decisions, typically making it more aggressive; others argue that the threat of bankruptcy makes debt financing costly, typically making a firm less aggressive. Our model integrates both ideas; moreover, unlike most previous work, we derive debt as an optimal contract. Compared with a situation in which both firms are unconstrained, the constrained firm produces less, while its unconstrained rival produces more; prices are higher for both firms. Both firms' outputs depend on the constrained firm's internal funds; the relationship is U-shaped for the constrained firm and inversely U-shaped for its unconstrained rival. The unconstrained rival has a higher market share, not because of predation but because of the cost disadvantage of the financially constrained firm.

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## 1. Introduction

In the study of how financial constraints affect a firm's output market decisions, two ideas play a central role. One is that *ex ante*, a firm incurring debt has an incentive to mitigate the risk of bankruptcy by limiting its borrowing, and hence behaves more cautiously in its output market.<sup>1</sup> The second idea is that *ex post*, debt alters a firm's incentives to invest. For example, "risk shifting" can arise because the firm is the residual claimant to high earnings but is protected from losses by limited liability. Similarly, bankruptcy costs provide an incentive to adopt strategies that generate cash and thus reduce the risk of bankruptcy. Some of these *ex post* effects lead to more aggressive output market behavior in the form of high output or low prices; others have the opposite effect.<sup>2</sup>

How *ex ante* and *ex post* effects work in isolation is well understood. However, in any realistic setting, we should expect both effects to be present, and little is known about how they interact. We study their interaction in a model in which a financially constrained firm competes in a Cournot market with a firm that is rich in cash. The constrained firm can raise funds from an investor to finance its production costs.

We follow the approach of Brander and Lewis (1986) and the subsequent industrial organization literature<sup>3</sup> in assuming that the financially constrained firm (hereafter "the firm") chooses how much money to raise from an investor *before* deciding how to spend its funds; this decision cannot be specified in a contract. We go beyond this literature in two ways. First, while debt is a key element of other models, its use is typically exogenously imposed. In reality, however, financial contracting and product market decisions are not separately made. We therefore derive debt as an optimal contract, and show that the resulting implications about firms' product market behavior are quite different from the predictions of models in which debt is exogenous.<sup>4</sup> Second, we explicitly account for variable production costs, which are typically ignored in the literature. We find that they play a central role.

Our main results are the following: (1) Debt finance necessarily entails a risk of bankruptcy and the loss of future profits. *Ex ante*, the firm has an incentive to limit this risk by reducing its borrowing. (2) The *ex post* distortions typically associated with debt vanish if debt is derived as an optimal contract; nevertheless, the firm's incentives after signing a debt contract differ from its incentives *ex ante*. (3) Because the firm must finance production out of its available funds, its *ex ante* incentive to produce less overrides its *ex post* incentive to produce more: A financially constrained firm "underinvests". (4) The firm's output is U-shaped, i.e., nonmonotonic, in its level of internal funds. (5) Variable costs are the critical link between a firm's financing and product market decisions; if costs are assumed to be zero, as is common in the literature, product market behavior does not

<sup>1</sup> See, e.g., Gale and Hellwig (1985), Bolton and Scharfstein (1990), or Stenbacka and Tombak (2002).

<sup>2</sup> Firms become more aggressive in, e.g., Brander and Lewis (1986), Maksimovic (1988), and Hendel (1996); they become less aggressive in, e.g., Glazer (1994) and Chevalier and Scharfstein (1996); either effect can occur in Showalter (1995). For a survey of the literature, see Maksimovic (1995).

<sup>3</sup> Cf. the references in the previous footnote.

<sup>4</sup> Maurer (1999) and Faure-Grimaud (2000) also derive debt as an optimal contract in models of product market competition with financial constraints. See Section 6.2 for a discussion of these papers.

depend on the firm's internal funds or debt. (6) Oligopoly interaction does not fundamentally change the effects of financial constraints on a firm's output market behavior; it merely amplifies them.

We conclude that the emphasis on the ex post effects of debt that prevails in the industrial organization literature is a result of ignoring production costs and of treating debt contracts as exogenously given securities. Under methodologically more appealing assumptions, the effects of financial constraints on product market behavior strongly differ from most predictions of the industrial organization literature (see the references in footnote 2), and instead resemble what simple models of debt-financed investment (which ignore ex post effects) would predict.<sup>5</sup>

The setup of the model is follows. Like in [Diamond \(1984\)](#), [Bolton and Scharfstein \(1990\)](#), and [Hart and Moore \(1998\)](#), the firm's earnings are not contractible. We assume that they are unobservable to the investor, which captures the idea that the firm can easily divert or hide its cash flow. The investor can threaten to liquidate the firm if it fails to repay, in which case its owners forfeit future profits. In this setting, a debt-like contract is optimal: It minimizes the probability of liquidation while inducing the firm to repay and allowing the investor to break even on average.

We extend the analysis in the papers mentioned by letting the output choice itself be unobservable, which introduces an additional moral hazard problem. Such moral hazard problems are key to industrial organization models that analyze ex post incentives. The contract affects the firm's output choice and hence its distribution of earnings, and an optimal contract must make sure that the firm both chooses the correct output level and has an incentive to repay. These goals may conflict, and therefore the design of the optimal contract cannot be separated from the output market incentives the contract induces. We show that in spite of these complications a simple debt contract is optimal.

As it turns out, once the contract is signed, the firm has first-best incentives; that is, the distortions emphasized in other models do not arise. By design of the optimal contract, the usual distortion caused by a debt-like repayment pattern is exactly offset by a probability of liquidation that increases with the extent of the firm's default. Ex ante, at the borrowing stage, the firm internalizes the cost of liquidation because the investor must break even in expected terms. At this stage, it prefers a smaller output level to limit its borrowing. Hence, the firm's incentives to produce are different ex ante and ex post.

With positive variable costs, the firm cannot produce more than it can finance using its own and borrowed funds. This implies that it can effectively commit itself to produce little by restricting its borrowing ex ante, although ex post the firm would want to produce more. Positive variable costs imply a close link between borrowing and investing that does not exist if marginal costs are zero or subsumed in a firm's earnings, as is often assumed in the industrial organization literature.<sup>6</sup> With zero variable costs and an optimal debt

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<sup>5</sup> For an extension of such models, see, e.g., [Stenbacka and Tombak \(2002\)](#), who assume that financing and investment decisions are simultaneous, and study how an oligopolistic firm's choice between debt and equity finance depends on its internal funds.

<sup>6</sup> See, e.g., [Brander and Lewis \(1986\)](#), [Glazer \(1994\)](#), [Showalter \(1995\)](#), or [Faure-Grimaud \(2000\)](#).

contract, the firm would ex post always produce the Cournot quantity, and there would be no link at all between financing and output decisions.

Next, we study how the firm's output varies with the internal funds that the firm can contribute to finance production. We find that output is a U-shaped function of the level of internal funds, which is driven by two effects. First, there is a "cost effect": A decrease in internal funds increases the probability of liquidation for any given level of production because the firm must borrow more. This increases the marginal cost of output expansion, which induces the firm to produce less. The second effect is a "revenue effect": Producing a high output allows the firm to generate revenue that it needs to repay the loan. This provides an incentive to increase output. For strongly negative levels of internal funds (which can occur if the firm must also incur fixed costs), the revenue effect dominates the cost effect, and output increases as the firm's internal funds decrease.

This nonmonotonicity implies that when looking at the output market effects of financial constraints, one has to distinguish between the *existence* of financial constraints and *changes* in the severity of those constraints. For example, it is often suggested that an increase in leverage leads a firm to produce more. In our model, the leveraged firm never produces more than a financially unconstrained firm does. On the other hand, if "more" means, *compared to the previous output level*, this effect can occur in our model if the level of internal funds is sufficiently negative.

Our analysis of duopoly competition with financial constraints yields several insights. First, financial constraints weaken a firm's competitive position: It produces less than the Cournot output, and in response its rival produces more, while total industry output decreases. Under Cournot competition with differentiated goods, the constrained firm's resulting market price is higher than the rival's, but both firms' prices are higher than with two unconstrained firms. Second, competition amplifies the effects of financial constraints: Our results hold for a monopoly, but are more pronounced in duopoly, because the rival's increase in output induces the constrained firm to reduce output even further. Thus, the output market effects of financial constraints are likely to be higher in industries in which competition is most intense. Third, we discuss to what extent financial predation can occur in our (static) model. We observe that the notion of financial predation itself is not necessarily well defined because a financially strong rival may produce more and have a lower price than a constrained rival simply because the firms' effective marginal costs are different, without any explicit "predatory scheme" used.

Our results are consistent with most empirical studies: Opler and Titman (1994), Chevalier (1995a), Phillips (1995), Kovenock and Phillips (1995, 1997), Khanna and Tice (2000), and Grullon et al. (2002) find that highly leveraged firms invest less and lose market share, in line with our underinvestment result. In addition, Chevalier (1995a) and Kovenock and Phillips (1995, 1997) find that for the less leveraged rivals of firms undergoing an LBO, both investments and share prices increase. Chevalier (1995b) finds that following an LBO, supermarkets charge higher prices if their rivals are also leveraged, but lower prices if the rivals are less leveraged and concentrated. The first effect is as predicted by our theory, the second possibly a result of predation. Phillips (1995) also finds that after LBOs, prices generally increase. Zingales (1997), in contrast, finds evidence of lower prices on part of overleveraged firms in the trucking industry.

**2. The model**

Two risk-neutral firms, 1 and 2, compete in quantities and produce  $q_1$  and  $q_2$ , respectively, at marginal cost  $c$ . Firm  $i$ 's revenue is  $R^i(q_1, q_2, \theta)$ , where  $\theta$  is a random variable distributed with density  $f(\theta)$  over some interval  $[\underline{\theta}, \bar{\theta}]$ . We assume the following about  $R^i$ :

- (1)  $R^1(0, q_2, \theta) = 0$  for all  $q_2$  and  $\theta$ .
- (2)  $R^1$  and  $R^2$  are twice differentiable in all arguments.
- (3)  $R_2^1$  and  $R_{12}^1$  are both negative.
- (4)  $R^1$  is strictly concave and has a unique maximum in  $q_1$  for each  $q_2$  and  $\theta$ .
- (5)  $R_{11}^1 R_{22}^2 > R_{21}^1 R_{12}^2$  for all  $q_1, q_2, \theta$ .
- (6)  $R^1$  and  $R^2$  are symmetric, i.e.,  $R^1(q, q', \theta) = R^2(q', q, \theta)$  for all  $q, q', \theta$ . Thus, all assumptions above about  $R^1$  hold mutatis mutandis for  $R^2$ .

Because both firms have the same constant marginal cost, these six assumptions also hold for the firms' net profits  $R^i(q_1, q_2, \theta) - cq_i$ . The first five assumptions are standard in Cournot models. For convenience, they are stated more restrictive than necessary. Together with the symmetry of the  $R^i$ , they guarantee the existence and uniqueness of a symmetric Nash equilibrium in  $q_1$  and  $q_2$  (which will serve as a reference point for the asymmetric equilibrium we derive below). That is, there exists  $q^*$  such that

$$\begin{aligned}
 q^* &= \arg \max_{q_1} \int_{\underline{\theta}}^{\bar{\theta}} R^1(q_1, q^*, \theta) f(\theta) d\theta - cq_1 \\
 &= \arg \max_{q_2} \int_{\underline{\theta}}^{\bar{\theta}} R^2(q^*, q_2, \theta) f(\theta) d\theta - cq_2.
 \end{aligned}
 \tag{1}$$

We shall refer to  $q^*$  as the Cournot quantity. Finally, if  $q_i^*(q_j)$  denotes firm  $i$ 's best response to  $q_j$  as specified in Eq. (1), we assume that

- (7) The derivatives  $R_{\theta}^i$  and  $R_{q_i \theta}^i$  are both positive for any  $q_i \leq q_i^*(q_j)$ .
- (8)  $R^i(q_1, q_2, \underline{\theta}) = 0$  for any  $q_1$  and  $q_2$ .

Assumption 7 states that higher values of  $\theta$  are “good” states of the world: They correspond to higher revenue and also a higher marginal return on output. A natural interpretation is to think of  $\theta$  as the state of demand. The last assumption ensures that a firm that borrows will default with positive probability. Together with  $R_{\theta}^i > 0$ , it implies that the probability of default converges to zero as the amount borrowed goes to zero.<sup>7</sup>

<sup>7</sup> Assumptions 7 and 8 imply  $R_{\theta}^i(q_1, q_2, \theta) > 0$  for all  $q_1 < q_1^*(q_2)$  and  $\theta > \underline{\theta}$  which in turn is equivalent to the assumption that increases in output lead to a first-order stochastic dominant shift in the distribution of realized revenues. That is, if  $G(R^i(q_1, q_2, \theta))$  is the c.d.f. of  $R^i$  induced by  $q_1, q_2$ , and  $\theta$ , then  $R_{\theta}^i(q_1, q_2, \theta) > 0$  is equivalent to  $\partial G(R^i(\cdot)) / \partial q_i < 0$ . Assumption 8 is not necessary for what follows; it merely serves to avoid tedious case distinctions that add little insight, see footnote 20.

Our model embodies the assumption that production and sales are separated in time. In many industries, firms choose capacities and inputs (e.g., employees who must be paid) before they learn the actual level of demand, and set or adjust prices afterward. In contrast, it seems much less common that firms commit to prices without knowledge of the level of demand and are unable to change them as information arrives.<sup>8</sup> We therefore believe that in a model with stochastic demand, it is not an arbitrary modeling choice whether firms compete in quantities or prices. To abstract from inventory building, we assume that products (or inputs) can be stored temporarily, but not beyond the current period. This assumption seems most appropriate for industries selling perishable goods, services, or durable goods with high market depreciation (e.g., cars).<sup>9</sup>

We assume that firm 1 is financially constrained, while firm 2 is not. More precisely, suppose that firm 1 has retained earnings  $r_0$  available and must finance both fixed costs  $F$  and variable costs  $cq$ . The fixed costs comprise both production startup costs and any outstanding liabilities that the firm may need to pay down before output is produced. We denote by “internal funds” the firm’s own funds that it can use to pay for variable production costs,  $w_0 = r_0 - F$ . Let  $w^* := cq^*$  denote the cost of producing the Cournot output  $q^*$ . Then we say that firm 1 is *financially constrained* if its internal funds are too small to finance the Cournot quantity, i.e., if  $w_0 < w^*$ . If the fixed costs exceed the firm’s retained earnings, the internal funds are negative. Because financing may still be feasible in this case, we allow for negative values of  $w_0$ . Negative internal funds are also empirically relevant: Cleary et al. (2003) study 20 years of annual Compustat data and find that different measures of internal funds are negative for approximately a quarter of all firm-year observations.

In addition to its internal funds, firm 1 can raise funds from an investor  $I$  in a competitive capital market. In a first-best world, firm 1 would promise to produce  $q^*$ , and it would agree with  $I$  on some form of profit sharing. However, we assume that neither  $q$  nor  $\theta$  can be observed by  $I$ , and that firm 1 cannot be forced to repay more than it earned because of limited liability. Feasible contracts then are ones in which firm 1 makes some (verifiable) payment to  $I$  and the contract specifies a probability with which the firm will be liquidated.<sup>10,11</sup>

If firm 1 is allowed to continue, its owners earn an additional payoff  $\pi_2 > 0$ . This payoff may represent future profits generated by the firm, and/or control rents that the firm’s owners enjoy. Only part of  $\pi_2$  can be transferred to  $I$ , who upon liquidation obtains an

<sup>8</sup> Instances of commitment to prices before demand is known include prices quoted in annual catalogs, on books, or at restaurants. Showalter (1995) analyzes a model similar to that of Brander and Lewis (1986), but in which firms set prices instead of quantities. He shows that firms will use strategic debt to commit to higher prices if demand is uncertain, but that firms will not use strategic debt if costs are uncertain. Showalter (1999) presents evidence in line with these predictions.

<sup>9</sup> In Section 7.1, we argue that our main results are likely to hold in a setting (similar to that of Kreps and Scheinkman, 1983) in which, upon observing the state of demand  $\theta$ , the firms compete in prices, taking their previously determined production levels as given.

<sup>10</sup> Alternatively, if the firm’s assets are divisible, the contract could stipulate partial liquidation of the assets. This would be formally equivalent to probabilistic liquidation of all assets if liquidation of a fraction  $\alpha$  of the assets yields a liquidation value  $\alpha L$  and a continuation value  $(1 - \alpha)\pi_2$ .

<sup>11</sup> We abstract from any agency problems that might exist within firm 1, e.g., among shareholders or between shareholders and managers. Such problems literally do not exist if the firm is run by a single entrepreneur; however, for reasons of symmetry, we prefer to speak of “firm 1” rather than an “entrepreneur” as competing with firm 2.

amount  $L < \pi_2$ . That is, transfer of ownership to  $I$  and subsequent liquidation of the firm leads to a loss of  $\pi_2 - L > 0$ . As we will see, the assumption that  $\pi_2 > L \geq 0$  is essential for deriving debt as an optimal contract and for the relevance of financial constraints: If  $\pi_2$  were zero, the firm would not have any incentive to repay, in which case debt-financed investment would not be feasible at all. And if  $L = \pi_2$ , there would not be any loss from transferring ownership of the firm to  $I$  (upon default), in which case external finance would be costless, and limits on the availability of internal funds irrelevant.<sup>12</sup> In a dynamic model, the firm's future profits are likely to depend on its revenue in the current period, and thus also on the firms' output choices as well as the state of realized nature. We abstract from this complication here by assuming that  $\pi_2$  is constant. The more general case in which  $\pi_2$  is a weakly increasing function of current revenue is discussed in Povel and Raith (in press); we comment in Section 5 to what extent our results carry over to this case.

The timing of the game is as follows:

- (1) Firm 1 can offer a financial contract to  $I$  to borrow  $w_1$ , which  $I$  accepts or rejects. Firm 2 knows  $w_0$  but cannot observe the contract between firm 1 and  $I$ .
- (2) The firms produce  $q_1$  and  $q_2$ , respectively, at constant marginal cost  $c$ . Firm 1's output is constrained by  $cq_1 \leq w_0 + w_1$ .  $I$  cannot observe either firm's quantity.<sup>13</sup>
- (3) The state of the world  $\theta$  is realized, and the firms earn revenue  $R^i(q_1, q_2, \theta)$  ( $i = 1, 2$ ). While the distribution of  $\theta$  is common knowledge, only firm 1, but not  $I$ , can observe  $\theta$  and its revenue.
- (4) Firm 1 makes some payment to  $I$ . Depending on this payment and the provisions of the contract, the firm is either liquidated or allowed to continue.

To abstract from adverse selection issues, we assume that at the beginning of the game, both firms and the investor have the same information, which also means that  $I$  and firm 2 know firm 1's financial position  $w_0$ . Firm 2, however, cannot observe the contract between firm 1 and  $I$ . This assumption implies that firm 1 cannot gain a first-mover advantage by committing itself to some output before firm 2 chooses its own.

More precisely, firm 1 might want to publicly commit itself, through a contract with  $I$  (or some other third party), to produce a higher output. Firm 2 would then respond by producing less, and firm 1 would obtain an advantage in its output market (see, e.g., Vickers, 1985; Fershtman and Judd, 1987). The problem with this idea, however, is that in the outcome of such a game, firm 1 is not using a best response against firm

<sup>12</sup> Our assumptions are technically equivalent to those of Bolton and Scharfstein (1990), where the firm requires additional funds from the investor in the future to continue its operation. As a referee pointed out, it is not necessary to assume that  $L$  represents a liquidation value;  $L$  could simply be the value of the firm in the hands of  $I$ . All that is necessary is that a transfer of ownership of the firm to  $I$  leads to a loss of  $\pi_2 - L > 0$  (otherwise, external finance would be costless in our model). However, we will nevertheless speak of liquidation whenever such a transfer of ownership occurs, to distinguish this case clearly from mere *bankruptcy* following default, where with some probability the firm remains in the hands of the current owners.

<sup>13</sup> We could allow for transfers or liquidation decisions at the end of stage 2; but as will become clear in Section 3, such provisions would not be included in an optimal contract.

2. Any provisions of a publicly announced contract can be undone by a second, secret contract whereby firm 1 produces less than announced, rendering the announcement noncredible.

If firm 2 cannot observe the contract, as we assume here, the parties play a simultaneous-moves game, although borrowing precedes and constrains the choice of output. This assumption does not per se rule out that firm 1 produces above the Cournot level and induces its rival to produce less. However, firm 1's contract and output must be a best response to firm 2's output. Any commitment effect vis-à-vis firm 2 generated by a contract between firm 1 and  $I$  must follow from the agency problem that creates the need for this contract, and not from attempts to influence firm 2's actions. For a discussion of these issues, see Katz (1991), Bolton, (1990), and Caillaud and Rey (1994).

While we do not allow firm 1 to commit itself by contract to a strategy that is not an optimal response to firm 2's strategy, we do assume that firm 1 and  $I$  can commit to any contract that is optimal at stage 2 of the game, taking firm 2's strategy as given. That is, we do not allow renegotiation of the contract (say, between stages 3 and 4) even if its fulfillment may call for liquidation of the firm, which by assumption is inefficient. We discuss the role of this assumption more fully in Section 5.

### 3. A simple debt contract

Our informational assumptions and the basic idea of the contract are similar to those in Diamond (1984) and Bolton and Scharfstein (1990): Because revenue is not observable, the threat of liquidation is necessary to induce the firm to repay any money. If the firm had no strategic decision to make (as the papers mentioned assume) or if the firm's output were contractible, it would then follow from the analyses of these papers that the optimal contract must have a debt-like structure.

Here, however, there is an additional agency problem: Firm 1 makes an unobservable quantity choice that affects the distribution of its revenue. This raises two questions, namely, what does an optimal financial contract look like in this setting, and how does the optimal contract affect firm 1's quantity choice? It turns out that these questions cannot be answered separately, which complicates the presentation of our analysis.

First, we derive the optimal contract for a *given* output choice (say, as if output were contractible); we call this a "simple" debt contract. Such a contract remains feasible in our setting, but may no longer be optimal because the details of the contract affect how much firm 1 decides to borrow and then to produce. Specifically, while a simple debt contract is optimal if firm 1 can commit itself to produce some  $q_1$ , firm 1 might prefer to choose some other  $q_1'$  after signing a simple contract if commitment to  $q_1$  is not possible. In this case, a different contract might be more efficient overall.

To answer this question, we analyze in Section 4 how the simple debt contract affects firm 1's incentives to produce both ex post and ex ante. Based on that analysis, we return to the question of contract design in Section 5 to show that a simple debt contract remains optimal in our setting with an additional moral hazard problem.



A preliminary and very general result is that when revenue is not observable, any optimal contract must resemble debt:

**Proposition 1.** *Any optimal contract between firm 1 and I has a debt-like structure: Firm 1 borrows  $w_1$  from I and promises to repay  $D \leq \pi_2$ . If firm 1 repays  $D$ , it is allowed to continue. If it repays  $r < D$ , it is liquidated with a probability that is decreasing in  $r$ .*

All proofs are in Appendix A. The basic idea of the proof is standard<sup>14</sup>: Because revenue is unobservable,  $I$  can induce the firm to repay only by threatening with liquidation upon default, in which case the firm would lose  $\pi_2$ . Moreover, whenever the firm is allowed to survive,  $I$  can obtain only some constant amount  $D$  (the “face value of debt”). More precisely, denote by  $\beta(r)$  the probability that the firm is allowed to continue, as a function of the repayment. The firm can be induced to repay  $D$  only if  $\beta(r)$  satisfies the incentive constraint

$$R - D + \pi_2 \geq R - r + \beta(r)\pi_2 \quad (2)$$

for any  $r < D$ . To minimize the expected loss from liquidation, the optimal contract induces the firm to pay out all of its revenue if it defaults. This requires that

$$\beta(R)\pi_2 \geq R - r + \beta(r)\pi_2, \quad (3)$$

which in turn implies that  $\beta$  must be increasing in  $r$ . That is, a defaulting firm is not liquidated with certainty, but with a probability that depends on the amount repaid: Failing to repay 99% of a debt obligation is “worse” than failing to repay 1%.

For Eq. (2) to hold requires that  $D \leq \pi_2$ , as stated in Proposition 1. We assume in what follows that  $\pi_2$  is sufficiently large, such that this constraint is not binding. As will become clear, this assumption for is convenience only: The constraint  $D \leq \pi_2$  may limit the amount  $w_1$  the firm can borrow, and therefore its output, but does not affect the structure of the optimal contract, and does not qualitatively affect any of our other results. Similarly, whether the firm or instead the investor has all bargaining power when offering a contract does not qualitatively matter for any of our results.

Proposition 1 is more general than previous results in that it also holds if the borrower’s investment is not contractible. As the proof shows, it is always possible to switch from an arbitrary contract to a debt-like contract that leads to a higher payoff for  $I$  while leaving firm 1’s net payoff in each state of the world, and hence its ex ante and ex post incentives, unchanged. This additional payoff can then be redistributed to the firm in an incentive-neutral way.

The optimal contract for any given  $q_1$  is the following:

**Proposition 2.** *If  $q_1$  is contractible, a contract with the structure described in Proposition 1 is optimal if for any repayment  $r < D$ , firm 1 is liquidated with probability  $1 - \bar{\beta}(r)$ , where  $\bar{\beta}(r) = 1 - (D - r)/\pi_2$ .*

<sup>14</sup> See, e.g., Diamond (1984) or Faure-Grimaud (2000) for the case of continuous revenue considered here, or Bolton and Scharfstein (1990) for the discrete case. For extensions to a multiperiod context, see Gromb (1994) and DeMarzo and Fishman (2000).

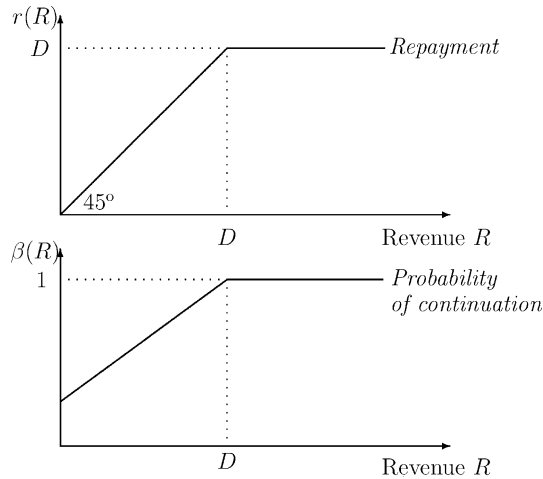


Fig. 1. Repayment and continuation probability as a function of revenue.

Firm 1’s repayment and survival probability as functions of its revenue are depicted in Fig. 1. The debt-like repayment structure follows from Proposition 1: Firm 1 owes  $I$  a fixed amount  $D$  and faces the possibility of liquidation if it repays less.  $D$ ,  $q_1$ , and  $q_2$  implicitly define a “bankruptcy” state  $\hat{\theta}$ :

$$D = R^1(q_1, q_2, \hat{\theta}). \tag{4}$$

If the realized state is  $\theta < \hat{\theta}$ , the firm is in default; if  $\theta \geq \hat{\theta}$ , it can repay  $D$  in full.<sup>15</sup>

For a given  $q_1$ , the optimal contract minimizes the expected net cost of liquidation subject to Eqs. (2) and (3). This is achieved by setting  $\beta(r) = \bar{\beta}(r)$  as defined in Proposition 2, such that Eqs. (2) and (3) hold with equality for any  $r < \min\{D, R\}$ . Firm 1 is then indifferent between paying  $D$  and paying less but suffering a loss of future profits with some probability, and weakly prefers to repay  $D$  or else all it has (notice that even if the repayment is zero,  $\beta$  may nevertheless be positive).

By contrast, a contract that calls for certain liquidation whenever the firm defaults is feasible but not optimal. If the decision to liquidate does not depend on the amount of repayment, a firm that is forced to default partially will always choose to default completely. When borrowing, it must promise a larger repayment, and hence is liquidated with higher probability than necessary.

One implication of the optimal contract is that after a default, the expected continuation value for the borrower is positive. In other words, the optimal contract specifies that absolute priority rules should be violated in bankruptcy. Such violations seem to be common in the United States. In practice, they may be better described by either partial losses of control, or partial liquidations. We could easily have adapted our

<sup>15</sup> Proposition 2 characterizes the structure of repayment and liquidation as functions of  $D$ , but leaves open how  $D$  is determined. In the next section, we close the model by assuming that  $I$  must break even on average, which allows us to determine  $D$  as a function of the anticipated duopoly equilibrium.

model to allow for certain but partial liquidation, instead of stochastic complete liquidation (cf. footnote 10).

The bankruptcy practice in many countries, particularly in the United States, may be regarded as a rough mechanism that makes the liquidation decision depend on the firm's financial situation. In our model, larger defaults make liquidation more likely. In practice, small defaults may be forgiven by lenders, or they may initiate one of several procedures that deal with insolvency. In the United States, firms can negotiate in private with their main lenders, to arrive at a so-called "workout". If a workout is not feasible because some lenders disagree, the majority can agree on a plan and file it with the bankruptcy court as part of a "prepackaged Chapter 11" (which can then be confirmed quickly). If negotiations prove even harder, it may be necessary to file for bankruptcy protection first, and then to negotiate with lenders under a bankruptcy judge's supervision (while in "Chapter 11"). If negotiations seem fruitless, the firm will have to agree to its least preferred procedure, a liquidation (under Chapters 7 or 11 of the Bankruptcy Code). Clearly, the legal process does not yield deterministic outcomes, and the extent of a default has an effect on which of these procedures will be used. If this effect is sufficiently strong, it may induce a defaulting borrower to fully cooperate in bankruptcy, ensuring a higher expected recovery rate, and therefore ex ante a lower promised repayment.

In Proposition 2, we have ignored the problem that firm 1 may prefer different levels of output before and after signing a contract. To address this issue, we now turn to the output market incentives implied by a simple debt contract.

#### 4. Output choice and duopoly equilibrium

In this section, we analyze firm 1's output incentives and the resulting product market equilibrium when firm 1 uses the contract described in Proposition 2 to obtain funds  $w_1$  from  $I$ . In Section 5, we show that this contract is indeed optimal in our setting.

##### 4.1. Ex post output choice

Our first result is that at stage 2 of the game, after signing the contract with  $I$ , firm 1 has first-best incentives at this stage, but is constrained by the funds borrowed:

**Proposition 3.** *Suppose that firm 1 has borrowed  $w_1$  from  $I$ , signing a debt contract according to Proposition 2. Then firm 1 has the same incentives as a financially unconstrained firm, but its output may be constrained by its available funds. Specifically, if  $w_0 + w_1 \geq w^*$ , firm 1 produces  $q^*$ , while if  $w_0 + w_1 < w^*$ , it produces  $q_1 = (w_0 + w_1)/c < q^*$ .*

The two cases of Proposition 3 are depicted in Fig. 2, which shows the firms' reaction curves at the output choice stage. Firm 1's reaction curve is truncated at the highest output that it can pay for. In panel (a), firm 1 has borrowed more than it needs to produce the Cournot output:  $w_1 > cq^* - w_0$ . Here, firm 1's financing constraint is not binding, and in the equilibrium of this subgame, both firms choose the Cournot output  $q^*$ . In panel (b), firm 1's own and borrowed funds are insufficient to produce  $q^*$ . Its reaction curve is truncated

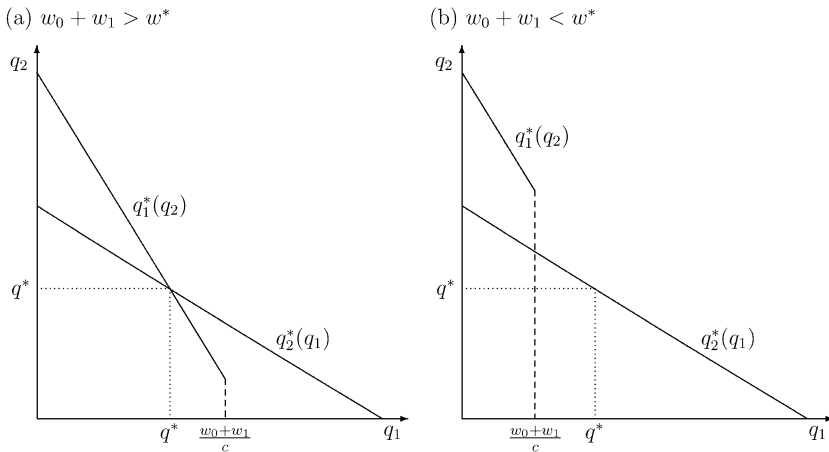


Fig. 2. Reaction curves at the output choice stage.

at a level below  $q^*$ , and the equilibrium is determined by the intersection of the two reaction curves, where firm 1 produces less than  $q^*$ , firm 2 more.

Proposition 3 establishes that with our simple debt contract, debt has *no strategic effect* on the borrower’s incentives when choosing an output level (although as panel (b) of Fig. 2 illustrates, there is a strategic effect on the rival’s output). This result stands in contrast to other models in which the repayment and liquidation provisions of debt are exogenous. In our model, unobservable revenue requires punishing default with possible liquidation, which mitigates the distortion of the firm’s output decision that might result from risk shifting or a fear of bankruptcy. If the contract is not only incentive compatible but also optimal (cf. the discussion of Proposition 2 above), the distortion is exactly eliminated: What the firm does not pay in money, it pays in expected loss of future profits. As a consequence, whatever the outcome, the firm loses a constant amount and is thus the residual claimant to its revenue.

Proposition 3 also demonstrates the significance of variable production costs. If  $c = 0$ , the firm may nevertheless have to borrow, e.g., to pay for fixed costs. In this case, we have  $w^* = 0$ , and according to Proposition 3, firm 1 just produces its Cournot output; that is, the firm’s financing and output decisions are unrelated.<sup>16</sup> In contrast, if production costs are positive and are incurred before the firm sells its goods, the firm’s financing and production decisions are linked directly: The firm cannot spend more than its available funds. Because the firm ex post has undistorted incentives, it produces  $q^*$  if  $cq^* \leq w_0 + w_1$ , or else as much as possible, i.e.,  $q_1 = (w_0 + w_1) / c$ . In particular, with the simple contract, a financially constrained firm never produces more than an unconstrained firm.

<sup>16</sup> See the results in Maurer (1999) and Faure-Grimaud (2000), discussed in Section 6.2.

4.2. Duopoly equilibrium and underinvestment

We now derive the equilibrium of the full game between firm 1, firm 2 and *I*. If  $w_0 + w_1 > w^*$ , Proposition 3 implies that firm 1 spends only  $w^*$  on production and holds  $\delta = w_0 + w_1 - w^*$  as cash. This part of the loan constitutes riskless debt; and firm 1 neither gains nor loses anything from borrowing in excess of  $w^*$ . Therefore, we can without loss of generality assume that firm 1 borrows exactly the amount needed to finance a desired level of  $q_1$ , after contributing its entire own funds; that is,  $w_1 = \max\{0, cq_1 - w_0\}$ . This establishes a one-to-one relationship between  $q_1$  and  $w_1$ .

Firm 1 then determines its output level when it decides how much to borrow. On the other hand, because firm 2 cannot observe the contract between firm 1 and *L*, it is as if firms 1 and 2 and *I* play a simultaneous-moves game. Formally, an equilibrium of the overall game is given by the  $q_1, q_2, D$ , and  $\hat{\theta}$  such that  $q_1$  and  $q_2$  maximize the profits of firms 1 and 2:

$$q_1 = \arg \max_{q_1} \int_{\underline{\theta}}^{\bar{\theta}} R^1(q_1, q_2, \theta) f(\theta) d\theta - D \quad \text{and} \tag{5}$$

$$q_2 = \arg \max_{q_2} \int_{\underline{\theta}}^{\bar{\theta}} R^2(q_1, q_2, \theta) f(\theta) d\theta - cq_2, \tag{6}$$

subject to the investor’s break-even constraint

$$\int_{\underline{\theta}}^{\hat{\theta}} \{R^1(q_1, q_2, \theta) + [1 - \beta(R^1(q_1, q_2, \theta))]L\} f(\theta) d\theta + \text{Prob}(\theta \geq \hat{\theta})D = cq_1 - w_0 \tag{7}$$

and Eq. (4), which defines  $\hat{\theta}$ . The right-hand side of Eq. (7) is the amount *I* lends to firm 1; the left-hand side is *I*’s expected payoff, which consists of firm 1’s repayment —*D* if firm 1 is solvent and  $R^1$  if it defaults—and the expected returns from liquidating firm 1’s assets in the case of default. In Appendix A, we show that the program above has a unique solution.

Because *I* must break even, firm 1 fully internalizes the costs of possible liquidation and trades off the benefits (higher current earnings) and costs of debt finance when choosing how much to borrow. Define

$$\underline{w} := - \left[ E_{\theta} [R_1(q^*, q^*)] \frac{\pi_2 - L}{\pi_2} + R_1(q^*, q^*, \bar{\theta}) \frac{L}{\pi_2} - cq^* \right] < 0. \tag{8}$$

The first two terms in the brackets in Eq. (8) are a weighted average of the expected revenue and the revenue for the highest level of demand when both firms set the Cournot quantity.

**Proposition 4.** *If firm 1 is financially constrained such that  $w_0 \in (\underline{w}, w^*)$ , then financing is feasible using a simple debt contract as described in Proposition 2; if  $c > 0$ , firm 1 produces strictly less than  $q^*$ .*

The first-order condition (A.12) derived in the proof can be equivalently expressed as

$$\int_{\underline{\theta}}^{\bar{\theta}} R_1^1(q_1, q_2, \theta) f(\theta) d\theta - c + \lambda \int_{\underline{\theta}}^{\hat{\theta}} [R_1^1(q_1, q_2, \theta) - c] f(\theta) d\theta = 0, \quad (9)$$

with  $\lambda > 1$ . Compared to an unconstrained firm, firm 1 places additional weight on the lower (default) states of demand, which are also states of lower marginal profit. It therefore produces less than the Cournot output. Put differently, because firm 1 may lose future profits, it has an incentive to reduce output below  $q^*$  to decrease the probability of default.

Ex post, the firm has first-best incentives, and if feasible, it would produce more (i.e., the Cournot level) than it would have wanted to commit to ex ante. With positive production costs, however, the firm faces a financing constraint, and by borrowing little, it can effectively commit to a lower output level.

Our result stands in contrast to the influential paper of [Brander and Lewis \(1986\)](#), who obtain the result that if a quantity-setting firm takes on debt, then it increases its output because of risk shifting. In a Cournot duopoly, the rival's best response is to cut output, leading to the conclusion that a firm may benefit from taking on debt purely for strategic reasons. The contrast results from three major differences between their paper and ours.

First, Brander and Lewis assume (as do some other authors) that a firm commits itself to some output before it learns about the level of demand, but can finance its production costs out of its later revenue. We argue that this is typically not feasible for a financially constrained firm. Whoever extends credit to pay for the production costs (banks, trade creditors, etc.) has to trust that the firm will repay the loan if its revenue is sufficient. In equilibrium, the parties will find it optimal to sign the debt-like contract derived here.

Second, in Brander and Lewis, bankruptcy is costless, whereas in our paper (part of) the firm's continuation value is lost if the firm is liquidated.<sup>17</sup> If one introduced a continuation value in the Brander–Lewis model, worry about survival could outweigh the limited-liability effect and hence lead to softer output market behavior, as in our paper. Nevertheless,  $\pi_2$  plays a very different role in this extended Brander–Lewis model than in ours: In the Brander–Lewis model, a firm's output would be decreasing in  $\pi_2$  because a higher  $\pi_2$  implies a higher cost of debt finance. In our model, output is independent of  $\pi_2$  as long as the firm is not credit-constrained, i.e., as long as  $\pi_2$  is large enough. For smaller values of  $\pi_2$ , firm 1 is credit-constrained, and its output is *increasing* in  $\pi_2$  because a greater  $\pi_2$  relaxes firm 1's credit constraint. Thus, while we assume for convenience that  $\pi_2$  is large, smaller values would lead to credit rationing and would only reinforce our underinvestment result.

More fundamentally, the continuation value  $\pi_2$  is necessary for debt finance to be feasible in the first place, because with unverifiable revenue, the firm has an incentive to repay its debt only if it has something to lose. In contrast, if (as in Brander and Lewis) the firm's revenue is verifiable, the firm and its investor would have no reason to write a debt contract (other than because of its expected effects on a third party, see the third point below). More generally, without some agency problem between investor and firm, there

<sup>17</sup> Brander and Lewis study the role of bankruptcy costs in their 1988 paper, cf. our discussion in Section 6.2.

would be no need to use debt. Any agency problem, however, entails some efficiency loss, which must be borne by the firm for an investor to break even.

Third, Brander and Lewis assume that a firm and its investor can publicly commit to a debt contract. The problem with this assumption is that in the resulting (Stackelberg-type) equilibrium, the chosen level of debt is not jointly optimal (for  $I$  and firm 1) if firm 2 cuts back its production in response. In our model, firm 1's contract with  $I$  is required to be a best response to firm 2's strategy, cf. our discussion in Section 2.

## 5. Optimality of the simple debt contract

Knowing how the simple debt contract of Proposition 2 affects output market incentives, we can now prove its optimality. It was designed to minimize the probability of liquidation subject to incentive compatibility. Because any optimal contract must have a debt-like repayment structure (cf. Proposition 1), an optimal contract that is not "simple" must specify a function  $\beta$  that lies below  $\bar{\beta}$  (as defined in Proposition 2), and an equal or smaller  $D$  (because  $I$  benefits from a higher probability of liquidation).

With a nonsimple contract, the firm's incentives when choosing an output level may not be first-best any more. If the firm is induced to choose an output no larger than  $q^*$ , then a nonsimple contract is strictly dominated because a simple contract can induce the same output choice at a lower expected liquidation loss.

The firm also cannot gain from a nonsimple contract that induces it to choose an output larger than  $q^*$ . Notice that Proposition 4 holds without any constraints on the level of output. That is, while Proposition 3 establishes that a  $q_1 > q^*$  cannot be implemented, the proof of Proposition 4 does not make use of this restriction. From Proposition 2 we know that if  $q_1$  were contractible, a simple debt contract would be optimal. But with a simple contract, the firm prefers to produce less than  $q^*$ . Thus, if a nonsimple contract makes financing even more expensive (due to the increased liquidation threat), the firm should limit its borrowing even more and thereby commit to producing less than  $q^*$ .

Thus, deviating from a simple contract would induce either an output smaller than Cournot that can be implemented more efficiently by a simple contract, or an output level larger than Cournot that the firm would ex ante not want to choose. Hence, we have:

**Proposition 5.** *A contract of the form given in Proposition 2 remains optimal if firm 1's output is not contractible.*

Notice that the simple contract is optimal even for small values of  $\pi_2$ , i.e., our assumption that  $\pi_2$  is large does not affect this result (see the discussion in Section 3).

In Povel and Raith (in press), we consider (within a single-firm model) an extension of the current setup where the firm's continuation payoff  $\pi_2$  is a weakly increasing function of its first-period investment (or equivalently, its expected first-period payoff). We show that in this more general setting, debt is still the optimal financial contract. The continuation function  $\beta$ , however, may take a more complicated form than that of  $\bar{\beta}$  shown to be optimal here. In particular, it may be necessary to punish default with a higher probability of liquidation (lower  $\beta$ ) to ensure that the firm will not simply run away with its borrowed funds. Also, the firm still underinvests, i.e., the equivalent of Proposition 4 above still

holds. However, because the exact form of the optimal debt contract can no longer be determined, it is also not possible to investigate how the firm's investment (or output) choice varies with its level of internal funds. Nevertheless it is still possible that a simple debt contract is optimal, in which case the results derived below should also generalize.

We have assumed that firm 1 and  $I$  can commit to any contract that is optimal ex ante, taking firm 2's strategy as given. That is, if the randomizing device employed in the optimal debt contract calls for liquidation of firm 1, then this decision is binding and is not renegotiated, although liquidation is ex post inefficient (because  $\pi_2 > L$ ). If, on the other hand, firm 1 expected to be able to renegotiate with  $I$ , it might want to withhold cash to buy its assets back from  $I$  and thus avoid liquidation, at least with some probability. In this case, there would be scope for renegotiation.

Following a standard approach, we assume that contracting parties can commit not to renegotiate in the future if this commitment is ex ante in their interest. Here, as in many other contexts (see, e.g., Bolton and Scharfstein, 1990; Hart and Moore, 1998), it is: while renegotiation leads to a higher surplus ex post, it also reduces the ex ante expected surplus. Because it is the threat of liquidation that induces the firm to repay the investor, it becomes more difficult for the investor to get her money back if provisions to liquidate the firm are renegotiated. She must then demand a higher repayment to break even, which reinforces the underinvestment result of Proposition 4, and leads to an overall less efficient outcome.

Reasons to rule out renegotiation include its costliness, e.g., because of asymmetric information or a multiplicity of lenders. Lenders may also refuse to renegotiate, to defend a reputation for nonforgiveness that is in their long-run interest. More fundamentally, Maskin and Tirole (1999) have argued that if the parties anticipate an incentive to renegotiate in the future, one would expect them to include rules that govern such situations in the original contract. That is, while contracts may be incomplete because of unforeseen contingencies, it is less plausible to assume that contracts are incomplete with respect to predictable events (such as, in our case, a decision to liquidate the firm).<sup>18</sup>

## 6. Internal funds and output choice

We now look at how firm 1's output depends on the internal funds  $w_0$  that the firm can contribute to cover variable production costs. The firm's internal funds may be negative if its fixed costs (including any outstanding bank loans the firm has to pay upfront) are high. We include this case in our analysis since up to some limit the firm can still obtain financing from  $I$ .

### 6.1. Nonmonotonicity of output

Denote by  $q_1(w_0)$  firm 1's equilibrium output when its internal funds are  $w_0$ .

<sup>18</sup> For a similar criticism, see DeMarzo and Fishman (2000). Harris and Raviv (1995) analyze a model of financial contracting where the parties, anticipating their incentive to renegotiate in the future, include rules that govern renegotiation in the original contract. The counterposition to the Maskin–Tirole argument is presented in Hart and Moore (1999).



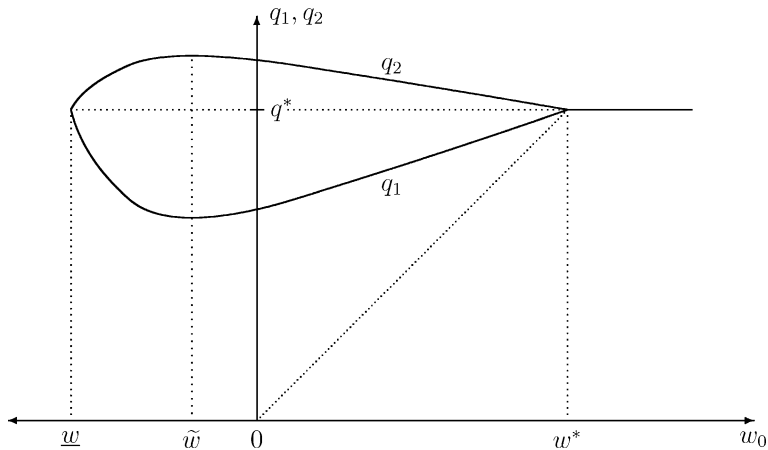


Fig. 3. Output as a function of firm 1’s internal funds.

**Proposition 6.** *Over the interval  $[\underline{w}, w^*]$ , firm 1’s equilibrium quantity  $q_1$  is a U-shaped function of  $w_0$ . More precisely,  $q_1(w^*) = q_1(\underline{w}) = q^*$ , and  $q_1(w_0)$  has a unique minimum at some  $\tilde{w} < 0$ .*

Proposition 6 is illustrated in Fig. 3.<sup>19</sup> For  $w_0 \geq w^*$ , both firms produce the Cournot quantity  $q^*$ . If firm 1 is financially constrained, its output is smaller than  $q^*$ , and firm 2’s is larger. Firm 1’s output reaches its minimum at a negative level of internal funds, which we denote by  $\tilde{w}$ . At  $w_0 = \underline{w}$ , the probability of a default reaches 1. Here, both firms produce the Cournot output again.<sup>20</sup> For the example of footnote 19,  $q_1(w_0)$  is slightly concave over some range of  $w_0 > \tilde{w}$ . Thus,  $q_1(w_0)$  is U-shaped or more precisely quasi-convex, but not necessarily convex throughout.

To understand this result, observe that the debt level  $D$  is implicitly determined by the investor’s break-even constraint (7). Its derivative  $\partial D / \partial q_1$  in turn is the marginal cost of debt-financed output at the financial contracting stage, which can be verified by inspection of the firm’s objective function at that stage (Eq. (5)). A change in  $w_0$  affects firm 1’s output by changing the marginal cost  $\partial D = \partial q_1$ . Because  $q_1$  enters in Eq. (7) in two ways, we can distinguish two direct effects of changing  $q_1$ . For given  $w_0$  and  $\hat{\theta}$ , an increase in  $q_1$  requires a larger loan  $cq_1 - w_0$  and hence larger debt  $D$ ; we call this the “cost effect”. On the other hand, given that  $q_1 < q^*$ , an increase in  $q_1$  leads to higher revenue and therefore to a higher expected repayment for  $I$ ; we call this the “revenue effect”.

<sup>19</sup> The curves are derived for a homogeneous-goods Cournot duopoly with inverse demand  $p = \theta(1 - q_1 - q_2)$ , where  $\theta$  is uniformly distributed on  $[0, 2]$ , and  $L = 0$ .

<sup>20</sup> Without assumption 8, two cases can arise that lead to a slightly different picture. First, if debt is risk-free up to some level, then the U-shaped and the  $q_1 = q^*$  segments of  $q_1(w_0)$  meet to the left of  $w^*$ . Second, the probability of bankruptcy may be strictly positive even with infinitesimal borrowing. For levels of internal funds slightly smaller than  $w^*$  the firm would then not borrow at all and just spend its internal funds, i.e., choose  $q_1 = w_0 / c$ . In Fig. 3,  $q_1(w_0)$  would have a third, middle segment along the dashed line, linking a lowered U-curve segment with the  $q_1 = q^*$  segment.

A decrease in internal funds requires a larger loan for any given quantity. How the firm responds to such a change depends on how the *marginal cost* of debt finance changes, which in turn depends on the cost and revenue effects. The cost effect leads to an increase in the marginal cost of debt and induces the firm to cut its output; the revenue effect leads to a decrease in the marginal cost of debt and induces the firm to increase output. For positive and moderately negative levels of internal funds, the cost effect dominates, and a drop in internal funds makes the firm choose a smaller output.

For sufficiently negative levels of internal funds, output must eventually increase if internal funds decrease, because otherwise financing becomes infeasible. The intuition behind this result is that if  $w_0$  is negative, the firm must already borrow just to pay its fixed costs. Only the difference between the amount the firm borrows and the sum of these expenses can be used to finance production. The investor, however, can break even only if production generates revenue that (on average) allows the firm to repay its loan. This leads to a negative relationship between internal funds and output: If  $w_0$  decreases and the firm needs to borrow a larger amount for purposes that do not generate revenue, the firm must eventually increase its output, for otherwise  $I$  would not provide funds. With internal funds  $\underline{w}$ , the firm produces the Cournot output, and defaults with probability 1. Thus, for sufficiently negative levels of internal funds, the firm's debt resembles a "junk bond" issue.<sup>21</sup>

Proposition 6 implies that it is important to be precise about what it means to say that a firm becomes "more aggressive" as its financial situation worsens. While a decrease in internal funds can lead a firm to produce more in our model, its output is still below the Cournot level. If the Cournot output is unknown, this prediction might be difficult to distinguish empirically from the prediction of other models that debt leads to an output *above* the Cournot level, if all that can be observed is the change in output in response to a change in internal funds. As Propositions 4 and 6 make clear, however, whether a firm produces less than an unconstrained firm and whether it produces less if its internal funds decrease are quite different questions.

## 6.2. Related results

A large body of work examines the effects of financial constraints on firms' investment behavior. Following the pioneering work of Fazzari et al. (1988), many studies have interpreted a high sensitivity of a firm's investment to changes in its internal funds as evidence of financial constraints. This approach was recently challenged by Kaplan and Zingales (1997). Cleary et al. (2003) study how a firm's optimal investment varies with two different measures of financial constraints: the firm's internal funds and the extent of asymmetric information between the firm and its investor. They show that more asymmetric information generally leads to lower investment, and investment becomes more sensitive to changes in internal funds. The relationship between internal funds and investment, in contrast, is U-shaped, according to a result similar to Proposition 6. An

<sup>21</sup> Consistent with this characterization, Fridson (1994) reports that the proceeds of a large fraction of junk bond issues in the early 1990s were used to pay down outstanding debt.

empirical analysis using a large data set lends strong support to this prediction. The results can explain seemingly contradictory findings in the recent empirical literature.

Brander and Lewis (1988) analyze a Cournot model in which firms strategically take on debt, writing standard debt contracts, and in which bankruptcy is costly. Both production costs and any ex post distortions of debt are ignored. For one version of the model in which bankruptcy costs are proportional to the extent of a firm's default, Brander and Lewis obtain that output is a U-shaped function of debt. Technically, our model has similar features, but these arise endogenously.

Moreover, in Brander and Lewis (1986, 1988), firms choose their output as a function of purely strategic debt. In our model, firms borrow (only) to finance costly production, which creates a feedback effect from output to financing needs. In particular, it can never pay for firm 1 to pay out cash to shareholders and thereby commit itself to be financially constrained, although for very low  $w_0$  a decrease in  $w_0$  leads to an increase in  $q_1$  and hence a decrease in  $q_2$ . To see this, define the "value" of an indebted firm as the sum of its expected profits and the expected future benefits. Then we can show:

**Proposition 7.** *The value of firm 1 is increasing in  $w_0$  for all levels of  $w_0$ , with a slope exceeding 1 if  $\underline{w} < w_0 < w^*$ , and a slope of 1 if  $w_0 \geq w^*$ . The value of firm 2 is decreasing in (firm 1's)  $w_0$  if  $w_0 > \tilde{w}$ , and increasing in  $w_0$  if  $w_0 < \tilde{w}$ .*

While the first part of Proposition 7 shows that firm 1 would never take on strategic debt, the second part has important implications for predatory behavior on part of firm 2, cf. Section 7.3 below: If firm 1 is severely constrained, then weakening firm 1's financial position even further can in fact be disadvantageous for firm 2, because it will make firm 1 a more aggressive competitor.

In Aghion et al. (2000), an agent borrows money and subsequently exerts unobservable effort that improves his output market performance. Lenders are willing to finance the agent's project only if his effort generates sufficiently large earnings. The larger the lender's share of earnings is, however, the lower is the agent's incentive to exert effort. In this situation, the agent can make a verifiable investment that credibly lowers his cost of effort. This induces him to increase effort, and hence enables him to raise more capital.

Our results differ from those in Aghion et al. (2000) in several ways. First, because of our optimal debt contract, firm 1 has first-best incentives ex post. Therefore, no contractible actions are needed to induce it to produce more if it decides to borrow more money: the lender *knows* that the firm will spend all money it has, up to  $q_1 = q^*$ . Second, the upward- and downward-sloping parts of the equilibrium effort in Aghion et al. correspond to two different regimes, a "shirking" regime and a "bonding" regime, distinguished by whether the agent is required to make a verifiable investment. In contrast, in our model, the entire U-curve is derived from the firm's first-order condition and the lender's break-even constraint (7). Finally, as in Brander and Lewis (1988), there is no feedback from output choice to financing needs in Aghion et al.; the latter are exogenous.

Maurer (1999) studies a model in which, as here, a financially constrained firm competes with an unconstrained one. Instead of choosing output, the manager of the financially constrained firm chooses how much effort to invest in an innovation with stochastic outcome. Maurer derives the optimal financial contract along the lines of Bolton

and Scharfstein (1990), and shows that the manager has first-best incentives to exert effort unless his rival stands to gain from predatory behavior. This situation is similar to the choice of output when variable costs are zero, because then financial decisions do not constrain subsequent operating decisions.

Faure-Grimaud (2000) revisits the model of Brander and Lewis (1986) and derives the debt contract between firms and their investors as an optimal contract. Faure-Grimaud assumes that a firm's output choice can be observed by investors but is not verifiable. However, an agreement to choose a certain output level is enforceable, because by assumption investors can withdraw their funds after the output decision is made, without incurring any losses. For any practical reasons, this makes output choice contractible, implying that moral hazard concerning a firm's output choice, which plays a central role in our paper, does not arise.<sup>22</sup>

## 7. Duopoly interaction and predation

In this section, we take a closer look at the duopoly interaction between the financially constrained firm 1 and its unconstrained rival, firm 2. Specifically, we discuss (1) equilibrium prices, (2) the role of the toughness of competition, and (3) financial predation.

### 7.1. Prices

If firm 1 produces lower output because of financial constraints, firm 2's response is to produce a higher output, as depicted in Fig. 3. As usual in Cournot models, total industry output decreases, because the slope of firm 2's reaction function is less than 1. The corresponding market price is higher than if both firms are unconstrained.

If firms 1 and 2 sell differentiated products, the firms' resulting prices are different. Consider a differentiated Cournot duopoly with the inverse demand  $p_i = \theta(1 - q_i - \sigma q_j)$ , where  $\sigma \in [0, 1]$  is the degree of product homogeneity. This is a generalized version of the example from above, and the resulting equilibrium quantities as a function of firm 1's internal funds look as depicted in Fig. 3 above. The corresponding average prices are shown in Fig. 4 for  $\sigma = 0.6$ . Firm 1's price is a mirror image of its quantity function; that is, a financially constrained firm charges a higher price than an unconstrained firm would. Firm 2 also charges more than it would without a financially constrained competitor (but

<sup>22</sup> In the discussion of his results, Faure-Grimaud observes that if in contrast to his assumption, the investors' funds are sunk after firms have chosen their output levels, the firms have first-best incentives. Because variable production costs are not considered, the firms end up in a standard Cournot equilibrium. In this case, financial constraints might still force a firm to shut down because external funds are too costly, but conditional on survival there would be no link between financial constraints and product market competition. As we have shown above, the predictions are quite different when production costs are modeled explicitly.

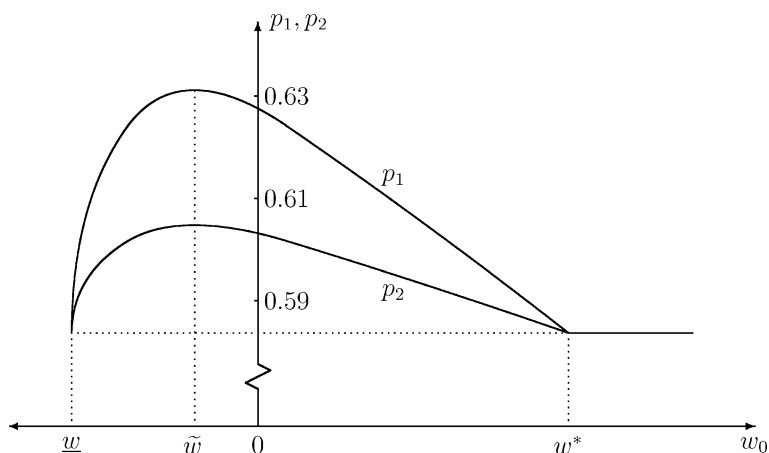


Fig. 4. Average prices in a differentiated Cournot market.

less than firm 1), because a decrease of firm 1’s output leads to an outward shift of firm 2’s residual demand function and hence to an increase of both its quantity and its price.<sup>23</sup>

Chevalier and Scharfstein (1996) offer a different explanation for why financially constrained firms might set higher prices: in the presence of switching costs, firms have an incentive to keep prices low in the long run, to attract new customers. In this sense, prices themselves are investments in market share. In the short run, however, firms in need of cash can increase profits by raising prices to exploit their locked-in customers. While this argument is very compelling for industries in which switching costs play a role, our theory shows that (1) switching costs are not a necessary assumption, and (2) prices need not be the firms’ main strategic variable for high debt to result in higher prices. Rather, firms may charge higher prices simply because they previously chose a lower level of production.

Phillips (1995) and Chevalier (1995b) study price changes in industries following large increases in debt by some of the firms. Phillips finds that prices rise in the fiberglass, tractor trailer, and polyethylene industries, but fall in the gypsum industry, in which several major competitors did not increase their leverage. Similarly, Chevalier finds that prices rise where the rivals of supermarkets undertaking an LBO are highly leveraged as well, but fall where the competitors are less leveraged, and concentrated. As Chevalier suggests, a price decrease could be a sign of predation (cf. also Section 7.3 below).

In the example above, prices are determined by the Cournot auctioneer. A more realistic setting would be one in which, upon observing the state of demand  $\theta$ , the firms compete in prices, taking their previously determined production as given. In contrast to the problems that arise in Kreps and Scheinkman (1983), Maggi (1996) has shown that a price equilibrium in such a game exists if the firms’ products are sufficiently differentiated and if the capacity constraints are not strict, i.e., allow for production above capacity at a

<sup>23</sup> See also Stenbacka and Tombak (2003), who study oligopoly interaction and compare the cases of strategic substitutes and complements. For strategic substitutes, their results are consistent with ours when the level of internal funds is nonnegative (as Stenbacka and Tombak assume).

higher marginal cost. While we have not formally analyzed this kind of model, we conjecture that in its reduced form, the (first stage) quantity game has the same features as the model presented here, and that equilibrium prices behave as in the model above.

### 7.2. Competition

If, in the example above,  $\sigma$  is decreased from 1 to 0, that is, as one moves from homogeneous to differentiated to independent products, the effects of financial constraints on the output market become less pronounced. If firm 1 is an independent monopolist,  $q_1(w_0)$  is still U-shaped, but its decrease in output below the monopoly level is smaller (in relative terms) than if a competitor is present. Thus, oligopolistic competition does not affect any of our results qualitatively, but rather amplifies them: If firm 1 reduces its output as a consequence of financial constraints, and firm 2 increases its output in response to this, then this second effect leads to further reduction of firm 1's output. Clearly, this additional effect depends on how competitive the market is.

Kovenock and Phillips (1997) find that debt has a significant effect on the product market only in relatively concentrated industries (see also Kovenock and Phillips, 1995). Their explanation for this is that concentrated industries are less competitive; hence, there is more scope for managers to spend cash flow in the output market in wasteful ways. In this case, debt is a useful disciplining device (cf. Jensen, 1986). In the light of the previous discussion, an alternative explanation is that with free entry, high concentration may not be a sign of a lack of competition, but rather the result of *intense* competition, as emphasized by Sutton (1991). With this interpretation, Kovenock and Phillips' finding is as expected: The effects of debt on product market behavior are larger the more competitive the industry is.<sup>24</sup>

### 7.3. Predation

According to the “long purse” story of predation, a financially strong firm can drive a financially weak firm out of the market by inflicting short-term losses on it, even if the firms are otherwise similarly strong on the output market. Long regarded as suffering from inconsistencies, this theory was given a rigorous formal foundation by Bolton and Scharfstein (1990). The essential features of that model are also present in ours: Even if firm 1 has a healthy position in the output market, the investor cannot bail out the firm if it goes bankrupt because the agency problems in the lender–borrower relationship make the threat of liquidation necessary.

One can define “financial predation” as any action taken by a financially healthy firm to reduce the profit of a financially constrained firm and drive it out of the market. With this definition, however, it is difficult to distinguish predation from the innocuous choice of a best response, both empirically and conceptually. On the one hand, firm 2 produces

<sup>24</sup> Cf. Raith (2003) for a formal presentation of this argument in a price-setting oligopoly model with stochastic costs. One result is that if products become more substitutable, price competition intensifies, profits fall, and some firms exit the market. In the new free-entry equilibrium, each firm serves a larger market, and fluctuations in costs therefore have a larger impact on demand and profits than before.

more than it would if firm 1 were not financially constrained. On the other hand, no “predatory scheme” is in use; firm 2 chooses a higher output *in response* to firm 1’s reduction in output, which in turn is a consequence of firm 1’s costs of borrowing. This situation seems no different from one in which firm 1 just has higher costs for some other reason, except that here, firm 1’s cost disadvantage is a result of its financial constraints, while the firms’ actual productions costs are the same.

Firm 2 plays a passive role in our model because it does not benefit from firm 1’s bankruptcy. If instead firm 2 stands to earn monopoly rents upon the exit of firm 1, its reaction curve lies further out compared to the case studied above. The outcome then seems closer to the intuitive meaning of “predation”: To increase the probability of firm 1’s exit, firm 2 behaves more aggressively than it would if firm 1’s presence in the market were certain (our model could be easily extended to analyze this idea; see also Maurer, 1999). Firm 2’s profit function remains continuous and single-peaked, however, which means that its output increase is not a drastic action that obviously constitutes a “predatory scheme”.<sup>25</sup>

An interesting extension of our model would be one in which next-period internal funds are this period’s net income, i.e., revenue minus debt repayment. Firm 2 could then increase output, not to drive firm 1 out of the market quickly, but just to hold its revenue to a low level. This would increase firm 1’s financing needs in the next period, lead to further reduction of its output, and possibly to bankruptcy in some later period. Recall from Proposition 6, though, that if firm 1 is already severely financially constrained, predatory behavior by firm 2 aimed at further weakening firm 1’s financial position could backfire, as it could lead firm 1 to produce *more* than before, reducing firm 2’s profit (cf. Proposition 7). Alternatively, if firm 1’s constraints are serious, firm 2 could just decide to “wait and see”: If this period’s low output leads to low retained revenue and even lower output, firm 1 might, over time, be forced to exit the market without any “help” from rivals.

## 8. Conclusion

It is well known in the corporate finance literature that if the threat of liquidation is an inefficient but necessary element of a debt contract, higher costs of debt financing lead to underinvestment, i.e., “softer” output market behavior, *in the absence of additional agency problems* regarding the choice of the investment. Independently, the industrial organization literature has explored how debt affects firms’ output market behavior by changing incentives *ex post*, e.g., by inducing risk shifting.

Each literature ignores what is central in the other: Papers studying firms’ *ex ante* incentives to borrow and invest ignore that often the use of borrowed funds cannot be specified in a contract, and that once a contract is signed, the incentives of a borrowing

<sup>25</sup> Drastic actions might be futile anyway: If  $w_0$  is positive, firm 1 can never be driven out of the market right away because it always has the option just to produce  $q_1 = w_0/c$ , i.e. not to borrow. Moreover, as before, driving the rival into certain bankruptcy does not ensure liquidation and exit from the market.

firm often change. In contrast, papers that emphasize the ex post effects of debt tend to ignore that firms normally raise external capital to finance their costs, including variable production costs.

In reality, firms' borrowing decisions depend on the intended use of borrowed funds and vice versa, as emphasized in the finance literature, whereas ex post distortions may arise because the use of borrowed funds cannot be specified in contracts, as emphasized in the industrial organization literature. To our knowledge, our paper is the first to combine the central themes of these two literatures. In contrast to most of the industrial organization literature, we explicitly account for production costs, and study the effects of debt in a context in which the use of debt is both feasible and optimal. Our results strongly differ from those obtained in the industrial organization literature, and instead resemble those of models of debt-financed investment in which ex post effects are ignored.

In our model, production costs are the critical link between a firm's financing and output decisions. With zero variable costs and an optimal debt contract, the firm's output is independent of its financial status. In contrast, if production costs must be incurred before any revenue is earned, the firm's output choice is ex post constrained by its available funds, and therefore in equilibrium determined by the ex-ante costs of borrowing. The result is softer output market behavior. Thus, the link between a firm's financing and output decisions is much closer than it appears in models in which the role of debt is reduced to its strategic effects.

The second contribution of our paper is to characterize output market behavior as a function of financial status. While a financially constrained firm produces less than an unconstrained firm, its output is not increasing, but rather U-shaped, in the degree of its financial constraints as measured by its internal funds. While for positive levels of internal funds, output decreases as a firm's financial situation gets worse, output increases again if the internal funds are sufficiently negative: If not all borrowed funds are invested in production, output must be large enough to generate the revenue that allow the firm to repay its loan.

Finally, our results suggest that the effects of financial constraints on a firm's output are reinforced, but not fundamentally altered, by oligopolistic interaction, in contrast to what has been suggested elsewhere. This brings the study of how financial constraints affect product market behavior, treated as a research field of its own for some time, very close to two more traditional lines of research: A firm's choice of debt-financed output can be analyzed in the same way as one would study investments in general. The resulting product market interaction, on the other hand, much resembles competition between firms that face different costs.

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**Appendix A. Proofs**

**Proof of Proposition 1.** Using the Revelation Principle, we can restrict attention to contracts in which firm 1 truthfully announces its level of total cash  $R$ , which leads to a required payment  $r(R)$  and a liquidation probability  $\beta(R)$ . It will later turn out that the optimal contract can be implemented simply through a function  $\beta(r)$  that specifies a liquidation probability as a function of the amount  $r$  that firm 1 repays. Firm 1 will truthfully announce its total cash  $R$  if

$$R - r(R) + \beta(R)\pi_2 \geq R - r(\hat{R}) + \beta(\hat{R})\pi_2 \quad \forall \hat{R}, R. \tag{A.1}$$

In addition, the required payment  $r(R)$  must be feasible, i.e.,  $r(R) \leq R$  for all  $R$ .

*Step 1.* For any  $R$ , either  $\beta(R) = 1$  or  $r(R) = R$ .

Consider an incentive compatible contract  $\langle r^0, \beta^0 \rangle$  for which this property does not hold, i.e., for which the set  $\rho := \{R | r^0(R) < R, \beta^0(R) < 1\}$  has a positive measure. Let a new contract  $\langle r^1, \beta^1 \rangle$  be defined by  $r^1(R) = r^0(R) + \delta(R)$  and  $\beta^1(R) = \beta^0(R) + \delta(R) / \pi_2$ , where  $\delta(R) = \min\{R - r^0(R), (1 - \beta^0(R))\pi_2\}$ . By construction,  $\langle r^1, \beta^1 \rangle$  satisfies the constraints  $r^1(R) \leq R$  and  $\beta^1(R) \leq 1$ . Moreover, for any  $R$ , firm 1’s payoff under  $\langle r^1, \beta^1 \rangle$  equals that under  $\langle r^0, \beta^0 \rangle$ :

$$\begin{aligned} R - r^1(R) + \beta^1(R)\pi_2 &= R - r^0(R) - \delta(R) + \left[ \beta^0(R) + \frac{\delta(R)}{\pi_2} \right] \pi_2 \\ &= R - r^0(R) + \beta^0(R). \end{aligned}$$

In particular, if  $\langle r^0, \beta^0 \rangle$  is incentive compatible, then so must be  $\langle r^1, \beta^1 \rangle$ . For any  $R \in \rho$ ,  $I$  is better off under  $\langle r^1, \beta^1 \rangle$  because for any  $R \in \rho$ ,

$$\begin{aligned} r^1(R) + [1 - \beta^1(R)]L &= r^0(R) + \delta(R) + \left[ 1 - \beta^0(R) - \frac{\delta(R)}{\pi_2} \right] L \\ &= r^0(R) + [1 - \beta^0(R)]L + \left( 1 - \frac{L}{\pi_2} \right) \delta(R). \end{aligned}$$

$I$ ’s gain in payoff can be redistributed to firm 1 (leaving  $I$ ’s payoff at zero) by decreasing all repayments  $r$  by a constant  $\varepsilon$  ( $r$  may become negative for some  $R$ );

this leaves *all* incentive constraints unchanged. Thus,  $\langle r^0, \beta^0 \rangle$  cannot have been optimal.

*Step 2.* There exists a constant  $D$  such that  $\beta(R)=1$  if and only if  $R \geq D$ .

For any two  $R, R', \beta(R)=\beta(R')=1$  if and only if  $r(R)=r(R')$ . Otherwise, Eq. (A.1) would be violated for either  $R$  or  $R'$ . Denote this constant repayment by  $D$ . Condition (A.1) then also implies that  $r(R) < D$  iff  $\beta(R) < 1$ . Then, for  $R \geq D$ , we must have  $\beta(R)=1$  because  $\beta(R) < 1 \Leftrightarrow r(R) < D \leq R$  would violate Step 1, and similarly for  $R < D$ , we must have  $\beta(R) < 1$  because  $\beta(R)=1 \Leftrightarrow r(R)=D > R$  would violate limited liability.  $\square$

**Proof of Proposition 2.** Suppose the optimal function  $\beta^0$  is *not*  $\bar{\beta}$  (as defined in Proposition 2). Then, in two steps, we can show that it is dominated by a simple contract that implements the same output choice [we omit the arguments  $(q_1, q_2, \theta)$  in the following].

Because of the truth-telling constraint for type  $R=D$ , any  $\beta^0$  must lie below  $\bar{\beta}$ . If the parties change to a simple contract without changing  $D^0$ , the investor’s break-even constraint is violated; therefore, the promised repayment must be increased to some  $D^1 > D^0$ . The net effect on the aggregate payoff of changing first  $\beta^0$  and then  $D^0$  is

$$\Delta^Z = \int_{\underline{\theta}}^{\hat{\theta}^0} \left[ -\frac{D^1 - R}{\pi_2} + 1 - \beta^0(R) \right] (\pi_2 - L) f(\theta) d\theta - \int_{\hat{\theta}^0}^{\hat{\theta}^1} \frac{D^1 - R}{\pi_2} (\pi_2 - L) f(\theta) d\theta$$

(where  $\hat{\theta}^0$  and  $\hat{\theta}^1$  are the bankruptcy thresholds with  $D^0$  and  $D^1$ , respectively). The net effect on the investor’s payoff is

$$\begin{aligned} \Delta^I &= \int_{\underline{\theta}}^{\hat{\theta}^0} \left[ \frac{D^1 - R}{\pi_2} - 1 + \beta^0(R) \right] L f(\theta) d\theta + \int_{\hat{\theta}^0}^{\hat{\theta}^1} \frac{D^1 - R}{\pi_2} L f(\theta) d\theta \\ &+ \int_{\hat{\theta}^0}^{\hat{\theta}^1} [R - D^0] f(\theta) d\theta + \int_{\hat{\theta}^1}^{\bar{\theta}} [D^1 - D^0] f(\theta) d\theta = 0. \end{aligned}$$

The last two terms are positive; therefore, the sum of the first two must be negative. Thus,  $\Delta^Z > 0$ , and because the investor’s expected payoff remains zero, the firm’s expected payoff must have increased, and the original contract  $\langle D^0, \beta^0 \rangle$  cannot have been optimal.

Two complications may arise, which do not affect the result, however. First, if  $w_1^0 > cq_1^0 - w_0$ , the firm borrows more than it needs for its production costs; if we apply the approach as outlined above, it could be that  $D^1 < D^0$ . In that case, instead of applying it directly, we first derive a contract that is equivalent and *then* apply the approach. If  $w_1^0 > cq_1^0 - w_0$ , the firm has at least an amount  $w_1 - (cq_1^0 - w_0)$  available, whatever revenue has been realized; this amount constitutes riskless debt that carries no risk premium. The equivalent contract is one in which both the borrowed amount and the promised repayment are reduced by  $w_1 - (cq_1^0 - w_0)$ ; it is equivalent because the firm can still finance the same output level, and its incentives must be unchanged (it would have had to part with that amount, anyway).

Second, if  $D^0 = \pi_2$ , we cannot construct the simple contract using the above method. Nevertheless, the firm would gain by switching to a simple contract while at the same time reducing output. The optimal loan size and output choice is such that the investor breaks even if  $D_1 = \pi_2$ . This is easy to see if we reverse the problem: Consider a simple contract that is meant to implement some output choice  $q_1$ . If  $D = \pi_2 + \varepsilon$  (for some small  $\varepsilon > 0$ ), the simple contract is not feasible. Reducing  $D$  by lowering  $\beta(R)$  below  $\beta(R)$  (using the liquidation value to help the investor break even) is not feasible because if  $D^0 = \pi_2$ , then already we have  $\beta(R(\cdot, \underline{\theta})) = 0$ , so that  $\beta$  cannot be reduced without violating the firm's incentive constraint. The remaining alternative is to reduce both  $D$  and  $w_1$ , and therefore  $q_1$ .  $\square$

**Proof of Proposition 3.** Denote by  $\tilde{R}^1(q_1, q_2, \theta)$  firm 1's total cash holdings after realization of  $\theta$ , which consist of  $R^1(q_1, q_2, \theta)$  and any unspent money. Under the contract of Proposition 1, firm 1 weakly prefers to repay  $\min \{D, \tilde{R}^1(q_1, q_2, \theta)\}$ . Then, for  $\theta \geq \hat{\theta}$ , firm 1 can repay  $D$  in full and is continued with certainty, whereas for  $\theta < \hat{\theta}$ , it pays  $\tilde{R}^1$  to  $L$  and is continued with probability  $1 - \frac{D - \tilde{R}^1}{\pi_2}$ . Hence, firm 1's expected payoff from choosing  $q$  is

$$\begin{aligned}
 E\pi(q_1, q_2) &= \int_{\underline{\theta}}^{\hat{\theta}} \left[ 1 - \frac{D - \tilde{R}^1(q_1, q_2, \theta)}{\pi_2} \right] \pi_2 f(\theta) d\theta \\
 &\quad + \int_{\hat{\theta}}^{\bar{\theta}} [\tilde{R}^1(q_1, q_2, \theta) - D + \pi_2] f(\theta) d\theta + w_0 + w_1 - cq_1 \\
 &= \int_{\underline{\theta}}^{\bar{\theta}} \tilde{R}^1(q_1, q_2, \theta) f(\theta) d\theta - D + w_0 + w_1 - cq_1 + \pi_2. \tag{A.2}
 \end{aligned}$$

This expression differs from the profit function in Eq. (1) only in the constant  $w_1 - D$ , once  $D$  is fixed in the contract. Firm 1 therefore has an objective function equivalent to that of an unconstrained firm, but it also faces the financing constraint  $cq_1 \leq w_0 + w_1$ . Then, if  $w_0 + w_1 \geq w^*$ , the standard Cournot equilibrium  $q_1 = q_2 = q^*$  results. On the other hand, if  $w_0 + w_1 < w^*$ , then the second-stage equilibrium in quantities is given where firm 1 spends all its available funds on production.  $\square$

**Lemma 8.** *The program defined by Eqs. (4–7) has a unique solution.*

**Proof.** Because  $R^i$  is strictly concave in  $q_i$ , it follows that the program has a unique solution if  $D$  as defined by Eq. (7) is convex in  $q_1$  for any given  $q_2$ . That is the case: write Eq. (7) as

$$\int_{\underline{\theta}}^{\hat{\theta}} \left[ R^1(\mathbf{q}, \theta) + \left( \frac{D - R^1(\mathbf{q}, \theta)}{\pi_2} \right) L \right] f(\theta) d\theta + \text{Prob}(\theta \geq \hat{\theta}) D - cq_1 + w_0 = 0,$$

where  $\mathbf{q}=(q_1, q_2)$ ; rearrange,

$$\frac{\pi_2 - L}{\pi_2} \int_{\theta}^{\hat{\theta}} [R^1(\mathbf{q}, \theta) - D]f(\theta)d\theta + D - cq_1 + w_0 = 0, \tag{A.3}$$

and differentiate twice with respect to  $q_1$ ; omitting arguments, writing  $R^1$  and  $\hat{R}^1$  instead of  $R^1(\mathbf{q}, \theta)$  and  $R^1(\mathbf{q}, \hat{\theta})$  and using subscripts to denote partial derivatives (e.g.,  $R^1_1 = \frac{\partial R^1(\mathbf{q}, \theta)}{\partial q_1}$ ), this yields

$$\begin{aligned} &\frac{\pi_2 - L}{\pi_2} \left[ \int_{\theta}^{\hat{\theta}} (R^1_{11} - D_{11})f(\theta)d\theta + 2(\hat{R}^1_1 - D_1)f(\hat{\theta})\hat{\theta}_1 \right] \\ &+ \frac{\pi_2 - L}{\pi_2} (\hat{R}^1 - D)[f'(\hat{\theta})(\hat{\theta}_1)^2 + f(\hat{\theta})\hat{\theta}_{11}] + D_{11} = 0. \end{aligned} \tag{A.4}$$

The second-last term is equal to zero because  $\hat{R}^1 = D$ ; substitute  $-\hat{R}^1_1\hat{\theta}_1$  for  $[\hat{R}^1_1 - D_1]$  (gained from implicit differentiation of  $\hat{R}^1 - D \equiv 0$ ), and rearrange to obtain

$$\frac{\pi_2 - L}{\pi_2} \int_{\theta}^{\hat{\theta}} R^1_{11}f(\theta)d\theta - 2\frac{\pi_2 - L}{\pi_2} f(\hat{\theta})\hat{R}^1_{\theta}(\hat{\theta}_1)^2 + \left[ 1 - \frac{\pi_2 - L}{\pi_2} \text{Prob}(\theta < \hat{\theta}) \right] D_{11} = 0.$$

Because  $R^1_{11} < 0$  and  $\hat{R}^1_{\theta} > 0$ , the sum of the first two terms must be negative; therefore,  $D_{11} > 0$ , i.e.,  $D$  is convex in  $q_1$  (the linearity of the cost function is not necessary for this result). □

**Proof of Proposition 4.** Substituting  $R^1(\mathbf{q}, \hat{\theta})$  for  $D$  into Eqs. (5) and (7) and setting up a Lagrangian for firm 1 leads to the first-order conditions

$$E[R^1_1] - \hat{R}^1_1 + \lambda \frac{\pi_2 - L}{\pi_2} \int_{\theta}^{\hat{\theta}} R^1_1 f(\theta)d\theta + \lambda \left[ 1 - \frac{\pi_2 - L}{\pi_2} \text{Prob}(\theta < \hat{\theta}) \right] \hat{R}^1_1 - \lambda c = 0 \tag{A.5}$$

$$-\hat{R}^1_{\theta} + \lambda \left[ 1 - \frac{\pi_2 - L}{\pi_2} \text{Prob}(\theta < \hat{\theta}) \right] \hat{R}^1_{\theta} = 0 \tag{A.6}$$

$$E[R^2_2(\mathbf{q}, \theta)] - c = 0 \tag{A.7}$$

and Eq. (7). Using Eq. (A.6), eliminate

$$\lambda = \frac{1}{1 - \frac{\pi_2 - L}{\pi_2} \text{Prob}(\theta < \hat{\theta})} \tag{A.8}$$

in Eq. (A.8), and the optimal  $q$  and  $\hat{\theta}$  are the solution to the system

$$g(\mathbf{q}, \hat{\theta}, w_0) = \frac{\pi_2 - L}{\pi_2} \int_{\underline{\theta}}^{\hat{\theta}} R_1^1(\mathbf{q}, \theta) f(\theta) d\theta + \left[ 1 - \frac{\pi_2 - L}{\pi_2} \text{Prob}(\theta < \hat{\theta}) \right] E[R_1^1(\mathbf{q}, \theta)] - c \tag{A.9}$$

$$h(\mathbf{q}, \hat{\theta}, w_0) = \int_{\underline{\theta}}^{\hat{\theta}} \left[ R^1(\mathbf{q}, \theta) + \frac{R^1(\mathbf{q}, \hat{\theta}) - R^1(\mathbf{q}, \theta)}{\pi_2} L \right] f(\theta) d\theta + \int_{\hat{\theta}}^{\bar{\theta}} R^1(\mathbf{q}, \hat{\theta}) f(\theta) d\theta - cq + w_0 = 0 \tag{A.10}$$

$$k(\mathbf{q}, \hat{\theta}, w_0) = E[R_2^2(\mathbf{q}, \theta)] - c = 0. \tag{A.11}$$

For any  $w_0$ , the firm’s optimal output is  $q_1$  if there exists a  $\hat{\theta}$  such that  $q_1, q_2, w_0$ , and  $\hat{\theta}$  jointly solve Eqs. (A.9–11). It is straightforward to establish that both  $(w_0, \mathbf{q}, \hat{\theta}) = (w^*, q^*, q^*, \underline{\theta})$  and  $(w_0, \mathbf{q}, \hat{\theta}) = (\underline{w}, q^*, q^*, \theta)$  are such solutions, because in both cases Eq. (A.9) reduces to the first-order condition of an unconstrained firm. If  $w_0 \in (\underline{w}, w^*)$ , in contrast, we have  $\underline{\theta} < \hat{\theta} < \bar{\theta}$ , and then Eq. (A.9) places relatively larger weight on the states  $\theta < \hat{\theta}$  with low  $R_1^1$ . Because  $R_{12}^1 < 0$ , it then follows that the solution to Eqs. (A.9) and (A.11) must satisfy  $q_1 < q^* < q_2$ . □

**Proof of Proposition 6.** (1) From the proof of Proposition 4, we have  $q_1 = q^*$  for both  $w_0 = \underline{w}$  and  $w_0 = w^*$ , which fixes the endpoints of the function  $q_1(w_0)$ . Next, we determine the slope of  $q_1(w_0)$ . The partial derivatives of  $g, h$ , and  $k$  with respect to  $q_1, q_2, \hat{\theta}$ , and  $w_0$  are (arguments omitted).

$$g_1 = \left[ 1 - \frac{\pi_2 - L}{\pi_2} \text{Prob}(\theta < \hat{\theta}) \right] E[R_{11}^1(\mathbf{q}, \theta)] + \frac{\pi_2 - L}{\pi_2} \int_{\underline{\theta}}^{\hat{\theta}} R_{11}^1(\mathbf{q}, \theta) f(\theta) d\theta$$

$$g_2 = \left[ 1 - \frac{\pi_2 - L}{\pi_2} \text{Prob}(\theta < \hat{\theta}) \right] E[R_{12}^1(\mathbf{q}, \theta)] + \frac{\pi_2 - L}{\pi_2} \int_{\underline{\theta}}^{\hat{\theta}} R_{12}^1(\mathbf{q}, \theta) f(\theta) d\theta$$

$$g_{\hat{\theta}} = -f(\hat{\theta}) \frac{\pi_2 - L}{\pi_2} \{ E[R_1^1(\mathbf{q}, \theta)] - R_1^1(\mathbf{q}, \hat{\theta}) \}$$

$$h_1 = \int_{\underline{\theta}}^{\hat{\theta}} \left[ R_1^1(\mathbf{q}, \theta) + (R_1^1(\mathbf{q}, \hat{\theta}) - R_1^1(\mathbf{q}, \theta)) \frac{L}{\pi_2} \right] f(\theta) d\theta + \text{Prob}(\theta \geq \hat{\theta}) R_1^1(\mathbf{q}, \hat{\theta}) - c$$

$$h_2 = \int_{\underline{\theta}}^{\hat{\theta}} \left[ R_2^1(\mathbf{q}, \theta) + (R_2^1(\mathbf{q}, \hat{\theta}) - R_2^1(\mathbf{q}, \theta)) \frac{L}{\pi_2} \right] f(\theta) d\theta + \text{Prob}(\theta \geq \hat{\theta}) R_2^1(\mathbf{q}, \hat{\theta})$$

$$h_{\hat{\theta}} = \int_{\underline{\theta}}^{\hat{\theta}} R_{\theta}^1(\mathbf{q}, \hat{\theta}) \frac{L}{\pi_2} f(\theta) d\theta + \text{Prob}(\theta \geq \hat{\theta}) R_{\theta}(\mathbf{q}, \hat{\theta})$$

$$k_1 = E[R_{12}^2]$$

$$k_2 = E[R_{22}^2]$$

$$g_w = 0, \quad h_w = 1 \quad \text{and} \quad k_w = 0. \tag{A.12}$$

Using  $g=0$ ,  $h_1$  can also be written as  $-[1 - \frac{\pi_2-L}{\pi} \text{Prob}(\theta < \hat{\theta})] (E[R_1^1(\mathbf{q}, \theta)] - R_1^1(\mathbf{q}, \hat{\theta}))$ . According to Cramer’s rule, we have  $dq_1/dw_0 = \det(\mathbf{M}_1)/\det(\mathbf{M})$ , where

$$\mathbf{M} = \begin{pmatrix} g_1 & g_2 & g_{\hat{\theta}} \\ h_1 & h_2 & h_{\hat{\theta}} \\ k_1 & k_2 & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{M}_1 = \begin{pmatrix} 0 & g_2 & g_{\hat{\theta}} \\ -1 & h_2 & h_{\hat{\theta}} \\ 0 & k_2 & 0 \end{pmatrix}.$$

Because  $\det(\mathbf{M}_1) = -k_2 g_{\hat{\theta}}$  and  $k_2 < 0$ ,  $\det(\mathbf{M}_1)$  has the same sign as  $E[R_1^1(\mathbf{q}, \theta)] - R_1^1(\mathbf{q}, \hat{\theta})$ .

(2) We now show that  $q_{ww} > 0$  when  $q_w = 0$ , which implies that  $q(w_0)$  has a unique extremal point, which is a minimum. Differentiate Eqs. (A.9–11) twice with respect to  $w_0$  to obtain

$$M \begin{pmatrix} \frac{d^2 q_1}{dw_0^2} \\ \frac{d^2 q_2}{dw_0^2} \\ \frac{d^2 \hat{\theta}}{dw_0^2} \end{pmatrix} = - \begin{pmatrix} \frac{dg_{\hat{\theta}}}{dw_0} \hat{\theta}_w + \frac{dg_1}{dw_0} \frac{dq_1}{dw_0} + \frac{dg_2}{dw_0} \frac{dq_2}{dw_0} \\ \frac{dh_{\hat{\theta}}}{dw_0} \hat{\theta}_w + \frac{dh_1}{dw_0} \frac{dq_1}{dw_0} + \frac{dh_2}{dw_0} \frac{dq_2}{dw_0} \\ \frac{dk_1}{dw_0} \frac{dq_1}{dw_0} + \frac{dk_2}{dw_0} \frac{dq_2}{dw_0} \end{pmatrix}. \tag{A.13}$$

When  $dq_1/dw_0 = 0$ , we also have  $dq_2/dw_0 = 0$ , and  $g_{\hat{\theta}} = h_1 = 0$ , according to Step 1. Then,  $\det(\mathbf{M})$  reduces to  $-h_{\hat{\theta}}(g_1 k_2 - g_2 k_1)$ , which is negative because  $R_{11}^1 R_{22}^2 > R_{12}^1 R_{12}^2$ . On the right-hand side of Eq. (A.16), all terms containing  $dq_i/dw_0$  drop out, and then (again using Cramer’s rule) we have  $d^2 q_1/dw_0^2 = k_2 h_{\hat{\theta}} (dg_{\hat{\theta}}/dw_0) \hat{\theta}_w / \det(\mathbf{M})$ , which has the same sign as  $(dg_{\hat{\theta}}/dw_0) \hat{\theta}_w$ . Here, we have

$$\frac{dg_{\hat{\theta}}}{dw_0} = g_{1\hat{\theta}} \frac{dq_1}{dw_0} + g_{\hat{\theta}\hat{\theta}} \hat{\theta}_w + g_{\hat{\theta}w} = g_{\hat{\theta}\hat{\theta}} \hat{\theta}_w$$

because the first and third terms vanish. Thus,  $d^2 q_1/dw_0^2$  has the same sign as  $g_{\hat{\theta}\hat{\theta}}(\hat{\theta}_w)^2$ , where

$$g_{\hat{\theta}\hat{\theta}} = \frac{\pi_2 - L}{\pi_2} \{ -f'(\hat{\theta}) [E[R_1^1(\mathbf{q}, \theta)] - R_1^1(\mathbf{q}, \hat{\theta})] + f(\hat{\theta}) R_{1\theta}(\mathbf{q}, \hat{\theta}) \},$$

which in turn is positive because the term in [ ] vanishes when  $dq_1/dw_0 = 0$ .

(3) Finally, we show that  $\tilde{w} < 0$  by proving that  $q_1(w_0)$  must be increasing at  $w_0 = 0$ , from which the claim follows because  $q_1(w_0)$  has a unique minimum. Define  $\hat{h}(\mathbf{q})$  as  $f$ ’s

profit as a function of  $q_1$  and  $q_2$  at  $w_0=0$ , holding  $\hat{\theta}$  fixed at the level where Eq. (7) is satisfied. That is,

$$\hat{h}(\mathbf{q}) = \int_{\underline{\theta}}^{\hat{\theta}} R^1(\mathbf{q}, \theta) f(\theta) d\theta + \text{Prob}(\theta \geq \hat{\theta}) R^1(q_1, q_2, \hat{\theta}) - cq_1.$$

Because  $\hat{h}(0, q_2) = 0$  and by construction  $\hat{h}(q(0), q_2) = 0$ , and because  $\hat{h}$  is concave in  $q$ , it follows that

$$\frac{\partial h(q(0), q_2)}{\partial q_1} = \int_{\underline{\theta}}^{\hat{\theta}} R^1_1(\mathbf{q}, \theta) f(\theta) d\theta + \text{Prob}(\theta \geq \hat{\theta}) R^1_1(\mathbf{q}, \hat{\theta}) - c < 0.$$

But this derivative equals  $h_1$  according to Eq. (A.15), and therefore equals  $-[1 - \frac{\pi_2 - l}{\pi} \text{Prob}(\theta < \hat{\theta})] \{E[R^1_1(\mathbf{q}, \theta)] - R^1_1(\mathbf{q}, \hat{\theta})\}$ . Thus, if  $h_1 < 0$  at  $w_0=0$ , then we must have  $E[R^1_1(\mathbf{q}, \theta)] > R^1_1(\mathbf{q}, \hat{\theta})$ , implying that  $q_1(w_0)$  must be upward-sloping at  $w_0=0$ .  $\square$

**Proof of Proposition 7.** The value of firm 1 equals its equity value, because the debt value is zero according to Eq. (7). Then, the first part follows because the marginal increase of the equity value with respect to  $w_0$  is given by the Lagrangian multiplier, which according to the proof of Proposition 4 equals  $\frac{1}{1 - \frac{\pi_2 - l}{\pi} \text{Prob}(\theta < \hat{\theta})}$ . The value of firm 2 depends on  $w_0$  only indirectly through  $q_1$ . With strategic substitutes, the result follows immediately from Proposition 6.  $\square$

## References

- Aghion, P., Dewatripont, M., Rey, P., 2000. Agency Costs, Firm Behavior and the Nature of Competition, mimeo, IDEI, Université de Toulouse.
- Bolton, P., 1990. Renegotiation and the dynamics of contract design. *European Economic Review* 34, 303–310.
- Bolton, P., Scharfstein, D.S., 1990. A theory of predation based on agency problems in financial contracting. *American Economic Review* 80, 93–106.
- Brander, J., Lewis, T., 1986. Oligopoly and financial structure: the limited liability effect. *American Economic Review* 76, 956–970.
- Brander, J., Lewis, T., 1988. Bankruptcy costs and the theory of oligopoly. *Canadian Journal of Economics* 21, 221–243.
- Caillaud, B., Rey, P., 1994. Strategic aspects of vertical delegation. *European Economic Review* 39, 421–431.
- Chevalier, J.A., 1995a. Capital structure and product–market competition: empirical evidence from the super-market industry. *American Economic Review* 85, 415–435.
- Chevalier, J.A., 1995b. Do LBO supermarkets charge more? An empirical analysis of the effects of LBOs on supermarket pricing. *Journal of Finance* 50, 1095–1112.
- Chevalier, J.A., Scharfstein, D.S., 1996. Capital market imperfections and countercyclical markups: theory and evidence. *American Economic Review* 86, 703–725.
- Cleary, S., Povel, P., Raith, M., 2003. The U-shaped Investment Curve: Theory and Evidence, mimeo, St. Mary's University, University of Minnesota and University of Rochester.
- DeMarzo, P.M., Fishman, M.J., 2000. Optimal Long-Term Financial Contracting with Privately Observed Cash Flows, mimeo, UC Berkeley and Northwestern University.

- Diamond, D., 1984. Financial intermediation and delegated monitoring. *Review of Economic Studies* 51, 393–414.
- Faure-Grimaud, A., 2000. Product market competition and optimal debt contracts: the limited liability effect revisited. *European Economic Review* 44, 1823–1840.
- Fazzari, S.R., Hubbard, G., Petersen, B., 1988. Financing constraints and corporate investment. *Brookings Papers on Economic*, 141–195.
- Fershtman, C., Judd, K.L., 1987. Equilibrium incentives in oligopoly. *American Economic Review* 77, 927–940.
- Fridson, M., 1994. The state of the high yield bond market: overshooting or return to normalcy? *Journal of Applied Corporate Finance* 7, 85–97.
- Gale, D., Hellwig, M., 1985. Incentive-compatible debt contracts: the one-period problem. *Review of Economic Studies* 52, 647–663.
- Glazer, J., 1994. The strategic effects of long-term debt in imperfect competition. *Journal of Economic Theory* 62, 428–443.
- Gromb, D. 1994. Renegotiation in Debt Contracts, mimeo, London School of Economics and Ecole Polytechnique, Paris.
- Grullon, G., Kanatas, G., Kumar, P. 2002. Financing Decisions and Advertising: An Empirical Study of Capital Structure and Product Market Competition, mimeo, Rice University and Vanderbilt University.
- Harris, M., Artur, R., 1995. The Role of Games in Security Design. *Review of Financial Studies*, 8, 327–367.
- Hart, O., Moore, J., 1998. Default and renegotiation: a dynamic model of debt. *Quarterly Journal of Economics* 113, 1–41.
- Hart, O., Moore, J., 1999. Foundations of incomplete contracts. *Review of Economic Studies* 66, 115–138.
- Hendel, I., 1996. Competition under financial distress. *Journal of Industrial Economics* 44, 309–324.
- Jensen, M.C., 1986. Agency costs of free cash flow, corporate finance and takeovers. *American Economic Review* 76, 323–339.
- Kaplan, S., Zingales, L., 1997. Do investment-cash flow sensitivities provide useful measures of financing constraints? *Quarterly Journal of Economics* 112, 169–215.
- Katz, M., 1991. Game-playing agents: unobservable contracts as precommitments. *Rand Journal of Economics* 22, 307–328.
- Khanna, N., Tice, S., 2000. Strategic responses of incumbents to new entry: the effect of ownership structure, capital structure, and focus. *Review of Financial Studies* 13, 749–779.
- Kovenock, D., Phillips, G., 1995. Capital structure and product-market rivalry: how do we reconcile theory and evidence? *American Economic Review* 85, 403–408.
- Kovenock, D., Phillips, G., 1997. Capital structure and product market behavior: an examination of plant exit and investment decisions. *Review of Financial Studies* 10, 767–803.
- Kreps, D.M., Scheinkman, J., 1983. Quantity precommitment and Bertrand competition yield Cournot outcomes. *Bell Journal of Economics* 14, 326–337.
- Maggi, G., 1996. Strategic trade policies with endogenous mode of competition. *American Economic Review* 86, 237–258.
- Maksimovic, V., 1988. Capital structure in repeated oligopolies. *Rand Journal of Economics* 19, 389–407.
- Maksimovic, V., 1995. Financial structure and product market competition, chapter 27. In: Jarrow, R.A., Maksimovic, V., Ziemba, W.T. *Handbooks in Operations Research and Management Science*, vol. 9. Finance, Elsevier North Holland.
- Maskin, E., Tirole, J., 1999. Unforeseen contingencies and incomplete contracts. *Review of Economic Studies* 66, 83–114.
- Maurer, B., 1999. Innovation and investment under financial constraints and product market competition. *International Journal of Industrial Organization* 17, 455–476.
- Opler, T.C., Titman, S., 1994. Financial distress and corporate performance. *Journal of Finance* 49, 1015–1040.
- Phillips, G.M., 1995. Increased debt and industry product markets: an empirical analysis. *Journal of Financial Economics* 37, 189–238.
- Povel, P., Raith, M., 2003. Optimal debt with unobservable investments. *Rand Journal of Economics* (in press).
- Raith, M., 2003. Competition, risk and managerial incentives. *American Economic Review* 93, 1425–1436.
- Showalter, D., 1995. Oligopoly and financial structure: comment. *American Economic Review* 85, 647–653.
- Showalter, D., 1999. Strategic debt: evidence in manufacturing. *International Journal of Industrial Organization* 17, 319–333.



- Stenbacka, R., Tombak, M., 2002. Investment, capital structure, and complementarities between debt and new equity. *Management Science* 48, 257–272.
- Stenbacka, R., Tombak, M. 2003. Capital Structure and Strategic Investment Competition: Theory and Empirical Evidence, mimeo, Swedish School of Economics (Helsinki).
- Sutton, J., 1991. *Sunk Costs and Market Structure*. M.I.T. Press, Cambridge, MA.
- Vickers, J., 1985. Delegation and the theory of the firm. *Economic Journal* 95, 138–147.
- Zingales, L., 1997. Survival of the fittest or fattest? Exit and financing in the trucking industry. *Journal of Finance* 53, 905–938.