Repo Collateral Fire Sales:

The Effects of Exemption from Automatic Stay*

Sebastian Infante†

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Abstract

What are the consequences of a potential fire sale stemming from repo's exemption from automatic stay? This paper shows that repo's exemption from stay alters firms' financing and investment decisions ex-ante. Specifically, stay exemption changes firms' investment opportunity set, enabling them to purchase defaulted collateral at fire sale prices. Fire sales arise endogenously because of limited capital available to purchase collateral posted by defaulted firms, i.e. *cash-in-the-market* pricing. This effectively creates a premium for holding on to *dry powder* and concentrates asset ownership with firms who have preferences to hold large risky asset positions and risk default. The premium reduces the initial asset price, potentially inducing more firms to take on risky positions, increasing the fraction of defaulting firms in the economy. When repo is subject to automatic stay, secured lenders do not receive their collateral immediately, eliminating the possibility of a fire sale and ex-ante price distortion.

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†Graduate School of Business, Stanford University. Please send comments to: *sinfante@stanford.edu*.

1 Introduction

The financial crisis of 2007-09 has spurred considerable debate over the role deregulation may have played in the instability of the financial system. One particular area of interest has been the treatment of derivatives and repurchase agreements (*repos*) in bankruptcy. For the most part, when firms in the United States file for bankruptcy a temporary hold is placed on the their assets known as *automatic stay*. The logic behind this bankruptcy provision is to prohibit creditors from collecting payments in a disorderly fashion, maintaining the firm as a going concern.[1] This is not the case for derivatives and repos which are exempt from automatic stay. Counterparties to these contracts have, among other things, the right to take immediate ownership of any collateral posted by the defaulted party. This is particularly important for repo contracts which in essence are secured loans backed by financial assets. The impact of repo's exemption from stay is a relatively broad question. This paper focuses on a particular aspect of the debate, namely the impact of a *fire sale* on firms' ex-ante incentives, which is triggered by the immediate sale of illiquid collateral when repo is exempt from automatic stay. To my knowledge, this is the first paper to address the impact of repos' special treatment in bankruptcy in a setting were fire sales arise endogenously.

This paper addresses the issue by examining agents' financing and investment decisions under two regulatory regimes: repo subject to, or exempt from, automatic stay. The model features a group of firms with limited capital that raise debt from a competitive lending sector to purchase a risky asset. In the stay regime, in case of a debtor's default secured lenders (i.e. repo lenders) must wait until the following period to receive their collateral. In the stay exemption regime, secured lenders have immediate ownership of the contract's collateral and have incentives to sell it to surviving firms, causing a fire sale. Fire sales occur because of limited liquidity available to solvent firms to purchase assets after a default event — a *cash-in-the-market* style pricing. Thus repo's exemption from stay alters firms' investment opportunity set, giving them the opportunity to purchase assets at discounted prices. This creates incentives for firms to hold their initial endowment (*dry powder*) to take advantage of the potential fire sale, creating a premium for having liquidity and reducing the initial price of the risky asset.

Specifically, in the stay exemption regime firms' incentives to exploit future fire sales attracts capital which would have otherwise participated in the market initially, thereby reducing the asset price before the fire sale. The incentives of firms to use their liquidity in the future places more assets in the hands of firms with preferences to take large positions and risk default, which is assumed to be costly. Moreover, the initial

[1] For the history of automatic stay in US bankruptcy see Skeel Jr. (2001).

price discount can be so severe, that even firms that do not have preferences to become large are motivated to choose high levels of debt, increasing the fraction of defaulting firms in the economy. This is not the case when repo is subject to automatic stay, since the lock-up of secured lenders' collateral does not allow them to sell it upon default, effectively eliminating any possibility of a fire sale.

The base setup consists of a group of firms — interpreted as dealer banks, hedge funds, or commercial banks — endowed with a fixed initial capital stock, that raise debt from a competitive lending sector — interpreted as money market funds and bond investors — to purchase a risky asset. Because of reasons outside the model a fixed fraction of firms enjoy *private benefits* for holding large risky asset positions, inducing them to take on higher leverage. This modeling device captures the fact that many financial firms increase their asset size by taking on debt, and that there is heterogeneity in financial firms' leverage.[2] Therefore, the quantity of assets each firm can purchase depends on its initial endowment, the amount of debt it can raise, and its willingness to increase its risky asset position. A well-capitalized lending market offers secured and unsecured loans to firms, where secured contracts set aside a fraction of the firm's risky asset position as collateral (i.e. repo contract). Firms are constrained as to how much debt they can raise, either due to regulatory constraints or because the lending sector demands firms to have some participation in the risky asset's outcome. It is also assumed that there are direct bankruptcy costs that must be paid from the defaulted firms' assets, which lenders price accordingly.

The main difference between regimes is that when repo is exempt from stay secured lenders receive the asset and sell it immediately to solvent firms. This assumption is paramount to the model, but is easily motivated. Once the repo lender is made whole, any additional upside on the collateral is property of the borrower.[3] For simplicity, I assume that repo contracts are overcollateralized to such an extent that the loan is absolutely safe, meaning that this contracting feature eliminates any incentives for the lender to hold the asset after the firm defaults. This makes the asset supply insensitive to the fire sale. Therefore the market clearing price for the collateral will depend on solvent firms' ability to raise funds and defaulting firms' initial asset holdings, potentially creating a market clearing price below the asset's expected future payoff, i.e. a cash-in-the-market pricing.

An important element of the model is the lack of initial firm capital committed to the market. This is the result of assuming market segmentation that only allows a fixed set of firms with limited capital to partic-

[2] He et al. (2010) present evidence of this during the 2007-09 financial crisis.

[3] This contracting feature is present in standard Master Repurchase Agreements.

ipate in the market initially. Market segmentation can arise because of a specific expertise needed to trade in the risky asset market, which only a limited number of firms possess. This also implies that more capital does not flood into the market after firms default, which could result from an unmodeled lemons problem that outside investors would suffer by participating in a market unfamiliar to them. Potentially because of slow moving capital, it takes time for new investors to come into the market, leaving only the original participants to take advantage of the price discount.[4] In the stay regime, since there is no "forced selling" of the asset, there is no fire sale. In that case, the asset price is set to its fundamental value, eliminating incentives for firms to trade amongst themselves.

More generally, since repo's exemption from stay places more of the asset in the hands of firms that risk default, there are higher default costs compared to the stay regime; though firms' private benefits are also larger. This paper also explores potential policy prescriptions that, in the context of the model, would eliminate the fire sale in the stay exemption regime; namely an asset purchase program by the government or a contracting change to repo arrangements. A brief subsection is dedicated to a setup in which agents do not internalize the cash-in-the-market pricing, which increases uncertainty and the magnitude of the fire sale. A subsection also studies a setting with an inflow of initial firm capital, which effectively eliminates the fire sale and any distortions.

The rest of the paper is structured as follows. The following section presents a literature review, with a brief overview of the current debate over derivatives' and repos' exemption from automatic stay. Section 3 introduces the model setup, detailing the main market frictions driving the results. The following section presents firms' optimal strategies, and section 5 characterizes the resulting equilibria in both regulatory regimes. Having detailed the model's mechanics, section 6 goes back to the stay versus stay exemption debate to put the model into context with the existing literature, highlighting the types of markets to which this paper is most relevant to and giving anecdotal evidence for the main mechanism of the model. Section 7 presents the cost benefit analysis, some policy considerations, and extensions to the original model. The final section concludes.

2 Related Literature

The paper's relevant literature can be divided into two broad themes. The first is the literature solely focused on the question concerning the costs and benefits of derivatives' and repos' exemption from automatic stay.

[4] See Duffie (2010).

The other is a broader literature related to financial frictions, limited capital, and financial intermediary incentives.

2.1 Stay vs. Stay Exemption Literature

The debate as to whether the exclusion of derivatives and repos from normal bankruptcy proceedings can have an impact on the stability of the financial system has interested economic and law scholars alike. In essence, the exemption from automatic stay gives these contracts higher priority in bankruptcy since it allows creditors to take immediate ownership of any collateral posted by the defaulted party. In contrast, counterparties/creditors of standard contracts must obtain approval from a bankruptcy court to collect any payments due, which may take time and effort. The notion behind automatic stay is to stop any inefficient dismantling of the defaulted party; that is, to conserve the value of the firm as a going concern. Duffie and Skeel (2012) provide a detailed discussion of the role of stay exemption and the main trade offs in the debate.

Interestingly, one of the initial reason behind the exclusion of derivatives and repos from stay was to increase financial stability.[5] The idea was that the collapse of a large debtor could have adverse consequences on it's counterparties, given the uncertainty of existing contracts between them. Be it the inaccessibility to any collateral posted by a debtor, or the effective protection of a particular hedging strategy with a failing counterparty, the stay may cause financial problems for the non defaulting agent. That is, because of the uncertainty embedded in the bankruptcy process the stay could potentially put the non defaulting party's solvency into question, creating a "domino effect".[6] The exemption from stay effectively reduces counterparty risk which was claimed to mitigate incentives for creditors to run, increase firms ability to rely on hedges, and free securities which could be efficiently used elsewhere rather than being trapped in a bankruptcy process.

The financial crisis highlighted some of the costs that the stay exemption of derivatives and repos have on firms during periods of market stress. Skeel Jr. and Jackson (2012) provide evidence from three of the most important episodes of the 2008 crisis: AIG, Bear Stearns, and Lehman Brothers. These authors argue that the (near) collapse of these institutions showed that exemption from stay reduced firms' incentives to file for bankruptcy in the first place. This is because part of the benefits of filing for bankruptcy is the safety provisions it provides, thus exclusions from automatic stay reduces incentives to do so. In addition, the crisis put into question the true effectiveness of the stay exemption to mitigate runs. Given the high degree

[5] See Skeel Jr. and Jackson (2012) for a discussion on the rationale behind the exclusion of these contracts from automatic stay.

[6] A version of this effect will be studied in Section 7.3.

of short term funding by some institutions, a bank run style phenomena manifested itself through lenders refusal to roll over contracts, eliminating any benefits from the stay exemption altogether.

There are also potential costs to stay exemption ex-ante. The reduction in counterparty risk associated with a stay exemption reduces firms incentive to monitor, which is the main point expressed by Roe (2011). It is unclear though why derivative and repo counterparties would be in a better position to monitor firms relative to others. It has also been argued that repo's special tretment in bankruptcy may cause firms to substitute away from traditional sources of financing since exemption from stay makes repurchase agreements safer. Although this argument is attractive it must also provide a reason as to why the firms total financing cost would be reduced. Essentially, the riskiness of the firm is borne by all it's creditors and if repos' priority reduces repo funding costs it would presumably be at the expense of other financing vehicles. Moreover, even if the firms total financing cost were reduced, it would have to be argued why firms increased reliance on repo funding would be an undesirable outcome.

An ex-ante view of firms incentives under both regimes is studied by Bolton and Oehmke (2011) in the context of a firm engaging in derivative contracts to hedge a portion of its operational risk. The authors study the effects of "super-seniority" and it's impact on a firm's debt claimants. They show that stay exemption creates incentives for firms to shift risk from derivative counterparties to debt holders, which in their model is inefficient. The main difference with my model is that I consider an economy-wide setup in which the price of the underlying asset is determined endogenously and can deviate from fundamentals because of the stay exemption.

Specifically, the focus of this paper is to study a particular externality that could arise from a mass default event, that would induce an asset sell off and a dramatic reduction in prices, i.e. fire sale. A fire sale could have adverse consequences on other firms during periods of market stress, since the price decrease could put their solvency into question. In addition, the existence of a fire sale could also have a an impact on firms' financing decisions ex-ante, which is the primary exercise of this paper. Oehmke (2012) studies the strategic interaction between lenders attempting to sell illiquid collateral after the default of a borrower. In that model lenders have to tradeoff the speed at which they sell collateral taking into account the impact they have on prices. The main difference with this paper is that I analyze firms financing decisions ex-ante, when agents internalize the possibility of a fire sale and when default arises endogenously.

2.2 Other Related Literature

This paper is related to the vast literature on capital constraints and their effects on prices. From the original paper by Kiyotaki and Moore (1997) where hard collateral constraints limit agents' funding, to the more recent literature that studies how different manifestations of capital constraints can influence intermediaries decisions (for example Gromb and Vayanos (2002), Brunnermeier and Pedersen (2009), and He and Krishnamurthy (2012)); many academics have characterized how financial constraints affect asset prices. An overview of different varieties of capital constraints and their consequences is provided by Krishnamurthy (2010). The recurring theme is how particular balance sheet constraints affect intermediaries capacity to trade assets, limiting their ability to exploit arbitrage opportunities, and allowing prices to deviate from fundamentals. Along the same lines recent research explores financial firms' financing decisions, in terms of maturity (Brunnermeier and Oehmke (2012)) and debt capacity (Acharya, Gale and Yorulmazer (2011)). This paper will adopt a similar type of funding constraints in the context of repo's exemption from automatic stay.

Another relevant strand of the literature is provided by the seminal work of Allen and Gale (1994), (1998), and (2005), which illustrates how agents ability to supply capital affects pricing. The mechanism, commonly referred to as cash-in-the-market pricing adopts the notion that an asset's price is the minimum between its fundamental value and the market clearing price induced by demand and supply, which could be restricted. That is, the price may differ from it's correct valuation because of limited demand to purchase it. This mechanism is key in my model: firms' limited capital is the source of the fire sale which has an impact on prices ex-ante. In effect, a firm's ability to purchase the asset at fire sale prices will reduce it's willingness to participate in the market before the fire sale occurs, limiting the amount of capital committed initially, resulting in a potential price discount.

This paper is also related to the literature on fire sales which are triggered when high valuation agents are forced to sell assets to low valuation buyers (for example Shleifer and Vishny (1992), (1997)). Stein (2012) present's a setup where banks create too much private money, which forces them to sell assets at fire sale prices. This attracts capital that could be used more efficiently in other investment projects. Acharya, Shin and Yorulmazer (2011) studies a mechanism similar to the one present in this paper: future fire sales create incentives for firms to hoard liquidity ex-ante. The main difference with this paper is that in my setting firms initial leverage choice is endogenous, lenders price the possibility of a bankruptcy, and repo lenders have clear incentives to sell the asset upon default. Moreover, the model in this paper considers a single risky asset that's priced through market clearing, which can create price distortions that motivate other firms to take on excessive leverage.[7] [8]

Geanakoplos (2010), Fostel and Geanakoplos (2011), and Simsek (2012) analyze the optimal contracting between parties with different beliefs in a general equilibrium framework. In these models agents disagree on the possible payoffs of an asset, and trade contracts and assets between them. They characterize the optimal contract traded in equilibrium under different scenarios in which risky and safe contracts can arise endogenously. The model in this paper considers a scenario where a subset of agents with limited capital will participate in a risky asset market, but they will receive funding from an unconstrained sector. Collateral constraints will limit the flow of capital and cause prices to deviate from fundamentals. This setup is inspired by the interaction between financial institutions participating in a relatively illiquid market that receive financing from agents with "deep pockets" via exogenously specified contract (i.e. money market funds in repo markets).[9]

The baseline model is inspired by the variable investment model of Holmstrom and Tirole (1997). Although the context is considerably different, the firm's problem in both papers have a similar structure: maximize the payoff of a residual claim, with potential private benefits, subject to linear funding restrictions. The linear setup in this model is particularly useful since it gives relatively simple characterizations of firms' demand and simplifies the exercise of clearing markets.

3 Model Set Up

The model consists of three periods $t \in \{0, 1, 2\}$. A continuum of firms raise one period loans from a competitive lending sector to finance their position in a single risky asset. Firms avoid default if the amount of cash they receive from the asset, or from additional funding, are sufficient to pay off existing creditors.

3.1 Asset

There is an initial fixed supply K of one risky asset, which can be purchased by firms, and its price is determined by market clearing whenever there is trade. The asset produces three possible cash flows in the final period: $\{C^H, C^{LH}, C^{LL}\}$, with $C^H > C^{LH} > C^{LL}$. After period zero, with probability α the asset has

[7] Shleifer and Vishny (2011) provide a complete overview of the existing fire sale literature.

[8] Stein (2012) also has endogenous prices, but they are determined by outside patient investors not present at the beginning of the game.

[9] More details on which market this paper is more relevant for is given in section 6

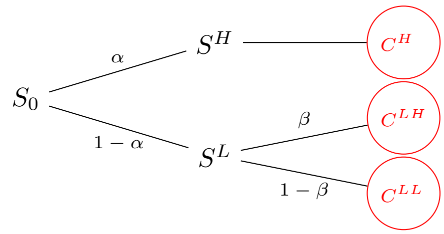

Figure 1: Risky Asset Process

a high outcome and will pay off a cash flow of C^H for sure in $t = 2$. With probability $1 - \alpha$ the asset has a low outcome and has some residual uncertainty: with probability β it will pay off C^{LH} and with probability $1 - \beta$ it will payoff C^{LL}. Thus, after a low outcome the expected future cash flow of the asset is denoted by $C^L = \beta C^{LH} + (1 - \beta)C^{LL}$. The initial expected cash flow of the asset is denoted by $\overline{C} = \alpha C^H + (1 - \alpha)C^L$. Final cash flows are received by firms and can be distributed among their claimants to settle contracts. The initial market clearing price is denoted by S, and the price will be denoted by S^H or S^L in the high and low outcome, respectively (see Figure 1). There is also a numeraire safe asset called "cash" and all agents in the economy have access to a storage technology, which will allow firms to deposit their initial cash endowments (i.e. dry powder), with a gross interest rate normalized to 1.

3.2 Agents

There are two types of agents in this economy: firms and lenders. The main participants in the model are a continuum of risk neutral firms who have a fixed initial endowment A^I and raise funds from a competitive lending sector to finance their risky asset position Q_t. All firms are risk neutral, but I assume a fraction θ also enjoy private benefits b for holding risky asset positions proportional to their size in the initial period. Private benefits can be interpreted as perks that firm managers enjoy for hold large risky asset positions, an unmodeled convenience yield perceived by these firms proportional to the portfolio size, or optimism on the final cash flow of the asset.[10] This last interpretation can involve private benefit firms believing the high asset outcome will be $\hat{C}^H$ with $\hat{C}^H - C^H = \frac{b}{\alpha}$, and agree to disagree with other agents in the economy. In this setting, if asset has a high payoff these firms update their beliefs, eliminating their optimism for the final period.The nature of these private benefits is not important. In essence it is a modeling device to ensure that a fraction of firms have incentives to increase their risky asset position by taking on risky debt which will be assumed to be costly.[11] Aggregate demand of private benefit and non private benefit firms at time t

[10] Alternative assumptions on the functional form of private benefits have also been explored. For example, that they are only perceived for the initial asset purchase but enjoyed in all periods, or that they are enjoyed in both periods for all the firm's risky asset holding. Each version employs slightly different parametric assumptions and additional restrictions, but the results are identical to this base setup: private benefit firms have incentives to take on risky levered positions initially.

[11] Details of these costs will be given in subsection 3.4.3.

are denoted $\overline{Q}_t^{PB}$ and $\overline{Q}_t^{NPB}$ respectively.

An important assumption in the model is that firms are the only agents in the economy who have the ability to initially participate in the risky asset market. This assumption will be paramount to induce a fire sale in the stay exemption regime: the asset's market clearing price will depend on solvent firms ability to raise more debt. This can be motivated by a prior participation decision that gave these firms the expertise to trade in the risky asset market. In the low state, because of slow moving capital, only firms that are already participating in the market can react to the fire sale return. An additional reason could be that in a crisis state only firms participating in the market beforehand would be willing to purchase the asset because of a potential lemons problem.

Lenders are risk neutral and their role is to provide firms with funds to increase their portfolio size beyond what they could purchase with their initial endowment. Lenders have no capital constraints, and they are split into two types: secured and unsecured. Secured lenders demand collateral as protection in case a firm defaults, which earmarks a fraction of the firm's asset position to which they have first lien on. Unsecured lenders have a claim to the firm's remaining assets. Lenders cannot purchase the risky asset directly, and will sell the asset if received before cash flows are realized. This last assumption is rather important in the model: in case of default, secured lenders' supply of the asset is insensitive to the market clearing price. In the model this behavior is optimal given the nature of the assumed repo contracts, since any gain above and beyond the promised debt payment is property of the original borrower. Thus, depending on the degree of overcollateralization (which will be assumed to give full protection) lenders who receive the asset upon default have little or no incentive to hold it till maturity.

3.3 Debt Contracts

Lenders use one-period secured and unsecured loans to satisfy firms' demand for funds. Secured contracts set aside a fraction of the firm's assets as protection in case of default. It is assumed that the only secured lending contracts transacted are absolutely safe ones, thus the loan repayment must be covered by the worst possible outcome of the asset (i.e. fully overcollateralized). This assumption avoids having to determine the haricut/interest rate trade off, simplifying the terms of trade between repo lenders and borrowers.[12] Alternatively, it could be assumed that secured lenders are relatively more risk averse and it is in the borrower's best interest to only accept absolutely safe loans. This implies that the initial amount of secured funding

[12] Martin et al. (2012) adopt a similar assumption for repo transactions, and in the general equilibrium framework of Geanakoplos (2010) the safe debt contract is the only one traded in equilibrium. This assumption is in line with the notion that repo loans are "relatively" safe investments.

that a firm can raise will differ under the two regimes, which will be detailed below.

The nature of a firms direct default costs (to be elaborated on in the following subsection), will make firms prefer secured funding before they take on any unsecured debt. I will assume an exogenous upper bound on the fraction of the risky asset position which can be earmarked to secured lenders $\overline{\varphi}$, to capture the notion that not all of the firm's risky asset position can be pledged to raise repo funding. [13] Labeling φ_t the fraction of the portfolio earmarked for secured lending and P_t^i as the worst case payment to secured lenders (which depends on regime $i \in \{\mathcal{S}, \mathcal{SE}\}$) the initial secured loan amount and repayment is

$$\varphi_t P_t^i Q_t.$$

Unsecured lenders set their lending terms to break even each period, internalizing secured lenders priority on earmarked assets and any potential deadweight loss. This additional funding channel allows intermediaries to increase their debt capacity, taking on positions that could make them insolvent in the following period. Denoting I_t^u the initial loan amount of unsecured secured funding, F_t^u the promised payment in the next period, D^i the value of the firm in case of default, and $1 - \lambda < 1$ the deadweight loss from liquidating any remaining dry powder, the lender solves

$$I_t^u = \mathbb{P}(\text{no default})F_t^u + (1 - \mathbb{P}(\text{no default}))\left(D^i Q_t + \lambda(A_t^I - A_t) - \varphi_t P_t^i Q_t\right) \tag{1}$$

where A_t^I is the amount of dry powder the firm holds at the beginning of period t. To ensure there is enough value in the firm's portfolio to make secured creditors whole, I need to impose $D^i Q_t + \lambda(A_t^I - A_t) - \varphi_t P_t^i Q_t \geq 0$.

The model also incorporates capital constraints that limit the amount of leverage firms can adopt. This can be interpreted as firms' capital requirements, or an unmodeled "skin in the game" constraint to ensure that firms' behave in the lender's best interests.[14] The constraint which creates an upper bound on the amount of debt firms can raise will depend on the asset's price. In the model capital requirements at time t take on the following form,

$$I_t^u + \varphi_t P_t^i Q_t \leq \eta(S_t^i Q_t + A_t^I - A_t) \tag{2}$$

where $\eta \in (0, 1)$ is the capital requirement parameter and S_t^i is the period t market clearing price, which

[13] In a more general setup, where a firm's risky portfolio is more diverse, there may be reasons why a firm may not want to use all of it's financial assets as repo collateral. Perhaps because of limited pledgeability of some asset classes or the need to have immediate access to assets in the firm's portfolio.

[14] Although I don't specify the nature of firms bad behavior, this type of friction has been commonly used in the literature. For example Acharya, Shin and Yorulmazer (2011), Acharya and Skeie (2011).

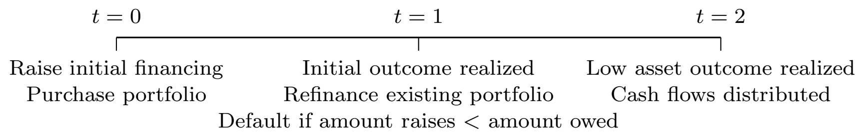

Figure 2: Timeline for Under Both Regimes

depends on regime i. Restriction (2) forces firms' equity to be at least $1 - \eta$ the value of the firm's assets. That is, the firm's debt capacity depends on the current period's market price. Therefore a depressed market value for the risky assets decreases the amount of financing a firm can raise. Note that the risky asset pays off in the final period, therefore firms must raise debt in $t = 1$ to payoff $t = 0$ claimants. That is, firms face roll over risk and the amount of financing will depend on their portfolio value because of capital constraints.

3.4 Stay vs. Stay Exemption: Timing & Defaults

The strategies and timing of the three period model depend on which regime is under consideration. In period 1 of the stay exemption regime a market for defaulted collateral opens, where secured lenders who receive defaulted collateral sell it to firms who have the capacity to buy it. In period 1 of the stay regime, when a firm defaults secured lenders must wait till the final period to receive their payment. This is the main difference between regimes. The exact timing of the model under both regimes can be summarized in Figure 2.

3.4.1 Stay Regime

In period 1 if a firm enters default, its claimants receive their payoff in the final period. Secured lenders get paid in full and unsecured lenders receive whatever is left in the firm. Therefore, in period 0, secured lenders lend up to

$$\varphi_0 C^{LL} Q_0$$

since in case of default they receive C^{LL} in the final period with certainty. Period 0 unsecured lenders, who lend to firms that risk default, solve

$$I_0^u = \alpha F_0^u + (1-\alpha)\left(D^{\mathcal{S}}(\varphi_0)Q_0 + \lambda(A_0^I - A_0) - \varphi_0 C^{LL} Q_0\right) \tag{3}$$

Period 1 unsecured contracts also take on this form with the appropriate change of subscript and where $A_1^I = A_0^I - A_0$. In that period surviving firms raise new debt to payoff period 0 creditors. In the final period firms receive the asset cash flow and payoff any outstanding claims.

3.4.2 Stay Exemption Regime

In period 1 if a firm defaults, secured creditors receive the asset and sell it immediately. In this case the asset supply is insensitive to the final market clearing price, but the market clearing price is internalized in the initial round of financing. Period 0 secured lenders lend up to,

$$\varphi_0 S^L Q_0$$

where S^L is the price of the low asset in period 1. Period zero unsecured lenders, who lend to firms that risk default, solve

$$I_0^u = \alpha F_0^u + (1-\alpha)\left(D^{\mathcal{SE}}(\varphi_0, S^L)Q_0 + \lambda(A_0^I - A_0) - \varphi_0 S^L Q_0\right) \tag{4}$$

Note that in period 1 secured creditors receive and sell their collateral, and solvent firms raise additional funds to purchase it. Market clearing between these agents will determine the period 1 price in the low state, denoted by S^L. The case when $S^L < C^L$ shall be considered a "fire sale", where firm's limited buying power induces a price that differs from the asset's expected future payoff.

3.4.3 Default Costs

The nature of firms' default costs are specifically related to direct costs of default. They represent the costs of litigation (experts, lawyers, etc.) to resolve disputes among claimants of a defaulted firm. They are not indirect costs due to the suboptimal use of an asset by a third party, nor are they costs associated to the firm as a going concern. In the model, there is no scope to restructure the firm since in the final period all uncertainty is resolved, the asset matures, and cash flows are distributed.

An important ingredient to the model is how these direct default costs affect firms under different regimes. In the stay exemption regime, collateralized assets leave the firm immediately, thus it can be argued that the cost to resolve these claims are relatively small.[15] This might not be the case for secured lenders in a stay regime, where disputes between various secured claimants could arise as to the specific asset to which they may be entitled to. To simplify the comparison between regimes, I will assume that in both regimes secured claims are resolved at no cost.[16] Although this is an extreme assumption, it will simplify the comparison of the total deadweight loss between regimes.

[15] Similar to leases in the context of Eisfeldt and Rampini (2009)

[16] Alternatively, a lump sum cost proportional to the size of the portfolio can also be assumed. Though this would not incorporate a preference for secured vs. unsecured funding, which would also have to be assumed.

Therefore in period 1, the default value of the firm in the stay regime takes on the following form,

$$D^{\mathcal{S}}(\varphi) = \varphi C^L + \lambda(1-\varphi)C^L$$

with $\lambda < 1$. Since in this regime there is no market for defaulted collateral the asset price is set to refelct fundamental value, which eliminates incentives for non constrained firms to trade. In the stay exemption regime the default value of the firm takes the following expression,

$$D^{\mathcal{SE}}(\varphi, S^L) = \varphi S^L + \lambda(1-\varphi)C^L.$$

Upon default $\varphi S^L Q_0$ go to secured creditors, leaving the reaming assets to bear the direct costs of default which is resolved in the following period.

An important consequence of this assumption is that secured funding will always be preferred in the stay regime since it induces lower default costs. This will also be true in the stay exemption regime as long as $S^L \geq \lambda C^L$, which creates a natural lower bound for the fire sale. In case the fire sale price were below the defaulted value of the asset, firms that take on risky positions would not be inclined to take on secured debt, leaving only unsecured debt holders and eliminating the fire sale altogether.

To simplify the analysis of agent's optimal strategies, and the resulting equilibrium, I shall assume there are no default costs in the final period. This dramatically reduces the number of cases to analyze in each regime since it simplifies solvent firms refinancing problem: in case of a fire sale, purchase as much of the asset as possible. Since there is no cost to default, firms will choose to raise as much as possible to take advantage of any price discount. One justification may be that it is less costly to resolve a bankruptcy proceeding involving matured assets, or alternatively one may think that the capital requirement bound is so strict that in the final period only safe debt can be raised (i.e. $\eta C^L < C^{LL}$). Both of these interpretations can be seen as extreme opposite cases which will impact the severity of the fire sale: easy resolution of mature assets increases debt capacity, mitigating the cash-in-the-market discount; or strict capital requirements limit funding, aggravating the mispricing.

In the remaining parts of the paper I adopt specific parameter assumptions to characterize agents optimal strategies and equilibrium outcomes. When introduced, the economic intuition behind them will be stated and the assumption will be expressed in economic terms. The precise parameter assumption will be detailed in the appendix.

4 Firm's Optimal Strategies

In each period solvent firms must decide how much debt to raise to refinance their existing risky asset position or finance a new asset purchase. This section characterizes the problem firms face and under what conditions they choose positions that may lead them to default (labeled *risky firms*), or avoid it altogether (labeled *safe firms*). It is important to note that there's no difference between secured and unsecured debt if firms' leverage doesn't put them at risk of default. Since there is no concern about a safe firm's bankruptcy cost, nor a change in the supply of defaulted collateral, unsecured and secured debt will turn out to be equivalent. All proofs are relegated to the appendix.

In addition, the refinancing problem in case of a high outcome in period 1 is straight forward. Firms do not default and there is no open market for defaulted collateral, nor do private benefit firms have incentives to purchase the asset from non-defaulting firms since they only enjoy b in the initial period.[17] Firms just refinance their portfolio, raising enough capital to pay off previous creditors using the high valued asset as collateral. The problem is trivial and will not be analyzed. Thus, all discussions and characterizations of firms' optimal strategies in period 1 will be assumed to be after a low outcome in the initial period. For generality, the analysis will be done from the perspective of a firm that enjoys private benefits. Strategies and payoffs for non private benefit firms will involve setting $b = 0$.

4.1 Stay Regime

The analysis of the stay regime is relatively simple. In period 1 there is no open market to purchase defaulted collateral, thus surviving firms only need to decide how much debt to raise to pay previous creditors, i.e. roll over their debt. Conditional on the firm's current balance sheet $(Q_0, A^I - A_0, F_0)$, where Q_0 is the initial period's risky asset position, $A^I - A_0$ the remaining dry powder, and F_0 the total amount of debt to be paid in period 1, the firm must decide how much to raise to pay off F_0. The firm is restricted to raise new debt up to its capital requirement, and may also use its remaining dry powder to pay off it's previous debt. It is in the firm's best interest to exhaust any remaining dry powder it may have to payoff F_0, since keeping it would only limit the total amount of cash available.[18] Also note that since there are no default costs in the final period, the firm is not concerned with maximizing its debt capacity. Therefore, the firms will be able

[17] In this case the price equals the final cash flow, $S^H = C^H$ thus there is no incentive to trade.

[18] In effect, if the firm retained all it's dry powder it would only be able to raise $\eta(C^L Q_0 + A^I - A_0)$ to pay off F_0. It can use it's dry powder more effectively by directly paying off creditors with it.

to avoid default in period $t = 1$ if

$$F_0 \leq \eta C^L Q_0 + A^I - A_0. \tag{5}$$

Knowing the default condition in the refinancing period, one can postulate the firm's initial financing decision. Firms have the choice to take on small levels of debt that will allow them to refinance their position in $t = 1$ (safe firms), or they can maximize their debt capacity risking default in $t = 1$ (risky firms). In case a firm decides to become risky, lenders will internalize this risk charging a higher interest rate on the loan. Therefore using lenders' debt pricing equation (1) to get the appropriate expression for F_0, firms' payoff under both strategies can be characterized. Given the size of firms' risky asset position, the amount of secured and unsecured debt, and the amount of dry powder committed purchase the risky asset, firms have the following possible payoffs:[19]

$$\begin{aligned}
\pi^S(Q_0, A_0, I_0^u, \varphi_0; A^I, b) &= \overline{C}Q_0 + A^I - A_0 - \varphi_0 C^{LL} Q_0 - I_0^u + bQ_0 \\
\pi^R(Q_0, A_0, I_0^u, \varphi_0; A^I, b) &= \underbrace{[\alpha C^H + (1-\alpha)(\varphi_0 C^L + (1-\varphi_0)\lambda C^L)]}_{:=C^D(\varphi_0)} Q_0 + (\alpha + (1-\alpha)\lambda)(A^I - A_0) \cdots \\
&\quad -\varphi_0 C^{LL} Q_0 - I_0^u + bQ_0
\end{aligned}$$

Using equation (5) I define the no default set for this regime: $N^{\mathcal{S}} = \{(Q_0, A_0, I_0^u, \varphi_0) : I_0^u + \varphi_0 C^L Q_0 \leq C^L Q_0 + A^I - A_0\}$. That is, if $(Q_0, A_0, I_0^u, \varphi_0) \in N^{\mathcal{S}}$ the firm will be able to roll over it's debt in $t = 1$. Thus, firms must solve the following optimization problem,

$$\max_{\{Q_0, A_0, I_0^u, \varphi_0\}} \pi^S(Q_0, I_0^u, \varphi_0, A_0; A^I, b) 1_{\{(Q_0, A_0, I_0^u, \varphi_0) \in N^{\mathcal{S}}\}} + \pi^R(Q_0, I_0^u, \varphi_0, A_0; A^I, b) 1_{\{(Q_0, A_0, I_0^u, \varphi_0) \in (N^{\mathcal{S}})^c\}}$$

subject to,

$$\begin{aligned}
A_0 &\leq A^I && \text{Dry Powder} \\
SQ_0 &\leq \varphi_0 C^{LL} Q_0 + I_0^u + A_0 && \text{Budget} \\
I_0^u + \varphi_0 C^{LL} Q_0 &\leq \eta(SQ_0 + (A^I - A_0)) && \text{Capital Requirement} \\
& I_0^u, A_0, Q_0 \geq 0, \ \varphi_0 \in [0, \overline{\varphi}]
\end{aligned}$$

The payoff of the firms is clear, conditional on being safe or risky it is the residual claim holder of the asset plus any additional private benefits it may perceive. The first restriction limits the amount of dry powder the firm can use to purchase the asset. The second restriction is the firm's budget constraint, i.e. the amount it raises from lenders plus the amount of dry powder it uses must be enough to purchase the

[19] Details in the appendix

risky asset. Finally, the third restriction is the firm's capital requirement which depends on the amount of debt raises and the value of the risky asset position.

In this regime a firm's decision to become risky or safe depends on the size of its private benefits. Firms that have relatively large private benefits would be willing to pay for their expected default costs to maximize their asset holdings and enjoy more of said rewards. To ensure that firms which adopt risky strategies actually default after a low asset outcome, the firm's capital requirements can't be too restrictive. For low values of η firms would be unable to increase their debt capacity to violate (5), thus I assume

Assumption A1. *The capital requirement restriction η is larger than the defaulted value of the firm.*

Assumption A1 states that the maximum amount the firm can raise is larger than the default value of the asset. This assumption is more restrictive than what may be necessary, nevertheless it will ensure risky firms default in both regimes and allow to characterize tractable solutions. The solution to the firm's initial financing problem in this regime is,

Proposition 1. *In the stay regime if $S > C^L$ and A1 holds, there exists a function $b^*(S)$ such that if $b < b^*(S)$ the solution to the firm's initial financing problem is*

$$Q_0^* = \begin{cases} 0 & \text{if } S > \overline{C} + b \\ Q_0^{undet} & \text{if } S = \overline{C} + b \\ \frac{A^I}{S - \eta C^L} & \text{if } S < \overline{C} + b \end{cases}$$

where $Q_0^{undet} \in \left[0, \frac{A^I}{\overline{C} + b - \eta C^L}\right)$ is undetermined. If $b \geq b^*(S)$ the solution to the firms initial financing problem is,

$$Q_0^* = \frac{A^I}{S(1-\eta)}.$$

These strategies lead to the following payoffs,

$$\pi_0(S; b) = \begin{cases} A^I & \text{if } S \geq \overline{C} + b \text{ and } b < b^*(S) \\ \frac{(\overline{C} + b - S)A^I}{S - \eta C^L} + A^I & \text{if } S < \overline{C} + b \text{ and } b < b^*(S) \\ \frac{(C^D(\overline{\varphi}) + b - S)A^I}{S(1-\eta)} + A^I & \text{if } b \geq b^*(S) \end{cases}$$

The solution to the firm's investment and financing problem can be thought to be split in two. Conditional on the decision to become safe or risky, which will depend on the size of it's private benefits, the firm's problem is linear. For relatively low private benefits firms take a safe strategy, where they either refrain from investing, take out the maximum amount of debt that will allow them to roll over their position in $t = 1$, or are indifferent between these two polar strategies. The price at which safe firms are indifferent is called the *break even price* S^{BE} which will be a recurring theme throughout the paper. It is the threshold that determines firms participation in the initial period. For relatively large private benefits firms will prefer to maximize their risky asset position, assuming the cost of potential default. An important ingredient of the analysis is to characterize for what prices firms decide to take on safe or risky strategies which ultimately pins down the amount of default in the economy. This will be relevant in the stay exemption regime and will be specified in the equilibrium analysis.

4.2 Stay Exemption Regime

The main difference with firm's optimal strategy in the stay exemption regime is that after the initial asset uncertainty is resolved there is an open market for defaulted firms' assets. Thus, surviving firms not only have to refinance their existing portfolios, but must also raise additional funds to purchase defaulted collateral. Because of cash-in-the-market pricing, the price for the defaulted asset may be lower than the expected value of its future cash flows. The potential price discount has two effects: first it generates a profit for non defaulting firms, creating incentives to underinvest in the initial period. Secondly, because of mark-to-market pricing, firms capital requirements will limit their ability to raise debt potentially aggravating the price discount. The defaulted asset's price is denoted by S^L and the focus of the model will be in a context where $S^L \leq C^L$. A fire sale is said to occur when the inequality is strict.

Given the state of the firms balance sheet $(Q_0, A^I - A_0, F_0)$, in $t = 1$ firms must choose how much debt to raise to pay off existing claims and finance a new asset purchase. Since there are no default costs in the final period, if the asset trading price is below its expected future cash flow, firms will want to maximize their debt capacity to purchase as much of the asset that they can. This will be restricted by their capital requirements. Therefore for $S^L < C^L$ firms will solve,

$$\begin{aligned} S^L Q_1 &= \varphi_1 C^{LL}(Q_0 + Q_1) + I_1^u + A^I - A_0 - F_0 \\ I_1^u + \varphi_1 C^{LL}(Q_0 + Q_1) &= \eta S^L (Q_0 + Q_1) \end{aligned}$$

where the first equality is the firm's budget constraint and the second is its capital requirement restriction. Firms raise secured funding earmarking its existing and new asset holdings, and also raise unsecured debt. As in the stay regime, it is optimal for the firm to use any of its remaining dry powder to purchase the asset (or refinance it's position) since keeping it only lowers the total amount of cash available. Therefore, in case of a fire sale, a firm's asset purchase in $t = 1$ is

$$Q_1^* = \frac{\eta S^L Q_0 + A^I - A_0 - F_0}{S^L(1-\eta)}. \tag{6}$$

Note that $Q_1^* < 0$ implies the firm is unable to pay off existing creditors and therefore must default. In case $S^L = C^L$ firms will have no incentive to purchase the asset, their demand will be undetermined, and they will receive no additional payoff from their asset purchase in $t = 1$. A firm with $(Q_0, A^I - A_0, F_0)$ which can roll over it's debt obtains the following expected payoff in the refinancing period,

$$\pi_1(Q_0, A^I - A_0, F_0; b) = \begin{cases} C^L Q_0 + A^I - A_0 - F_0 & \text{if } S^L = C^L \\ C^L Q_0 + A^I - A_0 - F_0 + (C^L - S^L)Q_1^* & \text{if } S^L < C^L \end{cases}$$

Knowing the firm's optimal asset choice, and refinancing condition in $t = 1$, one can postulate the firm's initial financing decision. Similar to the stay regime, firms have the choice between taking on small levels of debt that allow them to refinance their position and increase their risky asset position in $t = 1$ (safe firms), or they can maximize their debt capacity risking default (risky firms). The important difference with the stay regime is that in case of a fire sale safe firms can purchase the asset below it's fundamental value. Safe firm internalize that if they use too much of their debt capacity in the initial period, they will reduce their ability to purchase defaulted collateral at a discount. Specifically, if they expect a large price discount in the refinancing period it may be optimal to withhold their initial dry powder rather than participate in the initial market. This will determine the nature of resulting equilibrium discussed in the following section.

Using the firm's payoff in the refinancing period, and lenders debt pricing equation (1) to get the corresponding expression for F_0, one can characterize the firm's initial expected payoff. In effect, given the firm's asset position, it's initial debt structure, and the amount of dry powder committed to the initial purchase, the firm's expected payoffs are expressed as:[20]

$$\begin{aligned}
\pi^S(Q_0,, A_0, I_0^u, \varphi_0; A^I, b, S^L) &= \overline{C}Q_0 + A^I - A_0 + (1-\alpha)(C^L - S^L)\left(\frac{\eta S^L Q_0 + A^I - A_0 - \varphi_0 S^L Q_0 - I_0^u}{S^L(1-\eta)}\right)\cdots \\
&\quad -\varphi_0 S^L Q_0 - I_0^u + bQ_0 \\
\pi^R(Q_0, A_0, I_0^u, \varphi_0; A^I, b, S^L) &= \underbrace{[\alpha C^H + (1-\alpha)(\varphi_0 S^L + (1-\varphi_0)\lambda C^L)]}_{:=C^D(\varphi_0, S^L)} Q_0 + (\alpha + (1-\alpha)\lambda)(A^I - A_0)\cdots \\
&\quad -\varphi_0 S^L Q_0 - I_0^u + bQ_0
\end{aligned}$$

[20]Details in the appendix

From equation (6) the firm's default condition can be deduced (whenever $Q_1^* < 0$) and I can define the no default set for this regime: $N^{\mathcal{SE}} = \{(Q_0, A_0, I_0^u, \varphi_0) : I_0^u + \varphi_0 S^L Q_0 \leq S^L Q_0 + A^I - A_0\}$. That is, if $(Q_0, A_0, I_0^u, \varphi_0) \in N^{\mathcal{SE}}$ the firms will necessarily be able to roll over it's debt in $t = 1$, which depends on S^L. This gives the following optimization problem for $t = 0$,

$$\max_{\{Q_0,\varphi_0,I_0^u,A_0\}} \pi^S(Q_0, I_0^u, \varphi_0, A_0; A^I, b, S^L)1_{\{(Q_0,A_0,I_0^u,\varphi_0)\in N^{\mathcal{SE}}\}}+\pi^R(Q_0, I_0^u, \varphi_0, A_0; A^I, b, S^L)1_{\{(Q_0,A_0,I_0^u,\varphi_0)\in (N^{\mathcal{SE}})^c\}}$$

subject to,

$$\begin{aligned} A_0 &\leq A^I && \text{Dry Powder} \\ SQ_0 &\leq \varphi_0 S^L Q_0 + I_0^u + A_0 && \text{Budget} \\ I_0^u + \varphi_0 S^L Q_0 &\leq \eta(SQ_0 + (A^I - A_0)) && \text{Capital Requirement} \\ & Q_0, I_0^u, A_0 \geq 0,\ \varphi_0 \in [0, \overline{\varphi}] \end{aligned}$$

The firm's payoff is the residual claim from the initial portfolio, their private benefits, and in case of adopting a safe strategy the payoff from purchasing more of the asset in the refinancing period. The firm's restrictions are the same as in the stay regime; a dry powder bound, the firm's budget constraint, and the firm's capital requirements. It is important to note that the default condition depends on the fire sale. In this setting secured funding will be preferred over unsecured funding if $S^L \geq \lambda C^L$. Otherwise the cost of using secured funding would be too high and firms would only choose to raise unsecured funding. Using the same parameter assumption as in the stay regime to ensure that firms that take on full leverage actually default, the solution to the firms initial financing problem in this regime is given by,

Proposition 2. *In the stay exemption regime if $S > S^L$, $S^L \in (\lambda C^L, C^L]$ and A1 holds, there exists a function $b^{**}(S, S^L)$ such that if $b < b^{**}(S, S^L)$ the solution to the firm's initial financing problem is*

$$Q_0^* = \begin{cases} 0 & \text{if } S > S^{BE}(S^L, b) \\ Q_0^{undet} & \text{if } S = S^{BE}(S^L, b) \\ \frac{A^I}{S - \eta S^L} & \text{if } S < S^{BE}(S^L, b) \end{cases}$$

where $Q_0^{undet} \in \left[0, \frac{A^I}{S^{BE}(S^L,b)-\eta S^L}\right]$ is undetermined and

$$S^{BE}(S^L, b) = \frac{\overline{C} + b + (1-\alpha)\left(\frac{C^L - S^L}{S^L(1-\eta)}\right)\eta S^L}{1 + (1-\alpha)\left(\frac{C^L - S^L}{S^L(1-\eta)}\right)}. \tag{7}$$

If $b \geq b^{**}(S, S^L)$ the solution to the firm's initial financing problem is,

$$Q_0^* = \frac{A^I}{S(1-\eta)}$$

These strategies lead to the following payoffs,

$$\pi_0(S, S^L; b) = \begin{cases} (1-\alpha)\left(\frac{C^L - S^L}{S^L(1-\eta)}\right) A^I + A^I & \text{if } S \geq S^{BE}(S^L, b) \text{ and } b < b^{**}(S, S^L) \\ \frac{(\overline{C}+b-S)A^I}{S-\eta S^L} + A^I & \text{if } S < S^{BE}(S^L, b) \text{ and } b < b^{**}(S, S^L) \\ \frac{(C^D(\overline{\varphi}, S^L)+b-S)A^I}{S(1-\eta)} + A^I & \text{if } b \geq b^{**}(S, S^L) \end{cases}$$

As in the stay regime, the solution to the firm's investment and financing problem can be thought to be split in two. The firm's decision to become risky or safe ultimately resides on the size of its private benefits. In case firms have low private benefits and the initial asset price is relatively high ($S > S^{BE}(S^L, b)$), the firm's payoff consists of its initial endowment and its ability to purchase the defaulted asset in case of a low outcome. This strategy can be considered a "dry powder" strategy - the firm refrains from participating in the initial period, using its initial endowment to raise debt and purchase defaulted collateral in case of a fire sale. Relatively high asset prices induce low private benefit firms to take on larger risky asset positions, though still avoiding default in $t = 1$. As in the stay regime, firms with high private benefit firms will opt to take a risky strategy to maximize their risky asset position.

It can be easily verified that $S^{BE}(S^L, b)$ is increasing in S^L. This implies that whenever there is a severe fire sale, firm's incentives not to pariticipate in the initial market increases, which is expressed as a lower break even price. Specifically, if safe firms opt to increase their initial debt capacity, their ability to purchase defaulted collateral would be reduced. Therefore, these firms demand compensation for forfeiting any potential profits in case of a fire sale, which manifests itself through a lower break even price. It is important to note that if $S^L = C^L$, $S^{BE}(S^L, b)$ is equal to $\overline{C} + b$. That is, if there is no fire sale, safe firms' strategies and payoffs in the first period are exactly the same as in the stay regime.

With this in mind, whenever $b = 0$, $S^{BE}(S^L, 0)$ can be re arranged to give a more intuitive expression. After some algebraic manipulation $S^{BE}(S^L, 0)$ solves the following equality

$$1 + \frac{(1-\alpha)(C^L - S^L)}{S^L(1-\eta)} = \frac{\overline{C}}{S} + \frac{(1-\alpha)(C^L - S^L)}{S^L(1-\eta)} \frac{\eta S^L}{S} \tag{8}$$

for S. The left hand side is the gross return of holding on to dry powder to purchase defaulted collateral at a fire sale price, and the right hand side is the return from investing in the initial period. Dry powder receives a gross interest rate of 1 plus the expected net profit of $C^L - S^L$ of purchasing $1/(S^L(1-\eta))$ of the defaulted collateral. The right hand side gives a gross return of $\overline{C}/S$ plus the expected net profit of $C^L - S^L$ of purchasing $1/(S^L(1-\eta))$ of the defaulted collateral. This last purchase is realized using $\eta S^L Q_0$ of the original asset as collateral, which has an initial costs S.[21] Thus, for a non private benefit firm to be indifferent between investing in $t=0$ or $t=1$ condition (8) must hold.

As in the stay regime, for high enough private benefits firms would be willing to assume the ex-ante default costs. The equilibrium of interest will be one with parameter values such that private benefit firms will prefer to take on risky strategies, which will be the source of the asset supply in case of default. In the following section conditions will be set so that in equilibrium private benefit firms choose a risky strategy over a safe one.

5 Equilibrium

This section turns to analyze the equilibrium outcome in both regimes. The main setting under consideration will have private benefit firms taking on risky positions in the first period, and at least a fraction of non private benefit firms holding on to spare capacity for the second. These two conditions are necessary to have interesting equilibrium in the stay exemption regime: default is needed to generate a supply and surviving firms need spare capacity to generate demand in $t=1$.

The initial risky asset supply K is fixed and set exogenously. Firms' demand, the risky asset supply, and the fraction of private benefit firms θ pins down market clearing and the nature of the equilibrium. The relative size of the firm's initial endowment compared to the asset supply will be paramount to determine prices, especially to induce a fire sale in the stay exemption regime. The focus will be in a setting in which the stay regime's initial asset price is equal to its expected future cash flows. This is an economically reasonable starting point: with no forced sale in the refinancing period to cause a fire sale, demand and supply are such that prices reflect fundamental values. In the stay exemption regime, the same setting leads to a very different equilibrium outcome since the potential fire sale in the low state alters firms investment and financing decisions, increasing default costs by putting more of the asset in the hands of firms that adopt risky strategies.

The equilibrium concept employed in the model is a standard Walrasian equilibrium. Specifically, given

[21] The size of the defaulted asset purchase can be appreciated in the expression of Q_1^* in equation (6).

an initial endowment A^I, asset supply K, and a fraction θ of private benefit firms, an equilibrium is given by prices such that firms adopt optimal strategies and markets clear. The difference between regimes is that in the stay regime firms' optimal strategies are characterized by Proposition 1 and market clearing is only imposed in $t = 0$. In the stay exemption regime firms' strategies are given by Proposition 2 and market clearing is imposed in $t \in \{0, 1\}$. All proofs and parameter assumptions of this section are relegated to the appendix.

5.1 Stay Regime

The equilibrium of interest for this regime has $S = \overline{C}$. If private benefit firms adopt a risky strategy, from Proposition 1 firms' individual demands are given by,

$$Q_0^{PB} = \frac{A^I}{\overline{C}(1-\eta)}, \quad Q_0^{NPB} \in \left[0, \frac{A^I}{\overline{C} - \eta C^L}\right)$$

where Q_t^{PB} and Q_t^{NPB} denote private benefit and non private benefit firms' individual demand in t. Given non private benefit firms indifference when $S = \overline{C}$ their individual demand cannot be precisely specified. But integrating over all agents, and imposing market clearing, the resulting aggregate demand can be deduced. Thus, the initial fixed supply of the asset K that results in an equilibrium with $S = \overline{C}$ is

$$K \in \left[\frac{A^I\theta}{\overline{C}(1-\eta)}, \frac{A^I\theta}{\overline{C}(1-\eta)} + \frac{A^I(1-\theta)}{\overline{C} - \eta C^L}\right) := [K_{min}, K_{max})$$

which corresponds to private benefit firms taking on risky strategies and $\overline{Q}_0^{NPB} \in \left[0, \frac{A^I(1-\theta)}{\overline{C}-\eta C^L}\right)$. In effect, for $K < K_{min}$ private benefit firms would still choose the maximum leverage allowed, increasing the asset price above $\overline{C}$. For $K > K_{max}$, non private benefit firms would take on the maximum amount of leverage (though still avoiding default), reducing the asset price below $\overline{C}$.

To make sure private benefit firms adopt a risky strategy, the size of their position and magnitude of their benefits must compensate the cost of default. Specifically, from Proposition 1, private benefits must be set so that $b > b^*(\overline{C})$. This leads to the following theorem,

Theorem 1. *If parameter assumption A1 holds and $K \in [K_{min}, K_{max})$, then there exists a $b^* > 0$ such that $\forall b > b^*$ private benefit firms take on risky positions and non private benefit firms adopt safe strategies. In this setting, there exists a stay regime equilibrium with $S = \overline{C}$.*

Theorem 1 presents the benchmark setting to which the comparison of both regimes will take place. The initial capital committed to the market A^I, the market composition θ, and the supply of the asset K are such

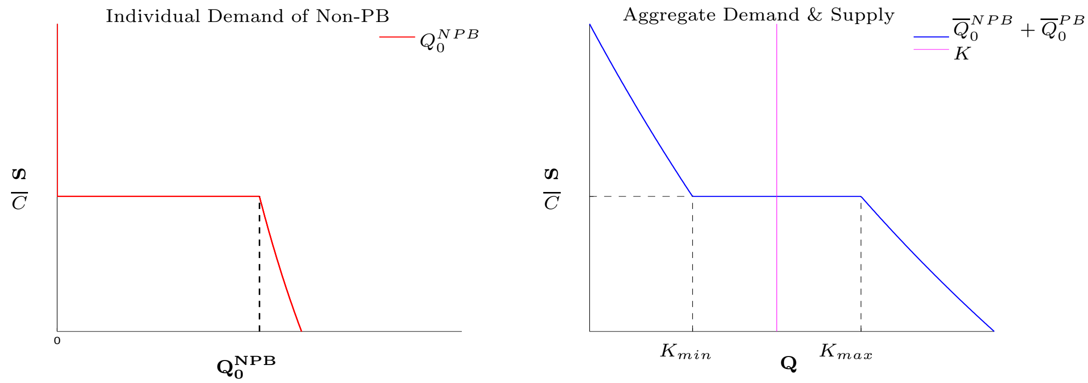

Figure 3: Stay Regime optimal strategies and equilibrium outcome - left hand side plot show the individual demand of a non private benefit firm in the stay regime when $S = \overline{C}$. Right hand side shows firms aggregate demand, and the relevant range where the asset supply implies prices equal to fundamental value. $A^I = 1, C^H = 110, C^L = 70, \alpha = .9, \lambda = .5, \eta = .75, \varphi = .5$.

that prices reflect fundamental values. Figure 3 shows non private benefit firm's individual demand and the resulting equilibrium in the stay regime. The following section will show that in the stay exemption regime prices deviate from fundamental values because capital must also be allocated to the refinancing period to purchase defaulted collateral.

5.2 Stay Exemption Regime

In this regime market clearing has to be imposed in both the initial and refinancing period. In the refinancing period, the supply of defaulted assets comes from secured lenders who received collateral from bankrupt firms and demand stems from solvent firms raising additional funds to purchase those assets.

The special feature of this regime is the possibility of a fire sale, which creates a premium for having spare capacity in period $t = 1$. When firms decide not to exhaust their debt capacity in $t = 0$ they have the potential to purchase the asset at a discounted price in period $t = 1$, increasing their expected profits. This premium for liquidity creates a reduction in the break even price for non private benefit firms to participate in the market initially: firms must be compensated in the initial period to give up the additional upside of holding on to dry powder.

Depending on the severity of the subsequent fire sale, the price discount in period $t = 0$ could also induce some non private benefit firms to take on risky positions initially. That is, the initial price discount could be large enough to compensate non private benefit firms to bear the cost of a potential default. This would give rise to an equilibrium where a fraction of non private benefit firms adopt a safe strategy and the remainder take a risky one. Considering the payoff functions characterized in Proposition 2, the indifference condition $\pi_0^S(S, S^L; 0) = \pi_0^R(S, S^L; 0)$ equates to, [22]

$$1 + (1-\alpha)\frac{C^L - S^L}{S^L(1-\eta)} = 1 + \frac{C^D(\overline{\varphi}, S^L) - S}{S(1-\eta)} \tag{9}$$

The left hand side of condition (9) is the gross expected return of holding on to dry powder to purchase the defaulted asset at a fire price. The right hand side it the expected return from taking on a levered position that results in default if the asset has a bad outcome. The additional leverage stemming from a depressed price is enough to compensate the loss from default and the missed opportunity of earning a fire sale profit. The asset price that satisfies this condition will be denoted S^M: if $S > S^M$ non private benefit firms will only undertake safe strategies, if $S = S^M(S^L)$ there will be mixing where a fraction ξ of these firms will take a risky strategy.

Under the assumption that private benefit firms adopt a risky strategy in the initial period, Proposition 2 characterizes their individual demand in $t = 0$. Integrating over all private benefit firms, their aggregate demand in the first and second period is

$$\overline{Q}_0^{PB} = \frac{A^I\theta}{S(1-\eta)}, \quad \overline{Q}_1^{PB} = -\frac{\overline{\varphi}A^I\theta}{S(1-\eta)} \tag{10}$$

where the negative sign in $\overline{Q}_1^{PB}$ indicates these firms are supplying the asset in the second period. Note that only $\overline{\varphi}$ of private benefit firms' risky asset position is supplied in $t = 1$, i.e. the fraction earmarked to secured lenders.

Conditional on S non private benefit firms have three potential strategies they could undertake in $t = 0$. They could take a relatively small position leaving some debt capacity for the refinancing period, adopt a fully levered position risking default in the down state, or refrain from investing altogether leaving all of their endowment to take advantage of the fire sale. These strategies, characterized by Proposition 2, lead to four potential aggregate strategies:

1. **Dry Powder Strategy (DP):** only private benefit firms participate in the initial period and non private benefit firms hold on to dry powder for the next. Non private benefit firms' aggregate demand is given by,

$$\overline{Q}_0^{NPB} = 0, \quad \overline{Q}_1^{NPB} = \frac{A^I(1-\theta)}{S^L(1-\eta)} \tag{11}$$

[22] In terms of Proposition 2 this occurs whenever $b^{**}(S, S^L) = 0$.

which occurs whenever $S > S^{BE}(S^L;0)$ and $S > S^M(S^L)$

2. **Break Even Strategy (BE):** private benefit firms and non private benefit firms participate in the initial asset purchase, but the latter maintain spare capacity to purchase the defaulted asset in $t = 1$. Non private benefit firms' demand aggregate is given by,

$$\overline{Q}_0^{NPB} = \overline{Q}_0^{undet} \in \left[0, \frac{A^I(1-\theta)}{S-\eta S^L}\right), \quad \overline{Q}_1^{NPB} = \frac{A^I(1-\theta) - (S-\eta S^L)\overline{Q}_0^{NPB}}{S^L(1-\eta)} \tag{12}$$

which occurs whenever $S = S^{BE}(S^L;0)$ and $S > S^M(S^L)$

3. **Dry Powder Mixing Strategy (DPmix):** private benefit firms and a fraction ξ of non private benefit firms take on risky positions. The remaining non private benefit firms hold on to dry powder to purchase the defaulted asset in the following period. Non private benefit firms' aggregate demand is given by,

$$\overline{Q}_0^{NPB} = \frac{A^I(1-\theta)\xi}{S(1-\eta)}, \quad \overline{Q}_1^{NPB} = \frac{A^I(1-\theta)(1-\xi)}{S^L(1-\eta)} - \frac{\varphi A^I(1-\theta)\xi}{S(1-\eta)} \tag{13}$$

which occurs whenever $S > S^{BE}(S^L;0)$ and $S = S^M(S^L)$

4. **Break Even Mixing Strategy (BEmix):** private benefit firms and a fraction ξ of non private benefit firms take on risky positions. The remaining non private benefit firms maintain spare capacity to purchase the defaulted asset in the following period. Non private benefit firms' aggregate demand is given by,

$$\overline{Q}_0^{NPB} = \overline{Q}_0^{undet} + \frac{A^I(1-\theta)\xi}{S(1-\eta)} \quad \text{with} \quad \overline{Q}_0^{undet} \in \left[0, \frac{A^I(1-\theta)(1-\xi)}{S-\eta S^L}\right) \tag{14}$$

$$\overline{Q}_1^{NPB} = \frac{A^I(1-\theta)(1-\xi) - (S-\eta S^L)\overline{Q}_0^{undet}}{S^L(1-\eta)} - \frac{\varphi A^I(1-\theta)\xi}{S(1-\eta)} \tag{15}$$

which occurs whenever $S = S^{BE}(S^L;0)$ and $S = S^M(S^L)$

5.2.1 Dry Powder - Dry Powder Mixing Strategy

In this sub section, the focus will be on a parameterization that gives rise to a simple and tractable equilibrium outcome: where non private benefit firms adopt a dry powder and a dry powder mixing strategy. In both these cases it isn't necessary to characterize non private benefit firm safe demand in $t = 0$, which reduces the number of equilibrium variables and simplifies the analysis. The main conclusions of the model will be illustrated for these specific equilibrium outcomes, but the results will not depend on the parameter assumptions adopted herein.

To compare the equilibrium of this regime to the benchmark case characterized in Theorem 1, where asset prices reflect fundamental values, I assume $K \in [K_{min}, K_{max})$. The main difference between regimes characterized in this paper stems from an environment where the potential shortage of capital in $t = 1$ induces a fire sale in the down state. To that effect, the fraction of private benefit firms is set such that firm demand in both periods is not enough for asset prices to reflect fundamental values. Thus, it will be assumed that θ is set such that for $K = K_{min}$ private benefit firm capital is enough so to ensure that $S = \overline{C}$, and non private benefit firm capital is sufficient for $S^L = C^L$. In other words, for the minimum asset supply non private benefit firms adopt a dry powder strategy, and their demand is high enough for prices to reflect the collateral's fundamental value in the down state. Arguably, this is a rather extreme case, since any increase in asset supply will entail a shortage of capital in the refinancing period yet it is useful to illustrate the model's main result. Accordingly, parameter assumptions are specified such that,

Assumption A2. *The fraction of private benefit firms θ is such that non private benefit firms take a DP strategy for $K = K_{min}$, and market clearing prices are $S = \overline{C}$ and $S^L = C^L$.*

For relatively low amounts of asset supply firms will take a dry powder strategy. For larger amounts of K, the initial asset price will be lower having the potential to induce some non private benefit firms to undertake risky strategies, i.e. dry powder mixing strategy. To ensure that non private benefit firms choose to not participate safely in $t = 0$, which is assumed for tractability, parameters must be set so that $S > S^{BE}(S^L, 0)$.

Assumption A3. *Non private benefit firms do not take a BE strategy: $S > S^{BE}(S^L, 0)$.*

As in the analysis of the stay exemption, conditions must be provided so that private benefit firms do in fact choose a risky strategy in $t = 0$. The same style of conditions must be applied in this setting; that is private benefit firms must perfer risky strategies over safe ones, whatever they may be. This is guaranteed if firms private benefits satisfy $b > b^{**}(S, S^L)$ of Proposition 2 for all (S, S^L) in equilibrium.

Note that a necessary condition of Proposition 2 if that the fire sale price is larger than the default value of the asset: $S^L > \lambda C^L$. This condition incentivizes risky firms to exhaust their secured funding first which should hold for the lowest fire sale price,

Assumption A4. *For a dry powder strategy $S^L > \lambda C^L$ for all $K \in [K_{min}, K_{max}]$.*

Finally, the interest of this subsection is to consider a setting in which S is low enough to induce some non private benefit take on risky positions, i.e. $S = S^M$. Thus I assume,

Assumption A5. *The Dry Powder price S is equal to S^M for $K^M \in [K_{min}, K_{max}]$.*

The aforementioned assumptions lead to the following result,

Theorem 2. *If parameter assumptions A1 - A5 hold then there exists* $b^{**} > 0, K^M$*, and* $K^{\mathcal{SE}}_{max} \leq K^{\mathcal{S}}_{max}$ *such that* $\forall b > b^{**}$ *private benefit firms take on risky strategies. For* $K \in [K_{min}, K^M]$ *non private benefit firms adopt a DP strategy, and for* $K \in (K^M, K^{\mathcal{SE}}_{max})$ *non private benefit adopt a DPmix strategy.*
In this setting, for $K \in [K_{min}, K^M]$*, there exists a stay exemption regime equilibrium with*

$$\begin{aligned} S &= \frac{A^I \theta}{K(1-\eta)} \\ S^L &= \frac{1-\theta}{\overline{\varphi}\theta} S. \end{aligned}$$

For $K \in (K^M, K^{\mathcal{SE}}_{max})$*, there exists a stay exemption regime equilibrium with*

$$\begin{aligned} S &= \frac{A^I(\theta + \xi(1-\theta))}{K(1-\eta)} \\ S^L &= \frac{(1-\theta)(1-\xi)}{\overline{\varphi}(\theta + \xi(1-\theta))} S \\ (1-\alpha)\left(\frac{C^L}{S^L} - 1\right) &= \frac{C^D(\overline{\varphi}, S^L)}{S} - 1 \end{aligned}$$

where $\xi > 0$ *is the fraction of private benefit firms that take on risky strategies.*

The equilibrium characterized in Theorem 2 has non private benefit firms take a dry powder strategy for relatively low levels of asset supply, and a dry powder mixing strategy for higher levels. This highlights one of the main results of the paper: the stay exemption can induce more firms to adopt risky strategies. This is caused by limited capital committed to purchase defaulted assets in period $t = 1$, resulting in a relatively large fire sale. The price distortion in $t = 1$ impacts pricing in $t = 0$, altering firms decisions, inducing more risk taking.

5.2.2 Break Even - Break Even Mixing Strategy: Numerical Exercise

It can be shown that similar conclusions result from alternative setups with different equilibrium outcomes. Many of the previously parameter assumptions can be dismissed, which were only adopted for tractability; but others are important for fire sales to manifest themselves and for additional risk taking to arise. In particular, it is necessary to have a shortage of capital in the refinancing period, which was adopted by assumption A2. Maintaining this assumption, and ensuring that private benefit firms prefer levered strategies in $t = 0$, Figure 4 shows the resulting first period price, fire sale price, and fraction of non private benefit firms that take a risky strategy.

For relatively low levels of asset supply, non private benefit firms invest some of their capital initially,

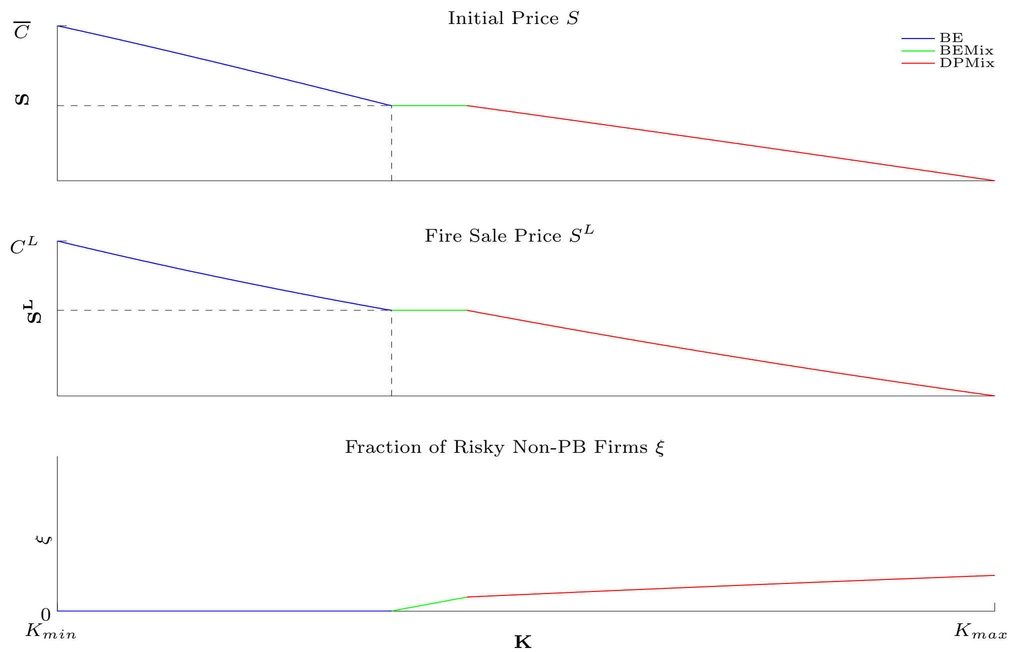

Figure 4: Break Even, Break Even Mixing, and Dry Powder Mixing Strategy equilibrium outcomes - plot shows the initial price, the fire sale price, and the fraction of firms that adopt risky strategies in period $t = 0$. For low levels of asset supply non private benefit firms take a break even strategy. Higher levels of supply depresses the initial price till some non private benefit firms mix between risky and safe strategies. For relatively high levels of asset supply non private benefit firms cease to participate safely in the initial market. Parameter values are: $A^I = 1, C^H = 110, C^L = 70, \alpha = .9, \lambda = .5, \eta = .75, \varphi = .5$

holding spare capacity for the fire sale. For even higher levels of asset supply, non private benefit firms reduce their initial safe investment which is picked up by a greater fraction of firms that take on risky strategies. Finally, for relatively large levels of asset supply, non private benefit firms either hold on to their entire endowment for the next period or invest in a risky manner.

5.3 Comparative Statics

This subsection explores how the equilibrium may be altered with small changes in underlying parameters. The main variables of interest are firms capital requirement η which effectively caps surviving firms demand in the stay exemption regime leading to a fire sale, the firms value upon default λ which measures the economic loss of a firm's default; and changes in the probability of of a good outcome α.

Changes in the equilibrium characterized in Theorem 1 are relatively straight forward. In this equilibrium, private benefit firms' portfolio remains largely unchanged: they take out the maximum amount of leverage allowed. The aggregate demand of non private benefit firms increases or decreases, adjusting their demand to maintain prices equal to expected future cash flows. Therefore, the equilibrium price remains the same for changes in λ and η; and changes in α are straight forward.

The more interesting exercise is to study how parameter changes can alter the stay exemption regime, particularly the fire sale and initial market premium. Considering the analytical characterization of Theorem 2 and using Lemma 1 in the appendix, I show that when non private benefit firms take a dry powder strategy the equilibrium changes as follows,

$$\frac{\partial S}{\partial \eta} > 0 \quad \frac{\partial S}{\partial \lambda} = 0 \quad \frac{\partial S}{\partial \alpha} = 0$$

The intuition behind these comparative statics are fairly simple to interpret. As capital requirements increase, firms ability to take on leverage is greater. This generates higher levels of demand in the initial financing period, increasing the S. The effect on S^L is only transmitted through changes in S. Though higher leverage in the initial period increases the supply of defaulted collateral in $t = 1$, solvent firms have larger purchasing power in the down state, canceling the effect. Thus, the only real effect is in the initial price increase. In this equilibrium, changes in λ and α don't have an impact on prices since firms decisions remain the same. Private benefit firms still insist on taking a risky position initially, and non private benefit firms still opt to hold on to their dry powder for the second.

The analysis when the equilibrium outcome involves non private benefit mixing into risky strategies is slightly more invovlved. Using the characterization of Theorem 2 for $K \in (K^M, K^{\mathcal{SE}}_{max})$, if the additional assumption of $\theta > (1 - \alpha)$ is adopted, Lemma 1 in the appendix gives the following comparative statics,[23]

$$\begin{array}{lll} \frac{\partial S}{\partial \eta} > 0 & \frac{\partial S}{\partial \lambda} > 0 & \frac{\partial S}{\partial \alpha} > 0 \\ \frac{\partial \xi}{\partial \eta} < 0 & \frac{\partial \xi}{\partial \lambda} > 0 & \frac{\partial \xi}{\partial \alpha} > 0 \end{array}$$

Changes in η causes the initial asset price to increase, since private benefit firms are able to increase their position, pushing up the price. The impact on ξ stems from the increase in S, which reduces non private benefit firms profits for taking risky strategies. The increase in portfolio size in the first period might have been a source of increased profits for taking on risky positions; but this is offset by the increased portfolio to take advantage of the fire sale.

The increase in λ is straight forward: a higher value for defaulted assets reduces financing costs for firms who take risky strategies. This increases the fraction of non private benefit firms who participate in

[23]Note that in the appendix $p = \theta + \xi(1 - \theta)$, and $\frac{\partial \xi}{\partial x} = \frac{\partial p}{\partial x} \frac{1}{(1-\theta)}$

the initial asset purchase and therefore increases the initial asset price. Since the increase in α implies an increase in profits for a risky strategy and a decrease in profits in a dry powder strategy, the fraction of non private benefit firms who adopt risky strategies increases, raising the market clearing price.

The effects on the fire sale price can be deduced from the interaction between S and ξ. Lemma 1 states,

$$\frac{\partial S^L}{\partial \eta} > 0 \quad \frac{\partial S^L}{\partial \lambda} < 0 \quad \frac{\partial S^L}{\partial \alpha} < 0$$

As λ and α increase, the payoff for undertaking risky strategies also increase, therefore more non private benefit capital migrates to the first period increasing the fire sale (i.e. reducing S^L). Increases in η result in an increase in the initial asset price and a reduction in the non private benefit firms who take risky strategies, which results in an increase in S^L.

6 Model Discussion

In this section I shall revisit the main trade offs in the stay vs. stay exemption debate for repos and highlight in which market the potential of a fire sale is likely to be a first order effect relative to other aspects of the discussion. Specifically, I will argue that a fire sale of repo collateral is more relevant for illiquid markets that may suffer from cash-in-the-market pricing, and for assets that do not have additional benefits beyond their pay off. In addition, I will provide anecdotal evidence of the main mechanism present in the model, namely firms' incentive to hold on to dry powder to exploit future fire sales.

6.1 Stay vs. Stay Exemption Discussion

The key issues surrounding the stay vs. stay exemption debate for derivatives and repos, along with the intuition behind them, where highlighted in subsection 2.1.[24] In the context of repo, the potential benefits of automatic stay exemption can be summarized as follows: a reduction in creditors' incentives to run, and the liberation of collateral that can be used more efficiently elsewhere. With regard to the first point, Skeel Jr. and Jackson (2012) provides anecdotal evidence that the stay exemption had little or no effect on lenders' incentives to renew their loans with debtors that were perceived to be in financial trouble. Even though creditors were certain to receive the underlying collateral in case of a firm bankruptcy, they preferred to avoid a default event altogether. This casts doubt on the effectiveness of repo's exemption from stay to mitigate runs on a firm.

[24]For more details on the discussion see Duffie and Skeel (2012).

The liberation of assets tied up in a bankruptcy proceeding may be a compelling argument if the assets in question have additional benefits above and beyond their pay off. In effect, US treasuries do provide added value given their flexibility to use as collateral to raise cheap financing, and their relatively low risk which helps firms fulfill regulatory capital requirements. These additional benefits have been documented for on-the-run treasuries, measured by the difference between the general collateral repo rate and the repo rate on a specific instrument, which is referred to as an asset's *specialness* (see Duffie (1996)). For these types of assets the effective "lock-up" of collateral could limit the additional benefits they may bring. But this issue may not be as relevant for assets in a relatively thin and illiquid market. In fact, during the financial crisis it was precisely the use of MBS as collateral which created difficulties for firms to raise financing (see Krishnamurthy et al. (2012) for evidence of the contraction of repo's using MBS). Thus, the efficient use of collateral is not likely to be an important factor for the types of markets this paper has in mind.

Key potential costs in the stay vs. stay exemption debate relevant for repos are: counterparties limited incentive to monitor, firm's inefficient substitution from traditional financing, and the potential to trigger a fire sale.[25] The first point, stressed by Roe (2011), may have limited importance in the context of money market funds financing large broker dealers whose operational complexities would be hard to gauge from an outsider's perspective. With regard to a firm's preferred financing vehicle, though it is true that repo's exemption from stay makes it less costly, an additional friction must be taken into account to argue a reduction in the firms total financing cost. Moreover, why would an increase in repo funding relative to other alternatives be "inefficient"? The default costs in this paper do give firms incentives to increase their repo funding which is beneficial since only non-earmarked assets suffer a default cost. But these features of the model are assumed. To address this issue a more extensive analysis is necessary taking into account firms' total debt capacity, incorporating a specific friction to explain firms' preference for repo funding, and specifying the inefficiency stemming from the excessive use of repo. Although this may be an interesting analysis, it is outside the scope of this paper.

Given the above, a fire sale stemming from the mass sale of collateral posted by defaulted firms is arguably an important aspect in the debate for relatively illiquid markets.[26] The model in this paper not only addresses the consequences during the fire sale, but also studies the effects on markets before the default

[25] Another potential cost is firms' reduced incentives to file for bankruptcy, given that exemptions from stay limit bankruptcy protection. But in the context of repo financing this is related to the potential withdrawal of funds from creditors - i.e. a run, which was addressed previously.

[26] Fire sales were a prime concern for regulators in the 2007-09 financial crisis. See Skeel Jr. and Jackson (2012) for details.

event. The partial evidence provided in subsection 6.2 suggest that the mechanism of hoarding liquidity and its impact on prices could have played an important role in the MBS market during the 2007-2009 financial crisis.

6.2 Supporting Evidence

The model presented in this paper points to a liquidity hoarding mechanism that ultimately results in an ex-ante price reduction, which changes firms' decisions. In essence, the main difference between regimes is that the stay exemption regime increases firms' investment opportunity set, attracting capital to exploit a future fire sale and causing the initial price to deviate from fundamentals.

In a related paper, Acharya, Shin and Yorulmazer (2011) provide anecdotal evidence of instances when firms strategically hoarded liquidity to take advantage of future price dislocations. They present the case of National City Bank, that later became Citibank, which foresaw the banking panic of 1893 and 1907, and strategically accumulated liquidity to take advantage of the future crisis. In both cases, before the crisis National City Bank accumulated significantly larger reserve ratios than other competing banks. This allowed the firm to increase deposits and expand their lending while other competitors were forced to contract. The motivation behind management's decisions was clear, as was made evident from a quote by the bank's President prior to the 1907 crisis, "If by able and judicious management we have money to help our dealers when trust companies have suspended, we will have all the business we want for many years."[27]

Acharya, Shin and Yorulmazer (2011) also provide anecdotal evidence of communications with bankers during the recent financial crisis, who claimed that the "...reasons for drying up of inter-bank lending markets has been the hoarding of liquidity by banks for acquisitions of troubled institutions at fire-sale prices, the other two reasons being precautionary motive from the risk of being distressed oneself and adverse selection about borrowing institutions." This suggests that during the crisis firms may have had the capacity to purchase assets, but refrained from doing so to await the collapse of another firm that would trigger a fire sale.

One interpretation of the model in the context of the financial crisis of 2007-09 is to consider private benefit firms as dealer banks and hedge funds holding large positions of MBS. Krishnamurthy et al. (2012) show that before the demise of Bear Stearns, 50% of repo transactions of four important dealer banks used non-Agency MBS as underlying collateral (see Figure 8 of that paper). Table 7 in He et al. (2010) provides evidence that these same firms had substantial positions in credit and mortgage related securities before the

[27] For more details of the case see Cleveland et al. (1985).

crisis. Non private benefit firms can be thought of as commercial banks, who also had significant positions in MBS but didn't rely as heavily on repo to finance them (see Gorton and Metrick (2012) pg 443). Arguably, the total leverage of dealer banks and hedge funds was larger than that of the commercial banking sector which at that time had considerably more regulatory restrictions. Evidence from the financial crisis suggest that the banking sector as a whole did have liquidity, yet opted not to use it. He et al. (2010) provide evidence of a mass sale of securitized products by hedge funds and dealer banks (approximately \$800 billion), much of which was picked up by commercial banks (approximately \$550 billion). But on aggregate the commercial banking sector actually increased their liquidity holdings, with support from the government.

Although the above analysis is subject to different interpretations, these stylized facts fit well with the arguments presented in this paper. The fact that commercial banks, which possessed the expertise to participate in the MBS market, held back some of their cash points to the mechanism of the paper. As was previously mentioned, this could have been for precautionary reasons or a potential lemons problem in the market, but liquidity hoarding is also a plausible explanation. The lack of demand for these assets in expectation of other firms defaulting, triggering a fire sale, could well have affected pricing throughout the crisis.

7 Equilibrium Analysis and Extensions

This section presents a cost-benefit analysis of the equilibrium in both regimes, some potential policy implications, and alternative settings of the model. Specifically, the policy subsection will consider alternatives in the stay exemption regime that would mitigate any distortions stemming from a fire sale. A myopic setting in which neither firms nor lenders internalize the potential fire sale will also be explored. Finally, I will briefly consider the case in which the initial capital committed to the market is allowed to increase, potentially eliminating all price deviations from fundamentals.

7.1 Cost Benefit Analysis

The main trade offs in the model are firms' private benefits for holding large risky asset positions vs. the deadweight cost brought on by default. Though it may be misleading to talk of "welfare" in such a stylized setting,[28] one can at least weight the positive and negative aspects within the model. Private benefits are proportional to the size of the risky asset position held by private benefit firms in $t = 0$, and the deadweight

[28] If would also be necessary to incorporate the impact on agents supplying the asset initially, which is outside the scope of this paper.

costs are the loss in asset value held by defaulting firms. Therefore, the expected benefits and costs are,

$$\text{PB gain} = b\frac{\theta A^I}{S(1-\eta)} \tag{16}$$

$$\text{Total Cost} = (1-\alpha)(1-\overline{\varphi})(1-\lambda)C^L\frac{(\theta+(1-\theta)\xi)A^I}{S(1-\eta)} \tag{17}$$

where a fraction $\xi \geq 0$ of non private benefit firms take risky strategies. Note that the costs are assumed by all firms that take risky strategies, which affects the non earmarked fraction of the firms risky portfolio after a bad asset outcome. It is clear from these expressions that whenever $\xi > 0$ the costs are proportionally larger than the potential benefits, since benefits are only borne by a fixed fraction of firms whereas the costs are borne by any firm who might choose a risky strategy.

In the stay regime, from Theorem 1, the economy's costs and benefits do not depend on $K \in [K_{min}, K^{\mathcal{S}}_{max})$. In effect,

$$\text{PB gain} = bK_{min}, \quad \text{Total Cost} = (1-\alpha)(1-\overline{\varphi})(1-\lambda)C^L K_{min}$$

Since the fraction of firms that undertake risky strategies remain the same and prices are fixed for all $K \in [K_{min}, K^{\mathcal{S}}_{max})$, firms' portfolios remain unchanged, making the costs and benefits constant. In effect, in this regime the same amount of the asset supply is held by private benefit firms which are the only firms that default.

This is not the the case in the stay exemption regime. Under a setting in which Theorem 2 holds for $K \in [K^{\mathcal{S}}_{min}, K^M]$ (when $\xi = 0$), replacing the initial market clearing price in equation (16) and (17) gives

$$\text{PB gain} = bK, \quad \text{Total Cost} = (1-\alpha)(1-\overline{\varphi})(1-\lambda)C^L K$$

As in the stay regime, the costs and benefits are borne by private benefit firms. But since they are the only type of firm participating in the market initially, their risky asset position becomes larger for higher levels asset supply. Thus they enjoy more benefits, but also pay for more costly default. When the equilibrium outcome has a fraction ξ of non private benefit firms mixing, from Theorem 2 when $K \in (K^M, K^{\mathcal{SE}}_{max})$, the private benefits and costs are,

$$\text{PB gain} = \frac{\theta}{\theta+\xi(1-\theta)}bK, \quad \text{Total Cost} = (1-\alpha)(1-\overline{\varphi})(1-\lambda)C^L K$$

The cost-benefit analysis for this equilibrium outcome is similar to the previous one: the initial asset supply is purchased by firms who adopt risky strategies - either private benefit or non private benefit firms. Note that in this case the total cost has the same expression as when $K \in [K^{\mathcal{S}}_{min}, K^M]$, but the benefits are only reaped by a fraction of those firms that risk default. Therefore this outcome has proportionally more costs than benefits because of mixing.

In case the equilibrium outcome consisted in non private benefit firms participating safely in the initial asset purchase (i.e. a break even or break even mixing strategy), the results would be qualitatively the same. In this scenario, for higher levels of K there would still be a price depreciation, increasing private benefit firms' asset position, and giving rise to higher private benefits and higher default costs. Moreover, whenever a fraction of non private benefit firms opt to take risky strategies the magnitude of private benefits remain intact, but the costs are still borne by all risky firms.

No matter what the equilibrium outcome in the stay exemption regime, the costs and benefits increase as cash-in-the-market pricing deviates prices from fundamentals. Whereas in the stay regime, since prices reflect fundamental values private benefit firms' participation is constant, thus the costs and benefits are held at a minimum.

7.2 Policy Recommendations

The motivation of the paper is to ask which regime is best when firms face a potential future fire sale. The analysis in subsection 7.1 shows that in the stay regime costs and benefits are held at a minimum, whereas in the stay exemption regime these trade offs will vary with the ratio of asset supply to initial capital committed to the market. Therefore, a reasonable policy implication may be to maintain excess benefits and costs to a minimum, which is equivalent to eliminate any cash-in-the-market effects that cause prices to deviate from fundamentals. In the very least, this would avoid non private benefit firms from undertaking risky strategies, unduly increasing default costs proportionately to benefits. From that perspective, the policy recommendation would be to have repo subject to automatic stay; at least for the type of collateral this paper has in mind.

But there would be other potential solutions to the cash-in-the-market phenomena in a stay exemption regime. Here I will consider two additional possibilities: government asset purchases and a contract modification that gives the lender the upside of the defaulted asset.

7.2.1 Government Asset Purchase

If a large outside investor without capital constraints were to commit to purchase the defaulted asset in case of a bad outcome, the fire sale would completely disappear along with any distortions it causes. The natural agent to undertake this task would be the government. In this context, agents would internalize that in the refinancing period the market clearing condition would take on the following form,

$$\overline{Q}_1^{PB} + \overline{Q}_1^{NPB} + G = 0$$

where G would be the government's demand for defaulted collateral. The role of the government would be to purchase the asset till $S^L = C^L$ eliminating any ex-ante fire sale premium. Thus non private benefit firm capital can be totally committed in the initial financing period (i.e $\overline{Q}_1^{NPB} = 0$), essentially reducing the setup to the stay regime.

Though this is a relatively simple exercise in the context of the model, this may not be straightforward in practice. Besides the commitment by the government to purchase assets in case of a cash-in-the-market discount, the government must be able to recognize wether the price reduction is in fact due to limited liquidity in the market or a deterioration in fundamentals. This may be difficult to do at a moment's notice. Moreover, the government's strategy does involve some degree of risk since $C^L > C^{LL}$, which may be undesirable. In spite of that, efforts made by the government during the 2007-09 financial crisis can be seen as exactly that: an attempt to purchase assets trading at prices below their economic value.

7.2.2 Lender Reception of Collateral Performance

An important ingredient in the model are lender's incentives to sell the asset upon default, irrespective of its trading price. The main driver behind this assumption is the fact that any upside from the collateral beyond the face value of the original loan is property of the borrower, mitigating incentives for the lender to retain the asset. This provision is currently present in repo arrangements. In the model, this is taken to an extreme since the initial face value on the loan is set to the fire sale price eliminating all incentives to hold the asset.

In a world in which the lender does in fact receive the asset upside after default, risk neutral lenders would have no incentive to sell it in the presence of a fire sale. In essence, the upside that was previously captured by solvent firms who purchased the defaulted collateral would now dissappear. The consequence of this contract change implies that trade only occurs whenever $S^L = C^L$, irrespective of the amount of capital held by firms. By eliminating the fire sale, there is no initial price discount and the resulting equilibrium is identical to the stay regime.

In the context of the model, this contract change must be considered carefully. An important ingredient of the model is market segmentation and limited capital committed to the market. Specifically, the standing assumption throughout the model is that lenders cannot participate in the risky asset market directly. This is motivated by some expertise necessary to actively trade in the market which firms are initially endowed with. Allowing lenders to hold on to collateral in case of default, effectively participating in the market in the down state, leads to the question why did they not participate initially? It could be argued that the initial asset purchase still involves some degree of expertise, due to an unmodeled lemons problem in the purchase of risky assets, which lenders need not concern themselves with if they receive defaulted collateral. From this perspective the "bad" assets were already filtered by the firms in the initial stage.

An important aspect of this policy prescription is that it does not rely on an active government. In this setting, firms and lenders would eliminate the potential fire sale themselves. Undoubtedly, this recommendation would have important effects on other aspects of the repo market, particularly in the determination of haircuts. But for the analysis of this paper the outcome is straightforward.

7.3 Myopic Agents

An interesting exercise is to consider a case in which agents in the economy do not internalize the potential fire sale that can arise in the stay exemption regime. That is, firms and lenders do not realize that due to limited capital in the down state, the market clearing price may be below the asset's expected future payoff. This may be because the total initial capital committed to the market is believed to be sufficient to satisfy any supply shock. This results in non private benefit firms committing their capital in the first period, disregarding the potential payoff from a future fire sale. In the very least, this implies less capital in the refinancing state and a more severe fire sale.

More formally, in the stay exemption regime non private benefit firms would adopt the strategy chacarterized by Proposition 2 when $S^L = C^L$. In this case, the break even price is equal to the fundamental value of the asset: $S^{BE}(C^L, 0) = \overline{C}$. Therefore, for $K \in [K_{min}, K_{max})$ non private benefit firms would increase aggregate demand till the first period price equals the asset's expected future cash flow. Given the linear structure of the problem whenever the trading price equals the break even price, non private benefit firms individual demand is undetermined. Thus, for a given asset supply K it is unclear how many non private benefit firms would have taken on debt that would induce them to default. This implies that in the refinancing period the aggregate demand and supply is undetermined since it cannot be established how many firms will default, nor how much spare capacity surviving firms will have to purchase defaulted collateral. What is clear is that surviving firms' aggregate demand in $t = 1$ will be lower compared to a setting where agents account for the fire sale, making the price distortion even larger. In the stay regime, since there is no fire sale there is no scope for firms to be myopic and the equilibrium outcome is the same as before. Therefore, the stay exemption regime would have a more severe fire sale and the stay regime would remain unchanged.

This setting arises naturally in scenarios where there may be a large degree of uncertainty as to the severity of any cash-in-the-market pricing. During March 2008, due to Bear Stearns financial troubles, regulators "...worried that a mass sale of repo collateral could drive down the values of mortgage-related securities and further destabilize the markets." [29] In the myopic model, because of repos' exemption from stay and the degree of uncertainty around the amount of capital available to withstand a supply shocks, the magnitude of the fire sale would be impossible to determine. Moreover, the severity of the fire sale could cause woes for other firms, exacerbating the illiquidity problem further by triggering more defaults.

7.4 Increase Initial Capital

The main theme of the model consists of limited capital in the refinancing period, causing prices to be below fundamental values and generating a fire sale premium. This mechanism shuts down if the initial capital committed to the market increases to satisfy demand in both periods. That is, if non private benefit firms recognize the initial fire sale premium and commit more capital to the risky asset market, the fire sale is eliminated, making distortions between the stay and stay exemption regime disappear.

This highlights the framework in which the analysis of this paper is most relevant: illiquid markets, with a fraction of highly levered players with capital constraints and a certain degree of expertise necessary to participate. It is unlikely that the effects of this model are first order in large, highly liquid markets, like for instance the US treasury market. Arguably, the depth and scope of that market would attract capital eliminating any price distortions. The model is more relevant to scenarios where the initial participation may be limited, perhaps because of high initial costs to become familiar with it's intricacies. Thus, this paper sheds like on stay vs. stay exemption regime discussion for less traditional asset classes, for example MBS market during the most recent crisis.

[29] Skeel Jr. and Jackson (2012)

8 Concluding Remarks

This paper explores the consequences of repo's exemption from automatic stay in the presence of potential fire sales that result from limited capital committed to a market. The main result of the paper is that future fire sales alter firms investment opportunity set, giving them incentives to hold their initial capital to take advantage of a possible price discount in the future. This implies that more of the risky asset is held by firms that have incentives to take on high levels of leverage, increasing the amount assets held by defaulting firms, which is assumed to be costly. The mechanism disappears when repo is subject to stay, since in the model secured lenders receive their payments when market turmoil has subsided and prices reflect fundamental values.

A mix of ingredients is necessary for fire sales to arise endogenously in the model. The firms who participate in the market can be considered as *insiders*, since they are the only agents in the economy able to purchase the asset. They receive funding from an unconstrained lending sector, but have limits to the amount of funding they can raise because of regulatory constraints or because the lending sector demand firms to have some skin in the game. Firms' funding constraints limit the size of their risky asset position, which induces cash-in-the-market pricing. Importantly a fraction of firms have incentives to increase their portfolio size, motivating them to risk default by raising more debt. In essence, this introduces leverage heterogeneity so that a fraction of firms default and the rest remain solvent. The incentive to have a large portfolio can be thought of as an unmodeled convenience yield for holding the risky asset or over optimism in the final asset pay off.

The setting described in the model is most relevant for scenarios where the asset market is relatively thin, leaving the possibility for cash-in-the-market pricing. I draw on anecdotal evidence from the 2007-09 financial crisis to motivate the model's setup and main mechanism. Firms in the model can be thought of as large broker dealers, hedge funds, or commercial banks with the expertise to participate in a relatively specialized market, like for example the MBS market. The lenders can be thought of as money market funds and regular bond investors catering to these firms financing needs. Arguably, money market funds and normal bond investors would not have the same expertise as broker dealers or specialized hedge funds to purchase these types of assets. Moreover, it is natural to think that the initial capital committed to a thin market is relatively small compared to the amount of capital held by regular cash investors, thus making firm capital relevant. In this setting, future fire sales create incentives for firms to hold on to their liquidity to take advantage of them.

This paper shows how exemption from automatic stay can distort incentives in the previously described setting. When highly levered firms participate in a relatively illiquid market prices can deviate from fundamentals, altering firms' decisions and the total costs of default. Though this paper cannot completely resolve the issue whether stay or stay exemption is an optimal bankruptcy regime, it does give insights as to how firms' behavior would change under different regulatory settings. The goal of this paper is to shed light on the role of potential fire sales in the automatic stay versus stay exemption debate for repos.

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Appendix

A Parameter Assumptions

A1 - The debt capacity restriction η is larger than the defaulted value of the firm: $\eta \geq \varphi_0 + \lambda(1-\varphi_0)$

A2 - The fraction of private benefit firms θ is such that firms in a DP equilibrium clear markets at $S = \overline{C}$ and $S^L = C^L$ for $K = K_{min}$: $\theta = \frac{\overline{C}}{\overline{C}+\overline{\varphi}C^L}$

A3 - In a Dry Powder equilibrium, non private benefit firms do not take a safe position in the initial market: $S > S^{BE}(S^L, 0)$: $\frac{\overline{C}}{C^L} > \frac{(1-\alpha)\eta}{\eta-\alpha}$

A4 - For a Dry Powder equilibrium, $S^L > \lambda C^L$ for all $K \in [K_{min}, K_{max}]$: $\frac{\overline{C}}{C^L} > \eta + \frac{\lambda(1-\eta)\overline{\varphi}}{1-\lambda}$

A5 - The Dry Powder price S is equal to S^M for $K^M \in [K_{min}, K_{max}]$: $\frac{(1-\overline{\varphi})(1-\lambda)}{\alpha\overline{C}-(1-\alpha)[\overline{\varphi}+(1-\overline{\varphi})(1-\lambda)]C^L} < \frac{\overline{\varphi}(1-\eta)}{\overline{C}-\eta C^L}$

B Proofs

Proof of Proposition 1:

The analysis of firms optimal behavior can be simply seen by splitting the problem in two. First, consider parameters such that firms optimally choose to take on a safe strategy. This involves solving their investment and financing problem imposing one additional restriction: $I_0^u + \varphi_0 C^L Q_0 \leq \eta C^L Q_0 + A^I - A_0$, i.e. $(Q_0, A_0, I_0^u, \varphi_0) \in N^{\mathcal{S}}$. Note that this condition is more restrictive than the firms capital requirements since $S > C^L$.[30]

Therfore, the safe firms optimization lagrangian takes the following form,

$$\begin{aligned}
\mathcal{L} &= (\overline{C} + b)Q_0 + A^I - A_0 - I_0^u - \varphi_0 C^{LL} Q_0 + \mu_B(\varphi_0 C^{LL} Q_0 + I_0^u + A_0 - SQ_0) + \\
&\quad \cdots \mu_D(\eta C^L Q_0 + A^I - A_0 - I_0^u - \varphi_0 C^{LL} Q_0) + \mu_A(A^I - A_0) + \mu_\varphi(\overline{\varphi} - \varphi_0)
\end{aligned}$$

note that $A_0 + I_0^u$ appear together in all expression except for the restriction on the firms dry powder. Calling $H = A_0 + I_0^u$ and solving for H ignoring the dry powder restriction. Taking FOC we have,

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial Q_0} &= \overline{C} + b - \varphi_0 C^{LL} + \mu_B(\varphi_0 C^{LL} - S) + \mu_D(\eta C^L - \varphi_0 C^{LL}) \leq 0 \\
\frac{\partial \mathcal{L}}{\partial H} &= -1 + \mu_B - \mu_D \leq 0 \\
\frac{\partial \mathcal{L}}{\partial \varphi_0} &= -C^{LL} Q_0 + \mu_B C^{LL} Q_0 - \mu_D C^{LL} Q_0 - \mu_{\varphi_0} \leq 0
\end{aligned}$$

Case $H^* > 0$ **&** $Q_0^* > 0$: implies $\mu_B = \mu_D + 1 > 0$, thus the budget constraint is active. Replacing μ_B in the first equation gives,

$$\mu_D = \frac{\overline{C} + b - S}{S - \eta C^L}$$

thus if $S < \overline{C} + b$ (since $S > \eta C^L$ by assumption) the firm takes on the maximum amount of leverage: $I_0^{u*} + \varphi_0^* C^{LL} Q_0^* = \eta C^L Q_0^* + A^I - A_0^*$. Together with the budget constraint one solves for Q_0^*. If $S = \overline{C} + b$, then the amount of debt and asset purchase are undetermined. Note that in both of these sub cases φ_0^* is not uniquely pinned down.

Case $H^* > 0$ **&** $Q_0^* = 0$: implies $\mu_B = \mu_D + 1$, thus the budget constraint is active and $H = 0$ leading to a contradiction.

Case $H^* = 0$: implies $\mu_B < \mu_D + 1$ which from the final equation implies $\varphi_0^* = 0$, since any positive φ_0^* would lead a contradiction. The budget constraint implies $Q_0^* = 0$ and the refinancing condition is slack. Using the first FOC and noting that $\mu_B < 1$ we have $\mu_B S > \overline{C} + b$, thus $S > \overline{C} + b$ pinning down the firm's strategies.

Having the firms strategies one can solve for their final payoffs.

Turning to the risky firm, if parameters as such that firms choose to undertake a risky strategy from equation (3), the price of risky debt alters the value of the firms risky asset position to

$$C^D(\varphi_0) = (\alpha C^H + (1-\alpha)(\varphi_0 C^L + (1-\varphi_0)\lambda C^L))$$

The optimizations problem's lagrangian takes the following form,

$$\begin{aligned}
\mathcal{L} &= (C^D(\varphi_0) + b)Q_0 + (\alpha + (1-\alpha)\lambda)(A^I - A_0) - \varphi_0 C^{LL} Q_0 - I_0^u + \mu_A(A^I - A_0) + \\
&\quad \cdots \mu_B(\varphi_0 C^{LL} Q_0 + I_0^u + A_0 - SQ_0) + \mu_D(\eta(SQ_0 + A^I - A_0) - I_0^u - \varphi_0 C^{LL} Q_0) + \mu_\varphi(\overline{\varphi} - \varphi_0)
\end{aligned}$$

[30] In case safe firms were to maximize their debt capacity (still avoiding default) it would be optimal for $A_0 = A^I$, making the refinancing condition tighter than the firm's capital requirements.

Taking FOC we have,

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial Q_0} &= C^D(\varphi_0) + b - \varphi_0 C^{LL} + \mu_B(\varphi_0 C^{LL} - S) + \mu_D(\eta S - \varphi_0 C^{LL}) \leq 0 \\
\frac{\partial \mathcal{L}}{\partial I_0^u} &= -1 + \mu_B - \mu_D \leq 0 \\
\frac{\partial \mathcal{L}}{\partial \varphi_0} &= (1-\alpha)(1-\lambda)C^L Q_0 - C^{LL}Q_0 + \mu_B C^{LL} Q_0 - \mu_D C^{LL} Q_0 - \mu_{\varphi_0} \leq 0 \\
\frac{\partial \mathcal{L}}{\partial A_0} &= -(\alpha + (1-\alpha)\lambda) - \mu_A + \mu_B - \eta\mu_D \leq 0
\end{aligned}$$

Since the proposition characterizes the optimal strategy of a levered firm the proof concentrates in the case when $\mu_D > 0$.

Case $\mu_D > 0$ **&** $I_u > 0$: implies $\mu_B = \mu_D + 1$ which from the final two equations implies $\mu_{\varphi_0}, \mu_A > 0$. Given that the budget constraint is active and $A_0^* = A^I$, necessarily $Q_0^* > 0$. Replacing μ_B in the first equation gives,

$$\mu_D = \frac{C^D(\overline{\varphi}) + b - S}{S(1-\eta)}$$

thus if $C^D + b > S$, the firm takes on maximal leverage $I_0^{*u} + \overline{\varphi}C^{LL}Q_0 = \eta S Q_0$. Together with budget constraint one solves for Q_0^*.

Case $\mu_D > 0$ **&** $I_u = 0$: the capital requirement restriction implies $\varphi_0 C^{LL} Q_0 = \eta(SQ_0 + A^I - A_0)$, therefore in the very least $\overline{\varphi}C^{LL} \geq \eta S$. But from assumption A1 and $S > C^L$ it can be deduced that $\eta S > \overline{\varphi}C^{LL}$, leading to a contradiction.

Case $\mu_D = 0$: implies $\mu_B \leq 1$. Since $S > \varphi_0 C^{LL}$, replacing the bound for μ_B in the first inequality gives $C^D(\varphi_0) + b \leq S$ violating one of the proposition's hypothesis.

It must be verified that the firm would in fact default in the down state, that is the firm cannot roll over it's debt. From equation (5) the maximum firm can raise after a bad outcome in the refinancing period is $\eta C^L Q_0 + (A^I - A_0)$. From equation (3) and the firms default costs, risky unsecured debt is,

$$\alpha F_0^u = I_0^u - (1-\alpha)\left((\varphi_0 C^L + \lambda(1-\varphi_0)C^L)Q_0 + \lambda(A_0^I - A_0) - \varphi_0 C^{LL} Q_0\right)$$

then for the firms to default $F_0^u + \varphi_0 C^{LL} Q_0 > \eta C^L Q_0 + (A^I - A_0)$ which is equivalent to,

$$I_0^u + \varphi_0 C^{LL} Q_0 > (\alpha\eta + (1-\alpha)[\varphi_0 + \lambda(1-\varphi_0)])\, C^L Q_0 + (\alpha + \lambda(1-\alpha))(A^I - A_0)$$

Under assumption A1, considering the optimal strategy $A^* = A^I$, and $S > C^L$ the risky firm actually defaults. In effect,

$$I_0^u + \varphi_0 C^{LL} Q_0 = \eta S Q_0 > \eta C^L Q_0 > (\alpha\eta + (1-\alpha)[\varphi_0 + \lambda(1-\varphi_0)])\, C^L Q_0$$

Having the firms strategies one can solve for their final payoffs.

Pinning down both types of firms optimal strategies and payoffs, the threshold function $b^*(S)$ must be characterized to determine down when firms take a risky or safe straegey. In effect, for risky firms to have a positive payoff it is necessary to consider $S \leq C^D(\overline{\varphi}) + b$, with $b > 0$, giving the first two restrictions for $b^*(S)$. In that price range, the optimal safe firm strategy is to increase debt capacity till it can roll over it's debt. Thus, a risky strategy will be preferred if,

$$\begin{aligned}\frac{C^D(\overline{\varphi})+b-S}{S(1-\eta)} &\geq \frac{\overline{C}+b-S}{S-\eta C^L}\\ b &\geq \frac{(\overline{C}-S)S(1-\eta)-(C^D(\overline{\varphi})-S)(S-\eta C^L)}{\eta(S-C^L)}:=b'(S)\end{aligned}$$

defining $b^*(S)=\max\{0,S-C^D(\overline{\varphi}),b'(S)\}$, for $b\geq b^*(S)$ firms will choose risky strategies, completing the proof.

■

Proof of Proposition 2:

The analysis of the problem is similar to the stay regime, it is simply seen by splitting the problem in two. First consider parameters such that firms optimally choose to take on a safe strategy. This involves solving their investment and financing problem on additional restriction: $I_0^u+\varphi_0 S^L Q_0\leq \eta S^L Q_0+A^I-A_0$, i.e. $(Q_0,A_0,I_0^u,\varphi_0)\in N^{\mathcal{SE}}$. Note that this condition is more restrictive than the firms capital requirements since $S^L>C^L$.

Therefore, the safe firms optimization problem takes the following form,

$$\begin{aligned}\mathcal{L} &= (\overline{C}+b)Q_0+(A^I-A_0)+(1-\alpha)(C^L-S^L)\left(\frac{\eta S^L Q_0+(A^I-A_0)-I_0^u-\varphi_0 S^L Q_0}{S^L(1-\eta)}\right)+\\ &\quad \cdots-\varphi_0 S^L Q_0-I_0^u+\mu_B(\varphi_0 S^L Q_0+I_0^u+A_0-SQ_0)+\mu_A(A^I-A_0)+\mu_\varphi(\overline{\varphi}-\varphi)+\\ &\quad \cdots\mu_D(\eta S^L Q_0+A^I-A_0-I_0^u-\varphi_0 S^L Q_0)\end{aligned}$$

note that $A_0+I_0^u$ appear together in all expressions except for the restriction on the firm's Dry Powder. Calling $H=A_0+I_0^u$ one solves for H ignoring the dry powder restriction. Taking FOC we have,

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial Q_0} &= \overline{C}+b-\varphi_0 S^L+(1-\alpha)\left(\frac{C^L-S^L}{S^L(1-\eta)}\right)(\eta-\varphi_0)S^L+\mu_B(\varphi_0 S^L-S)+\mu_D(\eta-\varphi_0)S^L\leq 0\\ \frac{\partial\mathcal{L}}{\partial H} &= -1-(1-\alpha)\left(\frac{C^L-S^L}{S^L(1-\eta)}\right)+\mu_B-\mu_D\leq 0\\ \frac{\partial\mathcal{L}}{\partial\varphi_0} &= -S^L Q_0-(1-\alpha)\left(\frac{C^L-S^L}{S^L(1-\eta)}\right)S^L Q_0+\mu_B S^L Q_0-\mu_D S^L Q_0-\mu_\varphi\leq 0\end{aligned}$$

Case $H^*>0$ & $Q_0^*>0$: implies $\mu_B=1+(1-\alpha)\left(\frac{C^L-S^L}{S^L(1-\eta)}\right)+\mu_D>0$, thus the budget constraint is active since $S^L\leq C^L$. Replacing μ_B in the first equation gives,

$$\mu_D=\frac{\overline{C}+b-S+(1-\alpha)\left(\frac{C^L-S^L}{S^L(1-\eta)}\right)(\eta S^L-S)}{S-\eta S^L}$$

thus if

$$S\left(1+(1-\alpha)\left(\frac{C^L-S^L}{S^L(1-\eta)}\right)\right)<\overline{C}+b+(1-\alpha)\left(\frac{C^L-S^L}{S^L(1-\eta)}\right)\eta S^L$$

since $S>S^L$ by assumption the firm takes on the maximum amount of leverage: $I_0^{u*}+\varphi_0^* S^L Q_0^*+A_0^*=\eta S^L Q_0^*+A^I$. Together with the budget constraint one solves for Q_0^*. If S is such that $\mu_D=0$, the amount of debt and asset purchase is undetermined and the above inequality is with equality, characterizing $S(S^L,b)$. Note that in both cases φ_0^* is not determined uniquely

Case $H^*>0$ & $Q_0^*=0$: implies $\mu_B=1+(1-\alpha)\left(\frac{C^L-S^L}{S^L(1-\eta)}\right)+\mu_D$. Since by assumption $C^L\geq S^L$, it follows that the budget constraint is active and thus $H^*=0$, leading to a contradiction.

Case $H^*=0$: implies $\mu_B<1+(1-\alpha)\left(\frac{C^L-S^L}{S^L-\eta S^L}\right)+\mu_D$ which from the final equation implies that $\varphi_0^*=0$, since any positive

φ_0^* would lead to a contradiction. The budget constraint implies $Q_0^* = 0$ and that the refinancing condition is slack: $\mu_D = 0$. Solving for $\mu_B < 1 + (1-\alpha)\left(\frac{C^L - S^L}{S^L(1-\eta)}\right)$ from the first FOC gives,

$$\mu_B S > \overline{C} + b + (1-\alpha)\left(\frac{C^L - S^L}{S^L(1-\eta)}\right)\eta S^L$$

therefore $S > S^{BE}(S^L, b)$.

Having the firms strategies pins down the expressions for the final payoffs.

Turning to the risky firm, if parameters as such that firms choose to undertake a risky strategy from equation (4) the price of risky debt alters the value of the firms risky asset position to,

$$C^D(\varphi_0, S^L) = (\alpha C^H + (1-\alpha)(\varphi_0 S^L + (1-\varphi_0)\lambda C^L))$$

Note that the lower bound on $S^L > \lambda C^L$ is to ensure that the firms decides to first exhaust it's secured funding before it raises unsecured funding. The optimization problem Lagrangean takes the following form,

$$\begin{aligned}
\mathcal{L} &= (C^D(\varphi_0, S^L) + b)Q_0 + (\alpha + (1-\alpha)\lambda)(A^I - A_0) - \varphi_0 S^L Q_0 - I_0^u + \mu_A(A^I - A_0) + \\
&\quad \cdots \mu_B(\varphi_0 S^L Q_0 + I_0^u + A_0 - SQ_0) + \mu_D(\eta(SQ_0 + (A^I - A_0)) - I_0^u - \varphi_0 S^L Q_0) + \mu_\varphi(\overline{\varphi} - \varphi_0)
\end{aligned}$$

Taking FOC we have,

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial Q_0} &= C^D(\varphi_0, S^L) + b - \varphi_0 S^L + \mu_B(\varphi_0 S^L - S) + \mu_D(\eta S - \varphi_0 S^L) \leq 0 \\
\frac{\partial \mathcal{L}}{\partial I_0^u} &= -1 + \mu_B - \mu_D \leq 0 \\
\frac{\partial \mathcal{L}}{\partial \varphi_0} &= (1-\alpha)(S^L - \lambda C^L)Q_0 - S^L Q_0 + \mu_B S^L Q_0 - \mu_D S^L Q_0 - \mu_{\varphi_0} \leq 0 \\
\frac{\partial \mathcal{L}}{\partial A_0} &= -(\alpha + (1-\alpha)\lambda) - \mu_A + \mu_B - \eta\mu_D \leq 0
\end{aligned}$$

since $S^L > \lambda C^L$ the arguments given for the proof of proposition 2 still hold. Thus the optimal strategies and final payoffs take the same form, replacing $C^D(\varphi)$ by $C^D(\varphi, S^L)$, C^{LL} by S^L and C^L by S^L in the FOC for φ_0.

It must be verified that the firm would in fact default in the down state, that is the firm cannot roll over it's debt. From equation (6) the maximum a firm can raise after a bad outcome is $\eta S^L Q_0 + (A^I - A_0)$. From equation (4) and the firms default costs, risky unsecured debt is,

$$\alpha F_0^u = I_0^u - (1-\alpha)\left(\lambda(1-\varphi_0)C^L Q_0 + \lambda(A_0^I - A_0)\right)$$

therefore the default condition is $F_0^u + \varphi_0 S^L Q_0 > \eta S^L Q_0 + (A^I - A_0)$ which is equivalent to,

$$I_0^u + \varphi_0 S^L Q_0 > \left(\alpha\eta S^L + (1-\alpha)[\varphi_0 S^L + \lambda(1-\varphi_0)C^L]\right)Q_0 + (\alpha + \lambda(1-\alpha))(A^I - A_0)$$

Assuming A1, considering the optimal strategy $A_0 = A^I$, and $S > S^L > \lambda C^L$ the risky firm actually defaults. In effect,

$$I_0^u + \varphi_0 S^L Q_0 = \eta S Q_0 > \left(\alpha\eta S^L + (1-\alpha)[\varphi_0 S^L + \lambda(1-\varphi_0)C^L]\right)Q_0$$

Pinning down both types of firms optimal strategies and payoff, the threshold function $b^{**}(S, S^L)$ must be characterized to determine when firms in fact take a risky or safe strategy. To do so, it must be that firms with a private benefit $b > 0$ prefer the risky strategy above both safe alternatives; where in one of them gives the firm a potential fire sale profit. In case the optimal safe strategy is to hold on to the firms dry powder, the following inequality must hold,

$$\begin{aligned}\frac{(C^D(\overline{\varphi}, S^L) + b - S)A^I}{S(1-\eta)} + A^I &\geq (1-\alpha)\left(\frac{C^L - S^L}{S^L(1-\eta)}\right)A^I + A^I \\ b &\geq S\left[(1-\alpha)\left(\frac{C^L - S^L}{S^L}\right) - \left(\frac{C^D(\overline{\varphi}, S^L) - S}{S}\right)\right] := b^{''}(S, S^L)\end{aligned}$$

In case the optimal safe strategy is to increase the firm's debt till it can roll over it's debt, firm's will choose a risky strategy if,

$$\begin{aligned}\frac{(C^D(\overline{\varphi}, S^L) + b - S)A^I}{S(1-\eta)} + A^I &\geq \frac{(\overline{C} + b - S)A^I}{S - \eta S^L} + A^I \\ b &\geq \frac{(\overline{C} - S)S(1-\eta) - (C^D(\overline{\varphi}, S^L) - S)(S - \eta S^L)}{\eta(S - S^L)} := b^{'''}(S, S^L)\end{aligned}$$

defining $b^{**}(S, S^L) = \max\{0, b^{''}(S, S^L), b^{'''}(S, S^L)\}$, for $b \geq b^{**}(S, S^L)$ firms will choose risky strategies, completing the proof.

■

Proof of Theorem 1:

With the conjectured equilibrium has $S = \overline{C}$, it is clear that non private benefit firms will adopt a safe strategy since otherwise their payoff would be lower than their initial endowment. For these firms, proposition 1 states that their demand is undetermined $Q^{undet} \in [0, A^I/(\overline{C} - \eta C^L))$ and final payoff is their initial endowment.

Private benefit firms have to be motivated to take on the risky strategy. From the proof of proposition 1, the restriction $b > b^*(\overline{C})$ ensures these firms take risky strategies. This amounts to impose,

$$b > (\overline{C} - C^D(\overline{\varphi}))\frac{\overline{C} - \eta C^L}{\eta(\overline{C} - C^L)} := b^*$$

since this condition is more restrictive than $\overline{C} - C^D(\overline{\varphi})$ and $b^* > 0$. Thus if $b > b^*$ private benefit firms will adopt a risky strategy.

Integrating over firms demand and imposing market clearing gives,

$$\frac{A^I\theta}{\overline{C}(1-\eta)} + \overline{Q}^{NPB} = K$$

which holds for all $\overline{Q}^{NPB} \in \left[0, \frac{A^I(1-\theta)}{\overline{C} - \eta C^L}\right)$, pinning down the supply interval in which the equilibrium holds.

■

Proof of Theorem 2:

Note that assumption A1 ensures that risky firms actually default in the low state. I conjecture that the equilibrium for relatively low levels of asset supply has firms adopt DP strategies. For firms do not participate in the initial period, the fire sale premium must be relatively large. In the conjectured equilibrium, the relation between the first and second period price stems from market clearing in the refinancing period (imposing the parameterization of θ from A2) , i.e.

$$\overline{\varphi}\frac{A^I\theta}{S(1-\eta)} = \frac{A^I(1-\theta)}{S^L(1-\eta)} \Longrightarrow S^L = \frac{1-\theta}{\overline{\varphi}\theta}S = \frac{C^L}{\overline{C}}S := \phi S$$

that is, in this setting, the initial gross return of the asset is equal to the gross fire sale return. If $S > S^{BE}(S^L, 0)$ non private benefit firms will refrain from investing in the initial period. This is equivalent to,

$$1+\frac{(1-\alpha)(C^L-S^L)}{S^L(1-\eta)} > \frac{\overline{C}}{S}+\frac{(1-\alpha)(C^L-S^L)}{S^L(1-\eta)}\frac{\eta S^L}{S}$$

imposing the relationship between gross returns gives,

$$(1-\alpha)(1-\eta\phi) > (1-\eta) \tag{18}$$

which, by replacing ϕ, is condition A3. Thus, non private benefit firms opt to only invest in the refinancing period for relatively low levels of asset supply. To conclude that private benefit firms take on risky strategies, it is necessary to determine the value of b^{**} such that $b^{**} \geq b^{**}(S, S^L)$ for all S and S^L in the conjectured equilibrium. This amounts to impose $b^{**} = b^{**}(S, \phi S)$ to deter private benefit firms from adopting a safe strategy, whichever that may be. From the proof of Proposition 2 there are two possible expressions for $b^{''}$ and $b^{'''}$. The first is so that private benefit firms prefer risky strategies over investing in a dry powder strategy, and the other is so that they prefer risky strategies over investing with low debt levels initially. Imposing the conjectured equilibrium gives,

$$\begin{aligned} b^{''}(S) &= (1-\alpha)\frac{C^L}{\phi}+\alpha S - C^D(\overline{\varphi}, \phi S) \\ b^{'''}(S) &= \frac{\overline{C}(1-\eta)-C^D(\overline{\varphi}, \phi S)(1-\phi\eta)}{\eta(1-\phi)}+S \end{aligned}$$

Therefore, choosing private benefits such that $b^{**} > \max\{b^{''}(S), b^{'''}(S)\}$ for all S in the conjectured equilibrium, private benefit firms will adopt a risky strategy.[31]

Having pinned down private and non private benefit firms strategies, market clearing in the first period is expressed as,

$$\frac{A\theta}{S(1-\eta)} = K$$

It must be verified $S^L > \lambda C^L$ for proposition 2 to hold. From market clearing and $S^L = \phi S$, this equates to

$$\lambda K < \frac{A\theta}{\overline{C}(1-\eta)}$$

thus to ensure that these conditions hold it is sufficient to prove that $K^{\mathcal{S}}_{max}$ is smaller than the RHS bounds,[32] which is equivalent to,

$$(1-\lambda)(\frac{1}{\phi}-\eta) > \lambda(1-\eta)\overline{\varphi}$$

This inequality is precisely assumption A4. Finally, to ensure the existence of K^M in which firms enter a DP-mixing equilibrium, is must be shown that there exists a price low enough for non private benefit firms to mix between safe and risky strategies. That is, from the mixing condition (equation (9)), we have

$$\alpha\frac{A\theta}{K(1-\eta)}+(1-\alpha)\overline{C} = C^D\left(\overline{\varphi}, \phi\frac{A\theta}{K(1-\eta)}\right)$$

which defines

$$K^M = \frac{A\theta}{\overline{C}(1-\eta)}+\frac{(1-\overline{\varphi})(1-\lambda)C^L}{\alpha\overline{C}-(1-\alpha)[\overline{\varphi}+(1-\overline{\varphi})(1-\lambda)]C^L}\frac{A\theta}{\overline{C}(1-\eta)}$$

[31] For a large set of primitive parameters of the model, the above expressions for $b^{''}(S)$ and $b^{'''}(S)$ are increasing in S. Thus, imposing private benefits to be equal to these functions when $S = \overline{C}$ (which coincides with the stay regime private benefits) will ensure the correct behavior throughout all the conjectured equilibrium.

[32] Note that they are trivially larger than $K^{\mathcal{S}}_{min}$.

Thus, K^M must be below $K^{\mathcal{S}}_{max}$, which is assumption A5.

The above proof ensures that for $K \in [K^{\mathcal{S}}_{min}, K^M]$ non private benefit firms adopt a DP strategies. For levels of supply greater than K^*, non private benefit firms mix between risky and safe strategies. Thus market clearing in the refinancing period takes on the following form,

$$\overline{\varphi}\frac{A^I(\theta+\xi(1-\theta))}{S(1-\eta)} = \frac{A^I(1-\xi)(1-\theta)}{S^L(1-\eta)} \Longrightarrow S^L = \frac{(1-\theta)(1-\xi)}{\overline{\varphi}(\theta+\xi(1-\theta))}S := \phi'(\xi)S$$

where ξ is the fraction of non private benefit firms that take on risky strategies. Note that $\xi = 0$ for $K = K^M$, $\phi'(0) = \phi$, and $\phi'(\xi)$ is decreasing in ξ. Given the conjectured strategies, market clearing in the initial period is given by,

$$\frac{A(\theta+\xi(1-\theta))}{S(1-\eta)} = K$$

and non private benefit firms mixing condition (equation (9)) is equivalent to,

$$\begin{aligned}
(1-\alpha)\left(\frac{C^L}{S^L}-1\right) &= \frac{C^D(\overline{\varphi}, S^L)}{S} - 1 \iff \\
S\left(\alpha-(1-\alpha)\frac{(1-\theta)(1-\xi)}{(\theta+\xi(1-\theta))}\right) &= \alpha C^H + (1-\alpha)(1-\overline{\varphi})\lambda C^L - (1-\alpha)\overline{\varphi}\frac{(\theta+\xi(1-\theta))}{(1-\theta)(1-\xi)}C^L
\end{aligned}$$

To verify that this in fact will constitute an equilibrium, it must be checked that private benefit and non private benefit firms adopt the corresponding DP-mixing strategies. This analysis is equivalent to the DP strategy analysis, replacing ϕ with $\phi'(\xi)$ for conditions stemming from 18, $S^L > \lambda C^L$, $b^{''}$, and $b^{'''}$. Note that conditions in 18 and $b^{'''}$ are relaxed as ξ increases, which hold for $\xi = 0$. Intuitively $b^{'''}$ is relaxed since for a more severe fire sale relative to the initial price, the return from a safe levered strategy decreases faster as the difference between S and ηS^L increases. Condition stemming from equation $S^L > \lambda C^L$ and $b^{''}$ become tighter as ξ increases, thus there exists a ξ^* such that for all $\xi < \xi^*$ both these inequalities continue to hold. Intuitively, the return from a dry powder strategy increases for more severe fire sales, which requires a higher $b^{''}$ for private firms to risk default. Define $K^{\mathcal{SE}}_{max}$ the minimum between $K^{\mathcal{S}}_{max}$ and the asset supply such that the fraction of mixing firms are below ξ^*. Thus, for all $K \in (K^*, K^{\mathcal{SE}}_{max})$ firms enter a DP-mixing equilibrium, completing the proof.

■

Lemma 1. *Under the assumptions of Theorem 2, the comparative statics of the Dry Powder equilibrium for $K \in [K^{\mathcal{S}}_{min}, K^M]$ depends on the equilibrium equation*

$$S = \frac{A^I\theta}{K(1-\eta)}$$

with $S^L = \frac{1-\theta}{\overline{\varphi}\theta}S$.

The comparative statics of the Dry Powder Mixing equilibrium for $K \in (K^M, K^{\mathcal{SE}}_{max}]$ depends on the equilibrium equations,

$$\begin{aligned}
T_1 &= \frac{A^I p}{K(1-\eta)} - S = 0 \\
T_2 &= \Gamma(C^H, C^L, \alpha, \overline{\varphi}, \lambda) - (1-\alpha)\overline{\varphi}\frac{p}{1-p}C^L - S\left(\alpha-(1-\alpha)\frac{1-p}{p}\right) = 0
\end{aligned}$$

where $\Gamma(C^H, C^L, \alpha, \overline{\varphi}, \lambda) = \alpha C^H + (1-\alpha)(1-\overline{\varphi})\lambda C^L$ and $p = \theta + \xi(1-\theta) > \theta$ with $S^L = \frac{1-p}{\overline{\varphi}p}S$. If $\theta > 1-\alpha$, for all parameters $x \in \{A^I, K, \eta, \lambda, \alpha, C^H, C^L\}$ either

$$\frac{\partial T_1}{\partial x} = 0 \quad and \quad sgn\begin{pmatrix}\frac{\partial p}{\partial x} \\ \frac{\partial S}{\partial x}\end{pmatrix} = sgn\begin{pmatrix}\frac{\partial T_2}{\partial x} \\ \frac{\partial T_2}{\partial x}\end{pmatrix}$$

with sgn $\left(\frac{\partial S^L}{\partial x}\right) = sgn\left(-\frac{\partial T_2}{\partial x}\right)$, *or*

$$\frac{\partial T_2}{\partial x} = 0 \quad and \quad sgn\begin{pmatrix} \frac{\partial p}{\partial x} \\ \frac{\partial S}{\partial x} \end{pmatrix} = sgn\begin{pmatrix} -\frac{\partial T_1}{\partial x} \\ \frac{\partial T_1}{\partial x} \end{pmatrix}$$

with sgn $\left(\frac{\partial S^L}{\partial x}\right) = sgn\left(\frac{\partial T_1}{\partial x}\right)$

Proof of Lemma 1

The first part of the Lemma is direct from the DP equilibrium characterization of Theorem 2 as well as the equilibrium equations T_1, T_2 for the DP Mixing equilibrium. Holding θ fixed, by inspection it can be appreciated that for any change in $x \in \{A^I, K, \eta, \lambda, \alpha, C^H, C^L\}$ results in either $\frac{\partial T_1}{\partial x} = 0$ or $\frac{\partial T_2}{\partial x} = 0$. Using the implicit theorem I can study how equilibrium variables S and p change with underlying parameters.

$$\begin{aligned}
\frac{\partial T_1}{\partial p} &= \frac{A}{K(1-\eta)} > 0 \\
\frac{\partial T_1}{\partial S} &= -1 < 0 \\
\frac{\partial T_2}{\partial p} &= -(1-\alpha)\left(\frac{\overline{\varphi}C^L}{(1-p)^2} + \frac{S}{p^2}\right) < 0 \\
\frac{\partial T_2}{\partial S} &= (1-\alpha)\frac{1-p}{p} - \alpha < 0
\end{aligned}$$

where the sign of the last partial derivative stems from the fact that $\theta > 1-\alpha$, for $\xi = 0\,(1-\alpha)\frac{1-p}{p} < \alpha$, and $\frac{1-p}{p}$ decreases with p. Denoting

$$M = \begin{bmatrix} \frac{\partial T_1}{\partial p} & \frac{\partial T_1}{\partial S} \\ \frac{\partial T_2}{\partial p} & \frac{\partial T_2}{\partial S} \end{bmatrix} = \begin{bmatrix} \frac{A}{K(1-\eta)} & -1 \\ -(1-\alpha)\left(\frac{\overline{\varphi}C^L}{(1-p)^2} + \frac{S}{p^2}\right) & (1-\alpha)\frac{1-p}{p} - \alpha \end{bmatrix}$$

Note that $det(M) < 0$. Thus applying the implicit function theorem,

$$\begin{pmatrix} \frac{\partial p}{\partial x} \\ \frac{\partial S}{\partial x} \end{pmatrix} = -M^{-1}\begin{pmatrix} \frac{\partial T_1}{\partial x} \\ \frac{\partial T_2}{\partial x} \end{pmatrix} = \frac{-1}{det(M)}\begin{bmatrix} (1-\alpha)\frac{1-p}{p} - \alpha & 1 \\ (1-\alpha)\left(\frac{\overline{\varphi}C^L}{(1-p)^2} + \frac{S}{p^2}\right) & \frac{A}{K(1-\eta)} \end{bmatrix}\begin{pmatrix} \frac{\partial T_1}{\partial x} \\ \frac{\partial T_2}{\partial x} \end{pmatrix}$$

Thus, for $x \in \{A^I, K, \eta\}$ we have $\frac{\partial T_2}{\partial x} = 0$ and

$$\begin{pmatrix} \frac{\partial p}{\partial x} \\ \frac{\partial S}{\partial x} \end{pmatrix} = \frac{-1}{det(M)}\begin{pmatrix} \left((1-\alpha)\frac{1-p}{p} - \alpha\right)\frac{\partial T_1}{\partial x} \\ (1-\alpha)\left(\frac{\overline{\varphi}C^L}{(1-p)^2} + \frac{S}{p^2}\right)\frac{\partial T_1}{\partial x} \end{pmatrix}$$

For $x \in \{\lambda, \alpha, C^H, C^L\}$ we have $\frac{\partial T_1}{\partial x} = 0$ and

$$\begin{pmatrix} \frac{\partial p}{\partial x} \\ \frac{\partial S}{\partial x} \end{pmatrix} = \frac{-1}{det(M)}\begin{pmatrix} \frac{\partial T_2}{\partial x} \\ \frac{A}{K(1-\eta)}\frac{\partial T_2}{\partial x} \end{pmatrix}$$

Finally, note that in both cases of the DP mixing equilibrium

$$\frac{\partial S^L}{\partial x} = \left(\frac{\partial S}{\partial x}\left(\frac{1-p}{p}\right) - \frac{\partial p}{\partial x}\frac{S}{p^2}\right)$$

therefore, in case $\frac{\partial T_2}{\partial x} = 0$ this implies,

$$\frac{\partial S^L}{\partial x} = \frac{-1}{det(M)\overline{\varphi}}\left((1-\alpha)\frac{\overline{\varphi}C^L}{p(1-p)} + \alpha\frac{S}{p^2}\right)\frac{\partial T_1}{\partial x}$$

and in case $\frac{\partial T_1}{\partial x} = 0$ this implies,

$$\frac{\partial S^L}{\partial x} = \frac{1}{det(M)\overline{\varphi}} \frac{S}{p} \frac{\partial T_2}{\partial x}$$

■