Unidentifiable Relationships in Conceptual Marketing Models

For fifty years methods have existed to diagnose whether a conceptual model is unidentifiable, but it appears marketing scholars do not regularly check identification before estimation. To confirm this, all conceptual models published in the Journal of Marketing from 1995 to 1999 are analyzed using the traditional diagnostic methods for identification. Two-thirds of the published conceptual models contain relationships that are unidentifiable. These relationships have been empirically estimated, although it is impossible to measure their parameters validly. The published empirical estimates are spurious and cannot be trusted to represent the behavior they claim to measure until the identification problem has been corrected. The theory, not the statistics, must change to validate the measurements, so the paper concludes with suggestions that can help avoid unidentifiable conceptual theories.

James Hess is Professor of Business Administration, University of Illinois, Urbana-Champaign. The author appreciates the helpful comments made by Jeongwen Chiang, Roderick McDonald, William Robinson, Jose Antonio Rosa, Jeffery Schmidt, and especially Franklin Fisher.
When researchers formulate models in the *Journal of Marketing*, they typically use graphs to record visually the variables and relationships between variables. These conceptual models represent constructs by nodes and represent relationships by directed arcs, whose tails begin at antecedent constructs and whose tips point to the consequent constructs that are influenced by the antecedents.\(^1\) Figure 1 illustrates with a conceptual model of advertising Message-type\(^2\) (comparative versus non-comparative) and consumers’ Attitude toward and Knowledge (awareness) about a brand.

\[
\text{Attitude } A = aK + bM \\
\text{Knowledge } K = cA + dM
\]

**Figure 1**  
**Conceptual Model**

Relationships between these constructs are captured in Figure 1 by four arrows. The figure tells us that Knowledge helps to establish Attitude toward the brand, because there is an arrow labeled “a” whose tail begins at the Knowledge box and whose tip points at the Attitude box. This is not the only determinant of Attitude since there is also an arrow labeled “b” from Message to Attitude. The labels on the arrows represent the strength of the relationship.

---

\(^1\) Conceptual models are also called path diagrams, structural models, or causal models. Similarly, constructs are abstract concepts, nodes are circles or boxes, directed arcs are arrows, antecedents are predecessors and consequences are successors. I will try to use the less formal terms, when appropriate.

\(^2\) The names of constructs will be capitalized in the text for ease of recognition. In graphs, boxes denote observed (manifest) constructs and circles denote unobserved (latent) constructs.
The arrows also tell us which variables are independent (exogenous) and which variables are dependent (endogenous) within the conceptual model. Every construct that has at least one arrow tip pointing at it is endogenous and is determined within the conceptual model. Every variable that has only arrows emanating from it and none pointing at it is an exogenous variable.

The challenge of such a conceptual model is to identify unique values of the parameters that characterize the strength of relationship between many variables. Even when the basic constructs are well measured, the estimation of the inter-variable relationships from data can be daunting. The basic statistics upon which these inter-relationship parameters, such as a, b, c, and d in Figure 1, are identified are the covariances between the measured variables in the model: COV(Attitude, Message), COV(Attitude, Knowledge) and COV(Knowledge, Message). There are four parameters to be identified but only three covariances to help. In fact, as we will verify in the next section, it is impossible to “identify” the true relationships in Figure 1’s conceptual model, no matter how much data has been gathered. This inability to uniquely determine the model parameters is the **identification problem**.

Listed in Table 1 and 2 are all forty-three *Journal of Marketing* papers that contain conceptual models published in the in the period 1995-1999 (see below for a detailed description of how the list of conceptual models was created and organized). Each of the twenty-eight papers in Table 1 has one or more equations that are unidentifiable. Of course, there are some conceptual models free of identification problems (Table 2), but two-thirds *Journal of Marketing*’s conceptual models have identification problems. That is, a majority of conceptual models are insufficiently structured theoretically to provide valid measures of a significant number of their parameters.
<table>
<thead>
<tr>
<th>Code</th>
<th>Authors</th>
<th>Title</th>
<th>Year</th>
<th>Vol.</th>
<th>No.</th>
<th>Pp.</th>
<th>Unidentified Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Miller, Chip E.; Reardon, James; McCorkle, Denny E.</td>
<td>The Effects of Competition on Retail Structure: An Examination of Intratype, Intertype, and Intercategory Competition</td>
<td>1999</td>
<td>63</td>
<td>4</td>
<td>107-20</td>
<td>Scale, Saturation</td>
</tr>
<tr>
<td>B</td>
<td>Handelman, Jay M.; Arnold, Stephen J.</td>
<td>The Role of Marketing Actions with a Social Dimension: Appeals to the Institutional Environment</td>
<td>1999</td>
<td>63</td>
<td>3</td>
<td>33-48</td>
<td>Support for organization</td>
</tr>
<tr>
<td>C</td>
<td>Menon, Anil; Bharadwaj, Sundar G.; Adidam, Phani Tej; Edison, Steven W.</td>
<td>Antecedents and Consequences of Marketing Strategy Making: A Model and a Test</td>
<td>1999</td>
<td>63</td>
<td>2</td>
<td>18-40</td>
<td>Creativity of strategy, Organization learning, Market performance</td>
</tr>
<tr>
<td>D</td>
<td>Garbarino, Ellen; Johnson, Mark S.</td>
<td>The Different Roles of Satisfaction, Trust, and Commitment in Customer Relationships</td>
<td>1999</td>
<td>63</td>
<td>2</td>
<td>70-87</td>
<td>Commitment, Trust</td>
</tr>
<tr>
<td>G</td>
<td>Dawes, Philip L.; Lee, Don Y.; Dowling, Grahame R.</td>
<td>Information Control and Influence in Emergent Buying Centers</td>
<td>1998</td>
<td>62</td>
<td>3</td>
<td>55-68</td>
<td>Buying center influence</td>
</tr>
<tr>
<td>H</td>
<td>MacKenzie, Scott B.; Podsakoff, Philip M.; Ahearne, Michael</td>
<td>Some Possible Antecedents and Consequences of In-Role and Extra-Role Salesperson Performance</td>
<td>1998</td>
<td>62</td>
<td>3</td>
<td>87-98</td>
<td>Job satisfaction</td>
</tr>
<tr>
<td>I</td>
<td>Siguaw, Judy A.; Simpson, Penny M.; Baker, Thomas L.</td>
<td>Effects of Supplier Market Orientation on Distributor Market Orientation and the Channel Relationship: The Distributor Perspective</td>
<td>1998</td>
<td>62</td>
<td>3</td>
<td>99-111</td>
<td>Trust, Cooperation, Commitment, Performance</td>
</tr>
<tr>
<td>L</td>
<td>Klein, Jill Gabrielle; Ettenso, Richard; Morris, Marlene D.</td>
<td>The Animosity Model of Foreign Product Purchase: An Empirical Test in the People’s Republic of China</td>
<td>1998</td>
<td>62</td>
<td>1</td>
<td>89-100</td>
<td>Willingness to buy</td>
</tr>
<tr>
<td>M</td>
<td>Grewal, Dhruv; Kavanoor, Sukumar; Fern, Edward F.; Costley, Carolyn; Barnes, James</td>
<td>Comparative Versus Noncomparative Advertising: A Meta-Analysis</td>
<td>1997</td>
<td>61</td>
<td>4</td>
<td>1-15</td>
<td>Affect</td>
</tr>
<tr>
<td>Page</td>
<td>Authors</td>
<td>Title</td>
<td>Year</td>
<td>Volume</td>
<td>Pages</td>
<td>Journal</td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>---------</td>
<td>-------</td>
<td>------</td>
<td>--------</td>
<td>-------</td>
<td>---------</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>Netemeyer, Richard G.; Boles, James S.; McKee, Daryl O.; McMurrian, Robert</td>
<td>An Investigation Into the Antecedents of Organizational Citizenship Behaviors in a Personal Selling Context</td>
<td>1997</td>
<td>61</td>
<td>85-98</td>
<td>OCB</td>
<td></td>
</tr>
<tr>
<td>O</td>
<td>Smith, J. Brock; Barclay, Donald W.</td>
<td>The Effects of Organizational Differences and Trust on the Effectiveness of Selling Partner Relationships</td>
<td>1997</td>
<td>61</td>
<td>3-21</td>
<td>Mutual satisfaction</td>
<td></td>
</tr>
<tr>
<td>Q</td>
<td>Brown, Tom J.; and Dacin, Peter A.</td>
<td>The Company and the Product: Corporate Associations and Consumer Product Responses</td>
<td>1997</td>
<td>61</td>
<td>68-84</td>
<td>Product evaluation</td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>Fornell, Claes; Johnson, Michael D.; Anderson, Eugene W.; Cha, Jaesung; and Bryant, Barbara Everitt</td>
<td>The American Customer Satisfaction Index: Nature, Purpose, and Findings</td>
<td>1996</td>
<td>60</td>
<td>7-18</td>
<td>Customer satisfaction, Perceived value, Customer loyalty</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>Lusch, Robert F.; Brown, James R.</td>
<td>Interdependency, Contracting, and Relational Behavior in Marketing Channels</td>
<td>1996</td>
<td>60</td>
<td>19-38</td>
<td>Contract explicitness, Normative contract, Relational behavior, Wholesaler performance</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>Singh, Jagdip; Verbeke, Willem; Rhoads, Gary K.</td>
<td>Do Organizational Practices Matter in Role Stress Processes? A Study of Direct and Moderating Effects for Marketing-Oriented Boundary</td>
<td>1996</td>
<td>60</td>
<td>69-86</td>
<td>Turnover intentions</td>
<td></td>
</tr>
<tr>
<td>W</td>
<td>Hui, Michael K.; Tse, David K.</td>
<td>What to Tell Consumers in Waits of Different Lengths: An Integrative Model of Service Evaluation</td>
<td>1996</td>
<td>60</td>
<td>81-90</td>
<td>Service evaluation</td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>Challagalla, Goutam N.; Shervani, Tasadduq A.</td>
<td>Dimensions and Types of Supervisory Control: Effects on Salesperson Performance and Satisfaction</td>
<td>1996</td>
<td>60</td>
<td>89-105</td>
<td>Satisfaction with supervisor, Performance</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>Green, Donna H.; Barclay, Donald W.; Ryans, Adrian B.</td>
<td>Entry Strategy and Long-Term Performance: Conceptualization and Empirical Examination</td>
<td>1995</td>
<td>59</td>
<td>1-16</td>
<td>Performance</td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>Anderson, Erin; Robertson, Thomas S.</td>
<td>Inducing Multiline Salespeople to Adopt House Brands</td>
<td>1995</td>
<td>59</td>
<td>16-31</td>
<td>Salesperson adoption of house brand</td>
<td></td>
</tr>
<tr>
<td>Code</td>
<td>Authors</td>
<td>Title</td>
<td>Year</td>
<td>Vol.</td>
<td>No.</td>
<td>Pp.</td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>---------</td>
<td>-------</td>
<td>------</td>
<td>------</td>
<td>-----</td>
<td>-----</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>Kuester, Sabine; Homburg, Christian; Robertson, Thomas S.</td>
<td>Retaliatory Behavior to New Product Entry</td>
<td>1999</td>
<td>63</td>
<td>4</td>
<td>90-106</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>Noble, Charles H.; Mokwa, Michael P.</td>
<td>Implementing Marketing Strategies: Developing and Testing a Managerial Theory</td>
<td>1999</td>
<td>63</td>
<td>4</td>
<td>57-73</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>Mittal, Vikas; Kumar, Pankaj; Tsiros, Michael</td>
<td>Attribute-Level Performance, Satisfaction, and Behavioral Intentions over Time: A Consumption-System Approach</td>
<td>1999</td>
<td>63</td>
<td>2</td>
<td>88-101</td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>Li, Tiger; Calantone, Roger J.</td>
<td>The Impact of Market Knowledge Competence on New Product Advantage: Conceptualization and Empirical Examination</td>
<td>1998</td>
<td>62</td>
<td>4</td>
<td>13-29</td>
<td></td>
</tr>
<tr>
<td>f</td>
<td>Moorman, Christine; Miner, Anne S.</td>
<td>The Convergence of Planning and Execution: Improvisation in New Product Development</td>
<td>1998</td>
<td>62</td>
<td>3</td>
<td>1-20</td>
<td></td>
</tr>
<tr>
<td>g</td>
<td>Tax, Stephen S.; Brown, Stephen W.; Chandrashekaran, Murali</td>
<td>Customer Evaluations of Service Complaint Experiences: Implications for Relationship Marketing</td>
<td>1998</td>
<td>62</td>
<td>2</td>
<td>60-76</td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>Grewal, Dhruv; Monroe, Kent B.; Krishnan, R.</td>
<td>The Effects of Price-Comparison Advertising on Buyers’ Perceptions of Acquisition Value, Transaction Value, and Behavioral Intentions</td>
<td>1998</td>
<td>62</td>
<td>2</td>
<td>46-59</td>
<td></td>
</tr>
<tr>
<td>i</td>
<td>Fisher, Robert J.; Maltz, Elliot; Jaworski, Bernard J.</td>
<td>Enhancing Communication Between Marketing and Engineering: The Moderating Role of Relative Functional Identification</td>
<td>1997</td>
<td>61</td>
<td>3</td>
<td>54-70</td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>Fein, Adam J.; Anderson, Erin</td>
<td>Patterns of Credible Commitments: Territory and Brand Selectivity in Industrial Distribution Channels</td>
<td>1997</td>
<td>61</td>
<td>2</td>
<td>19-34</td>
<td></td>
</tr>
<tr>
<td>l</td>
<td>Doney, Patricia M.; Cannon, Joseph P.</td>
<td>An Examination of the Nature of Trust in Buyer-Seller Relationships</td>
<td>1997</td>
<td>61</td>
<td>2</td>
<td>35-51</td>
<td></td>
</tr>
<tr>
<td>m</td>
<td>Bello, Daniel C.; Gilliland, David I.</td>
<td>The Effect of Output Controls, Process Controls, and Flexibility on Export Channel Performance</td>
<td>1997</td>
<td>61</td>
<td>1</td>
<td>22-38</td>
<td></td>
</tr>
<tr>
<td>o</td>
<td>DeCarlo; Thomas E.; Leigh, Thomas W.</td>
<td>Impact of Salesperson Attraction on Sales Managers’ Attributions and Feedback</td>
<td>1996</td>
<td>60</td>
<td>2</td>
<td>47-66</td>
<td></td>
</tr>
</tbody>
</table>
Many papers in *Journal of Marketing* have unnoticed identification problems. So what? Here is what. When an unidentifiable structural equation is estimated empirically, the reported measures of central tendency and accuracy are not those of the relationship parameters that they claim to measure. No amount of additional observations or data analysis will solve the identification problem. The estimators are not valid and the resulting substantive findings are, in fact, spurious.

I realize this is a very serious criticism of our marketing profession, so I want to make the issue completely self-contained in this paper to help avoid the identification trap in the future. Entirely graphical analysis has apparently not worked for us, so this paper uses more formulas than the typical paper published in *Journal of Marketing*: mathematics without apology! Treating the theory seriously, by expressing it as a system of equations rather than just a “guzinta”\(^3\) diagram, is critical for valid empirical research. The language of algebra makes the problem clear and its resolution most evident. For those readers who dislike formal algebraic versions of theories, please bear with me. At the end of the paper, there will be a geometric version spelled out to replace some of the algebra.

Identification is old news. It has been intensely studied since Working (1927) brought the “identification problem” to the attention of economists who were trying to measure supply and demand curves over seventy years ago. Identification is not a statistical issue, such as whether the error terms are normally distributed or whether two-stage least squares estimation is more appropriate than ordinary least squares. It is a theoretical issue. Therefore, only a modest amount of statistics is used in this paper. In fact, the complexity of the probabilistic versions of

\(^3\) Those of us from some parts of the United States describe our conceptual models by stating, “X guz in ta (‘goes in to’) determining Y.”
conceptual models may have distracted researchers from taking the simple pre-estimation step of checking that all relationships are identifiable. By focusing only on the theory of conceptual models, the issue will be less confusing. Identification is also an issue associated with the development of reliable and valid scales for the latent constructs. This measurement problem will not receive attention here. Naturally, excellent books exist which cover both the theoretical and statistical aspects of conceptual models (Bagozzi 1980 and Bollen 1989).

This paper is structured as follows. First, the problem of identification in conceptual models is reviewed. Existing methods allow us to diagnose whether a structural model has an identification problem, even before data collection begins. Second, all conceptual models published in the Journal of Marketing from 1995 to 1999 are analyzed for identification problems using these diagnostic tools. Each model falls into one of two categories: those with previously unrecognized identification problems and those that are free from such problems. Third, when relationships are not identifiable, the theory must change to make the measurements valid. Some suggestions are made for what types of adjustments can be made to correct the identification problem during the data collection phase of research.

Identification: Illustrations

Many conceptual models are designed to tell us succinctly the major constructs, along with a rough-and-ready roadmap of the theoretical underpinnings of the problem addressed in the paper. However, for every conceptual graph there also corresponds an algebraic version of the proposed theory. The simplest way to translate the graph to algebra is to assume all relationships are linear.
From Figure 1, the functional relationship that determines Attitude toward the brand is a linear form,

\[ A = aK + bM, \]  \hspace{1cm} (1)

where A, M, and K are values associated with Attitudes, Message and Knowledge. The coefficients of this linear relationship indicate the degree to which the driver variables, K and M, have an impact on the dependent variable, A. The bigger is the coefficient “a” relative to “b,” the more responsive Attitude is to Knowledge about the brand relative to advertising Message. Equation (1) is called a structural equation because it represents the structure of Figure 1.

Similarly, the relationship between Message and Attitude and the resulting Knowledge can be expressed as

\[ K = cA + dM. \]  \hspace{1cm} (2)

The coefficient “c” incorporates the degree to which brand Knowledge is influenced by the Attitude about the brand and “d” indicates the degree to which brand Knowledge is effected by comparative messages.

Conceptual graphs are clear and transparent, while algebra is a second language to us all. Why should one care whether the graph can be translated into corresponding equations? If most readers are better visual information processors, why should scholars ask them to be algebraic symbol manipulators? Journal editorial boards may even insist that algebraic exposition be minimized to make the research seem more practical.
Of course, there are many reasons that marketing scholars have developed a predilection for mathematical formulas. Moorthy (1993) gave an excellent exposition of many of them a few years ago in this Journal. Mathematical treatment of conceptual models is more than just an issue of expositional taste. There are times when equations shed light where a graph cannot illuminate.

The particular point I would like to make in this paper is that conceptual models provide theoretical structure for the important task of creating valid measures of the strength of the relationships between constructs. Drawing a conceptual model is important not just for the summarizing past studies. Informal graphical treatment has lead many empirical scholars to estimate the strength of relationships that are “unidentifiable” within their own conceptual model, even though they have been cautioned not to do this (Aaker and Bagozzi 1979).

To understand the fundamentals of the identification problem, consider the empirical identification of the conceptual model in Figure 1 above. This model explains the Attitude toward and the Knowledge about a brand. The endogenous constructs, Attitude and Knowledge, are influenced by the type of advertising message, as well as the other endogenous variable. One could attempt to estimate strength of the relationships in the conceptual model (coefficients a, b, c, d) from data gathered in an experiment. Subjects could be shown advertisements that had some degree of comparative versus non-comparative messages. The Message variable, M, is thus manipulated. Perhaps after a distracting task, the subjects could be asked about both their Attitude, A, toward the brand as well as their Knowledge, K, about the brand.

It would seem reasonably straightforward to estimate a statistical version of equations (1)-(2) via ordinary least squares regressions. Unfortunately, this will not produce valid estimates of the equations because neither of the two equations in the model can be “identified.” What does this
mean? The essence of the identification problem is that both dependent variables in the model of Figure 1 adjust in an interdependent way to satisfy both equations. It is impossible to establish independent causation from an observation of M, A, and K.

To see this, draw a graph with the two endogenous variables on the axes (K on the horizontal and A on the vertical in Figure 2a). The relationship that determines Attitude, A, is a line with an intercept equal to bM and a slope “a.” Similarly, the graph of relationship for Knowledge, K, is a line with an intercept on the K axis of dM and a slope 1/c (notice that the vertical axis A is the argument in the function determining K). The two lines intersect each other at the point \((\bar{K}, \bar{A})\). This point is the equilibrium Attitude and Knowledge of a brand for a particular Message type. Empirical observation of the experiment would not show us the lines—only the point \((\bar{K}, \bar{A})\). For a particular value of M, we would observe only Figure 2.b, where the lines determining the equilibrium \((\bar{K}, \bar{A})\) have vanished. Having only observed the manifest point \((\bar{K}, \bar{A})\), how can we identify the true locations of the latent lines responsible for generating it?

![Figure 2](image-url)
There are literally an infinite number of pairs of lines that would cross each other at the point \((\bar{K}, \bar{A})\), and yet this point is all we have to go on empirically. In Figure 2.b, there are two dotted lines that cross at the equilibrium \((\bar{K}, \bar{A})\), but they look nothing like the true lines found in Figure 2.a. What can we do? Enlarge the sample? Observe \((\bar{K}, \bar{A})\) for more than one value of the exogenous variable – manipulate the value of Message, \(M\), in an experiment?

In the case of equation (1), when \(M\) changes, not only is there a direct effect on Attitude (by an amount \(bM\)), but \(M\) also causes Knowledge to change (by \(dM\)) in equation (2) and hence there is an indirect effect on Attitude (by an amount \(adM\)). To see this explicitly, solve the two equations for the endogenous variables \(A\) and \(K\) in terms of the exogenous variable, \(M\):

\[
A = \frac{b + ad}{1 - ac} M, \quad (3)
\]

\[
K = \frac{d + bc}{1 - ac} M. \quad (4)
\]

These are called the “reduced-form” equations of the model. They show the complete (both direct and indirect) effect of changing the experimental control variable, \(M\), on the dependent variables, \(A\) and \(K\). The experimental manipulation of \(M\) allows us to measure the reduced form equations, and from these we can try to infer the precise values of the structural coefficients in equations (1) and (2). With empirical observations only, this cannot be done.

If we have empirical estimates of the coefficients of the reduced form equations (3) and (4) (call them \(\alpha_A\) and \(\alpha_K\)), then we would like to solve the two empirical equations,
\[ \alpha_A = \frac{b + ad}{1 - ac}, \]  
\[ \alpha_K = \frac{d + bc}{1 - ac}, \]

for the four coefficient a, b, c, and d. You need four equations, not two, to determine completely four variables, so a, b, c, and d are unidentifiable for the prespecified conceptual model.

This ought to stop researchers from trying to estimate unidentifiable coefficients (it does not, as demonstrated in Table 1). The empirical results may look excellent, when it is actually impossible to have valid measurements for an unidentifiable equation. To see this, let the true (but unknown) parameters of the model in Figure 1 be \( a=1, b=1, c=\frac{1}{2}, d=\frac{1}{2} \). If the Message variable had a value \( M=\frac{1}{2} \) in our experiment, then the equilibrium values of the endogenous variables would be \((\bar{A}=1, \bar{K}=1.5)\), from the reduced form equations. See the point labeled 1 in Figure 3a.

![Figure 3](image-url)
In our experiment, suppose the situation was observed for three different subjects, each with the same Message treatment, $M=\frac{1}{2}$. The subjects generate slightly different equilibrium values for \(K\) and \(A\), because there are unobserved disturbance terms in the structural equations (error terms cannot provide information to help identification because they are unobserved). In Figure 3a, there are two other observations because of unobserved errors: one when the Knowledge line shifted left and the Attitude line shifted up and one when the Knowledge line shifted right and the Attitude line shifted down. The observed data are the three points 1, 2, and 3 in Figure 3.b and Table 3 below.

<table>
<thead>
<tr>
<th>Observation</th>
<th>(K)</th>
<th>(A)</th>
<th>(M)</th>
<th>Disturbance in (A) equation</th>
<th>Disturbance in (K) equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1.5</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>2</td>
<td>0.5</td>
<td>1</td>
<td>-0.75</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>1</td>
<td>0.5</td>
<td>-1</td>
<td>0.75</td>
</tr>
</tbody>
</table>

If this was the only data, then the researcher might run a regression with \(A\) as the dependent variable and \(K\) and \(M\) as the independent variables. The result would be a “estimated” Attitude line, like that seen in Figure 3.b, that seems to indicate that Knowledge has a negative impact on Attitude and is primarily determined by Message. The statistical fit of this equation is perfect ($R^2=1.0$) and the standard errors for the unidentifiable coefficients are zero. The empirical researcher might falsely believe that the estimated values of the unidentifiable coefficients in the Attitude equation are very informative, when they are really spurious. This is precisely what many published papers have done.
The theory incorporated in the conceptual model is insufficient to identify the structural coefficients, no matter how much data is brought to bear on the problem. The theoretical structure of the conceptual model is at fault, not our ability to do algebra or our ability to do statistical estimation. The statistical estimation is made impossible because of, not in spite of, the theory. Without the analysis of the identification of the equations, the problem of identification might have remained undetected. The equations show the way, not the graph.

If the conceptual model was different, then identification might be possible. For example, suppose that our theory specified that Message had no a direct impact on Knowledge. In Figure 4 the conceptual model is redrawn, but where the arrow from Message to Knowledge is erased. This is equivalent to setting $d=0$ in equation (2).

Consider the estimated coefficients of the reduced-form equations under the adjusted conceptual model:

$$\alpha_A = \frac{b}{1 - ac}. \quad (7)$$
(8)

If we know the value of $\alpha_A$ and $\alpha_K$ from empirical measurement, then we can solve for $c$ by dividing equation (8) by equation (7) to get $c = \frac{\alpha_K}{\alpha_A}$, assuming $b \neq 0$. This demonstrates that a properly structured theory (conceptual model) can lead to valid estimation of some parameters: they are no longer unidentifiable coefficients.

Can we solve the empirical equations (7) and (8) for $a$ and $b$? No. We have already “used up” one of the equations to find “c,” but let us check to make sure. Substitute $\frac{\alpha_K}{\alpha_A}$ for “c” into both equation (7) and (8), cross-multiply, and rearrange to get, respectively:

\[
a \alpha_K + b = \alpha_A, \quad \text{from (7),}
\]

\[
a \alpha_K^2 + b \alpha_K = \alpha_A \alpha_K, \quad \text{from (8).}
\]

Clearly these are dependent linear equations: equation (10) is a multiple $\alpha_K$ of equation (9). With one independent equation and two variables, there are an infinite number of solutions, since any given value of coefficient “a” allows us to say only that coefficient “b” must equal $\alpha_A - a \alpha_K$.

Any empirical estimates of the Attitude coefficients lack validity because they represent some linear combination of the true but non-identifiable coefficients, at best.

This is demonstrated graphically in Figure 5, where $d=0$ and $b \neq 0$. As the Message variable takes on successively larger values, $M_1$, $M_2$, and $M_3$, the Attitude line shifts up, but the Knowledge line is unchanged because the theory specifies that $dM_i \equiv 0$. The equilibrium observations of $(\bar{K}, \bar{A})$, points labeled 1, 2, and 3, trace out the Knowledge line. A regression
explaining K in terms of A would produce a valid measure of the identifiable coefficient “c.” However, a regression explaining A in terms of K and M using the same data (the only data, of course) would produce a spurious line unrelated to the true but unknown parameters, “a” and “b.”

![Figure 5](image)

How did the model developer come to the theoretical restriction, d=0, that allows us to identify the coefficient “c” in the Knowledge equation? If it was based upon a specification search (see Leamer 1978, 1983) of the original model aided by preliminary “estimates” from the data, we still have a serious problem. Recall from the analysis of Figure 3.b that estimated unidentifiable coefficients may look statistically significant but still be spurious. In fact, from the preliminary specification search associated with Figure 3.b, we would falsely conclude that b=0 in the Attitude equation. If this becomes a data-driven, rather than theoretical, restriction on the model, then it might appear that the model is identifiable, even when this is clearly invalid. The punch line is, “Theoretical restrictions on the conceptual model aimed at identification cannot be data-driven from same source that is used to estimate the model.”
How can we diagnose from the structural model in Figure 4 that the Knowledge equation can be identified but the Attitude equation cannot? In the next section, a general answer will be given, but anticipating it, the critical fact is that the relationship determining Knowledge is “restricted” by the theory, while the relationship determining Attitude is not. In every conceptual model, the “missing” arrows are just as important the “visible” arrows. We all have a tendency to focus on what is before our eyes. If we are not careful, we will be fooled (like an audience watching the waving hand of the magician) and miss the critical elements of the theory.

More generally, how do we know when one equation, in a system of equations, is identifiable? Economists have studied this since Working first described the issue 73 years ago. What follows is a general procedure (explained in any graduate econometrics or structural equation textbook) for diagnosing when a conceptual model has an identification problem. Once the diagnostic procedure is explained, we apply it to Journal of Marketing conceptual models.

**General Analysis of Identification**

**Without Disturbances**

Generalizing the above example, suppose that there are $G$ endogenous variables, $y=(y_1,...,y_G)'$, that are related to one another and to $K$ predetermined, exogenous variables, $x=(x_1,...,x_K)'$, in a system of $G$ linear structural equations.\(^4\) The $i^{th}$ equation will be written

$$\beta_{i1} y_1 + \cdots + \beta_{iG} y_G + \gamma_{i1} x_1 + \cdots + \gamma_{iK} x_K = 0.$$  \hspace{1cm} (11)

\(^4\) $K$ will denote the number of exogenous variables, rather than the Knowledge construct, from this point forward in the paper.
The coefficients of the variables can be expressed in matrix notation as

$$
B = \begin{bmatrix}
\beta_{11} & \beta_{12} & \cdots & \beta_{1G} \\
\beta_{21} & \beta_{22} & \cdots & \beta_{2G} \\
\vdots & \vdots & \ddots & \vdots \\
\beta_{G1} & \beta_{G2} & \cdots & \beta_{GG} \\
\end{bmatrix},
$$

(12)

$$
\Gamma = \begin{bmatrix}
\gamma_{11} & \gamma_{12} & \cdots & \gamma_{1K} \\
\gamma_{21} & \gamma_{22} & \cdots & \gamma_{2K} \\
\vdots & \vdots & \ddots & \vdots \\
\gamma_{G1} & \gamma_{G2} & \cdots & \gamma_{GK} \\
\end{bmatrix}.
$$

(13)

Using these, the structural equations can be written in matrix notation as

$$
B y + \Gamma x = 0.
$$

(14)

Compare this compact notation to the notation in equations (1) and (2). The matrix $B$ is assumed to have $G$ independent rows, otherwise there are duplications of the same equations in the model.

We could do an experiment by manipulating the values of the exogenous variables, $x$, and measuring the resulting values of the endogenous variables, $y$. Because there are only $K$ exogenous variables, the maximum number of independent values of vector $x$ is also $K$. Assume then that the exogenous variables $x$ have been empirically manipulated with $K$ independent

---

5 Notice that we have not incorporated error or disturbance terms, $\varepsilon$, in equation (14): $By + \Gamma x = \varepsilon$. Because the error terms consist of the combined effect of all the unobservable or unmeasured variables, we typically know very little about them a priori. As Theil (1971, page 494) concludes, “Identification based on restrictions on the disturbance distribution is attractive only when there exists a sufficient knowledge of the process which generates the disturbances. This is usually not the case.” In the next sub-section, we discuss the identification of partially recursive models using restrictions on the disturbance terms.
values stored in a $K \times K$ matrix $X$, whose $t^{\text{th}}$ column contains the data for values of the manipulated variables, $x_{(t)}=(x_{1t},...,x_{Kt})'$, $t=1,..,K$. The corresponding equilibrium values of the endogenous variables $y$ would be stored in a $G \times K$ matrix $Y$ whose $t^{\text{th}}$ column contains the data for values of the $t^{\text{th}}$ endogenous variable vector, $y_{(t)}=(y_{1t},...,y_{Gt})$ , $t=1,..,K$. The empirical equations are therefore expressed as

$$BY + \Gamma X = 0.$$ \hspace{1cm} (15)

These equations could also be written

$$[B \quad \Gamma] \begin{bmatrix} Y \\ X \end{bmatrix} = AZ,$$

where the coefficient matrices $B$ and $\Gamma$ have been concatenated into $G \times (G+K)$ matrix $A$, and the data matrices $Y$ and $X$ have been concatenated into $(G+K) \times K$ matrix $Z$. The matrix $A$ has $G$ independent rows, since we assume that $B$ is of full rank.\(^6\)

If the equations were to be solved for $Y$ then we would have a “reduced form” of the model:

$$Y = \Pi X,$$ \hspace{1cm} (16)

where the reduced-form coefficients satisfy $\Pi = -B^{-1}\Gamma$ or

$$B\Pi + \Gamma = 0.$$ \hspace{1cm} (17)

The matrix $\Pi$ is of order $G \times K$. If we have experimentally manipulated $x$ so that $X$ is a nonsingular $K \times K$ matrix, $\Pi$ can be measured empirically from the reduced form equation (16), $\Pi = YX^{-1}$ (or if the data are not so well designed, then by the regression of $Y$ on $X$). Knowing the reduced form coefficients, $\Pi$, the equation (17) provides $GK$ empirical restrictions on the $G^2 + GK$ coefficients in the structural equations matrices $B$ and $\Gamma$. Therefore, estimation of the

---

\(^6\) The homogeneous structural equations are typically normalized by setting one of the coefficients in each row of $\Gamma$ equal to 1. That is, each equation is designed to explain just one of the endogenous variables.
reduced form does not by itself provide enough restrictions to determine $B$ and $\Gamma$ completely. Observational data is not sufficient to determine the true structural equations. The additional restrictions on the coefficients must come from “theory” not “data.”

Let us focus attention on the first structural equation (naturally the equations could be reordered so any equation is the first one) which might be written $\alpha_1 Z = 0$, where $\alpha_1$ is the first row of the concatenated matrix $A$,

$$\alpha_1 = e_1' A = e_1' \begin{bmatrix} B & \Gamma \end{bmatrix}, \quad (18)$$

($e_1$ is the 1$^{\text{st}}$ unit vector with 1 in the first row, 0 elsewhere). Notice that for notational simplicity $\alpha_1$ is a row vector, not the typical column vector.

The empirical restrictions on the structural coefficients, $\alpha_1$, come from measurement of $\Pi$. Equation (17) can be expressed

$$\alpha_1 \begin{bmatrix} \Pi \\ I \end{bmatrix} = \alpha_1 W = 0, \quad (19)$$

where $I$ is a $K \times K$ identity matrix and $W$ is the $G+K \times K$ concatenation of $\Pi$ and $I$. Clearly, there are $K$ independent rows in $W$ (the identity matrix has $K$ such rows), so the rank of $W$ is $K$. From equation (19), $\alpha_1$ is in the row-nullspace of the empirical matrix $W$. The Fundamental Theorem of Linear Algebra (the sum of the rank of a matrix and the dimension of its row null space equals the number of rows) tells us that the dimension of the row-nullspace of $W$ is $G$. The row vector $\alpha_1$ is in this row-nullspace, but so too are all the rows of the structural equations (from equation...
17). The rows of A are independent because B is of full rank. That is, the rows of A form a basis for the null space of W. This leads to the following important result.

**Theorem:** A proposed first row vector \( \alpha_1^* \) for the conceptual model cannot be distinguished from the true first row structural coefficients based upon empirical data if and only if it is a linear combination of the rows of the true \( A=[B \Gamma] \) matrix.\(^7\)

Conceptual models have theoretical restrictions on their equations that are typically of the form of “exclusion” restrictions: some arrow is missing from the graph or equivalently some variable does not appear in the equation. For example, if the variable \( x_2 \) does not appear in the first equation, this is equivalent to the statement that \( \gamma_{12}=0 \). This restriction could be expressed as a linear restriction on the coefficient vector \( \alpha_1 \):

\[
\begin{bmatrix}
\beta_{11} & \cdots & \beta_{1G} & \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1K} \\
0 & \ddots & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 1 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
\end{bmatrix} \cdot \begin{bmatrix}
\alpha_1 \\
\end{bmatrix} = 0. \tag{20}
\]

Other restrictions that state that two coefficients (the \( r^{th} \) and \( s^{th} \), for example) equal each other could be written by post-multiplying the coefficient row-vector \( \alpha_1 \) by \( e_re_s \) and setting this equal to zero. Nonhomogeneous restrictions, such as “the sum of the \( r^{th} \) and \( s^{th} \) coefficients equals 1,” could be handled by creating a restriction column \( e_r+e_s-e_n \), where \( n \) is the number of the

\(^7\) If \( \alpha_1^* \) cannot be distinguished from the true first row then it must fit the data like the true \( e_1'[B \Gamma] \) and hence it must be in the row nullspace of W. Since the true rows of A form a basis for the nullspace, this means that \( \alpha_1^* \) is a linear combination of the rows of A. On the other hand, if \( \alpha_1^* \) is a linear combination of the rows of A, which form a basis of the nullspace of W, then closure implies that \( \alpha_1^* \) is in the nullspace and hence satisfies all the empirical restrictions that the true first equation does.
endogenous variable whose coefficient is normalized to equal 1 (see Fisher (1966) for a complete discussion). If there were a column like that in equation (20) for each restriction on structural equation 1, then let’s collect all R linear restrictions given by the conceptual model into a \((G+K) \times R\) matrix \(\Phi_1\). The theoretical restrictions would then be written

\[ \alpha_1 \Phi_1 = 0. \]  

(21)

Combining empirical restriction (19) and the theoretical restriction (21), the restrictions we have on the parameters of the first structural equation can be written as

\[ \alpha_1 [W \, \Phi_1] = 0. \]  

(22)

That is, the values of the first structural equation are in the row kernel of the matrix \([W \, \Phi_1]\).

As we have seen above, a linear combination of the true rows of the structural coefficients \(A = [B \, \Gamma]\) cannot be distinguished from the true first row based upon data alone. However, such a linear combination could be distinguished if it violated the theoretical restrictions on the first row. For example, if the first row should have a zero in the \(j^{th}\) column, then a linear combination of rows that doesn’t have a zero in the \(j^{th}\) column is fundamentally different. Put more formally:

**Definition: Identifiable Equation.** The theoretical restrictions on the \(i^{th}\) row are sufficient to identify the true coefficients of the \(i^{th}\) equation if all linear combinations of rows of \(A = [B \, \Gamma]\) that satisfy the theoretical restrictions of the \(i^{th}\) row produce a vector that equals (up to a scalar multiplication) the true \(i^{th}\) row.

Write the linear combination of rows of \(A\) as \(f' A\), where \(f\) is a column \(G\)-vector. If the first equation is identifiable, then \(f' A \Phi_1 = 0\) implies that \(f = r e_1\) for some scalar \(r\). That is, the nullspace
of the $G \times R$ matrix $A\Phi_1$ is one-dimensional and by the Fundamental Theorem of Linear Algebra the Rank $(A\Phi_1) = G-1$.

**Theorem (Rank Condition):** The first equation in $By + \Gamma x = 0$ is identifiable if and only if $\text{Rank}([B \; \Gamma]\Phi_1) = G-1$.

Why do we see “G-1” in the Rank Condition instead of $G$? We must distinguish the first equation from the other “G-1” equations in the conceptual model.

**Illustration 1.** Consider the adjusted conceptual model described in Figure 4. The two equations, $A = aK + bM$ and $K = cA$, correspond to $B = \begin{bmatrix} 1 & -a \\ -c & 1 \end{bmatrix}$, $\Gamma = \begin{bmatrix} -b \\ 0 \end{bmatrix}$. There are no restrictions on the first equation; therefore it is unidentifiable. The linear restrictions on the second equation correspond to the 3-dimensional vector $\Phi_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

The second equation is identifiable according to the Rank Condition if and only if $A\Phi_2 = \begin{bmatrix} -b \\ 0 \end{bmatrix}$ has rank $G-1=1$. There is one independent row in this matrix if $b \neq 0$, so it is identified (just as we found from informal analysis in the previous sub-section). Of course, it may be true that $b$ is equal to zero, although this is not expected a priori. In this case, the rank condition holds except on a set of measure zero.

The Rank Condition provides necessary and sufficient conditions for the identification of one structural equation in a system of equations, but calculating the rank of $A\Phi_1$ is not trivial, specially for larger systems of equations. How could this be accomplished? One algorithm
would be to use elementary row and column transformations\(^8\) on the matrix \(A\Phi_1\). Any matrix can be manipulated by elementary row and column transformations to be equivalent to one of the following variations on the \(n\)-dimensional identity matrix \(I_n\):

\[
I_n, \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix}, [I_n, 0], \begin{bmatrix} I_n \\ 0 \end{bmatrix}.
\]

Since elementary row and column transformations do not change the rank of the matrix, the number of independent rows the original matrix must be equal to \(n\). By this procedure we can determine if the rank of \(A\Phi_1\) equals \(G-1\) in general, as required by the Rank Condition.

**Illustration 2.** Suppose we make a variant of the above conceptual model. In Figure 6, new endogenous and exogenous variables have been added: Purchase likelihood, \(P\), and Frequency of advertisements, \(F\). The adjusted theory specifies that Frequency will change the Knowledge that the customers have about the brand, but does not directly impact the Attitude toward the brand or the purchase likelihood. An arrow has been added, linking \(F\) to \(K\). The resulting equation for Knowledge is \(K=cA+eF\). Purchase likelihood is driven by a combination of Attitude and Knowledge about the brand, so two arrows have been added and the resulting equation is \(P=gA+hK\).

\[\text{Message} \quad \text{Frequency} \quad \text{Attitude} \quad \text{Knowledge} \quad \text{Purchase} \]

\[\text{M} \quad F \quad A=\alpha K+\beta M \quad K=cA+eF \quad P=gA+hK \]

\[\text{Illustration 2}: \quad \text{Suppose we make a variant of the above conceptual model. In Figure 6, new endogenous and exogenous variables have been added: Purchase likelihood, } P, \text{ and Frequency of advertisements, } F. \text{ The adjusted theory specifies that Frequency will change the Knowledge that the customers have about the brand, but does not directly impact the Attitude toward the brand or the purchase likelihood. An arrow has been added, linking } F \text{ to } K. \text{ The resulting equation for Knowledge is } K=cA+eF. \text{ Purchase likelihood is driven by a combination of Attitude and Knowledge about the brand, so two arrows have been added and the resulting equation is } P=gA+hK.\]

\[\text{Figure 6}: \quad \text{Conceptual Model}\]

\[\text{8 The three elementary row transformations of matrices (and analogous elementary column transformations) are:} \]
The structural equation coefficient matrices are
\[ B = \begin{bmatrix} 1 & -a & 0 \\ -c & 1 & 0 \\ -g & -h & 1 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} -b & 0 \\ 0 & -f \end{bmatrix}. \]

There are two restrictions on the first equation: \( F \) and \( P \) do not appear. These correspond to the restriction matrix:
\[ \Phi_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}. \]

The first equation is identifiable according to the Rank Condition if and only if
\[ A\Phi_1 = \begin{bmatrix} 0 & 0 \\ 0 & -f \\ 1 & 0 \end{bmatrix} \]
has rank \( G-1=3-1=2 \). Apply elementary row transformations (multiply second row by \( 1/(-f) \), \( f \neq 0 \), and interchange row one and three) to get
\[ A\Phi_1 \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}. \]

\( A\Phi_1 \) has rank 2, if \( f \neq 0 \), so the equation satisfies the Rank Condition and is identified.

---

1) Interchange rows i and j, 2) Multiply row i by \( k \neq 0 \), and 3) Add row j times \( k \neq 0 \) to row i. Clearly, the rank of a matrix is unchanged by these transformations: no new independent row vectors are created.
The matrix $A\Phi_1$ is $G\times R$, and hence its rank cannot exceed the smaller of the two dimensions, $G$ or $R$. If $R$ is less than $G-1$, then it is obvious the rank of $A\Phi_1$ cannot equal $G-1$. This provides a check of the lack of identification of an equation.

**Theorem (Order Condition):** A necessary condition for the $i^{th}$ equation to be identifiable is that $R_i \geq G-1$, where $R_i$ is the number of restrictions in the $i^{th}$ equation. That is, if there are less than $G-1$ independent linear restrictions on the coefficients in the $i^{th}$ equation, it cannot be identified.

If the restrictions are of the exclusion form, “variable $i$ does not appear in equation $j$,” then the Order Condition says that an identifiable equation must exclude as many variables from the equation as there are endogenous variables less one. For each endogenous variable in a conceptual model, count the number of missing arrows that do not point at this node (each is a restriction) and compare this to the number of endogenous variables less one. Because there are $G+K-1$ other nodes and we need $G-1$ not to have an influence on the endogenous variable, this implies that at most $K$ arrows can point at an endogenous node if it is identifiable. If there are $K+1$ or more arrows pointing at a node, then there is no hope that that equation can be identified in the conceptual model. This check can be done very quickly and does not require measurement of any of the variables in the conceptual model. It is a theoretical calculation, not an empirical calculation, and can be done prior to construct measurement and system estimation.\(^9\)

In the conceptual model of Figure 4, there are two endogenous variables and so we needed just one restriction. The Attitude variable has arrows pointing to it from all the other constructs, so it cannot be identified. In Figure 6, there are three endogenous variables, $A$, $K$, and $P$.

\(^9\) It is possible that the Order Condition is satisfied, but the Rank Condition is violated. The equation is not identified. This situation would hold only on a parameter set of measure zero, however. See Fisher (1966) pp.44-45.
(identified by the fact that arrow tips point at them). We need at least G-1=2 restrictions for each endogenous construct or equivalently since there are two exogenous variables, no more than two arrows can point at a node. Look at Attitude, A. There is no arrow from F to A and no arrow from P to A, so there are two restrictions: the equation “could” be identified (and will be if b≠0). Similarly, both Knowledge and Purchase each have two theoretical restrictions, so they, too, could be identifiable.

If there are more than G-1 restrictions on an equation’s parameters, the equation is over-identified. When an equation is over-identified, there are two or more ways to estimate the structural equations from the reduced form equations. This is a good problem, not a bad problem. Some statistical average of the multiple estimates must be made; that is exactly the purpose of two stage least squares and other simultaneous equation estimation methods.

Identification with Disturbance Terms: Partially Recursive Models

Even when the Order Condition fails for an equation, if we are very lucky there may be another way to identify the parameters of the equation. A priori information about the variables that make up the error or disturbance term of an equation helps to identify model parameters, as will be demonstrated here. However, theoretical knowledge about such variables is problematic because the variables are unobserved (obviously if they were observed, they would be incorporated in the conceptual model). In this sub-section we will deal with disturbances in two-endogenous-variable conceptual models; Fisher (1966) provides a general analysis.

Consider the conceptual model in Figure 7, where A=bM+eS+fC and K=cA+dM+gS+hC. Comparing this to the conceptual model of Figure 1, two things have changed: the feedback
relationship from Knowledge to Attitude has been eliminated by setting $a=0$ and there are two unobserved variables which make up to disturbance terms for the two endogenous variables.
The lack of a feedback means that the matrix $B$ that represents the relationships between the endogenous variables can be written so that the upper triangle (above the diagonal) has only zeroes, $B = \begin{bmatrix} 1 & 0 \\ -c & 1 \end{bmatrix}$. Whenever this reciprocal causation is missing, the conceptual model is called partially recursive. In the model of Figure 7, the Attitude equation is identified: clearly the reduced-form equation will identify coefficient “b” (but check the Order and Rank conditions). Without additional information, the Knowledge equation is not identified: it has no theoretical restrictions with respect to observable variables (and hence violates Order and Rank conditions).

The unobserved variables (enclosed in circles, not boxes like observable variables) are Situation, $S$, and Culture, $C$. For simplicity, assume that the unobserved variables are generated by independent random processes, which implies that $S$ and $C$ are uncorrelated, and

---

10 Most consumer behavior textbooks spell out a variety of possible meanings of these constructs, e.g. Chapters 3 and 7 of Engel, Blackwell, and Miniard (1990), but to provide specific instances for this sub-section, interpret
have mean zero and variance equal to 1.0 (achieved by normalizing units). The total unobserved disturbance for the Attitude equation is $\varepsilon_A = eS + fC$ and for the Knowledge equation is $\varepsilon_K = gS + hC$.

The degree to which the unobserved variables impact the endogenous variables cannot be directly measured, but it might be possible to measure their total impact as a residual difference between the endogenous variable and the explanatory part of the equation. In this partially recursive model, the residual $\varepsilon_A = A - bM$ for the equation with only one endogenous variable has an unbiased estimator because the ordinary least squares estimate of “$b$” is unbiased. This is not true of the other equation in the partially recursive system because the explanatory, but endogenous, variable $A$ is correlated with the disturbance term. Specifically, $\text{COV}(A, \varepsilon_K) = \text{COV}(bM + eS + fC, gS + hC) = eg + fh \neq 0$. When the regressors are correlated with the errors, ordinary least squares estimators are biased, which implies that ordinary least squares residuals are biased estimates of the true disturbances.

Rather than dealing with biased estimates the structural equation for Knowledge, suppose that we solve for the reduced form:

$$K = (d + cb)M + (g + ce)S + (h + cf)C. \quad (24)$$

An ordinary least squares estimate of $(d + bc)$ will be unbiased and hence the resulting estimate of the reduced form disturbance $\varepsilon_K^{RF} = (g + ce)S + (h + cf)C$ is unbiased (the superscript RF stands for Reduced Form). Combining, the least squares estimates of the two reduced forms of the partially

---

Situation as “mood states” (Mehrabian and Russell 1974) and Culture as “individualistic versus collectivistic” (Triandis 1995).
recursive system will provide unbiased estimates of the following statistics: \( E[b] \), \( E[d+bc] \), \( \text{VAR}[\varepsilon_A] \), \( \text{VAR}[\varepsilon_{KR-F}] \) and \( \text{COV}[\varepsilon_A, \varepsilon_{KR-F}] \). Under the normalization assumption that for the unobserved variables \( \text{VAR}[S]=\text{VAR}[C]=1 \), this gives us five equations for seven unknowns:

\[
E[b]=b, \tag{25}
\]
\[
E[d+bc]=d+bc, \tag{26}
\]
\[
\text{VAR}[\varepsilon_A]=e^2+f^2, \tag{27}
\]
\[
\text{VAR}[\varepsilon_{KR-F}]=(g+ce)^2+(h+cf)^2, \tag{28}
\]
\[
\text{COV}[\varepsilon_A, \varepsilon_{KR-F}]=e(g+ce)+f(h+cf). \tag{29}
\]

Without additional theoretical information, the partially recursive system cannot be identified because there is not a unique solution of (25)-(29) for \( b, c, d, e, f, g, \) and \( h \).

Suppose, however, we had \textit{a priori} information that the unobservable Situation had no impact on Knowledge, \( g=0 \), and that the unobservable Culture had no impact on Attitude, \( f=0 \). These theoretical restrictions imply that the resulting disturbance term of the Attitude equation, \( eS \), and the disturbance term of the Knowledge equation, \( hC \), are uncorrelated. When the partially recursive model is complemented by the restriction that there are no correlations between the disturbances of different equations, the conceptual model is called a \textit{recursive system}. In addition to empirical restrictions (25)-(29), we have the theoretical restrictions \( g=0 \) and \( f=0 \). This gives us seven equations for seven parameters and the recursive model is just identified. The unique solution of the equations for the model parameters is

\[
b=E[b], \tag{30}
\]

\(^{11}\) Assuming that the variables are expressed as differences from their means.
\[ c = \text{COV}[\varepsilon_A, \varepsilon_K^{RF}] / \text{VAR}[\varepsilon_A], \]  
(31)

\[ d = \text{E}[d + bc] - \text{E}[b] \text{COV}[\varepsilon_A, \varepsilon_K^{RF}] / \text{VAR}[\varepsilon_A], \]  
(32)

\[ e = \sqrt{\text{COV}[\varepsilon_A, \varepsilon_K^{RF}]} \]  
(33)

\[ f = 0, \ g = 0, \]  
(34)

\[ h = \text{VAR}[\varepsilon_K^{RF}] \text{COV}[\varepsilon_A, \varepsilon_K^{RF}]^2 / \text{VAR}[\varepsilon_A]. \]  
(35)

In summary, partially recursive conceptual models are not identifiable (although some equations within the system may be identifiable). However, if we have enough information about the unobserved variables in the disturbance term that we can honestly add the restriction “all error terms in the structural equation are uncorrelated,” the resulting recursive system is identifiable. In fact, each of the equations can then be estimated consistently using ordinary least squares (Fisher 1996, p. 97). Of course, making an ad hoc assumption about zero correlation is just as improper as making an ad hoc assumption about the structural parameters of the model. If either is done merely to achieve identification, the estimates lack validity since they only appear to be uniquely determined. Such a scientific practice is unacceptable, of course.

It is a well-known property of ordinary least squares estimators that the residuals are uncorrelated with the regressors in the sample. If the conceptual model of Figure 7 was estimated by ordinary least squares, then the empirical residuals of the structural equation for Knowledge, \( \hat{\varepsilon}_K = K - \hat{c}A - \hat{d}M \) where \( \hat{c} \) and \( \hat{d} \) are least squares estimators, would be uncorrelated in the sample with \( A \) and \( M \). The residual errors of the Attitude equation are calculated as \( \hat{\varepsilon}_A = A - \hat{b}M \), where \( \hat{b} \) is the least squares estimator. Since \( \hat{\varepsilon}_K \) is uncorrelated in the sample with both regressors \( A \) and \( M \), it is uncorrelated with \( \hat{\varepsilon}_A \) in the sample. That is, an effort
to test whether the errors of these two equations have correlated errors using ordinary least squares is guaranteed to produce the result “no correlation,” regardless of the true correlation in the population. This should produce healthy skepticism about claims of uncorrelated errors using estimates of the correlations drawn from a structural model that is otherwise not identifiable.\textsuperscript{12}

Tests of the theoretical restrictions of a conceptual model are only appropriate when it is over-identified (Fisher 1966). Because a recursive system is “just” identifiable (assuming no other restrictions on the model), the theory does not provide extra degrees of freedom that would allow us to “test” for identification looking at inter-equation covariances. However, if there are additional restrictions on the structural parameters (beyond the triangularity of $\mathbf{B}$), then it may be possible to test the assumed zero covariance assumptions using the reduced-form estimates.

For example if in the recursive version of Figure 7, we also knew that Message had no direct influence on Knowledge, $d=0$, then the equation for Knowledge is over-identified. Careful inspection of equations (25)-(26) give a second way to identify the coefficient “c” in addition to equation (31), namely, $c=E[d+bc]/E[b]$. If these two estimates of “c” are not equal, then one or more of the inter-disturbance covariances are different from zero, and the system is not recursive. The equality could be tested empirically.

Reduced form parameters have valid and unbiased estimates regardless of the identification problem, but this is not true of structural equations. It would be improper to assume zero correlations across equations (making a partially recursive system recursive) and then test the assumption by looking at the covariances of the residuals of estimated structural equations. Without the zero correlation assumption, such estimated structural residuals are not valid (Figure

\textsuperscript{12} I thank Franklin Fisher for bringing this to my attention.
3b makes this clear), so we would be caught in a vicious circle, testing the validity of assumptions using non-valid estimates of the assumptions.

**Diagnostic Procedure to Check Identification**

Order and rank conditions were developed over fifty years ago by Koopmans, Rubin, and Leipnik (1950). One would expect that the process is common knowledge. All marketing scholars ought to apply them early in the development of their conceptual models to avoid the validity issues discussed above, as recommended by Aaker and Bagozzi (1979, page 152):

The first step in constructing structural equation models is to propose relevant theoretical constructs, their indicators, and the structure or pattern of relationships. This set of procedures is known as specification. Specification depends fundamentally on the theory one hopes to develop and test, the observable variables available, the results of past research, the creativity and acumen of the researcher, and other factors. Once a theory has been specified, two additional steps are identification and estimation.

Let us review. A procedure to protect against estimating unidentifiable equations consists of the following steps.

---

**Diagnostic Procedure for Identification**

1. Before you gather data, draw a conceptual model and categorize variables as exogenous or endogenous (an endogenous variable would have an arrow pointing at it). **Count endogenous and exogenous variables.** Begin by inspecting the structure of observable variables. Specifically:

2. Using the **Order Condition**, establish whether an equation determining an endogenous variable is unidentifiable by counting arrows pointing at a node: if they exceed the number of exogenous variables, then the equation is unidentifiable. If they do not, then:

3. Create the Φ matrix whose columns describe the restrictions in the coefficients of the equation. Using the **Rank Condition** (and some elementary linear algebra), establish

---

13 Tjalling Koopmans won the Nobel Prize in Economic Sciences twenty-five years ago.
for certain whether the equation is identified by showing that \( \text{Rank}(\begin{bmatrix} \mathbf{B} & \mathbf{\Gamma} \end{bmatrix}\Phi) \) equals the number of endogenous variables less one. If either the Order or Rank condition is violated, then the equation for the endogenous variable cannot be identified using only the structure of the observable variables. If this is true, then

4. See if the structural model is **partially recursive** by looking to see that it lacks feedback loops or equivalently the matrix \( \mathbf{B} \) can be rearranged so that all the coefficients above the diagonal are zero. If the model is partially recursive, make a list, one for each endogenous construct, of all the unobserved variables that would be expected in theory to influence the construct. If the lists do not overlap, then assume the disturbance terms are uncorrelated: the system is **recursive** and thus is identified.

5. If all of the above fail, you have an identification problem and need to **rethink the conceptual model**, as discussed below. Perhaps a more sophisticated model would not have the problem.

6. Only after the conceptual model is free of unidentifiable endogenous variables should the researcher move on to the data-gathering and estimation phases of the project.

We now demonstrate this procedure with a study of the all the conceptual models published recently in the *Journal of Marketing*. Unfortunately, many marketing researchers do not follow the diagnostic procedure and skip the recommended *identification* steps in their haste to get to the *estimation* step. As a result, their estimates lack concept validity because of undetected identification problems.
Unidentifiable Conceptual Models in *Journal of Marketing*

All the papers published in the *Journal of Marketing* from January 1995 until October 1999 were classified into those with conceptual models designed for measurement and those without. To have a conceptual model for the purposes of this study, a paper had to 1) have a graphical representation of the constructs and their interlinkages, 2) have more than one endogenous variable, and 3) have some form of estimation of the model from data. Many published papers did have graphs, but these were used for illustrative purposes other than a conceptual model, and many others had just a single equation, which of course cannot be unidentifiable. There were a handful of thought-pieces that developed conceptual models but made no attempt to estimate the parameters of the implied relationships. Forty-three conceptual model papers were found among the four hundred or so papers published during this five-year period.

Each of the papers previously classified as having an estimated conceptual model was then analyzed using the diagnostic methods outlined above to see if there were unidentifiable equations within the conceptual model. The process will be illustrated below for two papers that have identification problems and one paper that does not. Fifteen papers were free of identification problems, but twenty-eight papers had one or more unidentifiable equation that was nonetheless estimated. Table 1 lists all the papers that had unidentifiable equations, along with the specific endogenous variables, the equations for which have parameter estimates are not valid. Table 2 lists all the papers that had no unidentified equations.
Two Examples of Unidentifiable Conceptual Models from JM

First, consider conceptual models that have an identification problem. I have selected two papers from Table 1 as examples, but all twenty-eight papers in the table have similar difficulties. The first paper was selected randomly: Brown and Dacin (1997). The second paper was selected to demonstrate that even the best research can run afoul of the identification problem: Lusch and Brown (1996) was awarded the 1997 Harold H. Maynard Award by the Editorial Review Board for its significant contribution to marketing theory and thought. Both papers have identification problems and estimates of relationship parameters that are not valid.

Brown and Dacin’s Unidentified Conceptual Model

Brown and Dacin (1997) created a conceptual model of consumers’ evaluation of new products as seen below in Figure 8 (this is a redrawing of their Figure 1).
There are four endogenous variables because Product Sophistication, Corporate Evaluation, Product Social Responsibility, and Product Evaluation have arrows pointing at them; G=4. These are denoted \( y_1, y_2, y_3, \) and \( y_4 \) in Figure 7. The other two constructs, Corporate Ability and Corporate Social Responsibility are exogenous and are denoted \( x_1 \) and \( x_2; K=2. \)

Following the process described above, first check the Order Condition of identification for each endogenous variable. Since the number of endogenous variables less one is 3, each endogenous variable must have three or more restrictions on its structural equation, or it will not be identifiable. Equivalently, any construct that has more arrows pointing at it than the system has exogenous variables (in this case, 2) has an unidentifiable equation. Counting arrows, constructs \( y_1, y_2, \) and \( y_3 \) satisfy the Order Condition: they have one, two, and one arrow pointing at them and these are all less than or equal to 2. This implies that they may be identified. On the other hand, there are three arrows pointing at \( y_4 \), and this exceeds the number of exogenous variables, in violation of the Order Condition. The equation determining \( y_4 \), Product Evaluation, is unidentifiable.

Now let us check the Rank Conditions for each endogenous variable. To do this, the canonical form of the linear conceptual model \((\mathbf{B} y + \mathbf{\Gamma} x = 0)\) implied by Figure 8 has a coefficient matrix \( \mathbf{A} = [\mathbf{B} \, \mathbf{\Gamma}] \):

\[
\mathbf{A} = [\mathbf{B} \, \mathbf{\Gamma}] = \begin{bmatrix}
1 & 0 & 0 & 0 & -\gamma_{11} & 0 \\
0 & 1 & 0 & 0 & -\gamma_{21} & -\gamma_{22} \\
0 & 0 & 1 & 0 & 0 & -\gamma_{32} \\
-\beta_{41} & -\beta_{42} & -\beta_{43} & 1 & 0 & 0
\end{bmatrix}.
\]

(36)

There are several exclusion restrictions for each equation (for example \( y_2 \) does not influence \( y_1 \)). The restriction matrices \( \Phi_1, \ldots, \Phi_4 \) for each of the four equations are:
According to the Rank Condition, the $i^{th}$ equation is identifiable if and only if the product of the coefficient matrix $A$ in equation (36) and the above restriction matrix $\Phi_i$ has $G-1=3$ independent rows. Multiplying these matrices is easy because the matrix $\Phi_i$ has only zeroes and ones. In fact, to get $A\Phi_i$ we only need to copy the entire column from $A$ associated with the excluded variables. For the first equation, this is

$$\Phi_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (37)$$

$$\Phi_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (38)$$

$$\Phi_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (39)$$

$$\Phi_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (40)$$
It is easy to see that by the elementary row (and column) operations that the matrix $A\Phi_1$ is equivalent to $\begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$. There are 3 independent rows in $A\Phi_1$, so its rank equals $3=G-1$. The first equation is identified. This will be true for $A\Phi_2$ and $A\Phi_3$, but $A\Phi_4$ equals

$$A\Phi_4 = \begin{bmatrix} -\gamma_{11} & 0 \\ -\gamma_{21} & -\gamma_{22} \\ 0 & -\gamma_{32} \\ 0 & 0 \end{bmatrix}. \quad (42)$$

One can easily show that there are just two independent columns in this matrix, so its rank equals $2 < 3=G-1$. The Rank Condition confirms that the equation for Product Evaluation, $y_4$, cannot be identified by empirical means.

The model is partially recursive, however. In equation (36) it is evident that $B$ has only zeroes above the diagonal. However, identification is only possible when the unobserved disturbance term for each construct is independent of the disturbance terms of other constructs. It is hard to believe that the unobserved factors that influence a Product’s Evaluation are completely different from those that influence the Corporate Evaluation, for example. The authors make no claim that this is true. Hence, the Corporate Evaluation equation is not recursive and not identifiable.
Even though the Product Evaluation parameters cannot be identified, Brown and Dacin nonetheless estimated all the coefficients in the conceptual model. What is the matter with the LISREL estimates they found (Figure 9 reproduces the estimates)? There is nothing wrong with the coefficients of the identifiable equations for $y_1$, $y_2$, and $y_3$, but the estimates of the coefficients of the equation for $y_4$ are spurious.

To understand, solve the structural equations for the endogenous variables, to get the reduced form equations for the endogenous variable $y_4$.

\[ y_4 = (\beta_{41}y_{11} + \beta_{42}y_{21})x_1 + (\beta_{43}y_{32} + \beta_{42}y_{22})x_2. \quad (43) \]
The estimates for $\gamma_{11}=.27$, $\gamma_{21}=.80$, $\gamma_{22}=.26$, and $\gamma_{32}=.09$ are valid, but the estimates of the $\beta_{ij}$s are not. Any values of $\beta_{41}$, $\beta_{42}$ and $\beta_{43}$ that satisfy

$$\beta_{41}.27 + \beta_{42}.80 = .34 * .27 + .16 * .80 = .22 \text{, and}$$

$$\beta_{43}.09 + \beta_{42}.26 = .09 * .09 + .16 * .26 = .05,$$

also satisfy all the theoretical and empirical restrictions. Brown and Dacin find that greater perceived Product Sophistication causes an increase in the Product Evaluation: $\beta_{41}=.34 > 0$. However, an equally appropriate estimate is $\beta_{41}=-0.9$ (with $\beta_{42}=0.5$) so the relationship could plausibly have the opposite sign. We can make an infinite number of lines pass through an observed point, but we cannot learn the truth about Product Evaluation from this conceptual model. The parameters are unidentifiable.

**Lusch and Brown’s Unidentified Conceptual Model**

Now consider the paper by Lusch and Brown (1996) who created a conceptual model of wholesaler-distributor performance as seen below in Figure 10 (this is a redrawing of their Figure 1). There are five endogenous variables ($G=5$) because arrows point to Long Term Orientation, Explicit Contract, Normative Contract, Relational Behavior and Wholesaler-Distributor Performance. These are denoted $y_1$, $y_2$, $y_2$, $y_4$, and $y_5$ in Figure 10. The other four constructs, Wholesaler More Dependent, Supplier More Dependent, Bilateral Dependency, and Relationship Length are exogenous ($K=4$) and are denoted $x_1$, $x_2$, $x_3$, and $x_4$.

The Order Condition implies that if more than $K=4$ arrows point at a construct, it cannot be identified. Four arrows point at Long Term Orientation; five arrows point at Explicit Contract;
five arrows point at Normative Contract; seven arrows point at Relational Behavior; six arrows point at Wholesaler-Distributor Performance. That is, all of the endogenous variables except \( y_1 \) are unidentifiable.

![Image of Lusch and Brown's Conceptual Model]

**Figure 10**

*Lusch and Brown’s Conceptual Model*

This is verified by checking the Rank Condition for each equation: calculate the rank of the matrix \( A \Phi_i \) and compare it to \( G-1 = 4 \). Below is a list of the matrices and their ranks.

\[
\begin{bmatrix}
0 & 0 & 0 \\
1 & 0 & 0 \end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
-\beta_{42} & -\beta_{43} & 1 & 0 \\
-\beta_{52} & -\beta_{53} & -\beta_{54} & 1
\end{bmatrix}
\]

\[\text{Rank}[A \Phi_i] = \text{Rank} \begin{bmatrix}
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
-\beta_{42} & -\beta_{43} & 1 & 0 \\
-\beta_{52} & -\beta_{53} & -\beta_{54} & 1
\end{bmatrix} = 4, \quad (46)\]
\[
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
1 & 0 & 0 \\
-\beta_{43} & 1 & 0 \\
-\beta_{53} & -\beta_{54} & 1
\end{bmatrix}
\]

\[
\text{Rank}[A\Phi_2] = \text{Rank}
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
1 & 0 & 0 \\
-\beta_{43} & 1 & 0 \\
-\beta_{53} & -\beta_{54} & 1
\end{bmatrix}
= 3, 
(47)
\]

\[
\begin{bmatrix}
0 & 0 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0 \\
-\beta_{42} & 1 & 0 \\
-\beta_{52} & -\beta_{54} & 1
\end{bmatrix}
\]

\[
\text{Rank}[A\Phi_3] = \text{Rank}
\begin{bmatrix}
0 & 0 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0 \\
-\beta_{42} & 1 & 0 \\
-\beta_{52} & -\beta_{54} & 1
\end{bmatrix}
= 3, 
(48)
\]

\[
\begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix}
\]

\[
\text{Rank}[A\Phi_4] = \text{Rank}
\begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix}
= 1, \text{ and } 
(49)
\]

\[
\begin{bmatrix}
1 & -\gamma_{14} \\
-\beta_{21} & -\gamma_{24} \\
-\beta_{31} & -\gamma_{34} \\
-\beta_{41} & -\gamma_{44} \\
0 & 0
\end{bmatrix}
\]

\[
\text{Rank}[A\Phi_5] = \text{Rank}
\begin{bmatrix}
1 & -\gamma_{14} \\
-\beta_{21} & -\gamma_{24} \\
-\beta_{31} & -\gamma_{34} \\
-\beta_{41} & -\gamma_{44} \\
0 & 0
\end{bmatrix}
= 2. 
(50)
\]

The last four equations violate the Rank Condition, verifying that they are unidentifiable.

The conceptual model in Figure 10 is partially recursive (there are no feedback loops).

As demonstrated above, it is well known that unless there are zero correlations between the unobserved variables that make up the disturbance terms for each endogenous variable, the system is not recursive. The paper does not specify the unobserved variables that make up the error terms for equations, so disturbances may be correlated. The estimated model of Lusch and Brown (reproduced below in Figure 11) does not provide valid measures of the parameters.
because four of the five equations are unidentifiable without theoretical justification of uncorrelated errors.

Figure 11
Lusch and Brown’s Estimates of the Conceptual Model

In fact, Footnote b of Table 2 in the paper states, “Estimated correlation between Equation 2 error and Equation 3 error (ψ32) is .378 (t=6.523).”¹⁴ This implies that equations 2 and 3, which explain choice of explicit and normative contracts, are unidentifiable. In their abstract, Lusch and Brown (1996) say, “The dependency structure between wholesale-distributors and their major

---

¹⁴ As discussed above, if the identification of a structural equation depends upon an assumption of zero correlation between error terms in equations, it would be improper to estimate the correlation from the structural equation.
suppliers is posited to influence the type of contract – explicit and normative – used… They obtain empirical support for many of the hypothesized linkages.” Since the equations for explicit and normative contracts cannot be identified, the empirical estimates do not represent the behavior claimed, and there is no valid support for the posited influence.

This is why identification needs to be taken so seriously. The two papers selected from the twenty-eight papers in Table 1 covered just one journal for just a five-year time frame. Each paper in that table has a similar unnoticed inability to distinguish true parameters from false parameters, and yet the parameters were estimated and claims made about their statistical and practical significance which are not valid.

The Brown and Dacin (1997) and Lusch and Brown (1996) conceptual models are fairly complicated, as are many of the others. In Table 4, I have tried to cut through the clutter to show geometrically the essence of why unidentifiable conceptual models are so prevalent. In Table 4 are uncomplicated models of the form

where the exogenous variable x is in a shaded box and the endogenous variables y's are enclosed in unshaded boxes. The above graph indicates that y_2 and y_3 are unidentifiable because each has only one of two required restrictions (y_2 does not have an arrow from y_3, and y_3 does not have an arrow from x). Each basic graph demonstrates an observed identification problem found in the papers listed in Table 1; the code letter from the first column of Table 1 indicates the particular papers. When constructing conceptual model, scholars who are uncomfortable with matrix algebra may simply want to avoid graphical structures like those in Table 4 (at minimum).
### Table 4
Basic Geometric Forms of Unidentifiable Relationships in *Journal of Marketing* Conceptual Models

<table>
<thead>
<tr>
<th>Conceptual Model</th>
<th>Unidentified equation for endogenous variable</th>
<th>Studies that have this conceptual model</th>
<th>Partially-Recursive Model?</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram 1" /></td>
<td>$y_2$</td>
<td>G, H, J, X, Z</td>
<td>Yes</td>
</tr>
<tr>
<td><img src="image2.png" alt="Diagram 2" /></td>
<td>$y_1, y_2$</td>
<td>M</td>
<td>No</td>
</tr>
<tr>
<td><img src="image3.png" alt="Diagram 3" /></td>
<td>$y_3$</td>
<td>C, F, T, AA</td>
<td>Yes</td>
</tr>
<tr>
<td><img src="image4.png" alt="Diagram 4" /></td>
<td>$y_3$</td>
<td>O, P, V, Y</td>
<td>Yes</td>
</tr>
<tr>
<td><img src="image5.png" alt="Diagram 5" /></td>
<td>$y_3$</td>
<td>U</td>
<td>Yes</td>
</tr>
<tr>
<td><img src="image6.png" alt="Diagram 6" /></td>
<td>$y_2, y_3$</td>
<td>A, D, I, W, Y, K</td>
<td>Yes</td>
</tr>
<tr>
<td>Diagram</td>
<td>Variables</td>
<td>Condition</td>
<td>Answer</td>
</tr>
<tr>
<td>---------</td>
<td>-----------</td>
<td>-----------</td>
<td>--------</td>
</tr>
<tr>
<td><img src="image1.png" alt="Diagram 1" /></td>
<td>$y_1, y_2$</td>
<td>AB</td>
<td>No</td>
</tr>
<tr>
<td><img src="image2.png" alt="Diagram 2" /></td>
<td>$y_2, y_3$</td>
<td>R, S</td>
<td>Yes</td>
</tr>
<tr>
<td><img src="image3.png" alt="Diagram 3" /></td>
<td>$y_2$</td>
<td>B, L</td>
<td>Yes</td>
</tr>
<tr>
<td><img src="image4.png" alt="Diagram 4" /></td>
<td>$y_3$</td>
<td>Q</td>
<td>Yes</td>
</tr>
<tr>
<td><img src="image5.png" alt="Diagram 5" /></td>
<td>$y_2$</td>
<td>E</td>
<td>No</td>
</tr>
</tbody>
</table>
What to Do When You Have an Identification Problem

The identification problem will not always reveal itself in the empirical measurement of the system of equations, as Figure 3b made clear. It may appear that the parameters of the relationships have been accurately measured. Empirical researchers who do not check the Rank and Order Conditions may therefore believe that they have reliable and valid measures of the interconnections in the model, when in fact their model is incapable of providing valid measures of these. More observations or better statistical data processing methods cannot solve the “validity” problem created by under-identification. It can only be solved by an improved theory. That is, additional restrictions on the current variables must be discovered (arrows must be erased in the conceptual model) or new variables must be added to the model (new constructs added to the conceptual model). When adjusting Figure 1 to create Figure 4, a new restriction was added: the arrow from Message to Knowledge was erased. We were then able to show that the Knowledge equation could be identified.

Needless to say, a theoretical restriction that cannot be justified except to address an identification problem is inappropriate - doubly so, if the restriction was based on preliminary specification searches on the dataset. Of course, estimating an unidentifiable equation to see if the parameters can be restricted is improper and circular because the estimates are invalid without the \textit{a priori} theoretical restriction. This questionable practice appears in some papers found in Table 1. In fact, there are papers whose initial conceptual model had an identification problem, but the final “data” model is identifiable after a specification search. Moreover, an equation with arbitrary restrictions is misspecified since a variable is falsely eliminated, and
therefore the estimates of the remaining coefficients will be biased (see Kennedy 1998 for details on specification analysis).

A better strategy is to seek other variables that have been overlooked in the theory development and add them to the model, or critically evaluate the reasoning behind each posited link between variables and drop the ones without strong \textit{a priori} justification. If the new variables do not appear in an equation that is unidentified, the new restrictions might help identification.

The conceptual model of Brown and Dacin (1997) (see Figure 8 above) has an unidentified equation for Product Evaluation. In the literature review, Brown and Dacin mentioned in passing that corporate associations might include “a person’s knowledge of his or her prior behaviors with respect to the company.” The two exogenous constructs, Corporate Ability and Corporate Social Responsibility, are distinct from “Prior Behaviors Toward the Company,” so it would seem reasonable to add this as a third exogenous variable, $x_3$. Assuming that this variable is mediated by Corporate Evaluation and has no direct impact on Product Evaluation, we have a new

![Figure 12: Brown and Dacin’s conceptual model](image-url)
conceptual model like that seen in Figure 12. Assuming $\gamma_{23} \neq 0$, the addition of this variable implies that Product Evaluation equation is now identifiable (check the Order and Rank Conditions).

The conceptual model of Lusch and Brown (1996) described above in Figure 10 had unidentifiable equations for Explicit Contract and Normative Contract. Both of these constructs include Relationship Length as an explanatory variable. However, Relationship Length has another impact on contracts: mediated by Long Term Orientation. A careful reading of Lusch and Brown’s rationale leading up to their Hypotheses 10a and 11a (Long Term Orientation influences Contracts) and Hypotheses 10b and 11b (Relationship Length influences Contracts) shows that there is no reason for the direct link when the mediated link is included. That is, the authors could safely drop the arrow connecting $x_4$ to $y_2$ and $y_3$, as seen in Figure 13.
The boxes for Explicit Contract and Normative Contract then have only four arrows pointing to them (the dotted arrows have been dropped from the model), so the equations satisfy the Order Condition. Rechecking, the Rank Conditions will verify that the equations for Explicit Contract and Normative Contract are identifiable in the adjusted conceptual model.

As with arbitrarily erasing arrows in a conceptual model to identify an equation, inappropriate ad hoc additions of variables to a model to achieve identification is a mistake. If the variable truly does not belong in the model, then the restriction that it does not appear in equation 1 does not help distinguish the parameters of equation 1 from those in Equations 2,…,G: the variable doesn’t belong in them, either. The suggested additional variable for the Brown and Dacin model seen in Figure 11 was based upon a reading of the literature and therefore seems appropriate. Dropping of the direct links in favor of the mediated links in Lusch and Brown’s model also is reasonable.

Since many conceptual models go hand-in-hand with the development of scales to measure latent constructs, there is another strategy suggested by the Order Condition. Recall that that a variable being explained by more variables than there are exogenous variables in the system cannot be identified. If a multi-item scale is summed to make a single exogenous variable, potential exogenous variables have disappeared. As a result, if the multi-item scale appears to measure two independent factors, the system identification may be helped if they are not collapsed together. I say, “may be helped,” because if the exogenous variable directly influences
the unidentified endogenous variable, splitting it in two does not add restrictions to the critical
node.

Finally, as can be seen from Table 4 almost all the papers with identification problems are
partially recursive. That is, the endogenous variables feed forward in determining other
endogenous variables but never feed back, or equivalently the matrix $B$ has zeroes above the
diagonal (both Brown and Dacin’s and Lusch and Brown’s models are partially recursive as seen
in Figure 8 and Figure 10). If the unobserved errors in the structural equations were
uncorrelated, then the system would be recursive and as a result it would be identified. This
requires that the researcher catalogue the variables that ought to influence each endogenous
variable. This task can be somewhat frustrating since many of these variables may be
unmeasured in this study. However, if there is essentially no overlap in the list of unmeasured
variables for two endogenous variables, then the disturbances of the variables are likely to be
uncorrelated. As stated above, ad hoc assumptions of zero covariance are unacceptable, but if the
researcher can convincingy demonstrate in this way that the major unobserved variables that
have been swept into the error terms of each equation do not overlap, then identification via
recursive systems is possible.

Table 5 provides a summary of recommended adjustments to marketing conceptual models to
produce identifiable relationships.

<table>
<thead>
<tr>
<th>Identification Tactic</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. Erase Theoretical Linkages</strong></td>
<td>The arrows missing in a conceptual model are equally as important arrows present or the predicted sign is of correlations. Superfluous linkages should be eliminated</td>
</tr>
<tr>
<td></td>
<td>Exogenous variables that “shift” other equations</td>
</tr>
<tr>
<td>2. Add an Exogenous Variable</td>
<td>around but do not effect the equation in question help identify this questionable one’s coefficients.</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>--------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>3. Split Exogenous Variables</td>
<td>If you have multi-item measures of an exogenous variable, you can create extra movement in some equations by not collapsing all the items into a single scale. Do this only if suggested by factor analysis.</td>
</tr>
<tr>
<td>4. Catalogue Missing Variables if the Model is Partially Recursive</td>
<td>If your model is partially recursive, then you need to think carefully about the missing variables that are “not” in your equations (and which make up the disturbance term). If the list of missing variables for each equation is unique, then you can plausibly assume that the disturbances are independent, making the model fully recursive and thus identified.</td>
</tr>
</tbody>
</table>

**Conclusion**

Conceptual models are the backbone of empirical and theoretical marketing. They must be treated with the same care as we do our questionnaire construction, purification of our measurement scales, design of experiments, and derivation of game theory equilibria. When they are constructed casually as a method of representing our hunches and instinct about the linkages between a variety of marketing constructs, we sometime put ourselves in the position of unwittingly believing that unidentifiable relationships can be measured. As Aaker and Bagozzi (1979) coached us, we need to creatively specify our models, check them for identifiability, and then estimate them with the best measurement and statistical methods available. Do not skip the middle step.
References


