Capital Budgeting Under Asymmetric Information and Product Market Competition

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Abstract

This paper studies how division managers' access to venture capital (VC) markets affects the internal capital allocation decision of a multi-division firm. Division managers may leave firms and seek venture financing if their project ideas are not funded by headquarters. A successful new venture poses a threat to the revenue of the incumbent firm through product market competition. The paper characterizes headquarters' decision to retain managers and allocate capital for different degrees of product market competition and fundamental parameters in the VC markets. Distortions in internal capital allocations arise as headquarters retains managers by allocating more capital to their divisions and increasing compensation. These distortions may lead to under-investment or over-investment in high NPV projects as well as creation of new ventures. The paper also shows that improving efficiency in the VC markets may induce incumbent firms to aggressively retain managers by over-investing in their ideas and may not necessarily lead to more VC-backed startups. The incentive to retain managers is higher in more concentrated industries.

JEL Codes: G30; G31; D82

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1 Introduction

Between 1986 and 1999, among the venture capital-backed start-ups, the share of entrepreneurs coming from public companies averaged approximately 45% per year (Gompers, Lerner, and Scharfstein (2005)). One of the main reasons employees decide to leave their firms is because the companies they work for refuse to finance their ideas (Klepper (2001)). The most prominent example is Xerox, whose employees decided to leave after their headquarters refused to finance their projects and subsequently founded highly successful companies such as Adobe, 3com, and DEC systems. The anecdotal example of Xerox suggests that the internal capital allocation decision of a firm is important in retaining employees with entrepreneurial ideas. It is also puzzling that companies with large financial resources decide not to pursue high NPV projects.

This paper theoretically explores the internal capital allocation decision of a firm when managers may leave and seek venture capital (VC) financing. The first contribution of the paper is to propose a model that shows how asymmetric information problems inside firms could lead to under-, over- or no investment at all in high NPV projects depending on the outside options that the VC markets can offer entrepreneurs. The second contribution of the paper is to link the structure of product market competition and efficiency in the VC markets to the internal capital allocation decision of an incumbent firm. The paper investigates theoretically how product market competition and parameters in the VC market might affect the headquarters' decision to let go of managers with entrepreneurial or innovative ideas. In an imperfectly competitive market, a new entrance to the product market could potentially bring down prices. Similarly, if the VC market is more efficient at developing entrepreneurial ideas, there is also a higher chance of successful start-ups entering the product market so firms may be more willing to keep related ventures in-house. Thus, parameters in the VC market feedback into the internal capital allocation decision of the firm through product market competition. Gompers, Lerner, and Scharfstein (2005) also documented that high spawner firms in Silicon Valley and Massachusetts tend to spawn less related ventures. More specifically, the paper addresses the following questions: how does the internal capital allocation decision of an incumbent firm change in the presence of VC markets? What types of projects are likely to get financed inside incumbent firms and under what conditions? Are firms likely to retain managers more aggressively if their project ideas pose a threat to the firms' revenue when successfully implemented outside? The paper addresses these questions by proposing a model with the following ingredients: managerial preference for capital, asymmetric information inside incumbent firms, and product market competition. The assumption that managers prefer more capital over less has also been employed in Harris and Raviv (1996, 1998) and Bernardo et al. (2001).

There are three types of agents in this model: headquarters or the incumbent firm, division managers, and venture capitalists. Division managers have private information about the quality of the project that they manage and report to the headquarters in order to receive capital allocation. Managers may leave their headquarters and seek venture financing if their utilities from capital and wages inside the firm is less than the utilities from the expected payoff under the VC. The fundamental difference between the

VC and the headquarters is that there is no asymmetric information between the VC and the manager, but there is uncertainty in the match formation. Once the project leaves the incumbent firm, the quality of the project may increase or decrease under the VC management. The rationale behind this modeling choice of the match formation between the VC and the entrepreneur is that VCs are generally actively involved in the development of projects under their management. Their active involvement may be overall value-reducing or value-adding depending on the complementarity between the entrepreneur's idea and the VC's expertise. Due to this uncertainty in the match formation, managers decide to leave their headquarters if the expected pay-off under VC management is higher than the payoff inside the incumbent firm. Since the VC's added productivity affects the project quality of the manager additively, managers with high quality projects from incumbent firms have higher expected outside offers than those with low quality projects.

The paper first shows how distortions in internal capital allocations arise due to the asymmetric information problem inside the incumbent firm and the pressure to match the outside VC offers. These distortions may take the form of under-investment, over-investment compared to the unconstrained first best level, or no investment at all in high NPV projects depending on the level of outside offers that headquarters needs to match. The paper also shows that increased efficiency in the VC markets affects the internal capital allocation decision of the incumbent firm through two channels: the outside offers of the division manager and the potential loss in revenue through product market competition. While increased efficiency in the VC markets leads to higher outside offers for the division managers, it also makes it more likely that the project will be successfully developed with a VC. Thus, the firm will lose revenue due to the entrance of a new start-up in the product markets. Increased VC offers encourage managers to leave the incumbent firm while product market competition makes the firm more willing to match the VC offer in order to keep managers within the firm. Whether or not a more efficient VC market leads to more venture creation depends on the relative increase in the payoffs VC can offer and what the incumbent firms are willing to match. The firms are willing to match the VC offer as long as the cost of distortions in internal capital markets is less than the potential loss in revenue due to product market competition.

Theoretically, this model adopts an optimal contracting approach. The standard trade-off in the asymmetric information models is the cost of information rent versus the loss in efficiency from distortions in allocations. In this model, besides the standard trade-offs, there is also the cost of satisfying participation constraints since headquarters needs to match the outside VC offers. Introducing different outside options also produces an endogenous separation in the contract, i.e., it may not be optimal for the firm to commit to funding all types of projects through a truth-telling mechanism. Hence, headquarters may optimally decide not to satisfy the participation constraints of managers with some project qualities. As a result, some division managers may leave the firm. In standard models, the principal solves the optimal contract while making sure that all the participation constraints are satisfied so that the agent never defaults or quits. In Bernardo et al. (2001), there is no investment in lowest quality projects, but the participation constraints of all managers are satisfied so that managers with all project qualities stay with the firm.

By linking parameters in the VC markets to the internal capital allocation decision of an incumbent firm, the model derives novel empirical implications for the investment behavior of firms and venture creation. The framework proposed in this paper is most relevant in understanding technology and research intensive industries where manager's human capital is key to innovation and where venture financing is more readily available. The model also implies that firms prefer to pay managers with capital instead of wages in order to induce truth-telling. A similar result has also been obtained in Scharfstein and Stein (2000). Their paper studies a two tier agency model, and shows that due to the agency problem between the CEO and outside investors, the CEO prefers to pay managers with low quality projects in capital rather than in cash. However, in this paper, there is only one level of agency problem between headquarters and division managers due to managerial preference for capital.

This paper is related to two different strands of literature. The first is the literature on the efficiency of internal capital markets. Harris and Raviv (1996) studies a model where headquarters trades off the auditing cost against the distortions in capital allocation due to managerial preference for capital. Their model finds regions for both under- and over-investment. In terms of the modeling set up, this model is closely related to Bernardo et al. (2001). Their model features both asymmetric information and moral hazard to study optimal capital allocation and managerial compensation. Their model assumes that managers with different qualities of projects have the same outside options and does not study the effect of product market competition on the internal capital allocation.

The second is the relation between internal or external capital markets and the boundaries of a firm. Inderst and Muller (2003) adopts an optimal contracting approach to study the conditions under which a financially constrained firm would prefer to borrow against one or multiple projects. In their model, there is asymmetric information between headquarters and outside investors but not between headquarters and division managers as in this model. The decision of the firm to finance one or multiple projects depends on the ability of the firm to credibly repay the debt; thus, the boundaries of the firm are determined by the degree of information asymmetry between the firm and the external capital markets. However, in this model, since firms are assumed to have access to unlimited capital, the decision to finance projects depends on its ability to incentivize the division managers and to match the VC offers; hence, the boundaries of the firm are determined jointly by the structure of the product market competition and the relative efficiency between internal capital markets and VC markets. Matsusaka and Nanda (2002) assumes a deadweight loss to external financing. In their model, firms trade off the cost of external financing against the over-investment problem in internal capital markets. The model in this paper implies that in evaluating whether to incorporate a project, firms tradeoff the cost of distortions in internal capital markets against the potential loss in market shares from product market competition. Amador and Landier (2003) also studies conditions under which innovative ideas are incorporated in existing firms. In their model, it is less costly to incorporate ideas in existing firms, but contractual advantage in new ventures allows truly innovative ideas to be implemented with the VCs. In this model, high quality projects are more likely to be undertaken outside incumbent firms as the degree of information asymmetry inside incumbent firms worsens and the VC markets become more efficient.

The paper is organized as follows. Section 2 describes the dynamics inside the incumbent firm while taking the outside options of the managers as exogenous. Section 3 introduces the VC markets and endogenizes the outside options. Section 4 concludes.

2 Model

This section presents a model of a multi-division firm's investment decisions. Section 2.1 describes the environment and the timing of the contract. Section 2.2 presents the full information model as a benchmark. Section 2.3 presents the model with asymmetric information.

2.1 Environment

There are two types of agents: an incumbent firm and division managers. In this section, the outside VC offers are taken as exogenous. The role of the VC will be modeled explicitly in Section 3 and the outside options will also be endogenized.

An incumbent firm is run by a headquarters. The model first explores how asymmetric information inside a multi-division firm can lead to distortions in internal capital allocation. The model abstracts from agency problems between shareholders and the CEO/headquarters and the ex-ante financial constraints that arise from such agency conflicts. The model assumes that headquarters acts in the interest of risk-neutral shareholders. Hence, in the rest of the paper, the term "headquarters" and the "incumbent firm" may be used interchangeably.

The role of headquarters is to allocate capital to the divisions under its control. Headquarters employs division managers who have access to projects but have no wealth of their own to run them independently. Headquarters in this model has unlimited access to capital. Due to the assumption of unconstrained capital, headquarters' decision to invest in projects is driven by maximization of total profits and not by the constraints from the ability to raise external funds. Therefore, the internal capital allocation decision of the incumbent firm does not feed back into the external financing cost of the firm (Inderst and Muller (2003)).

Project ideas occur to division managers. Project quality is denoted by t and can be high, t_H, with probability π and low, t_L, with probability $1-\pi$, where $t_H > t_L$. Headquarters knows the distribution of the project qualities but does not know the true quality of the project each manager runs. Since these projects require capital investment and managers are assumed to have no wealth of their own, they report the quality of their projects to the headquarters in order to receive capital investment. Projects in this model can be thought of as human capital since they are inseparable from the managers, i.e., if the headquarters refuses to fund the projects, managers may leave anytime and develop these projects with venture capitalists outside the firm. Hence, managers with different qualities of projects may also be thought of as different types of managers in this model.

Headquarters allocates capital to its divisions based on the project reports of the division managers. The cash flow specification in this model follows very closely that of Bernardo et al. (2001) and has a linear production function with convex cost of capital. In the absence of asymmetric information, for a project of quality t_θ with capital allocation k, the cash flow is given by the following:

$$V(t_\theta) = \delta t_\theta k - 0.5k^2, \tag{1}$$

where $\delta > 0$ and is known to both the headquarters and the managers. The parameter δ can be interpreted as prices or industry specific productivity that is commonly known. In the case of asymmetric information, k depends on the project reports of the managers.

Division managers receive capital allocation and compensation based on the project reports that they submit to the headquarters. Managers in this model are assumed to have an "empire building" tendency, i.e., they enjoy private benefits from controlling more capital and managing higher quality projects as in Harris and Raviv (1996, 1998) and Bernardo et al. (2001). In the absence of asymmetric information, manager's utility from running a project of quality t_θ and managerial compensation w under the headquarters is given by the following:

$$U(t_\theta) = w + \beta\delta t_\theta k, \tag{2}$$

where $\beta \in (0, 1)$ is a parameter that measures private benefits that managers enjoy from running a project with capital k and quality t_θ. Hence, managers have the incentive to overstate the quality of their projects in order to receive more capital from the headquarters. The assumption of managerial preference for capital is central to the over-investment result of the model. In the presence of asymmetric information, both wages and capital depend on reported project qualities. Managers with project quality t_θ have outside employment opportunities or expected value of being financed by a venture capitalist that gives managers utility u_θ. In this section, u_θ is treated as exogenous but will be endogenized in Section 3.

Since the optimal capital allocation depends on the quality of the project, headquarters designs a mechanism in order to induce managers to report the quality of their projects truthfully. Headquarters can implement this using both managerial compensation and capital allocation as both of these increase the utility of the division manager.

The sequence of the move is as follows. At date 0, headquarters and managers both observe the outside offers u_θ for $\theta = H, L$. At date 1, headquarters offers the manager a compensation package and capital allocation rule $\{w(t_\theta, \hat{t}_\theta), k(\hat{t}_\theta)\}$. At date 2, the manager privately observes the quality t_θ of the project. The manager decides to report $\hat{t}_\theta$ if $U(\hat{t}_\theta, t_\theta) \geq u_\theta$. Otherwise, the manager decides to leave the firm and take his outside option. At date 3, headquarters allocates capital $k(\hat{t}_\theta)$. At date 4, project cash flows are realized and distributed to the shareholders after $w(t_\theta, \hat{t}_\theta)$ is paid to the manager. Note that wages are paid after cash flow is realized and observed by both the manager and the headquarters. Since the actual project quality can be deduced from observed cash flow, headquarters may punish the manager if the manager is caught misreporting project quality. Thus, wages depend on both the true quality of the project and managers' reports. However, capital allocation depends only on the reported project quality as capital has to be disbursed before cash flow is realized.

It is important that the outside offers are observable to both the headquarters and the managers at date 0 before project quality reports are made. In solving for the optimal contract, headquarters needs to know u_θ in order to solve an optimization problem with and without some participation constraints. In other words, headquarters needs to know the levels of outside offers it needs to match so that it may decide wether or not it is worth keeping some managers in the firm by matching their outside offers. As is standard in the literature, this model assumes one sided lack of commitment on the side of the manager. In other words, headquarters cannot renege on the compensation and capital allocation that it proposes in date 1. Generally, this has been motivated in the literature through reputational concerns of the firm or rigid capital budgeting practices.

Definition 1 (**Manager's Contract**) Let $\mathbf{u}_\theta$ denote the set of outside options u_θ for $\theta \in \{H, L\}$ and $\hat{t}_\theta$ reported project quality. A contract specifies for each set of outside options $\{u_H, u_L\}$ and each reported project quality $\hat{t}_\theta$, managerial compensation w and a capital allocation k . Let $W = [0, \infty)$ denote the set of compensation w and $K = [0, \infty)$ the set of allocated capital k. A contract is a mapping $C : \mathbf{u}_H \times \mathbf{u}_L \times \{\hat{t}_H, \hat{t}_L\} \rightarrow W \times K$.

Note that when managers can commit or employment contracts are such that managers may not leave the firm with their project ideas, headquarters allocates the optimal level of capital by simply solving:

$$k_\theta^{UN} = \underset{\{k_\theta, w_\theta\}}{\arg\max}\ \delta t_\theta k_\theta - 0.5k_\theta^2 - w_\theta.$$

The unconstrained first best level of capital and wages are then given by:

$$\begin{aligned} k_\theta^{UN} &= \delta t_\theta, \\ w_\theta^{UN} &= 0. \end{aligned}$$

In order to provide a simple benchmark to the unconstrained optimum, in what follows, the optimal policies are normalized as follows.

Definition 2 (**Normalization**) Let $x_\theta = \beta\delta^2 t_\theta^2$; $\tilde{k}_\theta = \frac{k_\theta}{\delta t_\theta}$; $\tilde{w}_\theta = \frac{w_\theta}{x_\theta}$; $\tilde{U}_\theta = \frac{U_\theta}{x_\theta}$; $\tilde{u}_\theta = \frac{u_\theta}{x_\theta}$, $\tilde{H}_\theta = \frac{H_\theta}{x_\theta}$.

Note that under the normalization by definition 2, the managerial utility and the payoff to the headquarters have the following expression:

$$\begin{aligned} \tilde{U}_\theta &= \tilde{w}_\theta + \tilde{k}_\theta, \\ \tilde{H}_\theta &= \frac{1}{\beta}\left(\tilde{k}_\theta - 0.5\tilde{k}_\theta^2\right) - \tilde{w}_\theta. \end{aligned}$$

2.2 Full Information

This section solves for the case of full information where project qualities are observable to both the headquarters and the division managers. Headquarters maximizes firm value net of wage payments and costs of capital. Let k_θ denote capital allocated to the division with project quality t_θ, and w_θ denote wages paid to the corresponding division manager.

The headquarters' optimization problem from committing to fund a project of quality t_θ can be formally written as below:

$$H_\theta^{FB} = \max\{0, \bar{H}_\theta^{FB}\}.$$

where $\bar{H}_\theta^{FB}$ is given by:

$$\begin{aligned} \bar{H}_\theta^{FB} = \max_{\{k_\theta, w_\theta\}} \quad & \delta t_\theta k_\theta - 0.5 k_\theta^2 - w_\theta, \\ s.t \quad & \text{(i) } w_\theta + \beta \delta t_\theta k_\theta \geq u_\theta, \\ & \text{(ii) } w_\theta \geq 0. \end{aligned}$$

Headquarters chooses optimal capital k_θ and managerial compensation w_θ. Constraint (i) is the participation constraint that ensures that the division manager's utility inside the firm is at least as much as the utility he can obtain outside the firm. Constraint (ii) is the limited liability constraint that is fairly standard in the literature. For low levels of outside options u_θ, participation constraint might not bind, i.e., the perks that managers derive from controlling capital alone could potentially be larger than their outside options. In such cases, limited liability constraint prevents w_θ from becoming negative, i.e., headquarters cannot ask managers to pay out of their pockets in order to run projects.

Note that the optimization problem above can become negative for high values of u_θ. Headquarters would only commit to funding the project as long as $\bar{H}_\theta \geq 0$; otherwise, it chooses $k_\theta = 0$ and $w_\theta = 0$.

The following proposition summarizes the optimal policies for the full information model.

Proposition 1 (First Best Optimal Policy) *The optimal policy varies for different regions of outside offer u_θ, and is given by:*

Region I: $0 \leq \tilde{u}_\theta < 1$,

$$\begin{aligned} \tilde{k}_\theta^{FB} = 1 \quad &; \quad \tilde{w}_\theta^{FB} = 0; \\ \tilde{U}_\theta^{FB} = 1 \quad &; \quad \tilde{H}_\theta^{FB} = \frac{1}{2\beta}. \end{aligned}$$

Region II: $1 \leq \tilde{u}_\theta \leq (1+\beta)$,

$$\begin{aligned} \tilde{k}_\theta^{FB} = \tilde{u}_\theta \quad &; \quad \tilde{w}_\theta^{FB} = 0; \\ \tilde{U}_\theta^{FB} = \tilde{u}_\theta \quad &; \quad \tilde{H}_\theta^{FB} = \frac{1}{\beta}\tilde{u}_\theta - \frac{1}{2\beta}(\tilde{u}_\theta)^2. \end{aligned}$$

Region III: $(1+\beta) < \tilde{u}_\theta \leq \frac{(1+\beta)^2}{2\beta}$,

$$\tilde{k}_\theta^{FB} = 1+\beta \quad ; \quad \tilde{w}_\theta^{FB} = \tilde{u}_\theta - (1+\beta);$$
$$\tilde{U}_\theta^{FB} = \tilde{u}_\theta \quad ; \quad \tilde{H}_\theta^{FB} = \frac{(1+\beta)^2}{2\beta} - \tilde{u}_\theta.$$

Region IV: $\tilde{u}_\theta \geq \frac{(1+\beta)^2}{2\beta}$,

$$\tilde{k}_\theta^{FB} = 0 \quad ; \quad \tilde{w}_\theta^{FB} = 0;$$
$$\tilde{U}_\theta^{FB} = 0 \quad ; \quad \tilde{H}_\theta^{FB} = 0.$$

Proof: See Appendix.

Proposition 1 shows the optimal capital and wage policies for different regions of outside offers for managers with project quality t_θ. The optimal capital allocations in Proposition 1 are normalized by the unconstrained first best level of investment, δt_θ, and the optimal wages and payoffs are also normalized by the managerial perks from running the project at the unconstrained first best level, $\beta\delta^2 t_\theta^2$. Therefore, the optimal capital and utility to the managers derived in Proposition 1 have the convenient interpretation that they are proportional to the unconstrained first best level of capital and managerial utility.

There are four regions depending on the level of outside offers. Region I represents the unconstrained region. In this region, the outside offer, u_θ, is low enough so that headquarters does not need to match. The managerial perks from controlling capital inside the firm is already higher than the outside offer. In other words, the participation constraint does not bind in Region I. Hence, headquarters maximizes by investing the unconstrained first best level of capital and paying zero wages. Regions II and III represent constrained regions where the participation constraint is binding. In Region II, headquarters matches the outside offer by paying zero wages but allocating more capital to the division whereas in Region III, headquarters also pays positive wages to the manager. Both wages and capital increase the utility of the manager; however, headquarters initially commits to funding more capital instead of paying more wages because in this region, the managerial perks make the marginal cost of capital less expensive than wages. In this region, headquarters internalizes the positive externality of managerial perks in making the capital allocation decision. As headquarters commits more and more capital, at some level of outside offer, the convex cost of capital makes capital more expensive than wages. In Region III, headquarters retains managers by simply raising wages proportionately with the outside offer. Region IV represents the outside offers where the NPV of the project is negative. Therefore, headquarters does not undertake the project. Figure 1 and Figure 2 show graphically the optimal policies and payoffs from Proposition 1.

Figure 3 plots the Pareto Frontier. The outside offer u_θ determines where the payoff pair $(\tilde{U}_\theta, \tilde{H}_\theta)$ lies on the Frontier. As u_θ increases, the managerial utility increases and the payoff to the headquarters decreases; as a result, the payoff pair $(\tilde{U}_\theta, \tilde{H}_\theta)$ moves down along the Frontier. The concave part of

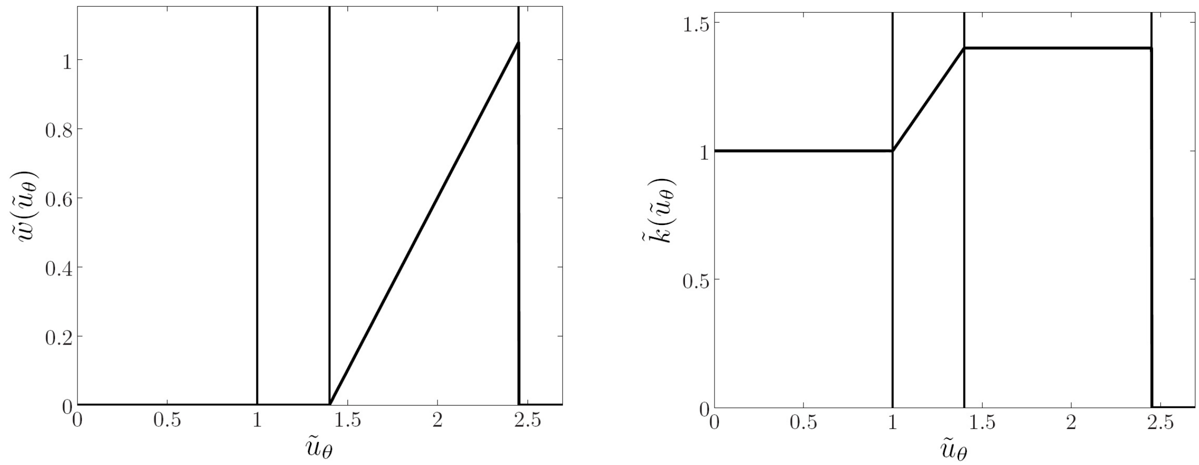

Figure 1: **Optimal Policies in Full Information Model.** The left panel plots the optimal wage policy for different values of outside options. The right panel plots the optimal capital policy. For this graph, $\beta = 0.4$ is used.

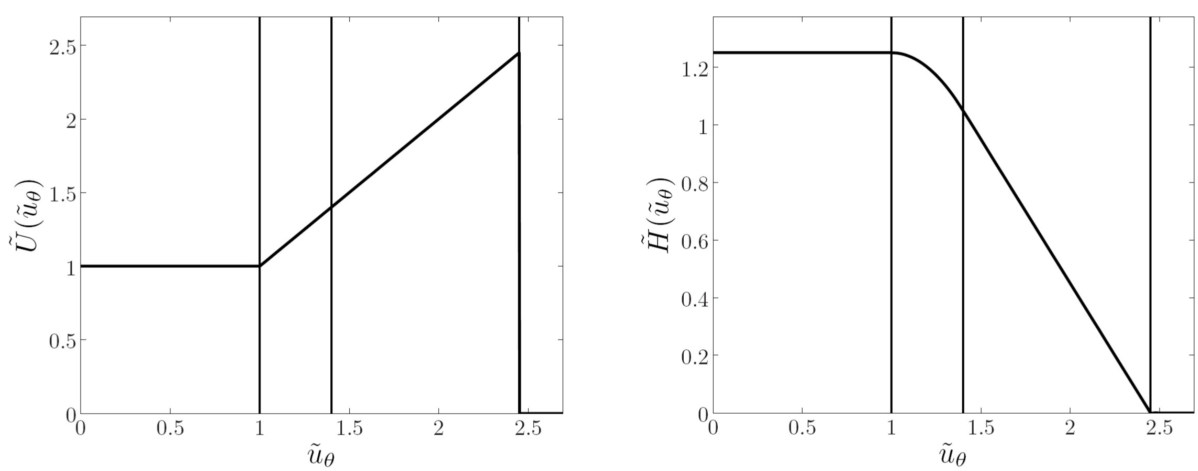

Figure 2: **Payoffs to Manager and Headquarters.** The left panel plots the payoff to the manager for different values of outside options. The right panel plots the payoff to the headquarters. For this graph, $\beta = 0.4$ is used.

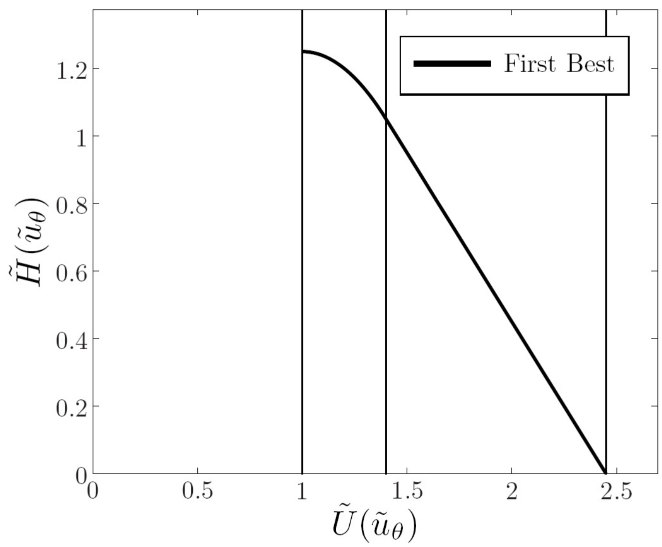

Figure 3: **The Pareto Frontier.** For this graph, $\beta = 0.4$ is used.

the Pareto Frontier corresponds to Region II where the headquarters retains the manager by allocating more capital. Since the cash flow to the headquarters is concave in capital k, the Pareto Frontier is concave in Region II. The Pareto Frontier in Region III has slope one since the headquarters simply transfers the payoff through wages to the division manager in this region.

2.3 Asymmetric Information

This section solves for the optimal mechanism when the headquarters does not observe the quality of the project that the division manager controls.

Assumption 1 $u_H > u_L$.

The model assumes that managers with high quality project ideas have better outside opportunities than managers with low quality project ideas, i.e., $u_H > u_L$. This may be because managers with high quality projects have better skills and are hence more likely to come up with better quality projects in the future or because there is a venture capital market that can distinguish different types of managers better than the incumbent firm. Section 3 introduces the venture capital markets and endogenizes the value of u_θ, $\theta \in \{H, L\}$.

Assumption 2 $u_L = 0$.

Assumption 2 normalizes the outside option of the manager with low quality project to be zero. As long as this outside option allows the low quality project to be implementable inside incumbent firms under first best, this assumption does not change the results either qualitatively or quantitatively.

Let $U(t_\theta, \hat{t}_\theta)$ denote the managerial utility when the division manager reports $\hat{t}_\theta$ when in fact the true quality of the project that he manages is t_θ. This is given by the following equation:

$$U(t_\theta, \hat{t}_\theta) = w(\hat{t}_\theta, t_\theta) + \beta\delta t_\theta k(\hat{t}_\theta).$$

Contingent on committing to fund both types of projects, t_H and t_L, headquarters solves an optimization problem similar to a monopolist's screening model. Headquarters solves for the optimal wage payments, $w(\hat{t}_\theta, t_\theta)$, and capital allocations, $k(\hat{t}_\theta)$, in order to induce managers to truthfully reveal the quality of their projects. By the Revelation Principle, we can restrict our attention to the direct revelation mechanism where division managers report truthfully, i.e., $\hat{t}_\theta = t_\theta$. Since the optimal policy only depends on true project quality, without loss of generality, let $k_\theta = k(t_\theta)$ and $w_\theta = w(t_\theta, t_\theta)$ in order to simplify notations. Let EH denote the payoff to the headquarters from committing to fund both types of projects.

$$EH = \max_{\{w_\theta, k_\theta\}_{\theta=H,L}} \pi\big[\delta t_H k_H - 0.5k_H^2 - w_H\big] + (1-\pi)\big[\delta t_L k_L - 0.5k_L^2 - w_L\big],$$

subject to:

$$\begin{aligned}
&\text{(i)} && U(t_H, t_H) \geq u_H,\\
&\text{(ii)} && U(t_L, t_L) \geq u_L,\\
&\text{(iii)} && U(t_H, \hat{t}_H) \geq U(t_H, \hat{t}_L),\\
&\text{(iv)} && U(t_L, \hat{t}_L) \geq U(t_L, \hat{t}_H),\\
&\text{(v)} && k_\theta \geq 0, w_\theta \geq 0, \theta \in \{L, H\}.
\end{aligned}$$

Headquarters solves for the optimal wage and capital allocation given a set of participation constraints, (i) and (ii), and incentive compatibility constraints, (iii) and (iv). Constraint (v) is the feasibility constraint for capital and the limited liability constraint for wages.

The model can be solved using standard techniques employed in models with asymmetric information. However, introducing $u_H > u_L$ produces non-trivial implications for internal capital allocations. The following proposition summarizes the optimal policy when the headquarters decides to fund both types of projects.

Proposition 2 (Optimal Policy under Screening) *Suppose that the quality premium is high enough, that is,* $\frac{t_H}{t_L} - 1$ *satisfies the following equation:*

$$\frac{t_H}{t_L} - 1 \geq \frac{\beta}{\pi}. \tag{3}$$

Contingent on committing to fund both types of projects, the optimal policies for wages and capital allocations are given by:

Region I': $0 \leq \tilde{u}_H \leq 1 - \epsilon_1$,

$$\tilde{k}_H = 1 - \epsilon_1 \quad ; \quad \tilde{k}_L = 1 + \beta;$$
$$\tilde{w}_H = 0 \quad ; \quad \tilde{w}_L = \frac{t_H}{t_L} - (1 + \frac{\beta}{\pi}),$$

where $\epsilon_1 = \beta \frac{(1-\pi)}{\pi} \frac{t_L}{t_H}$.

Region II': $1 - \epsilon_1 < \tilde{u}_H < 1 + \beta - \epsilon_1$,

$$\tilde{k}_H = \tilde{u}_H \quad ; \quad \tilde{k}_L = 1 + \beta;$$
$$\tilde{w}_H = 0 \quad ; \quad \tilde{w}_L = \frac{t_H}{t_L}\tilde{u}_H - (1 + \beta),$$

Region III': $\tilde{u}_H \geq 1 + \beta - \epsilon_1$,

$$\tilde{k}_H = 1 + \beta - \epsilon_1 \quad ; \quad \tilde{k}_L = 1 + \beta;$$
$$\tilde{w}_H = \tilde{u}_H - \left(1 + \beta - \epsilon_1\right) \quad ; \quad \tilde{w}_L = (1 + \beta)\frac{t_H}{t_L} - (1 + \frac{\beta}{\pi}).$$

Proof: See Appendix.

As shown in Proposition 2, there are three regions of optimal policy depending on the outside offer of managers with high quality projects, u_H. Recall that assumption 2 restricts $u_L = 0$, hence the optimal policy depends on u_H. Note that the spread between the low and high quality projects must satisfy equation (3) so that inducing truth-telling by paying information rent generates a higher profit than simply pooling the optimal capital policy using average productivity of the firm.

In Region I', the outside offer u_H is low enough so that headquarters does not need to match the outside VC offer. Due to the managerial perks for capital, the allocated capital in this region already gives the manager a utility higher than his outside option. The optimal policy in this region is reminiscent of the results in traditional models with asymmetric information. Since participation constraints of both types do not bind in this region, headquarters trades off information rent that it must give to the low type with the distortions in capital allocation that arise from asymmetric information. If more capital is allocated to the division with high quality project, more information rent must also be given to the manager with the low quality project in order to induce truth-telling. Hence, in this region, headquarters optimally chooses to under-invest in high quality projects.

Region II' represents the intermediate levels of u_H where headquarters matches the outside offer of the manager with the high quality project by allocating more capital to that division. Depending on the value of u_H, the optimal policy results in under- or over-investment compared to the unconstrained first best level. In this region, u_H is high enough so that headquarters needs to retain the division manager of the high quality project by matching his outside offer, i.e., the participation constraint of the high type binds in this region. Notice that the capital allocated to the division with high quality project increases proportionately with the outside offer. As in the full information model, allocating more capital increases the utility of the manager due to the managerial preference for capital. Paying one more unit in capital to the high type increases the high type utility by $\beta\delta t_H$ but increases the low type utility by $\beta\delta t_L$. While managerial perks from capital allows headquarters to retain managers by allocating more capital, it also makes it costly to do so. Since both the low and high quality managers derive perks from controlling more capital, the manager with the low quality project has the incentive to misreport in order to receive more capital. Recall that wages depend on both the true and reported project quality. Hence, if the manager is caught misreporting, he may be punished. However, due to the limited liability of the division manager, the worst punishment headquarters can impose is zero wages or to fire the manager. That is,

$$w(t_L, \hat{t}_H) = 0.$$

Therefore, the manager has the incentive to misreport if the difference in managerial perks from running larger capital is higher than the difference in wages. That is,

$$\beta\delta t_L(k_H - k_L) > w(t_L, \hat{t}_L) - w(t_L, \hat{t_H})$$

In order to prevent the manager of the low quality project from misreporting, headquarters needs to pay him more in wages. Thus, allocating more capital to a division with high NPV costs the headquarters not only the cost of capital for that division but also the information rent in the form of wages to the manager with the low quality project.

As capital increases, the convex cost of capital and information rent makes capital costlier to the headquarters than wages. When headquarters retains managers of high quality projects by paying only in more wages, it does not need to pay information rent to the low type. This is because wages can be contracted contingent on the actual realization of project cash flow. If the manager with the low quality project misreports in order to receive the wages for the high type, he will be caught lying after the cash flow is realized. In other words, the manager with the low quality project can never claim to have high quality in order to claim the high type's wages. Hence in Region III', headquarters incentivizes by paying in wages. Note that capital is constant in this region, but managers of high quality projects receive positive wages.

Figure 4 shows graphically the optimal policy under asymmetric information. The (red) dashed lines represent the optimal policy when managers report low quality projects while the (blue) solid lines represent the optimal policy for high quality projects. Note that in Region II', wages of managers with low quality projects are higher than those with high quality projects. This is because headquarters rewards managers who report high quality projects with more capital allocation but pays wages to those who report low quality projects in order to induce truth-telling. The right panel in Figure 4 plots the optimal capital as normalized by definition 2. Since the outside offer of the manager with the low quality project is assumed to be 0, the optimal capital under the first best is constant and is at 1 as given by Proposition 1. However, as shown in the right panel of Figure 4, under asymmetric information, the optimal capital allocated for the division manager with the low quality project is higher than its

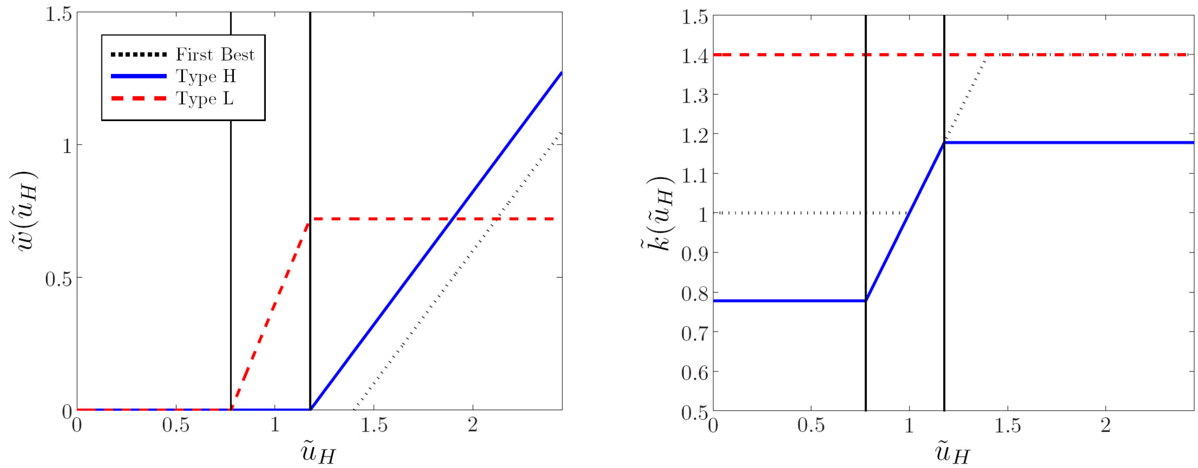

Figure 4: **Optimal Policy Under Asymmetric Information.** The left panel shows the optimal wage policy and the right panel plots the optimal capital policy. For this graph, $\beta = 0.4$, $\pi = 0.5$, and $t_H/t_L = 1.8$ is used.

unconstrained first best level of 1. However, it does not imply that the capital allocated for the low type is higher than that for the high type since the normalization is by the unconstrained first best level of each type of project. The graph also shows that the optimal capital allocated to the division with the low quality project is constant and is always socially efficient at $(1+\beta)$. This result can be explained as follows: as long as the spread between the two qualities satisfies equation (3), it is always the case that the optimal k_H is significantly higher than k_L^{FB}. Since managers of low quality projects may misreport in order to receive k_H, headquarters must pay information rent to the low type. Since increasing capital increases both the cash flow and managerial utility initially, headquarters exhausts the option of paying in capital before simply transferring the profits through wage payments. Therefore, managers with low quality projects always receive the socially efficient level of capital and some positive wages at higher levels of u_H.

Depending on the value of outside offer u_H, the optimal capital allocated for the manager with high quality projects also shows regions for under-investment and over-investment compared to the unconstrained first best level of 1. The under-investment regions correspond to low levels of u_H where headquarters optimally under-invests. However, as the outside offer u_H increases, headquarters retains managers by allocating more capital to their division. Thus, the optimal k_H increases. However, for very high levels of u_H, the convex cost of capital and information rent make capital more costly and headquarters retains managers by allocating more wages instead of capital. The (black) dotted line represents the optimal capital allocation under first best. Hence, as can been seen in the right panel of Figure 4, there is under-investment everywhere compared to the full information model. This is due to the fact that under full information, headquarters is able to retain managers by paying in capital due to managerial preference for capital until the convex cost of capital sets in. However, under asymmetric information, the information rent makes capital costlier since managers with low quality projects may misreport in order to receive a higher amount of capital. Thus, the benefit of paying in capital is decreased under asymmetric information.

It is also useful to compare the payoffs to the headquarters and to the division managers as they depend on the total wage payments and capital allocation, and each of these varies for different regions of u_H. The following proposition shows the payoffs in closed-form.

Proposition 3 (Optimal Payoff under Screening) *The corresponding managerial utility and headquarters profits for each region is also given by:*

Region I': $0 \leq \tilde{u}_H \leq 1 - \epsilon_1$,

$$\tilde{U}_H = 1 - \epsilon_1 \quad ; \quad \tilde{U}_L = \frac{t_H}{t_L}(1 - \epsilon_1);$$

$$\tilde{H}_H = \frac{1}{2\beta}(1 - \epsilon_1^2) \quad ; \quad \tilde{H}_L = \frac{1}{2\beta}(1+\beta)^2 - \frac{t_H}{t_L} + \frac{(1-\pi)\beta}{\pi},$$

where $\epsilon_1 = \beta \frac{(1-\pi)}{\pi} \frac{t_L}{t_H}$.

Region II': $1 - \epsilon_1 < \tilde{u}_H < 1 + \beta - \epsilon_1$,

$$\tilde{U}_H = \tilde{u}_H \quad ; \quad \tilde{U}_L = \frac{t_H}{t_L}\tilde{u}_H;$$

$$\tilde{H}_H = \frac{1}{\beta}\tilde{u}_H - \frac{1}{2\beta}\tilde{u}_H^2 \quad ; \quad \tilde{H}_L = \frac{(1+\beta)^2}{2\beta} - \frac{t_H}{t_L}\tilde{u}_H,$$

Region III': $\tilde{u}_H \geq 1 + \beta - \epsilon_1$,

$$\tilde{U}_H = \tilde{u}_H \quad ; \quad \tilde{U}_L = (1+\beta)\frac{t_H}{t_L} + 1 - \frac{\beta}{\pi};$$

$$\tilde{H}_H = \frac{(1+\beta)^2 - \epsilon_1^2}{2\beta} - \tilde{u}_H \quad ; \quad \tilde{H}_L = \frac{1}{2\beta}(1+\beta)^2 - (1+\beta)\frac{t_H}{t_L} + \frac{(1-\pi)}{\pi}\beta.$$

Proof: See Appendix.

Note that in Region I', the payoffs to the headquarters and managerial utilities do not depend on the outside offer u_H because the participation constraints of both types do not bind in this region. In Regions II' and III', the utility for the manager with high project quality is exactly equal to his outside option. The expected payoff to the headquarters decreases by as much as the outside offer u_H in Region III' as wages are used to match the outside offer in this region and for each dollar paid to retain the manager with the high quality project, headquarters must also pay the same to the manager with the low quality project because of the incentive compatibility constraint.

Figure 5 illustrates the utility to the division managers and the payoff to the headquarters. The (red) dashed lines represent the payoffs to the manager of low quality projects while the (blue) solid lines represent the manager of high quality projects. Due to the assumption that $u_L = 0$, the first best level of utility for the manager with low quality project is 1 as given by Proposition 1. However,

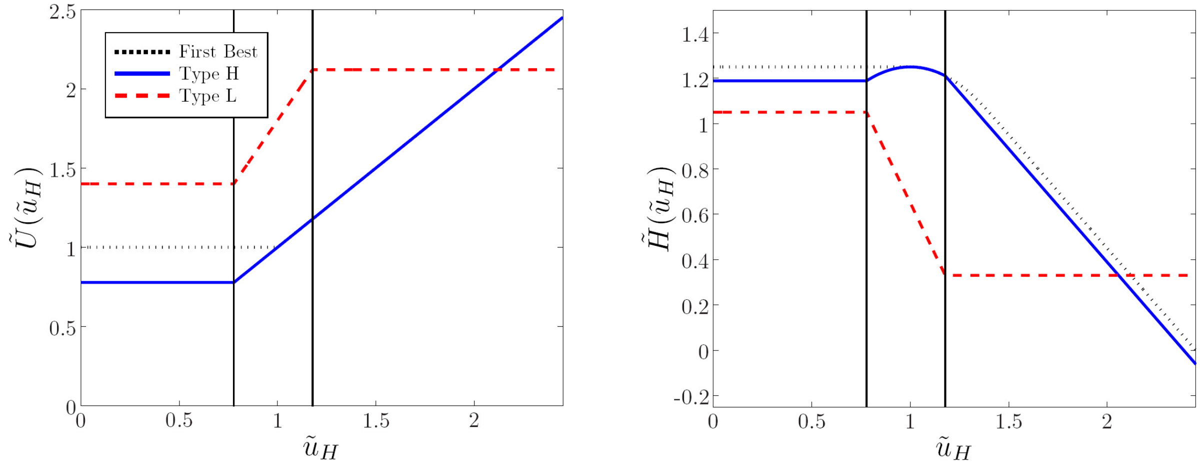

Figure 5: **Payoff under Asymmetric Information.** The left panel shows managerial utility for different levels of outside offer for the high type while the right panel shows the payoff to headquarters. For this graph, $\beta = 0.5$, $\pi = 0.5$, and $t_H/t_L = 2.5$ is used.

under asymmetric information, the division manager with the low quality project receives utility strictly higher than 1 as information rent. The utility of the division manager with the high quality project increases one for one with the outside offer in Regions II' and III', but is constant in Region I' where headquarters under-invests in high quality projects.

It is interesting to note that the payoff to the headquarters for the high project quality has a concave and increasing region. Recall that headquarters optimally chooses to under-invest in Region I'. As the outside option of the high type increases, headquarters increases capital allocation and therefore cash flows increase as well. Note that it does not imply that in the increasing concave region, headquarters is not optimizing. In fact, it maximizes the expected profits. Note also that the payoff to the headquarters from funding high quality projects under full information, represented in the graph by the dotted black line, is only slightly higher than that under asymmetric information. This is due to the fact that in the asymmetric information model, headquarters over-invests while underpaying in wages to the manager of high quality project. Due to the presence of the information rent, the payoff to the headquarters from the low quality projects can potentially be negative as shown in the right panel of Figure 5.

It is also important to note that the trade-offs headquarters faces in Regions II' and III' are similar to those in the full information model, i.e., headquarters optimally chooses to increase managerial utility by first committing more capital before paying in wages. In the asymmetric information model, the wedge between the marginal cost of capital and wages are amplified due to the presence of the information rent and managerial preference for capital. For every unit of increase in capital allocated to the manager with the high quality project, headquarters increases the utility of the manager by $\beta\delta t_H$ but only needs to compensate the manager with the low quality project by $\beta\delta t_L$. Therefore, the higher is the spreads in project qualities, $t_H - t_L$, the longer headquarters is able to use capital as a way to induce truth-telling. However, one unit increase in wages paid to the manager of the high quality projects also requires paying the same to the manager of the low quality ones. For high levels of the outside offer u_H, the information rent could potentially be too large. Therefore, the expected NPV of committing to fund both types of projects can become negative for some outside offers, and it may be optimal for the headquarters to decide to fund only low quality projects.

Let NPV_L denote the payoff to headquarters from committing to fund only low quality projects. Note that due to assumption 2, the optimal capital allocation in this case can be obtained from Proposition 1 and is given by $k_L = \delta t_L$. Hence, NPV_L can be derived to be:

$$NPV_L = 0.5(\delta t_L)^2.$$

The ex-ante optimal payoff to the headquarters is then given by:

$$H = \max\{(1-\pi)NPV_L, EH\}. \tag{4}$$

The left panel in Figure 6 shows graphically the Pareto Frontier for the high type in the (blue) solid line, the low type in the (red) dashed line, and the expected frontier in (black) dotted line. Note that headquarters maximizes expected payoff. If the expected payoff from committing to fund both types of projects is below the expected payoff from funding only the low quality projects, headquarters optimally chooses not to fund the high quality projects even though it may be positive NPV. In Figure 6, this can be seen by observing that headquarters will choose to develop only low quality projects where the (black) dotted line intersects the horizontal (black) line that represents $(1-\pi)NPV_L$. The right panel in Figure 6 contrasts the Pareto Frontier under full information with that in the asymmetric information model. The frontier is much lower in the asymmetric information model because the information rent makes it too costly for headquarters to fund some projects that are feasible under full information. In the full information model, headquarters is able to match any outside offer as long as the NPV of the corresponding project is positive. However, under asymmetric information, the cost of inducing truth-telling makes it costly to match these outside offers. As a result, in deciding whether to undertake a project or not, instead of using the NPV of each project type, headquarters uses the expected total NPV over two projects from the direct mechanism, which is given by EH in our model. Proposition 4 formalizes this intuition.

Note that $(1-\pi)NPV_L$ does not depend on the outside offer of the manager with the high quality project, u_H. However, EH, the expected profit from committing to fund both types of projects by inducing truth-telling, is strictly decreasing in u_H in Regions II' and III'. Let $\bar{u}_H$ denote the value of u_H that satisfies the following equation:

$$(1-\pi)NPV_L - EH = 0. \tag{5}$$

Proposition 4 (Threshold Outside Offers) *Denote by $\bar{u}_H^{FB}$ the highest level of outside option that gives non-negative profit under full information. If $\frac{t_H}{t_L}$ satisfies equation (3) and the following condition:*

$$\frac{1}{2\beta}\left[2\beta + \beta^2 - 1\right] > \frac{t_H}{t_L}(1 - \frac{1}{2\epsilon_1}), \tag{6}$$

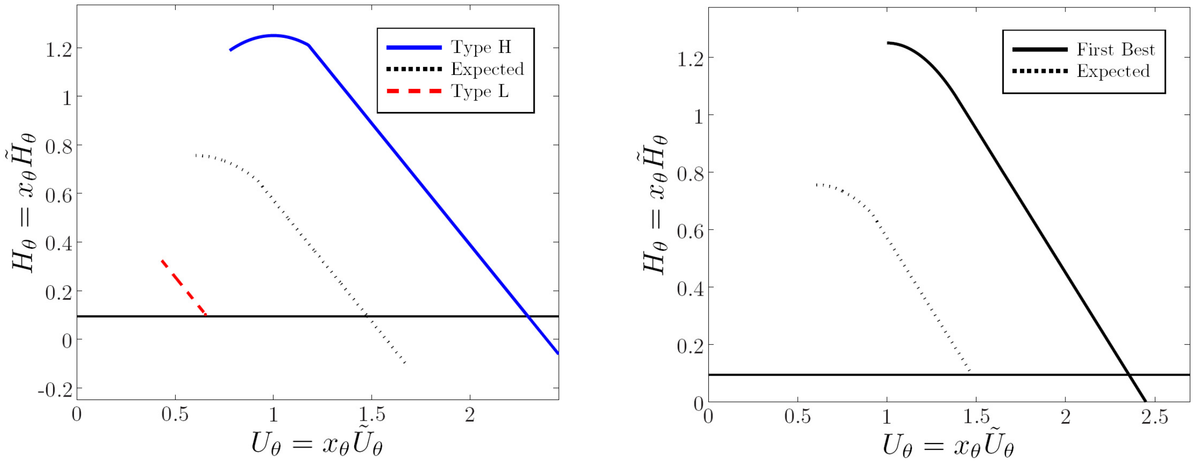

Figure 6: **Payoff to Headquarters and Pareto Frontier Under Asymmetric Information.** The left panel shows Pareto Frontier under asymmetric information and the right panel contrasts with the full information model. For this graph, $\beta = 0.4$, $\pi = 0.5$, and $t_H/t_L = 1.8$ is used.

then there exists $\bar{u}_H \in (0, \bar{u}_H^{FB})$ that satisfies equation (5). For all $u_H \in \mathbf{u}_H$, headquarters commits to screening projects if $u_H \leq \bar{u}_H$, headquarters commits to funding only low quality projects if $u_H > \bar{u}_H$. That is, headquarters' optimal policy is given by Proposition 2 when $u_H \leq \bar{u}_H$; otherwise, headquarters allocates:

$$k_L = \delta t_L \quad ; \quad w_L = 0\,;$$
$$k_H = 0 \quad ; \quad w_H = 0.$$

Moreover, $\bar{u}_H$ is decreasing in $\frac{t_H}{t_L}$.

Proof: See Appendix.

Proposition 4 summarizes the optimal policy of the headquarters. Equation (6) ensures the existence of $\bar{u}_H$ by requiring that the ratio between the high and low quality project ideas is not too high. Note that if $\frac{t_H}{t_L}$ satisfies equation (6), then the headquarters is at least able to match $u_H = \beta\delta^2 t_H^2$, the level of managerial perks from running the project at the unconstrained first best level. As far as the parameter for manager perks, β, is small, equation (6) is not a strong restriction.

Due to the information rent and distortions in capital allocation associated with the asymmetric information, headquarters is not able to match the level of outside offer that it was able to match under the first best. Therefore, $\bar{u}_H < \bar{u}_H^{FB}$. The headquarters is able to undertake any project as long as the NPV is positive in the first best scenario. However, in the asymmetric information model, in evaluating whether to undertake the high quality project or not, it is not the NPV of the high quality project that matters but the expected total NPV of both projects given by EH. Hence, the headquarters may forgo some positive NPV projects because of the information rent that is required to induce truth-telling.

In standard models of asymmetric information between headquarters/principal and the division manager/agent, firms can commit to funding all types of projects and subsequently retain all types of managers because under-investment in all qualities of projects also keeps the information rent low. In contrast, in this model, the information rent can become too high for the headquarters to commit to funding all types of projects. In Bernardo et al. (2001), there is no capital allocated to low quality projects. However, all managers stay within the firm because all participation constraints are satisfied. In this model, when the headquarters decides to allocate no capital to the division manager with high quality projects, the manager leaves the firm and develops the project with the venture capitalist.

3 Venture Capital Markets

In this section, the venture capital market is introduced and the outside offer, u_θ, is endogenized. In the previous section, it was assumed that if projects are not undertaken by headquarters, managers may seek financing from a venture capitalist (VC). Hence, it is important to establish how VCs are different from a multi-division incumbent firm. Section 3.1 lays out the assumptions on the VC market. Section 3.2 discusses how projects are developed under the VC and endogenizes u_θ. Section 3.3 introduces product market competition and shows how the internal capital allocation decision of the incumbent firm is affected.

3.1 Assumption on Venture Capitalists

The model makes some assumptions on the VC market based on recent surveys of the structure and governance of venture-backed organizations and the contract structure between the VC and the entrepreneurs. Assumption 3 lists the characteristics of the VC market assumed in the model.

Assumption 3 The VC market has the following characteristics:

1. The VC can verify the true project quality at a cost c;
2. VC's active involvement in the development of projects may be value-reducing or value-adding;
3. VC and entrepreneur share profits according to the sharing rule $\eta \in (0, 1)$, i.e., VC obtains $1 - \eta$ share of the profits and the entrepreneur η share of the profits.

The VCs generally perform an extensive evaluation of a potential project before agreeing to disburse funding. This evaluation process may involve analyzing the financial aspect of the partnership to hiring and seeking experts' advice from the industry. Characteristic 1 of the VC may be interpreted as the cost of performing due diligence on the part of the VC before funding a project.

It is also widely documented that VCs are generally actively involved in different stages of the project development. From monitoring the progress of the venture to recruiting key employees, VCs help the venture grow internally. While this participation of the VC may add value to the investment, not all ventures succeed. The VC's close monitoring could also lead to loss in productivity as entrepreneurs are more involved in preparing reports to meet targets established by the VC and are less devoted to working toward the long term health of the venture. Based on the complementarity in the industry expertise of the VC and the entrepreneurs' idea, a VC may or may not be able to set the right target or hire a suitable workforce. Characteristic 2 in the assumption captures this aspect of the VC. This part of assumption 3 could also be replaced by a search friction in meeting a VC with the right expertise. Qualitatively, it does not change the results of the model. Section 3.2 discusses in detail how the entrepreneur's project quality is affected under the VC management.

The model abstracts from the capital structure of the VC backed start-up. Since in the model there is no asymmetric information between the VC and the entrepreneur after due diligence is performed, the form of capital structure does not determine the success or the size of the venture. The capital structure also does not affect the division manager's decision to leave the headquarters. The profit sharing parameter η is taken as exogenous in this model. In Inderst and Muller (2004), this parameter depends on the bargaining powers of the entrepreneur and the VC.

3.2 Project Development Under Venture Capitalist

This section discusses the timing and contract structure of project development under the VC. After his project idea has been allocated no capital by the headquarters, the division manager leaves the incumbent firm, and meets a VC. The model assumes no search friction in meeting with a VC. In what follows, the division manager who leaves the incumbent firm to develop projects with a VC will be referred to as an entrepreneur.

After performing its due diligence, the VC decides whether to undertake the project or not. If the VC decides to enter the joint venture, a match specific parameter ρ is drawn from the distribution $F_\theta(\rho)$ where $\theta \in \{H, L\}$ and $\rho \in [\underline{\rho}, \bar{\rho}]$. The match specific parameter ρ can be thought of as resulting from the complementarity between the VC's expertise and the entrepreneur's project idea. The second characteristic of assumption 3 implies that ρ may be positive or negative. Therefore, $\underline{\rho} < 0 < \bar{\rho}$. For simplicity, it is assumed that ρ affects both low and high quality projects additively. However, the model assumes that $F_H(\rho)$ is different from $F_L(\rho)$. Let $t_{\theta\rho}$ denote the new quality of the project t_θ after the match specific parameter ρ is drawn and is given by:

$$t_{\theta\rho} = t_\theta + \rho.$$

Let $V(t_{\theta\rho})$ denote the cash flow under the management of the VC when the project quality under the headquarters is t_θ and match specific parameter is ρ. $V(t_{\theta\rho})$ is then given by the following equation:

$$V(t_{\theta\rho}) = \delta t_{\theta\rho} k - 0.5k^2. \tag{7}$$

Since both the entrepreneur and the venture capitalist can observe the match specific parameter ρ, the contract can be made contingent on ρ. After observing ρ, the VC chooses $k(t_{\theta\rho})$ to maximize the total cash flow.

$$k(t_{\theta\rho}) = \arg\max_{\tilde{k}(t_{\theta\rho})} [\delta t_{\theta\rho} \tilde{k}(t_{\theta\rho}) - 0.5\tilde{k}(t_{\theta\rho})^2]$$

Since ρ can potentially be low enough to allow $t_{\theta\rho}$ to be negative, for some values of ρ, it may be worthwhile to allocate no capital inside the venture. The model is static and hence does not consider the dynamic aspect of how failed entrepreneurs may return to their previous employment in incumbent firms.

The optimal capital allocation inside the venture is given by:

$$k(t_{\theta\rho}) = \max\{0, \delta(t_\theta + \rho)\}, \tag{8}$$

Hence, the VC will undertake the project as long as $\rho > -t_\theta$. By assumption 3, since the VC pays a sunk cost c to perform due diligence, the expected payoff to the VC of committing to fund an entrepreneur whose project quality under the headquarters is t_θ is given by:

$$\int_{-t_\theta}^{\bar{\rho}} (1-\eta)\big[V(t_{\theta\rho}) - c\big] dF_\theta(\rho).$$

The model retains the limited liability aspect of the entrepreneur and the managerial perks. Hence, the payoff to the entrepreneur with project quality t_θ is also given by:

$$u_\theta = \max\left\{0, \int_{-t_\theta}^{\bar{\rho}} \eta\big[V(t_{\theta\rho}) - c\big] + \beta\delta t_{\theta\rho} k(t_{\theta\rho}) dF_\theta(\rho)\right\},$$

Note that the payoff to the entrepreneur takes into account payments to the VC for the cost of initial evaluation of the project. It is common in venture contracts that VCs charge management fees besides taking on equity for bringing their expertise and their industry contacts. Therefore, the cost c may be interpreted as paying management fees or organizational expenses to the VC. Alternatively, c could also be considered as part of the convertible preferred stock where the investor recovers its costs and also realizes a portion of the gain in value through equity. While excluding c from the payoff to the entrepreneur does not change the results qualitatively, it simplifies the algebra in solving the model.

Assumption 4 The VC's cost of due diligence c satisfies the following:

$$c \geq \frac{1}{2}\left[\pi\delta^2 t_H^2 + (1-\pi)\delta^2 t_L^2\right]. \tag{9}$$

Assumption 4 ensures that headquarters will not be able to hire VCs to screen for projects inside the firm. It also implies that VCs will only enter an industry in which the expected payoff is higher than the cost of due diligence. Recall that in Section 2, assumption 2 normalizes $u_L = 0$. In this section, this assumption is replaced as follows.

Assumption 2′ The expected payoff from low quality project under VC management satisfies the following:

$$\left[\int_{\underline{\rho}}^{\bar{\rho}} V(t_{L\rho}) dF_L(\rho)\right] \leq 0. \tag{10}$$

It is realistic to assume that the VC can add more value to the project if it is of high quality. However, equation (10) assumes a stronger condition that the project under VC management is not profitable if it is of low quality. Relaxing this assumption to allow low quality projects to be profitable does not change the results of the model qualitatively. Since by assumption 3 VC can distinguish different qualities of projects after due diligence, they will not fund low quality projects of managers from incumbent firms. This gives zero expected payoff to the manager with the low quality project, i.e., $u_L = 0$ and is consistent with assumption 2.

Therefore, expected outside options for division managers with project qualities t_H and t_L before leaving the headquarters are given by:

$$\begin{aligned} u_L &= 0, \\ u_H &= \max\left\{0, \int_{-t_H}^{\bar{\rho}} \eta\big[V(t_{H\rho}) - c\big] + \beta\delta t_{H\rho} k(t_{H\rho}) dF_H(\rho)\right\}. \end{aligned}$$

Note that utility to the entrepreneur has the same functional form as that of the manager under the headquarters. The outside option treats the entrepreneur share of the cash flow analogously to the wage payments under the headquarters and the managerial preference over the project quality and capital enter the utility function analogously.

The timing of the contract from the model in section 2 is augmented to include the VC market as follows: At date 0, the VC market offers a sharing rule and an allocation rule, $\{\eta\big[V(t_{\theta\rho}) - c\big], k(t_{\theta\rho})\}$, which allows headquarters and division managers to realize u_θ. Note that it is always the case that $u_L = 0$ due to assumption 4. At date 1, headquarters proposes a compensation package and capital allocation rule, $\{w(t_\theta, \hat{t}_\theta), k(\hat{t}_\theta)\}$. At date 2, the manager privately observes the quality t_θ of the project and if $u_\theta \leq U(\hat{t}_\theta)$, he stays inside the incumbent firm and reports $\hat{t}_\theta$ to the headquarters and the timing continue as noted in section 2, but if $u_\theta > U(\hat{t}_\theta)$, he decides to leave the incumbent firm to become an entrepreneur and meets a VC. At date 3, if the VC decides to develop the project after performing due diligence, both the VC and the entrepreneur observe the match specific parameter ρ, and the VC allocates $k(t_{\theta\rho})$. At date 4, project cash flows are realized and η and $1-\eta$ shares of the cash flow given by equation (7) are distributed to the VC and the entrepreneur. Note that the VC makes the move first in announcing its allocation schedule as the headquarters' decision to retain managers is dependent on what the VC can offer.

3.3 Product Market Competition

In this section, product market competition is introduced. If a new venture is successfully implemented, the revenue to the incumbent firm may decrease depending on the structure of the product market competition. Recall that in the cash flow function, $\delta t_\theta k$, δ may be interpreted as publicly known industry returns or prices and is redefined as follows:

$$\delta = \frac{\delta_0}{n^\alpha},$$

where $n > 0$ is the number of firms and $\alpha \geq 0$ is a parameter for product market competition.

The expression for δ implies an inverse relation between prices and the number of firms. When $\alpha = 0$, the number of firms in the market does not affect prices and when α is positive, the number of firms in the market affects prices. For a fixed number of firms, n, a higher α implies a lower price or δ. Hence α can be interpreted as the degree of competition in the market: a higher α implies a more concentrated product market whereas a lower α implies a more competitive product market. Note that α also summarizes other aspects of the product market that the model does not consider explicitly such as substitutability of differentiated products. δ_0 may be interpreted as the industry specific productivity parameter that is common to all firms in the industry.

When the new venture is successfully implemented by a venture, a new start-up enters the product market, and the new δ denoted by δ' is given by:

$$\delta' = \frac{\delta_0}{(n+1)^\alpha}.$$

Denote by NPV_L^C cash flows to the headquarters when it develops only low quality projects in the presence of product market competition. Since the entrepreneur has $1 - F(-t_H)$ probability of successfully developing the project with a VC, NPV_L^C is given by:

$$NPV_L^C = F(-t_H)\Big[\delta t_L k_L - 0.5(k_L)^2\Big] + (1 - F(-t_H))\Big[\delta' t_L k_L - 0.5k_L^2\Big].$$

Hence, the payoff to the headquarters from equation (4) in section 2.3 is now modified as follows:

$$H = \max\{0, (1-\pi)NPV_L^C, EH\}.$$

Note that as in section 2.3, $(1-\pi)NPV_L^C$ does not depend on the outside offer of the manager with high project quality, i.e., on u_H. However, the expected profit from committing to fund both types of projects by inducing truth-telling, EH, is strictly decreasing in u_H in Regions II' and III'. Let $\bar{u}_H^C$ denote the value of u_H that satisfy the following equation:

$$(1-\pi)NPV_L^C - EH = 0. \tag{11}$$

It is important to note that introducing a VC market changes the internal capital allocation decision of the incumbent firm through product market competition. More importantly, the threshold outside offer $\bar{u}_H$ now changes with fundamental parameters in the VC market and in the structure of the product market competition, α and n. In order to express u_H in closed form, in what follows, it is assumed that $F_H(\rho)$ follows a uniform distribution with $\underline{\rho} = \mu - \frac{b}{2}$ and $\bar{\rho} = \mu + \frac{b}{2}$, where $b > 0$.

Assumption 5 $F_H(\rho) \sim U[\mu - b/2, \mu + b/2]$

Note that a uniform distribution function $F^1(\rho)$ with mean μ_1 first order stochastically dominates $F^2(\rho)$ with μ_2 if $\mu_1 > \mu_2$. Since a distribution function with a higher μ corresponds to a higher average productivity parameter $t_H\rho$ under the VC, in the rest of the paper, a higher μ signifies a more efficient VC market. Under assumption 5, the outside option for the manager with a high quality project can be rewritten as follows:

$$
\begin{aligned}
u_H &= \max\{0, \left[\left(\frac{\eta}{2}+\beta\right)\delta'^2 \int_{-t_H}^{\infty} (t_H+\rho)^2 dF_H(\rho) - \eta c\right]\}, \\
&= \max\{0, \left(\frac{\eta}{2}+\beta\right)\delta'^2 \left\{\frac{t_H^2}{b}(\mu+b/2) + \frac{t_H}{b}(\mu+b/2)^2 + \frac{1}{b}\left[\frac{(\mu+b/2)^3+t_H^3}{3}\right]\right\} - \eta c\}.
\end{aligned}
$$

Proposition 5 (Comparative Statics) *Let $\bar{u}_H^C$ denote the threshold level of outside offer that incumbent firms are willing to match under product market competition. For $0 < \alpha < 1$, the following holds:*

(i) $\bar{u}_H^C > \bar{u}_H$.

(ii) $\bar{u}_H^C$ is weakly increasing in α and μ, weakly decreasing in n, but does not vary with η.
In Region III',

$$
\frac{\partial \bar{u}_H^C}{\partial \mu} = \frac{\frac{(1-\pi)t_L^2\left[\delta^2-\delta'^2\right]}{2b}}{\pi} > 0.
$$

In Region II',

$$
\frac{\partial \bar{u}_H^C}{\partial \mu} = \frac{\frac{(1-\pi)t_L^2\left[\delta^2-\delta'^2\right]}{2b}}{\pi\left[\frac{\bar{u}_H^C}{\beta^2\delta^2 t_H^2} - \frac{1}{\beta}\right] + (1-\pi)\left(\frac{t_L}{t_H}\right)} > \frac{\frac{(1-\pi)t_L^2\left[\delta^2-\delta'^2\right]}{2b}}{\pi} > 0.
$$

$\frac{\partial^2 \bar{u}_H^C}{\partial \mu^2} = 0$ *everywhere.*

(iii) Entrepreneur's expected payoff from starting a venture, u_H, is strictly increasing in μ and t_H but decreasing in α and n. $\frac{\partial u_H}{\partial \mu} > 0$, $\frac{\partial^2 u_H}{\partial \mu^2} \geq 0$.

Proof: See Appendix.

The intuition for Proposition 5 is straightforward. When there is product market competition, prices decrease after a new start-up enters the industry. Therefore, the benefit of developing only low quality projects decreases. As a result, incumbent firms are more willing to match the outside offer to keep division managers inside the firm. Hence, $\bar{u}_H^C > \bar{u}_H$. Similarly, a higher α implies a less competitive product market, i.e., prices decrease more due to a new entrance. A higher μ also implies a higher expected productivity after signing up with a VC and hence a higher probability of the venture succeeding. Therefore, the cost of letting division managers leave increases and incumbent firms are more willing to retain managers. Note that when efficiency in the VC market is too low, the outside options to division managers are also low. In this case, the headquarters does not need to match the outside offers as the capital allocated already gives the managers a utility higher than his outside offer due to managerial preference for capital. A slight increase in α or μ in this region does not change the optimal policies inside the headquarters and $\bar{u}_H^C$ does not increase. Therefore, $\bar{u}_H^C$ is weakly increasing in both α and μ.

Proposition 5 also shows that the level of outside offers firms are willing to match, $\bar{u}_H^C$, is decreasing in n. This is in line with the intuition that prices decrease more as industries move from monopolistic competition with a few firms to one with more firms. In other words, if there were a small number of firms initially, prices would be affected more by a new entrance. If the industry is close to perfect competition with many firms initially, a new entrance has little effect on prices. Hence, the loss in revenue from developing only low quality projects is decreasing in n and so is $\bar{u}_H^C$. If η increases, the outside offer u_H of the division manager with high project quality increases. However, it does not affect the optimal policy of the firm on the level of the outside offer that they are willing to match since the entrepreneur's higher share of profits in the joint venture does not affect the probability of success of the venture and hence does not affect the inumbent firms' profits.

Part (ii) of Proposition 5 shows how the optimal threshold changes with efficiency in the VC market. Recall that in Region II' the incumbent firm uses capital allocation to screen different qualities of projects, but in Region III' it uses wage payments. As explained in section 2, the cost of the information rent to the headquarters in Region III' is higher. Therefore, the rate of increase in the threshold level of outside offers that headquarters can match is lower in Region III'. In other words, when headquarters is using capital allocation to retain managers, it is able to match higher VC offers than when it is using only wages.

The last part of Proposition 5 shows how division managers' outside offers change with parameters in the VC market. An increase in VC efficiency or a higher initial quality of project directly increases the productivity of the new venture and therefore increases the total profit of the new venture and the entrepreneur's share. Similarly, an increase in product market competition or the number of firms implies that the prices for all firms in the industry are lower; thus the total expected profit of the new venture is lower and so is the entrepreneur's share. Proposition 5 also shows that the increase in the division manager's outside option is convex in the efficiency of the VC market.

Corollary 6 (Venture creation) *In an imperfectly competitive industry,* $0 < \alpha < 1$*, a more efficient VC market does not always lead to the high quality projects being undertaken with VC. Depending on the level of initial* u_H*, improving the VC market leads to the following:*

(a) If $u_H > \bar{u}_H$*, i.e., if the project is initially done outside, improving the efficiency in the VC market leads to the projects being done with VC.*

(b) If $u_H \leq \bar{u}_H^C$*, improving efficiency in the VC market may or may not lead to the projects being done with a VC. If* $u_H(1 + \frac{\partial u_H}{\partial \mu}) \leq \bar{u}_H^C(1 + \frac{\partial \bar{u}_H^C}{\partial \mu})$*, then the high quality project is done inside incumbent firms. If not, the high quality project is done with a VC.*

Proof: See Appendix.

Corollary 6 summarizes the effect of the efficiency in the VC market on the creation of new ventures. The first part of Corollary 6 says that for a given level of product market competition, the headquarters' decision to let go of a manager with the high quality project is monotonic in the efficiency of the VC market. As shown in Proposition 5, the increase in the threshold level of outside offers is concave in VC efficiency whereas the increase in managers' outside offer is convex. Therefore, $\bar{u}_H$ and u_H can cross exactly once. In other words, division managers' outside offers increase faster than the level of offers that incumbent firms are willing to match as efficiency in the VC market improves. Hence, if a project is initially developed outside the incumbent firm, an increase in VC efficiency increases the outside offer more than the amount firms are able to match.

The second part of Corollary 6 shows that in the presence of product market competition, when projects are initially developed by the incumbent firm, improving the efficiency in VC markets has ambiguous effects on the rate of creation of the new ventures. There are two effects associated with improving the VC market: one direct effect is that it increases u_H; the other indirect effect comes from the product market competition that increases $\bar{u}_H^C$. Since the incumbent firm stands to lose revenue with a higher probability as the VC market improves, it is willing to match a higher level of threshold offer. Hence, $\bar{u}_H^C$ increases. Whether or not projects will be undertaken inside the incumbent firm or outside with a VC depends on the horse race between the marginal increase in the outside offers and the threshold offers.

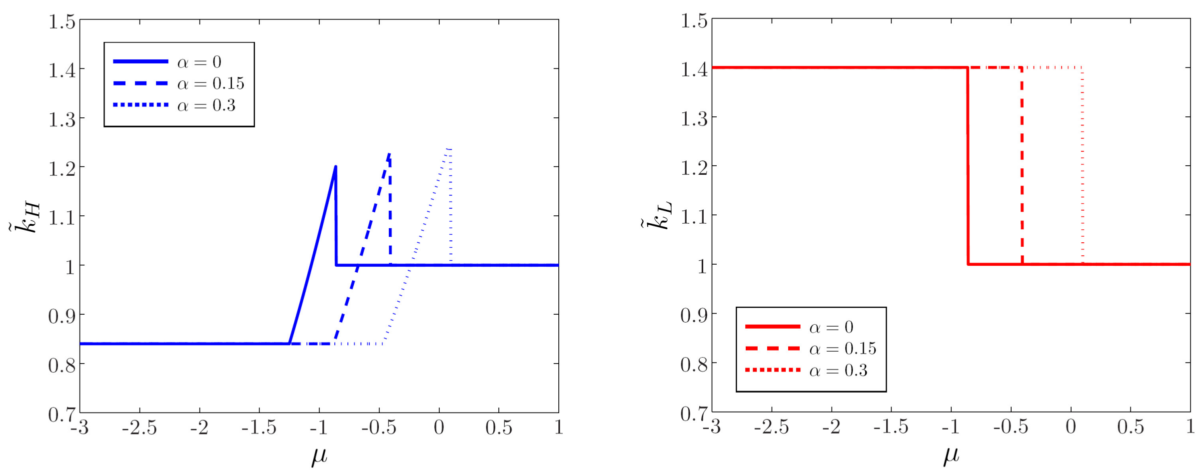

Figure 7: **Optimal Capital Allocation for Different Degrees of Product Market Competition and Efficiency in the VC Market.** The left panel shows the optimal capital allocation for high quality projects both under the incumbent firm and under the VC management. The right panel shows the optimal capital allocation for low quality projects under the incumbent firm.

Figure 7 shows graphically the optimal capital allocations both inside and outside headquarters for different degrees of product market competition, α, and efficiency in the VC market, μ. The left panel shows the optimal capital allocation in high quality projects while the right panel shows the case for low quality projects.

In the left panel, high quality projects are developed with VCs in the region where capital allocated is flat in μ and is equal to one. Since there is no asymmetric information between the VC and the entrepreneur, under the VC management, the optimal capital allocated is always equal to the unconstrained first best given the productivity $t_{H\rho}$. To the left of this region, projects are developed inside incumbent firms. Initially for low levels of VC efficiency, the VC market offers are too low. Headquarters does not need to try to retain managers in this region and hence does what is optimal for the firm. Due to asymmetric information inside headquarters, it is optimal to under-invest in this region. However, as the VC market improves, headquarters attempts to retain managers by allocating more capital. The highest peaks of k_H in the graph correspond to different cut-offs in outside VC offers that headquarters are willing to match. As α increases, the cut-off outside offer, $\bar{u}_H^C$, increases and the capital allocated increases as well. Note that as α increases, the curve for k_H moves to the right. In other words, as α increases, the under-investment region also increases. This is due to the fact that an increase in α also decreases δ'; that is, more competitive product markets imply a lower revenue or return for the new entrance as well. Hence, outside VC offers, u_H, are lower as well for higher α. Therefore, headquarters is able to retain the managers longer inside the firm and a higher VC efficiency is required in order for the headquarters to decide not to retain the manager.

In the right panel, the region where $k_L = 1$ corresponds to the case where headquarters decides to develop only low quality projects. In this case, since the headquarters knows that the only types of managers left inside the firms are those with low quality projects, it allocates the optimal unconstrained first best level of investment. However, to the left of this region, there is over-investment in low quality projects due to the information rent that headquarters needs to pay to the manager with low project quality in order to induce truth-telling. As a higher α implies that headquarters is more willing to retain managers and develop both low and high quality projects, the information rent to the manager with low quality projects persists for higher levels of VC efficiency.

4 Conclusion

This paper studies how division managers' access to venture capital markets affects the internal capital allocation decision of a multi-division firm. Division managers with high quality projects have better expected payoffs from signing up with a VC than managers with low quality projects. In the presence of asymmetric information between headquarters and division managers and managerial preference for capital, the optimal contract involves both under- and over-investment compared to the unconstrained first best level of investment. The under-investment regions correspond to low levels of efficiency in the VC market. When the VC market is inefficient, the expected payoffs to managers from starting a venture with a VC is low. In this region, headquarters need not try to retain division managers by matching the offer from the VC market since the managerial utility from the allocated capital inside the incumbent firm is already higher than the VC offer. There is under-investment in high quality projects inside the incumbent firm in this region due to the presence of asymmetric information. As the VC market improves, headquarters needs to match the outside offer by either allocating more capital or by raising wages. Due to the information rent required to induce truth-telling of private information, it is cheaper for the headquarters to over-invest instead of paying more in wages. Hence, the over-investment arises in incumbent firms correspond to relatively higher levels of efficiency in the VC market.

Due to the cost of the information rent that arises from the asymmetric information problem, incumbent firms are limited in their ability to match the offers from the VC. The optimal threshold level of outside offers that incumbent firms can match is endogenously determined by the spread in the project qualities, the severity of the asymmetric information problem inside the firm, and the degree of managerial preference for capital. Hence, even though some high quality projects are positive NPV, due to the cost of information rent, incumbent firms may not be able to commit to funding those projects. The paper also shows that when the VC market is sufficiently efficient, headquarters retain managers of high quality projects by first allocating more capital to their project ideas before raising wages. The incentive to retain managers is stronger in more monopolistic industries as the incumbent firm stands to lose more revenue if projects are successfully developed by the VC. Therefore, headquarters raises the level of the VC offer that they are willing to match in order to retain managers of high quality projects. The paper also derives novel empirical implications. Due to the feedback of product market competition into the internal capital allocation decision of the incumbent firm, the efficiency of the VC market has ambiguous effects on the creation of new ventures. Improving the efficiency of the VC market in a highly monopolistic industry may induce incumbent firms to aggressively retain managers of high quality projects by over-investing in their ideas and may not lead to new venture creation.

5 Appendix

Proof of Proposition 1

Let λ_1 and λ_2 denote the multipliers associated with constraint (i) and (ii) respectively. First order conditions are given by:

$$\begin{aligned} \delta t_\theta - k_\theta^{FB} + \lambda_1 \beta \delta t_\theta &= 0 \\ -1 + \lambda_1 + \lambda_2 &= 0 \end{aligned}$$

There are four cases depending on which constraints bind.

Case 1: (i) does not bind. (ii) does not bind. This is not optimal as reducing wages until (i) or (ii) binds make the headquarters strictly better off while still making sure that the manager stays with the firm.

Case 2: (i) does not bind. (ii) binds. $\lambda_1 = 0$, $\lambda_2 > 0$. Then $\mu = 1$, and $k_\theta^{FB} = \delta t_\theta$ and $w_\theta = 0$

Case 3: (i) binds. (ii) binds. $w_\theta + \beta \delta t_\theta k_\theta = 0 \Longrightarrow k_\theta^{FB} = \frac{u_\theta}{\beta \delta t_\theta}$, and $w_\theta = 0$.

Case 4: (i) binds. (ii) does not bind. $\lambda_1 > 0$, $\lambda_2 = 0$. Then $\lambda_1 = 1$ and $k_\theta^{FB} = \delta t_\theta + \beta \delta t_\theta$, $w_\theta^{FB} = u_\theta - \beta \delta t_\theta k_\theta^{FB}$

Plugging the optimal capital and wages back into Constraint (i) and (ii), the cut-off u_θ are as follows:

$$\begin{aligned} &\text{Case 2:} && 0 \leq u_\theta < \beta \delta^2 t_\theta^2 \\ &\text{Case 3:} && \beta \delta^2 t_\theta^2 \leq u_\theta \leq \beta \delta^2 t_\theta^2 + \beta^2 \delta^2 t_\theta^2 \\ &\text{Case 4:} && \beta \delta^2 t_\theta^2 + \beta^2 \delta^2 t_\theta^2 < u_\theta \leq 0.5(\delta t_\theta + \beta \delta t_\theta)^2 \end{aligned}$$

The cut-off value of u_H at which headquarters break even can be found by plugging the optimal policy into the objective function. The optimal polices and the payoffs can then be rewritten using the following normalization. Let $x_\theta = \beta \delta^2 t_\theta^2$; $\tilde{k}_\theta = \frac{k_\theta}{\delta t_\theta}$; $\tilde{w}_\theta = \frac{w_\theta}{x_\theta}$; $\tilde{U}_\theta = \frac{U_\theta}{x_\theta}$; $\tilde{u}_\theta = \frac{u_\theta}{x_\theta}$, $\tilde{H}_\theta = \frac{H_\theta}{x_\theta}$.

Proof of Proposition 2

To simplify notation,

$$w(t_\theta, \hat{t}_\theta) = w_\theta \text{ if } t_\theta = \hat{t}_\theta.$$

By the revelation Principle, we can restrict our attention to direct revelation mechanism without loss of generality. Hence, denote $k(\hat{t}_\theta) = k(t_\theta) = k_\theta$.

The headquarters' expected payoff from committing to fund both types of projects is given by:

$$EH = \max_{\{w_\theta, k_\theta\}_{\theta=H,L}} \pi\big[\delta t_H k_H - 0.5k_H^2 - w_H\big] + (1-\pi)\big[\delta t_L k_L - 0.5k_L^2 - w_L\big],$$

subject to

$$\begin{aligned}
&\text{(PCH)} \qquad w_H + \beta\delta t_H k_H \geq u_H, \\
&\text{(PCL)} \qquad w_L + \beta\delta t_L k_L \geq 0, \\
&\text{(ICH)} \qquad w_H + \beta\delta t_H k_H \geq \beta\delta t_H k_L, \\
&\text{(ICL)} \qquad w_L + \beta\delta t_L k_L \geq \beta\delta t_L k_H, \\
&\text{(LL)} \qquad k_\theta \geq 0, w_\theta \geq 0, \theta \in \{L, H\}.
\end{aligned}$$

- PCH may or may not bind.

- PCL never binds. Since it is always optimal to invest positive amount of capital ($k_L \geq 0$) and $w_L \geq 0$

- ICH never binds. $k_H \geq k_L$

- ICL always binds. Since $k_H \geq k_L$ always, in order for ICL to hold, $w_L > 0$. If ICL does not bind, w_L can be lowered until ICL binds, managers will still report the true project quality and headquarters will be strictly better off. Therefore,

$$w_L = \beta\delta t_L(k_H - k_L) \tag{12}$$

- LL
  - wages for low type does not bind. From equation 12, and $k_H > k_L$, $w_L > 0$
  - $k_\theta > 0$ for $\theta = H, L$

Hence Headquarters' problem can now be rewritten as follows:

$$EH = \max_{\{k_H, k_l, w_H\}} \pi\big[\delta t_H k_H - 0.5k_H^2 - w_H\big] + (1-\pi)\big[\delta t_L k_L - 0.5k_L^2 - \beta\delta t_L(k_H - k_L)\big],$$

subject to

$$\begin{aligned}
&\text{(PCH)} \quad w_H + \beta\delta t_H k_H \geq u_H, \quad (\lambda) \\
&\text{(LL)} \qquad\quad w_H \geq 0. \qquad\quad (\mu)
\end{aligned}$$

FOCs w.r.t k_H, k_L, and w_H

$$\begin{aligned}
\pi\big[\delta t_H - k_H\big] - (1-\pi)\big[\beta\delta t_L\big] + \lambda\beta\delta t_H &= 0 \\
\delta t_L - k_L + \beta\delta t_L &= 0 \\
-\pi + \lambda + \mu &= 0
\end{aligned}$$

Suppose that $\frac{t_H}{t_L}$ satisfies the quality premium: $\frac{t_H}{t_L} - 1 > \frac{\beta}{\pi}$

- Case 1: both PCH and LL do not bind.

  This cannot happen as w_H can be lowered until LL or PCH binds. Managers will still report the true project quality and Headquarters will be made better off. Mathematically, this means that both λ and μ cannot be zero at the same time, which is obvious by the FOC conditions.

- Case 2: PCH does not bind, but LL binds.

  For $u_H < \beta\delta t_H(\delta t_H - \frac{(1-\pi)}{\pi}\beta\delta t_L)$

  From FOC, $\lambda = 0$ and $\mu = \pi$

$$\begin{aligned}
k_H^* &= \delta t_H - \frac{(1-\pi)}{\pi}\beta\delta t_L \\
k_L^* &= (1+\beta)\delta t_L \\
w_H^* &= 0 \\
w_L^* &= \beta\delta t_L(k_H^* - k_L^*)
\end{aligned}$$

- Case 3: both PCH and LL binds.

  For $\beta\delta t_H(\delta t_H - \frac{(1-\pi)}{\pi}\beta\delta t_L) \leq u_H \leq \beta\delta t_H((1+\beta)\delta t_H - \frac{(1-\pi)}{\pi}\beta\delta t_L)$

  $\lambda > 0$ and $\mu > 0$

$$\begin{aligned}
k_H^* &= \frac{u_H}{\beta\delta t_H} \\
k_L^* &= (1+\beta)\delta t_L \\
w_H^* &= 0 \\
w_L^* &= \beta\delta t_L(k_H^* - k_L^*)
\end{aligned}$$

- Case 4: PCH binds, but LL does not bind.

  For $u_H > \beta\delta t_H((1+\beta)\delta t_H - \frac{(1-\pi)}{\pi}\beta\delta t_L)$

From FOCs, $\mu = 0$ and $\lambda = \pi$

$$\begin{aligned}
k_H^* &= (1+\beta)\delta t_H - \frac{(1-\pi)}{\pi}\beta\delta t_L \\
k_L^* &= (1+\beta)\delta t_L \\
w_H^* &= u_H - \beta\delta t_H k_H^* \\
w_L^* &= \beta\delta t_L (k_H^* - k_L^*)
\end{aligned}$$

The optimal polices and the payoffs can then be rewritten using the following normalization in definition 2.

Proof of Proposition 3

Using optimal policy from *Proof of Proposition 2*, it is straight forward to derive the payoffs.

Proof of Proposition 4

Equation (6) ensures that there exists a $\bar{u}_H > 0$ such that $EH > (1-\pi)NPV_L$. This condition can be derived by substituting $\bar{u}_H = 1 - \epsilon_1$ into $EH > (1-\pi)NPV_L$.

It is sufficient to show that at $\bar{u}_H^{FB}$, the payoff to headquarters from committing to fund both types of projects, EH, is less than zero.

The optimal policy does not depend on u_H in region I'. $\bar{u}_H^{FB}$ can lie either in region II' or III'.

Let u_H^3 denote the cutoff for region III' and is given by:

$$u_H^3 = (1+\beta)\beta\delta^2 t_H^2 - \frac{(1-\pi)}{\pi}\beta^2\delta^2 t_H t_L$$

$\bar{u}_H^{FB}$ lies in region III' if

$$\bar{u}_H^{FB} - u_H^3 = 0.5\delta^2 t_H^2 - \beta^2\delta^2 t_H^2 + \frac{(1-\pi)}{\pi}\beta^2\delta^2 t_H t_L > 0$$

The condition above can be rewritten as follows:

$$\frac{t_H}{t_L} > \frac{(1-\pi)}{\pi}\frac{2\beta^2}{(2\beta^2-1)}$$

If $\frac{t_H}{t_L}$ satisfies the above condition, then $\bar{u}_H^{FB}$ lies in region III'. Otherwise, it lies in region II'

It is straight forward to show that using the policy in regions II' and III' that the payoff to headquarter at $\bar{u}_H^{FB}$ is negative.

Let $F(\bar{u}_H, \frac{t_H}{t_L}) = (1-\pi)NPV_L - EH = 0$.

In region II',

$$\begin{aligned}
\frac{\partial F}{\partial u} &= -\pi\left\{\frac{1}{\beta} - \frac{\bar{u}_H}{\beta x_H}\right\} + (1-\pi)\frac{t_H}{t_L} \\
\frac{\partial F}{\partial \frac{t_H}{t_L}} &= (1-\pi)\frac{\bar{u}_H t_L}{t_H}
\end{aligned}$$

Since $\frac{u_H}{x_H} > 1 - \epsilon_1$, using implicit function theorem,

$$\frac{\partial \bar{u}_H}{\partial \frac{t_H}{t_L}} = (1-\pi)\frac{\bar{u}_H t_L}{t_H}\left[\pi\left\{\frac{1}{\beta} - \frac{\bar{u}_H}{\beta x_H}\right\} - (1-\pi)\frac{t_H}{t_L}\right]^{-1} < 0$$

Similarly, in region III',

$$\begin{aligned}
\frac{\partial F}{\partial u} &= -\pi \\
\frac{\partial F}{\partial \frac{t_H}{t_L}} &= -(1-\pi)x_H\epsilon_1(\frac{t_L}{t_H})^2 - \beta(1-\pi)\delta^2\epsilon_1 \\
\frac{\partial \bar{u}_H}{\partial \frac{t_H}{t_L}} &= -\frac{(1-\pi)}{\pi}x_H\epsilon_1(\frac{t_L}{t_H})^2 - \beta\frac{(1-\pi)}{\pi}\delta^2\epsilon_1 < 0
\end{aligned}$$

Proof of Proposition 5

(i) Recall the $\bar{u}_H^C$ satisfies the following equation:

$$(1-\pi)NPV_L^C - EH = 0$$

It follows directly from the observation that $NPV_L^C < NPV_L$, and that EH is strictly decreasing in u_H in region II' and III'. Let $f(u_H) = EH$. Since $\bar{u}_H$ solves $(1-\pi)NPV_L - EH$; $\bar{u}_H^C$ solves $(1-\pi)NPV_L^C - EH$; $f(u_H)$ is continuous and strictly decreasing in region II' and III' and $NPV_L^C < NPV_L$, $\bar{u}_H^C = f^{-1}((1-\pi)NPV_L^C) > f^{-1}((1-\pi)NPV_L) = \bar{u}_H$.

(ii) Let $G(\bar{u}_H^C, \mu) = (1-\pi)NPV_L^C - EH$. Using Implicit function theorem,

In Region II',

$$\begin{aligned}
\frac{\partial G}{\partial \bar{u}_H^C} &= -\pi\left[\frac{\bar{u}_H^C}{\beta^2\delta^2 t_H^2} - \frac{1}{\beta}\right] - (1-\pi)(\frac{t_L}{t_H}) < 0 \\
\frac{\partial G}{\partial \mu} &= \frac{(1-\pi)t_L^2\left[\delta^2 - \delta'^2\right]}{2b} \\
\frac{\partial \bar{u}_H^C}{\partial \mu} &= \frac{\frac{(1-\pi)t_L^2\left[\delta^2 - \delta'^2\right]}{2b}}{\pi\left[\frac{\bar{u}_H^C}{\beta^2\delta^2 t_H^2} - \frac{1}{\beta}\right] + (1-\pi)(\frac{t_L}{t_H})}
\end{aligned}$$

In Region III',

$$\begin{aligned}
\frac{\partial G}{\partial \bar{u}_H^C} &= -\pi \\
\frac{\partial G}{\partial \mu} &= \frac{(1-\pi)t_L^2\left[\delta^2 - \delta'^2\right]}{2b} \\
\frac{\partial \bar{u}_H^C}{\partial \mu} &= \frac{\frac{(1-\pi)t_L^2\left[\delta^2 - \delta'^2\right]}{2b}}{\pi} > 0
\end{aligned}$$

In region I', $\frac{\partial \bar{u}_H^C}{\partial \mu} = 0$ as in this region EH does not depend on u_H.

Note that $\pi\left[\frac{\bar{u}_H^C}{\beta^2\delta^2 t_H^2} - \frac{1}{\beta}\right] - (1-\pi)(\frac{t_L}{t_H}) < \pi$. Hence,

$$\frac{\partial \bar{u}_H^C}{\partial \mu} = \frac{\frac{(1-\pi)t_L^2\left[\delta^2 - \delta'^2\right]}{2b}}{\pi\left[\frac{\bar{u}_H^C}{\beta^2\delta^2 t_H^2} - \frac{1}{\beta}\right] + (1-\pi)(\frac{t_L}{t_H})} > \frac{\frac{(1-\pi)t_L^2\left[\delta^2 - \delta'^2\right]}{2b}}{\pi} > 0$$

Similarly, $\frac{\partial \bar{u}_H^C}{\partial \alpha} > 0$ in region II' and region III', but in region I', $\frac{\partial \bar{u}_H^C}{\partial \alpha} = 0$

Both NPV_L^C and EH do not depend on the entrepreneur's share of profits, η. Hence $\bar{u}_H^C$ does not depend on η.

(iii) Under the assumption of uniform distribution for ρ, u_H is given by:

$$\left(\frac{\eta}{2}+\beta\right)\delta'^2\left\{\frac{t_H^2}{b}(\mu+b/2)+\frac{t_H}{b}(\mu+b/2)^2+\frac{1}{b}\left[\frac{(\mu+b/2)^3+t_H^3}{3}\right]\right\}$$

Hence, $\frac{\partial u_H}{\partial \mu}$ and $\frac{\partial^2 u_H}{\partial \mu^2}$ are given by

$$\begin{aligned}
\frac{\partial u_H}{\partial \mu} &= \left(\frac{\eta}{2}+\beta\right)\delta'^2\left[\frac{t_H^2}{b}+\frac{2t_H(\mu+b/2)}{b}+\frac{(\mu+b/2)^2}{b}\right] > 0 \\
\frac{\partial^2 u_H}{\partial \mu^2} &= \left(\frac{\eta}{2}+\beta\right)\delta'^2\left[\frac{2t_H}{b}+\frac{2(\mu+b/2)}{b}\right] > 0
\end{aligned}$$

Similarly, $\frac{\partial u_H}{\partial t_H} > 0$ and $\frac{\partial^2 u_H}{\partial t_H^2} > 0$.

Since $\delta' = \frac{\delta_0}{(n+1)^\alpha}$, it is straight forward to show that $\frac{\partial u_H}{\partial \alpha} < 0$ and $\frac{\partial u_H}{\partial n} < 0$.

Proof of Corollary 6

(i) This is straight forward from observing that $\bar{u}_H^C$ and u_H cross only once. Notice that $\frac{\partial \bar{u}_H^C}{\partial \mu} > 0$ but $\frac{\partial^2 \bar{u}_H^C}{\partial \mu^2} = 0$; but $\frac{\partial u_H}{\partial \mu} > 0$, $\frac{\partial^2 u_H}{\partial \mu^2} > 0$.

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