Market Efficiency, Managerial Compensation, and Real Efficiency*

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Abstract

We examine how an exogenous improvement in market efficiency, which allows the stock market to obtain more precise information about the firm's intrinsic value, affects the shareholder-manager contracting problem, managerial incentives, and shareholder value. A key assumption in the model is that stock market investors do not observe the manager's pay-performance sensitivity ex ante. We show that an increase in market efficiency weakens managerial incentives by making the firm's stock price less sensitive to the firm's current performance. The impact on real efficiency and shareholder value varies depending on the composition of the firm's intrinsic value.

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Introduction

Understanding the real effects of stock market prices has been of long-standing concern to financial economists.[1] One important channel through which stock market prices may affect investment decisions is that decision makers within firms are often party to contracts that are contingent on secondary market prices. This is particularly true for managers whose compensation is often directly tied to their firm's share price. Moreover, shareholders who choose the managers' compensation contracts may themselves care for the firm's share price in order to preserve liquidity and to prevent dilution of their ownership stakes. Therefore, the shareholders's choice of compensation contracts and the managers' investment decisions could be affected by the manner in which the stock market aggregates information to determine prices. In this paper, we examine how an exogenous improvement in market efficiency, which allows the stock market to obtain more precise information about the firm's intrinsic value, affects the shareholder-manager contracting problem, managerial incentives, and shareholder value.

To fix ideas, it is helpful to consider a start-up firm that plans to undertake an IPO in the near future. The firm's intrinsic value consists of two components: a "managerial value added" (MVA) component that depends on the effort exerted by the current management to exploit the firm's technology or product market power, and a "core productivity" (CP) component that is independent of current effort and depends on the firm's innate productivity. The stock market prices the firm by aggregating two information signals: a noisy signal on the firm's current performance which is affected by the manager's effort ("performance signal"), and a noisy signal on the firm's underlying productivity that is independent of the manager's effort ("productivity signal"). As the productivity signal cannot be influenced by the manager's effort, an increase in the precision of this signal allows the stock market to obtain a better forecast of firm value. Hence, we interpret the precision of the productivity signal as our measure of market efficiency.

We assume that the shareholders of the firm care for the firm's market value apart from

[1] See Bond et al. (2012) for a survey of this literature.

its long-term intrinsic value. Hence, they have an incentive to influence the stock market's inference of the firm's productivity. A key assumption in our model that distinguishes it from earlier studies on the real implications of market efficiency (e.g., see Paul (1992)) is that the stock market does not observe the manager's pay-performance sensitivity ex ante, and hence, cannot infer the manager's true incentives.[2] Thus, shareholders may surreptitiously provide the manager with high-powered incentives in a bid to boost the performance signal. We believe that this is a realistic assumption because the contract between top executives and their firm may largely be an implicit, self-enforcing one. SEC rules only require firms to disclose their executive compensation policy in broad terms. Although firms are required to disclose stock and option grants *after the fact*, there is no requirement to precisely disclose the performance targets that would trigger fresh grants.[3] However, knowing the actual payment made to an agent is not the same as knowing the rule by which the compensation was calculated, and it does not allow one to infer what the agent's incentives are ex ante (Katz (1991)).

To illustrate the importance of non-observability of compensation contracts, we first analyze a benchmark equilibrium in which the stock market could observe the manager's pay-performance sensitivity. In this benchmark setting, we show that an increase in market efficiency, resulting from an increase in the precision of the productivity signal, unambiguously lowers real efficiency, and decreases shareholder value. Surprising as this may seem, this stark result is actually a reiteration of the key result in Paul (1992), who shows that improvement in market efficiency may worsen real efficiency, because the signals that are most informative about firm value may not be very informative about the manager's effort. In our model, as market efficiency improves, the stock market attaches more weight to the productivity signal and less weight to the performance signal, which is affected by the manager's effort. Thus, all else equal, an increase in market efficiency weakens managerial

[2]This is equivalent to assuming that shareholders may privately renegotiate the manager's compensation contract or offer the manager uncontracted side payments.

[3]The SEC rules on disclosure of executive compensation state that ".... companies are not required to disclose target levels with respect to specific quantitative or qualitative performance-related factors considered by the compensation committee or the board of directors, or any other factors or criteria involving confidential trade secrets or confidential commercial or financial information, the disclosure of which would result in competitive harm to the company."

incentives and increases the cost to shareholders of providing incentives to the manager, because the manager will exert a lower effort for the same level of incentive compensation as before. In equilibrium, shareholders respond to an increase in market efficiency by lowering the manager's pay-performance sensitivity, the manager exerts lower effort, and shareholder value decreases.

In a more realistic setting where the stock market does not observe the manager's pay-performance sensitivity, we find that the impact of enhanced market efficiency on real efficiency and shareholder value is more nuanced, and depends on the composition of the firm's intrinsic value. Specifically, we find that an improvement in market efficiency increases shareholder value in firms where a larger part of the firm's intrinsic value comes from the CP component ("high-CP" firms), that is unrelated to current effort. On the other hand, an improvement in market efficiency decreases shareholder value in firms where the intrinsic value largely comprises the MVA component ("low-CP" firms). In other words, an improvement in market efficiency benefits firms whose current performance is not a good measure of their intrinsic value, but is detrimental for firms whose current performance is a good measure of their intrinsic value.

To understand the intuition for this surprising result, note that the real inefficiency in our model arises because the stock market does not observe the manager's actual effort or pay-performance sensitivity, and prices the firm based on a *fixed* conjecture of the manager's effort. The manager's actual effort affects the stock market's inference of the firm's productivity. Given their desire for a higher stock price, the firm's shareholders have an incentive to induce a higher than efficient level of effort (by providing the manager with high-powered incentives) in a bid to positively influence the stock market's assessment of the firm's productivity.[4] Moreover, the shareholders' incentive to induce overinvestment in effort increases with the relative size of the CP component, because a larger CP component amplifies the effect of the manager's effort on the market's inference. Thus, in equilibrium, shareholders of high-CP firms induce overinvestment in effort, whereas shareholders of low-CP firms induce underinvestment in effort.

[4] The shareholders' incentives are similar to those studied in the signal-jamming literature (e.g., Fudenberg and Tirole (1986), Stein (1988) and Stein (1989)).

Given this differential nature of the investment distortion, an increase in market efficiency has a differential impact on real efficiency and shareholder value depending on the relative size of the CP component. Consider high-CP firms whose shareholders are more likely to induce overinvestment in effort. By making it costlier for shareholders to provide incentives to their managers, an increase in market efficiency corrects this overinvestment problem, and improves shareholder value. By a similar logic, however, an increase in market efficiency worsens the underinvestment problem of low-CP firms, and destroys shareholder value.

Although we have described a firm raising capital in the primary market, our model is valid for any firm with the following key features, which we believe are quite general: First, the welfare of the firm's shareholders (the principals) significantly depends on the firm's stock price. Second, stock price is the primary mechanism through which the firm's shareholders provide incentives to the firm's manager. This is because the manager's tenure is significantly shorter than that of his investment, and any short-term performance measures (burn-rate, investment in R&D, revenues, and operating profits) may be both manipulable and largely uninformative of long-term value. Third, stock market participants do not observe either the manager's effort of his true incentives ex ante.

In order to test our predictions empirically, it is important to identify good proxies for the relative size of the CP component in the firm's intrinsic value. We argue that one such proxy is the firm's market-to-book (M/B) ratio because it captures two key aspects of high-CP firms. First, current performance should not be a good indication of firm value for the high-CP firms because it only reflects the value of the MVA component but not of the CP component. Second, the market price of a high-CP firm should be more sensitive to changes in current performance, because the CP component amplifies the effect of the firm's current performance on the market's inference of the firm's innate productivity. We must also note that the M/B ratio is widely used as a proxy for future investment opportunities, which is consistent with our interpretation of a large CP component. Thus, the key empirical prediction of our model is that an exogenous improvement in market efficiency (as measured by an increase in the precision of the productivity signal) should increase shareholder value of firms with high M/B ratios, but should decrease shareholder value of firms with low M/B ratios. Moreover, an exogenous improvement in market efficiency should lower sensitivity of stock prices to earnings and should lead to a decrease in managers' pay-performance sensitivity, regardless of the firms' M/B ratio.

Our paper is related to the literature that examines the real effects of financial markets (see Bond et al. (2012) for a survey of this literature).[5] This literature identifies two broad channels through which financial markets may affect real decisions. First, managers may learn new information from secondary market prices, and use this information to guide their real decisions (e.g., Dow and Gorton (1997), Boot and Thakor (1997), and Subrahmanyam and Titman (1999)). Second, although managers do not learn new information from market prices, their incentives to take real actions will depend on the extent to which they will be reflected in stock prices. Our paper belongs to the stream of literature which emphasizes this second channel. Two closely related papers in this stream of literature are Fishman and Hagerty (1989) and Paul (1992).

In Fishman and Hagerty (1989), a share-price maximizing manager underinvests because prices do not fully reflect the value of the firm's cash flows. In their model, investors observe only one signal on firm value. Therefore, an increase in disclosure by the firm increases the sensitivity of share price to the manager's investment, and thus improves real efficiency by ameliorating the underinvestment. By contrast, in our paper (and in Paul (1992)), stock market investors observe multiple signals on different aspects of the firm. An improvement in market efficiency actually weakens managerial incentives by causing the stock market to attach a higher weight to the productivity signal, which does not depend on the manager's effort. Unlike Fishman and Hagerty (1989) and Paul (1992), we allow the shareholders to choose the manager's pay-performance sensitivity. Another important distinction is that our modeling of the real side allows for both underinvestment and overinvestment, depending on the composition of the firm's intrinsic value. Hence, the effect of improved market efficiency on real efficiency also varies depending on the composition of the firm's intrinsic value.

[5]Our paper is also related to the broader literature on public information, which argues that the availability of more precise public information may actually lower welfare by causing agents to ignore their own private information (Morris and Shin (2002)), by lowering the incentives of arbitrageurs to acquire information (Grossman and Stiglitz (1976), Grossman and Stiglitz (1980)), or by making insurance unviable (Hirshleifer (1971)). However, unlike our paper, these papers treat the real side of the firm as exogenous.

In terms of the core intuition, the paper closest to ours is Paul (1992) who demonstrates that stock market efficiency need not translate into real efficiency. The core point of his paper is that efficient markets weight information according to its informativeness about asset value, whereas for optimal incentives, information should be weighted according to its informativeness about the manager's actions. This mirrors the finding in our benchmark setting with observable contracts that an increase in market efficiency always worsens real efficiency, and lowers shareholder value due to its adverse impact on managerial incentives. However, once we make the more realistic assumption that the stock market cannot observe the manager's true incentives ex ante, we obtain more nuanced results as we highlighted above.

Our paper is also related to the literature that examines the impact of information revelation on the contracting problem between a principal and an agent. Cremer (1995) and Arya et al. (1998) argue that revelation of the agent's private information may be undesirable because it makes it less credible for the principal to commit to punishing the agent following poor performance. Gigler and Hemmer (2004) show that contracting in an opaque environment and using a risky contract to elicit the manager's private information is preferable to mandating disclosure of the manager's private information when the public signal is highly informative about the manager's action. Unlike in these papers, there is no revelation of private information in our model, and an increase in market efficiency only makes publicly available information more precise. Another related paper is Hermalin and Weisbach (2012), who show that allowing a firm's board of directors to obtain a more precise signal of the CEO's ability may lower firm value by increasing CEO compensation and by inefficiently increasing the rate of CEO turnover. In contrast, the key idea in our paper is that an increase in market efficiency weakens managerial incentives to expend effort, and thus, affects the shareholder-manager contracting problem. Moreover, we show that the effect of an increase in market efficiency on shareholder value is more nuanced, and depends on the composition of the firm's intrinsic value.

Our paper is also related to several papers in the accounting literature that examine the real effects of information disclosure/revelation; e.g., see Kanodia and Lee (1988), Dye and

Sridhar (2002), Sapra (2002), Kanodia et al. (2005), Dye and Sridhar (2007), and Langberg and Sivaramakrishnan (2010). However, none of these papers examine the impact of market efficiency on the contracting problem between the shareholders and the manager, which is the key focus of our paper.

The remainder of the paper is organized as follows. We describe our model and base assumptions in Section 1. We describe the market's valuation of the firm, and characterize managerial incentives in Section 2. We characterize the equilibrium in Section 3, discuss robustness of results in Section 4, and conclude the paper in Section 5.

1 The Model

Consider an all-equity firm that exists for four dates – 0, 1, 2 and 3. At date 0, the firm's current shareholders hire a professional manager and choose his compensation contract. At date 1, the manager takes an action that affects the value of the firm. Date 3 denotes the long term at which the intrinsic value of the firm, denoted $\tilde{V}$, is realized, whereas date 2 represents an intermediate time at which the firm's market value, denoted $\tilde{P}$, is determined by the investors in the stock market. For tractability, all agents in the model are assumed to be risk-neutral. We do not formally model the process by which firm-specific information is aggregated by dispersed stock market investors, and instead assume that the stock market fully and efficiently aggregates all the publicly available information. Our focus is on understanding how an exogenous increase in "market efficiency," which allows stock market investors to obtain more precise firm-specific information, affects the manager's incentives and the contracting problem between the manager and the firm's current shareholders.

At date 1, the manager takes a costly action, denoted k, at a personal cost of $\Psi(k) = \frac{1}{2}\psi k^2$. The action k may represent the physical or mental exertion of the manager in running the firm. Alternatively, k may be thought of as a discretionary investment made by the manager, in which case, $\Psi(k)$ is the opportunity cost to the manager of not diverting k into his own private benefits. Henceforth, we refer to k simply as the manager's effort. We assume that k is not observed by outsiders.

The firm's intrinsic value $\tilde{V}$ is increasing in the manager's effort k and the firm's marginal productivity (or return on investment), denoted $\tilde{\theta}$. For tractability, we assume that $\tilde{V}$ is linear in k and $\tilde{\theta}$, and has the form[6]

$$\tilde{V}(k, \theta) = \gamma_1 k \tilde{\theta} + \gamma_2 \tilde{\theta} \tag{1}$$

The first term $\gamma_1 k \tilde{\theta}$ is the component of firm value that is affected by the manager's effort k; we refer to this as the "managerial value added" (or MVA) component of intrinsic value.[7] On the other hand, the second term $\gamma_2 \tilde{\theta}$ denotes the component of firm value that is independent of current effort and depends only on the firm's underlying productivity $\tilde{\theta}$; we refer to this as the "core productivity" (or CP) component. This component captures the value of present and future investment opportunities that are unrelated to current effort and arise due to the firm's existing technology, market power, reputation, etc. We model this component as increasing in $\tilde{\theta}$ because more productive firms are more likely to generate value from such opportunities, regardless of managerial effort. The parameters $\gamma_1 > 0$ and $\gamma_2 > 0$ are common knowledge at date 0.

There is no information asymmetry between the manager, shareholders and stock market investors at any point of time regarding the firm's marginal productivity $\tilde{\theta}$. The true $\tilde{\theta}$ is not observed by any of the agents either at date 1 when the manager chooses his effort k or at date 2 when the stock market prices the firm. At date 1, it is commonly believed that $\tilde{\theta}$ is normally distributed with mean θ_m and precision (i.e., inverse of variance) τ.[8]

Manager's compensation and shareholder value: The current shareholders of the firm choose the manager's compensation contract at date 0. We assume that it is not possible to contract directly on the firm's intrinsic value $\tilde{V}$ because it is not observed except in the

[6] The linearity with respect to $\tilde{\theta}$ is important to obtain tractable expressions for the firm's market value at date 2. However, the linearity with respect to k is not necessary.

[7] This formulation assumes linear returns to scale from effort k. One possible interpretation is that effort k produces a perpetual stream of cash flows of $k\tilde{\theta}$ each period, and the present value of this cash flow stream is $\gamma_1 k \tilde{\theta}$.

[8] The advantage of the normality assumption is that it makes the model tractable. An obvious disadvantage is that $\tilde{\theta}$ can take a negative value. However, for large $\theta_m \sqrt{\tau}$ the probability of $\tilde{\theta}$ being negative would be very low.

very long run, and the manager's tenure is short compared to the life of the project/firm. For tractability, we restrict attention to compensation contracts that are linear in the firm's market value $\tilde{P}$ that is realized at date 2. Thus, the manager's compensation may take the form

$$\tilde{C}_m = w + \varphi \tilde{P} \tag{2}$$

where $w \geq 0$ denotes the manager's fixed wage, and $\varphi \in [0,1]$ denotes the fraction of the firm's stock held by the manager. We use the pair (w, φ) to denote the manager's compensation contract. Henceforth, we will refer to φ as the manager's pay-performance sensitivity.

The current shareholders of the firm care for the firm's market value $\tilde{P}$ in addition to its long-term intrinsic value $\tilde{V}$. We model this by assuming that current shareholders attach weights of α and $(1-\alpha)$ to $\tilde{P}$ and $\tilde{V}$, respectively, where $\alpha \in (0,1]$ is a given constant (see Miller and Rock (1985)). One reason why current shareholders may care for $\tilde{P}$ is because they expect to face liquidity needs at date 2, and have to sell some of their stock. As per this interpretation, α denotes the fraction of the stock sold by the current shareholders to outside investors. For instance, venture capitalists and other early-stage investors may use the firm's IPO as an opportunity to exit the firm, and to cash in on their investment.[9]

Recall that the current shareholders own a fraction $(1-\varphi)$ of the firm's equity, whereas the manager owns the remaining fraction φ. So the current shareholders' utility net of the manager's compensation is

$$\tilde{U}_s = \alpha (1-\varphi) \tilde{P} + (1-\alpha)(1-\varphi) \tilde{V} - w. \tag{3}$$

Henceforth, we refer to the net expected shareholder utility, $E\left[\tilde{U}_s\right]$, as shareholder value. We assume that preferences are common knowledge; that is, stock market investors know α.

[9] Alternatively, it may be that shareholders care for the firm's stock price because they want to minimize the dilution of their own stake in case the firm has to issue new stock to fund future investment opportunities. In case of institutional shareholders like mutual funds and pension funds, the concern for short-term stock price could arise because of mark-to-market regulations that tie the value of the institution to the stock prices of their portfolio firms.

A key feature of our model is that stock market investors do not observe the manager's pay-performance sensitivity φ ex ante. This is because, given that the welfare of shareholders depends on the realized stock price, they may surreptitiously skew the manager's incentives towards share-price maximization by privately renegotiating the manager's compensation contract or through uncontracted side payments. In practice, this can be achieved through equity grants (Core and Guay (1999)) or repricing of options (see e.g., Gilson and Vetsuypens (1993), Saly (1994), Acharya et al. (2000) and Brenner et al. (2000)). As we noted in the introduction, although the SEC requires firms to announce the amount of compensation paid to their top managers, knowing the actual payment made to an agent is not the same as knowing the rule by which the compensation was calculated, and it does not allow one to infer what the agent's incentives are ex ante (Katz (1991)).

Information structure and stock price:

Stock market investors observe two information signals at date 2 before they price the firm's stock. These signals relate to the two components of the firm's intrinsic value described above. The first signal $\tilde{y} = k\left(\tilde{\theta} + \tilde{\varepsilon}\right)$ is a noisy signal on the component of intrinsic value that is (stochastically) affected by the manager's effort, where the noise term $\tilde{\varepsilon}$ is independent of $\tilde{\theta}$, and is distributed normally with mean 0 and precision τ_ε. We refer to $\tilde{y}$ as the *"performance signal"* because it may be thought of as an aggregation of information obtained from the firm's performance metrics like sales and earnings. The second signal $\tilde{z} = \tilde{\theta} + \tilde{\delta}$ is a noisy signal on the firm's marginal productivity, where the noise term $\tilde{\delta}$ is independent of $\tilde{\theta}$ and $\tilde{\varepsilon}$, and is distributed normally with mean 0 and precision τ_δ. We refer to $\tilde{z}$ as the *"productivity signal."* We assume that τ_ε and τ_δ are common knowledge at date 0.

Observe that, unlike signal $\tilde{y}$, signal $\tilde{z}$ is not affected by the manager's effort k, and represents information on the firm's productivity independent of the information contained in performance metrics. As signal $\tilde{z}$ becomes more precise (i.e., as τ_δ increases), stock market investors obtain a more precise and independent estimate of the firm's productivity. Hence, we use τ_δ as a measure of market efficiency. Note that the manager's compensation contract and effort are endogenously determined in the model. Our analysis is focused on understanding how an increase in market efficiency τ_δ affects the manager's incentives and the contracting problem between the shareholders and the manager.

2 Market's valuation of the firm, and managerial incentives

Before we can characterize the equilibrium, it is important to understand the market's valuation of the firm. At date 2, the stock market observes the realization of signals $\tilde{y}$ and $\tilde{z}$, which we denote y and z, respectively. The stock market uses these realizations to update its expectation of $\tilde{\theta}$, and to price the firm. Recall that y depends on the manager's effort, k. Because the stock market does not observe k, it instead prices the firm based on its *fixed* conjecture of the manager's effort, denoted $\hat{k}$. So the firm's market value at date 2 is $\tilde{P}\left(y,z,\hat{k}\right) = E_{\theta}\left[V\left(\hat{k},\tilde{\theta}\right)|y,z\right]$. Given our assumption that current shareholders sell an exogenous fraction α of their stock to meet their liquidity needs, the market value $\tilde{P}$ is not affected by α.

We characterize $\tilde{P}\left(y,z,\hat{k}\right)$ in the next lemma. Define

$$f_y \equiv \frac{\tau_\varepsilon}{\tau + \tau_\delta + \tau_\varepsilon} \tag{4}$$

and

$$f_z \equiv \frac{\tau_\delta}{(\tau + \tau_\delta + \tau_\varepsilon)} \tag{5}$$

Lemma 1 *From the manager's perspective, the firm's market value at date 2 is*

$$\tilde{P}\left(y,z,\hat{k}\right) = \left(\gamma_1\hat{k} + \gamma_2\right)\left(\theta_m + \frac{f_y}{\hat{k}}\left(y - \hat{k}\theta_m\right) + f_z\left(z - \theta_m\right)\right), \tag{6}$$

where $\hat{k}$ denotes the stock market's (fixed) conjecture of the manager's effort.

We derive the expression for $\tilde{P}\left(y,z,\hat{k}\right)$ using Bayes' rule, and by exploiting the normality of $\tilde{\theta}$, $\tilde{y}$ and $\tilde{z}$ (see proof of Lemma 1 in the Appendix for details). The parameters f_y and f_z denote the weights the stock market attaches to signals $\tilde{y}$ (scaled by the level of effort) and $\tilde{z}$, respectively, in pricing the firm's stock. A key observation from Lemma 1 is that, all else equal, as the stock market becomes more informationally efficient (i.e., as τ_δ increases), the firm's market value becomes less sensitive to the performance signal $\tilde{y}$ (i.e., f_y decreases) and becomes more sensitive to the productivity signal $\tilde{z}$ (i.e., f_z increases).

Given the price function in equation (6), the manager's optimal effort at date 1 is derived from his incentive compatibility condition:

$$\max_k \; w + \varphi E_{y,z}\left[\tilde{P}\left(y, z, k, \hat{k}\right)\right] - \frac{\psi k^2}{2} \tag{7}$$

Taking expectations on both sides of (6), and substituting $E\left[\tilde{y}|k\right] = k\theta_m$ and $E\left[\tilde{z}|k\right] = \theta_m$, we obtain the following expression for the firm's expected market value:

$$E_{y,z}\left[\tilde{P}\left(y, z, k, \hat{k}\right)\right] = \left(\gamma_1 \hat{k} + \gamma_2\right)\left(\left(1 - f_y\right)\theta_m + \frac{f_y}{\hat{k}} k\theta_m\right) \tag{8}$$

For a given compensation contract, (w, φ), let $k(\varphi)$ denote the level of effort chosen by the manager (the fixed wage, w, does not affect the manager's investment decision on the margin). Applying the first-order condition to problem (7), we obtain that $k(\varphi)$ must satisfy the following equation:

$$\psi k(\varphi) = \frac{\varphi\left(\gamma_1 \hat{k} + \gamma_2\right) f_y \theta_m}{\hat{k}} \tag{9}$$

In equation (9), the left-hand side represents the marginal cost of effort to the manager, whereas the right-hand side represents the sensitivity of the manager's compensation to his effort. Equation (9) offers two important insights. First, all else equal, an increase in market efficiency τ_δ weakens the manager's incentives by lowering f_y, i.e., by making the firm's market value less sensitive to the manager's effort. Second, all else equal, an increase in the size of the CP component of value γ_2 strengthens the manager's incentives, which may seem surprising because the CP component does not depend on the manager's effort k. However, this effect arises because the manager's effort affects the firm's market value by influencing the stock market's inference of the firm's productivity θ. As the CP component increases with θ, the consequent impact on the firm's market value is larger when the size of the CP component γ_2 is high. In other words, γ_2 strengthens the manager's incentives by amplifying the effect of the manager's effort on the stock market's inference of θ.

Next, consider the shareholders' problem of choosing the manager's compensation contract at date 0. From the shareholders' point of view, their net expected utility ("shareholder value") from choosing a contract, (w, φ), given the pricing function (6) and the manager's effort choice, $k(\varphi)$, is

$$\begin{aligned} E_\theta\left[\tilde{U}_s(w,\varphi)\right] &= \alpha(1-\varphi)P\left(k(\varphi),\hat{k}\right) \\ &\quad +(1-\alpha)(1-\varphi)E_\theta\left[\tilde{V}\left(k(\varphi),\theta\right)\right]-w \end{aligned} \tag{10}$$

In equilibrium, the shareholders will choose a compensation contract (w^*, φ^*) that maximizes shareholder value subject to the manager's participation constraint:

$$\varphi E_{y,z}\left[\tilde{P}\left(y,z,\hat{k}\right)\right]-\frac{\psi k^2(\varphi)}{2}+w\geq 0 \tag{11}$$

Finally, given that the stock market has the same information regarding $\tilde{\theta}$ as the shareholders and the manager, it can conjecture the compensation contract chosen by the shareholders and the manager's effort choice *in equilibrium*, even if it cannot observe the actual compensation and the actual effort. Therefore, it must be that

$$\hat{k}=k(\varphi^*) \tag{12}$$

2.1 Benchmark Equilibrium with Observable φ

As a useful benchmark, we first characterize the equilibrium in an idealized setting in which the stock market could observe the manager's pay-performance sensitivity φ (i.e., it could deduce the manager's incentives perfectly ex ante). We use the superscript 'bm' to denote benchmark. In this idealized setting, the stock market perfectly conjectures the manager's effort for every φ (i.e., $\hat{k} = k(\varphi)$), and prices the firm accordingly. We have the following result:

Proposition 1 *Suppose the stock market could perfectly observe the manager's compensation contract, (w, φ). Then, in equilibrium under such an idealized setting, the manager chooses an effort*

$$k^{bm} = \frac{\gamma_1 f_y \theta_m}{2\psi}, \tag{13}$$

and shareholder value is

$$U_s^{bm} = \frac{\gamma_1^2 f_y \theta_m^2}{4\psi} + \gamma_2 \theta_m. \tag{14}$$

Both k^{bm} and U_s^{bm} decrease as market efficiency τ_δ increases.

We characterize the equilibrium fully in the proof of Proposition 1 in the Appendix. Recall that the firm's intrinsic value V has two components: an MVA component that is affected by the manager's effort, and a CP component that only depends on the firm's productivity $\tilde{\theta}$. Equation (13) illustrates that, under the benchmark setting, the manager's effort k^{bm} only depends on the size of the MVA component γ_1, and is not affected by the size of the CP component γ_2. The key force driving this result is the assumption that stock market observes φ. As the stock market can observe the manager's incentives perfectly, the shareholders cannot gain by trying to influence the manager to take an effort that does not optimize the component where the manager adds value, i.e., the MVA component. We show in Section 3 that if the stock market does not observe φ, then shareholders will try to influence the stock market's inference problem; in this case, the manager's effort in equilibrium will also depend on the size of the CP component γ_2.

The last part of Proposition 1 is perhaps most surprising, and states that an increase in market efficiency, as measured by an increase in the precision of the productivity signal, actually decreases shareholder value. This stark result is actually a reiteration of the key result in Paul (1992), who shows that market efficiency may worsen real efficiency, because the signals that are most informative about firm value may not be very informative about the manager's effort. In this benchmark setting, all else equal, an increase in τ_δ weakens managerial incentives by making the stock price less sensitive to the manager's effort. Therefore, shareholders will need to increase the manager's pay-performance sensitivity φ to obtain the same level of effort provision. As increasing φ is costly to shareholders, in equilibrium, shareholders demand a smaller effort provision by providing weaker incentives, which in turn, leads to lower shareholder utility. Put differently, the stock price is now a noisier signal on managerial effort, and thus, providing incentives for effort provision is costlier.

3 Characterizing the equilibrium

We now characterize the equilibrium in a setting where the stock market does not observe the manager's pay-performance sensitivity. We first characterize the optimal compensation contract, the manager's effort choice, and shareholder value for a given level of τ_δ. Then, we analyze how exogenous changes in τ_δ affect the equilibrium outcomes. For the ease of exposition, we characterize the equilibrium for the case of $\alpha = 1$, and relegate the more general case of $\alpha \in (0,1]$ to Section 4. Recall that α denotes the relative weight placed by the firm's existing shareholders on its market value, as opposed to its long-term intrinsic value. Therefore, $\alpha = 1$ means that shareholders only care for the firm's market value. We focus on the $\alpha = 1$ case because it enables us to derive closed-form solutions for φ^*, $k(\varphi^*)$ and U_s^*, and to build clearer intuition for the various trade-offs. However, as we show in Section 4, all our results hold more generally for any $\alpha \in (0,1]$.

3.1 Optimal compensation, effort, and shareholder value

In equation (9), we have already characterized the manager's effort choice as a function of compensation φ and the market's conjecture of effort $\hat{k}$. We now turn our attention to the shareholders' problem of choosing a compensation contract for the manager. Substituting $\alpha = 1$ and the expression for $P\left(k(\varphi), \hat{k}\right)$ from equation (8) into equation (10) yields the following expression for shareholder value:

$$E_\theta\left[\tilde{U}_s(w,\varphi)\right]=(1-\varphi)\left(\gamma_1\hat{k}+\gamma_2\right)\left((1-f_y)\,\theta_m+\frac{f_y k(\varphi)\,\theta_m}{\hat{k}}\right)-w \tag{15}$$

The shareholders' problem is to choose a compensation contract (w,φ) that maximizes $E_\theta\left[\tilde{U}_s(w,\varphi)\right]$, subject to the manager's incentive compatibility constraint (9) and participation constraint (11). Let (w^*,φ^*) denote the optimal compensation contract from shareholders' point of view. Given the manager's effort schedule, $k(\varphi)$, the shareholders' problem is equivalent to choosing the optimal effort $k^*\equiv k(\varphi^*)$.

Proposition 2 *In equilibrium, shareholders choose a compensation contract with $w^*=0$ and pay-performance sensitivity $\varphi^*=\frac{f_y}{1+f_y}$. The manager chooses an effort*

$$k^*=\frac{\gamma_1 f_y^2\theta_m+\sqrt{\left(\gamma_1 f_y^2\theta_m\right)^2+4\psi\gamma_2\theta_m f_y^2\,(1+f_y)}}{2\psi\,(1+f_y)} \tag{16}$$

Optimal shareholder value is

$$\begin{aligned}U_s^* &= (1-\varphi^*)\,(\gamma_1 k^*+\gamma_2)\,\theta_m\\ &= (\gamma_1 k^*+\gamma_2)\,\theta_m-\frac{\psi k^{*2}}{f_y}\end{aligned} \tag{17}$$

Given that the manager's reservation utility is zero, the manager's participation constraint (11) is satisfied whenever his incentive compatibility condition (9) is satisfied. Therefore, it is optimal to set $w=0$ because a positive w only lowers shareholder value without affecting the manager's effort.

In choosing a φ, the shareholders face the following trade-off: On the one hand, a high φ means giving away a higher fraction of the firm's shares to the manager, which hurts the shareholders because they are residual claimants. On the other hand, a high φ induces a higher effort from the manager (equation (9)) which benefits the shareholders by increasing the expected market value. We solve for the equilibrium k^* and φ^* as follows: First, we obtain a relationship between k^* and φ^* by using the fact that, in equilibrium, the stock market perfectly conjectures the manager's effort. Substituting $\hat{k} = k^*$ in equation (9) yields $\varphi^* = \frac{\psi k^{*2}}{(\gamma_1 k^* + \gamma_2) f_y \theta_m}$. Next, we apply the first-order condition to the shareholders' maximization problem, and use the relationship between k^* and φ^* to obtain the closed-form expressions in Proposition 2. The shareholder value, U_s^*, in equation (17) may be interpreted as firm value, $(\gamma_1 k^* + \gamma_2)\,\theta_m$, less a "contracting cost" of $\frac{\psi k^{*2}}{f_y}$.

A key difference from the benchmark equilibrium analyzed in Proposition 1 is that the manager's effort k^* depends on both γ_1 and γ_2, which denote the size of the MVA component and CP component of firm value, respectively. This difference arises because if the stock market does not observe φ, then shareholders can use φ to affect the stock market's inference of the firm's productivity θ. In the benchmark setting, shareholders cannot influence the stock market's inference through φ because the stock market observes φ, and can, therefore, derive the statistical properties of the conditional distribution of θ and update its beliefs accordingly. Thus, the unobservability of φ by the stock market affects the shareholder-manager contracting problem and managerial effort in equilibrium. We formalize this in our next result.

Define

$$\hat{\gamma}_2 = \frac{\gamma_1^2 \theta_m (1 - f_y)}{4\psi} \tag{18}$$

Proposition 3 *In equilibrium, k^* is increasing in γ_2. The manager overinvests with respect to the benchmark effort (i.e., $k^* > k^{bm}$) if $\gamma_2 > \hat{\gamma}_2$, and underinvests if $\gamma_2 < \hat{\gamma}_2$.*

Recall that both the components of the firm's intrinsic value increase with its productivity θ. Hence, given that shareholders care about the firm's market value, they have an incentive to positively influence the market's perception of the firm's productivity θ, and the only tool at their disposal is to provide the manager with high-powered incentives (high φ). As we showed in equation (9), for a given φ, an increase in γ_2 strengthens the manager's incentives because γ_2 amplifies the effect of the manager's effort on the market's inference of θ. Therefore, the shareholders' incentives to induce a higher effort increase as γ_2 increases, which explains why k^* increases with γ_2. Overall, $k^* < k^{bm}$ when the CP component of value is small (low γ_2), and $k^* > k^{bm}$ when the CP component is large (high γ_2). We derive the threshold $\hat{\gamma}_2$ in the appendix.

Our analysis complements that of Persons (1994), who shows that real efficiency is impossible to achieve when a manager's compensation contract can be privately renegotiated by the firm's shareholders. In Persons (1994), the sub-optimal investment arises because the manager exploits his private information about the firm's profits to renegotiate the compensation contract to mutually benefit him and the shareholders. In contrast, in our model, neither the manager nor the shareholders have any private information regarding the firm's profits. Rather, the inefficiency arises because of the shareholders' desire for a higher market value.

3.2 Impact of an increase in market efficiency on the shareholder-manager contracting problem

Our next result characterizes the impact of an increase in market efficiency on the contracting problem between the shareholders of the firm and the manager.

Proposition 4 *In equilibrium, the manager's pay-performance sensitivity φ^* and effort k^* decrease with an increase in market efficiency τ_δ.*

The channel through which τ_δ affects managerial incentives is the pricing function, $\tilde{P}\left(y, z, \hat{k}\right)$, specifically, the parameter f_y which denotes the weight the stock market attaches to the performance signal $\tilde{y}$ while pricing the firm. As noted in the discussion following Lemma 1, all else equal, an increase in τ_δ weakens the manager's incentives by lowering f_y. Hence, an increase in τ_δ lowers the marginal value to shareholders of providing incentives to the manager because the manager invests lower effort for any given φ. In equilibrium, shareholders respond to an increase in τ_δ by choosing a lower pay-performance sensitivity, and the manager responds with a lower effort.

Next, we examine how an increase in τ_δ affects shareholder value U_s^*. As $\frac{dU_s^*}{d\tau_\delta} = \frac{dU_S^*}{df_y} \times \frac{df_y}{d\tau_\delta}$, it is important to first characterize $\frac{dU_S^*}{df_y}$; that is, to understand if an improvement in the sensitivity of market value to current performance (f_y) is value-enhancing for shareholders or not. Differentiating equation (17) with respect to f_y yields

$$\begin{aligned} \frac{dU_S^*}{df_y} &= \frac{\partial U_s^*}{\partial k^*}\frac{dk^*}{df_y} + \frac{\partial U_s^*}{\partial f_y} \\ &= \left(\gamma_1\theta_m - \frac{2\psi k^*}{f_y}\right)\frac{dk^*}{df_y} + \frac{\psi k^{*2}}{f_y^2} \end{aligned} \tag{19}$$

The term $\frac{\partial U_s^*}{\partial f_y} = \frac{\psi k^{*2}}{f_y^2}$ in equation (19) may be interpreted as a decrease in contracting costs resulting from an improvement in managerial incentives; as f_y increases, the cost to shareholders of inducing the effort k^* decreases. The term $\frac{\partial U_s^*}{\partial k^*} = \left(\gamma_1\theta_m - \frac{2\psi k^*}{f_y}\right)$ in equation (19) denotes the marginal sensitivity of shareholder value (U_s^*) to effort (k^*) in equilibrium. Observe that $\frac{\partial U_s^*}{\partial k^*}$ is positive if, and only if, $k^* < k^{bm}$. Therefore, whether an increase in f_y improves shareholder value depends on whether the manager was underinvesting or overinvesting relative to the benchmark effort k^{bm}. Combining this with Proposition 3, it follows that the impact of an increase in market efficiency on shareholder value is itself likely to vary depending on the composition of the firm's intrinsic value; in particular, on the size of the CP component γ_2 relative to the size of the MVA component γ_1. Our next result formalizes this intuition.

Proposition 5 *An increase in market efficiency τ_δ decreases shareholder value U_s^* of firms with $\gamma_2 < \frac{\gamma_1^2\theta_m}{\psi}$ and increases shareholder value of firms with $\gamma_2 > \frac{\gamma_1^2\theta_m}{\psi}$.*

As we discussed above, an increase in τ_δ lowers the sensitivity of market value to the performance signal (i.e., f_y). Proposition 5 states that the decrease in f_y has a differential impact on shareholder value of firms with a relatively large CP component as compared to firms with a relatively large MVA component. To see why, first consider a firm with $\gamma_2 < \hat{\gamma}_2$, whose manager underinvests with respect to the benchmark effort k^{bm} (Proposition 3). This inefficiency arises because its market value is not very sensitive to the performance signal, which makes it very costly for shareholders to provide incentives to the manager. By further lowering f_y, an increase in τ_δ worsens this real inefficiency, thus causing a decrease in shareholder value.

Next, consider a firm with $\gamma_2 > \hat{\gamma}_2$, whose manager overinvests with respect to the benchmark effort k^{bm}. In this case, the inefficiency arises because the firm's market value is too sensitive to its current performance. Shareholders, who have an interest in maximizing the firm's market value, exploit the stock market's focus on current performance by inducing more managerial effort than what is efficient. In this case, an increase in τ_δ corrects the overinvestment problem by lowering the sensitivity of market value to the performance signal y. Of course, a decrease in f_y also increases the contracting cost $\left(\frac{\psi k^{*2}}{f_y}\right)$ by making it more costly for shareholders to provide incentives to the manager. However, for sufficiently large γ_2, specifically $\gamma_2 > \frac{\gamma_1^2 \theta_m}{\psi}$, the former effect prevails and shareholder value increases.

The upshot of Proposition 5 is that an increase in market efficiency, as measured by an increase in the precision of the productivity signal, will be detrimental to the shareholders of firms in which a relatively large part of value comes from the MVA component. However, for firms with a relatively large CP component, an increase in market efficiency improves real efficiency. Notice that the unobservability of compensation contracts (or equivalently, allowing shareholders to privately renegotiate the manager's compensation) completely overturns the insights that are obtained in the benchmark setting with observable compensation contracts, where an increase in market efficiency τ_δ necessarily worsens real efficiency (Proposition 1). Once we allow for the possibility that the stock market does not observe φ, the impact of market efficiency on real efficiency is no longer straight-forward.

4 Robustness of results

We derived the results in Sections 3 under the assumption that shareholders only care for the short-term market value (i.e., $\alpha = 1$). As we explained, this was mainly for ease of exposition. In this section, we show that our qualitative results hold more generally for any $\alpha \in (0, 1]$.

The expressions for $P\left(k, \hat{k}\right)$ and $k(\varphi)$ in equations (8) and (9) are unchanged from above as they do not depend on α. On the other hand, for a general $\alpha \in (0, 1]$, the expression for shareholder value becomes

$$E_\theta\left[\tilde{U}_s(w,\varphi)\right] = \alpha(1-\varphi)\left(\gamma_1\hat{k}+\gamma_2\right)\left(\theta_m(1-f_y)+\frac{f_y\theta_m}{\hat{k}}k(\varphi)\right) + (1-\alpha)(1-\varphi)(\gamma_1 k(\varphi)+\gamma_2)\theta_m \tag{20}$$

The shareholders' problem is to choose a compensation contract that maximizes $E_\theta\left[\tilde{U}_s(w,\varphi)\right]$ subject to the manager's incentive compatibility and participation constraints. We characterize the full equilibrium in Proposition 8, which is stated with proof in the Appendix. The intuition behind the equilibrium is very similar to that in the $\alpha = 1$ case that we characterized in Section 3. The main difference is that we cannot obtain closed-form expressions for φ^*, k^* and U_s^* that we were able to obtain with $\alpha = 1$. However, in Lemma 2 in the Appendix, we prove that k^* is increasing in the size of the CP component γ_2.

Proposition 6 *In equilibrium, an increase in market efficiency τ_δ leads to a decrease in pay-performance sensitivity (φ^*) and investment (k^*).*

Although the Proof of Proposition 6 is much more involved than that of Proposition 4, the underlying intuition is the same: all else equal, an increase in τ_δ lowers the sensitivity of stock price to earnings (f_y), which in turn, weakens managerial incentives and decreases the marginal value to shareholders of providing incentives to the manager. In equilibrium, shareholders respond to an increase in τ_δ by decreasing the manager's pay-performance sensitivity, and the manager responds with a lower effort.

Our next result is qualitatively similar to Proposition 5.

Proposition 7 *There exist parameter values γ_2^l and γ_2^h with $0 < \gamma_2^l < \gamma_2^h < \infty$, such that an increase in market efficiency τ_δ decreases shareholder value of firms with $\gamma_2 < \gamma_2^l$, and increases shareholder value of firms with $\gamma_2 > \gamma_2^h$.*

The intuition behind Proposition 7 is similar to that behind Proposition 5 for the $\alpha = 1$ case: an increase in market efficiency τ_δ hurts firms with a relatively large MVA component (low γ_2) by worsening their underinvestment problem, but benefits firms with a relatively large CP component (high γ_2) by partially correcting their overinvestment problem. Unlike in the $\alpha = 1$ case, we are unable to obtain closed-form expressions for the thresholds, γ_2^l and γ_2^h. However, we show that as $\alpha \to 1$, then $\gamma_2^l, \gamma_2^h \to \frac{\gamma_1^2 \theta_m}{\psi}$.

Thus, we have shown that the results we derived in Section 3 for the $\alpha = 1$ case hold more generally for any $\alpha \in (0, 1]$.

5 Concluding Remarks

In this paper, we examine how an exogenous improvement in market efficiency affects real efficiency and shareholder value through its impact on the shareholder-manager contracting problem and managerial incentives. In our setting, an improvement in market efficiency allows the stock market to obtain more precise information about the firm's innate productivity, and thus, improves the stock market's ability to forecast the firm's intrinsic value. A key assumption in our model is that stock market investors do not observe the manager's pay-performance sensitivity ex ante. Indeed, if shareholders could observe the manager's pay-performance sensitivity ex ante, then we show that an improvement in market efficiency unambiguously worsens real efficiency and decreases shareholder value. This stark result is a reiteration of the key result in Paul (1992), and follows because an improvement in market efficiency weakens managerial incentives by making the firm's stock price less sensitive to the firm's current performance, that is affected by the manager's effort.

In a more realistic setting where the stock market does not observe the manager's pay-performance sensitivity, we find that the impact of enhanced market efficiency on real efficiency and shareholder value is more nuanced, and depends on the composition of the firm's intrinsic value. Specifically, we find that an improvement in market efficiency improves real efficiency and increases shareholder value in firms where a larger part of the firm's intrinsic value is due to the firm's innate productivity and is unrelated to the current managerial effort. On the other hand, an improvement in market efficiency worsens real efficiency and decreases shareholder value in firms where the intrinsic value largely depends on the value generated by the manager's current actions.

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Appendix

Proof of Lemma 1: We know that $\tilde{P}\left(y,z,\hat{k}\right)=\left(\gamma_1\hat{k}+\gamma_2\right)E_\theta\left[\tilde{\theta}|y,z,\hat{k}\right]$. We use the following property (stated here without proof) of normally distributed random variables in deriving the formula for $P\left(y,z,\hat{k}\right)$: If x, y and z are normally distributed random variables, then:

$$E(x|y,z)=E(x)+\frac{\left(\sigma_z^2\sigma_{xy}-\sigma_{xz}\sigma_{yz}\right)}{\left(\sigma_y^2\sigma_z^2-\sigma_{yz}^2\right)}\left(y-E(y)\right)+\frac{\left(\sigma_y^2\sigma_{xz}-\sigma_{xy}\sigma_{yz}\right)}{\left(\sigma_y^2\sigma_z^2-\sigma_{yz}^2\right)}\left(z-E(z)\right) \tag{21}$$

In our problem, $\tilde{x}=\tilde{\theta}$. First, consider the coefficient on $y-E\left(y\right)$. Given the variance and covariance structure in our problem, it follows that

$$\begin{aligned}\sigma_z^2\sigma_{xy}-\sigma_{xz}\sigma_{yz} &= \left(\frac{1}{\tau}+\frac{1}{\tau_\delta}\right)\cdot\frac{\hat{k}}{\tau}-\frac{1}{\tau}\cdot\frac{\hat{k}}{\tau} \\ &= \frac{\hat{k}}{\tau_\delta\tau}\end{aligned} \tag{22}$$

and

$$\begin{aligned}\sigma_y^2\sigma_z^2-\sigma_{yz}^2 &= \hat{k}^2\left(\frac{1}{\tau}+\frac{1}{\tau_\varepsilon}\right)\left(\frac{1}{\tau}+\frac{1}{\tau_\delta}\right)-\left(\frac{\hat{k}}{\tau}\right)^2 \\ &= \hat{k}^2\left(\frac{1}{\tau}\left(\frac{1}{\tau_\varepsilon}+\frac{1}{\tau_\delta}\right)+\frac{1}{\tau_\varepsilon\tau_\delta}\right)\end{aligned} \tag{23}$$

Substituting from equations (22) and (23), and simplifying, it follows that

$$\frac{\sigma_z^2\sigma_{xy}-\sigma_{xz}\sigma_{yz}}{\sigma_y^2\sigma_z^2-\sigma_{yz}^2}=\frac{\tau_\varepsilon}{\hat{k}\left(\tau+\tau_\delta+\tau_\varepsilon\right)} \tag{24}$$

Using similar computations, it follows that

$$\frac{\sigma_y^2\sigma_{xz}-\sigma_{xy}\sigma_{yz}}{\sigma_y^2\sigma_z^2-\sigma_{yz}^2}=\frac{\tau_\delta}{\left(\tau+\tau_\delta+\tau_\varepsilon\right)} \tag{25}$$

Substituting the expressions in equations (24) and (25) back in equation (21) yields

$$E(\tilde{\theta}|y,z,\hat{k}) = \theta_m + \frac{\tau_\varepsilon}{\hat{k}\left(\tau+\tau_\delta+\tau_\varepsilon\right)}\left(y-\hat{k}\theta_m\right) + \frac{\tau_\delta}{\left(\tau+\tau_\delta+\tau_\varepsilon\right)}\left(z-\theta_m\right) \tag{26}$$

Therefore,

$$\begin{aligned}\tilde{P}\left(y,z,\hat{k}\right) &= \left(\gamma_1\hat{k}+\gamma_2\right)E_\theta\left[\tilde{\theta}|y,z,\hat{k}\right] \\ &= \left(\gamma_1\hat{k}+\gamma_2\right)\left(\theta_m + \frac{f_y}{\hat{k}}\left(y-\hat{k}\theta_m\right) + f_z\left(z-\theta_m\right)\right)\end{aligned} \tag{27}$$

where f_y and f_z are defined in equations (4) and (5), respectively. ■

Proof of Proposition 1 : Suppose the stock market observes φ. Then, it can conjecture the manager's effort $k(\varphi)$ for every φ (i.e., $\hat{k} = k(\varphi)$) and price the firm accordingly. Then, from the shareholders' point of view, the expected firm value from choosing a compensation contract (w, φ) is

$$\begin{aligned}E_{y,z}\left[\tilde{P}\left(y,z,k\left(\varphi\right)\right)\right] &= E_{y,z}\left[E_\theta\left[V\left(k\left(\varphi\right),\tilde{\theta}\right)|y,z\right]\right] \\ &= E_\theta\left[V\left(k\left(\varphi\right),\tilde{\theta}\right)\right],\end{aligned} \tag{28}$$

where the last equality is obtained by using the law of iterated expectations.

The shareholders' problem then is to choose a compensation package (w, φ) that maximizes

$$\begin{aligned}E\left[\tilde{U}_s\left(w,\varphi\right)\right] &= \left(1-\varphi\right)E_\theta\left[V\left(k\left(\varphi\right),\tilde{\theta}\right)\right] - w \\ &= \left(1-\varphi\right)\left(\gamma_1 k\left(\varphi\right)+\gamma_2\right)\theta_m - w,\end{aligned} \tag{29}$$

subject to the manager's incentive compatibility and participation constraints.

Next, substituting $\hat{k} = k(\varphi)$ in equation (9), it follows that $k(\varphi)$ is the only positive root of the following quadratic equation:

$$\begin{aligned} \psi k^2 - (\gamma_1 k + \gamma_2)\,\varphi f_y \theta_m &= 0 \\ \Rightarrow \quad \varphi &= \frac{\psi k^2}{(\gamma_1 k + \gamma_2)\cdot f_y \theta_m} \end{aligned} \tag{30}$$

Substituting $k(\varphi) = k$ and $\varphi = \frac{\psi k^2}{(\gamma_1 k+\gamma_2)\cdot f_y \theta_m}$ in equation (29), the shareholders' problem can be written as a choice of k as follows:

$$\max_{k,w} \; (\gamma_1 k + \gamma_2)\cdot \theta_m - \frac{\psi k^2}{f_y} - w \tag{31}$$

Let $\left(w^{bm}, \varphi^{bm}\right)$ denote the optimal compensation contract chosen by the shareholders, k^{bm} denote the manager's effort, and $U_s^{bm} \equiv E\left[\tilde{U}_s\left(w^{bm}, k^{bm}\right)\right]$ denote the optimal shareholder value under the efficient equilibrium.

It is optimal to set $w = 0$, because a positive w only lowers shareholder value without affecting the manager's effort. We obtain the expression for k^{bm} in equation (13) by applying the first-order condition (note that the second-order condition is also met). Substituting $\varphi = \frac{\psi k^2}{(\gamma_1 k+\gamma_2)\cdot f_y \theta_m}$ yields the optimal pay-performance sensitivity

$$\varphi^{bm} = \frac{\gamma_1^2 f_y \theta_m}{2\left(\gamma_1^2 f_y \theta_m + 2\psi\gamma_2\right)} \tag{32}$$

Substituting $k = k^{bm}$ in equation (31), and simplifying, yields the expression for U_s^{bm} in equation (14).

It is obvious from equation (14) that U_s^{bm} is increasing in f_y. Since f_y is decreasing in τ_δ, it follows that U_s^{bm} is decreasing in τ_δ ■

Proof of Proposition 2: The proof involves two steps.

Step I: We show that any contract (w, φ) that satisfies the manager's incentive compatibility constraint (9) will also satisfy the manager's participation constraint (11).

It is evident from the expression for $P\left(k,\hat{k}\right)$ in equation (8) that the manager will get a positive net expected payoff even if he chooses $k=0$. Therefore, the manager's net expected payoff evaluated at $k=k(\varphi)$ has to be positive. Because we have assumed that the manager's reservation utility is zero, the incentive-compatible effort will also satisfy the manager's participation constraint (11).

Step II: Solving for the optimal contract.

An immediate implication of Step I is that the manager's participation constraint can be ignored while solving the shareholder's optimization problem. Moreover, it is optimal to set $w=0$.

Applying the first-order condition to the shareholders' problem, it follows that φ^* must satisfy

$$(1-\varphi^*)\frac{f_y k_\varphi(\varphi^*)\theta_m}{\hat{k}} - \left(\theta_m(1-f_y)+\frac{f_y\theta_m}{\hat{k}}k(\varphi^*)\right)=0 \tag{33}$$

where

$$k_\varphi(\varphi)=\frac{\left(\gamma_1\hat{k}+\gamma_2\right)f_y\theta_m}{\psi\hat{k}} \tag{34}$$

Next, in equilibrium, the market's conjecture is perfect; i.e., $\hat{k}=k^*\equiv k(\varphi^*)$. Substituting in equation (9), and rearranging yields,

$$\varphi^*=\frac{\psi {k^*}^2}{(\gamma_1 k^*+\gamma_2)f_y\theta_m} \tag{35}$$

Substituting $\hat{k}=k^*$, $k_\varphi(\varphi)=\frac{(\gamma_1 k^*+\gamma_2)f_y\theta_m}{\psi k^*}$ and the expression for φ^* in equation (33), and simplifying, we obtain that k^* must satisfy the following condition:

$$(\gamma_1 k^*+\gamma_2)f_y^2\theta_m-\psi k^{*2}(1+f_y)=0 \tag{36}$$

Solving this quadratic equation yields the expression for k^* in Proposition 2. Combining the above two equations yields

$$\frac{\psi {k^*}^2}{(\gamma_1 k^* + \gamma_2) f_y \theta_m} = \varphi^* = \frac{f_y}{1+f_y}.$$

Finally, the expression for U_s^* is obtained by substituting $\hat{k} = k^*$ and $\varphi^* = \frac{\psi {k^*}^2}{(\gamma_1 k^* + \gamma_2) f_y \theta_m}$ in the expression for $E_\theta \left[\tilde{U}_s (w, \varphi) \right]$. ■

Proof of Proposition 3: It is evident from equation (16) that k^* is increasing in γ_2. The threshold, $\hat{\gamma}_2$, is obtained by solving for the value of γ_2 at which $k^* = k^{bm}$. ■

Proof of Proposition 4: We showed in the proof of Proposition 2 that k^* satisfies the following equation:

$$(\gamma_1 k^* + \gamma_2) f_y^2 \theta_m - \psi k^{*2} (1 + f_y) = 0 \tag{37}$$

Implicitly differentiating the above equation with respect to f_y yields

$$\begin{aligned} \frac{dk^*}{df_y} &= -\frac{2 (\gamma_1 k^* + \gamma_2) f_y \theta_m - \psi k^{*2}}{\gamma_1 f_y^2 \theta_m - 2\psi k^* (1 + f_y)} \\ &= \frac{(2 - \varphi^*) (\gamma_1 k^* + \gamma_2) f_y \theta_m}{2\psi k^* (1 + f_y) - \gamma_1 f_y^2 \theta_m}, \end{aligned} \tag{38}$$

where the second equality is obtained by substituting $\varphi^* = \frac{\psi k^{*2}}{(\gamma_1 k^* + \gamma_2) f_y \theta_m}$. Because $\varphi^* = \frac{f_y}{1+f_y} < 1$ and $k^* > \frac{\gamma_1 f_y^2 \theta_m}{2\psi(1+f_y)}$ (see equation (16)), it follows that $\frac{dk^*}{df_y} > 0$.

Next, differentiating φ^* with respect to f_y yields $\frac{d\varphi^*}{df_y} = \frac{1}{(1+f_y)^2} > 0$.

The result in Proposition 4 follows by noting that $\frac{\partial f_y}{\partial \tau_\delta} < 0$. ■

Proof of Proposition 5: *Step I: We show that* $\frac{dU_s^*}{df_y} > 0 \Leftrightarrow k^* < \frac{\gamma_1 f_y \theta_m}{\psi}$.

Differentiating equation (17) with respect to f_y, we obtain

$$\begin{aligned}\frac{dU_s^*}{df_y} &= \frac{\psi k^{*2}}{f_y^2} + \left(\gamma_1\theta_m - \frac{2\psi k^*}{f_y}\right)\frac{dk^*}{df_y} \\ &= \frac{\psi k^{*2}}{f_y^2} + \left(\frac{\gamma_1 f_y\theta_m - 2\psi k^*}{f_y}\right)\frac{(2-\varphi^*)(\gamma_1 k^* + \gamma_2) f_y\theta_m}{\left(2\psi k^*(1+f_y) - \gamma_1 f_y^2\theta_m\right)},\end{aligned} \tag{39}$$

after substituting for $\frac{dk^*}{df_y}$ from equation (38) in the proof of Proposition 4.

Multiplying both sides of the equation with $\frac{f_y(1+f_y)}{(\gamma_1 k^*+\gamma_2)f_y\theta_m}$, and exploiting the fact that $\frac{\psi k^{*2}}{(\gamma_1 k^*+\gamma_2)f_y\theta_m} = \varphi^* = \frac{f_y}{1+f_y}$, yields

$$\begin{aligned}\frac{f_y(1+f_y)}{(\gamma_1 k^* + \gamma_2) f_y\theta_m}\cdot\frac{dU_s^*}{df_y} &= 1 + \left(\frac{\gamma_1 f_y\theta_m - 2\psi k^*}{2\psi k^*(1+f_y) - \gamma_1 f_y^2\theta_m}\right)(2+f_y) \\ &= \frac{2(\gamma_1 f_y\theta_m - \psi k^*)}{2\psi k^*(1+f_y) - \gamma_1 f_y^2\theta_m}\end{aligned} \tag{40}$$

Therefore, it follows that

$$\frac{dU_s^*}{df_y} > 0 \iff k^* < \frac{\gamma_1 f_y\theta_m}{\psi} \tag{41}$$

Step II: We show that $k^* < \frac{\gamma_1 f_y\theta_m}{\psi} \iff \gamma_2 < \frac{\gamma_1^2\theta_m}{\psi}$.

It follows from equation (16) that

$$\begin{aligned}k^* < \frac{f_y\gamma_1\theta_m}{\psi} &\Leftrightarrow \sqrt{\left(\gamma_1 f_y^2\theta_m\right)^2 + 4\psi\gamma_2 f_y^2\theta_m(1+f_y)} < (2+f_y) f_y\gamma_1\theta_m \\ &\Leftrightarrow \gamma_2 < \frac{\gamma_1^2\theta_m}{\psi}\end{aligned} \tag{42}$$

Combining conditions (41) and (42), we conclude that $\frac{dU_s^*}{df_y} > 0 \Leftrightarrow \gamma_2 < \frac{\gamma_1^2\theta_m}{\psi}$.

The results in Proposition 5 follow by noting that $\frac{\partial f_y}{\partial\tau_\delta} < 0$. ∎

Proposition 8 *In equilibrium, shareholders choose a compensation contract with* $w^* = 0$

and a φ^* that satisfies

$$\varphi^* = \frac{\psi k^{*^2}}{(\gamma_1 k^* + \gamma_2) f_y \theta_m}, \tag{43}$$

where the manager's effort, k^* is a solution to the following equation:

$$((\alpha f_y + (1-\alpha)) \gamma_1 k + \alpha \gamma_2 f_y) \left((\gamma_1 k + \gamma_2) f_y \theta_m - \psi k^2\right) - \psi k^2 (\gamma_1 k + \gamma_2) = 0. \tag{44}$$

Optimal shareholder value is

$$U_S^* = (\gamma_1 k^* + \gamma_2) \theta_m - \frac{\psi k^{*2}}{f_y} \tag{45}$$

Proof of Proposition 8: By a similar logic as in the proof of Proposition 2, it follows that $w^* = 0$ and that any contract that satisfies the manager's incentive compatibility constraint will also satisfy the manager's participation constraint (11).

So the shareholders' problem reduces to

$$\begin{aligned} \max_{\varphi} \ & \alpha (1-\varphi) \left(\gamma_1 \hat{k} + \gamma_2\right) \left(\theta_m (1 - f_y) + \frac{f_y \theta_m}{\hat{k}} k(\varphi)\right) \\ & + (1-\alpha)(1-\varphi)(\gamma_1 k(\varphi) + \gamma_2) \theta_m \end{aligned} \tag{46}$$

subject to

$$k(\varphi) = \frac{\varphi}{\psi} \left(\gamma_1 + \frac{\gamma_2}{\hat{k}}\right) f_y \theta_m \tag{47}$$

Substituting the equilibrium condition $k^* = k(\varphi^*)$ in equation (47), we obtain

$$\varphi^* = \frac{\psi k^{*^2}}{(\gamma_1 k^* + \gamma_2) f_y \theta_m} \tag{48}$$

Next, applying the first-order condition, it follows that φ^* must satisfy

$$\begin{aligned} & \left((\alpha f_y + (1-\alpha)) \gamma_1 + \alpha \gamma_2 \frac{f_y}{\hat{k}}\right) (1-\varphi) \theta_m k_\varphi(\varphi) \\ & -\alpha \left(\gamma_1 \hat{k} + \gamma_2\right) \left(\theta_m (1-f_y) + \frac{f_y \theta_m}{\hat{k}} k(\varphi)\right) - (1-\alpha)(\gamma_1 k(\varphi) + \gamma_2) \theta_m = \quad 0 \end{aligned} \tag{49}$$

where

$$k_{\varphi}(\varphi) = \frac{1}{\psi}\left(\gamma_1 + \frac{\gamma_2}{\hat{k}}\right) f_y \theta_m \tag{50}$$

Substituting $\hat{k} = k^*$, $k_{\varphi}(\varphi) = \frac{(\gamma_1 k^* + \gamma_2) f_y \theta_m}{\psi k^*}$ and $\varphi^* = \frac{\psi {k^*}^2}{(\gamma_1 k^* + \gamma_2) f_y \theta_m}$ in equation (49), and simplifying, we obtain that k^* must satisfy the following condition

$$((\alpha f_y + (1-\alpha))\gamma_1 k + \alpha\gamma_2 f_y)\left((\gamma_1 k + \gamma_2) f_y \theta_m - \psi k^2\right) - \psi k^2 (\gamma_1 k + \gamma_2) = 0 \tag{51}$$

Finally, the expression for U_s^* is obtained by substituting $\hat{k} = k(\varphi) = k^*$ and $\varphi^* = \frac{\psi {k^*}^2}{(\gamma_1 k^* + \gamma_2) f_y \theta_m}$ in the expression for $E\left[U_s\left(w, \varphi, \hat{k}\right)\right]$.

Observe that when $\alpha = 1$, equation (44) simplifies to the quadratic equation $(\gamma_1 k + \gamma_2) f_y^2 \theta_m - \psi k^2 (1 + f_y) = 0$, which yields the expression for k^* in equation (16) and implies that $\varphi^* = \frac{f_y}{1+f_y}$. ■

Proof of Proposition 6: As in the proof of Proposition 4, all we need to show is that $\frac{dk^*}{df_y} > 0$ and $\frac{d\varphi^*}{df_y} > 0$. The result in Proposition 6 then follows by noting that $\frac{df_y}{d\tau_\delta} > 0$.

Define,

$$A(k, f_y, \gamma_2) \equiv (\rho\gamma_1 k + \alpha\gamma_2 f_y)\left((\gamma_1 k + \gamma_2) f_y \theta_m - \psi k^2\right) - (\gamma_1 k + \gamma_2)\psi k^2 \tag{52}$$

where

$$\rho = \alpha f_y + (1-\alpha) \tag{53}$$

(1) Proving that k^* is increasing in f_y.

From (44), we know that k^* satisfies the equation $A(k, f_y, \gamma_2) = 0$. Implicitly differentiating this equation with respect to f_y, we obtain that $\frac{dk^*}{df_y} = -\frac{\partial A/\partial f_y}{\partial A/\partial k^*}$. Now,

$$\begin{aligned} \frac{\partial A}{\partial f_y} &= (\rho\gamma_1 k + \alpha\gamma_2 f_y)(\gamma_1 k + \gamma_2)\theta_m + \alpha(\gamma_1 k + \gamma_2)\left((\gamma_1 k + \gamma_2) f_y \theta_m - \psi k^2\right) \\ &> \alpha(\gamma_1 k + \gamma_2)\left((\gamma_1 k + \gamma_2) f_y \theta_m - \psi k^2\right) \end{aligned} \tag{54}$$

But $A = 0$ implies that

$$\left(\left(\gamma_1 k + \gamma_2\right) f_y \theta_m - \psi k^2\right) = \frac{\left(\gamma_1 k + \gamma_2\right) \psi k^2}{\rho \gamma_1 k + \alpha \gamma_2 f_y} > 0 \tag{55}$$

Therefore, it follows that $\frac{\partial A}{\partial f_y} > 0$. We show below in Claim 1 that $\frac{\partial A}{\partial k^*} < 0$. So $\frac{dk^*}{df_y}$ has the same sign as $\frac{\partial A}{\partial f_y}$; i.e., $\frac{dk^*}{df_y} > 0$.

(2) Proving that $\varphi^* = \frac{\psi {k^*}^2}{(\gamma_1 k^* + \gamma_2) f_y \theta_m}$ *is increasing in* f_y.

Rearranging the terms in equation (52) yields

$$\frac{\psi k^2}{\left(\gamma_1 k + \gamma_2\right) f_y \theta_m} = \frac{\left(\rho \gamma_1 k + \alpha f_y \gamma_2\right)}{\left(\rho \gamma_1 k + \alpha f_y \gamma_2\right) + \left(\gamma_1 k + \gamma_2\right)} \tag{56}$$

Therefore,

$$\varphi^* = \frac{\psi {k^*}^2}{\left(\gamma_1 k^* + \gamma_2\right) f_y \theta_m} = \frac{1}{1 + \frac{(\gamma_1 k^* + \gamma_2)}{(\rho \gamma_1 k^* + \alpha f_y \gamma_2)}} \tag{57}$$

So we only need to show that the term $B \equiv \frac{(\gamma_1 k^* + \gamma_2)}{(\rho \gamma_1 k^* + \alpha f_y \gamma_2)}$ is decreasing in f_y. Differentiating B with respect to f_y yields

$$\frac{dB}{df_y} = \frac{\partial B}{\partial k^*} \frac{dk^*}{df_y} + \frac{\partial B}{\partial f_y} \tag{58}$$

Now,

$$\begin{aligned} \frac{\partial B}{\partial k^*} &= \frac{\left(\rho \gamma_1 k^* + \alpha f_y \gamma_2\right) \gamma_1 - \left(\gamma_1 k^* + \gamma_2\right) \rho \gamma_1}{\left(\rho \gamma_1 k^* + \alpha f_y \gamma_2\right)^2} \\ &= \frac{\left(\alpha f_y - \rho\right) \gamma_1 \gamma_2}{\left(\rho \gamma_1 k^* + \alpha f_y \gamma_2\right)^2} < 0, \text{ because } \rho = \alpha f_y + (1 - \alpha) \end{aligned} \tag{59}$$

and

$$\frac{\partial B}{\partial f_y} = -\frac{\alpha \left(\gamma_1 k^* + \gamma_2\right)^2}{\left(\rho \gamma_1 k^* + \alpha f_y \gamma_2\right)^2} < 0 \tag{60}$$

Because $\frac{dk^*}{df_y} > 0$ (by part 1 of the proposition), it follows from (58), (59) and (60) that $B = \frac{(\gamma_1 k^* + \gamma_2)}{(\rho \gamma_1 k^* + \alpha f_y \gamma_2)}$ is decreasing in f_y. Hence, $\varphi^* = \frac{1}{1+B}$ is increasing in f_y. ■

Claim 1 $\frac{\partial A}{\partial k^*} < 0$

Proof of Claim 1: Differentiating (52) w.r.t. k, we obtain

$$\frac{\partial A}{\partial k^*} = \left(\rho\gamma_1 k^* + \alpha\gamma_2 f_y\right)\left(\gamma_1 f_y \theta_m - 2\psi k^*\right) + \rho\gamma_1\left(\left(\gamma_1 k^* + \gamma_2\right) f_y\theta_m - \psi k^{*2}\right) - \gamma_1\psi k^{*2} - 2\psi k^*\left(\gamma_1 k^* + \gamma_2\right) \tag{61}$$

where ρ is as defined in equation (53). Multiplying equation (61) with k^*, and subtracting equation (52) from it, we obtain

$$k^*\frac{\partial A}{\partial k^*} - A = -\left(\rho\gamma_1 k^* + \alpha\gamma_2 f_y\right)\left(\psi k^{*2} + \gamma_2 f_y\theta_m\right) - \gamma_1\psi k^{*3} + \rho\gamma_1 k^*\left(\left(\gamma_1 k^* + \gamma_2\right) f_y\theta_m - \psi k^{*2}\right) - \psi k^{*2}\left(\gamma_1 k^* + \gamma_2\right) \tag{62}$$

Clearly,

$$k^*\frac{\partial A}{\partial k^*} - A < \rho\gamma_1 k^*\left(\left(\gamma_1 k^* + \gamma_2\right) f_y\theta_m - \psi k^{*2}\right) - \psi k^{*2}\left(\gamma_1 k^* + \gamma_2\right) \tag{63}$$

because the remaining terms are negative. But, from equation (52), $A = 0$ implies that

$$\rho\gamma_1 k^*\left(\left(\gamma_1 k^* + \gamma_2\right) f_y\theta_m - \psi k^{*2}\right) - \psi k^{*2}\left(\gamma_1 k^* + \gamma_2\right) = -\alpha\gamma_2 f_y\left(\left(\gamma_1 k^* + \gamma_2\right) f_y\theta_m - \psi k^{*2}\right) \tag{64}$$

It follows from (63) and (64) that

$$\begin{aligned} k^*\frac{\partial A}{\partial k^*} - A &< -\alpha\gamma_2 f_y\left(\left(\gamma_1 k^* + \gamma_2\right) f_y\theta_m - \psi k^{*2}\right) \\ &< 0 \end{aligned} \tag{65}$$

where the last inequality follows by noting that $\left(\gamma_1 k^* + \gamma_2\right) f_y\theta_m - \psi k^{*2} > 0$ (as we showed in equation (55)). Setting $A = 0$, the above inequality implies that $\frac{\partial A}{\partial k^*} < 0$. ■

Lemma 2 *In equilibrium, the manager's effort, k^*, is increasing in the size of the CP component γ_2. Moreover, $k^* \to \infty$ as $\gamma_2 \to \infty$.*

Proof of Lemma 2 By a similar logic as in the proof of Proposition 6, $\frac{dk^*}{d\gamma_2} = -\frac{\partial A/\partial \gamma_2}{\partial A/\partial k^*}$. Note that

$$\frac{\partial A}{\partial \gamma_2} = \alpha f_y \left(\left(\gamma_1 k^* + \gamma_2 \right) f_y \theta_m - \psi k^{*2} \right) + \left(\rho \gamma_1 k^* + \alpha \gamma_2 f_y \right) f_y \theta_m - \psi k^{*2} \tag{66}$$

But $A = 0$ implies that

$$\begin{aligned} \left(\rho \gamma_1 k^* + \alpha \gamma_2 f_y \right) f_y \theta_m - \psi k^{*2} &= \frac{\left(\rho \gamma_1 k^* + \alpha \gamma_2 f_y \right) \psi k^{*2}}{\left(\gamma_1 k^* + \gamma_2 \right)} \\ &> 0 \end{aligned} \tag{67}$$

It follows from (66) and (67) that $\frac{\partial A}{\partial \gamma_2} > 0$. Because $\frac{\partial A}{\partial k^*} < 0$ (by Claim 1), this implies that $\frac{dk^*}{d\gamma_2} > 0$. Moreover, $\lim_{\gamma_2 \to \infty} \frac{\partial A}{\partial \gamma_2} = \infty$ and so it must be that $\lim_{\gamma_2 \to \infty} \frac{dk^*}{d\gamma_2} > 0$. Therefore, it follows that $\lim_{\gamma_2 \to \infty} k^* = \infty$. ■

Proof of Proposition 7: As in the proof of Proposition 5, we characterize how $\frac{dU_s^*}{df_y}$ varies with γ_2. In doing so, we make use of Lemma 2 above where we show that $\frac{dk^*}{d\gamma_2} > 0$. The result in Proposition 7 then follows by noting that $\frac{df_y}{d\tau_\delta} < 0$.

Differentiating the expression for U_s^* in equation (45) with respect to f_y yields

$$\frac{dU_S^*}{df_y} = \left(\gamma_1 \theta_m - \frac{2\psi k^*}{f_y} \right) \frac{dk^*}{df_y} + \frac{\psi \left(k^* \right)^2}{f_y^2} \tag{68}$$

(1) We prove the existence of a $\gamma_2^l > 0$ such that $\frac{dU_s^*}{df_y} > 0$ if $\gamma_2 < \gamma_2^l$.

Because $\frac{dk^*}{df_y} > 0$ (Proposition 6), it follows that $\frac{dU_S^*}{df_y} > 0$ if $k^* < \frac{\gamma_1 \theta_m f_y}{2\psi}$.

We showed in Lemma 2 that $\frac{dk^*}{d\gamma_2} > 0$. Substituting $\gamma_2 = 0$ in equation (44), and solving for K, we obtain the following expression for $[K^*]_{\gamma_2=0}$:

$$[K^*]_{\gamma_2=0} = \frac{\gamma_1 f_y \theta_m}{\psi}\left(\frac{(\alpha f_y + (1-\alpha))}{1+(\alpha f_y + (1-\alpha))}\right) \tag{69}$$
$$< \frac{\gamma_1 f_y \theta_m}{2\psi}, \text{ because } \alpha f_y + (1-\alpha) < 1.$$

So it follows that there exists a $\gamma_2^l > 0$ such that $k^* = \frac{\gamma_1\theta_m f_y}{2\psi}$ if $\gamma_2 = \gamma_2^l$ and $k^* < \frac{\gamma_1\theta_m f_y}{2\psi}$ if $\gamma_2 < \gamma_2^l$. Further, it follows from (68) that $\frac{dU_s^*}{df_y} > 0$ if $\gamma_2 < \gamma_2^l$.

(2) We prove the existence of a γ_2^h such that $\frac{dU_s^*}{df_y} < 0$ if $\gamma_2 > \gamma_2^h$.

Although the proof of this step is long, it involves showing that $\frac{dU_s^*}{df_y} < 0$ for sufficiently high values of k^*. Because $\frac{dk^*}{d\gamma_2} > 0$ and $\lim_{\gamma_2\to\infty} k^* = \infty$, it will then follow that there exists a γ_2^h such that $\frac{dU_s^*}{df_y} < 0$ for firms with $\gamma_2 > \gamma_2^h$.

The expression for $\frac{dU_S^*}{df_y}$ can be re-written as follows:

$$\frac{dU_s^*}{df_y} = \left(\gamma_1\theta_m - \frac{\psi k^*}{f_y}\right)\frac{dk^*}{df_y} - \frac{\psi k^*}{f_y}\left(\frac{dk^*}{df_y} - \frac{k^*}{f_y}\right) \tag{70}$$

Because $\frac{dk^*}{df_y} > 0$ (Proposition 6), the term $\left(\gamma_1\theta_m - \frac{\psi k^*}{f_y}\right)\frac{dk^*}{df_y} < 0$ for $k^* > \frac{\gamma_1 f_y\theta_m}{\psi}$.

Next, consider the term $\frac{\psi k^*}{f_y}\left(\frac{dk^*}{df_y} - \frac{k^*}{f_y}\right)$. We showed in the proof of Proposition 6 that $\frac{dk^*}{df_y} = \frac{\partial A/\partial f_y}{(-\partial A/\partial k^*)}$, where $A = 0$ is defined in (52). Therefore,

$$\frac{dk^*}{df_y} - \frac{k^*}{f_y} = \frac{1}{f_y\left(-\partial A/\partial k^*\right)}\left(f_y\frac{\partial A}{\partial f_y} + k^*\frac{\partial A}{\partial k^*}\right) \tag{71}$$

Because $(-\partial A/\partial k^*) > 0$ (by claim 1), $\frac{dk^*}{df_y} - \frac{k^*}{f_y}$ has the same sign as $f_y\frac{\partial A}{\partial f_y} + k^*\frac{\partial A}{\partial k^*}$.

Now, $\frac{\partial A}{\partial f_y}$ and $\frac{\partial A}{\partial k^*}$ are defined in (54) and (61), respectively. Although the expression for $f_y\frac{\partial A}{\partial f_y} + k^*\frac{\partial A}{\partial k^*}$ is long, by combining terms of the form

$$A \equiv (\rho\gamma_1 k^* + \alpha\gamma_2 f_y)\left((\gamma_1 k^* + \gamma_2) f_y\theta_m - \psi k^{*2}\right) - (\gamma_1 k^* + \gamma_2)\psi k^{*2} \tag{72}$$

and setting them equal to 0 (because $A = 0$ by definition of k^*), we obtain

$$f_y \frac{\partial A}{\partial f_y} + k^* \frac{\partial A}{\partial k^*} = \gamma_2 \psi k^{*2} - \gamma_2 f_y \theta_m \left(\alpha f_y \left(\gamma_1 k^* + \gamma_2\right) + (1-\alpha)\gamma_1 k^*\right) + \left(\left(\gamma_1 k^* + \gamma_2\right) f_y \theta_m - \psi k^{*2}\right) \alpha f_y \gamma_1 k^* \tag{73}$$

It follows from equation (55) that the term $\left(\left(\gamma_1 k^* + \gamma_2\right) f_y \theta_m - \psi k^{*2}\right) \alpha f_y \gamma_1 k^*$ in equation (73) is positive. Moreover,

$$\begin{aligned} \gamma_2 \psi k^{*2} &- \gamma_2 f_y \theta_m \left(\alpha f_y \left(\gamma_1 k^* + \gamma_2\right) + (1-\alpha)\gamma_1 k^*\right) \\ &= \gamma_2 k^* \left(\psi k^* - f_y \theta_m \left(\alpha f_y \gamma_1 + (1-\alpha)\gamma_1\right)\right) - \alpha \gamma_2^2 f_y^2 \theta_m \\ &> 0, \text{ for } k^* \text{ sufficiently large} \end{aligned}$$

So it follows that there exists a k^t such that $\frac{dk^*}{df_y} - \frac{k^*}{f_y} > 0$ for $k > k^t$. Combining with equation (70), it follows that $\frac{dU_s^*}{df_y} < 0$ for $k^* > \max\left\{\frac{\gamma_1 f_y \theta_m}{\psi}, k^t\right\}$. Because k^* is increasing in γ_2, and $\lim_{\gamma_2 \to \infty} k^* = \infty$ (by Proposition 6), it follows that there exists a γ_2^h such that $k^* = \max\left\{\frac{\gamma_1 f_y \theta_m}{\psi}, k^t\right\}$ if $\gamma_2 = \gamma_2^h$, and $k^* > \max\left\{\frac{\gamma_1 f_y \theta_m}{\psi}, k^t\right\}$ if $\gamma_2 > \gamma_2^h$. Moreover, because $\max\left\{\frac{\gamma_1 f_y \theta_m}{\psi}, k^t\right\} > \frac{\gamma_1 f_y \theta_m}{2\psi}$, it follows that $\gamma_2^h > \gamma_2^l$ defined in (1) above. Finally, $\frac{dU_s^*}{df_y} < 0$ for $\gamma_2 > \gamma_2^h$. ■