Analyst Monitoring and Managerial Incentives

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Abstract

In this paper, we investigate whether an increase in analyst monitoring, which improves the informational efficiency of stock prices, necessarily translates into firms’ managers taking more value-enhancing decisions (“real efficiency”). We show that when the manager’s compensation is tied to stock prices, then an increase in analyst monitoring weakens managerial incentives by making the firm’s stock price less sensitive to the firm’s current performance. If the manager’s compensation contract is observed by stock market participants, then an increase in analyst monitoring is always detrimental to real efficiency. However, if market participants do not observe the manager’s compensation contract, then an increase in analyst monitoring improves real efficiency for growth firms and reduces real efficiency for value firms.
Introduction

Financial analysts play an integral role in ensuring that stock prices are “informationally efficient” in that they accurately forecast the value of firms’ future cash flows (Dow and Gorton (1997)). Apart from relying on information disclosed by firms, financial analysts are able to use their specific industry expertise to also generate independent information on firm fundamentals; e.g., by studying a firm’s competitors, investigating its supplier relationships, or by visiting plants and retail outlets. However, the real effects of analyst monitoring are not very well understood. In particular, it is not clear whether the improvement in informational efficiency from better analyst monitoring necessarily translates into firms’ managers taking more value-enhancing decisions (“real efficiency”). In this paper, we investigate one potential channel through which analyst monitoring may affect real efficiency, namely, its impact on the shareholder-manager contracting problem and managerial incentives.

To fix ideas, it is helpful to consider a start-up firm that plans to undertake an IPO at the end of the year. The market will price the firm by aggregating two types of information. The first type of information relates to the firm’s current state of affairs, and is an aggregation of the market’s predictions on the firm’s production, sales, earnings, and costs (“performance signal”). The second type of information generated as a result of monitoring by financial analysts is more intrinsic and fundamental to the firm, and relates to the firm’s productivity and growth opportunities. We model this as a noisy signal on the firm’s productivity (“productivity signal”). The key distinction between the performance signal and the productivity signal is that the former is a signal on the current output of the manager’s effort, whereas the latter is an independent signal generated by financial analysts on the firm’s productivity. We conjecture that an increase in analyst monitoring should lead to more independent information gathering, and hence, a more precise signal on the firm’s productivity.

We first analyze the benchmark case in which market participants can observe the manager’s compensation contract (or equivalently, the shareholders can credibly
commit not to privately renegotiate the compensation contract with the manager). In this benchmark setting, we show that an increase in analyst monitoring unambiguously lowers real efficiency and decreases shareholder value. Surprising as this may seem, the result is actually reminiscent of the one obtained by Paul (1992). The key to this result is that an increase in analyst monitoring, which makes the productivity signal more precise, ends up weakening managerial incentives. This is because the stock market attaches less weight to the performance signal as the productivity signal becomes more precise, thus lowering the sensitivity of the firm’s stock price to the manager’s effort. Put differently, an increase in analyst monitoring makes it costlier for shareholders to provide incentives to the manager, as the manager will require a higher pay-performance sensitivity to expend the same effort as before. In equilibrium, shareholders respond to an increase in analyst monitoring by lowering the manager’s pay-performance sensitivity, and shareholder value decreases.

Note that the logic described above for why informational efficiency lowers real efficiency crucially depends on the assumption that market participants can observe the manager’s compensation contract ex ante. Although this assumption is convenient and is widely used in the literature, it is quite unrealistic. In practice, it is not clear how market participants would be able to observe the actual compensation contract given to the manager of a firm undertaking an IPO. Even publicly listed firms make compensation disclosures via their 10-K statements only at the end of the financial year, and estimates of pay-performance sensitivity computed using these disclosures tend to be noisy (see Core and Guay (2002)). Therefore, we depart from much of the existing literature, and assume that market participants do not observe the manager’s compensation contract (or equivalently, that shareholders can privately renegotiate the compensation contract with the manager).\footnote{In practice, shareholders can easily change the manager’s pay-performance sensitivity through equity grants or repricing of options.} With this more realistic assumption, we show that the impact of an increase in analyst monitoring on real efficiency is more nuanced: it improves real efficiency for growth firms and lowers real efficiency for value firms.
To see the intuition for our result, note that the real inefficiency in our model arises because stock market participants do not observe the manager’s actual effort or pay-performance sensitivity, and price the firm based on a fixed conjecture of the manager’s effort. Therefore, the shareholders of the firm, who desire a higher stock price, have an incentive to induce a higher than efficient level of effort (using a compensation contract with a higher pay-performance sensitivity) in a bid to improve the stock market’s assessment of the firm’s marginal productivity. The shareholders’ incentives to induce overinvestment in effort increase with the firm’s growth opportunities, because the stock price is more sensitive to the market’s assessment of the firm’s marginal productivity when a larger part of the firm value comes from growth opportunities. In equilibrium, shareholders of firms with high growth opportunities (“growth firms”) induce overinvestment in effort, whereas shareholders of firms with low growth opportunities (“value firms”) induce underinvestment in effort.

Therefore, an increase in analyst monitoring has a differential impact on growth firms and value firms, because of differences in the nature of their real inefficiency, overinvestment vs. underinvestment. Consider growth firms whose shareholders are more likely to induce overinvestment in effort in equilibrium. By making it costlier for shareholders to provide incentives to their managers, an increase in analyst monitoring corrects the overinvestment problem and improves shareholder value. By the same logic, however, an increase in analyst monitoring worsens the underinvestment problem for value firms, and actually destroys shareholder value.

Although we have described a firm raising capital in the primary market, our model is valid for any firm with the following key features: First, the welfare of the firm’s shareholders (the principals) significantly depends on the firm’s stock price. Second, stock price is the primary mechanism through which the firm’s shareholders provide incentives to the firm’s manager. This is because the manager’s tenure is significantly shorter than that of his investment, and any short-term performance measures (burn-rate, investment in R&D, revenues, and operating profits) may be

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2 The shareholders’ incentives are similar to those studied in the signal-jamming literature (e.g., Fudenberg and Tirole (1986), Stein (1988) and Stein (1989)).
both manipulable and largely uninformative of long-term value. Third, stock market participants do not observe the manager’s incentive contract.

Our paper is related to the literature that examines the real effects of financial markets (see Bond et al. (2011) for a survey of this literature). This literature identifies two broad channels through which financial markets may affect real decisions. First, managers may learn new information from secondary market prices, and use this information to guide their real decisions (e.g., Dow and Gorton (1997), Boot and Thakor (1997), and Subrahmanyam and Titman (1999)). Second, although managers do not learn new information from market prices, their incentives to take real actions will depend on the extent to which they will be reflected in stock prices. Our paper belongs to the stream of literature which emphasizes this second channel. Two closely related papers in this stream of literature are Fishman and Hagerty (1989) and Paul (1992).

In Fishman and Hagerty (1989), a share-price maximizing manager underinvests because prices do not fully reflect the value of the firm’s cash flows. In their model, investors observe only one signal on firm value. Therefore, an increase in disclosure by the firm increases the sensitivity of share price to the manager’s investment, and thus improves real efficiency by ameliorating the underinvestment. By contrast, in our paper (and in Paul (1992)), stock market investors observe multiple signals on different aspects of the firm. An improvement in analyst monitoring actually worsens managerial incentives by causing the stock market to attach a higher weight to the productivity signal, which does not depend on the manager’s effort. Unlike Fishman and Hagerty (1989) and Paul (1992), we allow the shareholders to choose the manager’s pay-performance sensitivity. Another important distinction is that our modeling of the real side allows for both underinvestment and overinvestment, depending on the

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3Our paper is also related to the broader literature on public information, which argues that the availability of more precise public information may actually lower welfare by causing agents to ignore their own private information (Morris and Shin (2002)), by lowering the incentives of arbitrageurs to acquire information (Grossman and Stiglitz (1976), Grossman and Stiglitz (1980)), or by making insurance unviable (Hirshleifer (1971)). However, unlike our paper, these papers treat the real side of the firm as exogenous.
firm’s growth prospects. Hence, the effect of improved analyst monitoring on real efficiency also varies depending on the value of the firm’s growth opportunities.

In terms of the core intuition, the paper closest to ours is Paul (1992) who demonstrates that stock market efficiency need not translate into real efficiency. The core point of his paper is that efficient markets weight information according to its informativeness about asset value, whereas for optimal incentives, information should be weighted according to its informativeness about the manager’s actions. This mirrors the finding in our benchmark setting with observable contracts that an increase in analyst monitoring always worsens real efficiency and lowers shareholder value, due to its adverse impact on managerial incentives. However, once we make the more realistic assumption that the stock market cannot observe the manager’s compensation contract, we obtain more nuanced results as we highlighted above.

There are other related papers in the accounting literature that examine the real effects of information disclosure/revelation. In a setting where corporate managers have superior information on firm profitability than the capital markets, and the investments undertaken by firms are publicly observable, Kanodia and Lee (1988) show that periodic performance reports discipline the manager’s investment incentives and allow the firm’s observable investment to emerge as a credible signal of the manager’s information. Sapra (2002) shows that mandatory hedge disclosures may induce the manager to take extreme positions in the futures market in a bid to convince the stock market that he has received a high private signal on the spot price. In a signaling framework, Kanodia et al. (2005) show that noise in the investment signal may actually enhance value by correcting the manager’s tendency to overinvest in a bid to convince the stock market that the firm is highly profitable. Dye and Sridhar (2002) and Langberg and Sivaramakrishnan (2009) study the real effects of disclosure in a setting where disclosure can be used to extract information available to capital market participants that is not known to the firm’s management. Dye and Sridhar (2007) show that firms choose more precise disclosures and invest more when the precision of disclosures is not publicly observable. However, none of these papers
examine the impact of informational efficiency on the contracting problem between the shareholders and the manager, which is the key focus of our paper.

Our paper is also related to the literature that examines the impact of information revelation on the contracting problem between a principal and an agent. Cremer (1995) and Arya et al. (1998) argue that revelation of the agent’s private information may be undesirable because it makes it less credible for the principal to commit to punishing the agent following poor performance. Gigler and Hemmer (2004) show that contracting in an opaque environment and using a risky contract to elicit the manager’s private information is preferable to mandating disclosure of the manager’s private information when the public signal is highly informative about the manager’s action. Unlike in these papers, there is no revelation of private information in our model, and an increase in analyst monitoring only makes publicly available information more precise. Another related paper is Hermalin and Weisbach (2007), who show that allowing a firm’s board of directors to obtain a more precise signal of the CEO’s ability may lower firm value by increasing CEO compensation and by inefficiently increasing the rate of CEO turnover. In contrast, the key idea in our paper is that an increase in analyst monitoring weakens managerial incentives to expend effort, and thus, affects the shareholder-manager contracting problem. Moreover, we show that an increase in analyst monitoring has a differential impact on value and growth firms.

The remainder of the paper is organized as follows. We describe our model and base assumptions in Section 1. We describe the market’s valuation of the firm, and characterize managerial incentives in Section 2. We characterize the equilibrium in Section 3, and conclude the paper in Section 4.

1 The Model

We model the investment problem of a firm that trades on the stock market, and investigate how exogenous changes in analyst monitoring affects the aggregation of firm-specific information by investors, and the contracting problem between the firm’s
shareholders and its manager. Consider an all-equity firm that exists for four dates – 0, 1, 2 and 3. At date 0, the firm’s shareholders hire a professional manager and choose his compensation contract. At date 1, the manager takes an action that affects the value of the firm. Date 2 represents an intermediate time at which the firm’s market value, denoted \( \tilde{P} \), is determined by the investors in the stock market. Date 3 denotes the long term at which the intrinsic value of the firm, denoted \( \tilde{V} \), is realized.

At date 1, the manager takes a costly action, denoted \( k \), at a personal cost of \( \Psi (k) = \frac{1}{2} \psi k^2 \). The action \( k \) may represent the physical or mental exertion of the manager in running the firm. Alternatively, \( k \) may be thought of as a discretionary investment made by the manager, in which case, \( \Psi (k) \) is the opportunity cost to the manager of not diverting \( k \) into his own private benefits. Henceforth, we refer to \( k \) simply as the manager’s effort.

The firm’s intrinsic value \( \tilde{V} \) is increasing in the manager’s effort \( k \) and the firm’s marginal productivity (or return on investment), denoted \( \tilde{\theta} \). Following Myers (1977), we assume that \( \tilde{V} (k, \theta) \) consists of two additive components: the value of the assets-in-place and the value of growth opportunities. The value of the assets-in-place may be thought of as the total value generated by the manager’s effort, \( k \). We assume linear returns to scale from effort \( k \) that will produce a perpetual stream of cash flows of \( k \tilde{\theta} \). So the value of the assets-in-place is the present value of this cash flow stream, i.e., \( \gamma_1 k \tilde{\theta} \), where \( \gamma_1 > 0 \) is a given constant.

The second component of the firm’s intrinsic value is the value of growth opportunities, which may be thought of as the value generated by the investments that the firm is expected to make in future. This component of value will depend on future investment opportunities available to the firm, and its ability to raise capital to pursue these opportunities. We do not explicitly model the firm’s future investments, but instead assume that the value of growth opportunities is increasing in \( \tilde{\theta} \). This assumption is trying to capture the fact that more productive firms are likely to have more profitable investment opportunities in future, and each of these investment is more likely to have a higher value. We denote the value of growth opportunities \( \gamma_2 \tilde{\theta} \).
where $\gamma_2 > 0$ is a given constant.

Therefore, the firm’s intrinsic value is given by $V(k, \hat{\theta}) = (\gamma_1 k + \gamma_2) \hat{\theta}$. We assume that the parameters $\gamma_1$ and $\gamma_2$ are common knowledge at date 0. The linearity with respect to $\hat{\theta}$ is important to obtain tractable expressions for the firm’s market value at fate 2. However, the linearity with respect to $k$ is not necessary.

**Stock market, analyst monitoring, and information structure:**

We do not formally model the process by which firm-specific information is aggregated by dispersed stock market investors. Instead, we assume that the stock market fully and efficiently aggregates all the publicly available information. Thus, the firm’s market value is the expected intrinsic value conditional on all information available to the stock market at date 2.

The stock market uses a variety of information signals to price the firm. Some of the information may relate to the firm’s current state of affairs, whereas some may relate to its productivity and growth opportunities. Accordingly, we differentiate between two types of information signals available to the market participants.

The first type of information relates to the firm’s current state of affairs, and is an aggregation of the market’s predictions on the firm’s production, sales, earnings, and costs. The distinguishing feature is that the realization of this kind of information stochastically depends on managerial effort. Let $\tilde{y} = k (\bar{\theta} + \tilde{\varepsilon})$ denote the aggregation of all such signals that are affected by the manager’s effort. We refer to $\tilde{y}$ as the “performance signal.” The noise term $\tilde{\varepsilon}$ is independent of $\bar{\theta}$, and is distributed normally with mean 0 and precision (i.e., inverse of variance) $\tau_\varepsilon$. Observe that $\tilde{y}$ is a noisy signal on the value of assets-in-place.

The second type of information that we consider is more intrinsic and fundamental to the firm’s productivity. Most market participants will not be able to obtain information on the firm’s productivity independent of the information obtained from performance metrics. However, financial analysts that specialize in specific industries generate information on firm fundamentals by visiting plants and retail outlets, inves-
tigating supplier relationships and studying competitors. This information generated due to independent investigations by financial analysts, relates to the firm’s productivity and the value of its growth opportunities. Let $\tilde{z} = \tilde{\theta} + \tilde{\delta}$ denote the aggregation of all such signals regarding the firm’s productivity. We refer to $\tilde{z}$ as the “productivity signal.” Observe that, unlike $\tilde{y}$, the signal $\tilde{z}$ is not affected by the manager’s effort $k$.

The noise term $\tilde{\delta}$ is independent of $\tilde{\theta}$ and $\tilde{\varepsilon}$, and is distributed normally with mean 0 and precision $\tau_\delta$. An improvement in analyst monitoring leads to an increase in $\tau_\delta$, which makes the signal $\tilde{z}$ more informative. Hence, we use $\tau_\delta$ as a measure of analyst monitoring.

There is no information asymmetry between the manager, the shareholders and the stock market investors at any point of time regarding $\tilde{\theta}$, the firm’s marginal productivity. The true $\tilde{\theta}$ is not observed by any of the agents at date 1 when the manager chooses his effort $k$, or at date 2 when the stock market prices the firm. At date 1, it is commonly believed that $\tilde{\theta}$ is normally distributed with mean $\theta_m$ and precision $\tau$. We also assume that $\tau_\varepsilon$ and $\tau_\delta$ are common knowledge at date 0.

Manager’s compensation and shareholder value:

The shareholders of the firm choose the manager’s compensation contract at date 0. For tractability, we assume that the manager is risk-neutral, and we restrict attention to compensation contracts that are linear in the firm’s market value. Thus, the manager’s compensation may take the form

$$\tilde{C}_m = w + \varphi \tilde{P}$$

where $w \geq 0$ denotes the manager’s fixed wage, $\varphi \in [0, 1]$ denotes the fraction of the firm’s stock held by the manager that vests at date 2, and $\tilde{P}$ is the firm’s market value conditional on all the information that market participants have at date 2. We use the pair $(w, \varphi)$ to denote the manager’s compensation contract. Henceforth, we will

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4The advantage of the normality assumption is that it makes the model tractable. An obvious disadvantage is that $\tilde{\theta}$ can take a negative value. However, for large $\theta_m \sqrt{\tau}$ the probability of $\tilde{\theta}$ being negative would be very low.
refer to $\varphi$ as the manager’s pay-performance sensitivity. Note that, by contracting on the firm’s market value, we have implicitly assumed that the tenure of the manager is short compared to the life of the project or the firm.

We also assume that the firm’s shareholders are risk-neutral, and that they maximize the firm’s market value net of the manager’s compensation. Recall that the shareholders own a fraction $(1 - \varphi)$ of the firm’s equity, while the manager owns the remaining fraction $\varphi$. So the shareholder’s utility net of the manager’s compensation is

$$\tilde{U}_s = (1 - \varphi) \tilde{P} - w. \quad (2)$$

Henceforth, we refer to the net expected shareholder utility, $E[\tilde{U}_s]$, as shareholder value.

Note that the shareholders’ welfare in our model is tied to the firm’s market value, and not to its intrinsic value $\tilde{V}$.\(^5\) This is equivalent to assuming that the shareholders’ investment horizon is short compared to life of the firm. This assumption can be justified on a variety of grounds. Shareholders could expect to face liquidity needs at date 2, and may have to either sell their stock or borrow using their stock as collateral. Alternatively, it may be that shareholders prefer a higher market value because they are concerned about the threat of a takeover, or because they want to minimize the dilution of their stake in case the firm has to issue new stock to fund future investment opportunities. In case of institutional shareholders like mutual funds and pension funds, the concern for market value could arise because of mark-to-market regulations that tie the value of the institution to the stock prices of their portfolio firms.

A key feature of our model is that stock market investors do not observe the manager’s compensation contract. This feature is motivated by the observation that,

\(^5\)The qualitative results in the paper hold even if we assume that shareholders attach weights of $\alpha$ and $(1 - \alpha)$ to market value $\tilde{P}$ and long-term intrinsic value $\tilde{V}$, respectively, where $\alpha \in (0, 1]$ is a given constant (see Miller and Rock (1985)). Please contact the authors for the proof of results with this more general set up.
in practice, outsiders are not privy to the dealings between shareholders and man-
gers. Given that the welfare of shareholders depends on the firm’s market value, they
may surreptitiously skew the manager’s incentives towards share-price maximization
by privately renegotiating the manager’s compensation contract or through uncon-
tracted side payments. In practice, this can be achieved through equity grants (Core
and Guay (1999)) or repricing of options (see e.g., Gilson and Vetsuypens (1993),
Saly (1994), Acharya et al. (2000) and Brenner et al. (2000)).

2 Market’s valuation of the firm, and managerial
incentives

Before we can characterize the equilibrium, it is important to understand the market’s
valuation of the firm. At date 2, the stock market observes the realization of signals \( \bar{y} \)
and \( \bar{z} \), which we denote \( y \) and \( z \), respectively. The stock market uses these realizations
to update its expectation of \( \tilde{\theta} \), and to price the firm. Recall that \( y \) depends on the
manager’s effort, \( k \). Because the stock market does not observe \( k \), it instead prices
the firm based on its fixed conjecture of the manager’s effort, denoted \( \hat{k} \). So the firm’s
market value at date 2 is

\[
\tilde{P}(y, z, \hat{k}) = E_{\theta} \left[ V(\hat{k}, \tilde{\theta}) | y, z \right].
\]

We characterize \( \tilde{P}(y, z, \hat{k}) \) in the next lemma. Define

\[
f_y \equiv \frac{\tau_{\varepsilon}}{\tau + \tau_{\delta} + \tau_{\varepsilon}}
\]

and

\[
f_z \equiv \frac{\tau_{\delta}}{(\tau + \tau_{\delta} + \tau_{\varepsilon})}
\]
Lemma 1 \textit{From the manager’s perspective, the firm’s market at date 2 is given by}

\[ \tilde{P}(y, z, \hat{k}) = (\gamma_1 \hat{k} + \gamma_2) \left( \theta_m + \frac{f_y}{\hat{k}} (y - \hat{k} \theta_m) + f_z (z - \theta_m) \right) \]  

(6)

where $\hat{k}$ denotes the stock market’s (fixed) conjecture of the manager’s effort.

We derive the expression for $\tilde{P}(y, z, \hat{k})$ using Bayes’ rule, and by exploiting the normality of $\tilde{\theta}$, $\tilde{y}$ and $\tilde{z}$ (see proof of Lemma 1 in the Appendix for details). The parameters $f_y$ and $f_z$ denote the weights the stock market attaches to signals $\tilde{y}$ (scaled by the level of effort) and $\tilde{z}$, respectively, in pricing the firm’s stock. The higher the $f_y$, the more sensitive is the firm’s market value to the output signal $\tilde{y}$; similarly with $f_z$ and the productivity signal.

A key observation from equation (6) is that, all else equal, as the productivity signal $\tilde{z}$ becomes more precise (i.e., as $\tau_\delta$ increases), the firm’s market value becomes less sensitive to the performance signal $\tilde{y}$ (i.e., $f_y$ decreases) and becomes more sensitive to the productivity signal $\tilde{z}$ (i.e., $f_z$ increases). By a similar logic, as the performance signal becomes more precise (i.e., as $\tau_\varepsilon$ increases), then, all else equal, the firm’s market value becomes more sensitive to the performance signal $\tilde{y}$ ($f_y$ increases) and less sensitive to the productivity signal $\tilde{z}$ ($f_z$ decreases). Also, observe that both $f_y$ and $f_z$ are decreasing in $\tau$: when the uncertainty regarding the firm’s productivity is low to begin with, the firm’s market value is less sensitive to the signals realized at date 2.

Taking expectations on both sides of (6), and substituting $E[\tilde{y}|k] = k \theta_m$ and $E[\tilde{z}|k] = \theta_m$, we obtain the following expression for the firm’s expected market value:

\[ E_{y,z} [\tilde{P}(y, z, k, \hat{k})] = (\gamma_1 \hat{k} + \gamma_2) \left( (1 - f_y) \theta_m + \frac{f_y}{\hat{k}} k \theta_m \right) \]  

(7)

Observe that the term with $f_z$ in equation (6) drops off when we take expectations, because $E[\tilde{z}|k] = \theta_m$. This, however, does not mean that the expected market value does not depend on the statistical properties of the productivity signal, $\tilde{z}$. As we
noted above, the weight that the stock market attaches to the earnings signal \( (f_y) \) itself depends on the precision of signal \( \tilde{z} \).

Next, consider the manager’s choice of effort at date 1. Incentive compatibility requires that the manager choose an effort level that maximizes his expected utility, \( w + \varphi E_{y,z} E_{y,z} \left[ \tilde{P} \left( y, z, k, \hat{k} \right) \right] \), net of his cost, \( \psi k^2 \). For a given compensation contract, \((w, \varphi)\), let \( k(\varphi) \) denote the level of effort chosen by the manager (the fixed wage, \( w \), does not affect the manager’s investment decision on the margin). Applying the first-order condition, we obtain that \( k(\varphi) \) must satisfy the following equation:

\[
\psi k(\varphi) = \varphi \left( \gamma_1 \hat{k} + \gamma_2 \right) f_y \theta_m \hat{k} \tag{8}
\]

In equation (8), the left-hand side represents the marginal cost of effort to the manager, whereas the right-hand side represents the sensitivity of the manager’s compensation to his effort. Observe that \( k(\varphi) \) increases with \( f_y \), because the higher the \( f_y \), the greater is the sensitivity of the firm’s market value to the manager’s effort.

Next, consider the shareholders’ problem of choosing the manager’s compensation contract at date 0. From the shareholders’ point of view, their net expected utility (“shareholder value”) from choosing a contract, \((w, \varphi)\), given the pricing function (3) and the manager’s effort choice, \( k(\varphi) \), is

\[
E_\theta \left[ \tilde{U}_s (w, \varphi) \right] = (1 - \varphi) E_{y,z} \left[ \tilde{P} \left( y, z, \hat{k} \right) \right] - w \tag{9}
\]

In equilibrium, the shareholders will choose a compensation contract \((w^*, \varphi^*)\) that maximizes shareholder value subject to the manager’s participation constraint:

\[
\varphi E_{y,z} \left[ \tilde{P} \left( y, z, \hat{k} \right) \right] - \frac{\psi k^2(\varphi)}{2} + w \geq 0 \tag{10}
\]

Finally, given that the stock market has the same information regarding \( \tilde{\theta} \) as the shareholders and the manager, it can conjecture the compensation contract chosen
by the shareholders and the manager’s effort choice in equilibrium, even if it cannot observe the actual compensation and the actual effort. Therefore, it must be that

$$\hat{k} = k(\varphi^*)$$  \hspace{1cm} (11)

## 2.1 Benchmark Equilibrium with Observable Compensation Contracts

As a benchmark, we first characterize the equilibrium in a setting in which the stock market perfectly observes the manager’s compensation contract. We refer to this idealized setting as the “efficient” equilibrium. In the efficient equilibrium, the market perfectly conjectures the manager’s effort for every $\varphi$; i.e., $\hat{k} = k(\varphi)$. Substituting in equation (8), it follows that $k(\varphi)$ is the only positive root of the following quadratic:

$$\psi k^2 - (\gamma_1 k + \gamma_2) \varphi f_y \theta_m = 0.$$ \hspace{1cm} (12)

Next, consider the shareholders’ problem. Substituting $k(\varphi) = k$ and $\varphi = \frac{k^2}{(\gamma_1 k + \gamma_2) f_y \theta_m}$ from equation (12), the shareholders’ maximization problem (9) simplifies to

$$\max_{k, w} (\gamma_1 k + \gamma_2) \cdot \theta_m - \frac{\psi k^2}{f_y} - w$$ \hspace{1cm} (13)

Let $(w_{eff}, \varphi_{eff})$ denote the optimal compensation contract chosen by the shareholders, $k_{eff}$ denote the manager’s effort, and $U_{s_{eff}} \equiv E \left[ \hat{U}_s (w_{eff}, k_{eff}) \right]$ denote the optimal shareholder value under the efficient equilibrium.

**Proposition 1** Suppose the stock market could perfectly observe the manager’s compensation contract, $(w, \varphi)$. Then, in equilibrium, shareholders choose a compensation contract with $w_{eff} = 0$ and

$$\varphi_{eff} = \frac{\gamma_1^2 f_y \theta_m}{2 (\gamma_1^2 f_y \theta_m + 2 \psi \gamma_2)}$$ \hspace{1cm} (14)
The manager chooses an effort

\[ k^{\text{eff}} = \frac{\gamma_1 f y \theta m}{2 \psi} \]  

(15)

Shareholder value is

\[ U^{\text{eff}}_s = \frac{\gamma_1^2 f^2 y^2 \theta^2 m}{4 \psi} + \gamma_2 \theta m \]  

(16)

In such an idealized setting, an increase in analyst monitoring \( \tau_\delta \) will lead to a decrease in the manager’s equilibrium effort \( k^{\text{eff}} \) and shareholder value \( U^{\text{eff}}_s \).

As we have assumed that the manager’s reservation utility is zero, it follows that the manager’s participation constraint is satisfied whenever the incentive compatibility condition is satisfied. Therefore, it is optimal to set \( w = 0 \) because a positive \( w \) only lowers shareholder value without affecting the manager’s effort. The expression for \( k^{\text{eff}} \) is obtained by applying the first-order condition to the maximization problem (13); it is easily verified that the second-order condition is satisfied. The expressions for \( \varphi^{\text{eff}} \) and \( U^{\text{eff}}_s \) are obtained by substituting \( k^{\text{eff}} = \frac{\gamma_1 f y \theta m}{2 \psi} \) in equations (12) and (13), respectively.

The intuition behind Proposition 1 is similar to that in Paul (1992), who shows that stock market efficiency may worsen real efficiency because the signals that are most informative about firm value may not be very informative about the manager’s effort. In our model, signal \( \tilde{z} \) is highly informative about the firm’s value of growth opportunities, but is not affected by the manager’s effort. Hence, an improvement in analyst monitoring, which increases the precision of signal \( \tilde{z} \), weakens managerial incentives. If the stock market observes the manager’s pay-performance sensitivity \( \varphi \), then it will correctly conjecture that the manager will exert a lower effort in equilibrium as analyst monitoring improves, and will price the firm accordingly. Therefore, in equilibrium, as analyst monitoring improves, shareholders will choose a lower \( \varphi \), and overall shareholder value decreases.
3 Characterizing the equilibrium

The stark result in Proposition 1 is obtained under the assumption that stock market investors observe the manager’s pay-performance sensitivity $\varphi$. However, as we argued in section 1, this assumption is unrealistic. To understand the important role played by independent analysts, we analyze a more realistic setting. In this section, we analyze the benefits, or lack thereof, of analyst monitoring when the stock market investors do not observe the details of the manager’s compensation contract.

3.1 Optimal compensation, effort, and shareholder value

In this section, we take the level of analyst monitoring $\tau_\delta$ as given and characterize the optimal compensation contract, the manager’s investment choice, and shareholder value. In equation (8), we have already characterized the manager’s effort choice as a function of compensation $\varphi$ and the market’s conjecture of effort $\hat{k}$. We now turn our attention to the shareholders’ problem of choosing a compensation contract for the manager. Substituting the expression for $E_{y,z} \left[ \tilde{P} \left( y, z, k(\varphi), \hat{k} \right) \right]$ from equation (7) into equation (9) yields the following expression for shareholder value:

$$E_{\theta} \left[ \tilde{U}_s(w, \varphi) \right] = (1 - \varphi) \left( \gamma_1 \hat{k} + \gamma_2 \right) \left( (1 - f_y) \theta_m + \frac{f_y k(\varphi) \theta_m}{\hat{k}} \right) - w$$ (17)

The shareholders’ problem is to choose a compensation contract $(w, \varphi)$ that maximizes $E_{\theta} \left[ \tilde{U}_s(w, \varphi) \right]$, subject to the manager’s incentive compatibility constraint (8) and participation constraint (10). Let $(w^*, \varphi^*)$ denote the optimal compensation contract. Given the manager’s effort schedule, $k(\varphi)$, the shareholders’ problem is equivalent to choosing the optimal effort $k^* \equiv k(\varphi^*)$.

Our next result characterizes the optimal compensation contract, the optimal effort, and shareholder value.

Proposition 2 (Optimal compensation and shareholder value): In equilib-
rium, shareholders choose a compensation contract with $w^* = 0$ and

$$\varphi^* = \frac{f_y}{1 + f_y} \tag{18}$$

The manager chooses an effort

$$k^* = \gamma_1 f_y^2 \theta_m + \sqrt{(\gamma_1 f_y^2 \theta_m)^2 + 4\psi \gamma_2 \theta_m f_y^2 (1 + f_y)} \over 2\psi (1 + f_y) \tag{19}$$

Optimal shareholder value is

$$U_s^* = (1 - \varphi^*) \left( \gamma_1 k^* + \gamma_2 \right) \theta_m$$
$$= (\gamma_1 k^* + \gamma_2) \theta_m - \frac{\psi k^{*2}}{f_y} \tag{20}$$

Given that the manager’s reservation utility is zero, the manager’s participation constraint (10) is satisfied whenever his incentive compatibility condition (8) is satisfied. Therefore, it is optimal to set $w = 0$ because a positive $w$ only lowers shareholder value without affecting the manager’s effort.

In choosing a $\varphi$, the shareholders face the following trade-off: On the one hand, a high $\varphi$ means giving away a higher fraction of the firm’s shares to the manager, which hurts the shareholders because they are residual claimants. On the other hand, a high $\varphi$ induces a higher effort from the manager (equation (8)) which benefits the shareholders by increasing the expected market value. We solve for the equilibrium $k^*$ and $\varphi^*$ as follows: First, we obtain a relationship between $k^*$ and $\varphi^*$ by using the fact that, in equilibrium, the stock market perfectly conjectures the manager’s effort. Substituting $\hat{k} = k^*$ in equation (8) yields $\varphi^* = \frac{\psi k^{*2}}{(\gamma_1 k^* + \gamma_2) f_y \theta_m}$. Next, we apply the first-order condition to the shareholders’ maximization problem, and use the relationship between $k^*$ and $\varphi^*$ to obtain the closed-form expressions in Proposition 2.

The shareholder value, $U_s^*$, in equation (20) may be interpreted as firm value, $(\gamma_1 k^* + \gamma_2) \theta_m$, less a “contracting cost” of $\frac{\psi k^{*2}}{f_y}$. 

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Observe that as opposed to the idealized setting characterized in Section 2.1, in our setting, the stock market does not observe the manager’s actual pay-performance sensitivity, \( \varphi \), or investment, \( k(\varphi) \), even though it can infer the equilibrium outcome perfectly. Therefore, as we show in our next result, the firm’s shareholders have an incentive to deviate from the efficient effort, \( k^{\text{eff}} \).

Define

\[
\hat{\gamma}_2 = \frac{\gamma_2^2 \theta_m (1 - f_y)}{4 \psi}
\]

Lemma 2 (Effort and growth opportunities): In equilibrium, \( k^* \) is increasing in the firm’s growth component, \( \gamma_2 \). The manager overinvests with respect to the efficient effort (i.e., \( k^* > k^{\text{eff}} \)) if \( \gamma_2 > \hat{\gamma}_2 \), and underinvests if \( \gamma_2 < \hat{\gamma}_2 \).

To understand why \( k^* \) may differ from the efficient effort, \( k^{\text{eff}} \), consider the following thought experiment: Suppose the market values the firm based on the conjecture that the manager’s effort is \( k^{\text{eff}} \). Then, the firm’s shareholders, who want to maximize the firm’s market value, have an incentive to induce a higher (lower) effort than \( k^{\text{eff}} \) if the sensitivity of the market value to effort at \( k = k^{\text{eff}} \) is higher (lower) than the sensitivity of intrinsic value to effort. Because the sensitivity of market value to effort is higher for firms with a higher growth component, \( \gamma_2 \), the effort induced in equilibrium, \( k^* \), is increasing in \( \gamma_2 \). Intuitively, the market value of a firm that has a higher growth component, \( \gamma_2 \), is more sensitive to the market’s perception of its \( \tilde{\theta} \) because a larger fraction of its intrinsic value depends on growth opportunities, which are increasing in \( \tilde{\theta} \). All else equal, shareholders of high growth firms benefit more, on the margin, from an increase in managerial effort. Therefore, the equilibrium effort, \( k^* \), is increasing in the firm’s growth component, \( \gamma_2 \). Firms with low growth opportunities (low \( \gamma_2 \)) underinvest with respect to the efficient effort, \( k^{\text{eff}} \), whereas firms with high growth opportunities overinvest.

Our analysis complements that of Persons (1994), who shows that investment efficiency is impossible to achieve when a manager’s compensation contract can be privately renegotiated by the firm’s shareholders. In Persons (1994), the sub-optimal
investment arises because the manager exploits his private information about the firm’s profits to renegotiate the compensation contract to mutually benefit him and the shareholders. In contrast, in our model, neither the manager nor the shareholders have any private information regarding the firm’s profits. Rather, the inefficiency arises because of the shareholders’ desire for a higher short-term stock price, coupled with the fact that the firm’s investment decisions are not perfectly observed by the stock market.

3.2 Impact of an increase in analyst monitoring on the shareholder-manager contracting problem

Now that we have characterized the equilibrium, we analyze how exogenous changes in analyst monitoring $\tau_5$ affect managerial incentives, effort and shareholder value. The channel through which analyst monitoring affects managerial incentives is the pricing function, $\tilde{P}(y, z, \hat{k})$, specifically, the parameter $f_y$ which denotes the weight the stock market attaches to the output signal in pricing the firm. Recall that the manager’s effort affects only signal $\tilde{y}$, but not signal $\tilde{z}$. Therefore, the manager’s incentives are strengthened as $f_y$ increases. Now, as we noted in the discussion following Lemma 1, as the productivity signal $\tilde{z}$ becomes more precise (i.e., as $\tau_5$ increases), the firm’s market value becomes less sensitive to the output signal $\tilde{y}$ (i.e., $f_y$ decreases). Hence, overall, it follows that an increase in analyst monitoring $\tau_5$ weakens managerial incentives by lowering $f_y$.

Our next result characterizes the impact of an increase in analyst monitoring on the contracting problem between the shareholders of the firm and the manager.

Proposition 3 (Analyst monitoring and pay-performance sensitivity): In equilibrium, the manager’s pay-performance sensitivity $\varphi^*$ and effort $k^*$ decrease with an increase in analyst monitoring $\tau_5$.

To understand the intuition behind Proposition 3, consider the impact of an in-
crease in the sensitivity of stock price to earnings \((f_y)\) on the contracting problem between the shareholders and the manager. As \(f_y\) increases, the manager’s incentives are strengthened, which means that the marginal value to shareholders of providing incentives to the manager increases (because the manager invests greater effort for any given \(\phi\)). In equilibrium, shareholders respond by choosing a higher pay-performance sensitivity, \(\phi^*\), and the manager responds with a higher effort.

Next, we examine how an increase in analyst monitoring affects shareholder value \(U^*_s\). As \(\frac{dU^*_s}{d\tau_d} = \frac{dU^*_s}{df_y} \times \frac{df_y}{d\tau_d}\), it is important to first characterize \(\frac{dU^*_s}{df_y}\), i.e., to understand if an improvement in the sensitivity of market value to current performance \((f_y)\) is value-enhancing for shareholders or not. Differentiating equation (20) with respect to \(f_y\) yields

\[
\frac{dU^*_S}{df_y} = \frac{\partial U^*_S}{\partial k^*} \frac{dk^*}{df_y} + \frac{\partial U^*_S}{\partial f_y} = \left(\gamma_1 \theta_m - \frac{2\psi k^*}{f_y}\right) \frac{dk^*}{df_y} + \frac{\psi k^{*2}}{f_y^2}
\]

(22)

The term \(\frac{\partial U^*_S}{\partial k^*} = \frac{\psi k^{*2}}{f_y^2}\) in equation (22) may be interpreted as a decrease in contracting costs resulting from an improvement in managerial incentives; as \(f_y\) increases, the cost to shareholders of inducing the effort \(k^*\) decreases. The term \(\frac{\partial U^*_S}{\partial k^*} = \left(\gamma_1 \theta_m - \frac{2\psi k^*}{f_y}\right)\) in equation (22) denotes the marginal sensitivity of shareholder value \((U^*_S)\) to effort \((k^*)\) in equilibrium. Observe that \(\frac{\partial U^*_S}{\partial k^*}\) is positive if, and only if, \(k^* < k_{eff}\); all else equal, an increase in \(f_y\) improves shareholder value if, and only if, the manager was underinvesting relative to the efficient effort. Combining this with Lemma 2, it follows that the impact of an increase in analyst monitoring on shareholder value is itself likely to vary depending on the firm’s growth opportunities \((\gamma_2)\). Our next result formalizes this intuition.

**Proposition 4 (Analyst monitoring and shareholder value):** An increase in analyst monitoring \(\tau_d\) decreases shareholder value \(U^*_s\) of firms with \(\gamma_2 < \frac{\gamma_2 \theta_m}{\psi}\) ("value firms") and increases shareholder value of firms with \(\gamma_2 > \frac{\gamma_2 \theta_m}{\psi}\) ("growth firms").
As we discussed above, an increase in analyst monitoring lowers the sensitivity of market value to the performance signal (i.e., $f_y$). Proposition 4 states that the decrease in $f_y$ has a differential impact on shareholder value of value firms and growth firms. To see why, first consider a firm with $\gamma_2 < \hat{\gamma}_2$, whose manager underinvests with respect to the efficient effort $k^{\text{eff}}$ (Lemma 2). This inefficiency arises because its market value is not very sensitive to current performance, which makes it very costly for shareholders to provide incentives to the manager. By further lowering $f_y$, an increase in analyst monitoring worsens this inefficiency, thus causing a decrease in shareholder value.

Next, consider a firm with $\gamma_2 > \hat{\gamma}_2$, whose manager overinvests with respect to the efficient effort $k^{\text{eff}}$. In this case, the inefficiency arises because the firm’s market value is too sensitive to its current performance. Shareholders, who have an interest in maximizing the firm’s market value, exploit the stock market’s focus on current performance by inducing more managerial effort than what is efficient. In this case, an increase in analyst monitoring corrects the overinvestment problem by lowering the sensitivity of market value to the performance signal $y$. Of course, a decrease in $f_y$ also increases the contracting cost $\left(\frac{\psi k^{\text{eff}}}{f_y}\right)$ by making it more costly for shareholders to provide incentives to the manager. However, for sufficiently large $\gamma_2$, specifically $\gamma_2 > \frac{\gamma_2^2 \theta_m}{\psi}$, the former effect prevails and shareholder value increases.

The upshot of Proposition 4 is that an increase in analyst monitoring, which makes stock prices more informationally efficient, will be detrimental to the shareholders of value firms only. However, for firms with large growth prospects, an increase in analyst monitoring improves both the informational efficiency of stock prices as well as real efficiency.

4 Concluding Remarks

In this paper, we analyze how an increase in analyst monitoring affects real efficiency and shareholder value through its impact on the shareholder-manager contracting
problem and managerial incentives. In our setting, an increase in analyst monitoring allows the stock market investors to obtain a more precise signal of the firm’s underlying productivity, causing them to attach less weight to the firm’s current performance while valuing the firm. Therefore, when the manager’s compensation is tied to the firm’s stock price, an increase in analyst monitoring weakens managerial incentives by making the stock price less sensitive to the manager’s effort. We show that if stock market investors observe the manager’s compensation contract, then an increase in analyst monitoring will always worsen real efficiency and lead to a decrease in shareholder value.

In a more realistic setting in which stock market investors do not observe the manager’s compensation contract, we show that the impact of an increase in analyst monitoring on real efficiency is more nuanced. In this setting, real efficiency arises because shareholders of the firm, who desire a higher stock price, have an incentive to induce a higher than efficient level of effort (using a compensation contract with a higher pay-performance sensitivity) in a bid to improve the stock market’s assessment of the firm’s marginal productivity. The shareholders’ incentives to induce overinvestment in effort increase with the firm’s growth opportunities, because the stock price is more sensitive to the market’s assessment of the firm’s marginal productivity when a larger part of the firm value comes from growth opportunities. In equilibrium, shareholders of growth firms induce overinvestment in effort, whereas shareholders of value firms induce underinvestment in effort.

Therefore, an increase in analyst monitoring has a differential impact on growth firms and value firms, because of differences in the nature of their real inefficiency, overinvestment vs. underinvestment. Consider growth firms whose shareholders are more likely to induce overinvestment in effort in equilibrium. By making it costlier for shareholders to provide incentives to their managers, an increase in analyst monitoring corrects the overinvestment problem and improves shareholder value. By the same logic, however, an increase in analyst monitoring worsens the underinvestment problem for value firms, and actually destroys shareholder value.
References


Appendix

Proof of Lemma 1: We know that \( \tilde{P}(y, z, \hat{k}) = (\gamma_1 \hat{k} + \gamma_2) E_{\theta} [\tilde{\theta}|y, z, \hat{k}] \). We use the following property (stated here without proof) of normally distributed random variables in deriving the formula for \( P(y, z, \hat{k}) \): If \( x, y \) and \( z \) are normally distributed random variables, then:

\[
E(x|y, z) = E(x) + \frac{(\sigma_x^2 \sigma_y - \sigma_x \sigma_y)}{(\sigma_x^2 - \sigma_z^2)} (y - E(y)) + \frac{(\sigma_y^2 \sigma_x - \sigma_y \sigma_z)}{(\sigma_y^2 - \sigma_z^2)} (z - E(z)) \tag{23}
\]

In our problem, \( \tilde{x} = \tilde{\theta} \). First, consider the coefficient on \( y - E(y) \). Given the variance and covariance structure in our problem, it follows that

\[
\sigma_z^2 \sigma_y - \sigma_x \sigma_y = \left( \frac{1}{\tau} + \frac{1}{\tau_\delta} \right) \cdot \frac{\hat{k}}{\tau} - \frac{1}{\tau} \cdot \frac{\hat{k}}{\tau} \tag{24}
\]

and

\[
\sigma_y^2 \sigma_z^2 - \sigma_y^2 = \hat{k}^2 \left( \frac{1}{\tau} + \frac{1}{\tau_\epsilon} \right) \left( \frac{1}{\tau} + \frac{1}{\tau_\delta} \right) - \left( \frac{\hat{k}}{\tau} \right)^2 \tag{25}
\]

Substituting from equations (24) and (25), and simplifying, it follows that

\[
\frac{\sigma_z^2 \sigma_y}{\sigma_y^2 \sigma_z^2 - \sigma_y^2} = \frac{\tau_\epsilon}{\hat{k} (\tau + \tau_\delta + \tau_\epsilon)} \tag{26}
\]

Using similar computations, it follows that

\[
\frac{\sigma_y^2 \sigma_z^2 - \sigma_y \sigma_y}{\sigma_y^2 \sigma_z^2 - \sigma_y^2} = \frac{\tau_\delta}{(\tau + \tau_\delta + \tau_\epsilon)} \tag{27}
\]

Substituting the expressions in equations (26) and (27) back in equation (23)
yields

\[ E(\tilde{\theta}|y,z,\hat{k}) = \theta_m + \frac{\tau_\varepsilon}{\hat{k}(\tau + \tau_\delta + \tau_\varepsilon)} (y - \hat{k}\theta_m) + \frac{\tau_\delta}{(\tau + \tau_\delta + \tau_\varepsilon)} (z - \theta_m) \]  

(28)

Therefore,

\[
\tilde{P}(y,z,\hat{k}) = \left(\gamma_1 \hat{k} + \gamma_2\right) E_{\theta} \left[ \tilde{\theta}|y,z,\hat{k} \right]
\]

\[ = \left(\gamma_1 \hat{k} + \gamma_2\right) \left( \theta_m + \frac{f_y}{k} (y - \hat{k}\theta_m) + f_z (z - \theta_m) \right) \]  

(29)

where \( f_y \) and \( f_z \) are defined in equations (4) and (5), respectively.

Proof of Proposition 2: The proof involves two steps.

Step I: We show that any contract \((w,\varphi)\) that satisfies the manager’s incentive compatibility constraint (8) will also satisfy the manager’s participation constraint (10).

It is evident from the expression for \( P(k,\hat{k}) \) in equation (7) that the manager will get a positive net expected payoff even if he chooses \( k = 0 \). Therefore, the manager’s net expected payoff evaluated at \( k = k(\varphi) \) has to be positive. Because we have assumed that the manager’s reservation utility is zero, the incentive-compatible effort will also satisfy the manager’s participation constraint (10).

Step II: Solving for the optimal contract.

An immediate implication of Step I is that the manager’s participation constraint can be ignored while solving the shareholder’s optimization problem. Moreover, it is optimal to set \( w = 0 \).

Applying the first-order condition to the shareholders’ problem, it follows that \( \varphi^* \) must satisfy

\[
\left(1 - \varphi^*\right) \frac{f_y k^* \varphi^*}{k} \theta_m - \left( \theta_m (1 - f_y) + \frac{f_y \theta_m}{k} k^* \varphi^* \right) = 0
\]  

(30)
where
\[ k_{\varphi}(\varphi) = \frac{(\gamma_1 k + \gamma_2) f_y \theta_m}{\psi k} \]  

(31)

Next, in equilibrium, the market’s conjecture is perfect; i.e., \( \hat{k} = k^* \equiv k(\varphi^*) \). Substituting in equation (8), and rearranging yields,
\[ \varphi^* = \frac{\psi k^{*2}}{(\gamma_1 k^* + \gamma_2) f_y \theta_m} \]  

(32)

Substituting \( \hat{k} = k^* \), \( k_{\varphi}(\varphi) = \frac{(\gamma_1 k^* + \gamma_2) f_y \theta_m}{\psi k^*} \) and the expression for \( \varphi^* \) in equation (30), and simplifying, we obtain that \( k^* \) must satisfy the following condition:
\[ (\gamma_1 k^* + \gamma_2) f_y^2 \theta_m - \psi k^{*2} (1 + f_y) = 0 \]  

(33)

Solving this quadratic equation yields the expression for \( k^* \) in Proposition 2. Also, rearranging the above equation yields that
\[ \frac{\psi k^{*2}}{(\gamma_1 k^* + \gamma_2) f_y \theta_m} = \frac{f_y}{1 + f_y} \]
\[ \text{i.e., } \varphi^* = \frac{f_y}{1 + f_y}. \]  

(34)

Finally, the expression for \( U^*_s \) is obtained by substituting \( \hat{k} = k^* \) and \( \varphi^* = \frac{\psi k^{*2}}{(\gamma_1 k^* + \gamma_2) f_y \theta_m} \) in the expression for \( E_\theta \left[ \hat{U}_s(w, \varphi) \right] \).

Proof of Lemma 1: Suppose the stock market observes \( \varphi \). Then, it can conjecture the firm’s investment, \( k(\varphi) \), for every \( \varphi \), and price the firm accordingly. Then, from the shareholders’ point of view, the expected firm value from choosing a compensation contract \( (w, \varphi) \) is
\[ E_{y,z} \left[ \hat{P}(y, z, k(\varphi)) \right] = E_{y,z} \left[ E_\theta \left[ V(k(\varphi), \bar{\theta}) \middle| y, z \right] \right] \]
\[ = E_\theta \left[ V(k(\varphi), \bar{\theta}) \right], \]  

(35)
where the last equality is obtained by using the law of iterated expectations.

The shareholders’ problem then is to maximize

$$E \left[ \tilde{U}_s (w, \varphi) \right] = (1 - \varphi) E_{\tilde{\theta}} \left[ V \left( k (\varphi), \tilde{\theta} \right) \right] - w$$

$$= (1 - \varphi) (\gamma_1 k (\varphi) + \gamma_2) \theta_m - w,$$  \hfill (36)

subject to the manager’s incentive compatibility and participation constraints.

Substituting $\varphi = \frac{\psi k^2}{(\gamma_1 k + \gamma_2) f_y \theta_m}$, the shareholders’ problem can be written as

$$\max_{k, w} E \left[ \tilde{U}_s (w, k) \right] \equiv \frac{1}{f_y} ((\gamma_1 k + \gamma_2) f_y \theta_m - \psi k^2) - w$$  \hfill (37)

As in the proof of Proposition 2, it is optimal to set $w = 0$. Applying the first-order condition to equation (37), it follows that shareholder value is maximized by choosing $k = k_{eff}$.

**Proof of Lemma 2:** It is evident from equation (19) that $k^*$ is increasing in $\gamma_2$. The threshold, $\hat{\gamma}_2$, is obtained by solving for the value of $\gamma_2$ at which $k^* = k_{eff}$.  \hfill ■

**Proof of Proposition 3:** We showed in the proof of Proposition 2 that $k^*$ satisfies the following equation:

$$(\gamma_1 k^* + \gamma_2) f_y^2 \theta_m - \psi k^{*2} (1 + f_y) = 0$$  \hfill (38)

Implicitly differentiating the above equation with respect to $f_y$ yields

$$\frac{dk^*}{df_y} = - \frac{2 (\gamma_1 k^* + \gamma_2) f_y \theta_m - \psi k^{*2}}{\gamma_1 f_y^2 \theta_m - 2 \psi k^* (1 + f_y)}$$

$$= \frac{(2 - \varphi^*) (\gamma_1 k^* + \gamma_2) f_y \theta_m}{2 \psi k^* (1 + f_y) - \gamma_1 f_y^2 \theta_m},$$  \hfill (39)

where the second equality is obtained by substituting $\varphi^* = \frac{\psi k^{*2}}{(\gamma_1 k^* + \gamma_2) f_y \theta_m}$. Because
\[ \varphi^* = \frac{f_y}{1 + f_y} < 1 \text{ and } k^* > \frac{\gamma_1 f_y^2 \theta_m}{2 \psi(1 + f_y)} \text{ (see equation (19)), it follows that } \frac{dk^*}{df_y} > 0. \]

Next, differentiating equation (18) with respect to \( f_y \) yields

\[ \frac{d\varphi^*}{df_y} = \frac{1}{(1 + f_y)^2} > 0. \]

The result in Proposition 3 follows by noting that \( \frac{\partial f_y}{\partial \tau} < 0. \]

**Proof of Proposition 4:**  
**Step I:** We show that \( \frac{dU^*_s}{df_y} > 0 \Leftrightarrow k^* < \frac{\gamma_1 f_y \theta_m}{\psi}. \)

Differentiating equation (20) with respect to \( f_y \), we obtain

\[
\frac{dU^*_s}{df_y} = \frac{\psi k^*}{f_y} + \left( \frac{\gamma_1 \theta_m - 2 \psi k^*}{f_y} \right) \frac{dk^*}{df_y}
\]

\[
= \frac{\psi k^*}{f_y^2} + \left( \frac{\gamma_1 f_y \theta_m - 2 \psi k^*}{f_y} \right) \frac{(2 - \varphi^*) (\gamma_1 k^* + \gamma_2) f_y \theta_m}{(2 \psi k^* (1 + f_y) - \gamma_1 f_y^2 \theta_m)},
\]

after substituting for \( \frac{dk^*}{df_y} \) from equation (39) in the proof of Proposition 3.

Multiplying both sides of the equation with \( \frac{f_y (1 + f_y)}{(\gamma_1 k^* + \gamma_2) f_y \theta_m} \), and exploiting the fact that \( \frac{\psi k^*}{(\gamma_1 k^* + \gamma_2) f_y \theta_m} = \varphi^* = \frac{f_y}{1 + f_y} \), yields

\[
\frac{f_y (1 + f_y)}{(\gamma_1 k^* + \gamma_2) f_y \theta_m} \cdot \frac{dU^*_s}{df_y} = 1 + \left( \frac{\gamma_1 f_y \theta_m - 2 \psi k^*}{2 \psi k^* (1 + f_y) - \gamma_1 f_y^2 \theta_m} \right) (2 + f_y)
\]

\[
= \frac{2 (\gamma_1 f_y \theta_m - \psi k^*)}{2 \psi k^* (1 + f_y) - \gamma_1 f_y^2 \theta_m}.
\]

Therefore, it follows that

\[
\frac{dU^*_s}{df_y} > 0 \iff k^* < \frac{\gamma_1 f_y \theta_m}{\psi}
\]

**Step II:** We show that \( k^* < \frac{\gamma_1 f_y \theta_m}{\psi} \iff \gamma_2 < \frac{\gamma_2 \theta_m}{\psi}. \)

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It follows from equation (19) that

\[ k^* < \frac{f_y \gamma_1 \theta_m}{\psi} \iff \sqrt{(\gamma_1 f_y^2 \theta_m)^2 + 4\psi \gamma_2 f_y^2 \theta_m (1 + f_y)} < (2 + f_y) f_y \gamma_1 \theta_m \]

\[ \iff \gamma_2 < \frac{\gamma_1^2 \theta_m}{\psi} \] \hspace{1cm} (43)

Combining conditions (42) and (43), we conclude that \( \frac{dU^*_x}{df_y} > 0 \iff \gamma_2 < \frac{\gamma_1^2 \theta_m}{\psi} \).

The results in Proposition 4 follow by noting that \( \frac{\partial f_y}{\partial \tau_5} < 0 \).  \( \blacksquare \)