Asset Pricing with Endogenous Disasters*

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Abstract

We propose a model with endogenous disasters generated through a labor dynamics mechanism. The model is parsimonious, having only one continuous state variable as well as CRRA agents with reasonable risk aversion. In such a simple setting we solve for prices in closed form and show that we can account for the high equity premium and volatility observed in the U.S. stock market as well as for a low riskfree rate. Excess returns and volatility are predictable and dividend yields implied by our model constitute stronger predictors than the observed dividend yield or *cay*. Having generated disasters through a labor mechanism, we are able to validate our model by calibrating it to labor-specific data, such as labor's share of income, while testing its asset pricing predictions, such as the magnitude of the consumption drop in an economic collapse. In this vein, we find support for our model's implication that more capital intensive economies experience larger disasters.

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1 Introduction

Three major puzzles continue to receive the attention of financial economists: first, that expected excess returns of stocks relative to bonds are high relative to consumption growth (the "equity premium puzzle"; Mehra and Prescott [1985]); second, that stocks' volatility is higher than the volatility of dividends (the "excess volatility puzzle"; Shiller [1981]; Le Roy and Porter [1981]; Keim and Stambaugh [1986]; Campbell and Shiller [1988]; and Hodrick [1992]); and third, that the riskfree rate is small (the "riskfree rate puzzle"; Weil [1989]). One possibility to address the high equity premium, described in models by Reitz [1988], Veronesi [2004], and Barro [2006], is that the equity premium should allow for rare catastrophic events.[1]

We propose a model whose contribution to the Rietz-Barro literature is fourfold. First and foremost, in contrast to previous research, in our model disasters are *endogenous*. More precisely, disasters in our model are explained by labor dynamics and as such, the model may elucidate how asset prices and employment are related. Second, our setting is *parsimonious*: the agents have simple CRRA preferences with reasonable risk aversion, and there is only one state variable which is a continuous diffusion without exogenous jumps. Third, our model is *unifying* as our parsimonious setting addresses all the puzzles enumerated above. Fourth, as in Veronesi [2004], by using conditional moments of returns we are able to *empirically test* our model regardless of whether an actual disaster has been observed in the data.

Our economic setting shares a number of features with Diamond [1982] and Mastuyama [1991]. Like in Diamond [1982], our economy can be illustrated using a tropical island setting, and like in Matsuyama [1991] it has two sectors. In the first sector, labor climbs trees to pick coconuts. Trees are not a public good as in Diamond [1982] but represent the capital asset as in Lucas [1978]. The production technology in this sector is Cobb-Douglas, but as in Matsuyama [1991] and Diamond [1982] is directly proportional to total employment in the sector. This latter feature alludes to an economy producing specialized, non-homogeneous goods which must be first traded with other laborers in this sector before they are consumed. We further assume that the production technology is directly proportional to an exogenous productivity variable which on our island may be the weather. Productivity is the only state variable in our model and does not have jumps.

[1] These models are also referred to as "peso models." The reason lies in the collapse of the Mexican peso in 1994. The high peso premium observed prior to the collapse was explained by the small probability of a huge out-of-sample devaluation, which was eventually observed (see Danthine and Donaldson [1999]).

There is also an alternative production sector that does not require capital and is not subject to productivity shocks, which we call home production.

Laborers, as well as the owners of capital, are price takers and act in order to maximize their CRRA utility. With this specification, laborers may sometimes optimally elect to shift to home production. To see why this is the case, suppose that productivity decreases (i.e., trees produce less coconuts for the same amount of labor because the weather worsens). If this drop is large enough, as output will be lower, wages in the coconut picking sector may decrease to levels smaller than the wage paid in home production, and consequently the marginal laborer may optimally decide to stop working in coconut picking and move to home production instead. This will decrease employment in coconut picking, and will in turn further decrease output. Thus wages will decrease further, triggering a further decrease in employment in coconut picking in favor of home production. This cascading mechanism will unfold throughout the entire first sector until all laborers move into home production.[2] With such a move, rents on capital drop to zero, triggering a jump down in overall consumption. In other words, we generate an economic *disaster* in the sense understood in the peso literature. In contrast to the previous peso research, however, this negative jump in consumption occurs endogenously and via a well specified mechanism, while our state variable does not have any jumps. To further strengthen the endogenous character of the economic disaster in our model and prevent "sunspots" or self-fulfilling cataclysmic events (Cass and Shell [1983]; and Azariadis [1981]) from happening in our economy, we assume away the possibility that workers shift into home production arbitrarily.

In this setting, our state variable−the productivity−affects asset prices through three concurrent mechanisms. The first mechanism is as in Lucas [1978]: a reduction in output reduces prices because the level of expected cash flows is lower. The second mechanism is that lower productivity means that a disaster is more imminent, and hence the duration at which dividends are received is shorter.[3] The third mechanism stems from the fact that through a risk aversion effect, when a disaster is closer, the pricing kernel of the model becomes lower. From the fundamental pricing equation we have that $P \approx \mathbb{E}[MDT]$ (where P is price, M the discount factor, D the level of dividends, and T the duration dividends are paid). A decline in productivity induces a decline in

[2] A "cascade" would happen in reality but on our island, the shift of labor to home production is instantaneous.

[3] This is similar with having probabilities of disaster that are perfectly negatively correlated with consumption as in Gourio [2008a] or Wachter [2009].

all three D, T, and M, thereby decreasing prices substantially. Since a small change in D may result, through the coordination of these three mechanisms, in a big change in P, our model can accommodate small volatility of dividends with a high volatility of prices, thus addressing the volatility puzzle. Additionally, consumption and equity prices are correlated and in turn, because of the high volatility of prices relative to their fundamentals, the equity premium will be high. Moreover, our analysis shows that the sensitivity of prices to dividends is stronger when the economy is weaker, leading to counter-cyclical volatility and equity premium. Finally, we obtain these effects with a small riskfree rate.

We calibrate the model by following Barro [2006] or Mehra and Prescott [1985] by assuming an annual volatility of consumption of 3.57% and a relative risk aversion under 10. With these parameters we are able to provide a full resolution to both the excess volatility puzzle and the equity premium puzzle. In addition, given the fact that our disaster-inducing mechanism is fully specified, we are able to check the validity of our model in several independent ways in contrast to using only asset pricing data. For example, our model produces the simple implication that more capital intensive economies experience larger disasters. We use cross-sectional country data reported in Gollin [2002] for labor's share of income and disaster magnitudes from Barro and Ursua [2009] to check this prediction, and find support for it. For US data, in particular, since in our model the drop in consumption when a disaster occurs is determined only by the capital's share of income, we predict this drop to be 36%. This is in the ballpark of the calibrations in Barro [2006]. Moreover, when calibrating recoveries in our model, once we fix the median time till recovery, the growth from the trough is fixed as opposed to remaining a free parameter. We observe that when calibrating our model to match observed durations of disasters, the growth from the trough is consistent with what is reported, for example, in Gourio [2008b]. Finally, we are agnostic on whether the Great Depression was actually a disaster or not. From this perspective, we present two distinct calibrations of our model. In one of the calibrations, the economy recovers as soon as recovery is feasible (we call this the "social planner" calibration, since it assumes that a social planner keeps the economy playing the equilibrium which is socially optimal when multiple static equilibria are feasible). In this calibration, the Great Depression is in fact a disaster, consistent with Barro [2006]. In the other calibration, the economy needs a "big push" (Murphy, Shleifer and Vishny [1989]) in order to recover from a disaster. In this latter calibration, in the Great Depression the U.S. economy got very close to, but has not experienced a disaster.

Like in Veronesi [2004], by using conditional moments of returns we are able to test our model regardless of whether an actual disaster has been observed in the data. We test whether conditional volatility and returns follow our predictions and find strong support for our model. In fact, the dividend yield implied by our model completely subsumes the predictive power of the observed dividend yield and of another traditional predictor, namely, the *cay* variable of Lettau and Ludvigson [2001]. Predictability of returns in our model does not make the agents irrational, as it is driven by predictable changes in expected returns.

The paper is organized as follows: Section 2 presents the model of the economy and solves for dividends, consumption and asset prices levels. Section 3 discusses the testable predictions generated from our model. Section 4 calibrates the model and tests it, and Section 5 offers our conclusions.

2 The Model

In this section we introduce our model, which is that of a dynamic production economy. This setting is a simplified version of Matsuyama [1991], which is in turn inspired by Diamond [1982]. In our economy, labor and capital meet at the firm level and output is generated. Consistent with empirical observations on wages rigidity in the labor literature (e.g., Kramarz [2001], Dickens et al. [2006]), we assume that laborers in our economy have a reservation wage. The production function in the first sector depends on an externality and, as in Diamond [1982], our economy exhibits multiple static equilibria in each time period. These equilibria differ by labor's choice to work with current technologies, or realize the reservation wage by working in a different sector. Since labor's share of income is countercyclical (e.g. Gomme and Greenwood [1995]), this alternative economic sector ought to be less capital-intensive. Consistent with this stylized fact, and for tractability reasons, we assume that when labor works for the reservation wage, capital is absent from the production function. The dynamics of our economy are driven by productivity which we assume to a continuous diffusion, i.e., a stochastic process *without jumps*. In this parsimonious model, in which random equilibrium choices are assumed away, we show that at times, labor may shift entirely out of the capital-intensive productive sector. When this happens, as capital does not enter the production function, output decreases and consumption exhibits a jump down. Thus, in our economy, a disaster, or equivalently a *peso* phenomenon occurs: this effect consists of a shift from the "good" static equilibrium, in which all of the labor force is employed by the firms which produce, to the "bad" equilibrium in which labor is working for reservation wages.[4] This peso effect is generated without any assumptions of jumps in the state variable. Finally, we assume that a social planner (the "invisible hand") keeps the economy playing that particular static equilibrium which is socially optimal. This prevents arbitrary disasters from happening in our model.

We start by presenting our model's assumptions.

2.1 Assumptions

There is a single perishable consumption good, which also serves as the numeraire.

There are infinitely many households in the economy, indexed by $j \in \mathcal{J}$, all infinitely lived. Households are endowed with a flow of labor services according to a measure $L_t^{\mathcal{E}}$ defined on $\mathcal{J}$ with a total mass of 1. Households are also endowed with a capital asset K with a total mass of 1. In our model, K cannot be accumulated and does not depreciate.

There are two sectors in the economy. In one sector there are infinitely many firms, indexed by $i \in \mathcal{I}$. Firms rent labor services and the use of capital goods and produce the consumption good. Firms do not own any assets and do not trade in financial assets. Firms can only rent labor services and technology rights at the prevailing spot prices and sell their output at the spot price of the consumption good (which is also the numeraire). The other sector is a "home production" technology. The production functions characterizing both sectors will be detailed in the next section.

The assumption that households supply both labor and capital is an oversimplification that does not reduce the generality of our model. As will become apparent below, the structure of ownership is irrelevant in our model.

[4]The designation of the equilibrium in which labor is employed in home production as "bad" reflects the disaster associated with the occurrence of this equilibrium. Inherently, this equilibrium is not a "bad" equilibrium as when it is played by the economy, it is the only feasible equilibrium.

There are securities $s \in S$. An example is the capital asset (i.e., $K \in S$), which is the only security with a total supply of 1. All the other securities (e.g., bonds) are in zero supply.[5]

Finally, our economy is intertemporal. We proceed now to describe the technologies employed in each sector of the economy.

2.2 Technologies

In the first sector of the economy, each firm i uses the same production technology at time t. The production function is modeled to have a Cobb-Douglas form, with a externality originating in Diamond [1982]. As outlined in Diamond [2011], the particular functional form employed in the production function allows for an interaction between the labor market and the output market. To create such an interaction, the standard Cobb-Douglas production function is multiplied by a function of labor. Since the employment rate is procyclical,[6] the simplest such function is the linear one. With these considerations, for each firm i this production function is given by:

$$\begin{aligned} F_t(L_{it}, K_{it}) &= \theta_t \bar{L}_t L_{it}^{1-a} K_{it}^a \\ \bar{L}_t &= \textstyle\sum_{i \in \mathcal{I}} L_{it}, \end{aligned} \tag{1}$$

where L_{it} and K_{it} respectively are the amounts of labor and capital supplied to firm i at time t. The production function of each firm is directly proportional with the amount of labor $\bar{L}_t$ supplied in aggregate to the first sector. Intuitively, when less labor is supplied in aggregate, the trading costs each firm incurs to sell its output are higher, and therefore the usable output generated by each firm is lower.[7] a is the capital's share of income. An economy where the services sector is preponderant would serve as a good example for understanding the externality in this

[5]The model ignores the possibility that households may trade their current and future labor endowment. Extending the set of securities to allow such trades is relatively straightforward but notationally cumbersome.

[6]See, for example, Barro [2007], Figure 9.7.

[7]The steps to arrive at this functional form of the production functions are as follows. We first start with a simple Cobb-Douglas production function, $F_t^1(L_{it}, K_{it}) = L_{it}^{1-a} K_{it}^a$. This is the "gross" output of the firm. However, as in Diamond [1982] each firm needs to find a trade partner (other firms) in order to sell its output. This is possible to the extent that other firms are operating. We assume that this search cost is a fraction of the output of the firm that is proportional with total employment in firms. We assume that the search cost is a fraction $(1 - \bar{L}_t)$ of the gross output. This results in a net output equal to $F_t^2(L_{it}, K_{it}) = \bar{L}_t L_{it}^{1-a} K_{it}^a$. To this we add an exogenous productivity factor θ_t (modeling the fact that sometimes the coconut trees make more coconuts for the same amount of labor and capital invested in the tree) and thus obtain the production function of equation (1).

production function. In such an economy, demand for the production good is naturally lower when unemployment is higher, and in equilibrium production adjusts accordingly.

The variable θ, the sole state variable of our model, is a continuous, positive diffusion. We shall remain vague about the modeling choices for θ for now, but we will discuss these choices in a future section. We further point out that, in contrast to Weitzman [2007], in our model there is no uncertainty about the parameters, or the value of θ.

We denote the reservation wage for labor by $Z > 0$. Consistent with evidence that the labor's share on income is countercyclical,[8] the technology employed to generate the reservation wage for labor must be less capital intensive than the production function in our first productive sector. For tractability, we assume that when labor works for the reservation wage, the technology employed requires no capital, i.e., a production function of the form:

$$F(L) = ZL. \tag{2}$$

We emphasize that both technologies produce the *same* type of consumption good, in particular, the goods generated through home production are part of the overall consumption.

Next, we will describe the agents' preferences.

2.3 Preferences

The agents in the economy have time-separable utilities and are not satiated. The utility of the representative agent j from a consumption stream $C = (C_t)_{t\geq 0}$ is given by:

$$U(C) = \mathbb{E}\left[\int_0^\infty e^{-\delta t} u\left(C_t\right) dt\right], \tag{3}$$

where δ is a discount factor. In the analysis of the real side of the economy, all we require is that $u(\cdot)$ is a strictly increasing function. We shall use constant relative risk aversion (CRRA) preferences, i.e., $u(C) = (1-\gamma)^{-1}C^{1-\gamma}$ with $\gamma > 1$. Constantinides [1982] shows that in an economy with heterogenous agents with different risk aversion coefficients and different initial endowments there exists a single representative agent, and we shall use his result to abstract away from modeling individual preferences.

[8]For example, Gomme and Greenwood [1995] cite this as a stylized fact about the labor's share of income. They report a correlation of -0.37 between labor's share of income and GNP (both detrended using a Hodrick-Prescott filter).

We continue by describing the households' and firms' optimization problems.

2.4 Households' optimization problem

Each household j is endowed at time t with an amount of labor $L^{\mathcal{E}}_{jt}$ of which it decides to supply $L^{\mathcal{S}}_{jt}$ to the first sector by working for firms. The remainder $L^{\mathcal{E}}_{jt} - L^{\mathcal{S}}_{jt}$ works in the technology which pays the reservation wage. Each household may supply capital and may hold the securities $s \in S_j$, in proportion of π_{jst} (one of these securities may be the capital asset K). S_j is a subset of S, the set of all securities.

The total flow of income that household j receives at time t is the sum of wages paid by firms, the output of labor assigned to home production, and the dividends paid by the securities it holds:

$$D_{Lt} L^{\mathcal{S}}_{jt} + Z \left(L^{\mathcal{E}}_{jt} - L^{\mathcal{S}}_{jt} \right) + \sum_{s \in S_j} D_{st} \pi_{jst}. \tag{4}$$

This income is used for consumption at a rate C_{jt} or to finance rebalancing of the household's portfolio. Therefore, at each time t the budget constraint faced by a household is

$$C_{jt} dt + \sum_{s \in S_j} P_{st} d\pi_{jst} \leq \left[D_{Lt} L^{\mathcal{S}}_{jt} + Z \left(L^{\mathcal{E}}_{jt} - L^{\mathcal{S}}_{jt} \right) + \sum_{s \in S_j} D_{st} \pi_{jst} \right] dt. \tag{5}$$

Households are price takers. In that respect, a household observes the current wage D_{Lt}, the prices P_{st}, as well as the dividends generated by ownership in the capital asset D_{Kt}, and maximizes its utility from consumption, as defined in equation (3), by selecting the amount of labor supplied to the firms $L^{\mathcal{S}}_{jt}$ and by deciding what securities (including the capital asset K) to hold in its portfolio. As the utility u is increasing, the household must first maximize the upper bound on consumption, which it controls, for example, through the labor supply $L^{\mathcal{S}}_{jt}$. The following proposition describes the amount of optimal amount of labor supplied by a household:

Lemma 1 *The optimal decision of the representative household may be described as follows. At any time t:*

1. *When $D_{Lt} > Z$, each household prefers to rent out all its labor endowment to the first sector (i.e., resulting in $\bar{L}_t = 1$ in the entire economy).*

2. *When $D_{Lt} < Z$, each household uses all of its labor endowment in the technology producing the reservation wage (i.e., $L_{jt} = 0$ for all j).*

3. *When $D_{Lt} = Z$, a household may rent out any amount a labor $L^{\mathcal{S}}_{jt} \in \left[0, L^{\mathcal{E}}_{jt}\right]$ to the first sector and use the remainder $L^{\mathcal{E}}_{jt} - L^{\mathcal{S}}_{jt}$ to generate the reservation wage (i.e., any $\bar{L}_t \in [0,1]$ may result as a solution to the households' optimization problem).*

We now turn to describing the firms' optimization problem.

2.5 Firms' optimization problem

Any firm's objective is to maximize profits from production at each point in time. The firm takes as exogenous the level of rents it has to pay the production factors (capital and labor) and chooses its levels of demand for labor $L^{\mathcal{D}}$ and respectively capital $K^{\mathcal{D}}$ that maximize its output:

$$(L^{\mathcal{D}}_{it}, K^{\mathcal{D}}_{it}) = \underset{L_{it}, K_{it}}{\arg\max}\ F_t(L_{it}, K_{it}) - K_{it} D_{Kt} - L_{it} D_{Lt}. \tag{6}$$

The firm decides on $L^{\mathcal{D}}_{it}$ and $K^{\mathcal{D}}_{it}$ at time t. At the time the decision is made, D_{Kt}, D_{Lt}, and θ_t are observable. The firm also has an expectation of the aggregate labor supplied to the first economic sector, which we denote by $\bar{L}^e_{it}$.

The firm's first-order conditions are derived from equations (1) and (6):

$$\frac{L^{\mathcal{D}}_{it}}{K^{\mathcal{D}}_{it}} = \left((1-a)\frac{\theta_t \bar{L}^e_{it}}{D_{Lt}}\right)^{1/a} \tag{7}$$

$$D_{Kt} = a\theta_t \bar{L}^e_{it}\left(\frac{L^{\mathcal{D}}_{it}}{K^{\mathcal{D}}_{it}}\right)^{1-a}. \tag{8}$$

Note that our technology exhibits constant returns to scale. Consequently, wages and dividends paid to capital are driven up until the firms' profit is zero. This makes the firm ownership structure irrelevant.

2.6 Equilibrium

A rational expectations equilibrium comprises security prices P_{st}, $\ s \in S$, wages D_{Lt}, dividends paid to the capital asset D_{Kt}, and expected labor level dedicated to the first sector $\bar{L}^e_{it}$, such that at any time t:

(A) Demand and supply of labor in the first sector are equal:

$$\sum_{i\in\mathcal{I}} L_{it}^{\mathcal{D}} = \sum_{j\in\mathcal{J}} L_{jt}^{\mathcal{S}} \qquad \text{for all } t.$$

(B) The expected aggregate labor dedicated to the first sector is realized:

$$\bar{L}_t = \bar{L}_{it}^e \qquad \text{for all } i, t.$$

(C) Demand and supply of capital are equal, provided some capital is needed in the first sector (recall that no capital is required for home production):

$$\sum_{i\in\mathcal{I}} K_{it}^{\mathcal{D}} = 1 \qquad \text{for all } t \text{ for which } \sum_{i\in\mathcal{I}} K_{it}^{\mathcal{D}} > 0.$$

(D) The security market clears:

$$\begin{aligned} \sum_{j\in\mathcal{J}} \pi_{jKt} &= 1 \qquad \text{for all } t \\ \sum_{j\in\mathcal{J}} \pi_{jst} &= 0 \qquad \text{for all } t, \text{and for all } s \neq K. \end{aligned}$$

With only the conditions (A) - (D), Matsuyama [1991] shows that multiple equilibria may exist. In order to avoid multiple equilibria when this case is possible, we are making an assumption designed to pin down the equilibrium:

(E) When there exist multiple solutions $(P_{st}, D_{Lt}, D_{Kt}, \bar{L}_t)$ satisfying (A)−(D), a "social planner" selects the Pareto-optimal solution, i.e., that equilibrium for which total consumption[9] is maximized:

$$(P_{st}, D_{Lt}, D_{Kt}, \bar{L}_t) = \arg\max \left\{ \bar{L}_t D_{Lt} + (1 - \bar{L}_t^e) Z + D_{Kt} \right\} \text{ for each } t.$$

[9] Total consumption $C(t)$ is equal to the sum of dividends paid to the capital asset and wages. Wages are paid by firms (in proportion of $\bar{L}_t$) and by home production (in proportion of $1 - \bar{L}_t$).

We proceed to characterizing the equilibrium in two stages. The first stage characterizes the rents D_{Lt} and D_{Kt}, and an aggregate expected labor level dedicated to the first sector $\bar{L}^e_{it}$, so that conditions (A)−(D) are met. As we shall show, conditions (A)−(D) yield multiple static equilibria and (E) offers a mechanism to select among them in a way that makes the dynamic equilibrium unique. The second stage characterizes security prices such that the security market clears.

We start by first exploring the possible static equilibria, i.e., those triplets $(D_{Lt}, D_{Kt}, \bar{L}^e_{it})$ satisfying conditions (A)−(D) above.

First, since the firms may scale the production arbitrarily, we may assume that they do so as long as there is capital to be raised in order to enable production. Since the total mass of available capital is 1, condition (C), which states that the market for K should clear, is always met. Furthermore, condition (D) will be useful when we calculate security prices, but since securities do not influence the real economy in our model we may assume that condition (D) is met. Thus, in order to characterize those equilibria for which conditions (A)−(D) are met, it is enough to focus on (A) and (B) only.

Using Lemma 1 we can now characterize the set of equilibria satisfying conditions (A) - (D). The following result is proved in the Appendix:

Lemma 2 *At any time t the following three static equilibria satisfy conditions (A) to (D) above:*

1. *An equilibrium in which all labor is working in the sector generating the reservation wage (and none in the first sector), i.e., $\bar{L}_t = 0$, always exists. In this equilibrium $D_{Kt} = 0$ and $0 < D_{Lt} < Z$.*

2. *An equilibrium in which all labor is dedicated to the first sector (and none to the sector generating the reservation wage), i.e., $\bar{L}_t = 1$, exists if and only if $\theta_t \geq (1-a)^{-1}Z$. In this equilibrium $D_{Kt} = a\theta_t$ and $D_{Lt} = (1-a)\theta_t$.*

3. *A mixed equilibrium in which some but all labor is dedicated to the first sector (i.e., $0 < \bar{L}_t < 1$) exists if and only if $\theta_t > (1-a)^{-1}Z$. In this equilibrium $D_{Kt} = a\theta_t$ and $D_{Lt} = Z$.*

We contrast the existence of multiple static equilibria satisfying conditions (A)−(D) with the case of a single equilibrium encountered in a single sector production economy. In the classical production economy with one productive sector (i.e., no reservation wage) and without labor externalities, workers are employed in the productive sector and total output θ_t gets divided proportionally between labor and capital. That is,

$$\begin{aligned} D_{Kt}^{baseline} &= a\theta_t \\ D_{Lt}^{baseline} &= (1-a)\theta_t, \ \forall t. \end{aligned} \tag{9}$$

By contrast, the mechanism responsible for the existence of multiple static equilibria is similar to the one in Diamond [1982]: the production function of *each* firm depends on the aggregate level of labor supplied to the first sector. If the marginal supplier of labor decides not to rent out her services to firms, the result is a general dropout in the firms' production. In turn, there are fewer wages to be paid to the next marginal laborer, and hence, the wages paid to labor in the first sector drop to the levels that are below the reservation wage.

After characterizing those equilibria satisfying conditions (A)$-$(D), we can now turn to finding those which in addition satisfy (E). In order to do so, we first observe that at any time t for which $\theta_t < (1-a)^{-1}Z$, only the equilibrium in which all labor is employed in home production is feasible, and thus this is also the Pareto-dominating equilibrium. For those times t when $\theta_t \geq (1-a)^{-1}Z$, however, all three equilibria described in Lemma 2 are feasible. Criterion (E) selects that particular equilibrium in which the rents paid out to labor and capital combined are maximized. The smallest total rent is paid out in the equilibrium in which all labor works in the home production technology; this total rent is equal to Z. The next smallest rent is paid out when the partial equilibrium is played; in this case, labor and capital combined receive $\bar{L}_t a\theta_t + (1-\bar{L}_t)Z$. Since in this case $\bar{L}_t < 1$ (that is, not all labor works in the first sector) and $\theta_t \geq (1-a)^{-1}Z$ (a condition for the partial equilibrium to exist), we observe that the total rent paid in the partial equilibrium is smaller than θ_t. However, θ_t is the total rent paid out in the equilibrium in which all labor is dedicated to the first sector. Thus, when $\theta_t \geq (1-a)^{-1}Z$, the Pareto-dominating equilibrium is the equilibrium in which all laborers are employed in the first sector. We can thus completely characterize the equilibria satisfying (A)$-$(E) as follows. As long as $\theta_t \geq (1-a)^{-1}Z$, all labor is employed in the first sector. The rent on capital is $D_{Kt} = a\theta_t$, and the rent on labor is $D_{Lt} = (1-a)\theta_t$. When $\theta_t \leq (1-a)^{-1}Z$, all labor is employed in home production. The rent on capital is $D_{Kt} = 0$, and the rent on labor is the reservation wage, $D_{Lt} = Z$. Therefore, the nature of the equilibrium is described by the position of the state variable θ relative to the barrier $(1-a)^{-1}Z$.

Having detailed our equilibrium, we turn to modeling choices for the productivity θ.

2.7 Productivity models and dynamic equilibrium

In our model, the economic dynamics are represented by the sole state variable, the productivity θ_t. Therefore, modeling the productivity variable is ultimately responsible for the dynamics of our economy, and for that of security prices. Before making a modeling choice, we recall first that total consumption in our model is θ_t when all labor is employed in the first economic sector, and is equal to Z when labor works for the reservation wage. This ease the burden of a choice as we turn to standard modeling assumptions on either consumption or productivity.

To start with, Nelson and Plosser [1982] postulate that productivity is log-normal. Given our equilibrium, detailed in the preceding section, this in turn implies that conditional on the labor working in the productive sector, as well as in the baseline case, consumption is log-normal as well. This is consistent with traditional modeling choices in the asset pricing literature[10] With mean consumption growth of μ and volatility of consumption σ, this yields the process $\theta_t = \theta_0 \exp\left((\mu - \sigma^2/2)t + \sigma W_t\right)$, where W is a standard Brownian motion. With this standard choice of the productivity variable, however, relevant asset pricing quantities turn non-stationary in our model.[11] The reason for this non-stationarity is that while conditional on disasters not happening, consumption growth is stationary as in the standard asset pricing setting, its variance increases in time and thus the frequency of disasters in our economy decreases.

In order to design a model in which asset pricing quantities are stationary, we need to employ a state variable that besides being continuous and positive is also stationary. As shown in Doob [1942], such a process must be a function of an Ornstein-Uhlenbeck diffusion. That is,

$$\begin{aligned} \theta_t &= \theta_0 e^{X_t}, \\ dX_t &= k(\overline{X} - X_t)dt + \sigma dW_t. \end{aligned} \tag{10}$$

[10] See, for example, Cochrane [2008].

[11] We wish to thank the editor, Pietro Veronesi, as well as an anonymous referee for pointing this out to us.

We select the normalizing constant θ_0 so that a peso event (characterized by θ_t hitting the barrier $(1-a)^{-1}Z$) coincides with the state variable X hitting zero. This amounts to

$$\theta_0 = (1-a)^{-1}Z. \tag{11}$$

We note than in the case of $k = 0$, we retrieve the case of log-normal productivity. If $k = 0$ and there are no labor externalities, our model reduces to the standard asset pricing model, where prices of the risky assets are claims on log-normal dividend payouts.

Having selected the state variable determining the dynamics of our economy, we can

Proposition 1 *There exists a unique equilibrium satisfying conditions (A)−(E). Employment and rents are characterized as follows:*

- *If $X_t \geq 0$, all labor is employed in the first economic sector. The rent on capital is $D_{Kt} = a\theta_t$, and the rent on labor $D_{Lt} = (1-a)\theta_t$. Total consumption is equal to θ_t.*
- *If $X_t < 0$, all labor is employed in home production. The rent on capital is $D_{Kt} = 0$, and the rent on labor is $D_{Lt} = Z$. Total consumption is equal to Z.*

From the proposition above we can conclude that our economy exhibits a *peso* characteristic: as long as $X_t \geq 0$, labor and capital meet in a productive economy and the consumption good is produced. Total consumption is $D_{Kt} + D_{Lt} = \theta_t = (1-a)^{-1}Ze^{X_t}$. When X_t is close to zero but positive, consumption gets close to $(1-a)^{-1}Z$, which is strictly greater than Z as a is between zero and one. As soon as X_t reaches zero, labor shifts to the less productive sector generating a reservation wage, and total consumption becomes Z. Thus, as X_t hits the zero barrier, total consumption experiences a drop of $(1-a)^{-1}Z - Z$. Therefore, consistent with the peso literature, X_t reaching zero triggers a *disaster*, or a *peso event*. In contrast to previous work, we generate a disaster without modeling jumps in our state variable. For notational convenience, we will keep track of the random times when labor switches economic sectors. We assume that initially, our economy starts with labor being employed in the capital intensive sector. We denote by T_1 the first time X_t hits zero, i.e., the first time labor shifts into the sector generating only reservation wages; in other words, T_1 is the time of the first disaster. We denote by $T_2 > T_1$ the time the economy recovers from the first disaster, by $T_3 > T_2$ the time of the second disaster, and so on.

Thus, T_{2n+1} are the random times when the economy shifts into the disaster state, while T_{2n} are the recovery times.

We continue by presenting security prices, risk premia and volatilities in our setting.

2.8 Security prices

In this subsection we compute closed form security prices of any security s at any time t using the Euler equation:

$$P_{st}u'(C_t) = \mathbb{E}_t\left[\int_t^{\infty} e^{-\delta(l-t)}u'(C_l)\,D_{sl}dl\right], \tag{12}$$

where C_t is the equilibrium consumption level of the representative agent at time t. Alternatively we shall employ the notation $P_{st} = P_s(X_t)$, to reflect dependence of the state variable X_t underlying our system's dynamics.

The calculation in equation (12) is complicated by the fact that the stream of cash flows D_{st} depends on the state of the economy, specifically, on whether labor works in the first sector or makes the reservation wage.

We start by deriving the term structure in our economy, then the price of the capital asset.

2.8.1 Term structure

We denote by $B(\tau, W_t)$ the price of a bond of maturity τ and by $y(\tau, W_t) = -\ln(B(\tau))/\tau$ its annualized yield at time t. We then have the following:

Proposition 2 *Define:*

$$\begin{aligned} m_{t,\tau} &:= (\overline{X} - X_t)(1 - e^{-k\tau}) \\ v_{\tau} &:= \frac{\sigma\sqrt{e^{2k\tau}-1}}{\sqrt{2k}e^{k\tau}}. \end{aligned} \tag{13}$$

Then the following describes the term structure of our model:

Conditional on labor being employed in the first sector (i.e., $\bar{L}_t = 1$), we have the following:

1. *The yield of the bond with maturity τ is:*

$$y(\tau, X_t) \quad = \delta - \frac{1}{\tau} \; \log \left[B \; e^{\gamma X_t} \; \mathcal{N}(-(m_{t,\tau} + X_t)/v_\tau) + e^{-m_{t,\tau}+(\gamma\sigma)^2/2} \; \mathcal{N} \left(\frac{X_t + m_{t,\tau} - \gamma v_\tau^2}{v_\tau} \right) \right]. \tag{14}$$

where $B = (1-a)^{-\gamma}$ and $\mathcal{N}(\cdot)$ represents the cumulative normal distribution.

2. *The riskfree rate is given by*

$$r(X_t) := \lim_{\tau \downarrow 0} \; y(\tau, X_t) = \delta - \frac{(\sigma\gamma)^2}{2} + k\gamma(\overline{X} - X_t). \tag{15}$$

Conditional on the labor working in the less-capital intensive sector, we have the following:

1. *The yield of a bond with maturity τ if given by:*

$$y(\tau, X_t) \quad = \delta - \frac{1}{\tau} \; \log \left[\mathcal{N}(-(X_t + m_{t,\tau})/v_\tau) + \tfrac{1}{B} \; e^{-X_t - m_{t,\tau}+(\gamma\sigma)^2/2} \; \mathcal{N} \left(\frac{X_t + m_{t,\tau} - \gamma v_\tau^2}{v_\tau} \right) \right]. \tag{16}$$

2. *The riskfree rate is constant and equal to δ.*

We continue by calculating the price of the risky asset.

2.8.2 The price of the risky asset

For notational simplicity, when we calculate the price of the capital asset we shall drop the subscript K from the expression of the price, that is, we will denote $P_{Kt} = P_K(X_t)$ by P_t or $P(X_t)$.

In order to calculate the price of the capital asset, we proceed as follows. First, from the Euler equation observe that the prices P_s of any security s at the current time t and prices at any future time $t' > t$ are linked by:

$$P_s(X_t)u'(C_t) = \mathbb{E}_t \left[\int_t^{t'} e^{-\delta(l-t)} u'(C_l) \, D_{sl} dl + e^{-\delta(t'-t)} u'(C_{t'}) \, P_s(X_{t'}) \right]. \tag{17}$$

The formula above can be applied by taking the times t and t' to be those when the labor transitions between the productive sector and the sector ensuring the reservation wage. Recall that $T_{1,3,5,\ldots}$ are the random times when labor crosses from the productive sector into the sector generating the reservation wage, and $T_{2,4,\ldots}$ the times in which labor crosses back to the productive sector. Applying the formula above for the stopping times $t = T_{1,3,\ldots}$, $t' = T_{2,4,\ldots}$ and then for $t = T_{2,4,\ldots}$, $t' = T_{1,3,\ldots}$, and observing that $P^g := P_{T_{2n}}$ and $P^b := P_{T_{2n+1}}$ are independent of n, we can first calculate the prices at the times when labor shifts between sectors.[12] The following is proved in the Appendix:

Proposition 3 *Let the constant $B = (1-a)^{-\gamma}$, and let $g(\cdot)$ be the solution of Equation (24) described in the Appendix. Then the price of the risky asset at the time when labor shifts out of the capital intensive sector is given by:*

$$P^b = \begin{cases} \dfrac{B}{B^2-1}\;\dfrac{aZ}{1-a}\;[g(0)-1] & \text{if } k > 0 \\[2ex] \dfrac{aZ[\sigma(1-\gamma)+\sqrt{2\delta}]}{2B(1-a)[\delta-\sigma^2(\gamma-1)^2/2]\sqrt{2\delta}} & \text{if } k = 0. \end{cases} \tag{18}$$

Applying formula (17) again, for any time t and for $t' = T_1$ (and noting that $P_{T_1} = P^b$), we can readily obtain the price of the capital asset for those times t when labor works in the first sector. Applying the pricing formula for any time t when labor works in home production and for $t' = T_2$, we can obtain the price in the state of the economy in which all labor is dedicated to the home production sector. The following is proved in the Appendix:

Proposition 4 *Let $B = (1-a)^{-\gamma}$, P^b defined in Proposition 3, $\ell(\cdot)$ the function described in Lemma A.1 and $g(\cdot)$ the function described in Lemma A.2 and equation (24).*

Conditional on labor being employed in the first sector at time t (i.e., $T_{2n} \le t < T_{2n+1}$ for some $n = 0, 1, 2, \ldots$, with $T_0 = 0$), the following hold:

1. *The price of the risky asset is:*

[12] Our notation $P^b = P_{1,3,\ldots}$ is explained by the fact that at $T_{1,3,\ldots}$ the economy shifts to the *bad* equilibrium where labor only makes the reservation wage. Similarly at $T_{2,4,\ldots}$ the economy shifts to the *good* equilibrium where all labor is employed in the first economic sector, and this switch to recovery explains the notation P^g.

$$P(X_t) = \begin{cases} \left[\dfrac{aZ}{1-a}g(0) + BP^b\right] e^{\gamma X_t}\ell(X_t) - e^{X_t}g(X_t) & \textit{if } k > 0 \\ \\ \left[\delta - \sigma^2(\gamma-1)^2/2\right] e^{X_t} - \left(\delta - \sigma^2(\gamma-1)^2/2 - BP^b\right) e^{(\gamma-\sqrt{2\delta}/\sigma)X_t} & \textit{if } k = 0 \end{cases} \tag{19}$$

2. *Let $D_K(X_t)$ be the rate of dividends paid to the risky asset.*[13] *The volatility of the returns of the risky asset is given by:*[14]

$$Vol(X_t) = \begin{cases} \sigma\gamma + \dfrac{e^{\gamma X_t}\left[\dfrac{aZ}{1-a}g(0) + BP^b\right]\ell'(X_t) - e^{X_t}g'(X_t) - (1-\gamma)e^{X_t}g(X_t)}{\left[\dfrac{aZ}{1-a}g(0) + BP^b\right] e^{\gamma X_t}\ell(X_t) - e^{X_t}g(X_t)} & \textit{if } k > 0 \\ \\ (\sigma\gamma - \sqrt{2\delta}) + \dfrac{2}{\sigma(\gamma-1)+\sqrt{2\delta}}\ \dfrac{D_K(X_t)}{P(X_t)} & \textit{if } k = 0 \end{cases} \tag{20}$$

3. *The expected excess return (the drift) of the risky asset is given by:*

$$\mu(X_t) - r(X_t) = \sigma\gamma\ Vol(X_t) \quad \textit{for all } k \geq 0. \tag{21}$$

4. *The Sharpe ratio of the returns of the risky asset is constant and equal to $\sigma\gamma$.*

Conditional on labor working only for reservation wages at time t (i.e., $T_{2n-1} \leq t < T_{2n}$ for some $n = 1, 2, ...$), the following hold:

1. *The price of the risky asset is:*

$$P(X_t) = \begin{cases} P^b\ \ell(X_t) & \textit{if } k > 0 \\ \\ P^b\ e^{\frac{\sqrt{2\delta}}{\sigma}X_t} & \textit{if } k = 0. \end{cases} \tag{22}$$

[13]From Proposition 1, $D_K(X_t) = a\theta_t = a\theta_b e^{X_t}$ when labor is employed in the first sector.

[14]The Appendix details closed form expressions for the derivatives $\ell'(\cdot)$ and $g'(\cdot)$ of the functions $\ell(\cdot)$ and $g(\cdot)$.

2. *The volatility of the risky asset is:*

$$Vol(X_t) = \begin{cases} \sigma \dfrac{\ell'(X_t)}{\ell(X_t)} & \text{if } k > 0 \\ \\ \sqrt{2\delta} & \text{if } k = 0. \end{cases} \tag{23}$$

3. *The expected excess return of the risky asset is equal to zero.*

4. *The Sharpe ratio is equal to zero.*

We continue now by deriving empirical implications of our model and by showing that the asset prices we derived are consistent with a variety of asset pricing stylized facts.

3 Empirical predictions

Having solved for prices of securities, we now turn to the predictions generated by our model. In the first section, we investigate the validity of our labor mechanism. We then demonstrate that our parsimonious model delivers many asset pricing stylized facts in a unified setting.

3.1 Labor's share of income and disasters

From Proposition 1, we can infer the size of a consumption drop in disasters. Precisely, a disaster translates into a decline of $Z/(1-a) - Z$ in aggregate consumption. In relative terms, this represents a $[Z/(1-a) - Z]/[Z/(1-a)] = a$ drop in total consumption during a disaster. Therefore, our model has a very simple testable implication:

Proposition 5 *More capital intensive economies experience larger disasters.*

3.2 Volatility

The next results can be easily (but tediously) derived from formula (20). The complete proofs are in the Appendix.

Proposition 6 *Conditional on labor being fully employed in the first sector:*

1. *The volatility of returns is higher than the volatility of dividends for any degree of risk aversion, including risk neutrality.*

2. *The volatility is a non-increasing function of prices.*

3. *When the state variable is a random walk (i.e., $k = 0$), the volatility is an affine function of dividend yields.*

4. *Volatility is persistent.*

The model predicts that prices are more volatile than dividends, thereby addressing the excess volatility puzzle (see Shiller [1981]; Leroy and Porter [1981]; and West [1988]). Furthermore, the endogenous volatility generated by the model decreases with prices, consistent with empirical observations reported in the ARCH literature. This explains both the persistence observed in volatility and the asymmetric property of it. Since both prices and volatility are endogenous in our model, we are not proposing a volatility feedback mechanism (Campbell and Hentschel [1992]; and Bekaert and Wu [2000]), in which an anticipated increase in volatility leads to a price decline. Neither should this result be interpreted as a leverage effect (Black [1976]; and Christie [1982]), in which lower prices drive the increased volatility; this effect should be expected even with no leverage in the capital structure. It is also worth mentioning that our model contrasts with Barro [2006] or Rietz [1988]: while in these peso models the volatility of consumption equals the volatility of prices, in our model the latter is several times higher. Wachter [2009] generates a similar effect by modeling probabilities of disaster that are time varying. Similarly to hers, our model also exhibits time-varying probabilities of disaster.[15] In contrast to her model, ours has only a single factor and agents with CRRA preferences.

It has been noted that volatility is very persistent−French, Schwert and Stambaugh [1987] note that autocorrelation of volatility remains high even after 12 monthly lags and conclude that volatility is not stationary. We show that in our model, when the state variable is a random walk, the volatility is a function of this random walk and therefor is persistent. When the state variable is mean-reverting, we show that volatility has more autocorrelation than the state variable. These implications of our model are consistent with the empirical observations on volatility stationarity.

[15] Specifically, the lower the value of the state variable W_t (and thus the lower the consumption), the higher the probability of disaster.

Volatility is known to be higher in recessions. As recessions are periods in which equity prices are low, and since the volatility is a decreasing function of price, our model is consistent with this stylized fact.

Volatility has been noted to react differently to a positive returns innovation as compared to a negative returns innovation: volatility tends to decrease after a realization of positive returns and increase after a realization of negative returns (Nelson [1991]). A positive realization of returns in our model is the result of a positive innovation to the state variable. From equation (20) we can show that an increase in the state variable results in reduced volatility. Likewise, a negative realization of returns in our model is the result of a negative innovation to the state variable. A decline in the state variable will thus result in increased volatility.

3.3 Expected returns

In this section we show how our model is consistent with several empirical observations on the expected excess returns of the risky asset.

Proposition 7 *Conditional on labor being fully employed in the first sector:*

1. *Expected excess returns are a decreasing function of prices.*
2. *When the state variable is a random walk (i.e., $k = 0$), expected excess returns are an affine function of dividend yields.*

The model predicts that when the state variable is a random walk, both expected excess returns and volatility are linearly related to dividend yields. Fama and French [1988] and Campbell and Shiller [1988] report that conditional excess returns are predicted by linear regressions with dividend yield as the explanatory variable. The model here gives a theoretical justification to these findings. Furthermore, as the model is based on rational agents, this result does not imply any market inefficiency. The result is not driven by any change in the agent's risk aversion as, for example, in habit formation models (Campbell and Cochrane [1999]). Risk in the model moves in tandem with dividend yields: when the risk is higher, the agent requires a higher premium to hold the asset. Thus, dividend yields predict returns because they are positively related to expected returns.

In the general case of a mean-reverting state variable (i.e., $k > 0$), volatility and expected excess returns are not affine functions of dividend yields. However, as they are both functions of the state variable, as are the dividend yields, it follows that expected excess returns as well as volatilities are (nonlinear) functions of dividend yields. For low values of the mean reversion speed k, expected excess returns and volatilities admit close approximations by affine functions of dividend yields, and thus the logic above, that dividend yields predict returns lends itself to the general case when the state variable is a slow mean reverting diffusion.

3.4 The riskfree rate

Our model is parsimonious and does not include any monetary component, and as such we do not attempt to resolve any interest rates puzzles. However, several facts are to be noted. Specifically, conditional on labor being fully employed in the first sector:

1. The riskfree rate is bounded up by $\delta - (\sigma\gamma)^2/2 + k\overline{X}$.

2. For values of the coefficient of risk aversion that are greater than $k\overline{X}/\sigma^2$, the riskfree rate is decreasing with the coefficient of relative risk aversion.

Unlike Weil [1989], our model allows for lower riskfree rates associated with *higher* coefficients of risk aversion. As Gourio [2008a] and Wachter [2009] show that one needs Epstein-Zin preferences in order to match the low magnitude of the riskfree rate in a peso model, it is worth noting the difference between our results and theirs. Like ours, these studies model a time-varying probability of disaster. Unlike their models, however, in which the disaster probabilities and the changes in consumption are modeled by two distinct state variables, in our one factor model they are correlated as implied by the real economic mechanism we postulate: prior to a disaster, when all labor is engaged in the first economic sector, along with a drop in the aggregate consumption θ_t comes also an increase in the risk of disaster. Thus, unlike these studies, in our model stylized term structure facts, such as a low riskfree rate, are generated even if agents do not have Epstein-Zin preferences.[16]

[16] Our model also produces a variety of term structure implications. Specifically, we can show the following: (i) Medium-maturity bonds have yields that are increasing in the state variable. This is "flight-to-quality" property observed in bonds; (ii) Term structure is U-shaped. Such a structure has been observed in the UK (Brown and Schaefer [1984]. A downward slopping term structure (as we have at low maturities) has been documented by Evans [1998]. These results are available upon request.

4 Calibration and empirical tests

In this section we calibrate our model, test its empirical implications, and compare our results with empirical studies that tested similar predictions.

As mentioned in the introduction, we present two distinct calibrations. In one of the calibration, we assume that the state variable driving our model is mean reverting. In this calibration, because the state variable is pulled back to its mean, our economy has a chance to collapse even if the current value of the state variable is high. This is in contrast with the second calibration, where we assume that the state variable is a random walk. In this second calibration, because this non-stationarity of the state variable, economies which experience much growth in the past has a very small chance to collapse. Consequently, in such economies, the conditional risk premium as well as conditional volatilities are small.

4.1 Data

We use the monthly returns of the value weighted CRSP index for the period 1927−2008, the riskfree rate of the Federal Reserve's Publication H.15 and the Bureau of Labor Statistics Consumer Price Index and productivity series. In order to reconstruct a real return series for the risky asset in our model, we have to adjust by the economic growth relative to the growing in the security base that has been observed in the U.S. economy. The estimated difference between the deterministic trend in GDP growth (which proxies for the change in our numeraire) and the deterministic trend in the growth of the security base is estimated at 8.6 basis points. We produce a series of real returns by subtracting 8.6 basis points from the time series of the CRSP value weighted market returns. While this detrending procedure is standard, it is worth mentioning that our results show little to no sensitivity to this transformation.

4.2 Parameter calibration

The parameters of the model are $\Psi = (\delta, k, \overline{X}, \sigma, \gamma, a, X_0)$. Note that Z is not among the parameters we need to fit: the reason is that from equation (19) it can be inferred that prices are linear functions of Z. Together with the fact that dividends (to both labor and the risky asset) are also linear functions of Z and that the riskfree rate is independent of Z, this fact implies that expected returns, volatilities, and Sharpe ratios are independent of Z as well.

We start by choosing parameters that tackle the equity premium puzzle as it was exposed by Mehra and Prescott [1985]. Thus we select their value for the volatility of consumption, conditional on the economy not being in a disaster state. As in these states of the world the consumption process is given by $\theta_t = \theta_b e^{X_t}$, its volatility is equal to σ. Following Mehra and Prescott [1985], we use $\sigma = 3.57\%$. We note that this may be an oversimplification as the volatility of the time series of consumption varied over time and, in particular, decreased following World War II. Volatility of real GDP is 4.74% for the entire sample period, and is 2.99% for the post-war period. Volatility of dividends is 12.3% for the entire sample period, and 5.8% for the post-war period. Consumption of non-durable goods is the least volatile: Mehra and Prescott [1985] find its volatility to be 3.57% for their entire sample period, and according to Chapman [2002], it may be as low as 1% for the post-war period[17]. Alternatively, Barro [2006] and Wachter [2009] calibrate this volatility to 2%. For the calibration where the state variable is mean-reverting, we select a long-run mean of consumption growth of 2%. Nordhaus [2005] reports average consumption growth rates calculated in different periods and using different methods. For United States, the rates he reports range from 1.24% to 2.53% per annum.[18] In order to select the speed of mean reversion, we rely on estimates from Storesletten et al. [2004], who assess that autocorrelation in quarterly

We select risk aversion so that the Sharpe ratio implied by our model, conditional on the economy not being in a disaster state, matches the observed Sharpe ratio over the entire period of our data. This amounts to selecting γ such that $\sigma\gamma = 0.28$. We thus select $\gamma = 7.84$. Note that Mehra and Prescott [1985] consider that a model with $\gamma < 10$, with the observed volatility of consumption and the observed average equity premium, offers a resolution to the equity premium puzzle. Therefore, our model tackles the equity premium puzzle in the sense defined by Mehra and Prescott [1985].

[17]Aït-Sahalia, Parker and Yogo [2004] argue that consumption of luxury goods is what should matter, since equity holders are typically rich and satiated with the consumption of basic goods. They find that the volatility of luxury goods consumption is an order of a magnitude higher than the volatility of overall consumption.

[18]Since in the "good" states, aggregate consumption is θ_t, it results that with a productivity long run mean of 2% annually, consumption growth also averages to 2% in the long run. This is consistent with consumption data from Robert Shiller's website, for example.

We then set the value of a to correspond to the labor share of $1 - a = 0.64$ observed in the U.S. economy.[19] Note that there is a direct link between the size of the decline in aggregate consumption as the economy experiences a disaster and the labor share of income. Precisely, a disaster translates into a decline of $Z/(1-a) - Z$ in aggregate consumption. Thus, when the disaster occurs, aggregate consumption experiences a relative drop of $a = 36\%$. This is a larger drop than the 31% reported by Barro [2006] for the Great Depression; however, we find it supportive of our model that the consumption drop suggested by our setting is in the ballpark of what has been observed in the Great Depression.

Additionally, we set the value of the discount factor δ in a standard fashion. The savings literature, for example Hubbard, Skinner and Zeldes [1995], uses values consistent with a discount factor $e^{-\delta} = 0.97$. Following this, we set $\delta = 3.1\%$.

Finally, we fit the remaining parameters W_0 and W^g. We start first by observing some complementarity between W_0 and W^g: one could generate the same value of the starting price $P(W_0)$ using different combinations of the values W_0, W^g. We thus make the choice to calibrate W^g directly. In the "social planner" case, an economy recovers as soon as W_t reaches zero, so in this case $W^g = 0$. In order to select W^g for the "big push" case, we rely on the notion of *time to recovery*. The time to recovery is the time labor spends working in home production. Since labor shifts to home production when $W_t = 0$ and shifts back to the first sector as soon as $W_t = W^g$, the time to recovery is equal with the time taken by a Brownian motion W to reach W^g after it started at zero. The median of this time[20] is $m(W^g) = 2.1981(W^g)^2$.

To form an idea about what this value may be in the data, we rely on a compilation of "disasters" by Barro [2006]. Barro [2006] compiles data on twenty OECD countries and seven Latin American and Asian countries where there were contractions larger than minus 15% in per-capita GDP. The length of these contractionary periods then gives an idea about time to recovery. From Barro [2006] it seems that the median duration of those contractions is around six years, and accordingly we set $W^g = 1.6522$. The validity of this calibration may be checked against the magnitude of growth from the trough that is observed during a recovery. With our calibration, the growth from the trough is, by the time of recovery, $[(1-a)^{-1}Ze^{\sigma W^g} - Z]/Z$. With our choice of W^g the growth from the trough is 65.74%, which seems consistent with the values reported for

[19] See Kydland and Prescott [1982].

[20] This is the median of the hitting time of a Brownian motion.

a six-year period of growth from the trough in Gourio [2008b].[21] We then select W_0 such that the first moments of the Brownian innovations $W_{t+dt} - W_t$, as implied from real prices, correspond to a normal distribution $\mathcal{N}(0, dt)$. In order to obtain these innovations we use the time series of real returns on the risky asset. Precisely, we observe that by setting W_0, we can calculate the price $P(W_0)$ at the start of calibration period. Using the returns in the data Ret_1 for the first time period, we may thus infer the price $P(W_1) = P(W_0)e^{Ret_1}$ at time 1. Since prices as given by Proposition 4 are one-to-one mappings with respect to the Brownian motion W we can thus infer W_1, and so on.

Therefore, using the returns from the data and W_0 we can re-create the time series $(W_t)_t \geq 0$. We can then estimate the first and second moments of this time series innovations. For the "social planner" case, the first and second moment of the Brownian shocks are matched to those of a normal distribution when $W_0 = 1.4800$, while for the "big push" case the calibration yields $W_0 = 3.9629$. Calibrating this model to price data follows a trend in disaster models (e.g., Barro [2006]; Gourio [2008a]; and Wachter [2009]), but this is not a unique choice. For example, Balvers and Huang [2007] calibrate a productivity-based model to productivity data while Barro and Ursua [2009] show that these models may be calibrated to consumption data. Finally, in our model a disaster occurs immediately: consumption drops suddenly and stays low until recovery. Whether a disaster model is calibrated to match the cumulative consumption drop or not is however important, as Juilliard and Ghosh [2009] argue. Since the observed disasters were never instantaneous, it is important to calibrate a model in which such instantaneous drops were not observed in the US data. Thus, our "big push" calibration, in which a disaster has not yet happened in the available time series in the US, is important.

A list of the parameters resulting from calibration is presented in Table 1.

4.3 Empirical tests: labor's share of income and disasters

In this subsection we will test our labor mechanism. In order to do so, we note that Proposition 5 assesses that more capital intensive societies should experience larger consumption drops in a disaster. This is precisely the implication we will test in this subsection.

[21] Gourio [2008b] reports a growth from the trough of 52.2% for five years. Our estimate for six years seems in the ballpark of the values reported by Gourio [2008b].

In order to test this implication, we obtain data on labor's share of income from Gollin [2002] and on size of economic disasters from Barro and Ursua [2009]. Intersecting the data presented in these two studies produces a small sample of 12 countries for which both labor's share of income as well as size of disasters are available.

Figure 1 presents the labor's share of income (on the x-axis) plotted against observed magnitudes of disasters (on the y-axis). Gollin [2002] reports both naively calculated labor's shares as well as adjustments designed to include the operating surplus of private unincorporated enterprises into the income share. Following his work we plot the naive labor's share as well as the adjustments. While Gollin [2002] reports three different adjustments we only use his first two as using the third one would further decrease our already small sample. Barro and Ursua [2009] report magnitudes of consumption and GDP during recessions across the world. We use four different measures of disasters magnitude. We use both consumption as well as GDP declines observed during recessions. We also use the declined that occurred closest to the time when labor's share of income was measured as well as the largest observed drop in a country's available data. Using three measures of labor's share and four measures of disaster size yields the 12 plots of Figure 1.

From Figure 1, we observe that the relationship between labor's share of income and magnitude of disasters is negative for all measures used, supporting our model's empirical implication as well attesting to its robustness.[22]

4.4 Empirical tests: matching stylized facts of returns

In this subsection we will discuss the empirical implications of our model as it was calibrated in the previous subsection. While some stylized asset pricing facts are matched theoretically by our model, in this section we explore the quantitative insights offered by calibration.

4.4.1 Behavior of the state variable and distance to disaster

As detailed in the previous section, from the time series of returns we may infer the values of the state variable W. In turn, at each point in time, we can calculate the median time to disaster. Figure 2 presents the values of the state variable when the "big push" calibration is used. Since $W_0 > 0$, we observe that our economy starts in January 1927, with labor fully employed in the first

[22] The power of these tests is however small given that the sample contains only 12 data points.

sector and having a 50% probability to shift to home production in about 34 years. The median time to disaster was about 100 years in 1929, then bottomed to around 1 year in 1932, which is the trough of the Great Depression. The race-to-the-moon decade had the median time to shift reaching 1,000 years. There was an increased uncertainty in 1973−1990, measured at a median of around 200 years. In the 90's, this median went up to 400 years. The burst of the dot-com bubble increased the levels of anxiety, and as of the end of 2002, the economy was pricing a 50% chance of a break in the next 100 years, a level similar to 1929. Finally, in December 2008, the median time to a disaster was 89 years. As previously discussed, this interpretation assumes that the median time for a recovery is about six years. If we chose a longer value for the median duration of the time the economy spends in a state of disaster once the disaster occurs, we would get an estimate in which disasters are less probable, and vice versa. Similarly, if we chose a larger risk aversion coefficient, disasters would be less probable. Finally, in this particular calibration the economy does not encounter a disaster (i.e., the state variable gets close to but never reaches zero). Our alternative calibration, using the "social planner" model, does encounter a disaster in the trough of the Great Depression and has a median time to collapse of 22.5 years in December 2008.[23]

4.4.2 Conditional volatility and the equity premium

In this section we discuss how our model fits conditional moments of the observed equity premium in the U.S. economy.

The conditional equity premium as implied by our "big push" calibration is presented in Figure 3, while Table 2 presents statistics on the equity premium and volatility of the risky asset for both calibrations. Panel A presents the first two moments of the equity premium as they are implied by Proposition 4. The "social planner" calibration implies an average conditional equity premium of 5.15% over the entire sample and a volatility of 18.91%, while the "big push" calibration suggests an average equity premium of 4.09% with an average conditional volatility of 14.62%. When we calibrate relative to the "social planner" model, the equity premium was 4.58% in December 2008 while the median time to a disaster was 89 years. According to the calibration relative to the "social planner" model, the equity premium was higher at 6.29% in December 2008, pricing in a much shorter median time to disaster of 22.5 years. Our estimates

[23]Similar qualitative properties of the time series of our state variable are obtained if we used the "social planner" calibration. Results for the case when the "social planner" calibration is used are available upon request.

seem consistent with previous estimates of the conditional ex-ante equity premium (e.g., by Fama and French [2002]; and Pastor and Stambaugh [2001]). Additionally, a 95% confidence interval for monthly returns is the expected excess returns plus or minus twice the conditional volatility, derived in equations (20) and (21). Figure 4 plots this confidence interval, along with the realized monthly excess returns of U.S. stocks in this period. With the realized expected returns within the confidence interval, the plot shows that the model predicts well both the absolute level of the conditional volatility and its dynamics.

However, the ex-post returns in a "peso" environment have a positive survival bias as in Brown, Goetzmann and Ross [1995]. In order to estimate the predicted returns for a surviving economy as suggested by our initial calibration, we simulate 1,000,000 sample paths of 984 months (i.e., 82 years) for our economy. The results of these simulations are presented in Panel B of Table 2. For the "big push" calibration, for example, 65% of these economies experience a disaster within these 82 years. The average simulated equity premium for the remaining 35% paths was 4.28% with a standard deviation of 0.97%. The simulated value is one standard deviation away from the observed equity premium in the U.S. markets, which is 5.24%. The average volatility in the paths where a disaster did not occur was 15.30%, with a standard deviation of 3.43%. The simulated value is within one standard deviation away from the volatility level that has been observed for the US economy, which is 18.97%. In comparison, the volatility in a Mehra-Prescott [1985] setting, assuming, as they did, a relative risk aversion of 10 and volatility of consumption of 3.57%, is about 1%. For similar parameters, the volatility that arises in our model is about twenty times larger. This high volatility delivers in turn a high equity premium. These simulations show that the concept of a "peso" economy, along with the induced survival bias, may offer a complete resolution to both the equity premium puzzle and the excess volatility puzzle.

We now turn to examining returns and volatility predictability.

4.5 Empirical tests: returns predictability

We now test the hypothesis that the dividend yield implied by our model predicts conditional excess returns. To see why this is the case, recall that the excess returns in our model are proportional to the volatility (because the Sharpe ratio is constant). Since Vol is an affine function of ID/P

from Proposition 6, part 3, the expected excess returns must also be an affine function of dividend yields. Hence, we expect the latter to predict the former.[24]

In Table 3 we report the results of such predictive regressions for 1953−2008. We present results when the calibration employed is the "big push" one, although the results are similar for the alternative calibration. Predictive variables studied in the literature, such as Campbell and Shiller [1988], Cochrane [1992], Fama and French [1988] and Keim and Stambaugh [1986], include the *observed* dividend yield and the Lettau and Ludvigson [2001] *cay* variable. We understand that given the persistence shown by dividend yield ratios and documented by Lettau and Wachter [2007], these results on predictability should be interpreted with caution. However, our variable ID/P is shown to have more predictive power than realized dividend yield, both in terms of the resulting R^2 and in terms of the statistical significance: we get an R^2 of 3.56% for the month-ahead prediction, 9.85% for the quarter-ahead prediction, and 32.70% for the year-ahead prediction. Even though the correlation between the two variables is 0.83, the combined regressions of Model 3 show that our variable remains significant at 99% confidence, while the observed dividend yield becomes insignificant. The *cay* variable improves predictability in the monthly regressions, loses some of its statistical significance in the quarterly regressions, and becomes subsumed in the annual regressions. The high t-statistics we obtain (more than 4) allow us to reject the null hypothesis of no predictability with more than 99% confidence even after the bias in predictive regressions (Stambaugh [1999]) is accounted for.

4.6 Empirical tests: volatility predictability

A similar predictability test is performed for conditional volatility, and the results are reported in Table 4. The first column reports the result for the entire sample period of 1926-2008, and the following four columns report the results from various sub-periods.[25] The results show that our variable predicts conditional volatility better than realized dividend yield−for the entire sample, implied dividends explain 21.60% of the changes in volatility, compared with 9.92% explained with realized dividends. For the 1946−1965 subperiod, our variable is significant with the correct

[24] We are aware that predictability at intervals as long as a month in real data may not be driven by predictability of excess returns but by market "fads" (Lehmann [1990]).

[25] Schwert [1989] notes that the relation between volatility and dividend yield is not stable over time, and consequently we took precautions to document any test using various sub-periods.

sign, whereas when realized dividend yields are used as the explanatory variable, the coefficient is negative.

4.7 Time series properties of Sharpe ratios

The ex-ante Sharpe ratio in the model is constant conditional on no disaster (see Proposition 4), and equal to $\sigma\gamma$. This might seem at odds with a stylized fact of the U.S. data, namely, that volatility and returns do not move together. Glosten, Jagannathan and Runkle [1993] actually find a negative relation between risk and return. Whitelaw [1997] documents that Sharpe ratios vary considerably over time. Whitelaw [2000] shows that in an economy with time-varying transition probabilities, the *ex-post* time series of volatility and returns, under constant relative risk aversion (i.e., constant *ex-ante* Sharpe ratio), exhibit a complex time varying relation, which is negative in the long run. As the model we develop also has time varying transition probabilities into disaster, we will show similar properties of the ex-post Sharpe ratio.

In order to study the time series properties of the realized Sharpe ratios in our model, we simulate daily paths of monthly histories, each one 984 months long. We draw paths of our model until we reach a number of 250,000 paths that did not exhibit a disaster, consistent with the path observed in the U.S. economy, given our "big push" calibration. For each month, we compute the resulting volatility and the realized excess returns of that month. Dividing the two gives us the Sharpe ratio of that month. In order to document their time variability, we regress the 984 monthly Sharpe ratios on the dividend yield at the beginning of the month and record the statistical significance (i.e., the t-statistics) of this regression.

The null hypothesis of constant Sharpe ratios would imply that they are not predictable by any variable. In that case, the t-statistics would be normally distributed with a zero mean. Clearly, this is not the case: the mean t-statistic is 2.88, and 93% of the simulated histories result in a statistically significant relation between realized Sharpe ratios and dividend yield.[26]

[26] Two issues drive this spurious predictability. The first is driven by the interaction between realized returns and expected returns. Suppose there is a large negative shock at the beginning of the month. This causes volatility and expected returns to increase for the rest of the month. However, the change in expected returns is relatively small, and most likely the end result for the month would be high volatility and negative returns. This effect weakens the relation between realized returns and volatility and makes realized Sharpe ratios seem to co-move with returns (and hence with dividend yield). The second issue is the survival bias, which is driven by the fact that ex-post the possible break in the economy did not happen. This bias is larger in the "bad" states of the economy, as the ex-ante probability of a disaster is greater. This results in a counter-cyclical bias, and therefore returns and Sharpe ratios appear counter-cyclical in a surviving economy.

Finally, we note that the model could be extended to include sources of volatility that are orthogonal to our pricing kernel. For example, such volatility could arise if the division of output between labor and capital would change. This would affect asset prices, but since it does not change overall consumption, it would not command a price premium. The existence of such additional sources of volatility would cause Sharpe ratios to be even more counter-cyclical. Likewise, having σ decrease with W_t (i.e., a setting in which being farther away from the disaster triggers an increase in the stability of production) would result in even stronger counter-cyclical Sharpe ratios.[27]

5 Conclusions

Peso models solve many asset pricing puzzles and are testable. We add to this literature by proposing a parsimonious, one-factor model with CRRA agents in which the disasters occur endogenously.

Our model produces the simple implication that more capital intensive economies experience larger disasters. This model is also capable of matching several stylized asset pricing facts. Three in the list of these facts address the puzzles we already mentioned on the onset: namely, in our model, the equity premium is adequately high, the prices of the risky asset are volatile relative to consumption growth and the riskfree rate is low.

In addition, our model goes beyond just these facts. For example, not only price volatility in the model is high, but it also decreases with prices. Because both prices and volatility are endogenous, this is not a volatility feedback effect, nor is this a leverage effect because in our model there is no leverage.

Since expected returns as well as volatility of the risky asset in our model are analytically shown to be affine functions of dividend yields conditional on no disasters, we offer a rationale for why dividend yields are excess returns and volatility predictors.

While our model is similar to disaster models in which the probability of disaster is time varying, in contrast to those models ours is simpler, having only one factor and agents with constant relative risk aversion.

[27] Such an assumption also has the potential to generate pro-cyclical short term rates. It is not surprising that disaster models have the potential to resolve a multitude of bond pricing puzzles. For example Gabaix [2009] shows that different exogenous disaster specifications may solve a multitude of asset pricing puzzles.

In order to create our setting, we relied on existing economic models, which we extended by adding dynamics and then by calculating asset prices. From this perspective *none* of the modeling assumptions are new, but their integration is. Given the minimality of our model as well as its ability to match several asset pricing facts, we see our study as moving the peso literature toward a unified asset pricing model.

Appendix

Proof of Lemma 2:

If firms (rationally) expect that labor works in their sector, then $\bar{L}^e_{it} = 1$ for each firm i. If also $D_{Lt} = (1-a)\theta_t$, then by equation (7) it results that $K_{it} = L_{it}$ (for each firm). Because the aggregate supply of capital is 1, the aggregate demand of labor will also be one, and the labor market will clear. This is also consistent with laborers being willing to supply labor to firms, as $D_{Lt} = (1-a)\theta_t > Z$.

When $\theta_t < (1-a)^{-1}Z$, labor working in the first sector is no longer feasible. Assume by absurd that firms expect labor will work for them, that is, $\bar{L}^e_{it} = 1$. We clearly cannot have that $D_{Lt} \leq (1-a)\theta_t$, because this will result in $D_{Lt} < Z$, which means that the laborers will prefer to work in home production, which pays the higher wage of Z. We must then have that $D_{Lt} > (1-a)\theta_t$. From equation (7), it results that $L^{\mathcal{D}}_{it} < K^{\mathcal{D}}_{it}$ for each firm i. This however results in the supplied labor aggregating to $\sum_i L^{\mathcal{D}}_{it} < \sum_i K^{\mathcal{D}}_{it} = 1$, which means that the labor market does not clear. This is a contradiction with the equilibrium definition.

Labor working in home production is always possible: if each firm expects that all labor will be working in home production, as clearly $D_{Lt} \geq Z > 0$, from equation (7) it results that $L^{\mathcal{D}}_{it} = 0$ for each firm. With no labor, the firms' output is zero, and hence, the rents paid to capital are zero as well, i.e., $D_{Kt} = 0$. Both equations (7) and equations (8) are satisfied regardless of the value of $K^{\mathcal{D}}_{it}$, in particular, we can select these values so that condition (C) of the equilibrium is met.

Finally, we address the case of mixed equilibrium. In the mixed equilibrium, the wages paid by the home production technology must be equal with the wages paid by firms, that is, $D_{Lt} = Z$. From equation (7), $L^{\mathcal{D}}_{it} = K^{\mathcal{D}}_{it}\left[(1-a)\theta_t\bar{L}^e_{it}Z^{-1}\right]^{1/a}$ for each firm i, and in order to sustain the condition (C) of the equilibrium as well as the condition (B), we must have that $\bar{L}_t = \left[(1-a)\theta_t\bar{L}_tZ^{-1}\right]^{1/a}$, or that $\bar{L}_t = [Z(1-a)^{-1}\theta_t^{-1}]^{1-a}$. Since in the mixed equilibrium $\bar{L}_t < 1$, this is sustainable if and only if $\theta_t > (1-a)^{-1}Z$.

We continue by presenting a few facts regarding functionals of Ornstein-Uhlenbeck processes, which are useful for our calculations.

Functionals of Ornstein-Uhlenbeck processes

The following lemma is from Borodin and Salminen [1996]:

Lemma A.1 *Let X_t be an Ornstein-Uhlenbeck process, i.e.*

$$dX_t = k(\overline{X} - X_t)dt + \sigma dW_t.$$

Let $X_0 = x$ and denote by T the first time that X reaches zero. Then

$$\ell(x) := \mathbb{E}\left[e^{-\delta T}|X_0 = x\right] = \begin{cases} \frac{e^{k(x-\overline{X})^2/2\sigma^2}}{e^{-k\overline{X}^2/2\sigma^2}} \frac{D_{-\delta/k}(-(x-\overline{X})\sqrt{2k}/\sigma)}{D_{-\delta/k}(\overline{X}\sqrt{2k}/\sigma)} & , x < 0 \\ \frac{e^{k(x-\overline{X})^2/2\sigma^2}}{e^{-k\overline{X}^2/2\sigma^2}} \frac{D_{-\delta/k}((x-\overline{X})\sqrt{2k}/\sigma)}{D_{-\delta/k}(-\overline{X}\sqrt{2k}/\sigma)} & , x \geq 0 \end{cases},$$

where D is the parabolic cylinder function.

Lemma A.2 *Let X_t be an Ornstein-Uhlenbeck process like above, and let T the first time X reaches zero. Then:*

$$\Updownarrow(x) := \mathbb{E}\left[\int_0^T e^{-\delta t+(1-\gamma)X_t}dt|X_0 = x\right] = \ell(x)g(0) - e^{(1-\gamma)x}g(x).$$

where $\ell(\cdot)$ is the function from Lemma A.1 and g is any (closed form) solution of the Laplace equation:

$$g_{xx} + (\overline{b} - \overline{a}x)g_x + (\overline{d} - \overline{c}x)g = \overline{e}, \tag{24}$$

with the constants $\overline{a}, \overline{b}, \overline{c}, \overline{d}, \overline{e}$ defined as:

$$\overline{a} = -2k/\sigma^2 \tag{25}$$

$$\tag{26}$$

$$\overline{b} = 2(1-\gamma) + 2k\overline{X}/\sigma^2 \tag{27}$$

$$\tag{28}$$

$$\overline{c} = -2k(1-\gamma)/\sigma^2 \tag{29}$$

$$\tag{30}$$

$$\overline{d} = (1-\gamma)^2 + 2[k(1-\gamma) - \delta]/\sigma^2 \tag{31}$$

$$\tag{32}$$

$$\overline{e} = 2/\sigma^2. \tag{33}$$

We continue by presenting a closed form solution for g in Lemma A.2.

Closed form solution for equation (24):

Equation (24) is a version of the *Laplace ordinary differential equation*, and techniques to solve it in close form are described in Davies [1985], pp. 342. Following Davies [1985], one starts by looking for a solution of the form

$$g(x) = \int_{c_1}^{c_2} S(s) e^{sx} ds,$$

for some constants $c_{1,2}$. Substituting in (24) and then integrating by parts, we are left with a first order ordinary differential equation for S, which can be solved in closed form. We select c_1, c_2 so that one of them is equal to $\pm\infty$ and the other constant is zero. The choice of $+\infty$ or $-\infty$ is such that the function $f(x) = ax + c$ does not have a root in the interval $[c_1, c_2]$. In our case, we select $c_1 = -\infty,\ c_2 = 0$.

Finally, the solution g is given by:

$$g(x) = \overline{e}|\overline{c}|^{\overline{d}+\overline{a}-\overline{b}\overline{c}/\overline{a}+\overline{c}^2/\overline{a}^2-1} \int_{-\infty}^{0} |\overline{a}s + \overline{c}|^{1-(\overline{c}+\overline{a}-\overline{b}\overline{c}/\overline{a}+\overline{c}^2/\overline{a}^2)} e^{-s^2/2\overline{a}-(\overline{b}/\overline{a}-\overline{c}/\overline{a}^2-x)s} ds. \tag{34}$$

The derivative of g is given by:

$$g_x(x) = \overline{e}|\overline{c}|^{\overline{d}+\overline{a}-\overline{b}\overline{c}/\overline{a}+\overline{c}^2/\overline{a}^2-1} \int_{-\infty}^{0} |\overline{a}s + \overline{c}|^{1-(\overline{c}+\overline{a}-\overline{b}\overline{c}/\overline{a}+\overline{c}^2/\overline{a}^2)} s e^{-s^2/2\overline{a}-(\overline{b}/\overline{a}-\overline{c}/\overline{a}^2-x)s} ds. \tag{35}$$

Proof of Proposition 2

Assume first that all labor is employed in the capital-intensive sector. We can thus write that the price $B(\tau, X_t)$ of the bond satisfies:

$$u'(\theta_t)B(\tau, X_t) = e^{-\delta\tau}\mathbb{E}_t\left[u'(\theta_{t+\tau})1_{\{X_{t+\tau}>0\}} + u'(Z)1_{\{X_{t+\tau}\leq 0\}}\right].$$

Since X is an Ornstein-Uhlenbeck diffusion, its distribution is normal, with the following paramters:

$$X_{t+\tau} \sim \mathcal{N}\left(X_t e^{-k\tau} + \overline{X}(1 - e^{-k\tau}),\ \frac{\sigma\sqrt{e^{2k\tau} - 1}}{\sqrt{2k}e^{k\tau}}\right).$$

We observe that v_τ in the Proposition is the standard deviation of $X_{t+\tau}$ while $m_{t,\tau} + X_t$ is the mean of $X_{t+\tau}$. Thus, taking logarithm, dividing by τ and expressing the expectations above as integrals,

$$\begin{aligned} y(\tau, X_t) = \delta - \frac{1}{\tau}\log\ & \Big[Be^{\gamma X_t}\mathcal{N}\left(-\mathrm{mean}(X_{t+\tau})/\mathrm{stdev}(X_{t+\tau})\right) \\ & + \frac{1}{\sqrt{2\pi}v_\tau}\int_{-X_t}^{\infty} e^{-\gamma(x - X_t)} e^{-\dfrac{(x - \mathrm{mean}(X_{t+\tau}))^2}{2v_\tau^2}} dx\Big] \end{aligned} \tag{36}$$

Making the substitutions $z = x - X_t$, then $u = [z - (m_{t,\tau} - \gamma v_\tau^2)]/v_\tau$ in the second term inside the brackets, and completing the square in of the exponential inside the integral, we get the formula for the bond yield.

To obtain the expression for the risk free rate, note that the first term inside the brackets converges to zero as $\tau \downarrow 0$. Thus, near $\tau = 0$, the yield $y(\tau, X_t)$ behaves, near $\tau = 0+$, like:

$$\delta - \gamma X_t \frac{1 - e^{-k\tau}}{\tau} + \gamma\overline{X}\frac{1 - e^{-k\tau}}{\tau} - \frac{\gamma^2\sigma^2(e^{2k\tau} - 1)}{4ke^{2k\tau}}\frac{e^{2k\tau} - 1}{\tau}.$$

Pushing $\tau \downarrow 0$, we obtain the expression for the risk free rate:

$$r = \delta - (\gamma\sigma)^2/2 + k\gamma(\overline{X} - X_t).$$

Making $\tau \downarrow 0$, we readily obtain the formula for the riskfree rate.

In order to calculate yields when all labor works in the less-capital intensive sector, note that the expression for the bond price becomes:

$$u'(Z)B(\tau, X_t) = e^{-\delta\tau}\mathbb{E}_t\left[u'(\theta_{t+\tau})1_{\{X_{t+\tau} > 0\}} + u'(Z)1_{\{X_{t+\tau} \leq 0\}}\right].$$

Converting expectations into integrals using that $X_{t+\tau}$ is normally distributed and completing the squares inside the exponentials in integrals we obtain, just like before, the expression for the bond yield.

In order to calculate the riskfree rate in this case, observe that in the Euler equation above, describing the price of the bond, the first term inside the brackets converges to zero as τ approaches zero. This is because the chance that $X_{t+\tau} > 0$ in the next infinitesimal time interval is very small when the starting value is $X_t < 0$ (using the dominated convergence theorem, the same intuition translates to expectations). Thus, in the limit, only the second term inside the brackets matters when we calculate the riskfree rate, and thus we readily obtain that the riskfree rate is δ.

Proof of Proposition 3

First, as mentioned previously, we observe that the "bad" prices $P_{T_n^b}$ (i.e., the prices at the times when $\theta_{T_n^b} = \theta_b$ and the economy collapses) as well as the "good" prices $P_{T_n^g}$ (i.e. the prices at the times when $\theta_{T_n^g} = \theta_b e^{\sigma W^g} =: \theta_g$ while the economy recovers by labor moving from home production into the first economic sector) are independent of n. This is because the state variable W_t is solely determining the price. Thus we can drop the subscripts for the "good", respectively the "bad" prices and we shall simply call them P^b and P^g. We will treat first the case $W^g > 0$.

We shall use equation (17) in order to prove this proposition. If $t' = T_2^b$ and $t = T_1^g$, equation (17) becomes

$$u'(\theta_g)P^g = a\mathbb{E}_{T_1^g}\left[\int_{T_1^g}^{T_2^b} e^{\delta(l-t)}u'(\theta_l)e^{\sigma W_l}dl + e^{-\delta(T_2^b - T_1^g)}u'(Z)P^b\right].$$

where $\theta_g = \theta_b e^{\sigma W^g}$. If T is the time at which a Brownian motion started at w hits zero, then $E[e^{\lambda T}] = e^{-\sqrt{2\lambda}x}$. Using this in the case of the Brownian motion W_t which starts at W_g at time T_1^g and hits zero after $T_2^b - T_1^g$, we have that

$$\mathbb{E}_{T_1^g}[e^{-\delta(T_2^b - T_1^g)}] = e^{-\sqrt{2\delta}W^g}.$$

In order to calculate $\mathbb{E}_{T_1^g}\left[\int_{T_1^g}^{T_2^b} e^{\delta(l-t)}u'(\theta_l)e^{\sigma W_l}dl\right]$, we observe that

$$M_\tau := \int_{T_1^g}^{\tau} e^{-\sigma(\gamma-1)W_l - \delta(l - T_1^g)}dl + \frac{e^{-\sigma(\gamma-1)W_\tau - \delta(\tau - T_1^g)}}{\delta - \frac{1}{2}\sigma^2(\gamma-1)^2} \tag{37}$$

is a martingale. By the optional sampling theorem, $\mathbb{E}_{T_1^g}[M_{T_2^b}] = \mathbb{E}_{T_1^g}[M_{T_1^g}]$, which implies that

$$\mathbb{E}_{T_1^g}\left[\int_{T_1^g}^{T_1^b} e^{-\sigma(\gamma-1)W_l-\delta(l-T_1^g)}dl\right] = \frac{e^{-\sigma(\gamma-1)W^g} - e^{-\sqrt{2\delta}W^g}}{\delta - \frac{1}{2}\sigma^2\left(\gamma-1\right)^2}. \tag{38}$$

With these calculations we obtain that:

$$P^g = \frac{aZ}{1-a}\frac{e^{\sigma W^g} - e^{(\sigma\gamma-\sqrt{2\delta})W^g}}{\delta - \sigma^2(\gamma-1)^2/2} + \frac{1}{(1-a)^\gamma}e^{(\sigma\gamma-\sqrt{2\delta})W^g}P^b.$$

If we denote

$$A = \frac{aZ}{1-a}\frac{1}{\delta - \sigma^2(\gamma-1)^2/2} \text{ and } B = \frac{1}{(1-a)^\gamma},$$

then we have that

$$P^g = Ae^{\sigma W^g} - (A - BP^b)e^{(\sigma\gamma-\sqrt{2\delta})W^g}. \tag{39}$$

If we use equation (17) for $t = T_1^b$ and $t' = T_1^g$ since between T_1^b and T_1^g the economy plays the equilibrium in which all labor is dedicated to the home production technology and thus the capital asset pays zero dividends, we obtain that

$$u'(Z)P^b = \mathbb{E}_{T_1^b}[e^{-\delta(T_1^g-T_1^b)}u'(\theta_{T_1^g})P^g].$$

Using again that $\mathbb{E}_{T_1^b}[e^{\delta(T_1^g-T_1^b)}] = e^{-\sqrt{2\delta}W^g}$, we obtain that

$$BP^b = e^{-(\sigma\gamma+\sqrt{2\delta})W^g}P^g. \tag{40}$$

Note that equations (39) and (40) form a system with unknowns P^b and P^g that can be solved. The solution is the "bad" price $P^b = P_{T_n^b}$ in the proposition, for the case in which $W^g > 0$. To obtain the bad price in the case $W^g = 0$ we push W^g to zero in the equation of the bad stock price. In order to take the limit observe first that for $W^g > 0$ we can write that:

$$P^b = \frac{A}{B}\frac{e^{[\sigma(1-\gamma)+\sqrt{2\delta}]W^g} - 1}{e^{2\sqrt{2\delta}W^g} - 1}.$$

We then use that as $x \to 0$, $\dfrac{e^x - 1}{x} \to 1$, and thus we obtain the formula for the bad price when $W^g = 0$.

To obtain $P_{T_n^b}$ for the case when $W^g = 0$, we simply calculate the limit if the first case as $W^g \downarrow 0$.

Proof of Proposition 4:

We can then apply equation (17) one more time with t being the current time and $t' = T_1^b$, and we can thus obtain the equation for the stock price (19), conditional of all labor being employed in the first sector at time t:

$$u'(\theta_t)P(W_t) = \mathbb{E}_t\left[\int_t^{T_1^b} u'(\theta_l)e^{-\delta(l-t)}D_l + e^{-\delta(T_1^b - t)}u'(Z)P^b\right].$$

In the above equation D_l represents the dividend to the risky asset paid at time l if all labor is employed in the first sector. Thus $D_l = a\theta_l$. Using again the martingale arguments in the Proof of Proposition 3 as well as the formula for the expectations involving exponential of Brownian motion hitting times, , we can obtain that:

$$e^{-\sigma\gamma W_t}P(W_t) = A\left(e^{-\sigma(\gamma-1)W_t} - e^{-\sqrt{2\delta}W_t}\right) + BP^b e^{-\sqrt{2\delta}W_t}.$$

This proves the first part of the proposition.

In order to prove point 2, note that conditional on labor working fully in the first economic sector, $ID(W_t) = a\theta_t$. In order to obtain the volatility of the returns on the risky asset conditional on labor being employed in the first sector, note that applying Itô's lemma to $P(W_t)$ the returns R are given by:

$$dR_t = \frac{dP(W_t) + ID(W_t)dt}{P(W_t)} = \left(\frac{\frac{1}{2}P''(W_t) + ID(W_t)}{P(W_t)}\right)dt + \frac{P'(W_t)}{P(W_t)}dW_t. \tag{41}$$

Thus the volatility of returns is thus equal to

$$Vol(W_t) = \frac{P'(W_t)}{P(W_t)} = \frac{A\sigma e^{\sigma W_t} - (A - BP^b)(\sigma\gamma - \sqrt{2\delta})e^{(\sigma\gamma - \sqrt{2\delta})W_t}}{A\sigma e^{\sigma W_t} - (A - BP^b)e^{(\sigma\gamma - \sqrt{2\delta})W_t}}.$$

This can be rearranged after some algebra into:

$$Vol(W_t) = (\sigma\gamma - \sqrt{2\delta}) + \frac{2}{\sigma(\gamma-1)+\sqrt{2\delta}}\ \frac{ID(W_t)}{P(W_t)}.$$

From formula (41), we can also calculate the expected returns as

$$\mu(W_t) = \frac{\frac{1}{2}P''(W_t) + ID(W_t)}{P(W_t)}$$

$$= \frac{\left[A\sigma^2/2 + aZ(1-a)^{-1}\right] e^{\sigma W_t} - (1/2)(A - BP^b)(\sigma\gamma - \sqrt{2\delta})^2 e^{(\sigma\gamma-\sqrt{2\delta})W_t}}{Ae^{\sigma W_t} - (A - BP^b)e^{(\sigma\gamma-\sqrt{2\delta})W_t}}.$$

After some tedious algebra it can be shown that

$$\mu(W_t) = r + \sigma\gamma Vol(W_t).$$

Note that we also showed that the Sharpe ratio is constant conditional on labor being employed in the first economic sector.

Prices at times when labor exclusively works in home production are calculated in a similar fashion, using the pricing equation (17) and noting that the capital asset does not pay any dividends when labor works in home production. The expected returns and volatility expressions in this case are immediate from Itô's lemma.

Proof of Proposition 6:

Part 1 is trivial for those parameter values for which $\sigma\gamma - \sqrt{2\delta} \geq \sigma$, because $D(W_t)$ and $P(W_t)$ are positive. Thus we will concentrate on the case in which $\sigma\gamma - \sqrt{2\delta} < \sigma$.

If we denote

$$\begin{array}{llll} \lambda & := [\sigma(\gamma-1) - \sqrt{2\delta}] & x & := e^{\lambda W^g} \\ y & := e^{\lambda W_t} & p & := e^{2\sqrt{2\delta}W^g}, \end{array}$$

we observe that $\sigma\gamma - \sqrt{2\delta} < \sigma$ means $\lambda < 0$. Also, after some (tedious but straightforward) algebra, we observe that with the notations above,

$$\frac{1}{Vol(W_t) - \sigma} = \frac{1}{\lambda y}\left(y - \frac{p-1}{p-1/x}\right).$$

Since we are in the case $\lambda < 0$, proving that $Vol(W_t) > \sigma$ is equivalent with proving that

$$y - \frac{p-1}{p-1/x} < 0.$$

Note first that the highest value that can be reached by $y - \dfrac{p-1}{p-1/x}$ is when $y = 1$, because when $\lambda < 0$ $y \leq 1$. That highest value is obtain by making thus $y = 1$ and it is

$$1 - \frac{p-1}{p-1/x} = \frac{1-1/x}{p-1/x}.$$

Now, the fraction above is negative: when $\lambda < 0$, we have that $x < 1$, while $p - 1/x = (px-1)/x = \left[e^{(\sigma(\gamma-1)+\sqrt{2\delta})W^g} - 1\right]/x > 0$. At $\lambda = 0$, $Vol(W_t) = \sigma$. We thus proved that $Vol(W_t) \geq \sigma$. To prove Part 2, observe that with the notations above,

$$\frac{1}{Vol(W_t) - \sigma} = \frac{1}{\lambda}\left(1 - \frac{1}{y}\frac{p-1}{p-1/x}\right).$$

When $\lambda > 0$, y as a function of W_t is increasing and then it is straightforward that the righthandside above is increasing in W_t. Therefore, Vol is decreasing in W_t. If $\lambda < 0$ then y is decreasing in W_t and thus the righthandside is again increasing in W_t as $\lambda < 0$. Thus, in this case, we also have that Vol decreases in W_t. If $\lambda = 0$ then the volatility is constant.

Parts 3 is immediate from the expression of Vol. Part 4 follows as explained after the proposition.

Proof of Proposition 7:

Since conditioned on no disasters $\mu(W_t) = r + \sigma\gamma Vol(W_t)$ and from Proposition 6 we have that Vol is affine in dividend yields and non-increasing in W_t it results that $\mu(W_t)$ has the same properties.

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Tables and Figures

Table 1: Parameters resulting from calibration.

The Table presents the model parameters resulting from the calibration procedure.

Symbol	What it represents	Value
σ	Volatility of consumption conditional on full employment	3.57%
γ	Relative risk aversion	7.84
δ	Preferences discount factor	3.10%
a	Cobb-Douglas coefficient	0.36
W_0	Initial value of the state variable	
	– "social planner" case	1.4800
	– "big push" case	3.9629
W^g	Value of the state variable triggering the recovery	
	– "social planner" case	0.0000
	– "big push" case	1.6522

Table 2: Alternative calibrations.

The Table presents alternative calibrations of our model. The results are presented in two categories, Conditional and respectively Simulated. The conditional results are obtained in the following way: using the model parameters and the observed U.S. returns, a time series of the state variable is constructed and the conditional equity premium as well as volatility are calculated as described in Proposition 4. The averages of these values are reported, as well as end-of-sample values of the conditional equity premium and the median time to disaster. If the fitted state variable drops under zero, we report that in the respective calibration the economy experienced a disaster. The period in which the economy is in a disaster state is, for the "social planner" calibration, November 1931−April 1933. In order to obtain the simulated results, we simulate 1,000,000 Brownian paths starting at W_0 and calculate the time series average equity premium and volatility as in Proposition 4. We then report the average of these values across all the simulations as well as their standard errors (in parentheses). We also report the percentage of paths experiencing a disaster.

	"Big Push" Calibration	"Social Planner" Calibration
Panel A: Conditional		
Av. Eq. Prem.	4.09%	5.15%
Av. Vol.	14.62%	18.91%
Periods of disaster	No	Yes
Eq. Prem in 12/2008	4.58%	6.29%
Median time to disaster in 12/2008	89 yrs	22.5 yrs
Panel B: Simulated		
Av. Eq. Prem. – no disaster	4.28% (0.97%)	4.25% (0.79%)
Av. Eq. Prem. – entire population	4.75% (1.75%)	3.87% (1.81%)
Av. Vol. – no disaster	15.30% (3.43%)	15.18% (2.82%)
Av. Vol. – entire population	22.81% (6.80%)	23.02% (4.93%)
% of paths without a disaster	35%	15%

Table 3: Return predictability, 1953–2006.

The Table presents predictive regressions of returns of the CRSP value weighted portfolio. The independent variables are as follows: ID/P are the lagged dividend yield as implied by our model (a nonlinear function of the observed price constructed using the "big push" calibration); D/P are the observed dividend yield of the CRSP value weighted portfolio; the 30 day T-bill rate are the rates of returns on a portfolio of 30 days maturity T-Bills; *cay* is the Lettau and Ludvigson [2001] variable. t-stats are in parentheses.

	Model 1	Model 2	Model 3	Model 4	Model 5
Panel A: Monthly predictability, July 1953 – December 2006					
Intercept	-0.02	-0.01	-0.02	0.01	-0.01
	(-2.48)	(-1.03)	(-2.45)	(3.29)	(0.89)
ID/P	1.79		1.57		0.83
	(4.33)		(2.58)		(1.16)
D/P		0.67	0.12		0.17
		(3.46)	(0.44)		(0.61)
30 day T-bill	-3.12	-2.46	-3.12		-2.41
	(-3.84)	(-3.21)	(-3.80)		(-2.82)
cay				0.59	0.42
				(4.73)	(2.73)
R^2	3.56%	2.65%	3.59%	3.38%	4.86%
Panel B: Quarterly predictability, July 1953 – December 2006					
Intercept	-0.02	-0.01	-0.02	0.01	-0.01
	(-2.84)	(-1.26)	(-2.79)	(3.08)	(1.18)
ID/P	1.85		1.63		0.93
	(4.91)		(2.65)		(1.25)
D/P		0.70	0.12		0.17
		(3.74)	(0.42)		(0.57)
30 day T-bill	-2.97	-2.35	-2.96		-2.35
	(-3.94)	(-3.20)	(-3.93)		(-2.93)
cay				0.59	0.39
				(4.26)	(2.31)
R^2	9.84%	7.37%	9.92%	8.48%	12.66%
Panel C: Annual predictability, 1953 – 2006					
Intercept	-0.02	-0.01	-0.02	0.01	-0.02
	(-2.61)	(-1.08)	(-2.59)	(3.30)	(1.87)
ID/P	1.71		1.69		1.53
	(4.53)		(2.83)		(2.34)
D/P		0.60	0.01		0.03
		(3.26)	(0.04)		(0.13)
30 day T-bill	-2.47	-1.76	-2.47		-2.35
	(-3.78)	(-2.17)	(-3.67)		(-3.53)
cay				0.31	0.08
				(3.00)	(0.57)
R^2	32.7%	22.2%	32.7%	10.0%	33.2%

Table 4: Volatility prediction: model implied versus true dividend yields.

The Table presents monthly predictive regressions of volatility on the lagged dividend yields implied by our model (we used the "big push" calibration) and the true dividend yields. Panel A presents predictive regressions where the predictor is the lagged dividend yield implied by our model, ID/P. Panel B presents predictive regressions where the predictor is the lagged true dividend yields on the CRSP portfolio, denoted by D/P. 30-day T-Bill represents the interest rate on 30 day Treasury Bills. The volatility data used start with January of the beginning year and end with December of the ending year. t-stats are in parentheses.

	1927–2008	1927–1945	1946–1965	1966–1985	1986–2008
Panel A: Predicting volatility with implied dividend yields					
Intercept	0.01	-0.02	0.03	0.01	0.08
	(2.48)	(-2.17)	(4.71)	(2.22)	(4.24)
ID/P	1.14	1.76	0.26	0.82	-1.96
	(9.87)	(8.75)	(1.67)	(3.77)	(6.42)
30-day T-Bill	-0.17	13.00	-4.01	-0.73	1.11
	(0.63)	(4.37)	(2.36)	(1.60)	(1.07)
R^2	21.60%	21.41%	10.55%	18.14%	15.26%
Panel B: Predicting volatility with the true dividend yield					
Intercept	0.03	0.02	0.05	0.01	0.06
	(6.40)	(1.70)	(6.42)	(2.63)	(14.36)
D/P	0.51	0.95	-0.09	0.47	-1.02
	(5.07)	(5.60)	(-0.85)	(3.05)	(-5.62)
30-day T-Bill	-1.53	2.67	-6.54	-0.78	1.85
	(-0.63)	(0.99)	(-3.70)	(-1.53)	(1.63)
R^2	9.92%	12.98%	10.13%	16.66%	12.95%

Figure 1: Labor's share of income and disaster magnitude

The Figure presents the labor's share of income (on the x-axis) and the magnitude of consumption/GDP shocks during observed disasters for the following countries: Australia, Belgium, Finland, France, Italy, Japan, Netherlands, Norway, Portugal, Sweden, United Kingdom and United States. The data is obtain from the intersection of Barro and Ursua [2009] (decrease in consumption/GDP) and from Gollin [2002] (labor's share of income). We report results for four measures of disaster magnitude (from Barro and Ursua [2009]): GDP decline during a disaster (we use both the most recent observation as well as the largest decline) and consumption drop during a disaster (we use both the most recent observation as well as the largest decline). We use three measures of labor's share of income (from Gollin [2002]): naively calculated labor's share of income, labor's share of income calculated adding the operating surplus of private unincorporated enterprises (OSPUE) to labor income and labor's share of income calculated by proportionally diving the operating surplus of private unincorporated enterprises between labor and capital. A fitted linear relationship is reported for each plot.

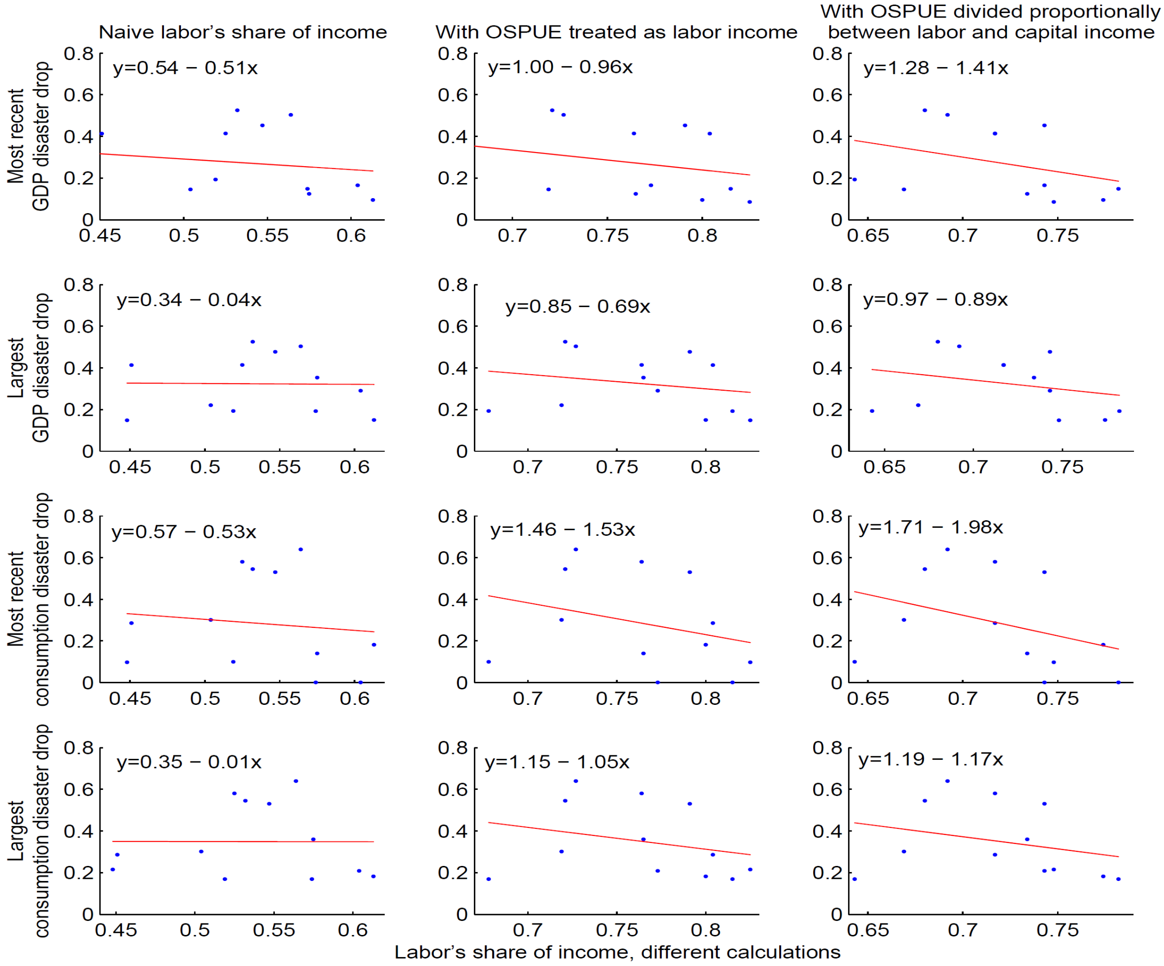

Figure 2: The state variable and corresponding median time to disaster.

The Figure presents the state variable W_t as well as the median time till disaster. The time period is January 1927 to December 2008. The state variable is backed up from prices using the “big push” calibration.

Figure 3: The conditional equity premium

The figure presents the ex-ante equity risk premium in the U.S., as implied by our model. The model is calibrated assuming the "big push" model.

Figure 4: Observed and predicted equity premium

The Figure presents the observed equity premium in the U.S. (thin line) along with a 95% confidence interval for the equity premium implied by our model using the "big push" calibration (thick line). Each month from January 1927 to December 2008, the confidence interval is obtained by plotting the equity risk premium as implied by equation (21) plus and respectively minus two standard deviations of equation (20).