Optimal Leverage and Investment under Uncertainty*

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Abstract

This paper studies the effects of changes in uncertainty on optimal financing and investment in a dynamic firm financing model in which firms have access to complete markets subject to collateral constraints. Entrepreneurs finance projects with their net worth and by issuing state-contingent securities, which have to be collateralized with physical capital. An increase in uncertainty leads to deleveraging, as entrepreneurs reduce their demand for external financing and fund a larger share of their investment from net worth. Upon an increase in uncertainty, investment initially falls as entrepreneurs decrease the scale of their projects. Investment recovers as entrepreneurs build up net worth and transition into an environment with high uncertainty. Quantitatively, changes in uncertainty have large effects on optimal leverage and investment dynamics.

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1 Introduction

During the recent financial crisis the U.S. economy has experienced a significant increase in measured uncertainty as documented in Bloom, Floetotto, and Jaimovich (2009). At the same time the economy suffered a severe recession with sharp contractions in investment and credit, among other indicators. Figure 1 plots the evolution since 2006 of the implied volatility index, VIX, as a measure of uncertainty, real investments, and real commercial loans advanced to U.S corporations. Furthermore, as can be see from Figure 1, in the aftermath of the crisis non-financial corporations sharply increased their holdings of liquid assets, prompting the question of why firms are not investing more given that they have access to so much liquidity?[1] Motivated by these observations, in this paper I study the effects of changes in uncertainty on financing and investment decisions in a dynamic firm financing model, where firms have access to complete markets subject to collateral constraints.

The main contribution of this paper is the study of the effects of changes in uncertainty in an environment where financing is based on an optimal long-term contract. Specifically, I derive comparative statics results and using calibrated parameter values, I then provide quantitative results.

Dynamic firm financing is modeled as in Rampini and Viswanathan (2010a), who derive collateral constraints endogenously from limited enforcement constraints. In this setting, uncertainty jointly determines firms' optimal capital structure and investment decisions. Specifically, collateral constraints impose limits on borrowing, which forces entrepreneurs to finance their projects with both net worth and external funds. In turn, limits to borrowing affect firms' investment choices, as available net worth determine the investment that entrepreneurs can afford.

Entrepreneurs use physical capital as their only factor of production. Investment in physical capital is funded from the existing net worth and external financing. External financing has benefits, but comes with a risk for entrepreneurs. On the one hand, ex-ante higher leverage allows entrepreneurs to increase their investment

[1] See "Show us the money," The Economist, July 1, 2010 and "The cost of repair," The Economist, October 7th, 2010.

and achieve faster growth if ex-post returns on investments are high. On the other hand, external financing carries a risk for entrepreneurs, as the repayment of debt in periods of low returns reduces entrepreneurs' net worth. As reductions in net worth constrain future investment decisions, the scale of the project determines the trade-off between faster growth when realized returns are high and the risk of losing net worth in states with low returns. Thus, entrepreneurs have to choose not only their investment policy but also their financing policy.

In this paper I show that increases in uncertainty amplify the risk of borrowing. With an increase in uncertainty, the variance of the returns that entrepreneurs face on their investment increases. As a result, in periods when returns are low, repaying debt leads to larger reductions of net worth. Consequently, upon an increase in uncertainty entrepreneurs will decrease the scale of their projects and will delever; that is, entrepreneurs will reduce their demand for external financing and fund a larger share of investments from their net worth. Thus, an increase in uncertainty initially leads to a fall in optimal investment. Investment recovers as entrepreneurs build up their net worth and transition into an environment with higher uncertainty and lower leverage.

It is instructive to relate this result to the standard result on the effect of uncertainty on investment when firms face convex adjustment costs. As shown in Abel (1983), an increase in uncertainty induces a precautionary savings behavior, and since capital is the only vehicle through which firms can save, increased uncertainty leads to an increase in investment. In this paper, in addition to investment, entrepreneurs also choose their financing policy. Consequently, the precautionary savings motive can manifest either through an increased investment or through a decrease in external financing. This paper shows that when all collateral constraints bind before and after an increase in uncertainty, entrepreneurs can only save by increasing their investment, just as in Abel (1983). However, when some collateral constraints are slack, entrepreneurs can also save by borrowing less at the margin, which may reduce their investment.

In the long run, the change in uncertainty will be reflected in firms' capital structure. Upon an increase in uncertainty, firms decrease their demand for external financing and will finance a larger share of their investment from their net worth. In the new environment with high uncertainty, firms will have larger net worth and lower leverage and will be able to operate the firm at the initial scale. Thus this paper highlights the importance of capital structure as the main mechanism through which uncertainty affects firm dynamics.

The model has several important implications. First, the paper has implications for corporate risk management practices. The main prediction of the model is that upon an increase in uncertainty, risk management concerns override firms' financing needs, and as a result investment decreases. The need to hedge fluctuations in net worth implies that entrepreneurs issue fewer claims against lower states; however this comes at the expense of their financing needs, resulting in reduced investments.

Furthermore, the predictions of this paper are in line with the observed increase in liquid assets held by non-financial corporations. The model features complete markets, subject to collateral constraints, which allow firms to engage in risk management. Firms can hedge idiosyncratic risk by issuing fewer claims against lower states, but also by conserving net worth in all states to take advantage of future investment opportunities. Conserving net worth against all states in this context can be thought of as hoarding cash. The results in this paper show that upon an increase in uncertainty firms will increase their cash holdings, thus providing a potential explanation for the observed increase in liquid assets holdings.

Additionally, it is important to note that leverage and collateral are determined in equilibrium. This is so despite the fact that collateral constraints are derived endogenously from limited enforcement constraints,[2] where the tightness of the constraint is governed by one parameter. Models that feature collateral constraints typically assume that collateral constraints are always binding and thus the leverage ratio is exogenously fixed.[3] Indeed, the occasionally binding nature of collateral constraints is crucial to the results in this paper.

[2]See Rampini and Viswanathan (2010a) and Kehoe and Levine (1993). For an alternative environment with endogenously incomplete markets, see Geanakoplos (1997), Geanakoplos (2003), Dubey, Geanakoplos, and Shubik (2005), Geanakoplos (2009).

[3]See Kiyotaki and Moore (1997), Iacoviello (2005), with the notable exception of Mendoza (2010) and Khan and Thomas (2010).

Finally, it is instructive to compare an economy with complete markets, subject to collateral constraints, to an economy with incomplete markets that is subject to the same constraints. When markets are incomplete, entrepreneurs insure against fluctuations in productivity by conserving net worth. Thus, under incomplete markets firms tend to have higher capitalization. While entrepreneurs are less able to insure against risk in the economy, higher capitalization allows entrepreneurs to weather unexpected changes in uncertainty. In economies with complete markets, entrepreneurs can hedge states with low returns. Since hedging improves risk sharing, entrepreneurs need not conserve as much of their net worth. But this also implies that entrepreneurs will be thinly capitalized in the face of unexpected shocks. As a result in economies with complete markets, subject to collateral constraints, shocks tend to be amplified, while economies with incomplete markets, also subject to collateral constraints, tend to dampen the effects of uncertainty shocks.[4]

This paper builds on Rampini and Viswanathan (2010a), who study risk-neutral entrepreneurs subject to limited liability, whereas this paper assumes that entrepreneurs are risk-averse. The main implication of the assumption of risk-averse entrepreneurs can be found in the optimal firm size. Specifically, in the model with collateral constraints, well-capitalized (high net worth) entrepreneurs will operate at the same optimal size as in the frictionless economy. Crucially, this result allows for the analytical derivation of comparative statics results. Furthermore, by using calibrated values, I compute the the effects of uncertainty shocks and show that the mechanism presented in the model is quantitatively significant.

The paper is related to several lines of research. First, I follow the literature that considers dynamic incentive problems as the main determinant of firm financing and capital structure. Specifically, I consider limited enforcement problems between financiers and investors as in Albuquerque and Hopenhayn (2004), Lorenzoni and Walentin (2007), Rampini and Viswanathan (2010a), and Rampini and Viswanathan (2010b). Albuquerque and Hopenhayn (2004) consider the case of a firm which needs financing for a project with an initial non-divisible investment, whereas here I con-

[4]Cooley, Marimon, and Quadrini (2004) also find that complete markets tend to amplify the shocks in the economy.

sider a standard neoclassical investment problem; moreover the limits of enforcement differ in the two specifications. Lorenzoni and Walentin (2007) are the first to derive endogenously collateral constraints from limited enforcement constraints and study its implications on investment and Tobin's q. Because of constant returns to scale, in their setup firm-level net worth does not matter; moreover, they assume that all collateral constraints bind. The focus in this paper is on the effect of uncertainty on the capital structure, and the interaction between net worth and demand for external financing. Aggregate implications of limited enforcements are further studied in Cooley, Marimon, and Quadrini (2004) and Jermann and Quadrini (2007). None of these papers analyze the effect of changes in uncertainty on capital structure and investment dynamics.

Implications of incentive problems due to private information about cash flows or moral hazard on capital structure and investment dynamics are studied in Quadrini (2004), Clementi and Hopenhayn (2006), DeMarzo and Fishman (2007), DeMarzo et al. (2009). However they do not consider the effect of changes in the level of uncertainty on leverage and investment dynamics.

Second, there is a recent literature that looks at the aggregate effect of uncertainty shocks. In the presence of adjustment costs,[5] Bloom (2009) and Bloom, Floetotto, and Jaimovich (2009) argue that uncertainty shocks represent important driving forces for business cycles,[6] although the quantitative importance of this mechanism is debated in the literature, see Bachmann and Bayer (2009). As in the above-mentioned papers, my focus is also on the effect of uncertainty on capital accumulation; however I do not consider the presence of adjustment costs.

Third, uncertainty shocks also have been considered in models with financing frictions; this literature considers models in which entrepreneurs have private information about their cash flows and obtain financing through optimal, one-period debt contracts as in Townsend (1979) and Bernanke, Gertler, and Gilchrist (1999). In this setup, changes in uncertainty affect credit spreads, thus influencing the cost of fi-

[5]There is a large literature on the role of uncertainty on capital accumulation, but the focus is on the long run effects of uncertainty, see Abel (1983), Dixit (1989), Caballero (1991), Dixit and Pindyck (1994), Bertola and Cabellero (1994), Abel and Eberly (1996), Abel and Eberly (1999).

[6]This idea initially was developed in Bernanke (1983).

nancing. The first paper that formalized this insight is Williamson (1987); recently it received further attention in Christiano, Motto, and Rostagno (2009) and Gilchrist, Sim, and Zakrajšek (2009). In contrast to the above literature, this paper considers optimal long-term contracts where the agency friction is limited enforcement, and furthermore assumes that the cost of financing is constant, so the mechanism described above is absent.

Finally, Arellano, Bai, and Kehoe (2010) study fluctuations in uncertainty in an economy without capital, whereas Fernández-Villaverde et al. (2010) consider the effect of fluctuations in uncertainty on small, open economies.

The outline of this paper is as follows. The next section presents the model and characterizes the solution. Section 3 contains the main analytical results of the paper, while Section 4 contains the quantitative results. The last section concludes.

2 The Model

This section presents a neoclassical investment model where entrepreneurs have access to a complete set of state-contingent securities, subject to collateral constraints, as in Rampini and Viswanathan (2010a). Due to the collateral constraints, entrepreneurs have to finance their investment from their net worth and external funds. External funds are provided by lenders who have access to a limitless supply of capital. Credit markets are subject to limited enforcement; that is, entrepreneurs can default on their loan obligations and divert cash flows and a fraction of their capital holdings. Lenders discount the future at the rate $\tilde{\beta} \equiv R^{-1}$, and are willing to supply funds as long as, in net present value terms, the loans are repaid.

There is a measure one of risk-averse, relatively impatient entrepreneurs, who discount the future at the rate, $\beta < \tilde{\beta}$. Entrepreneurs have access to a production technology with decreasing returns to scale.[7] Capital, k, is the only factor of production, which depreciates at a constant rate $\delta \in (0, 1)$.

Assumption 1 *The production function, f, is strictly increasing, strictly concave*

[7]This assumption can alternatively be motivated by a decreasing industry demand function.

and differentiable, $f'(k) > 0$, $\lim_{k \to 0} f'(k) = \infty$, $\lim_{k \to \infty} f'(k) = 0$.

The return on capital, k', is subject to stochastic shocks $A(s')f(k')$, where $A(s')$ is the realization of the total factor productivity in state $s' \in S$. Let the history of events up to time t be denoted by $s^t = [s_0, \ldots, s_{t-1}, s_t]$, where $s_t \in S^t$, $\forall t$. Furthermore, assume that the state s follows a Markov chain process with transition matrix, $\pi(s_t, s_{t+1})$, with $s_t \in S, \forall t$.

Assumption 2 *For all $s, \hat{s} \in S$, where $\hat{s} > s$, $A(\hat{s}) > A(s)$ and $A(s) > 0$, $\forall s \in S$*

Entrepreneurs have the possibility to default. Upon default they can divert the cash flow and $(1 - \theta) \in (0, 1)$ fraction of available capital, whereas creditors can seize the remaining θ fraction of the resale value of capital. A crucial assumption of the model is that defaulting entrepreneurs are not excluded from either capital nor physical goods markets.

2.1 Limited Enforcement

Entrepreneurs enter into long-term contracts with risk-neutral lenders who have unlimited capital. The contract specifies payments, $p_t(s^t)$ between entrepreneurs and lenders. These payments can be negative or positive, depending on whether entrepreneurs need financing or pay back their loans. In order for lenders to participate in this contract, the present value of net payments must be non-negative:

$$\sum_{t=0}^{\infty} \sum_{s^t} R^{-t} \pi(s_0, s^t) p_t(s^t) \geq 0 \tag{1}$$

where $\pi(s_0, s^t) = \pi(s_0, s_1) \times \ldots \times \pi(s_{t-1}, s_t)$.

Additionally, since entrepreneurs might default in any future period or state, lenders must ensure that in an eventual case of default, the value of the assets that they recoup will cover the present value of their net payments. Since in the present context, the value that lenders can recoup equals θ fraction of the resale value of the capital stock, the enforcement constraint is:

$$\theta k_{t+1}(s^t)(1-\delta) \geq \sum_{j=t}^{\infty} \sum_{s^j} R^{t-j} \pi(s_t, s^j) p_j(s^j), \ \forall s^j \in S \tag{2}$$

Notice that the enforcement constraint takes a very simple form; the present value of capital holdings serves as collateral for the entrepreneurs' future promised payments. Because of the possibility of default, entrepreneurs can credibly issue promises against state s^{t+j}, of up to the θ fraction of the resale value of undepreciated capital in that state.

Denote $Rb_1(s_0, s_1)$ as the present value of all future payments from the entrepreneur to the lender in state s_1. Then (1) can then be written as follows

$$\begin{aligned} Rb_1(s_0, s_1) &= \sum_{t=0}^{\infty} \sum_{s^t} R^{-t} \pi(s_0, s^t) p_t(s^t) \\ &= p_0(s_0) + R \sum_{s_1|s_0} R^{-1} \pi(s_1, s_2) b_2(s_1, s_2) \\ &= p_0(s_0) + \sum_{s_1|s_0} \pi(s_1, s_2) b_1(s_1, s_2) \end{aligned} \tag{3}$$

Notice that (3) implies that entrepreneurs issue state-contingent, one-period securities; however these securities are priced by the lenders with the probability that particular states occur. This is intuitive; since lenders are risk-neutral, they price state-contingent assets only with the probability of that state occurring; that is, without correcting for any risk factor. With this notation, the enforcement constraint (2) can be written:

$$\theta k_{t+1}(s^t)(1-\delta) \geq Rb_{t+1}(s_t, s_{t+1}), \ \forall s_{t+1} \in S \tag{4}$$

Furthermore, conditions (4) makes it clear that the long-term contract can be implemented by a sequence of one-period contracts, where entrepreneurs issue state-contingent claims that are subject to state-contingent collateral constraints. Notice that, in general enforcement constraints depend on the value of default, and these enforcement constraints have to hold in all future periods. This implies that for entrepreneurs to make credible promises to repay the loan, lenders need to know all preference parameters and to keep track of the whole history of repayments. In the present context, lenders only have to observe the current level of entrepreneurs' physical capital, and thus the informational requirements on the lenders' knowledge is greatly reduced. Specifically, an important implication of the present model is that lenders need to know only the per-period publicly available asset holdings of entrepreneurs. Thus, the value of the default, in general a value function itself, now depends only on the level of physical assets. This simplifies the problem considerably[8].

We now turn to the entrepreneurs' problem.

2.2 Entrepreneurs' Problem

Entrepreneurs choose dividends, investment and financing to maximize the expected utility of their future dividend consumption. I assume entrepreneurs are risk-averse over their dividend payments.

Assumption 3 *The utility function, u, is strictly increasing, strictly concave, and differentiable:* $u'(d) > 0$, $\lim_{d\to 0} u'(d) = \infty$, $\lim_{d\to\infty} u'(d) = 0$.

Using the collateral constraints (4), the entrepreneurs' problem can be written in recursive form. Furthermore, the problem can be substantially simplified with the introduction of an additional variable, net worth. Define net worth in state s' as $w(s') = z' f(k') + k' (1 - \delta) - Rb(s')$, the return on investment and resale value of capital less the state-contingent debt to be repaid. The introduction of net worth allows the reduction of the number of potential state variables from at least three $(k', b(s'), s')$ (where, notice, debt in every state of the economy is part of the state variables), to only two $(w(s'), s')$, significantly simplifying the problem. I suppress notation by assuming that every variable depends implicitly on (w, s). The en-

[8] See for example the treatment in Marcet and Marimon (1992), Kehoe and Levine (1993), Alvarez and Jermann (2000), and Marcet and Marimon (2009)

trepreneurs' problem can then be written as:

$$V(w,s)=\max_{\{d,k',b(s'),w(s')\}}\left\{u(d)+\beta\sum_{s'\in S}\pi(s,s')V(w(s'),s')\right\} \tag{5}$$

subject to

$$w+\sum_{s'\in S}\pi(s,s')b\left(s'\right)\geq d+k' \tag{6}$$

$$A(s')f(k')+k'\left(1-\delta\right)\geq w(s')+Rb(s'), \qquad \forall s'\in S \tag{7}$$

$$\theta k'\left(1-\delta\right)\geq Rb(s'), \qquad \forall s'\in S \tag{8}$$

and

$$d\geq 0, \qquad k'\geq 0.$$

Entrepreneurs in each period use their net worth and potential borrowing to fund gross investments, k', and pay out dividends, d, as can be seen from the budget constraint (6). To obtain funding, entrepreneurs issue state-contingent securities that they promise to buy back in the next period. Next period's net worth depends on the amount of investment, the realized state of the economy and the cost of financing, as can be seen in equation (7).

Given the possibility of default, entrepreneurs' promises to repay their debt are not credible and they need to secure their borrowing with their physical capital. Lenders are willing to provide financing only if, in case of default, entrepreneurs' assets can cover the provided funds. This is encoded in the collateral constraints (8), which need to hold in every state of the world.

Entrepreneurs issue state-contingent claims, secured with their capital holdings. Obtained financing must be repayed at a cost $Rb(s')$. Entrepreneurs have to trade off their need for investment with the cost of financing. Borrowing against state s' reduces next period's net worth in that state. This implies that borrowing against state s' carries a risk for entrepreneurs, as investment in state s' will be constrained by the available net worth. Notice also that the above collateral constraints are similar to the one used in Kiyotaki and Moore (1997), with the exception that the collateral constraints here are derived endogenously from a limited enforcement problem, and that borrowing is state-contingent.

Next, I turn to the characterization of the recursive problem. The below proposition states that the entrepreneurs' problem is well-defined and there exists a unique value function V satisfying (5) - (8).

Proposition 1 ***(i)*** *There is a unique V, satisfying (5) - (8).* ***(ii)*** *V is continuous, strictly increasing, and strictly concave in w.* ***(iii)*** $\forall\ \hat{s}, s \in S$ *such that* $\hat{s} > s$, $\pi(\hat{s}, s')$ *strictly first order stochastically dominates* $\pi(s, s')$, *V is increasing in s.*

The proofs for Parts (i) - (iii) are relatively standard. The concavity of the production function, and the risk-aversion of entrepreneurs guarantee that V is a unique, strictly increasing and strictly concave function of net worth.

Denote the Lagrange multipliers on the constraints (6), (7), (8) as λ, $\beta\pi(s, s')\lambda(s')$, $\beta\pi(s, s')\lambda(s')\mu(s')$. The first-order conditions for the entrepreneur are:

$$\lambda = u'(d) \tag{9}$$

$$\lambda = \beta \sum_{s' \in S} \pi(s, s')\lambda(s')(A(s')f'(k') + 1 - \delta + \mu(s')\theta(1 - \delta)) \tag{10}$$

$$\lambda = \beta R\lambda(s')(1 + \mu(s')), \qquad \forall s' \in S \tag{11}$$

$$\mu(s')(\theta k'(1 - \delta) - Rb(s')) = 0, \mu(s') \geq 0 \qquad \forall s' \in S \tag{12}$$

The envelope condition is $V_w(w, s) = \lambda$. Due to the assumptions on the production and utility functions, capital and dividends is always positive; thus I do not include that constraint in the above Kuhn-Tucker conditions.

Condition (9) governs the dividend payout policy of entrepreneurs, and the envelope condition makes it clear that dividend payout depends on the marginal valuation of net worth. Condition (10) governs the optimal investment of entrepreneurs. Notice that in states when the collateral constraint (8) does not bind for any state $s' \in S$ next period, $\mu(s') = 0$, (10) reduces to the standard Euler equation, where optimal investment is governed by the marginal revenue of capital weighted by entrepreneurs' stochastic discount factor. A binding collateral constraint (8), $\mu(s') > 0$, drives a wedge between the marginal product of capital and the relative marginal utilities of wealth. Specifically, binding collateral constraints imply that entrepreneurs use physical capital both as a factor of production and as an asset that can be used for collateral. Thus, internal funds require a premium in the presence of binding collateral constraints.

Equation (11) governs the evolution of entrepreneurial net worth. Optimal next period net worth depends on the financing need of entrepreneurs. In states where the collateral constraint does not bind, $\mu(s') = 0$. In states however when the collateral constraint binds, the use of capital for collateral purposes is encoded in the value of $\mu(s')$.

The next proposition shows that the problem (5) - (8) has a unique solution.

Proposition 2 *Denote* $x_0 \equiv [d_0, k'_0, b_0(s'), w_0(s')]$. *The optimal policy* x_0 *is unique.*

Next, I discuss the solution of the frictionless problem, when entrepreneurs are not relatively impatient and borrowing is not subject to collateral constraints.

2.3 Frictionless Case

In this section I consider the frictionless case, when there are no collateral constraints and entrepreneurs have the same discount factor as lenders. In that case, entrepreneurs can perfectly insure against idiosyncratic productivity shocks, $\lambda = \lambda(s')$ for all $s' \in S$,[9] and operate on the optimal scale. Indeed, the optimal capital stock then is given by:

$$1 = \beta \sum_{s' \in S} \pi(s, s')(A(s')f'(\bar{k}') + 1 - \delta)$$

Since markets are complete, firms' capital structure is indeterminate, and firms operate at the optimal scale, $\bar{k}'$, at all levels of net worth. Next, I turn to the case when entrepreneurs are relatively impatient and face collateral constraints.

[9]See Chapter 8, in Ljungqvist and Sargent (1994).

2.4 Characterization of the Optimal Policy

In this section I characterize the optimal financing and investment policies of entrepreneurs. Due to the presence of collateral constraints, entrepreneurs must finance part of their investments with their internal funds. Depending on their level of net worth, entrepreneurs must accumulate enough internal funds to be able to afford levels of investment that maximize the return on their project. Entrepreneurs' optimal policies determine their financing demand, investment choice and their optimal accumulation of internal funds. Throughout this section I derive the results under the assumption of constant investment opportunities, $\pi(s, s') = \pi(s')$.

The optimal financing policy can be best characterized by analyzing the shadow value of collateral, $\mu(s')$.

Proposition 3 (Optimal Financing Policy) ***(i)*** *There exists* $\underline{w} > 0$ *such that if* $w < \underline{w}$*, then the collateral constraint in all states bind* $\mu(s') > 0$ $\forall s' \in S$. ***(ii)*** *The marginal value of net worth is (weakly) decreasing in the state* s'*, whereas the multipliers on collateral constraints are (weakly) increasing in the state* s'*;* $\forall s', s'_+ \in S$*, such that* $s'_+ > s'$, $\lambda(s'_+) < \lambda(s')$ *and* $\mu(s'_+) > \mu(s')$*.* ***(iii)*** *There exist* $\bar{w} > 0$ *such that if* $w \geq \bar{w}$ *then* $\mu(s') = 0$, $\forall s' \in S$

To understand the implications of the model for financing demand, recall that entrepreneurs must finance part of their investment with their own net worth. Thus entrepreneurs' investment decisions are constrained by their available net worth. The first part of the proposition states that when the net worth of entrepreneurs is low enough, entrepreneurs will exhaust their debt capacity against all future states. Of course, entrepreneurs do this because, in all future states, the marginal return on their investment will be higher then their cost of financing, which in the current model is R.

The second part of the proposition characterizes the shadow value of collateral in different states of nature. The proposition says that in states when returns are low the shadow value of collateral is lower. Intuitively, entrepreneurs would always like to borrow less against states with low returns, as in such states repaying the debt leads to losses of net worth. Entrepreneurs will always want to issue more claims against states with high returns, but the collateral constraints restrict the amount of funds that can be borrowed. Thus entrepreneurs' shadow value of collateral is higher against states with high returns.

The last part of the proposition states that when entrepreneurs have accumulated enough net worth, they will choose to issue fewer claims than the value of their collateral, against all future states. The intuition for this result is that when the level of net worth is high enough, entrepreneurs will be able to perfectly hedge the idiosyncratic fluctuations in productivity, and they will no longer value physical assets for the purpose of collateral.

Turning now to the optimal investment policy,

Proposition 4 (Optimal Investment Policy) *There exists* $\bar{w} > 0$ *such that* ***(i)*** *if* $w, w_+ < \bar{w}$*, for* $w < w_+$ *such that* $w < w_+$ *then* $k'(w) < k'(w_+)$*.* ***(ii)*** *If* $w \geq \bar{w}$*, then* $k' = \bar{k}$*.*

Entrepreneurs with low levels of net worth will be constrained in their investment opportunities, as part of their investments need to be financed by net worth. As entrepreneurs increase their net worth, their investment decisions become less constrained. Entrepreneurs keep accumulating net worth until the return on their investment is greater or equal to their cost of financing R. Thus, as long as long as net worth is low enough, in that entrepreneurs are constrained in their investment choice, entrepreneurs' optimal investment policies are increasing in their net worth. However, once optimal investment reaches the level at which the marginal return on investment equals the opportunity cost of investment, R, entrepreneurs stop accumulating further capital.

Notice that the maximal level of investment, $\bar{k}$, is also the solution to the neoclassical investment problem with complete markets, same discount factor, and no collateral constraints. Since all risk is idiosyncratic, entrepreneurs can perfectly insure against this risk, and at all levels of net worth they will invest $\bar{k}$. In the presence of collateral constraints, when net worth is low, investment will be constrained by the available net worth. Thus entrepreneurs will have to build up their net worth in order to afford the same level of investment as in the problem without limits to borrowing.

To understand the optimal dividend policy recall the optimality conditions (9) and the envelope condition: $\lambda = u'(d) = V_w(w, s)$. Intuitively risk aversion ensures that entrepreneurs increase their dividend payout in line with accumulation of internal funds.

The evolution of optimal net worth is presented in the proposition below.

Proposition 5 (Net worth transition dynamics) *Suppose* $\pi(s, s') = \pi(s')$, $\forall s, s' \in S$. ***(i)*** $\forall s', s'_+ \in S$, *such that* $s'_+ > s'$, $w(s'_+) \geq w(s')$, *with equality if* $\mu(s'_+) = \mu(s') = 0$. ***(ii)*** $w(s')$ *is increasing in* w, $\forall s' \in S$; *for* w *sufficiently small,* $w(s') > w$, $\forall s' \in S$; *and for* w *sufficiently large,* $w(s') < w$, $\forall s' \in S$. ***(iii)*** $\forall s' \in S$, $\exists w$ *dependent on* s' *such that* $w(s') = w$.

Part (i) of Proposition 5 states that the higher the returns on the projects are in a state, the larger will net worth be in the next period. To understand Part (ii), recall the optimality condition (11). If the level of initial net worth is low enough, entrepreneurs will be able to grow by levering up against future states. That is, at levels of initial net worth at which $\beta R(1 + \mu(s')) > 1$, entrepreneurial net worth increases. However when net worth is high enough, entrepreneurs, being relatively impatient, have no incentive to save and thus they will pay out net worth as dividends. Therefore, next periods' net worth decreases. Part (iii) states that there exists a unique level of net worth in each state at which net worth stays constant.

The equilibrium outcome will be a stationary distribution of firms, in terms of their net worth. The next proposition shows the existence of a stationary distribution and characterizes its support.

Proposition 6 (Existence of a Stationary Distribution) *There exists a unique stationary distribution of net worth. Define* w_l, $\underline{s}$, w_u, *and* $\bar{s}$, *where* $s \geq \underline{s}$, *and* $s \leq \bar{s}$, $\forall s \in S$, *such that* $\mu(w_l, \underline{s}') = 1/(\beta R) - 1$ *and* $\mu(w_u, \bar{s}') = 1/(\beta R) - 1$. *Then the support of the stationary distribution is* $w \in [w_l, w_u]$.

The partial equilibrium framework allows for the characterization of the stationary distribution and to provide sharp bounds on its support. From (11), notice that whenever $\mu(\underline{s}') < 1/(\beta R) - 1$, $\lambda < \lambda(\underline{s}')$ which implies that $w > w(\underline{s}')$. Thus, entrepreneurs in state $\underline{s}'$ choose to have lower net worth. However, since entrepreneurs were already constrained in their investment choices, a further decline in capitalization will further constrain their investment possibilities. This implies that with a decline in net worth, the Lagrange multiplier on the collateral constraint, μ, next period will have to rise. When $\mu(\underline{s}')$ increases such that $\mu(\underline{s}') > 1/(\beta R) - 1$, from (11) we have that $\lambda > \lambda(\underline{s}')$; thus entrepreneurs will increase their net worth. The symmetric argument applies when $\beta R(1 + \mu(\bar{s}')) > 1$.

Levels of net worth, at which $\mu(\underline{s}') > 1/(\beta R) - 1$ and $\mu(\bar{s}') < 1/(\beta R) - 1$ are transient. Whenever, net worth is low enough, $w < w_l$, regardless of the realization of the shocks entrepreneurs' net worth in next period increases. Similarly, when net worth is high enough, $w > w_u$, entrepreneurs prefer to pay out net worth as dividends and thus next period's net worth decreases. As a result levels of net worth outside of the support $w \in [w_l, w_u]$ are transient.

In the more general case, with autocorrelated shocks, investment and financing policies will depend both on the current state and net worth. In Section 4, I study the quantitative implications when investment opportunities are stochastic. There the properties of the technology shocks will be calibrated to empirically plausible measures of autocorrelation and volatility.

2.5 Risk-Averse Entrepreneurs

Let me now turn to the discussion of the importance of risk-averse entrepreneurs. As I have shown above, the assumption of risk aversion implies that investment equals frictionless investment when net worth is sufficiently high. In contrast, with risk-neutral entrepreneurs this is not the case. This result has several implications.

Above a threshold level of net worth, entrepreneurs will hedge all future states; that is, entrepreneurs will conserve net worth against all states to be able to take advantage of future investment opportunities. This happens despite the fact that conserving net worth is costly, as entrepreneurs are relatively impatient. Accumulating net worth against all states can also be interpreted as firms holding onto cash or liquid assets, which I will discuss in the next section. If entrepreneurs are risk-neutral, as shown in Rampini and Viswanathan (2010b) entrepreneurs will not hedge states with high returns when investment opportunities are constant.

The assumption of risk aversion makes also it also convenient to derive comparative statics results. When entrepreneurs' net worth is high enough, then the project is operated at the same scale as in the frictionless economy. And since this scale of operation does not depend on the level of uncertainty, the bounds for the stationary distribution can be exactly pinned down. This significantly simplifies the derivation of comparative static results.

Finally, the results in this paper crucially depend on whether the collateral constraints bind. When agents are risk averse using calibrated parameters, I find that collateral constraints will bind in some regions of the state variable, while not in others. In a model with risk-neutral agents, under the current parameterization all collateral constraints bind and thus from a quantitative point of view, the effects discussed in this paper will not be present.

2.6 Collateral Constraints and Borrowing Constrained States

In this section I discuss the nature of entrepreneurs' collateral constraints and how financing depends on them. Since investment is constrained by the available net worth, entrepreneurs want to accumulate net worth as fast as possible. However entrepreneurs also want to insure against states with low realization of shocks. That is, they want to transfer net worth from high states to low states. Collateral constraints (8) imply that entrepreneurs cannot promise to pay more than the value of their collateral in the subsequent period. As a result, collateral constraints impose a limit on how much insurance entrepreneurs can achieve. Consequently, collateral constraints tend to bind against states with high realizations of shocks, and be slack against states with low returns. However, this does not mean that entrepreneurs are borrowing constrained in states with high returns. After all, both their investment and financing policies are choice variables. It simply means that in the absence of collateral they cannot shift enough wealth from states with high realizations of productivity to states with low realizations of productivity.

In fact, the lower their net worth is, the more constrained entrepreneurs become. To understand this, notice that collateral constraints imply that part of investment must to be funded from entrepreneurs' available net worth. The lower the net worth, the lower the down payment entrepreneurs can afford, and the more constrained their investment choices becomes. And since collateral constraints impose a limit on firms' maximum leverage, they tend to be borrowing constrained precisely at lower levels of net worth, which can happen after a series of realizations of low productivity shocks.

3 Uncertainty, Financing, and Investment Decisions

In this section I analyze the effects of an increase in uncertainty on investment and financing decisions in the presence of collateral constraints. All results are derived under the assumption of constant investment opportunities; that is $\pi(s, s') = \pi(s')$.

The effect of uncertainty can be modeled in two equivalent ways. On the one hand one can assume two stochastic probability distributions, in which case one probability distribution second order stochastically dominates the other probability distribution. The two probability measures will impact the optimal investment and financing decision through their impact on the prices of state-contingent securities. Since lenders are risk neutral, the state-contingent securities are priced according to their probability measures.

Changes in uncertainty are modeled as a mean preserving spread over the magnitude of productivity shocks. As such, the price of state-contingent securities remains the same, however entrepreneurs' demand for state-contingent debt will change as the magnitude of shocks change. The effect of uncertainty is modeled by comparing the stationary distribution of net worth, and the resulting investment and financing decision under two total factor productivity processes, A_L, A_H. The two productivity processes have the same mean, but differ only in their variance. Denote the mean productivity level as $\bar{A}_i = \sum_{s\in S} \pi(s)A_i(s)$, $\forall\ i \in \{L, H\}$. Then

Assumption 4 *Define the mean of the two productivity levels as* $\bar{A}_i = \sum_{s\in S} \pi(s)A_i(s)$, *and define the variances as* $\sigma_i^2 = \sum_{s\in S} \pi(s)(A_i(s) - \bar{A}_i)^2$, $\forall\ i \in \{L, H\}$. *Assume that* $\bar{A}_L = \bar{A}_H$, *but* $\sigma_L < \sigma_H$.

Let us turn to the effect of uncertainty on firm investment and financing decisions. The next proposition shows that dividend payout decreases with uncertainty.

Proposition 7 (Uncertainty and Dividend Payout) *Dividend payout decreases with an increase in uncertainty; that is* $d_L > d_H$ *for all* w.

The intuition behind this result can be understood as follows. Since the value function is strictly concave, an increase in uncertainty induces a precautionary savings behavior for entrepreneurs; as a result entrepreneurs want to save more. Thus, at any given level of net worth, entrepreneurs decrease their dividends payout in order to be able to save more.

The next proposition states that when uncertainty is high, firms decide to hedge at lower levels of net worth, whereas firms reach the level of net worth that allows their investment choice to be unconstrained at higher levels of net worth.

Proposition 8 (Uncertainty and Financing Demand) *Denote* $\underline{s}, \bar{s} \in S$ *such that* $s \geq \underline{s}$, *and* $s \leq \bar{s}$, $\forall s \in S$. ***(i)*** *Denote* $\underline{w}_L$ *and* $\underline{w}_H$ *such that* $\mu_L(\underline{w}_L, \underline{s}') = 0$ *and* $\mu_H(\underline{w}_H, \underline{s}') = 0$. *Then* $\underline{w}_H < \underline{w}_L$. ***(ii)*** *Denote* $\bar{w}_L$ *and* $\bar{w}_H$ *such that* $\mu_L(\bar{w}_L, \bar{s}') = 0$ *and* $\mu_H(\bar{w}_H, \bar{s}') = 0$. *Then* $\bar{w}_H > \bar{w}_L$.

Intuitively, uncertainty affects the risk of external financing. On the one hand, when uncertainty is high, entrepreneurs may want to borrow more against states with high returns; however the collateral constraints limit the amount of external financing provided by lenders. Thus, entrepreneurs cannot hedge the larger risks by borrowing more against states with high returns. On the other hand, entrepreneurs' in low states now face lower returns on their project. As a consequence in periods with low returns, servicing the debt leads to larger losses of net worth, thus rendering entrepreneurs more constrained in next periods' investment decisions. As a result, entrepreneurs' incentive is to hedge more states with low returns, by borrowing less against those states.

The next proposition states the effect of uncertainty on investment decisions.

Proposition 9 (Uncertainty and Investment) ***(i)*** *Assume $\underline{w}_H$ the level of net worth such that $\mu_H(\underline{w}, \underline{s}') = 0$. If $w < \underline{w}$ then $k'_L < k'_H$.* ***(ii)*** *Assume $\bar{w}_H$ the level of net worth such that $\mu_H(\bar{w}_H, \bar{s}') = 0$. If $w \geq \bar{w}_H$ then $\bar{k}'_L = \bar{k}'_H$.* ***(iii)*** *There exists $\underline{w}_H < \hat{w} < \bar{w}_H$, such that if $w \leq \hat{w}$ then $k'_L \leq k'_H$. If $w \geq \hat{w}$ then $k'_L \geq k'_H$.*

To understand the above result, remember that the value function is concave in net worth. The concavity of the value function induces a precautionary motive for entrepreneurial savings. When all collateral constraints bind entrepreneurial savings can happen only through an increase in capital accumulation.

Moreover, the maximum level of investment does not depend on the level of uncertainty. This is intuitive, since investors would never invest such that the marginal return on capital would be lower then the cost of financing, R. Or put it differently, since entrepreneurs can save using state-contingent securities with return R as well, state-contingent securities represent an opportunity cost for entrepreneurs. Thus, they will never accumulate levels of capital at which the marginal return on capital is less than the opportunity cost, R. Alternatively, when net worth is high enough, entrepreneurs can perfectly insure against idiosyncratic shocks and thus the level of risk does not matter for their optimal decision.

The last part of the proposition states that there is a threshold level of net worth $\hat{w}$, below which entrepreneurs invest more when uncertainty is high, and above which, entrepreneurs decrease their investment with an increase in uncertainty. The intuition behind this result is the following. When net worth is low enough entrepreneurs will borrow to the maximum extent of their collateral. With an increase in uncertainty, as long as collateral constraints bind, entrepreneurs choose to invest more due to precautionary reasons. Thus there is a region of net worth where entrepreneurs' investment increases with uncertainty. However with an increase in uncertainty, firms start hedging at lower levels of net worth, invest less and thus lower their capital growth. But when uncertainty is high, entrepreneurs reach the maximum scale of their project at higher levels of net worth. However with lower growth, there must be a threshold level of net worth, $\hat{w}$, above which entrepreneurs will operate on a lower scale, as compared to when uncertainty was low. Thus, when the level of net worth is high enough, entrepreneurs' investment decreases with uncertainty.

It is instructive to relate these findings to the result on the effect of uncertainty on investment in the presence of convex adjustment costs, as in Abel (1983). In that model, firms' value function is concave because of the assumed constant returns to scale production functions and convex adjustment costs. Upon an increase in uncertainty, entrepreneurs' precautionary motive for savings increases, but the only vehicle through which entrepreneurs can save is capital, so they invest more. In this paper, when all collateral constraints bind, savings can only increase through more investment in physical capital. However, when some collateral constraints are slack, entrepreneurs can also save by borrowing less at the margin, which may reduce their investment. Indeed, with an increase in uncertainty, entrepreneurs with high enough net worth chose to save more by decreasing their demand of external financing. With lower funds entrepreneurs invest less and operate at a lower scale.

Let us now turn to the quantitative results.

4 Quantitative Results

In this section I show that quantitatively the effects presented in the previous section are significant. First, I look at comparative statics; that is, how levels of uncertainty and the collateral constraints affect the stationary distribution, especially the leverage ratio. Then, I compute the effects of uncertainty shocks in a calibrated economy.

The idiosyncratic shock process is modeled as a two-state Markov Chain process, with a symmetric transition matrix. Specifically, assume that the productivity level can be written as

$$A(s_t) = \begin{cases} A_L = \overline{A} - \sigma \\ A_H = \overline{A} + \sigma \end{cases} \tag{13}$$

where $\overline{A}$ is the unconditional value of the productivity process, $\overline{A} = (A_L + A_H)/2$

and the variance is such that $\sigma = (A_H - A_L)/2$.

The literature on estimating the properties of firm level total factor productivity processes does not provide uniformly accepted values for the autocorrelation and unconditional variance. In fact, estimates for both parameters differ widely across studies. For example Veracierto (2002) finds the unconditional volatility of the technology shock to be $\sigma = 0.056$, while Cooper and Haltiwanger (2006) finds that the unconditional volatility, $\sigma = 0.30$. Conditional volatility estimates are even harder to find in the literature. The exception is Bloom (2009), who derives the estimates from stock market data. In this paper I follow Bloom, Floetotto, and Jaimovich (2009), who calibrate the volatility process of idiosyncratic and aggregate productivity to match moments of the cross-sectional dispersion of the inter-quartile sales growth and moments based on a GARCH(1,1) estimated conditional heteroscedasticity of GDP growth.[10] In this paper I consider only idiosyncratic productivity, and I will follow the parametrization in Bloom, Floetotto, and Jaimovich (2009). For the specific values, I assume that when uncertainty is low, the standard deviation of productivity is $\sigma_L = 0.067$, while when uncertainty is high $\sigma_H = 0.13$, thus uncertainty increases twofold.

As with the volatility of the idiosyncratic productivity process, there is no consensus on the estimate for the autocorrelation parameter either. For example Veracierto (2002) estimates the autocorrelation of the idiosyncratic shock to be 0.83, while Cooper and Haltiwanger (2006) estimate a higher autocorrelation parameter of 0.885. Gomes (2001) and Khan and Thomas (2010) calibrate the autocorrelation parameter to be 0.65, to match the persistence of the investment process. Here too I follow Bloom, Floetotto, and Jaimovich (2009), and assume the autocorrelation to be 0.86, resulting in the following transition matrix for the stochastic process:

$$\pi(s, s') = \begin{bmatrix} 0.93 & 0.07 \\ 0.07 & 0.93 \end{bmatrix}$$

I take relatively standard values for the remaining parameters. The values of the

[10] These moments in both cases are: mean, standard deviation, skewness, and serial correlation of the annual IQR sales growth rates and GDP growth.

parameters are summarized in Table 1. Specifically, following Bernanke, Gertler, and Gilchrist (1999), I assume a yearly discount factor for lenders to be equal to $\tilde{\beta} = 0.95$, which implies a yearly gross interest rate of $R = 1/0.95 = 1.0524$. Entrepreneurs are assumed to have a CRRA utility function; that is $u(d) = d^{(1-\gamma)}/(1-\gamma)$ with the coefficient of risk aversion, $\gamma = 1$. Turning to the production function, I assume that the capital share in the production function is $\alpha = 0.33$, while the yearly depreciation is 10%.

The two relatively unconventional values are the magnitude of the relative discount factor and the collateral constraint parameter θ. For the relative impatience parameter I assume $\beta = 0.93$, which implies a yearly premium on internal funds of 2.2%. In comparison, Iacoviello (2005) assume the quarterly premium on internal funds to be 1.1%, which gives a yearly premium of 4.4%. Finally, I assume that the share of physical capital that can be pledged as collateral is 70%; that is $\theta = 0.7$. Depending on the level of uncertainty, this results in a book leverage ratio between 0.53 and 0.59 in line with the book leverage of 0.587, as found in Covas and Den Haan (2010) and Covas and Den Haan (2011) using Compustat data. Using Flow os Funds data Jermann and Quadrini (2010) report a somewhat lower book leverage ratio; they find that the ratio of debt to capital over the period of 1984-2009:1 for the Nonfinancial Business Sector is 0.46.

Define $Z = W \times S$ and $\phi(Z)$ as the cross sectional distribution of firms over net worth and idiosyncratic shocks. Now define the leverage ratio as total liabilities over net worth:

$$L = \int_Z \frac{\sum_{s' \in S} \pi(s, s') b(s')}{w} \phi(z) \tag{14}$$

Table 1 summarizes the parameter values.

Table 1: Parameter Calibration

β	γ	R	α	δ	θ	$\bar{A}$	ρ	σ_L	σ_H
0.93	1	1/0.95	0.33	0.1	0.7	1	0.86	0.067	0.13

The next section presents the comparative statics.

4.1 Comparative Statics

In this section I present the descriptive statistics of the stationary distributions for two levels of uncertainty. Table 2 contains the results.

Table 2: Stationary Distribution

Variables	$\sigma = 0.067$	$\sigma = 0.13$
Output, y	1.42	1.42
Investment, k'	2.90	2.90
Dividend, d	1.04	1.05
External Financing, $\sum_{s' \in S} \pi(s, s')b(s')$	1.70	1.56
Net Worth, w	2.23	2.38
Book Leverage	0.58	0.53
Leverage Ratio, L	0.76	0.67

The primary difference across levels of uncertainty is the difference between the equilibrium levels of net worth, debt, and leverage ratio. Notice that the real variables remain equal across different levels of uncertainty, however the capital structure changes dramatically. In the stationary distribution when uncertainty is high, firms choose to increase their internal funds by more then 7.1% just to be able to afford the same level of investment and reach the same output. The consequence of this change in the capital structure is that the leverage ratio in the economy decreases by 12%.

4.2 Transitional Dynamics

This section contains the results of how firms respond to unanticipated changes in uncertainty. Studying the effects of unexpected shocks is important in light of the recent financial crisis of 2007-2009 that was widely thought of as not having been anticipated by most market participants or economists. Indeed, there was a widespread belief that house prices could not fall and developments in the financial sector allowed risks to be spread widely. Even the current Chairman of the Federal Reserve Bank, Ben Bernanke, declared on March 28, 2007: “At this juncture, however, the impact on the broader economy and financial markets of the problems in the subprime market seems likely to be contained."[11] Thus, it seems warranted as a baseline scenario to investigate the effects of unanticipated changes in uncertainty. However, in the next section I also present results for the stochastic volatility case.

Basically, by allowing for an unanticipated, permanent uncertainty shock, I trace the transitional dynamics from a low to a high volatility environment. In order to compute the transitional dynamics, I first solve the entrepreneurs' problem (5) - (8) for the optimal decisions. To compute the stationary distribution, I simulate a large number of firms and trace them over time, until based on some metric, the stationary distribution converges. I choose 100000 firms and track them over time until the mean of net worth converges.

Having computed the stationary distribution, I increase the volatility of shocks. Then I trace the evolution of firms for 20 periods. I repeat this last step 500 times and chose the mean value of the variables. This last step is required to rule out the dependence of transitional dynamics on a particular draw of idiosyncratic shocks.

In order to understand the effects of uncertainty shocks, first, I describe the optimal policies in a constant investment opportunity environment. Figures 2 and 3 show the optimal policies for investment (k'), Lagrange multipliers on the collateral constraints ($\mu(s')$), optimal dividend policy, and optimal net worth in next period ($w(s')$), in both cases when uncertainty is low and high.

Notice that when uncertainty is high, entrepreneurs start to hedge at lower levels of net worth as compared to when uncertainty is lower. This is so because as the ex-post risk of losing net worth in the economy increases, entrepreneurs issue fewer claims against the state with lower returns. Moreover, entrepreneurs reach the unconstrained scale of their projects at higher levels of net worth. However, costly hedging crowds out investment, and as such entrepreneurs increase the size of their project at a lower rate. This can be seen by comparing the optimal investment policies (k') of firms, and noticing that the slope is lower in the case of high uncertainty, under the region where entrepreneurs hedge. Finally, notice that the stationary dis-

[11] See "The Economic Outlook", March 2007, Report before the joint economic committee, U.S. Congress.

tribution widens with an increase in uncertainty, as can be seen from the bottom right panel in the two figure. The bounds of the stationary distribution increase, as the bounds are at levels of net worth at which the optimal policy crosses the 45 degree line.

Figure 4 presents the transitional dynamics for investment, debt, net worth, and output for the case of autocorrelated productivity shocks; that is, when investment opportunities are stochastic. The parameters are listed in Table 2. Notice that the effect of an unexpected, permanent increase in uncertainty is large; net investment drops by more than 40%. Furthermore, upon an increase in uncertainty dividend payout is reduced by more than 2% as entrepreneurs use their net worth for investment rather than pay it out as dividends. In order to hedge the increased risk, entrepreneurs reduce their demand for external financing, which decreases the leverage ratio.

The main difference, however, resides in the different behavior of real and financial variables. Notice that after 5 periods (years), entrepreneurs manage to build up enough net worth to essentially operate at the same scale at which they operated when uncertainty was low. However, the balance sheet undergoes significant change for 15 more periods, as entrepreneurs conserve enough net worth to hedge the higher risk in the economy. With constant investment opportunities, entrepreneurs will never hedge the highest state, as they are able to insure against risks just by hedging states with low realizations of returns. This is not the case anymore with stochastic investment opportunities. Now, with an increase in uncertainty, entrepreneurs will hedge all states; that is, they will conserve net worth against all states in order to be able to take advantage of future investment opportunities. Thus with autocorrelated shocks, an increase in uncertainty will increase entrepreneurs' holdings of cash.

4.3 Stochastic Volatility

In this section I present the results for the case of stochastic volatility. With stochastic volatility, entrepreneurs are aware that uncertainty in the economy can change and will hedge accordingly. To model stochastic volatility, I assume, just as before, that productivity shocks follow a Markov process. However, now the magnitude of the variance changes as well. Thus, I assume that

$$A(s_t) = \begin{cases} A_L = \overline{A} - \sigma_{t-1} \\ A_H = \overline{A} + \sigma_{t-1} \end{cases} \tag{15}$$

where $\sigma_t \in \{\sigma_L, \sigma_H\}$. I assume that σ_t follows a Markov process with the transition probabilities

$$\pi_\sigma(s, s') = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$$

This implies that the yearly autocorrelation in the stochastic volatility process is 0.4.

To study the effects of an uncertainty shock in this environment, I follow Arellano, Bai, and Kehoe (2010). First, I compute the stationary distribution in this economy when uncertainty changes as well. Next, I force uncertainty to be low for 10 periods, and then I increase the level of uncertainty from then on for 25 periods. Figure 5 presents the results from this experiment.

As a result of the low uncertainty environment, leverage in the economy is increased by more than 4%. This implies that relative to steady state, entrepreneurs decrease their net worth by 1.5%. Upon an increase in uncertainty entrepreneurs delever in order to hedge the larger shocks in the economy. Hedging, however, comes at the expense of financing needs and thus results in an initial reduction in investments. As entrepreneurs build up net worth, investment recovers and dividend payout increases. Furthermore, notice that now the leverage ratio falls below the steady state level; thus, as before, changes in uncertainty in the long run will be reflected in entrepreneurs' capital structure.

The main message of this section however is that even in the presence of stochastic volatility, changes in uncertainty have large effects on optimal financing and investment dynamics. Let me now turn to some related implications of this model and a discussion of the importance of complete markets for these results.

4.4 Why Do Firms Save So Much?

As Figure 1 documents, by the third quarter of 2010, US nonfinancial corporations amassed approximately \$1.95 trillion in liquid assets.[12] Furthermore Bates, Kahle, and Stulz (2009) provide evidence that firms' cash to assets ratio more than doubled between 1980 and 2006, while recent reports[13] suggest that firm level cash holdings have increased significantly after the recent recession. In this section I show that an increase in uncertainty induces firms to conserve more net worth against all states. Conserving net worth against all states can be interpreted as cash, and thus this paper potentially can explain why, in the aftermath of the financial crisis, US nonfinancial firms appear to have increased their holdings of cash and liquid assets on their balance sheets.

With higher uncertainty, firms start hedging states with low returns at lower levels of net worth. Moreover, firms can fully hedge fluctuations in productivity at larger levels of net worth, as compared to an environment with low uncertainty. Thus, with increases in uncertainty, firms engage in more risk management. Furthermore, if one interprets as cash the net worth that entrepreneurs conserve against all future states, then the results show that entrepreneurs save more cash when uncertainty in the economy is high.

Table 3 presents the ratio of cash and net worth under the two uncertainty regimes.

Table 3: Holdings of Liquid Assets

	$\sigma = 0.067$	$\sigma = 0.13$
Liquid assets, $\int_Z (w - \bar{w})/w)\phi(z)$	0%	0.93%

Notice that when uncertainty is low firms do not hold cash. However when uncertainty in the economy increases, entrepreneurs choose to hold cash worth almost

[12] See the Flow of Funds data from the Federal Reserve Board, Table L102.

[13] On October 26 Moody's Investor Service estimated that nonfinancial U.S companies are hoarding \$943 billion of cash, an increase from \$775 billion since the end of 2008. See http://www.reuters.com/article/idUSN2614487020101027.

1% of their net worth. As such, these results are qualitatively in line with the documented increase in liquid assets after the financial crisis of 2007-2009. As documented in Bloom, Floetotto, and Jaimovich (2009), measured uncertainty increased during this period, and thus increasing holdings of cash for risk management purposes might be a rational response from firms.

The predictions of this model are also in line with empirical results documenting the gradual increase in cash holding by U.S firms.[14] Campbell et al. (2001) and Comin and Philippon (2005) document a secular increase in various measures of firm level volatility. According to the results in this paper, firms experiencing an increase in uncertainty hedge more, and thus will increase their holdings of liquid assets in order to take advantage of future investment opportunities.

4.5 Collateral Constraints and Asymmetric Responses

The presence of collateral constraints implies that shocks can have asymmetric effects, depending on whether collateral constraints bind.

To understand the embedded asymmetry, take for example the case of a permanent increase in uncertainty. With increased risk, entrepreneurs want to hedge more states with lower realizations of shocks. However, collateral constraints restrict the issuance of state-contingent securities against states with higher realizations of shocks, thus making it more difficult for entrepreneurs to hedge. Moreover, since entrepreneurs are risk-averse, upon an increase in uncertainty firms do not cut their dividend payout enough to meet the financing needs required to hedge the larger shocks. Instead entrepreneurs cut their investments and external financing. Thus, an increase in uncertainty will lead entrepreneurs to decrease the scale of their project, which leads to a recession.

The response of the economy to an unexpected decrease in uncertainty, however, is very different, as can be seen in Figure 6. Upon an increase in uncertainty, entrepreneurs will have too much net worth accumulated relative to the risks in the economy, and as such they have an incentive to pay out the extra net worth as divi-

[14]See, for example Bates, Kahle, and Stulz (2009).

dends. Risk aversion, however, implies that it is not optimal to pay out all the extra net worth in one payment, and as such they will gradually pay out the extra net worth until their balance sheet reflects the risks in the economy. Notice, however, that when entrepreneurs need to pay out their net worth, collateral constraints will be less binding, as entrepreneurs' hedging and financing needs will be reduced. As a result investment will react less to a decrease in uncertainty.

4.6 Uncertainty, Incomplete Markets and Collateral Constraints

To better understand the importance of complete markets, subject to collateral constraints, in this section I compute a model with exogenously incomplete markets where financing is subject to collateral constraints. The key question is how do results change if one assumes instead of complete markets, markets that are exogenously incomplete?

The entrepreneurs' problem now becomes:

$$V(w,s) = \max_{\{d,k',b',w(s')\}} \left\{ u(d) + \beta \sum_{s' \in S} \pi(s,s') V(w(s'),s') \right\}$$

subject to

$$w + b' \geq d + k'$$

$$A(s')f(k') + k'(1-\delta) \geq w(s') + Rb', \qquad \forall s' \in S$$

$$\theta k'(1-\delta) \geq Rb'.$$

When markets are exogenously incomplete, entrepreneurs can insure against fluctuations in productivity only by conserving net worth. This is the only way for entrepreneurs to be able to take advantage of future investment opportunities. This happens despite the fact that entrepreneurs are relatively impatient; thus, it is costly for them to save net worth. Relative impatience, however, also means that entrepreneurs will not save enough to become forever unconstrained. Upon a series of low realizations of shocks, entrepreneurs' net worth may be reduced so much that their investment choices become constrained and the collateral constraint will bind again.

The precautionary savings motive implies that the collateral constraint will bind as long as entrepreneurs have net worth below a threshold level. Above this threshold level, entrepreneurs can choose the unconstrained level of investment. Importantly, however, the unconstrained level of investment will be below the investment level achieved under complete markets. The reason is that, as shown in Angeletos and Calvet (2006), when markets are incomplete, entrepreneurs cannot insure against the idiosyncratic production risk and thus will charge a risk premium on net worth.

Upon an increase in uncertainty, in the region of net worth where the collateral constraint binds in both regimes of uncertainty, entrepreneurs will choose to accumulate capital at a faster rate. This is due to the precautionary savings motive, just as in Abel (1983). The maximum level of investment, however, now will be lower, since an increase in uncertainty leads to an increase in the risk premium on net worth. Figure 7 plots the optimal investment policy in both cases of constant and stochastic investment opportunities.

What do these results imply for the overall effect of a permanent increase in uncertainty? For the numerical results, I use the same parameter value as in Table 1. The next table presents descriptive statistics of the stationary distributions under different levels of uncertainty.

Table 4: Stationary Distribution under Incomplete Markets

Variables	$\sigma = 0.067$	$\sigma = 0.13$
Output, y	1.42	1.42
Investment, k'	2.91	2.93
Dividend, d	1.04	1.04
External Financing, $\sum_{s' \in S} \pi(s, s') b(s')$	1.74	1.74
Net Worth, w	2.21	2.23
Book Leverage	0.59	0.59
Leverage Ratio, L	0.78	0.77

From the table above, one can see that in economies with incomplete markets the long run effect of an increase in uncertainty is to increase the capital stock. The intuition behind this result can be found in the shape of the stationary distribution. In the case of stochastic investment opportunity the precautionary savings effect overcomes the risk premium effect and a larger mass of entrepreneurs will have higher investments. This is in stark contrast to the comparative statics results under complete markets.

Additionally, results from this section also imply that incomplete markets tend to dampen the effects of uncertainty shocks. Intuitively, under incomplete markets conserving net worth is the only way to insure against risks in the economy. But this implies that entrepreneurs become overly capitalized. And since changes in uncertainty effect capital structure, uncertainty shocks have a small impact on investment decisions.

Under complete markets subject to collateral constraints, results are both qualitatively and quantitatively different. With complete markets, entrepreneurs can hedge risks by issuing state-contingent securities subject to collateral constraints. Thus entrepreneurs are now better able to hedge risk and are not required to conserve as much net worth. But this implies that in the face of an increase in uncertainty firms find themselves undercapitalized and thus their reaction to the shock is larger. As a result, complete markets subject to collateral constraints tend to amplify shocks in the economy.

These results highlight the importance of how different choices of modeling of financial frictions can have profoundly different implications.

5 Conclusion

In this paper I studied the effect of uncertainty on financing demand and investment in the context of a model where firms face collateral constrains. The main innovation of this paper is to study the effect of uncertainty in a dynamic model of firm financing, where financing is advanced based on an optimal long-term contract.

Collateral constraints limit the amount that firms can borrow. Thus, investment needs to be financed both with internal funds and external funds. In this setting, uncertainty affects the capital structure of firms, specifically the optimal mix of internal and external financing that firms use to fund their investment projects. Uncertainty implies that external financing is risky, in the sense that if the return on investment is low, servicing the debt leads to a loss of internal funds, which constrains future investment choices.

Upon an increase in uncertainty, entrepreneurs reduce their demand for external financing and consequently the scale of their production. Investment and output rebound as entrepreneurs build up internal funds and transition to an equilibrium with high uncertainty. Quantitatively, an unexpected increase in uncertainty has large effects on optimal leverage and investment dynamics.

In this paper I assume that the price of capital does not change. An important extensions would be to endogenize the price of capital, since then collateral constraints then would depend on the resale value of capital. Fluctuation in the price of capital could potentially amplify the mechanism presented in this paper. Furthermore, it would be important to consider the effect of uncertainty on credit spreads, as fluctuations in the cost of financing will affect the maximum scale of investment and thus could further amplify the mechanism presented above. These lines of research are left for future work.

Appendix: Proofs

To prove Propositions (1)-(6), I adapt the proofs from Rampini and Viswanathan (2010a), to the case of risk averse entrepreneurs. Define x as the set of choice variables $x \equiv \{d, k', b(s'), w(s')\}$, where $x \in \mathbb{R}_+^{2+S} \times R^S$, and the constraint set $\Gamma(w,s)$ is given by the state variables such that (6) - (8) is satisfied. Lemma 1 shows that the choice set is convex

Lemma 1 ***(i)*** $\Gamma(w,s)$ *is convex, given* (w,s)*, and convex in* w *and monotone in the sense that* $w \leq \hat{w}$ *implies* $\Gamma(w,s) \subseteq \Gamma(\hat{w},s)$.

Proof of Lemma 1. Suppose $x, \hat{x} \in \Gamma(w,s)$. For $\phi \in (0,1)$, let $x_\phi \equiv \phi x + (1-\phi)\hat{x}$. Then $x_\phi \in \Gamma(w,s)$ since equations (6), (8) are linear, and from (7) as f is concave, we have $f(k'_\phi) \geq \phi f(k') + (1-\phi) f(\hat{k}')$.

Let $x \in \Gamma(w,s)$ and $\hat{x} \in \Gamma(\hat{w},s)$. For $\phi \in (0,1)$, let $x_\phi \equiv \phi x + (1-\phi)\hat{x}$. Since (7), (8) does not include w, and $\hat{w}$, and $\Gamma(w,s)$ is convex given w, x_ϕ satisfies these equations. Moreover, since x and $\hat{x}$ satisfy equation (6) at w, and $\hat{w}$, respectively, and equation (6) is linear in x and w, x_ϕ satisfies the equation at w_ϕ. Thus, $x_\phi \in \Gamma(\phi w + (1-\phi)\hat{w}, s)$. In this sense, $\Gamma(w,s)$ is convex in w.

If $w < \hat{w}$, then $\Gamma(w,s) \subseteq \Gamma(\hat{w},s)$ ■

Proof of Proposition 1. Part **(i)**. Define the operator T as follows:

$$(Tg)(w,s) = \max_{x \in \Gamma(w,s)} u(d) + \beta \sum_{s' \in S} \pi(s,s') g(w(s'),s')$$

where x and $\Gamma(w,s)$ is defined above. To show that the problem (5) has a unique solution V, it is enough to show that the operator T satisfies Blackwell's sufficient conditions for a contraction.

Suppose $g(w,s) \geq f(w,s)$, for all $(w,s) \in \mathbf{R}_+ \times S$. Then, for any $x \in \Gamma(w,s)$

$$(Tg)(w,s) \geq u(d) + \beta \sum_{s' \in S} \pi(s,s') g(w(s'),s') \geq u(d) + \beta \sum_{s' \in S} \pi(s,s') f(w(s'),s').$$

Thus,

$$(Tg)(w,s) \geq \max_{x \in \Gamma(w,s)} u(d) + \beta \sum_{s' \in S} \pi(s,s') f(w(s'),s') = (Tf)(w,s)$$

for all $(w,s) \in \mathbf{R}_+ \times S$. Thus, T satisfies monotonicity.

Next, we show that T satisfied discounting

$$T(f+a)(w,s) \leq \max_{x\in\Gamma(w,s)} u(d) + \beta \sum_{s'\in S} \pi(s,s')(f+a)(w(s'),s') = (Tf)(w,s) + \beta a$$

Thus T is a contraction and by the contraction mapping theorem, has a unique fixed point V.

Part **(ii)**. Let $x_0 \in \Gamma(w,s)$ and $\hat{x}_0 \in \Gamma(\hat{w},s)$ attain $(Tf)(w,s)$ and $(Tf)(\hat{w},s)$, respectively. Suppose f is increasing in w and suppose $w \leq \hat{w}$. Then,

$$\begin{aligned}(Tf)(\hat{w},s) &= u(\hat{d}_0) + \beta \sum_{s'\in S} \pi(s,s') f(\hat{w}_0(s'),s') \geq u(d) + \beta \sum_{s'\in S} \pi(s,s') f(w(s'),s') \\ &\geq \max_{x\in\Gamma(w,s)} u(d) + \beta \sum_{s'\in S} \pi(s,s') f(w(s'),s') = (Tf)(w,s)\end{aligned}$$

Hence, Tf is increasing in w. Moreover, suppose $w < \hat{w}$, then

$$\begin{aligned}(Tf)(\hat{w},s) &\geq u((\hat{w}-w)+d_0) + \beta \sum_{s'\in S} \pi(s,s') f(w_0(s'),s') \\ &\geq u(d_0) + \beta \sum_{s'\in S} \pi(s,s') f(w_0(s'),s') > (Tf)(w,s)\end{aligned}$$

implying that Tf is strictly increasing. Hence, T maps increasing functions into strictly increasing functions, which implies that V is strictly increasing.

Suppose f is concave. Then, for $\phi \in (0,1)$, let $x_{0,\phi} \equiv \phi x_0 + (1-\phi)\hat{x}_0$ and $w_\phi \equiv \phi w + (1-\phi)\hat{w}$, we have

$$\begin{aligned}(Tf)(w,s) &\geq u(d_{0,\phi}) + \beta \sum_{s'\in S} \pi(s,s') f(w_{0,\phi}(s'),s') \\ &> \phi u(d_0) + (1-\phi)u(\hat{d}_0) + \beta \sum_{s'\in S} \pi(s,s') f(w_{0,\phi}(s'),s') \\ &= \phi(Tf)(w,s) + (1-\phi)(Tf)(\hat{w},s)\end{aligned}$$

Thus, the concavity of u ensures that Tf is strictly concave, and T maps concave functions into strictly concave functions, which implies that V is strictly concave. Since V is increasing and strictly concave in w, it must be continuous in w.

Part **(iii)**. Let $S = s_1, \ldots, s_n$, with $s_{i-1} < s_i$, for all $i = 2, \ldots, n$ and $N = 1, \ldots, n$. Define the step function on the unit interval $b : [0,1] \to \mathbf{R}$ as $b(\nu) = \sum_{i=1}^n b(s_i')\mathbf{1}_{B_i}(\nu)$, $\forall \nu \in [0,1]$, where $\mathbf{1}$ is the indicator function, $B_1 = [0, \pi(s,s_1')]$, and

$$B_i = \left[\sum_{j=1}^{i-1} \pi(s,s_j'), \sum_{j=1}^{i} \pi(s,s_j')\right], \quad i = 2, \ldots, n.$$

For $\hat{s}$, define $\hat{B}_i$, $\forall i \in N$, analogously. Let $B_{ij} = B_i \cap \hat{B}_j$, $\forall i, j \in N$, of which at most $2n-1$ are non-empty. Then, we can define the step function $\hat{b} : [0,1] \to \mathbf{R}$ as

$$\hat{b}(\nu) = \sum_{j=1}^{n} \sum_{i=1}^{n} b(s_i') \mathbf{1}_{B_{ij}}(\nu), \qquad \nu \in [0,1]$$

We can then define the stochastic debt policy for $\hat{B}_j$, $\forall j \in N$, with positive Lebesgue measure ($\lambda \hat{B}_j > 0$), as $\hat{b}(s_i'|s_j') = b(s_j')$ with conditional probability $\chi(s_i'|s_j') = \lambda(B_{ij})/\lambda(\hat{B}_j)$. Moreover, this implies a stochastic net worth

$$\begin{aligned} \hat{w}(s_i'|s_j') &= A(s_j')f(k') + k'(1-\delta) - R\hat{b}(s_i'|s_j') \\ &\geq A(s_i')f(k') + k'(1-\delta) - Rb(s_i') = w(s_i'), \quad a.e. \end{aligned}$$

with strict inequality when $i < j$, since $\lambda(B_{ij}) = 0$, whenever $i > j$. Moreover, $\hat{w}(s_i'|s_j') > w(s_i')$ with positive probability given our assumption stated in the Proposition.

Now suppose $\hat{s} > s$ and $f(w, \hat{s}) \geq f(w, s)$, $\forall w \in \mathbf{R}_+$. Let x_0 attains $(Tf)(w, s)$. Then

$$\begin{aligned} (Tf)(w, \hat{s}) &\geq u(d_0) + \beta \sum_{\hat{s}' \in S} \pi(\hat{s}, \hat{s}') \sum_{s' \in S} \chi(s'|\hat{s}'), \hat{s}') \\ &> u(d_0) + \beta \sum_{s' \in S} \pi(s, s') f(w_0(s'), s') = (Tf)(w, s) \end{aligned}$$

Thus, T maps increasing functions into strictly increasing functions, implying that V is strictly increasing in s. ■

Proof of Proposition 2. We now show that x_0 that attains $V(w, s)$ is unique. To see this, we first show that $w_0(s')$ is unique $\forall s' \in S$. Suppose that there exist $\tilde{x}_0$ with $\tilde{w}_0(s') \neq w_0(s')$ for some $s' \in S$ that also attains $V(w, s)$. Then a convex combination $x_{0,\phi}$ is feasible and attains a strictly higher value due to strict concavity of $V(w, s)$, a contradiction. Thus $x_{0,\phi}$ is unique in terms of $w_0(s')$, $\forall$ w and s.

To see that the choice of the optimal capital stock k_0' is unique, suppose that x_0 and $\tilde{x}_0$ both attain $V(w, s)$, but $k_0' \neq \tilde{k}_0'$. Since $\Gamma(w, s)$ is convex, by taking the convex combination of x_0 and $\tilde{x}_0$, note that

$$\begin{aligned} A(s')f(k_{0,\phi}') + k_{0,\phi}(1-\delta) >& \phi[A(s')f(k_{0,\phi}') + k_{0,\phi}(1-\delta)] \\ &+ (1-\phi)[A(s')f(k_{0,\phi}') + k_{0,\phi}(1-\delta)] \end{aligned}$$

However this implies that at $x_{0,\phi}$, (7) is slack, and hence there exists a feasible choice that attains a strictly higher value, a contradiction. Thus $x_0(w,s)$ is unique in terms of k'_0, for all w and s.

Given k'_0 and $w'_0(s')$, for all $s' \in S$, $b_0(s')$ is uniquely determined by (7), and the payout policy, d_0, by (6). ■

Proof of Proposition 3. Part **(i)**: Using (6)

$$
\begin{aligned}
w &= d + k' - \sum_{s' \in S} \pi(s,s') b(s,s') > k' - \sum_{s' \in S} \pi(s,s') b(s,s') \\
&> k' - \theta k'(1-\delta) = (1-\theta)(1-\delta)k'
\end{aligned}
$$

where, for the first inequality we use that $\lim_{d \to 0} u(d) = \infty$, which implies that $d > 0$, and the second inequality uses the collateral constraint (8). Notice however, that the above results implies that as $w \to 0$, investment $k' \to 0$. Now using (10) and substituting out $\mu(s')$ from (11)

$$
1 \geq \beta \sum_{s' \in S} \pi(s,s') \frac{\lambda(s')}{\lambda} \left(\frac{A(s')f'(k') + (1-\delta)(1-\theta)}{1 - \theta(1-\delta)/R} \right)
$$

As $k \to 0$ implies that $f'(k') \to \infty$, but from the above equation then it must be that $\lambda(s')/\lambda \to 0$. But then using the collateral constraint (8): $\mu(s')\lambda(s')/\lambda \to (\beta R)^{-1}$, resulting that $\mu(s') > 0 \ \ \forall s' \in S$. Thus, by continuity, $\exists \underline{w} > 0$, such that $\forall w < \underline{w}$: $\mu(s') > 0 \ \ \forall s' \in S$.

Part **(ii)**: If $w(s') \leq w(s'_+)$, then $\lambda(s') \geq \lambda(s'_+)$ by concavity. Moreover, from (11), $\lambda(s')(1+\mu(s')) = \lambda(s'_+)(1+\mu(s'_+))$, thus $\mu(s') \leq \mu(s'_+)$. Suppose instead that $w(s') > w(s'_+)$. Then, from (7), $\mu(s') = 0$, but then $\lambda(s') = \lambda(s'_+)(1+\mu(s'_+))$, resulting that $\lambda(s') \geq \lambda(s'_+)$. But this implies that $w(s') \leq w('s_+)$, which is a contradiction.

Part **(iii)**: From part **(ii)**, it is enough to show that $\mu(\bar{s}') = 0$ if $w > \bar{w} > 0$, where $\bar{s}' \geq s' \ \forall s' \in S$. Since the marginal product of capital in states when the collateral constraint binds is $A(s')f'(k') + (1-\delta)(1-\theta)$, while the marginal product of capital in states when the collateral constraint doesn't bind is $A(s')f'(k') + (1-\delta)$, and noting that the opportunity cost of investment is R it has to be that $\exists \ \bar{k}$ such that $k' < \bar{k}' \forall \ \ w > 0$.

Assume that w is high enough, in the sense that $\bar{k}'$ is the optimally chosen level of investment. I prove the statement by contradiction. First, assume that $w(\bar{s}')$ is chosen optimally such that $\mu(\bar{s}') = 0$. But this implies that $w(\bar{s}') = A(\bar{s}')f(\bar{k}') + \bar{k}'(1-\delta) - Rb(\bar{s}')$. Then (11) implies that $\lambda = R\beta\lambda(\bar{s}')$.

Now take $w_1(\bar{s}')$ such that $\mu_1(\bar{s}') > 0$. Then

$$\begin{aligned} w_1(\bar{s}') &= A(\bar{s}')f(\bar{k}') + \bar{k}'(1-\delta)(1-\theta) \\ &< A(\bar{s}')f(\bar{k}') + \bar{k}'(1-\delta) - Rb(\bar{s}') = w(\bar{s}') \end{aligned}$$

Thus $w(\bar{s}') > w_1(\bar{s}')$. However, from (11), $\lambda = R\beta\lambda_1(\bar{s}')(1+\mu_1(\bar{s}'))$ and $\lambda = R\beta\lambda(\bar{s}')$ implying that $\lambda_1(\bar{s}') < \lambda(\bar{s}')$, resulting that $w_1(\bar{s}') > w(s')$. But this is a contradiction, and thus it cannot be that at the optimum $\mu(\bar{s}') > 0$. Then $\exists\ \bar{w} > 0$ such that $\mu(s') > 0\ \forall s' \in S$. ■

Proof of Proposition 4. Part **(i)**. Suppose that $w < \bar{w}$. If $\mu(s') > 0\ \forall s' \in S$, then $k' = (w-d)/(1-\theta(1-\delta)/R)$. Take any w^+, w such that $\mu(s') > 0\ \forall s' \in S$. Assuming that $w^+ > w$, implies that $\lambda^+ < \lambda$. Now proceed by assuming that $k'^+ < k'$. From (7) $w^+(s') < w(s')\ \forall s' \in S$. Then from (10)

$$\begin{aligned} \lambda\left(1 - \frac{\theta(1-\delta)}{R}\right) &= \beta \sum_{s' \in S} \pi(s,s')\lambda(s')(A(s')f'(k') + (1-\delta)(1-\theta)) \\ &< \beta \sum_{s' \in S} \pi(s,s')\lambda^+(s')(A(s')f'(k'^+) + (1-\delta)(1-\theta)) \\ &< \lambda^+\left(1 - \frac{\theta(1-\delta)}{R}\right) \end{aligned}$$

However this implies that $\lambda < \lambda^+$, which is a contradiction. As a result $k'^+ > k'$ whenever $\mu(s') > 0\ \forall s' \in S$.

Suppose now that $\mu(s') = 0$, for some $s' \in S$. Then from (10) and (11) we get

$$\begin{aligned} 1 &= \beta \sum_{s' \in S} \pi(s')\frac{\lambda(s')}{\lambda}(A(s')f'(k') + (1-\delta) + \theta\mu(s')(1-\delta)) \\ &= \beta \sum_{s'|\mu(s')>0} \pi(s')\frac{\lambda(s')}{\lambda}(A(s')f'(k') + (1-\delta)(1-\theta)) + \sum_{s'|\mu(s')>0} \pi(s')\theta(1-\delta)/R \\ &\quad + \beta \sum_{s'|\mu(s')=0} \pi(s')\frac{\lambda(s')}{\lambda}(A(s')f'(k') + (1-\delta)) \end{aligned}$$

Take as before $w^+ > w$ and $k'^+ < k'$. Then $f'(k'^+) \geq f'(k')$. Moreover, for $\{s'|\mu(s') = 0\}, \lambda(s')/\lambda = (\beta R)^{-1}$. Since $\lambda^+ < \lambda$, the above equation implies that $\exists s'$ such that $\lambda^+(s') < \lambda(s')$. But $k'^+ < k'$ implies that for $\{s'|\mu(s') > 0\}$, $w^+(s') < w(s')$ and hence $\lambda^+(s') > \lambda(s')$, a contradiction. Hence, k' and $w(s')$ are strictly increasing in w for $w \leq \bar{w}$.

Part **(ii)**. If $w > \bar{w}$ then $\mu(s') = 0\ \forall s' \in S$. Combining (10) and (11) we get

$$R = \sum_{s' \in S} \pi(s')(A(s')f'(\bar{k}) + 1 - \delta)$$

Thus the maximum level of capital, $\bar{k} = f'^{-1}((R - 1 + \delta)/\sum_{s' \in S} \pi(s')A(s'))$. ■

Proof of Proposition 5. Part **(i)** follows directly from Proposition 2 **(ii)**.
Part **(ii)**. To show that $\exists w$ such that $w(s') > w$, $\forall s' \in S$ note that Proposition 2 implies that for w sufficiently small $\mu(s') > 0$, $\forall s' \in S$. Since $w(s') = A(s')f(k') + k'(1 - \delta)(1 - \theta)$, we have that:

$$\frac{\partial w(s')}{\partial w} = A(s')f'(k')\frac{\partial k'}{\partial w} + (1 - \delta)(1 - \theta)\frac{\partial k'}{\partial w} > 0$$

where we use that the production function is increasing in its argument, $f' > 0$, and Proposition 3 **(i)**. To show that $\exists w$ such that $w(s') < w$, $\forall s' \in S$ note that Propositions 2 also implies that for w sufficiently large $\mu(s') = 0$, $\forall s' \in S$. From (11) then $\lambda = \beta R \lambda(s') > \lambda(s')$, $\forall s' \in S$, which implies that if $w > \bar{w}$ then $w(s') < w$, $\forall s' \in S$.
Part **(iii)**: By the theorem of maximum $w(s')$ is continuous in w, and the intermediate value theorem and part **(ii)** hence imply the result. ■

Proof of Proposition 6. Define the induced state space $W = [w_l, w_u] \in \mathbf{R}$ with its Borel subsets $\mathcal{W}$. Take P to be the induced transition function on $(W, \mathcal{W})$, with the associated operator on bounded continuous functions $T : B(W, \mathcal{W}) \to B(W, \mathcal{W})$ and the associated operator on probability measures $T^* : P(W, \mathcal{W}) \to P(W, \mathcal{W})$.

First, I show that P is monotone (that is, for any bounded, increasing function f, the function Tf defined by $(Tf)(w) = \int f(w')P(w, dw'), \forall w$, is also increasing) and has the Feller property (that is, for any bounded, continuous function f, the function Tf is also continuous). Take any bounded, increasing function f. Then $(Tf)(w) = \sum_{s' \in S} \pi(s')f(w(s')(w))$ is increasing since $w(s')(w)$ is increasing by part (ii) of Lemma 2. For any bounded, continuous function f, $(Tf)(w)$ is moreover continuous as $w(s')(w)$ is continuous by the theorem of maximum.

Next, I show that $\exists w^0 \in \mathcal{W}, \varepsilon > 0$, and $N \geq 1$, such that $P^N(w_l, [w^0, w_h]) \geq \varepsilon$ and $P^N(w_h, [w_l, w^0]) \geq \varepsilon$.

First, notice that all levels of net worth outside of $[w_l, w_h]$ are transient. To see this, suppose that $w < w_l$. Then $\mu(w, s') > \mu(w_l, s') = 1/(\beta R) - 1$, $\forall s' \in S$. But then from (11) if $\mu(w, s') > 1/(\beta R) - 1$, $\forall s' \in S$, then $w < w(s')$, $\forall s' \in S$. Similarly, assume that $w > w_h$. Then $\mu(w, s') < 1/(\beta R) - 1$, $\forall s' \in S$, and from (11) if $\mu(w, s') < 1/(\beta R) - 1$, $\forall s' \in S$, results that $w > w(s')$, $\forall s' \in S$.

Since $\mu(w_l, \bar{s}') > 1/(\beta R) - 1$, take w^0 such that $w_0 = w(\bar{s}')(w_l)$. Then $P(w_l, [w^0, w_h]) \geq \pi(\bar{s}')$ and $N_1 = 1$ and $\exists \varepsilon_1 > 0$ such that $\pi(\bar{s}') > \varepsilon_1 > 0$.

Now given $w^0 = w(\bar{s}')(w_l)$, I show that $\exists N_2 \geq 1$, such that $P^{N_2}(w_h, [w_l, w^0]) \geq \varepsilon_2$. The idea is that given a sufficiently long sequence of the lowest productivity realization results in a net worth lower than w^0. Notice that $\mu(w, \underline{s}') < 1/(\beta R) - 1, \forall w > w_l$. This implies that $w(\underline{s}')(w) < w, \forall w > w_l$. Thus $\exists N_2 < \infty$ and $\varepsilon_2 > 0$ such that $P^{N_2}(w_h, [w_l, w^0]) \geq \varepsilon_2$, where $\pi^{N_2}(\underline{s}') > \varepsilon_2$. ■

Proof of Proposition 7. First, we show that when in both regimes of uncertainty all collateral constraints bind, $d_L > d_H$. Assume that the the optimal choice variables are constant across regimes of uncertainty. That is, assume that w is low enough such that $\mu_i(s') > 0$, for all $i \in \{L, H\}$ and $s' \in S$. Then $d_L = d_H$, $k'_L = k'_H$, $\mu_L(s') = \mu_H(s')$, $b_L(s') = b_H(s')$, and $V_L = V_H$. But then from (7) and since (8) holds with equality: $w_i = A_i(s')f(k'_i) + k'_i(1-\delta)(1-\theta)$, $\forall i \in \{L, H\}$. But Assumption 4 implies that $Var(w_L(s')) < Var(w_H(s'))$. And since V is concave, $\sum_{s' \in S} \lambda_H(s') > \sum_{s' \in S} \lambda_L(s')$, which implies that (10) cannot hold for $A_H(s')$, which is a contradiction. Thus, then $d_L \neq d_H$, $k_L \neq k_H$, and $V_L \neq V_H$ when w is low enough such that $\mu_i(s') > 0$ for all $i \in \{L, H\}$, $s' \in S$.

Assume instead that $d_L < d_H$, and $\mu_i(s') > 0$ for all $i \in \{L, H\}$, $s' \in S$ still holds. Then $k'_L > k'_H$. But this implies that $\lambda_L > \lambda_H$. But $k'_H < k'_L$ implies that $A_H(s')f'(k'_H) + (1-\delta)(1-\theta)$ increases for all $s' \in S$ and such, $\lambda_H(s')$ must decrease for all $s' \in S$. To see that this cannot happen, take w such that $0 < \mu(w, \underline{s}') < 1/(\beta R) - 1$, for all $i \in \{L, H\}$. But from (11) results that $\lambda_H = \beta R \lambda_H(\underline{s}')(1 + \mu_H(\underline{s}'))$. Now using the assumption $\mu(w, \underline{s}') < 1/(\beta R) - 1$, results that $\lambda_H < \lambda_H(\underline{s}')$; that is $w > w_H(\underline{s}')$. But this is a contradiction, as from the concavity of the value function $w_H(\underline{s}') < w$ implies that $d_H(\underline{s}') < d_H$ and $\lambda_H(\underline{s}') > \lambda_H$. As a result it must be that $d_L > d_H$ and $k'_L < k'_H$ whenever $\mu_i(s') > 0$ for all $i \in \{L, H\}$, $s' \in S$.

Next, we show that when all collateral constraints are slack, $d_L < d_H$. Assume that w is high enough such that $\mu_i(s') = 0, \forall i \in \{L, H\}$, $s' \in S$. First, note that optimal investment is given by $R = \sum_{s' \in S} \pi(s')(A(s')f'(\bar{k}') + 1 - \delta)$, thus the optimal capital stock does not change with uncertainty, $\bar{k}'_L = \bar{k}'_H = \bar{k}'$. Now assume that $d_L = d_H$. Note, that when $\mu(s') = 0$, for all $s' \in S$, then $\lambda_i = \beta R \lambda_i(s')$ has to hold for all $i \in \{L, H\}$, and $s' \in S$. From (10) however since $Var(A_L(s')) < Var(A_H(s'))$ it must be that, states with more extreme realizations of the shocks have to be weighed less. For example it has to be that $\lambda_H(\underline{s}') < \lambda_L(\underline{s}')$. But $\lambda_L(s') = \lambda_H(s')$ for all $s' \in S$, thus a contradiction. As a result it cannot be that $d_L = d_H$ when w is high enough such that $\mu_i(s') = 0, \forall i \in \{L, H\}$.

Now take $d_L < d_H$. This implies that $\lambda_L > \lambda_H$. From (11) it has to be that $\lambda_H = \beta R \lambda_H(s')$ for all $s' \in S$. However this implies that $\lambda_H < \lambda_H(s')$, for all $s' \in S$. But then (10) cannot hold, contradiction. Thus is must be that $d_L > d_H$ when w is such that $\mu_i(s') = 0$ for all $i \in \{L, H\}$ and $s' \in S$.

Finally, since V is strictly concave function it then must be that $d_L > d_H$ for all w. ■

First, I show that the collateral constraint binds against the lowest state when the marginal return on capital in that state is greater than the cost of financing. Then, I prove Proposition 8.

Lemma 2 *$\mu(s') > 0$ if $A(s')f'(k') + (1 - \delta) > R$, for all $s' \in S$.*

Proof of Lemma 2. Take $\underline{s}' \in S$ such that $\mu(\underline{s}') > 0$. Now I show that $\mu(\underline{s}') > 0$ if $A(\underline{s}')f'(k') + (1 - \delta) > R$. Suppose otherwise; assume that $\mu(\underline{s}') = 0$ and $A(s')f'(k') + (1 - \delta) > R$. Denote the optimal debt against state $\underline{s}'$ as $b(\underline{s}')$. Then increase borrowing against state $\underline{s}'$ by $\varepsilon > 0$, furthermore increase investment by $\hat{k}' = k' + \pi(\underline{s}')\varepsilon$, such that $A(\underline{s}')f'(\hat{k}') + (1 - \delta) > R$ still holds and d stays the same. But then (6) still holds, $\hat{w}(s') > w(s')$ for all $s' \in S$. Thus there exist a allocation that achieves higher utility and satisfies the budget constraints, contradiction. As such it cannot be that $\mu(\underline{s}') = 0$ and $A(s')f'(k') + (1 - \delta) > R$. ■

Proof of Proposition 8. Part (i). From Proposition 7 we know that for all w such that if $\mu_i(s') > 0$ for all $i \in S$ then $k'_L < k'_H$. But then $A_H(\underline{s}')f'(k'_H) + 1 - \delta < A_L(\underline{s}')f'(k'_L) + 1 - \delta$. Moreover $A_H(\underline{s}')f'(k'_H) + 1 - \delta$ is decreasing in k'_H. And since, from Proposition 4, k'_H is increasing in w_H then it must be that there is $0 < \underline{w}_H < \underline{w}_H$ such that $A_H(\underline{s}')f'(k'_H(w_H)) + 1 - \delta = R$, $A_L(\underline{s}')f'(k'_L(w_H)) + 1 - \delta > R$, and $A_L(\underline{s}')f'(k'_L(w_l)) + 1 - \delta = R$. But then from Lemma 2, at $\underline{w}_H$, $\mu_H(\underline{w}_H, \underline{s}') = 0$, whereas $\mu_L(\underline{w}_H, \underline{s}') > 0$. Thus $\underline{w}_H < \underline{w}_L$.

Part (ii). Now I show that if $\mu_H(\bar{w}_H, \bar{s}') = 0$ and $\mu_L(\bar{w}_L, \bar{s}') = 0$ then $\bar{w}_H > \bar{w}_L$. First take the case when the level of net worth is high enough that entrepreneurs can fully insure in both regimes of uncertainty. Then from (6) and since $d_L(w) > d_H(w)$ results that $\sum_{s' \in S} \pi(s')b_L(s') < \sum_{s' \in S} \pi(s')b_H(s')$. Moreover, since $Var(A_L(s')) < Var(A_H(s'))$ and from (11) results that $b_H(\bar{s}') > b_L(\bar{s}')$. But recall that from (8), $\theta k'(1 - \delta) \geq Rb(s')$. Thus, take $\bar{w}_L$ such that at $w = \bar{w}_L - \varepsilon$, for $\varepsilon > 0$ very small, $\theta k'_L(w)(1 - \delta) = Rb_L(w, \bar{s}')$, but $\theta \bar{k}'_L(1 - \delta) > Rb_L(\bar{w}_L, \bar{s}')$. But then entrepreneurs in the high uncertainty regime cannot afford to borrow $b_H(\bar{w}_L, \bar{s}')$ such that $\theta \bar{k}'_H(1 - \delta) > Rb_H(\bar{w}_L, \bar{s}')$, as $b_H(\bar{w}_L, \bar{s}') > b_L(\bar{w}_L, \bar{s}')$. As such entrepreneurs can not be perfectly insured at $\bar{w}_L$. As a result at $\bar{w}_L$, when $\mu_L(\bar{w}_L, \bar{s}') = 0$,

$\mu_H(\bar{w}_L, \bar{s}') > 0$. ■

Proof of Proposition 9. Part **(i)**. See Proposition 7 above.
Part **(ii)** See Proposition 7 above.
Part **(iii)** Using the results from Proposition 8, we have that $\mu_H(\underline{s}') < \mu_L(\underline{s}')$, whenever $\mu_L(\underline{s}') > 0$. Thus $\exists \omega > 0$ such that $\mu_H(\underline{s}') = 0$ and $\mu_L(\underline{s}') > 0$. Moreover, since $\mu_H(\bar{s}') > \mu_L(\bar{s}')$, if $\mu_H(\bar{s}') > 0$, then there is $\hat{\omega}_L < \hat{\omega}_H$. This implies that at $\hat{\omega}_L$), $k'_H < k'_L$. But then since $\exists \underline{\omega} > 0$ such that $k'_H > k'_L$, and $\bar{\omega}$ such that $k'_H \leq k'_L$, and that k' is monotone and continuous, it results that $\exists \hat{\omega} > 0$ such that if $\omega < \hat{\omega}$ then $k'_H > k'_L$, whereas if $\omega > \hat{\omega}$ then $k'_H < k'_L$. ■

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Figure 1: Aggregate Economic Indicators

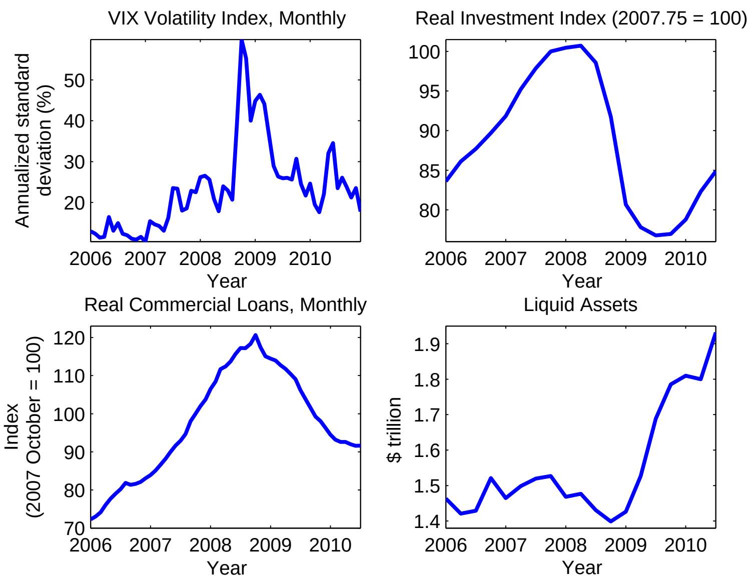


Figure 2: Optimal Policy - Low Volatility, $\pi(s,s') = \pi(s')$

Parameter values are:
$\beta = 0.93$, $R = 1/0.95$, $\gamma = 1$, $\alpha = 0.33$, $\delta = 0.1$, $\theta = 0.7$, $\sigma_H = 0.067$

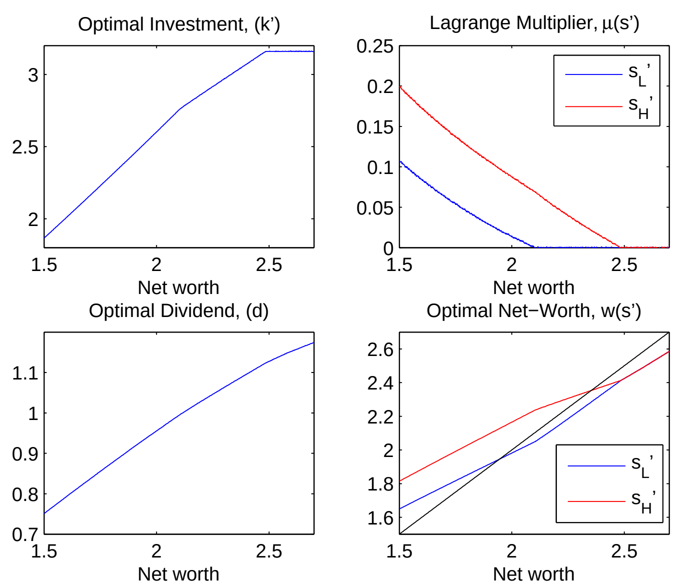


Figure 3: Optimal Policy - High Volatility, $\pi(s, s') = \pi(s')$

Parameter values are:
$\beta = 0.93$, $R = 1/0.95$, $\gamma = 1$, $\alpha = 0.33$, $\delta = 0.1$, $\theta = 0.7$, $\sigma_H = 0.13$

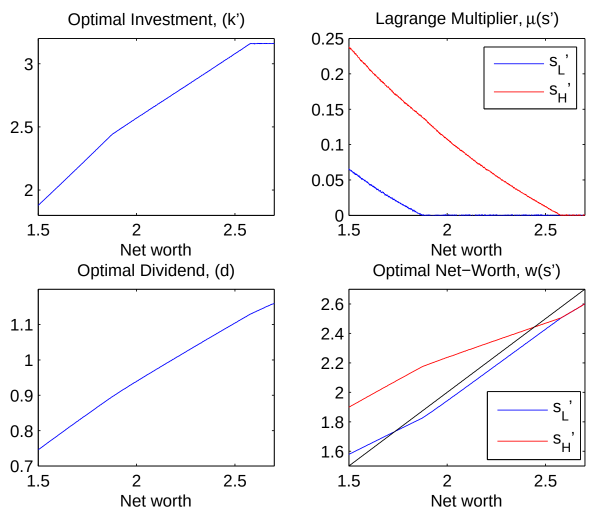


Figure 4: Increase in Uncertainty, Transitional Dynamics

The figure depicts the impact of an unexpected increase in uncertainty that occurs in period 0. All variables are relative to their steady-state values under low uncertainty. Parameter values are: $\beta = 0.93$, $R = 1/0.95$, $\gamma = 1$, $\alpha = 0.33$, $\delta = 0.1$, $\theta = 0.7$, $\rho = 0.86$, $\sigma_L = 0.067$, $\sigma_H = 0.13$.

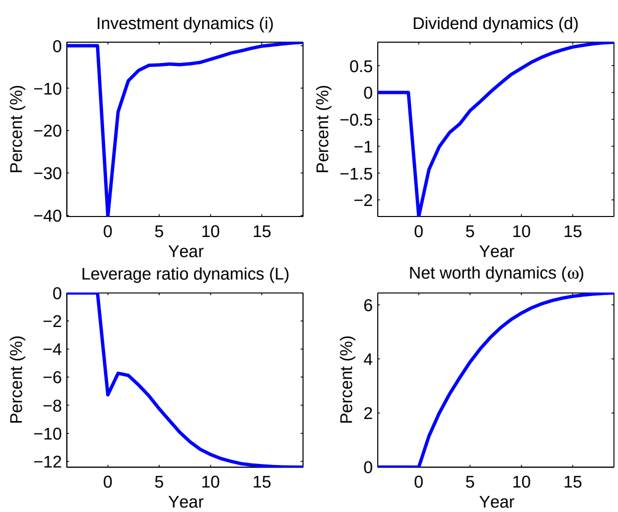


Figure 5: Uncertainty Shock, Stochastic Volatility

All variables are relative to their steady-state values. Parameter values are: $\beta = 0.93$, $\gamma = 1$, $R = 1/0.95$, $\alpha = 0.33$, $\delta = 0.1$, $\theta = 0.7$, $\rho_A = 0.86$, $\rho_\sigma = 0.4$, $\sigma_L = 0.067$, $\sigma_H = 0.13$.

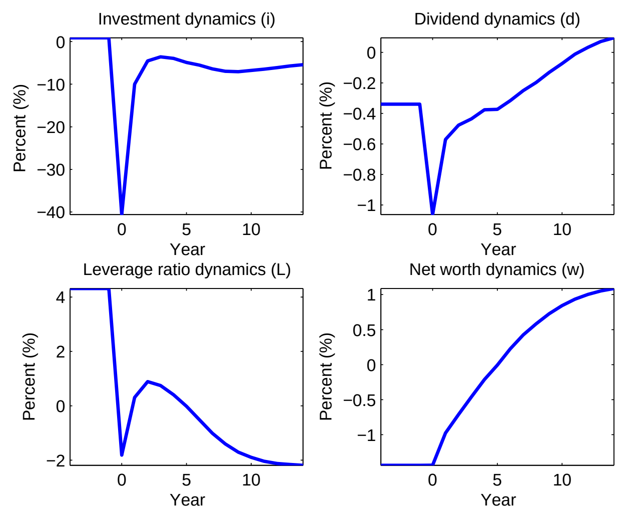


Figure 6: Decrease in Uncertainty, Transitional Dynamics

The figure depicts the impact of an unexpected increase in uncertainty that occurs in period 0. All variables are relative to their steady-state values under high uncertainty. Parameter values are: $\beta = 0.93$, $\gamma = 1$, $R = 1/0.95$, $\alpha = 0.33$, $\delta = 0.1$, $\theta = 0.7$, $\rho = 0.86$, $\sigma_L = 0.067$, $\sigma_H = 0.13$.

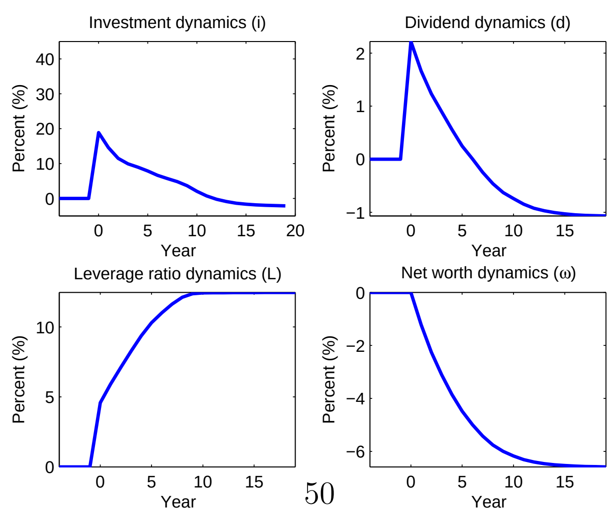
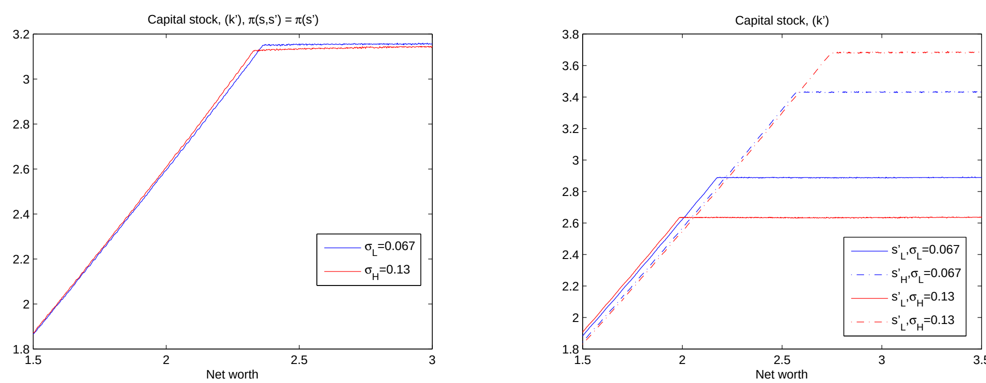


Figure 7: Optimal Scale of Production in Incomplete Markets

Panel (a) shows the optimal scale with iid shocks, and (b) shows the optimal scale with correlated shocks. Parameter values are: $\beta = 0.93$, $\gamma = 1$, $R = 1/0.95$, $\alpha = 0.33$, $\delta = 0.1$, $\theta = 0.7$, $\sigma_L = 0.067$, $\sigma_H = 0.13$.



(a) Constant investment opportunity (b) Stochastic investment opportunity