Lecture 5

Predictability

Traditional Views of Market Efficiency (1960-1970)

- CAPM is a good measure of risk
- Returns are close to unpredictable
  - (a) Stock, bond and foreign exchange changes are not predictable
  - (b) Constant equity premium
- Market volatility does not change much through time
- Professional managers do not reliably outperform simple indices and passive portfolios once one corrects for risk
- Jensen (1978): “I believe there is no other proposition in economics which has more solid evidence supporting it than the Efficient Markets Hypothesis.”
Modern Empirical Research  
(1980-to-present)

- Returns are predictable
  1. Valuation ratios (D/P, E/P, B/M ratios and )
  2. Interest rates (term spread, short-long T-bill rates, etc.)
  3. Decision of market participants (corporate financing, consumption).
  5. Bond and foreign exchange returns are also predictable.
- Some funds seem to outperform simple indices, even after controlling for risk through market betas
- Development of equilibrium models with time-varying equity premium.

Predictive Regressions I
Fama and French (1989), JFE: economic questions

- Economic questions:
  1. Do the expected returns on bonds and stocks move together? Do the same variables forecast bond and stock returns?
  2. Is the variation in expected returns related to business cycles?

- Motivation:
  1. Mounting evidence that stock and bond returns are predictable
  2. Interpretations: market inefficiency versus rational variation in expected returns
• Framework: Regress future returns on variables $X(t)$ known at time $t$.

$$ r(t, t + \tau) = \alpha(\tau) + \beta(\tau) X(t) + \varepsilon(t, t + \tau) \quad (1) $$

where $\tau$ can be one month, one quarter, and one to four years.

• $r(t, t + \tau)$: value- and equal-weighted market portfolios of NYSE; value-weighted corporate bond portfolios.

• $X(t)$ variables:
  − Dividend yields $D(t)/P(t)$: summing monthly dividends for the year preceding time $t$ divided by the value of the portfolio at time $t$ (Discount rate intuition)
  − Term Premium $TERM(t)$ – from Keim and Stambaugh (1986)
  − Default premium $DEF(t)$ – from Keim and Stambaugh (1986)

• Fama and French (1989) sample non-overlapping data for quarterly and annual frequencies (240 quarterly and 60 annual observations) and use traditional OLS standard errors. For longer horizons, they use annual overlapping observations and modify the standard errors.
### Table 1

Slopes, t-statistics, and $R^2$ from multiple regressions of excess returns on the term spread ($TERM_t$) and the value-weighted dividend yield ($D/P_t$) on the default spread ($DEF_t$), 1941-1987.*

<table>
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<th>Portfolio</th>
<th>Aaa</th>
<th>Aa</th>
<th>A</th>
<th>Baa</th>
<th>LG</th>
<th>VW</th>
<th>EW</th>
<th>Aaa</th>
<th>A</th>
<th>Aa</th>
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<th>LG</th>
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<th>EW</th>
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<td>$r(t, t + 1) = a + b(DEF_t) + c(TERM_t) + et(t, t + 1)$</td>
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*The regressions for T = one month (M), one quarter (Q), and one year use nonoverlapping returns. The regressions for two to four-year returns use overlapping annual observations. The numbers of observations in the regressions are: M, 732; Q, 246; (1 yr), 60; (2 yr), 60; (3 yr), 59; and (4 yr), 58. The standard errors in the t-statistics for the slopes are adjusted for heteroscedasticity and (for two- to four-year returns) the sample autocorrelation of overlapping residuals with the method of Hansen (1982) and White (1980). See note 1 to Table 1 for definition of portfolio.
• D/P has strongest effect (high t-stats and high R²)

• Regression coefficients and R² rise with the forecast horizon.

• Rational time-variation of expected return:
  – time-varying risk aversion
  – time-varying amount of risk

• A parallel explanation based on investor sentiment
  – Evidence does not distinguish among potential explanations
Predictive Regressions II

- Lamont also uses a predictive regression framework. He adds Earning Yield and Payout Ratio to the Dividend Yield.
- No interest rate data. Tests for unit roots in levels but not in ratios(?!)
- Findings: Dividends and earnings contain information about future returns.
- Interpretation: Dividends contain information about future dividends (dividends measure the permanent component of stock prices?) and earnings contain information on business conditions
- Cross-sectional predictability with the dividend payout ratio.

| Table IV |
| Regressions of current stock returns on lagged dividend yields, earnings yields, and dividend payout ratios, 1947Q1-1994Q4. The dependent variable, $R_{t+1} - E_{t+1}$, is quarterly log excess returns on the S&P Composite Index. $d_t - p_t$ is the log dividend yield, $e_t - g_t$ is the log earnings yield, and $q_t - q_t$ is the log dividend payout ratio. $BEL_t$ is the relative bill rate. Forecast returns are the projected excess returns in 1994Q1, using 1994Q4 values of the regressors. Estimation error is the standard error of the point estimate, based on sampling error in the coefficients. Total forecast error includes both sampling error and residual variance. OLS standard errors are in parentheses below the coefficient estimates. |

<table>
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<tr>
<th>Constant</th>
<th>$d_{t+1}$</th>
<th>$q_{t+1}$</th>
<th>$e_{t+1}$</th>
<th>$R^2$</th>
<th>Forecast Return 1994Q4</th>
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<td>-0.004 Forecast</td>
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<td>(0.024) Total forecast error</td>
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<td>0.012</td>
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<td>-0.07 Forecast</td>
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<td>(0.015) Estimation error</td>
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<td>(0.020) Total forecast error</td>
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<td>(0.020) Total forecast error</td>
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<td>(0.083) Total forecast error</td>
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Methodological Concerns with Predictive Regression Framework (90’s)

- Data snooping? Are D/P, TERM, Payout Ratios the only variables used in those regressions? Pre-testing Bias very likely.
- Regressors are only predetermined, but not exogenous. OLS slopes have a small bias: Stambaugh (1986). Traditional OLS S.E. are only appropriate asymptotically --if there is no serial correlation of the error term and if it is conditionally homoskedastic. Hodrick (1992).
- Valuation ratios are persistent and their innovations are correlated with returns, causing
  - biased predictive coefficients: Stambaugh (1999)
  - over-rejection by standard $t$ test: Cavanagh-Elliott-Stock (1995)
- These problems are less relevant for interest rates and recently proposed predictor variables (persistent, but less correlated with returns).
D/P is persistent, with a constant mean (mean reverting?) and no trend.

Issue: How persistent is D/P?
- D/P is likely to be persistent: it reflects long-run expectations.
- But, is D/P stationary? unit root? explosive?

Recall the Campbell-Shiller log-linear return formulas:

\[ d_t - \rho_t = \frac{k}{1 - \rho} + E_t \sum_{j=0}^{\infty} \rho^j [r_{t+1+j} - \Delta d_{t+1+j}] \]

D/P is stationary if dividend growth and returns are stationary.

J.Y. Campbell thinks that D/P might have a unit root, but
- It should not be explosive
- It should not have a trend (mean change = 0)

Any return predictability that is not perfectly correlated with dividend predictability will show up in D/P.

Side literature: Dividend Growth Predictability

- Based on the Campbell-Shiller log-linear approximation, \( \log(D/P) \) depends on future dividend growth and returns.
- General finding: aggregate stock returns are predictable by the D/P, not but dividend growth. Dividend growth rate is not predictable by D/P.
- Chen (2008) points out that results are sample dependent (some predictability before WWII) and also affected by the construction of the dividend growth data (with dividends reinvested or not).
- Potential Problem: Measurement error in \( d_t \) - too smooth. The observed data may not be the “true” series of interest. IVE: Earnings growth.
Stambaugh Bias
Stambaugh (1999): motivation

- One econometric problem in Fama and French (1989): the regressors are only predetermined but not exogenous.

- Start with predictive regression for returns, $r(t+1)$:
  \[ r(t+1) = \alpha + \beta x(t) + u(t+1) \]  
  (2)
  $x(t)$: $D(t)/P(t)$ – i.e., the dividend price ratio

- $x(t)$ depends on the price at the beginning of $t$, the change of $x$ at the end of $t+1$ reflects changes in price from $t$ to $t+1$, as does $r(t+1)$; $E[u(t+1)|x(t+1),x(t)] \neq 0$, more generally, $E[u(t)|x(s),x(w)] \neq 0$, $s<t<w$

- Stambaugh further assumes that
  \[ x(t) = \theta + \rho x(t-1) + v(t) \]
  where $(u(t), v(t))$’ follows a $N(0, \Sigma)$, independent across $t$.

- Results: $b$ (OLS estimate) is biased upward, positively skewed, and has higher variance and kurtosis than the normal sampling distribution of the OLS estimator.

- Stambaugh bias:
  \[ E(b - \beta) = \left( \frac{\sigma_{uv}}{\sigma_v^2} \right) E(p - \rho) \]
  where $p$ is the OLS estimator of $\rho$.

- It turns out $p$ has a downward bias and $\sigma_{uv}$ is negative
  \[ \Rightarrow b \text{ shows an upward bias.} \]

- Valuation ratios are sufficiently persistent. Conventional t-tests are misleading.
• The exact finite-sample moments and $p$-values in Table 1 depend on $\rho$ and $\Sigma$ (both unknown in practice). That is, in practice, we cannot know precisely the exact bias of $b$.

• The finite-sample properties in Table 1 are computed using the values of $\rho$ and $\Sigma$ obtained in the OLS estimation. (Many of the computations are relatively insensitive to small changes in the parameters.)

• Correcting the bias weakens the predictability evidence.

• **Result from Hodrick (1992) and Kim and Nelson (1993).** In a (1)-(2) framework Newey-West standard errors are not reliable in small samples.
**Side Note about Long Horizon Results**

- Recall that D/P and other ratios forecast excess returns on stocks. Regression coefficients and $R^2$ rise with the forecast horizon.
- This is a result of the fact that the forecasting variable is persistent

Consider the stylized model relating returns to a persistent valuation ratio like dividend/price

$$r_{t+1} = \beta s_{t+1} + \varepsilon_{t+1}, \beta > 0$$

$$s_{t+1} = \rho s_t + \eta_{t+1}, 0 < \rho < 1$$

The relationship between $r_{t+2}(2) = r_{t+2} + r_{t+1}$ and $s_t$ is

$$r_{t+2}(2) = \beta s_{t+1} + \beta s_t + \varepsilon_{t+2} + \varepsilon_{t+1}$$

$$= \beta (\rho s_t + \eta_{t+1}) + \beta s_t + \varepsilon_{t+2} + \varepsilon_{t+1}$$

$$= \beta (1 + \rho) s_t + w_{2t}$$

$$= \beta_2 s_t + w_t, \beta_2 > \beta.$$

In general,

$$r_{t+k}(k) = \beta (1 + \rho + \ldots + \rho^{k-1}) s_t + w_{kt}$$

$$= \beta_k s_t + w_{kt}, \beta_k > \beta_{k-1}$$

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**Recent Contributions**
Baker and Wurgler, JF (2000)

- Baker and Wurgler (2000) also use a predictive regression framework. They add to the Fama and French (1989) variables B/M$_t$, S$_{t-1}$ (equity share in new issues), and lagged R$_{t-1}$.
- S$_{t-1}$ is the most consistent and negative predictor of future returns.
- The S$_{t-1}$ coefficient is over 20 times too large to be due to MM leverage effect: New issues represent only a small fraction of outstanding capital. Not enough influence on aggregate leverage to change expected returns.
- S$_{t-1}$ could be related to future returns through investment, but in the aggregate investment is essentially unrelated to subsequent aggregate returns.
- They discuss the Stambaugh bias, with a lukewarm approach.
Dealing with Stambaugh Bias
Lewellen (2004), JFE


$$E(b - \beta | \rho=1,p) = (\sigma_{uv}/ \sigma_{v}^2) (p - 1)$$

$$b_{adj} = b - (\sigma_{uv}/ \sigma_{v}^2) (p - 1)$$

- Estimated persistence is very close to one. The bias correction is small. Predictability survives:
  - D/P predicts market returns from 1946–2000 and sub-samples.
Dealing with Stambaugh Bias
Campbell and Yogo (2006), JFE

- Known result: Conventional t-test (when $\rho$ is unknown) has good large-sample properties when $x_t$ is I(0).
- But, even if the predictor variable is I(0), first-order asymptotics can be a poor approximation in finite samples when $\rho$ is close to one.
- Two approaches with persistent regressors:
  - local-to-unity asymptotics (largest root is modeled as $\rho=1+c/T$, with $c$ constant): Elliot and Stock (1994), Campbell and Yogo (2006).
- Propose a Q-statistics:
  $$Q(\beta_u, \rho) = \frac{\sum_{t=1}^{T} x_t^2 \left[ r_t - \beta u \delta (x_t - \rho x_{t-1}) \right]}{\sigma^2 (1 - \delta^2)^{1/2} \left( \sum_{t=1}^{T} x_{t-1}^2 \right)^{1/2}}$$

where $\beta_u = \sigma_u / \sigma_u^2$. If we estimate $\rho$, then we can write:
  $$Q(\beta_u, \rho) = \frac{\bar{\beta} - \beta_u - \beta_u (\hat{\rho} - \rho)}{\sigma_u (1 - \delta^2)^{1/2} \left( \sum_{t=1}^{T} x_{t-1}^2 \right)^{-1/2}}.$$

- Under the null of stationarity, the test is asymptotically normal. However, it is infeasible, since it depends on $\rho$ –or in $c=T(\rho-1)$ and $\Sigma$. But, it can be easily made feasible (correcting for size, since we use $\alpha_1$ to test $\rho$ and $\alpha_2$ to test $\beta(\rho)$, using Bonferroni bounds):
  $$\beta(\rho) = \frac{\sum_{t=1}^{T} x_t^2 \left[ r_t - \beta u \delta (x_t - \rho x_{t-1}) \right]}{\sum_{t=1}^{T} x_{t-1}^2},$$
  $$\beta(\rho, \alpha_2) = \beta(\rho) - z_{\alpha_2/2} \sigma_u \left( \frac{1 - \delta^2}{\sum_{t=1}^{T} x_{t-1}^2} \right)^{1/2},$$
  $$\bar{\beta}(\rho, \alpha_2) = \beta(\rho) + z_{\alpha_2/2} \sigma_u \left( \frac{1 - \delta^2}{\sum_{t=1}^{T} x_{t-1}^2} \right)^{1/2},$$
  $$C_\rho(\alpha) = [\beta(\bar{\rho}(\alpha_1), \alpha_2), \bar{\beta}(\rho(\alpha_2), \alpha_2)].$$
Lewellen’s (2004) test is a special case, when $\rho$ is known.

- Note, that first we need to estimate $\rho$. For this, we can use the ADF test or the better behaved DF-GLS –see Elliot and Stock (1994).

- Campbell and Yogo (2006) compare the power of their Q-test to the Bonferroni t-test of Cavanagh et al. (1995) and Lewellen’s (2004). The Bonferroni Q-test has good power over the other feasible tests.

- Findings:
  - Over the full sample, E/P has predictive power at various frequencies (annual to monthly), while D/P only at annual frequency. In the post-1952 sample, results are weaker. If we can rule out explosive root, D/P has predictive power.
  - t-test leads to valid inference for the interest rate data. Persistence is not a problem for interest rate variables because their innovations have low correlation with the innovations to stock returns.
Understanding D/P: Back to the Gordon Growth Model – Campbell and Thompson (2007), RFS

- Assume, as in the Gordon growth model, that the dividend is known one period in advance.
- Assume that the log(D/P) follows a RW with normal innovations.
- Assume that the two-period ahead dividend growth rate is conditionally normal.
  - \( D_{t+1}/P_t = \exp(x_t) \)
  - \( D_{t+1}/D_t = 1 + G_t = \exp(g_t) \)
  - \( x_t = x_{t-1} + \epsilon_t \) \( \epsilon_t \sim N(0,\sigma^2) \)
- Use the definition of return and the formula for the conditional expectation of a lognormal R.V. \( -E(Y) = \exp(\mu + \sigma^2/2) \)

\[
1 + R_{t+1} = (P_{t+1} + D_{t+1})/P_t = D_{t+1}/P_t + (D_{t+2}/D_{t+1})(D_{t+1}/P_t) \left( D_{t+2}/P_{t+1} \right)^{-1} = \exp(x_t)[1+\exp(g_{t+1} - x_{t+1})]
\]
\[
E_t[1 + R_{t+1}] = \exp(x_t)[1+E_t[\exp(g_{t+1} - x_{t+1})]]
\]
\[
= \frac{D_{t+1}}{P_t} + \exp(E_t[g_{t+1}]-0)\exp(\sigma^2_g/2 + \sigma^2_x/2 - \sigma_{gx})
\]
\[
= \frac{D_{t+1}}{P_t} + \exp(E_t[g_{t+1}])\exp(\text{Var}(p_{t+1}-p_t)/2)
\]
\[
\approx \frac{D_{t+1}}{P_t} + \exp(E_t[g_{t+1}]) + \frac{1}{2} \text{Var}(r_{t+1})
\]

- As in the Gordon model, the expected return is the level of D/P (not the log) plus expected dividend growth.

- The variance effect is subtle:
  - In the original Gordon model, returns and dividend growth have the same volatility.
  - In that case the expected return is level of D/P plus arithmetic average dividend growth.
  - In the data, stock returns are much more volatile.
  - In that case the expected return is level of D/P plus geometric average dividend growth plus one-half stock return volatility (not dividend volatility).

- Equivalently, the level of D/P plus geometric average dividend growth predicts the log stock return (instead of the simple stock return).

- Empirically, this approach has the advantage that we do not have to estimate the unconditional mean stock return from the noisy historical data.

- Instead, we can use historical average growth, along with the current level of D/P.