Lender Moral Hazard and Reputation in Originate-to-Distribute Markets*

Andrew Winton[†] Vijay Yerramilli[‡]

March 2015

Abstract

In a dynamic model of originate-to-distribute lending, we examine whether reputation concerns can incentivize a bank to monitor loans it has sold. Investors believe that banks with fewer recent loan defaults are more likely to monitor ("have higher reputation"). In equilibrium, banks monitor more and retain a smaller loan fraction when their reputations are high. Monitoring is harder to sustain in periods with uncommonly large spikes in loan demand ("booms"), especially for low-reputation banks, which are more likely to accommodate boom demand and forgo monitoring. Increased likelihood of facing a rival with reputation concerns also weakens monitoring incentives.

*We thank Viral Acharya (discussant), Andres Almazan, Thomas Chemmanur, Thierry Foucault, Paolo Fulghieri, Thomas Gehrig (discussant), Bruno Gerard, Dirk Hackbarth (discussant), Charles Kahn, Rich Mathews (discussant), Elvira Sojli, Ajay Subramanyam, Zaime Zender (discussant), and seminar participants at the 2012 SFS Finance Cavalcade (University of Virginia), 2012 Paris Spring Corporate Finance Conference, 2012 Western Finance Association meetings (Las Vegas), 2012 European Finance Association meetings (Copenhagen), 2012 Summer Conference at the Indian School of Business, 2012 University of Oxford Reputation Symposium, 1st Oxford Financial Intermediation Theory Conference, Carlos III University, Erasmus University, Federal Reserve Banks of New York and Philadelphia, Georgia State University, London School of Economics, Norwegian Business School in Oslo, Oxford University, Pompeu Fabra University, Rotterdam Business School, Tilburg University, and the University of Minnesota, for their helpful comments or discussions on issues examined in the paper. All remaining errors are our responsibility.

[†]Carlson School of Management, University of Minnesota; email: awinton@csom.umn.edu

[‡]C. T. Bauer College of Business, University of Houston; email: vyerramilli@bauer.uh.edu

Introduction

Traditional theories of financial intermediation emphasize that banks must hold the loans they make so as to maintain their incentives to screen and monitor them, but present-day lenders increasingly sell off the loans that they originate.[1] Although this "originate-to-distribute" (OTD) model can improve risk-sharing and the lender's liquidity position, it also undermines the traditional mechanism for maintaining monitoring incentives. Up until the recent financial crisis, the typical response of market participants to such concerns was that the lender's concern for its reputation would provide it with the incentives to monitor even after it had laid off its exposure to credit risk, but subsequent revelations of poor credit underwriting even by highly-reputable institutions cast doubt on this.[2] The natural question that follows is when and to what extent can such reputation concerns sustain monitoring by lenders?

In this paper, we address this question in a model of repeated OTD lending, in which a lender ("bank") originates a loan but wishes to sell the loan immediately to investors, possibly because of binding minimum-capital constraints or liquidity needs (see DeMarzo and Duffie (1999)). Afterwards, the bank can improve loan outcomes by monitoring the borrower at a cost.[3] Monitoring lowers the probability of default, but does not eliminate default entirely. Thus, default is a noisy signal of whether the bank has monitored or not, because it can occur due to bad luck. The main friction in the model is that the bank cannot

[1]The idea of delegated monitoring is that banks monitor and enforce loan terms on borrowers on behalf of the bank's own depositors and shareholders. See Leland and Pyle (1977), Diamond (1984) and Holmström and Tirole (1997) for traditional theories of delegated monitoring, and Boyd and Prescott (1986) and Ramakrishnan and Thakor (1984) for models of delegated screening. Gorton and Pennacchi (1995) are among the first to highlight the trend towards selling off originated loans.

[2]Keys et al. (2010) and Purnanandam (2011) provide empirical evidence that securitization led to lax screening in the mortgage market. Piskorski et al. (2010) and Agarwal et al. (2011) show that securitization affects servicing of loans, in particular, renegotiation of delinquent loans.

[3]In OTD markets, there is an expectation that the bank originating the loan will continue to service the loan even after it has sold the loan. In loan securitization, the originating bank often acts as loan servicer on behalf of the investors, and its rights, duties, and compensation are set out in a "Pooling and Servicing Agreement" (PSA). In loan syndication, the lead arranger is expected to retain a portion of the loan and service the loan.

commit to monitor unless monitoring is incentive-compatible.

It follows that, in a single-period setting, the bank would not monitor loans it sold off, reducing the expected value of its loans and overall welfare. But we consider a repeated setting, in which the bank faces a new borrower and a new set of investors each period. Now, market participants can use the history of defaults on the bank's past loans to form their beliefs about the bank's monitoring choice ("reputation") in the current period. Note that the reputation mechanism does not reflect any learning about the bank because the bank has no innate type in our model; instead, it is a sanctioning mechanism that operates purely through the threat of future punishment for poor performance. We analyze the circumstances under which monitoring can be sustained by such reputation concerns, and the factors that may undermine monitoring, both in a simple baseline model and in extensions that address what happens when the bank can retain some of its loans as a possible commitment to monitor, when the bank faces varying levels of loan demand, and when the bank faces potential competition from a rival lender with its own reputation concerns.

As in any infinitely repeated game, there are many equilibria. We examine a class of equilibria where borrowers and investors condition their beliefs about the bank's monitoring intensity based only on the performance of the bank's most recent loan. Hence, in any period, the bank is in one of two possible reputation states: "low" or "high" depending on whether its most recent loan defaulted or did not default, respectively.[4] The present value of the stream of future rents accruing to the bank ("bank value") in each reputation state is determined endogenously in the model, and depends on the market's beliefs about bank monitoring. In such equilibria, reputation concerns may affect monitoring because the bank realizes that if it shirks on monitoring in the current period, it increases the likelihood that the loan will default, hurting the bank's reputation next period. If the incremental value

[4] We choose one-period reputation for analytical tractability, but show that our qualitative results extend to settings where reputation depends on longer performance histories. However, the analysis becomes less tractable with longer performance histories, because the number of states that we must consider increases exponentially with the length of the performance history.

from a higher future reputation is sufficiently high, then the bank will monitor more in the current period.

To fix ideas, we start with a baseline model in which the bank faces a constant loan demand each period, and cannot credibly commit to retain any exposure to the loan after origination. We show that if there is some monitoring in equilibrium, then the probability of monitoring is strictly higher when the bank is in the high-reputation state compared to the low-reputation state. Thus, the secondary loan market price also depends on the bank's performance in the most recent period, and is higher if the bank's most recent loan did not default. A "full monitoring" equilibrium in which the bank always monitors in the high-reputation state but monitors with a lower probability in low-reputation states is more likely to hold as the bank's inter-temporal discount rate is lower, its liquidity needs are stronger, and as monitoring is less costly or has greater impact on default probability. An increase in these parameters also increases the value of a high reputation. In addition, the value of a high reputation also increases as the default probability in the absence of any monitoring ("baseline default probability") decreases, because a decrease in the baseline probability of default lowers the probability of default by bad luck.

Suppose that the bank can commit to retain a fraction of the loan it makes in a given period; e.g., the commitment might take the form of loan sale restrictions in the loan contract. Obviously, the bank could then commit to monitor simply by retaining a large fraction of the loan, but this may be suboptimal from the bank's perspective, given its preference for immediate liquidity. However, in a repeated setting, the bank can rely both on loan retention and reputation to preserve its monitoring incentives. We show that loan retention and reputation are substitutes; that is, if the bank's reputation has been damaged due to a default in the most recent period, then it must retain a higher fraction of its current loan in order to guarantee monitoring. This is consistent with the empirical evidence in Gopalan et al. (2011) that a lead arranger that experiences large defaults is likely to retain a larger fraction of the loans that it underwrites in the subsequent year. As market participants attach higher weight to reputation in forming their beliefs about bank monitoring, the bank value in the high-reputation state always increases, whereas the bank value in the low-reputation state increases only if the bank's inter-temporal discount rate is low and its liquidity needs are strong.

The recent financial crisis was preceded by a massive lending boom in the housing sector, during which many reputable institutions relaxed their lending standards and underwrote large volumes of loans. This experience raises the concern that the reputation mechanism may break down during lending booms if reputable institutions are tempted to increase loan volumes without adequate monitoring so as to earn a large one-off surplus. To examine this issue, we introduce stochastic loan demand into our baseline model, as follows. In each period, the loan market may either be in a "normal demand" state or a "boom demand" state. If the boom demand is realized, the bank may either continue with its normal level of lending or choose to increase its lending to meet the boom demand. The other market participants do not observe the level of loan demand directly; they only observe the bank's lending volume and whether the loans did or did not default at the end of the period. In this setting, we examine how bank lending and monitoring vary with reputation across normal and boom periods.

The equilibrium depends on the frequency of booms and on how much loan demand increases in booms. The more interesting scenario is one in which booms are relatively uncommon events. In this scenario, we show that a bank that decides to meet the boom demand will necessarily shirk on monitoring: the marginal cost of monitoring the boom-time loan volume exceeds its marginal value, because the value of monitoring depends on the future value of reputation, and when booms are uncommon, the future largely consists of normal loan periods and volumes. Whether the bank meets the boom-time demand and shirks on monitoring or sticks to its normal lending volume depends on the size of the boom and the bank's current reputation. The bank is more likely to accommodate the boom demand and shirk on monitoring when it is in the low reputation state, where it has less to lose from subsequent default. Specifically, the threshold size of the boom above which the bank decides to accommodate the boom demand is higher if the bank is in the high reputation state than in the low reputation state. Overall, monitoring is weakly higher in normal times than in booms, and is negatively correlated with bank loan volume.

The final extension we consider is the case when the bank has a fixed chance of facing a rival lender in any given period. We assume that the rival also has reputation concerns, and examine when full-monitoring equilibria can be sustained. Unsurprisingly, the presence of a rival reduces the rate a bank can charge its borrower in the current period. Nevertheless, although a high reputation bank could potentially withdraw from the market when faced with a rival so as to save its high reputation for a future period when it does not face competition, this never occurs in equilibrium. Banks that have the same reputation as one another compete fiercely for the borrower, to the extent that they are indifferent between staying to compete or withdrawing in hopes of not facing a rival the following period; however, when faced with a low-reputation rival, a high-reputation bank can always set a rate that the borrower prefers to the rival's offer while maintaining positive rents for the high-reputation incumbent. Intuitively, such a full-monitoring equilibrium is less likely to be feasible as either the probability that the bank faces a rival increases or as the probability that a rival (when present) has a high reputation increases.

The type of equilibria that we analyze are similar to that in Dellarocas (2005), who explores reputation mechanism design in an online trading environment with pure moral hazard on the seller's part and imperfect monitoring. Adapting this approach to model the interaction between bank reputation and monitoring in an OTD setting leads to a number of significant differences between his work and ours. First, we incorporate the role of bank liquidity needs, which in turn lets us examine tradeoffs between fractional loan retention and reputation as mechanisms that sustain monitoring. Second, we allow for stochastic loan demand and allow the bank to choose both its lending volume and monitoring intensity. This allows us to examine how the effectiveness of the reputation mechanism varies between lending booms and normal periods.

As noted above, the reputation mechanism in our model operates purely through the threat of future punishment for poor performance. While this is reminiscent of equilibria supported by grim-trigger strategies, which feature randomized punishment (Green and Porter (1984), Abreu (1986)), there are some important differences. First, in equilibria with randomized punishments, the bank either has reputation or does not (i.e., the market prices its loans under the belief that the bank will not monitor), whereas our approach allows for multiple reputation states and more nuanced behavior: a low reputation now can improve later if subsequent defaults are fewer, and low-reputation banks may monitor with some intensity, albeit lower than that of high-reputation banks. Second, our approach only requires that market participants coordinate on loan prices, which is simpler than requiring coordination to impose a randomized punishment.

A few recent financial intermediation papers use reputation models that, like ours, operate in a world of pure moral hazard. Bolton et al. (2007) examine whether a financial intermediary's concern for its reputation can alleviate conflicts of interest between the intermediary and its customers (see also Bolton et al. (2012)). Both papers assume that the intermediary suffers an *exogenous reputation loss* when a lie told by the intermediary results in the customer purchasing an unsuitable financial product. By contrast, we endogenize the value of reputation and examine its sensitivity to a number of complicating factors. Bar-Isaac and Shapiro (2013) use a reputation model with pure moral hazard to understand how the value of reputation and the quality of ratings issued by credit rating agencies vary over the business cycle. Unlike us, they focus on grim-trigger-strategies where investors never purchase an investment rated by a rating agency that is known to have issued a faulty good rating at *any* point in the past.[5] As noted above, this does not allow the more nuanced

[5]Another banking paper that employs a pure moral hazard model of reputation with grim-trigger strategies is Dinc (2000), although his focus is very different from ours. Dinc examines whether a bank can commit to rescue distressed entrepreneurs in order to preserve its reputation for being a relationship lender, and how this commitment varies with the level of competition from other arm's length and relationship lenders.

behavior of our model, where reputations can be recovered. Moreover, in our model of imperfect monitoring, defaults are a noisy signal of whether the bank has monitored or not. As a result, the second-best solution cannot support full monitoring indefinitely: defaults eventually occur, damaging bank reputation, which reduces the bank's incentives to monitor. Finally, their focus on ratings agencies abstracts from issues connected with loan origination, such as lender liquidity needs, loan retention decisions, and variations in loan demand.

A much larger literature models reputation in settings where an agent's actions are dictated by innate type as well as strategic concerns. Here, reputation arises from learning over time about an agent's innate type, but the agent can adjust his or her behavior to affect the learning process (e.g., Kreps and Wilson (1982), Milgrom and Roberts (1982), Boot et al. (1993), and Holmström (1999)). It is common to assume that the agent is either an "honest" type that is committed to acting in the first-best manner or a "strategic" type that always acts to maximize his or her utility. Diamond (1989), Benabou and Laroque (1992), Chemmanur and Fulghieri (1994b), Chemmanur and Fulghieri (1994a), and Fulghieri et al. (2014) build on this literature to model reputation formation of borrowers in credit markets, financial gurus in the stock market, banks in credit markets, investment banks in equity markets, and credit rating agencies, respectively. In such models, incentive problems are most severe for agents with short track records and become less severe as the agent accumulates a good reputation following a good track record. A recent paper that features both a type and a moral hazard problem without assuming the existence of an "honest" type is Chen et al. (2015). In their model of investment banking, the threat of punishment by counterparties mitigates the bank's moral hazard only when it has a high type reputation, which makes it worthwhile for banks to build their type reputation. By contrast, in our model of reputation with pure moral hazard, a long track record will not necessarily improve incentives because the bank has no innate type and can choose to either monitor or not monitor in each period. In fact, the value of reputation does not depend on the length of past performance history observed by borrowers and investors.

Among reputation papers using this "mixed" approach, the paper with the topic that is closest to ours is Hartman-Glaser (2011). Hartman-Glaser models a securitization game with reputation concerns, where the issuer can credibly signal the asset's quality by retaining a portion of the asset. In his model, reputation concerns arise due to asymmetric information over the issuer's innate preference for "honesty" (truthfully reporting a bad asset's type). This difference affects his results. Although, like us, Hartman-Glaser finds that the issuer retains less of the asset when she has a higher reputation, the impact of reputation on the issuer's moral hazard problem is the opposite of ours: in his model, as the opportunistic issuer's reputation improves, she *decreases* the probability that she truthfully reveals asset quality, whereas in our model, as the bank's reputation improves, it *increases* the probability that it monitors.

Another related paper using this approach is Mathis et al. (2009). They examine a credit rating agency's incentives to inflate ratings in a model of endogenous reputation, assuming the existence of an "honest" type that always reports truthfully. In addition, they assume that the ratings agency obtains some of its profits from another (unmodeled) line of business, and that this exogenous profit stream is lost if the ratings agency's reputation is hurt. Like Hartman-Glaser (2011), they find that ratings agencies that only get income from ratings activities subject to moral hazard lie more as reputation increases. As the stream of profits from other non-strategic activities increases, the ratings agency's incentives to behave honestly improves. By contrast, we show that even in a setting where all activities are subject to moral hazard, increased reputation can improve lender behavior.

The rest of the paper is organized as follows. We describe our baseline model in Section 1, and characterize the equilibrium in Section 2. In Section 3, we allow the bank to retain a portion of the loan on its books, and examine how the retention decision varies with the bank's reputation. We introduce stochastic loan demand in Section 4, where we examine how bank monitoring and lending activity varies between periods of normal loan demand and lending booms. We examine the effect of bank competition in Section 5. Section 6 concludes the paper.

1 Baseline Model: Assumptions and Framework

Consider a monopolist long-run lender ("bank") that exists for an infinite number of discrete periods (denoted $t = 0, 1, \ldots$), and in each period, faces a new borrower and a new set of secondary loan market investors who only exist for one period. All agents are risk neutral. Let δ denote the bank's per-period discount factor, which may reflect time value of money or the bank's impatience; the higher is δ, the more patient is the bank. The bank's objective is to maximize the discounted value of its expected future payoffs.

At the beginning of each period, a borrower obtains a loan of one unit from the bank to fund its project. By the end of the period, the project either succeeds, yielding X, or fails, yielding C, where $C < 1 < X$. The cash flows from the project are verifiable. Thus, default occurs only if the project fails; C represents the collateral value that can be seized in the event of a default. Let $R \leq X$ denote the *endogenous* loan repayment amount if the project succeeds. Thus, $R - C$ is the risky component of the loan that the bank obtains only if the project succeeds. We describe below how R is determined in equilibrium.

The bank can improve loan outcomes by monitoring borrowers at a cost of $m > 0$. The project succeeds with probability p if the bank does not monitor, and with probability $p + \Delta$ if it does, where $\Delta > 0$ denotes the impact of monitoring. Monitoring can be thought of as keeping an eye on the firm and enforcing covenants so as to keep the firm from engaging in moral hazard.[6] The bank's monitoring effort is unobservable, and cannot be contracted upon. We refer to $1 - p$ as the "baseline default probability" because it denotes the probability of

[6]Our reduced-form model of monitoring is consistent with a more detailed structure in which the underlying moral hazard problem involves the borrower's project choice. In the more complex model, the borrower chooses between two projects, one of which has a higher chance of success but lower private benefits than the other; monitoring allows the bank to intervene and prevent choice of the project with lower chance of success. For details of this model, which is itself a simplified form of that in Holmström and Tirole (1997), see Parlour and Winton (2013).

default in the absence of any monitoring.

The borrower will undertake the project only if its expected payoff from the project exceeds the value of its outside option, $u \geq 0$. Let q denote the borrower's conjecture regarding the probability with which the bank monitors ("monitoring intensity"). Therefore, the borrower's expected payoff from undertaking the project is $(p + q\Delta)(X - R)$. As the bank is a monopolist, it will set the loan repayment at the lowest value at which the borrower is indifferent between undertaking the project and pursuing the outside option. Let $R(q)$ denote this indifference value; it must satisfy

$$(p + q\Delta)(X - R(q)) = u. \tag{1}$$

We assume that the bank has an incentive to raise immediate cash by selling the loan in the secondary loan market. Formally, we assume that the bank values immediate cash at $1+\beta$ per dollar for some $\beta > 0$, whereas if it waits to collect loan payments, it only values those payments at 1 per dollar. This benefit from immediate cash could reflect the shadow cost of binding minimum capital requirements: if the bank retains the (risky) loan, it must raise costly equity capital, whereas it will not face this cost if it sells the loan (see Dewatripont and Tirole (1995), DeMarzo and Duffie (1999), and Parlour and Winton (2013)).

Given the belief q regarding the bank's monitoring choice, the price of the loan in the secondary market is

$$\begin{aligned} P(q) &= (p + q\Delta)(R(q) - C) + C \\ &= (p + q\Delta)(X - C) + C - u, \end{aligned} \tag{2}$$

where the second equation follows from equation (1). For simplicity, we begin by assuming that the bank cannot credibly commit to hold a fraction of the loan either because borrowers and investors cannot observe whether the bank has sold the loan or not, or else because they can observe this only after a significant delay. We relax this assumption in Section 3.

We impose some parametric restrictions to focus on situations of economic interest, and to make the model tractable.

Assumption 1: $0 < \Delta < 1 - p$*; monitoring lowers the probability of default but does not eliminate it completely.*

Because monitoring does not completely eliminate the possibility of default, defaults are a noisy signal of whether the bank has monitored or not. As a result, no equilibrium can support full monitoring indefinitely: defaults eventually occur, damaging bank reputation. A decrease in the baseline probability of default $1 - p$ lowers the probability that the loan defaults by bad luck even when the bank monitors.

Assumption 2: $\Delta(X - C) > m$*; monitoring is socially optimal.*

To see why, note that firm value net of monitoring cost is $(p + \Delta) \cdot (X - C) + C - m$ if the bank monitors, and $p(X - C) + C$ if it doesn't. Therefore, for monitoring to be socially optimal, it must be that $\Delta(X - C) > m$.

Assumption 3: $p(X - C) + C \geq 1 + u$*; the borrower and the bank break even on the project even if the bank does not monitor.*

Assumption 3 implies that the secondary loan price $P(q) \geq 1$ even if market participants do not believe that the bank will not monitor the loan (i.e., $q = 0$). This assumption is not necessary, but it simplifies analysis by ensuring that even a bank with a bad reputation is not completely excluded from the loan market.

Let

$$v \equiv (1 + \beta) \cdot [(p + \Delta) \cdot (X - C) + C - 1 - u] \tag{3}$$

denote the bank's total current period surplus if it monitors the borrower.

2 Equilibrium in the Baseline Model

We now characterize equilibria in our baseline model. Although we focus on a simple case of what we call "reputational" equilibria, it illustrates the key tradeoffs that are our focus. At the end of this section, we discuss some alternative equilibria and their relationship to those on which we focus; detailed derivation of these alternatives and their properties may be found in Appendix B.

Because monitoring cannot be contracted upon, it is clear that if the bank lived for only one period, it would not have any incentive to monitor the borrower once it had sold the loan. Anticipating this, the investors would then price the loan at $p(X - C) + C$. However, this need not to be true for a long-lived bank. As long as borrowers and investors can observe the performance history—default versus no default—of the loans made by the bank in previous periods, these market participants can condition their beliefs about how intensively the bank will monitor in the current period based on its past performance. In other words, the bank's past performance may affect its current reputation, and consequently the prices it can charge in the secondary loan market. In this section, we examine whether and to what extent such a reputation mechanism can incentivize the bank to monitor the borrower. Note that the reputation mechanism operates purely through the threat of future punishment for poor performance, and does not reflect any learning about the bank; indeed, there is nothing to learn here because the bank has no innate type in our model.

As in any infinitely repeated game, there are many potential equilibria. We examine equilibria where borrowers and investors condition their beliefs about the bank's monitoring intensity based on the number of defaults $d(N)$ that the bank has sustained in the previous N periods (see Dellarocas (2005)). Hence, we refer to $d(N)$ as the bank's reputation, and use the notation $q(d(N))$ to denote the market participants' conjecture of the bank's monitoring in the current period. Throughout our analysis, given a a class of equilibria defined by the reputation period N, we focus on those in which borrowers and investors hold the highest beliefs about the bank's monitoring intensity that are consistent with such monitoring being incentive compatible. Also, although we allow the bank to use randomized strategies, we assume that the many investors in the market cannot coordinate on such strategies. This rules out randomized "grim-trigger" punishments of the sort found in Green and Porter (1984) and Abreu (1986). We discuss the features and drawbacks of such equilibria at the end of this section.

To simplify illustration, the bulk of our analysis focuses on equilibria with $N = 1$; i.e., equilibria in which participants condition their beliefs about the bank's monitoring based on whether the bank's most recent loan defaulted ($d = 1$) or not ($d = 0$). Thus, the bank may be in one of two possible reputation states: "high" which corresponds to $d = 0$ or "low" which corresponds to $d = 1$. For expositional convenience, we use the term "high-reputation bank" ("low-reputation bank") to denote that the bank is in the high (low) reputation state. We discuss equilibria with $N > 1$ at the end of this section. As we explain there, our assumption that $N = 1$ entails almost no loss of generality.

2.1 Reputation Equilibria with $N = 1$

Let $V(d)$ denote the expected discounted value of the bank's profits in equilibrium, as a function of d. To characterize $V(d)$, it is important to understand how the bank's reputation transitions from one time period to another. In an equilibrium with $N = 1$, the bank's reputation next period will depend only on whether its current loan defaults or not; its reputation would be $d = 1$ if the current loan defaults, and $d = 0$ otherwise.

The monitoring decision affects the transition probabilities of the bank's reputation as follows. If the bank monitors the current borrower, then its reputation at the end of the current period is $d = 0$ with probability $p+\Delta$, and $d = 1$ with probability $1-p-\Delta$. On the other hand, if the bank shirks on monitoring, then its reputation at the end of the current period is $d = 0$ with probability p, and $d = 1$ with probability $1-p$. Therefore, ignoring the current period surplus from selling the loan (which is sunk when monitoring is chosen), the bank's expected payoff if it monitors is $V_{mon} \equiv -m + \delta\left[(p+\Delta)V(0) + (1-p-\Delta)V(1)\right]$, and its expected payoff if it shirks on monitoring is $V_{shirk} \equiv \delta\left[pV(0) + (1-p)V(1)\right]$. Monitoring is incentive compatible if and only if $V_{mon} \geq V_{shirk}$.

Upon inspection, it is clear that the bank faces the following tradeoff in its choice of monitoring: monitoring costs m, but it increases the probability of the bank being in the high reputation state by Δ, which is worth $\delta\Delta(V(0) - V(1))$ in present value terms. It follows that the incentive compatibility condition $V_{mon} \geq V_{shirk}$ is equivalent to

$$\Lambda \equiv V(0) - V(1) \geq \frac{m}{\delta\Delta}, \tag{4}$$

where Λ denotes the incremental discounted value of a high reputation over that of a low reputation.

The current surplus from selling the loan is $S(d) = (1+\beta)(P(q(d)) - 1)$. This can be written as $S(d) = q(d) \cdot A + B$, where

$$A \equiv \Delta(1+\beta)(X - C), \tag{5}$$

and

$$B \equiv (1+\beta)(p(X - C) + C - 1 - u). \tag{6}$$

Here, B is the "base level" surplus created by an unmonitored loan, and A is the additional value (gross of costs) created by monitoring the loan with probability 1. Note that $S(d)$ is increasing in $q(d)$. Assumption 3 ensures that $B \geq 0$, and so $S(d) \geq 0$ for all d. Also, if the market believes that the bank will certainly monitor the borrower (i.e., $q(d) = 1$), then the bank's current surplus is $A + B = v$, where again v has been defined in Equation (3).

We can now write the Bellman equation as

$$
\begin{aligned}
V(d) = {} & S(d) - mq(d) + \delta q(d) \cdot ((p+\Delta)\Lambda + V(1)) \\
& + \delta(1-q(d))(p\Lambda + V(1))
\end{aligned}
\tag{7}
$$

Substituting for $S(d) = q(d).A + B$ and rearranging yields

$$
V(d) = q(d) \cdot (A - m) + B + \delta[(p + \Delta q(d)) \cdot \Lambda + V(1)] \tag{8}
$$

Equation (8) states that the bank's expected value in equilibrium, $V(d)$, is the sum of two components: its net current period surplus, $q(d) \cdot (A - m) + B$, and the present value of its expected value next period, $\delta[(p + \Delta q(d)) \cdot \Lambda + V(1)]$.

We have the following result. (Detailed proofs of all results are in Appendix A.)

Lemma 1 *In any reputational monitoring equilibrium, the bank's incentive compatibility constraint (4) binds: $\Lambda = \frac{m}{\delta\Delta}$. Also, the probability of monitoring is strictly higher if there was no default last period than if there was a default last period: $q(0) > q(1)$.*

Suppose the bank's incentive compatibility constraint holds strictly; i.e., $\Lambda > \frac{m}{\delta\Delta}$. Then the bank will strictly prefer to monitor in both the high- and low-reputation states, so that $q(0) = q(1) = 1$. But if the bank monitors with the same intensity in both states, then it follows from the Bellman equation (8) that $\Lambda = V(0) - V(1) = 0$, violating the incentive compatibility condition. Therefore, we must have $\Lambda = \frac{m}{\delta\Delta} > 0$ in any monitoring equilibrium. Making this substitution in the Bellman equation and simplifying, it follows that $\Lambda = (q(0) - q(1))A$, and so $q(0) > q(1)$ in such an equilibrium. Combining with equation (1), an immediate implication of Lemma 1 is that $R(0) > R(1)$; the promised loan repayment is higher when the bank is in the high reputation state.

Lemma 1 implies that in any reputational monitoring equilibrium, the monitoring intensities $q(0)$ and $q(1)$ must satisfy the condition $(q(0) - q(1))A = \frac{m}{\delta\Delta}$. Rearranging this yields $q(1) = q(0) - \frac{m}{\delta\Delta A}$. But for the monitoring equilibrium to be well defined, we must also have $q(1) \geq 0$. Clearly, this feasibility condition is most likely to be satisfied when the bank always monitors the loan in the high-reputation state (i.e., $q(0) = 1$), but monitors with a strictly lower probability $q(1) = \hat{q}$ in the low-reputation state. We formalize this intuition in the next result, and also fully characterize the full monitoring equilibrium. Define

$$V^* = (1-\delta)^{-1} \cdot \left(v - \frac{m(1-p)}{\Delta}\right) \tag{9}$$

Proposition 1 *Any reputational monitoring equilibrium is feasible if, and only if,*

$$m \leq \delta(1+\beta)\Delta^2(X - C). \tag{10}$$

If Condition (10) is satisfied, then the full monitoring equilibrium described above is characterized by

$$\hat{q} = 1 - \frac{m}{\delta(1+\beta)\Delta^2(X-C)}, \tag{11}$$

and the value functions are given by $V(0) = V^*$ *and* $V(1) = V^* - \frac{m}{\delta\Delta}$.

To understand the necessity of condition (10), note that any reputational monitoring equilibrium is feasible only if $q(1) = q(0) - \frac{m}{\delta\Delta A} \geq 0$. Substituting $A = \Delta(1+\beta)(X-C)$ and noting that $q(0) \leq 1$ yields the feasibility condition (10). Sufficiency follows by noting that the full monitoring equilibrium is feasible when this condition is satisfied. It is easily verified that this condition is more likely to hold as the monitoring cost m is lower, the discount factor δ is higher, the value of liquidity β is higher, the impact of monitoring Δ is higher, or project risk $X - C$ is higher.

Substituting $d = 0$ and $V(1) = V(0) - \frac{m}{\delta\Delta}$ in equation (8), and solving for $V(0)$ yields $V(0) = V^*$; the expression for $V(1)$ follows by incentive compatibility. It is easily verified that V^* increases as the impact of monitoring Δ increases, and decreases as the base default probability of the loan, $1 - p$, increases.

A key feature of our model is that default is a noisy signal of bank monitoring, because monitoring does not completely eliminate the possibility of default. If instead monitoring completely eliminated the possibility of default (i.e., $p + \Delta = 1$), the value function would be $(1-\delta)^{-1}(v - m)$. In our setting, it is lower because of the chance that, even if the bank monitors, there may be a default due to bad luck; thus, defaults eventually occur, damaging bank reputation.

For a bank in the high reputation state, let n_{high} denote the number of periods it spends in the high reputation state before its reputation is damaged. Similarly, for a bank in the low reputation state, let n_{low} denote the number of periods it spends in the low reputation state before its reputation improves. Clearly, n_{high} and n_{low} are random variables whose probability distribution depends on the monitoring choices of the bank in the high and low reputation states, respectively. Our next result characterizes the expected durations in the high and low reputation states.

Lemma 2 *In a full monitoring equilibrium, if a bank is in the high reputation state, its expected duration in the high reputation state is* $E[n_{high}] \equiv \frac{1}{1-p-\Delta}$ *periods. On the other hand, if it is in the low reputation state, its expected duration in the low reputation state is* $E[n_{low}] \equiv \frac{1}{p+\Delta\hat{q}}$ *periods.*

Observe that $E[n_{high}]$ is increasing in p and Δ, and that $E[n_{high}]$ approaches ∞ as $p+\Delta$ approaches 1. On the other hand, after substituting for $\hat{q}$ from equation (11), it is evident that $E[n_{low}]$ is increasing in monitoring cost m, and is decreasing in p, Δ, risky cash flow $(X - C)$, value of liquidity β, and the bank's discount rate δ.

2.2 Alternative Equilibria

We now discuss two alternative types of equilibria in our model: those in which market participants base their beliefs about the bank's monitoring probability on its loan performance over more than just the most recent period, and those in which participants make use of jointly randomized "grim-trigger" strategies. As we will see, using longer default histories does not change our qualitative results; indeed, far from improving matters, it makes it *less* likely that the constrained first-best (full-monitoring) equilibrium is feasible. By contrast, using correlated grim-trigger strategies makes it more likely that a full-monitoring equilibrium can be achieved, but this requires a level of coordination that seems implausible in practice.

Although it may seen intuitive that allowing participants to base the bank's reputation on a longer history would help matters, this intuition assumes there is learning going on—in which case allowing a longer history helps discern the bank's type. That is not true in our model, where the bank's abilities and motives are publicly-known, and reputation deters pure moral hazard through the threat of future punishment for poor performance. Instead, a longer-term reputation means that it is harder for a bank that has had one or more defaults to restore itself to the best (no defaults) reputation level. For example, if $N = 2$, and the bank has a default in the most recent period, then even if it monitors 100%, it will take two periods before the bank has a chance of being back at a 2-period history of no defaults—and two periods means there are twice as many chances for defaults due to bad luck to occur than there are in a one-period reputation setting. In Appendix B.1, we show that this intuition is in fact true, and that the feasibility condition for a full-monitoring equilibrium in the case of $N = 2$ is more stringent than that for $N = 1$, which is Equation (10) above. In addition, the equilibrium monitoring intensity of the bank is still increasing in its reputation, so that a bank with no defaults monitors more than one with one default, etc. Nor is this intuition restricted to the case where $N = 2$; in a model isomorphic to our baseline setting, Dellarocas (2005) shows that monitoring is always strictly increasing in reputation for high quality, and full monitoring equilibria become less likely to be feasible as the length of the reputation history increases.

Of course, in a more general model with hidden information about some of the bank's attributes, longer reputation histories would allow more learning about these attributes, providing an offset to the advantages of a one-period reputation equilibrium. Our point is not that using a one-period reputation history is always optimal; rather, it is that this equilibrium class captures the key qualitative features of reputation in our setting in a relatively simple fashion.

The literature on infinitely-repeated games with moral hazard has shown that one can do even better in supporting (constrained) optimal outcomes if players can make use of optimal punishments that involve randomized retaliation. For example, Green and Porter (1984) and Abreu (1986) show that better outcomes follow from randomly applying "grim-trigger" strategies in which the offending player is excluded for the rest of the game. In our setting, this would correspond to market participants responding to a default on the most recent bank loan by doing the following: with some probability $\pi > 0$, all participants believe that the bank will never monitor both now and in the future; otherwise(i.e, if there either was no default or the coordination threat didn't materialize), they believe that the bank will monitor this period with some positive probability q. Both π and q would have to be chosen to be consistent with equilibrium behavior.

In Appendix B.2., we analyze this type of equilibrium in detail, and show that the condition for this equilibrium to sustain the full-monitoring outcome $q = 1$ is actually *less* stringent than the condition for a one-period reputation equilibrium to do so (Equation ((10)). This is in line with similar results in Dellarocas (2005). The intuition is as follows. In a one-period reputation equilibrium, any default is followed with certainty by the bank monitoring less intensively the next period; however, this in turn increases the odds of another default, which would cause yet another period of inefficient monitoring. By contrast, in the grim-trigger equilibrium the punishment is harsher, but is only imposed with an endogenously chosen probability; this allows a degree of fine-tuning which the reputation equilibrium does not.

Nevertheless, such optimal punishments require a randomization device that all market participants observe and coordinate on. This seems unlikely in practice. It is one thing to argue that numerous participants think that a bank with given reputation will monitor with some endogenous probability; it is another to argue that these participants collectively construct a credible randomization device.

It is worth noting that the qualitative features of the grim-trigger equilibrium with full monitoring are similar to those of the reputation equilibria already discussed. The bank is more likely to monitor if its past loans did not default than if these loans did default. Moreover, just as before, the full-monitoring feasibility condition is more likely to be met if its monitoring cost is lower, its discount factor is higher, the impact of monitoring is higher, the value of immediate liquidity is higher, or the risk of the loan is higher.

Thus, alternative equilibria with multiple-period reputations or coordinated random punishments do not change the basic flavor of our results from the one-period reputation case. Because full monitoring under multiple-period reputations involves more stringent feasibility conditions than one-period reputation does, and randomized punishments require implausible coordination of diffuse investors and other participants, focusing on one-period reputation equilibria involves no loss of generality. Accordingly, in our analysis of extensions of the baseline model, we will focus on equilibria that use one-period reputations.

3 Reputation and Loan Retention

In the base model, we assumed that the bank could not credibly commit to retain a fraction of the loan on its balance sheet. In this section, we depart from the base model and assume that the bank can credibly commit to retain a fraction $\alpha \in [0, 1]$ of the loan on its books. An immediate implication of this assumption is that monitoring may be sustained even in a one-period setting without any reputation considerations if α is sufficiently high, specifically if $\alpha\Delta(R - C) \geq m$. Let

$$\alpha_{sp} \equiv \frac{m}{\Delta\left(X - C - \frac{u}{p+\Delta}\right)} \tag{12}$$

denote the critical threshold level of α above which monitoring can be sustained in a single period setting.

We now explore how reputation, retention (α), and monitoring interact in a multi-period setting. For tractability, we focus on equilibria in which the bank's reputation only depends on the performance $d \in (0, 1)$ of the bank's most recent loan.[7] For the current period loan, let $q(d, \alpha)$ denote the market's conjecture of the bank's monitoring intensity, given the bank's reputation d and its retention α.

Given reputation d, let $\alpha(d)$ denote the fraction of the loan that the bank will hold in equilibrium, and let $V(d)$ denote the expected discounted value of the bank's profits in equilibrium. As before, denote $\Lambda = V(0) - V(1)$. If a bank with reputation d holds a fraction α of the loan with repayment value R, then its payoffs from monitoring and shirking, respectively, are

$$V_{mon}(d) = -m + \delta[(p + \Delta)\Lambda + V(1)] + \alpha \cdot ((p + \Delta)(R - C) + C), \tag{13}$$

and

$$V_{shirk}(d) = \delta[p\Lambda + V(1)] + \alpha \cdot (p(R - C) + C). \tag{14}$$

Therefore, for there to be some monitoring in equilibrium, it must be that

$$\delta\Delta\Lambda + \alpha\Delta(R - C) \geq m \tag{15}$$

In equilibrium, the market conjectures the bank's monitoring perfectly; i.e., $q(d, \alpha) = q$. Therefore, by the logic established in equation (1), the loan repayment value must satisfy

[7] In principle, the bank's reputation could depend on its past retention decisions as well as its past loan performance (default vs. no default). However, this would make the analysis much less tractable.

$R = R(q) = X - \frac{u}{p+\Delta q}$ (where we have suppressed the arguments of q for convenience). Substituting for $R(q)$ in condition (15) yields the following condition which must be satisfied in equilibrium:

$$\delta\Delta\Lambda + \alpha\Delta\left(X - C - \frac{u}{p+\Delta q}\right) \geq m \tag{16}$$

Note that if the bank holds a fraction α_{sp} of the loan regardless of its reputation (i.e., if $\alpha(d) = \alpha_{sp}$ for $d \in \{0,1\}$), then the incentive compatibility condition (16) is satisfied. In such a "pure retention" equilibrium, the bank relies entirely on loan retention to maintain its monitoring incentives, and its reputation is irrelevant because $q(0) = q(1) = 1$ and $V(0) = V(1) \Rightarrow \Lambda = 0$. Our focus will be on equilibria where the bank also relies on its reputation to maintain its monitoring incentives. In such reputational equilibria, $\alpha < \alpha_{sp}$, which in turn, implies that $\alpha\Delta(R - C) < m$ and $\delta\Delta\Lambda > 0$.

Next, let us examine the bank's choice of α. The bank's current period surplus $S(\alpha, d) = (1+\beta)\cdot((1-\alpha)P(q) - 1) + \alpha P(q)$, where $P(q)$ is set after the bank announces α. We can substitute for $P(q)$ from equation (2), which allows us to rewrite $S(\alpha, d)$ as follows:

$$S(\alpha, d) = (1 + \beta(1-\alpha))\cdot[(p+\Delta q)(X - C) + C - u] - (1+\beta) \tag{17}$$

The value function V can then be written as follows:

$$V(\alpha, d) = S(\alpha, d) - mq + \delta(p + \Delta q)\cdot\Lambda + \delta V(1) \tag{18}$$

Lemma 3 *In any reputational monitoring equilibrium, the incentive compatibility constraint (15) binds for all d. Also, the bank monitors with a strictly higher probability and holds a strictly lower fraction of the loan if there was no default last period than if there was a default last period: $q(0) > q(1)$ and $\alpha(0) < \alpha(1)$.*

As the bank values immediate liquidity at $\beta > 1$, it incurs a liquidity cost by retaining a fraction $\alpha > 0$ of the loan. Therefore, in equilibrium, it will hold the lowest possible α at which the incentive compatibility constraint (15) binds with equality, because otherwise it can improve its expected value by retaining a slightly lower fraction $\hat{\alpha} = \alpha - \varepsilon$ while still maintaining the incentives to monitor.

Next, if the condition (16) holds with equality, then it must be that

$$\alpha(d) = \frac{m - \delta\Delta\Lambda}{\Delta\left(X - C - \frac{u}{p+\Delta q(d)}\right)}. \tag{19}$$

The above equation implies that the higher the q, the lower is the α. Moreover, the Bellman equation can be rewritten as follows (see the proof of Lemma 3 for details):

$$\begin{aligned} V(d) &= (1-\alpha(d)) \cdot \{(1+\beta)[(p+\Delta q(d))(X-C)-u] + \beta C\} \\ &\quad + C - (1+\beta) + \frac{mp}{\Delta} + \delta V(1) \end{aligned} \tag{20}$$

It is evident from equation (20) that $V(.)$ is increasing in q and decreasing in α. Therefore, for the incentive compatibility condition to be satisfied, it is necessary that $q(0) > q(1)$, which implies that $\alpha(0) < \alpha(1)$.

Let $\delta\Delta\Lambda = \rho m$ for $\rho \in [0,1]$; therefore, $\alpha(d)\Delta(X - \frac{u}{p+\Delta q(d)}) = (1-\rho)m$ by condition (16). Note that $\rho = 0$ corresponds to the pure retention equilibrium, whereas $\rho = 1$ corresponds to the pure reputation equilibrium with no retention that we characterized in the base model. If $0 < \rho < 1$, then the equilibrium involves both reputation effects and retention. For each $\rho \in (0,1]$, we now characterize the "full monitoring" equilibrium in which the bank always monitors the loan in the high-reputation state ($d = 0$), but monitors with a strictly lower probability $\hat{q}(\rho) < 1$ in the low-reputation state ($d = 1$). In general, it is difficult to obtain tractable, closed-form expressions for $q(d)$ and $\alpha(d)$. Hence, we are able to do a full characterization of the equilibrium only for the case where the collateral value $C = 0$. We have the following result.

Define

$$\hat{q}(\rho) = 1 - \frac{\rho m}{(1+\beta)\delta\Delta \cdot (\Delta X - (1-\rho)m)} \tag{21}$$

and

$$\alpha(1,\rho) = \frac{m(1-\rho)(p+\Delta\hat{q}(\rho))}{\Delta((p+\Delta\hat{q}(\rho))X - u)} \tag{22}$$

Proposition 2 *Suppose $C = 0$. Then, for a given $\rho \in (0,1]$, the full monitoring equilibrium is feasible if, and only if,*

$$m \leq \frac{\delta(1+\beta)\Delta^2 X}{\rho + (1-\rho)(1+\beta)\delta\Delta}. \tag{23}$$

If condition (23) is satisfied, then in equilibrium, a bank in the high-reputation state always monitors the loan and retains a fraction $\alpha(0,\rho) = (1-\rho)\alpha_{sp}$ of the loan, whereas a bank in the low-reputation state monitors with probability $\hat{q}(\rho)$ and retains a fraction $\alpha(1,\rho)$ of the loan. Under this equilibrium, the value function is given by

$$\begin{aligned} V(0,\rho) &= V^* - \frac{m\beta(1-\rho)}{(1-\delta)\Delta}, \\ \textit{and } V(1,\rho) &= V(0,\rho) - \frac{\rho m}{\delta\Delta}. \end{aligned} \tag{24}$$

We solve for the full monitoring equilibrium as follows. First, we obtain expressions for $\alpha(0,\rho)$ and $\alpha(1,\rho)$ using equation (19), after substituting $q(0,\rho) = 1$ and $q(1,\rho) = \hat{q}(\rho)$. Next, we use the Bellman equation (20) to obtain an expression for $\Lambda = V(0) - V(1)$ in terms of $\hat{q}(\rho)$. Finally, we set $\Lambda = \frac{\rho m}{\delta\Delta}$ and solve for $\hat{q}(\rho)$. The full monitoring is equilibrium is feasible only if, and only if, $\hat{q}(\rho) \geq 0$. Rearranging the expression for $\hat{q}(\rho)$ yields the feasibility condition (23) in the Proposition.

Lemma 4 *(Comparative statics w.r.t ρ):*

1. *The bank's monitoring in the low reputation state, $\hat{q}(\rho)$, is lower for higher ρ.*

2. *If $(1+\beta)\delta\Delta \geq 1$, then the full monitoring equilibrium is feasible for all $0 < \rho \leq 1$. Otherwise, the full monitoring equilibrium is less likely to be feasible for higher ρ.*

3. *The value of high reputation $V(0,\rho)$ is higher for higher ρ. The value of low reputation $V(1,\rho)$ increases with ρ if $\delta(1+\beta) \geq 1$, and decreases with ρ otherwise.*

Recall that higher values of ρ correspond to equilibria in which the bank relies more on reputation and less on retention to maintain its monitoring incentives. Therefore, it is not surprising that the bank's monitoring in the low reputation state, $\hat{q}(\rho)$, is lower for higher ρ. In an equilibrium with $\rho = 0$ (the pure retention equilibrium), $\alpha(1,\rho) = \alpha(0,\rho) = \alpha_{sp}$ and $\hat{q}(\rho) = 1$; i.e., the bank always retains α_{sp} portion of the loan and fully monitors the loan. At the other extreme of $\rho = 1$ (pure reputation equilibrium), $\alpha(1,\rho) = \alpha(0,\rho) = 0$ and $\hat{q}(\rho) = 1 - \frac{m}{(1+\beta)\delta\Delta^2 X}$.

Part 2 of the proposition follows by noting that the expression on the right-hand side of the feasibility condition (23) increases with ρ $(1+\beta)\delta\Delta \geq 1$, and decreases with ρ otherwise. Observe that condition (23) is satisfied for $\rho = 0$ because $\Delta X \geq m$ (by Assumption 2). Therefore, if $(1+\beta)\delta\Delta \geq 1$, then the feasibility condition is satisfied for all ρ. By the converse logic, if $(1+\beta)\delta\Delta < 1$, then the feasibility condition is less likely to be met for higher ρ.

Finally, consider the comparative statics on $V(d,\rho)$. Note that there are two countervailing effects on the value of high reputation $V(0,\rho)$ as ρ increases. On the one hand, a higher ρ allows the bank to maintain its monitoring incentives with lower retention, which lowers its liquidity costs. On the other hand, there is also lower monitoring in the low-reputation state for higher values of ρ, which must lower $V(0,\rho)$. Overall, the former effect dominates, and $V(0,\rho)$ increases as ρ increases.

A natural corollary to Lemma 4 is that the bank will prefer the reputation equilibrium with the lowest amount of retention that is feasible.

Corollary 1 *(to Lemma 4): The value of high reputation $V(0,\rho)$ is maximized at the highest value of $\rho \in (0,1]$ at which the feasibility condition (23) is satisfied.*

4 Reputation and Lending Booms

In the baseline model, we assumed that the bank faces a constant demand for loans each period, which we normalized to 1 unit. In this section, we allow for stochastic "lending booms" (i.e., periods during which there is a sharp spike in the demand for bank loans), and examine how the bank's monitoring incentives and reputation considerations vary between boom periods and periods of normal loan demand.

4.1 Assumptions

We make the following modifications to the baseline case. In each period, the level of loan demand from the entrepreneur is now stochastic: the entrepreneur requires only 1 unit of loan ("normal" demand) with probability ϕ, but can use up to $\gamma > 1$ units of loans ("boom" demand) with probability $1 - \phi$, where $\phi \in (0, 1)$. Let $s \in \{n, b\}$ denote the state of the economy/loan demand (where n denotes normal and b denotes boom). For simplicity, we assume that the probabilities of the two economic states in any given period are independent of the realized economic state in previous periods. We also assume that, if the entrepreneur borrows more than 1 unit, her project returns and the bank's monitoring costs increase proportionately.

At the beginning of each period, the bank observes the state of the economy $s \in \{n, b\}$ and then decides on its loan volume, denoted ℓ. For tractability, we restrict the bank to choose between either 1 or γ as its loan amount in a boom; interior choices are not permitted. If the normal loan demand is realized, the bank simply lends 1 unit as in the baseline model. As in the baseline case, the bank cannot commit to retaining part of the loans it makes. After the loans have been sold, the bank decides whether or not to monitor; then, as in the baseline model, loan outcomes occur and are observed by all agents in the economy.

Although investors observe the bank's loan volume $\ell \in \{1, \gamma\}$, they never observe the loan demand s. This assumption simplifies the possible links between bank actions, states, and investor beliefs about bank monitoring; in particular, the market cannot differentiate between the bank lending 1 unit in a normal economy from the bank lending 1 unit in a boom. Also, following a similar approach to that in the previous section, we focus on equilibria in which investors' beliefs about bank monitoring depend only on its current reputation (i.e., last period's default history $d \in \{0, 1\}$) and on its current loan volume ℓ.[8] Thus, as we detail below, the price of the bank's loans in the secondary market also depend only on its current reputation d and loan volume ℓ.

Let $V(d)$ denote the bank's expected discounted value at the beginning of any given period, before the state $s \in \{n, b\}$ is realized. This in turn can be written as

$$V(d) = \phi V_n(d) + (1 - \phi)V_b(d), \tag{25}$$

where $V_s(d)$ represents the bank's discounted expected profits given that it has a reputation of d and the current state of the economy is s. Because investors know that the bank may vary its choice of monitoring based on its current loan volume, we denote the bank's equilibrium choice of monitoring by $q(d, \ell)$ and its equilibrium choice of loan volume by $\ell_s(d)$. Thus, although monitoring choice depends only on reputation and current loan volume, the current loan volume depends on reputation and the state of the loan demand. Note that, if $s = n$, then $\ell_n(d) = 1$ regardless of the bank's reputation d.

As before, we solve for equilibrium backwards, beginning with the bank's monitoring decision.

[8] In principle, investors could base their beliefs on past loan volumes as well as past defaults. Similarly, even though the current state of loan demand reveals nothing about future loan demand in our Markov setting, one could allow the bank to choose a different monitoring level in a boom than in normal times even if it chose the same loan volume in both. Again, we impose our simplifying assumptions to maintain tractability while capturing the key elements of the stochastic loan demand setting.

4.2 The Bank's Monitoring Decision and the Bellman Equation

Consider the situation once the bank has sold its loans ℓ in the secondary market, and needs to choose its monitoring intensity. The current period surplus from selling the loans is sunk at this point, and is thus, irrelevant for the bank's monitoring choice. Given that the bank's reputation next period depends only on the performance of its current loan, there are only three cases that we need to consider: (1) the economy is in the normal state ($s = n$); (2) the economy is in the boom state but the bank has chosen to lend only 1 unit ($s = b$ and $\ell = 1$); and (3) the economy is in the boom state and the bank has chosen to fulfill the boom demand ($s = b$ and $\ell = \gamma$).

Consider case (1). Because the economy is normal, the banks has made only 1 unit of loans. Hence, by a similar logic as in the base model, it follows that the bank monitors if, and only if, the following incentive compatibility constraint holds:

$$V(0) - V(1) \equiv \Lambda \geq m/\delta\Delta. \tag{IC(1)}$$

Note that although condition IC(1) is the same as condition (4) in the base model, the discounted value functions $V(0)$ and $V(1)$ are more complex now.

Next, consider case (2), where the bank chooses to lend 1 unit in a boom state. Because investors do not observe the state of the loan market, the bank's reputation next period depends only on whether the current loan defaults or not. Moreover, we have assumed that next period's state does not depend on this period's. Hence, it is easy to see that the bank's payoffs from monitoring and shirking in case (2) are just as in case (1). Thus, the bank will monitor if, and only if, condition IC(1) holds.

Finally, consider case (3), which corresponds to $s = b$ and $\ell = \gamma$. Because of the assumption that monitoring costs are proportional to loan volume, the bank's net payoff from monitoring its loans is $V_{mon}(\gamma) \equiv -\gamma m + \delta[(p + \Delta) \cdot \Lambda + V(1)]$, whereas its payoff from shirking is $V_{shirk}(\gamma) \equiv \delta(p \cdot \Lambda + V(1))$. Hence, the bank monitors if, and only if, the following condition holds:

$$\Lambda \geq \gamma m / \delta \Delta. \tag{IC(γ)}$$

Note that, because Λ does not depend on the current state of loan demand, IC(γ) is stricter than IC(1).

Moving back to the point at which the bank makes and sells off its loans, let $S(d, \ell)$ denote current period surplus from a unit of loan sales given reputation d and current loan volume ℓ. We have $S(d, \ell) = q(d, \ell) \cdot A + B$, where A and B are just as in Section 2.

We can now write the bank's value function in each state of the economy. First, if the economy is in a normal state, we have

$$V_n(d) = S(d, 1) - q(d, 1) \cdot m + \delta[(p + q(d, 1) \cdot \Delta)\Lambda + V(1)]. \tag{26}$$

If instead the economy is in a boom, we have

$$V_b(d) = \ell_b(d) \cdot [S(d, \ell_b(d)) - q(d, \ell_b(d)) \cdot m] + \delta[(p + q(d, \ell_b(d)) \cdot \Delta)\Lambda + V(1)]. \tag{27}$$

We can now establish the following result:

Lemma 5 *In any equilibrium in which monitoring occurs with positive probability, either IC(1) binds and IC(γ) fails to hold, or else IC(γ) binds and IC(1) holds strictly.*

The intuition is straightforward: if one or the other incentive compatibility constraint does not bind, then one can show that a bank earns the same expected future profits regardless of its reputation; but then Λ equals zero, and there is no gain to monitoring, which is a contradiction. Because IC(γ) is stricter than IC(1), one of the two possibilities described in the lemma must hold if there is any monitoring in equilibrium.

It follows that there are two broad classes of monitoring equilibria in this setting: those in which the bank's monitoring incentives hold when it lends 1 unit but not more, and those in which its monitoring incentives hold when it lends either 1 unit or γ units (and hold strictly when it lends 1 unit). However, as we show in Appendix B, the second class of equilibria only exists if ϕ is sufficiently low, that is, when booms are very common. In much of our analysis, we will assume that this is *not* the case, by imposing the following assumption:

Assumption 4: $m > \delta\Delta(1-\phi)A = \delta\Delta^2(1-\phi)(1+\beta)(X-C)$.

Essentially, this assumption requires that the probability of a boom next period $(1-\phi)$ cannot be too high. Proposition 1 showed that having a full monitoring equilibrium requires that $m \leq \delta\Delta A$; thus, if the probability of a boom is too close to 1, Assumption 4 must be violated. We return to the case where booms are likely at the end of this section.

4.3 Equilibria when Booms are Uncommon

Assume that booms are relatively uncommon, so that Assumption 4 holds. This means that, in a monitoring equilibrium, the incentive constraint IC(1) binds and IC(γ) fails; thus, if a bank lends γ units in a boom, it will not monitor its loans.

As already noted, the bank must choose how much it lends in a boom: $\ell_b(d) = 1$ or γ. It must then choose the probability with which it monitors its loans, $q(d, \ell_s(d))$, based on its reputation d and its loan volume $\ell_s(d)$. There are four possible loan volume configurations to analyze, based on how a bank chooses between lending 1 or γ units in a boom, given its current reputation (high vs. low). In equilibrium, three incentive compatibility conditions must hold: (a) IC(1) must bind, so we must have $\Lambda = \frac{\gamma m}{\delta\Delta}$; (b) in a boom, a high-reputation bank must not prefer to switch its loan volume choice, $\ell_b(0)$; and (c) in a boom, a low-reputation bank must not prefer to switch its loan volume choice $\ell_b(1)$.

We have the following result:

Proposition 3 *Equilibria in which monitoring is incentive-compatible for the bank only when it lends one unit (i.e., IC(1) is binding) may be feasible as follows:*

1. *An equilibrium in which the bank lends 1 unit in both booms and normal economies, regardless of its reputation state is feasible only if* $m \leq \delta\Delta \cdot (A - (\gamma - 1)B)$*. In this equilibrium, the bank monitors with probability 1 in the high reputation state, and with probability* $\widehat{q} = 1 - m/\delta\Delta A$ *in the low reputation state.*

2. *An equilibrium in which a high-reputation bank lends 1 unit during booms, whereas a low-reputation bank lends* γ *units is feasible only if* $A \geq (\gamma - 1)B$ *and* $m/\delta\Delta \leq A - (1-\phi)(\gamma-1)B$*. In this equilibrium, a high-reputation bank monitors with probability* $q_0 \equiv \min\{1, [(\gamma-1)B/A + m/\delta\Delta A]\}$ *in both booms and normal economies. By contrast, a low-reputation bank monitors with probability* $q_0 - m/\delta\Delta A - (1 - \phi)(\gamma - 1)B/A$ *in normal economies and does not monitor in booms.*

3. *An equilibrium in which the bank lends* γ *units during booms, regardless of its reputation, is feasible only if* $m \leq \phi\delta\Delta A \cdot \min\{1, (\gamma - 1)B/A\}$*. In this equilibrium, the bank never monitors in a boom, regardless of its reputation. In a normal economy, the bank monitors with with probability* $\min\{1, (\gamma - 1)B/A\}$ *in the high reputation state, and probability* $\min\{1, (\gamma - 1)B/A\} - m/\phi\delta\Delta A$ *in the low reputation state.*

4. *An equilibrium in which a low-reputation bank lends 1 unit during booms, whereas a high-reputation bank lends* γ *units is infeasible.*

While seemingly complex, the proposition's results have an intuitive explanation. The key is to understand that, when the boom demand is realized, a low-reputation bank has strictly higher incentive than a high-reputation bank to lend γ units and shirk on monitoring instead of lending 1 unit and monitoring with some positive probability. Intuitively, in equilibrium, a low-reputation bank monitors less and creates less value than a high-reputation bank, and so it has less to lose by deviating to a higher loan volume and not monitoring at all. Hence, equilibria in which a high-reputation bank lends γ in a boom whereas a low-reputation bank lends 1 unit are infeasible (part (4) of the proposition). As we detail below, depending on the boom volume γ and the probability of a boom $1 - \phi$, one of three possible types of equilibria emerge: bank never deviates to the higher loan volume, regardless of its reputation (part (1)); bank deviates to higher loan volume only in the low reputation state (part (2)); or the bank deviates to the higher loan volume in both the high and low reputation states (part (3)).

The equilibrium in part (1) is identical to the full-monitoring equilibrium from Proposition 1 in the base model, but is feasible less often in the presence of booms: the feasibility condition $m/\delta\Delta \leq A - (\gamma - 1)B$ in the present case is more restrictive than that in the earlier proposition, which is $m/\delta\Delta \leq A$. The more limited feasibility is due to the added constraint that a low-reputation bank must weakly prefer lending 1 unit in a boom and monitoring with intensity $\widehat{q}$ to lending γ units and not monitoring at all (as noted above, this constraint is sufficient to guarantee that a high-reputation bank will also not deviate). Such an equilibrium is feasible only if the boom volume γ is not too high, so that additional gross value created by monitoring (i.e., A) is sufficiently large in comparison to the incremental current surplus that can be obtained by deviating to the higher loan volume (i.e., $(\gamma - 1)B$).

Because a low-reputation bank has strictly higher incentive to lend more in a boom, there can be an equilibrium as in part (2) of the proposition, in which a low-reputation bank lends γ in a boom whereas a high-reputation bank still lends 1. Note that the feasibility condition in part (1) is stricter than the conditions in part (2): the equilibrium in part (2) is feasible whenever the equilibrium in part (1) is feasible, but not the other way round. In particular, this alternative equilibrium is feasible even when the equilibrium in part (1) is not feasible, provided booms are not very likely (i.e., $1 - \phi$ is low) and the boom volume γ is not so high that even a high-reputation bank prefers to deviate to high volumes during booms.

Finally, there can be an equilibrium as in part (3), in which the bank accommodates boom loan demand, regardless of its reputation. For such an equilibrium to be feasible, booms must be sufficiently unlikely (ϕA exceeds $m/\delta\Delta$, so ϕ is sufficiently close to 1); otherwise, because reputation doesn't affect profits in booms, it would be impossible to support incentive compatibility based on the difference in the values of high and low reputations in normal times. Also, boom volumes must be sufficiently high ($\phi(\gamma - 1)B$ exceeds $m/\delta\Delta$, so γ is sufficiently high), because otherwise a high-reputation bank would prefer to lend only 1 unit in a boom and reap the gains of its reputation for monitoring intensively.

To summarize, the equilibrium is likely to be the same as in our baseline model for low boom volumes. As booms become less likely or have higher levels of loan demand, the bank accommodates boom demand and shirks on monitoring only in the low reputation state. For more extreme boom volumes, the bank accommodates boom demand and shirks on monitoring, regardless of its reputation. Overall, monitoring is weakly higher in normal times than in booms, and negatively correlated with bank loan volumes.

4.4 Equilibria when Booms are More Common

We now discuss what happens when Assumption 4 does not hold. In this case, equilibria are possible in which IC(γ) binds, that is, the bank has incentives to monitor even when it lends γ units. An immediate implication is that (IC(1) holds strictly, so that banks monitor with certainty whenever they make 1 unit of loans. This, in turn, implies that, the bank always monitors in a normal economy, regardless of its reputation.

However, in contrast to the results in Section 4.3, such equilibria are feasible only when the probability ϕ of a normal loan demand being realized is low. To see why, note that for IC(γ) to bind, we must have $\Lambda = V(0) - V(1) = \gamma m/\delta\Delta > 0$, which means that the incremental value of a high reputation (Λ) must be even larger than in the case when IC(1) binds. But if banks always monitor with certainty whenever they lend 1 unit, there is no incremental value to a high reputation during a normal economy: $V_n(0) = V_n(1)$. This means that all the incremental difference Λ must come from the incremental value of a high reputation during booms, weighted by the probability of a boom:

$$V(0) - V(1) = (1 - \phi)[V_b(0) - V_b(1)] = \gamma m/\delta\Delta.$$

In Proposition 7 in Appendix B.3, we show that this requires that $(1-\phi)\delta\Delta A \geq m$; i.e., Assumption 4 must be violated, and so booms must be reasonably common. Moreover, both the boom-time loan demand γ and the maximum loan surplus $A+B-m$ must be sufficiently high. When these conditions hold, there are two types of equilibria. In the first, the bank lends γ units in a boom, regardless of its reputation; it monitors with probability 1 in the high reputation state, and strictly less in the low reputation state. In the second equilibrium, a low-reputation bank *always* lends 1 unit and monitors with certainty; by contrast, a high-reputation bank lends γ units in a boom and monitors with probability *less* than 1.

These results are more intuitive if one associates the loan demand of 1 unit with a *downturn* and the boom in loan demand with an *expansion*. Then, the first equilibrium requires that, in the most common state of the world (expansions), banks always accommodate total demand, and high-reputation banks monitor more intensively than low-reputation banks. In the less common state of the world (downturns), loan demand is unusually low, and all banks monitor intensively. This is consistent with the stylized fact that banks monitor more in downturns, and is reasonably intuitive.

The second equilibrium requires that, in the most common state (expansions), a high-reputation bank meets high loan demand and generically monitors with less than full intensity, whereas a low-reputation bank has to scale back to loan volume 1 and monitor with full intensity. This is actually consistent with the evidence from the syndicated loan market in Gopalan et al. (2011) that, following large-scale defaults on their loans, lead arrangers experience a significant drop in their total volume of loan origination. If one considers their sample as belonging to a situation where robust credit growth was considered normal (the period prior to the financial crisis of 2007–2009), then this second equilibrium seems very applicable. Indeed, one can go further: in that equilibrium, high-reputation banks actually monitor less intensively, which should lead to worse performance in a downturn. Thus, if the second equilibrium explains behavior in the syndicated loan market in the early and mid 2000s, then one should find that syndicated loans from more reputable banks performed relatively worse during the crisis.[9]

5 Reputation with Competition

In our baseline model, we assumed that the bank is a monopolist. As the bank captures the entire surplus from the loan, this assumption effectively guarantees the bank a stream of positive rents if it makes and sells loans. In this section, we allow for the possibility of competition from other lenders, and examine the impact on the bank's incentives to maintain a reputation for monitoring. As we will see, this reduces the value to having a high reputation, which in turn makes any monitoring harder to sustain in equilibrium.

We modify the baseline model as follows: in each period, there is a probability λ that a rival bank will compete with the incumbent bank for the period's borrower. Conditional on arrival, the rival has a high reputation with probability φ and a low reputation with probability $1-\varphi$. Thus, in each period, the incumbent bank's scenarios are as follows: no competition with probability $1-\lambda$, competition from a low-reputation rival with probability $\lambda(1-\varphi)$, and competition from a high-reputation rival with probability $\lambda\varphi$. The probabilities λ and φ are common knowledge, and are independent across periods.

As in the baseline model, if a rival does not appear, then the incumbent bank captures all the surplus from monitoring by quoting the loan rate $R(q)$; the borrower's surplus equals its reservation utility u. Let $V(d, none)$ denote the bank's expected discounted profits when no rival is present. Let $V(d, rival_0)$ and $V(d, rival_1)$ denote expected discounted profits when faced with rival with reputation $d=0$ and $d=1$, respectively. So the expected value of having a reputation d is

$$V(d) = (1-\lambda) V(d, none) + \lambda\varphi V(d, rival_0) + \lambda(1-\varphi) V(d, rival_1) \tag{28}$$

[9] On the other hand, it is worth noting that this second equilibrium requires more restrictive conditions than the first does, so it may apply less broadly in practice.

Given that the probability of competition is independent across periods, the bank's monitoring decision in any period does not depend on the rival's arrival or reputation during that period, but only depends on the difference $V(0) - V(1)$. Hence, monitoring is incentive compatible if, and only if,

$$\Lambda \equiv V(0) - V(1) \geq \frac{m}{\delta \Delta}. \tag{29}$$

We assume that banks have the option to temporarily withdraw from the loan market (i.e., to not compete), and by doing so, can preserve their reputation till the next period. Therefore, a bank in the reputation state $d \in \{0, 1\}$ will compete only if its expected value from competing exceeds $\delta V(d)$. For a bank with reputation d, let $\bar{S}(d)$ denote the level of current surplus at which the bank is indifferent between competing and withdrawing. Therefore, $\bar{S}(d)$ must satisfy the following condition:

$$\delta V(d) = \bar{S}(d) - m \cdot q(d) + \delta \left[V(1) + (p + \Delta q(d)) \Lambda \right] \tag{30}$$

Let $\bar{R}(d)$ denote the corresponding "break-even loan rate" at which the bank is indifferent between competing and withdrawing. Recall that the bank's current-period surplus increases with the loan rate. Hence, for any $R > \bar{R}(d)$, the bank strictly prefers competing over withdrawing.

In the event of competition, the borrower will accept the loan offer that provides it the higher expected surplus, $(p + q\Delta)(X - R)$. Therefore, in equilibrium, either the incumbent or the rival (or both) will offer its break-even loan rate. To resolve ties, we assume that if the borrower's surplus is the same under the two competing loan offers, then it will go with the bank that has quoted a rate above its break-even loan rate (i.e., $R > \bar{R}(d)$); this is a reasonable assumption because such a bank can strictly dominate the competing bank's offer by offering $R - \varepsilon$ for some infinitesimally small $\varepsilon > 0$. If both the competing banks have quoted their break-even loan rates, we assume that the borrower randomly chooses between them with equal probability.

Lemma 6 *In any reputational monitoring equilibrium, the incentive compatibility condition (29) binds with equality;* $\Lambda = \frac{m}{\delta\Delta}$*. Moreover, the bank monitors with a strictly higher probability if there was no default last period than if there was a default last period;* $q(0) > q(1)$*.*

The intuition is similar to that in the baseline model: if $\Lambda > \frac{m}{\delta\Delta}$, then one can show that a bank earns the same expected future profits regardless of its reputation; but then Λ equals zero, and there is no gain to monitoring, which is a contradiction. Similarly, if $q(1) \geq q(0)$, then one can show that $\Lambda = V(0) - V(1)$ is negative, which is a contradiction.

Using lemma 6, it is easy to show that the borrower's expected surplus at the high-reputation bank's break-even rate is

$$\bar{U}(0) \equiv (p + \Delta q(0))(X - C) + C - 1 - \frac{\delta\Lambda(1-p)}{(1+\beta)}.$$

Similarly, the borrower's expected surplus at the low-reputation bank's break even rate is

$$\bar{U}(1) \equiv (p + \Delta q(1))(X - C) + C - 1 + \frac{\delta p \Lambda}{1+\beta}.$$

When the two competing banks are in different reputation states, then the resolution of competition will depend on how $\bar{U}(0)$ compares with $\bar{U}(1)$. If $\bar{U}(0) > \bar{U}(1)$, then the bank in the high reputation state should be able to win the loan by making an offer that can't be matched by the bank in the low reputation state, and vice versa. Our next result characterizes how competition is resolved in a reputational monitoring equilibrium.

Lemma 7 *In any reputational monitoring equilibrium, the borrower strictly prefers the high-reputation's break-even offer over the low-reputation bank's break-even offer (i.e.,* $\bar{U}(0) > \bar{U}(1)$*). Competition is resolved as follows:*

1. *If both the competing banks are in the same reputation state* d*, then both banks bid the loan rate* $\bar{R}(d)$ *at which they are indifferent between competing and withdrawing, and*

win the loan with 0.5 probability. Hence, either bank's continuation value is $\delta V(d)$ *whether or not it wins the loan.*

2. *If the competing banks are in different reputation states, then the bank in the high reputation state* ($d = 0$) *wins the loan and has a continuation value of* $[q(0) - q(1)]A + \delta V(1)$ *(which strictly exceeds the value of* $\delta V(0)$ *it could have obtained by withdrawing). On the other hand, the bank in the low reputation state* ($d = 1$) *has a continuation value of* $\delta V(1)$.

We prove, by contradiction, that it is not feasible to have a reputational monitoring equilibrium in which $\bar{U}(0) \leq \bar{U}(1)$. To see why not, notice that $\bar{U}(0) \leq \bar{U}(1)$ is equivalent to the condition $[q(0) - q(1)]A \leq \delta\Lambda$. One can show that any equilibrium that meets this requirement will violate the incentive compatibility constraint (29). Hence, $\bar{U}(0) > \bar{U}(1)$ in any feasible monitoring equilibrium. This, in turn, implies that when a high-reputation bank competes with a low-reputation bank, the former can offer the borrower a deal that's more attractive than that offered by the latter, and yet be strictly better off lending to the borrower versus withdrawing.

Our next result characterizes the conditions under which a reputational monitoring equilibrium is feasible. Define

$$V^*_{compete} \equiv (1 - \lambda) V^* + \frac{\lambda m (1 - \varphi)}{\delta \Delta (1 - \lambda\varphi)}, \tag{31}$$

where V^* is defined in equation (9) and denotes the value of high reputation in a full-monitoring equilibrium with no competition.

Proposition 4 *A reputational monitoring equilibrium is feasible if, and only if,*

$$m \leq \frac{\delta \Delta A (1 - \lambda\varphi)}{(1 - \lambda\delta\varphi)}. \tag{32}$$

1. *If Condition (32) is satisfied, then the full-monitoring equilibrium with* $q(0) = 1$ *and* $q(1) = 1 - \frac{m(1-\lambda\delta\varphi)}{\delta\Delta A(1-\lambda\varphi)}$ *is feasible. Under this equilibrium,* $V(0) = V^*_{compete}$ *and* $V(1) = V^*_{compete} - \frac{m}{\delta\Delta}$.

2. *Monitoring equilibria are less likely to be feasible as either the probability* λ *of a rival appearing or the probability* φ *such a rival has a high reputation increases. Whenever monitoring equilibria are feasible, the value of a high reputation also decreases as either* λ *or* φ *increases.*

Using the results derived in lemmas 6 and 7, we show that $\Lambda = \frac{A[q(0)-q(1)](1-\lambda\varphi)}{(1-\lambda\delta\varphi)}$. Setting $\Lambda = \frac{m}{\delta\Delta}$, and solving for $q(1)$, yields that $q(1) = q(0) - \frac{m(1-\lambda\delta\varphi)}{\delta\Delta A(1-\lambda\varphi)}$. As $q(0) \leq 1$, the requirement $q(1) \geq 0$ is met only if the feasibility condition (32) is satisfied. It is easily verified that condition (32) is more likely to be met when competition is less likely (low λ) and when the competition is less likely to be from a bank in the high reputation state (low φ). In fact, in the absence of competition (i.e., if $\lambda = 0$), condition (32) is the same as the feasibility condition (58) that we obtained in the baseline model with a monopolist bank.

Turning to the value function for having a high reputation under the full-monitoring equilibrium, $V^*_{compete}$, it is easy to show that this decreases as the probability φ that the rival bank has a high reputation increases. Intuitively, the bank faces stronger competition whenever it has a rival, and this decreases average profits. Similarly, we can show that, whenever a monitoring equilibrium is feasible, an increase in the probability λ of facing any rival also reduces the value of having a high reputation.[10]

Overall, the main takeaway from Proposition 4 is that competition lowers the value of reputation, and makes it less likely that the bank's monitoring incentives can be sustained.

[10] $V^*_{compete}$ can actually increase in λ if $\lambda\varphi$ is sufficiently high, but in these cases Condition (32) is violated: the probability of facing competition from a high-reputation rival is too high to support monitoring incentives in the first place.

6 Concluding Remarks

In this paper, we analyze a dynamic model of OTD lending in which there is no uncertainty about the monitoring ability or honesty of the bank that originates the loan, but the bank may not have incentives to monitor after it has sold the loan. In this setting, we examine whether the bank can maintain its incentives to monitor out of concern for its reputation, which is endogenously determined. In the spirit of Dellarocas (2005), we examine equilibria where the market participants' beliefs regarding the bank's monitoring choice (i.e., the bank's reputation) depends on the number of defaults in the bank's recent performance history. As the bank's past performance does not contain any information regarding the bank's monitoring choice, the reputation mechanism works by punishing the bank for defaults. In equilibrium, a bank that has caused more defaults in the past obtains a lower secondary market price on its current loan, monitors less intensively, and retains a larger fraction of the loan on its books.

We then examine how the bank's monitoring incentives are affected by stochastic lending booms, during which demand for loans increases. We show that, when such booms are relatively uncommon, the equilibrium outcome depends on the size of the boom. Smaller booms tend to support the same full-monitoring equilibrium as in our baseline model. As booms become larger, however, low-reputation banks are likely to accommodate increased boom-time demand and shirk on monitoring; and for even larger booms, all banks shirk and accommodate the higher demand, regardless of reputation.

By contrast, if booms are very common, it becomes more natural to interpret them as "normal" expansions, whereas periods of lower demand are "downturns". In this case, bank behavior in expansions is similar to that in our baseline model: banks accommodate all loan demand, and high-reputation banks monitor more intensively than low-reputation banks. In periods of low demand, however, all banks monitor intensively, regardless of reputation.

Finally, we examine how the interplay between reputation and monitoring is affected by the arrival of a rival lender that competes with the incumbent bank for that period's loan. Although matters are complicated by the rival's own reputation concerns, it is still true that an increased probability of competition makes monitoring harder to sustain and reduces the value of having a high reputation. Moreover, all else equal, increasing the probability that a rival bank has a high reputation also makes monitoring harder to sustain and reduces the value of a high reputation.

Overall, our analysis sheds light on the effectiveness of reputation mechanisms in sustaining monitoring in OTD markets even when banks can commit to retaining part of their loans, and how monitoring incentives are affected by variations in loan demand and by competition among lenders.

References

Abreu, D. (1986). Extremal Equilibria of Oligopolistic Supergames. *Journal of Economic Theory 39*, 191–225.

Agarwal, S., G. Amromin, I. Ben-David, S. Chomsisengphet, and D. Evanoff (2011). The Role of Securitization in Mortgage Renegotiation. *Journal of Financial Economics 102*, 559–578.

Bar-Isaac, H. and J. Shapiro (2013). Ratings Quality over the Business Cycle. *Journal of Financial Economics 108*, 62–78.

Benabou, R. and G. Laroque (1992). Using Privileged Information to Manipulate Markets: Insiders, Gurus, and Credibility. *Quarterly Journal of Economics 107*.

Bolton, P., X. Freixas, and J. Shapiro (2007). Conflicts of Interest, Information Provision, and Competition in the Financial Services Industry. *Journal of Financial Economics 85*, 297–330.

Bolton, P., X. Freixas, and J. Shapiro (2012). The Credit Ratings Game. *Journal of Finance 67*, 85–112.

Boot, A. W., S. Greenbaum, and A. V. Thakor (1993). Reputation and discretion in financial contracting. *American Economic Review 83*, 1165–1183.

Boyd, J. H. and E. C. Prescott (1986). Financial Intermediary-Coalitions. *Journal of Economic Theory 38*, 211–232.

Chemmanur, T. and P. Fulghieri (1994a). Investment Bank Reputation, Information Production, and Financial Intermediation. *Journal of Finance 49*, 57–79.

Chemmanur, T. and P. Fulghieri (1994b). Reputation, Renegotiation, and the Choice between Bank Loans and Publicly Traded Debt. *Review of Financial Studies 7*, 475–506.

Chen, Z., A. D. Morrison, and W. J. Wilhelm, Jr. (2015). Traders vs. Relationship Managers: Reputational Conflicts in Full-Service Investment Banks. *Review of Financial Studies 28*, 1153–1198.

Dellarocas, C. (2005). Reputation Mechanism Design in Online Trading: Environments with Pure Moral Hazard. *Information Systems Research 16*, 209–230.

DeMarzo, P. and D. Duffie (1999). A Liquidity-Based Model of Security Design. *Econometrica 67*, 65–99.

Dewatripont, M. and J. Tirole (1995). *The Prudential Regulation of Banks*. MIT Press.

Diamond, D. (1984). Financial Intermediation and Delegated Monitoring. *Review of Economic Studies 51*, 394–414.

Diamond, D. (1989). Reputation Acquisition in Debt Markets. *Journal of Political Economy 97*, 826–862.

Dinc, S. (2000). Bank Reputation, Bank Commitment, and the Effects of Competition in Credit Markets. *Review of Financial Studies 13*, 781–812.

Fulghieri, P., G. Strobl, and H. Xia (2014). The Economics of Solicited and Unsolicited Credit Ratings. *Review of Financial Studies 27*, 484–518.

Gopalan, R., V. Nanda, and V. Yerramilli (2011). Does Poor Performance Damage the Reputation of Financial Intermediaries? Evidence from the Loan Syndication Market. *Journal of Finance 66*, 2083–2120.

Gorton, G. and G. Pennacchi (1995). Banks and Loan Sales: Marketing Non-Marketable Assets. *Journal of Monetary Economics 35*, 389–411.

Green, E. J. and R. H. Porter (1984). Noncooperative Collusion Under Imperfect Price Information. *Econometrica 52*, 87–100.

Hartman-Glaser, B. (2011). Reputation, Signaling, and Security Issuance. Working Paper, Duke University.

Holmström, B. (1999). Managerial Incentive Problems: A Dynamic Perspective. *Review of Economic Studies 66*, 169–182.

Holmström, B. and J. Tirole (1997). Financial Intermediation, Loanable Funds, and the Real Sector. *Quarterly Journal of Economics 112*, 663–691.

Keys, B. J., T. Mukherjee, A. Seru, and V. Vig (2010). Did Securitization Lead to Lax Screening? Evidence from Subprime Loans. *Quarterly Journal of Economics 125*, 307–362.

Kreps, D. M. and R. Wilson (1982). Reputation and Imperfect Information. *Journal of Economic Theory 27*, 253–279.

Leland, H. E. and D. H. Pyle (1977). Informational Assymetries, Financial Structure and Financial Intermediation. *Journal of Finance 32*, 371–387.

Mathis, J., J. McAndrews, and J.-C. Rochet (2009). Rating the Raters: Are Reputation Concerns Powerful Enough to Discipline Rating Agencies? *Journal of Monetary Economics 56*, 657–674.

Milgrom, P. and J. Roberts (1982). Predation, Reputation, and Entry Deterrence. *Journal of Economic Theory 27*, 280–312.

Parlour, C. A. and A. Winton (2013). Laying Off Credit Risk: Loan Sales versus Credit Default Swaps. *Journal of Financial Economics 107*, 25–45.

Piskorski, T., A. Seru, and V. Vig (2010). Securitization and Distressed Loan Renegotiation: Evidence from the Subprime Mortgage Crisis. *Journal of Financial Economics 97*, 369–397.

Purnanandam, A. (2011). Originate-to-Distribute Model and the Subprime Mortgage Crisis. *Review of Financial Studies 24*(6), 1881–1915.

Ramakrishnan, R. T. S. and A. Thakor (1984). Information Reliability and a Theory of Financial Intermediation. *Review of Economic Studies 51*, 415–432.

Appendix A

This appendix contains the proofs of all formal results stated in the paper.

Proof of Lemma 1: Suppose $V(0) - V(1) > \frac{m}{\delta\Delta} \Rightarrow V_{mon} > V_{shirk}$. Then the bank always strictly prefers to monitor, so $q(0) = q(1) = 1$. But, substituting $q(0) = q(1) = 1$ in the Bellman equation (8) yields $V(0) - V(1) = 0$, which contradicts incentive compatibility. Therefore, it must be that $V(0) - V(1) = \frac{m}{\delta\Delta}$.

Substituting $\delta\Delta\Lambda = m$ in the Bellman equation (8), and using that to compute the difference $\Lambda = V(0) - V(1)$, we obtain that $\Lambda = (q(0) - q(1))A$. As $\Lambda = \frac{m}{\delta\Delta} > 0$, it must be that $q(0) > q(1)$. ■

Proof of Proposition 1: Substituting $q(0) = 1$ and $q(1) = \hat{q}$, it follows that $\Lambda = V(0) - V(1) = (1 - \hat{q})A$ in a full monitoring equilibrium. Combining this with the incentive compatibility condition, $\Lambda = \frac{m}{\delta\Delta}$, it follows that $\hat{q} = 1 - \frac{m}{\delta\Delta A}$. Substituting $A = \Delta(1+\beta)(X - C)$ yields the expression for $\hat{q}$ in equation (11). For the equilibrium to be well defined, it must be that $\hat{q} > 0$, which yields the feasibility condition (10) in the Proposition.

We can now solve for the value function $V(d)$. Substituting $q(0) = 1$, $V(0) - V(1) = \frac{m}{\delta\Delta}$, and $V(1) = V(0) - \frac{m}{\delta\Delta}$ in equation (8) yields

$$V(0) = (A - m) + B + \delta \left(V(0) - (1 - p_H) \frac{m}{\delta\Delta} \right) \tag{33}$$

Substituting for A and B from equations (5) and (6), and solving for $V(0)$, yields $V(0) = V^*$. Next, since $V(0) - V(1) = \frac{m}{\delta\Delta}$, it must be that $V(1) = V^* - \frac{m}{\delta\Delta}$. ■

Proof of Lemma 2: *(1) Characterizing $E[n_{high}]$.*

As a high reputation bank monitors with probability 1, the probability of default and no default are $1 - p - \Delta$ and $p + \Delta$, respectively. Therefore, it follows that for any $t \geq 1$,

$\Pr(n_{high} = t) = (1 - p - \Delta) \cdot (p + \Delta)^{t-1}$. Hence,

$$E[n_{high}] = (1 - p - \Delta) \cdot \left(\sum_{t=1}^{\infty} t \cdot (p + \Delta)^{t-1} \right) = \frac{1}{1 - p - \Delta}, \tag{34}$$

where the last equation follows by noting that $\sum_{t=1}^{\infty} t \cdot (p + \Delta)^{t-1} = \frac{1}{(1-x)^2}$.[11]

(2) As a low reputation bank monitors with probability $p + \Delta\hat{q}$, it follows that for any $t \geq 1$, $\Pr(n_{low} = t) = (p + \Delta\hat{q}) \cdot (1 - p - \Delta\hat{q})^{t-1}$. Hence

$$E[n_{low}] = (p + \Delta\hat{q}) \left(\sum_{t=1}^{\infty} t \cdot (1 - p - \Delta\hat{q})^{t-1} \right) = \frac{1}{p + \Delta\hat{q}}. \tag{35}$$

The comparative statics follow by substituting $\hat{q} = 1 - \frac{m}{\delta(1+\beta)\Delta^2(X-C)}$ in the above equation. ∎

Proof of Lemma 3: (1) We prove part (1) by contradiction. Suppose the inequality in condition (15) is strict for some $d \in \{0, 1\}$. Then, it must be that $q(d) = 1$. Consider the following cases:

Case (a): $\alpha(d) > 0$. In this case, it is possible to choose an alternative $\hat{\alpha} = \alpha(d) - \varepsilon$ where $\varepsilon > 0$ such that the condition (15) still holds. Therefore, $q(d, \hat{\alpha}) = 1$, which means that $P(d, \hat{\alpha}) = p_H(R - C) + C = P(d, \alpha(d))$. But if $P(d, \hat{\alpha}) = P(d, \alpha(d))$, then it must be that $V(d, \hat{\alpha}) > V(d, \alpha(d))$ because the bank places a higher value on immediate liquidity. Hence, in equilibrium, it cannot be that $\alpha(d) > 0$ and condition 15 is strict.

Case (b): Suppose $\alpha(d) = 0$. Then, it must be that $\delta\Delta \cdot (V(0) - V(1)) > m$. Hence, condition (15) holds strictly for both $d = 0$ and $d = 1$, which implies that $q(0) = q(1) = 1$. But then, it follows from the Bellman equation that $V(0) - V(1) = 0$, which contradicts the assumption that $\delta\Delta \cdot (V(0) - V(1)) > m$.

Hence, in equilibrium, the incentive compatibility constraint binds with equality.

[11] To see why, let $Y = \sum_{t=1}^{\infty} tx^{t-1}$. Then $Y - xY = \sum_{t=0}^{\infty} x^t = \frac{1}{1-x}$, which implies that $Y = \frac{1}{(1-x)^2}$.

(2) If the IC holds with equality, then $\alpha(d)$ is given by equation (19). Moreover, the Bellman equation simplifies to:

$$V(d) = (1+\beta)(1-\alpha(d)) \cdot P(q(d)) + \alpha(d) \cdot C - (1+\beta) + \frac{mp}{\Delta} + \delta V(1) \tag{36}$$

Substituting $\alpha C = C - (1-\alpha)C$ and the expression for $P(q)$ in the above equation yields the expression in equation (20). Computing $V(0)$ and $V(1)$ using the Bellman equation (20), and differencing them yields:

$$\begin{aligned} V(0) - V(1) &= \Delta(1+\beta)(X-C) \cdot [(1-\alpha(0)) \cdot q(0) - (1-\alpha(1)) \cdot q(1)] \\ &\quad +(\alpha(1) - \alpha(0)) \cdot [(1+\beta)(p(X-C) - u) + \beta C] \end{aligned} \tag{37}$$

Recall that $\delta\Delta\Lambda > 0$ in a reputation equilibrium. Suppose $q(0) \leq q(1)$ in a reputation equilibrium. Then, it follows from equation (19) that $\alpha(0) \geq \alpha(1)$. But that would imply $V(0) - V(1) \leq 0$ (because $\Delta(1+\beta)(X-C) > 0$ and $p(X-C) - u > 0$), which contradicts the incentive compatibility requirement that $\delta\Delta\Lambda > 0$. Therefore, in a reputation equilibrium, it must be that $q(0) > q(1)$, which in turn, implies that $\alpha(0) < \alpha(1)$. ■

Proof of Proposition 2: Note that if the IC condition (16) binds with equality, then it must be that

$$\alpha(d) \cdot [(p + \Delta q(d)) \cdot (X-C) - u] = (p + \Delta q(d)) \cdot \frac{(1-\rho)m}{\Delta} \tag{38}$$

Making this substitution in the Bellman equation (20), and simplifying yields,

$$\begin{aligned} V(d) &= (1+\beta)\left[(p + \Delta q(d))\left(X - C - \frac{(1-\rho)m}{\Delta}\right) - u - 1\right] \\ &\quad +(1 + \beta[1-\alpha(d)])C + \frac{mp}{\Delta} + \delta V(1) \end{aligned} \tag{39}$$

Using the above equation for $V(d)$, we can characterize $\Lambda = V(0) - V(1)$ as follows:

$$\begin{aligned} \Lambda &= (1+\beta)\Delta \cdot (q(0) - q(1)) \cdot \left(X - C - \frac{(1-\rho)m}{\Delta} \right) \\ &\quad + [\alpha(1) - \alpha(0)] \cdot \beta C \end{aligned} \tag{40}$$

(1) In a full monitoring equilibrium, $q(0) = 1$ and $q(1) = \hat{q}(\rho)$, where $\hat{q}(\rho)$ is to be characterized. Then, it follows from equation (19) that $\alpha(0, \rho) = (1-\rho)\alpha_{sp}$ and

$$\alpha(1, \rho) = \frac{m(1-\rho)(p + \Delta\hat{q}(\rho))}{\Delta[(p + \Delta\hat{q}(\rho))(X - C) - u]} \tag{41}$$

Substituting $\Lambda = \frac{\rho m}{\delta \Delta}$ and the expressions for $\alpha(0, \rho)$ and $\alpha(1, \rho)$ in equation (40), and simplifying, yields

$$\begin{aligned} \frac{\rho m}{\delta \Delta} &= (1+\beta)\Delta \cdot (1 - \hat{q}(\rho)) \cdot \left(X - C - \frac{(1-\rho)m}{\Delta} \right) \\ &\quad + \left[\frac{m(p + \Delta\hat{q}(\rho))}{\Delta((p + \Delta\hat{q}(\rho))(X - C) - u)} - \alpha_{sp} \right] \cdot (1-\rho)\beta C \end{aligned} \tag{42}$$

In general, it is difficult to obtain tractable, closed-form solutions for $\hat{q}(\rho)$. However, if $C = 0$, then $\hat{q}(\rho)$ is a solution to the following equation:

$$(1+\beta)\delta\Delta \cdot (1 - \hat{q}(\rho)) \cdot (\Delta X - (1-\rho)m) - \rho m = 0. \tag{43}$$

Solving the above equation for $\hat{q}(\rho)$ yields the expression in equation (21). For the equilibrium to be well-defined, it is necessary that $\hat{q} \geq 0$, which is equivalent to condition (23) in Proposition 2.

(2) Substituting $q(0) = 1$, $C = 0$ and $\delta V(1) = \delta V(0) - \frac{\rho m}{\Delta}$ in equation (39) yields

$$\begin{aligned}
(1-\delta)V(0,\rho) &= (1+\beta)\left[(p+\Delta)\left(X - \frac{(1-\rho)\,m}{\Delta}\right) - u - 1\right] + \frac{mp}{\Delta} - \frac{\rho m}{\Delta} \\
&= v - \frac{m}{\Delta}\left(1 - p + \beta\left(1-\rho\right)\right),
\end{aligned}$$

where the second equation is obtained by substituting $v = (1+\beta)[(p+\Delta)X - u - 1]$. Solving the above equation for $V(0,\rho)$, and substituting $V^* = \frac{1}{(1-\delta)}(v - \frac{m}{\Delta}(1-p))$, yields the expression for $V(0,\rho)$ in equation (24). ■

Proof of Lemma 4: (1) Let LHS denote the expression on the left-hand side of equation (43) that characterizes $\hat{q}(\rho)$. By implicit differentiation, is a solution to equation (43). By implicit differentiation, we obtain:

$$\begin{aligned}
\frac{d\hat{q}}{d\rho} &= -\frac{\partial LHS/\partial\rho}{\partial LHS/\partial\hat{q}} = \frac{m[(1+\beta)\delta\Delta\cdot(1-\hat{q}) - 1]}{(1+\beta)\delta\Delta\cdot(\Delta X - (1-\rho)m)} \\
&= -\frac{m(\Delta X - m)}{(1+\beta)\delta\Delta\cdot(\Delta X - (1-\rho)m)^2},
\end{aligned}$$

where the last equation is obtained by substituting $(1+\beta)\delta\Delta\cdot(1-\hat{q}) = \frac{\rho m}{\Delta X - (1-\rho)m}$ and simplifying. As $\Delta X > m$ by Assumption 2, it follows that $\frac{d\hat{q}}{d\rho} < 0$.

(2) Observe that condition (23) is satisfied for $\rho = 0$ because $\Delta X \geq m$ (by Assumption 2). If $(1+\beta)\delta\Delta \geq 1$, then the right-hand side of condition (23) increases with ρ, and hence, will exceed m for all $\rho > 0$. By the converse logic, if $(1+\beta)\delta\Delta < 1$, then the feasibility condition is less likely to be met for higher ρ.

Next, the full monitoring equilibrium is feasible if, and only if, $\hat{q}(\rho) \geq 0$. As $\hat{q}(\rho)$ is decreasing in ρ, the feasibility condition is less likely to be met as ρ increases.

(3) It is obvious from equation (24) that $V(0,\rho)$ increases as ρ increases. Next,

$$\frac{dV(1,\rho)}{d\rho} = \frac{m\beta}{(1-\delta)\Delta} - \frac{m}{\delta\Delta} = \frac{m}{\delta\Delta}\left(\frac{\delta(1+\beta) - 1}{1-\delta}\right) \tag{44}$$

Therefore, $V(1, \rho)$ is increasing in ρ if $\delta(1+\beta) \geq 1$, and is decreasing in ρ otherwise. ■

Proof of Lemma 5: Because IC(γ) is stricter than IC(1), there are five cases to consider: (1) $\Lambda > \gamma m/\delta\Delta \implies$ IC(1) and IC(γ) both hold strictly; (2) $\Lambda = \gamma m/\delta\Delta \implies$ IC(γ) binds, IC(1) holds strictly; (3) $\gamma m/\delta\Delta > \Lambda > m/\delta\Delta \implies$ IC(γ) fails, IC(1) holds strictly; (4) $\Lambda = m/\delta\Delta \implies$ IC(γ) fails, IC(1) binds; (5) $m/\delta\Delta > \Lambda \implies$ IC(γ) and IC(1) both fail.

The proof of the lemma boils down to showing that Cases (1), (3), and (5) above are inconsistent with any monitoring in equilibrium.

(a) If Case (1) holds, then $q(d, \ell) = 1$ for all d and ℓ, so the bank monitors with certainty regardless of its reputation or its lending volume. In a normal economy, the bank lends 1 unit and sells this for surplus $S(d, 1) = A + B$, so its discounted profits are

$$V_n(d) = A + B - m + \delta[(p + \Delta)\Lambda + V(1)] \equiv V_n, \tag{45}$$

regardless of its reputation d. If the economy is in a boom and the bank lends 1 unit, it also gets $V_b(d) = V_n$; if it lends γ, it gets

$$V_b(d) = \gamma(A + B - m) + \delta[(p + \Delta)\Lambda + V(1)] \equiv V_{b,\gamma} > V_n. \tag{46}$$

It follows that the bank always lends γ in a boom. But then we have $V(d) = \phi V_n + (1 - \phi)V_{b,\gamma}$ regardless of reputation d. Thus, $\Lambda = V(0) - V(1) = 0$, contradicting the assumption of Case (1).

(b) If Case (3) holds, then $q(d, 1) = 1$ and $q(d, \gamma) = 0$ for any reputation d. It follows immediately that, in a normal economy, the bank gets $V_n(d) = V_n$ as in Case (1).

If the economy is in a boom and $\ell_b(d) = 1$, the bank also monitors with certainty, so it gets $V_b(d) = V_n$, too, regardless of reputation. If instead $\ell_b(d) = \gamma$, the bank does not monitor, so its surplus per unit is B, and it gets

$$V_b(d) = \gamma B + \delta[p\Lambda + V(1)] = V_{b,nm} \tag{47}$$

regardless of reputation.

If $\ell_b(0) = \ell_b(1)$, an analysis similar to that in (a) shows that $V(0) = V(1)$, and so $\Lambda = 0$, contradicting the assumption of Case (3). Thus, we must have $\ell_b(0) \neq \ell_b(1)$.

If $\ell(0) = \gamma$ and $\ell(1) = 1$, then $V_b(0) = V_{b,nm}$ and $V_b(1) = V_n$. But if $V_{b,nm} < V_n$, a bank with reputation $d = 0$ strictly prefers to switch to lending 1, breaking the proposed equilibrium. If $V_{b,nm} > V_n$, a bank with reputation $d = 1$ strictly prefers to switch to lending γ, also breaking the equilibrium. Finally, if $V_{b,nm} = V_n$, then $V(d) = \phi V_n + (1 - \phi)V_b(d)$ has the same value for both $d = 0$ and $d = 1$, which leads to $\Lambda = 0$, contradicting the assumption of Case (3).

If instead $\ell(0) = 1$ and $\ell(1) = \gamma$, then the analysis is essentially the same.

(c) If Case (5) holds, then the bank never finds it incentive compatible to monitor, so $q(d, \ell) = 0$ for all d and ℓ, and monitoring never takes place in equilibrium. ■

Proof of Proposition 3: If IC(1) binds, IC(γ) must fail, so that if the bank lends γ in a boom, it will not monitor its loans. There are four possible cases to consider, based on how the bank chooses its lending volume $\ell \in \{1, \gamma\}$ in boom, in the high and low reputation states. In each case, we must check three necessary conditions: (a) IC(1) must bind; (b) in a boom, a high-reputation bank must not want to switch its loan volume choice, $\ell_b(0)$; and (c) in a boom, a low-reputation bank must not want to switch its loan volume choice, $\ell_b(1)$.

Case (1): $\ell_b(0) = \ell_b(1) = 1$. Condition (a) is $\Lambda = V(0) - V(1) = m/\delta\Delta$. First, it is easy to see that, because loan volumes and thus monitoring choices are the same in normal and boom states, we will have $V_b(d) = V_n(d) = V(d)$. Making use of IC(1) being binding, it follows that $V(0) = q(0,1)A + B + \frac{pm}{\Delta} + \delta V(1)$ and $V(1) = q(1,1)A + B + \frac{pm}{\Delta} + \delta V(1)$. Therefore,

$\Lambda = [q(0,1) - q(1,1)]A = \frac{m}{\delta\Delta}$; thus, as in the main paper, we must have $q(0,1) > q(1,1)$ to ensure that $\Lambda > 0$.

To examine condition (b), note that if the high-reputation bank switches to a loan volume of γ, it won't monitor, and so it will receive $\gamma B + \frac{pm}{\Delta} + \delta V(1)$. This cannot exceed $V_b(0) = V(0)$; comparing with the expression for $V(0)$ and rearranging, we have $(\gamma - 1)B \leq q(0,1)A$.

Similarly, if the low-reputation bank switches to a loan volume of γ , it won't monitor, and so it will receive $\gamma B + \frac{pm}{\Delta} + \delta V(1)$. Condition (c) requires that this cannot exceed $V_b(1) = V(1)$; similar steps show that this is equivalent to $(\gamma - 1)B \leq q(1,1)A$.

Because $q(0,1) > q(1,1)$, it is clear that condition (c) is always more binding than condition (a). Furthermore, it is most likely to hold when $q(1,1)$ is as high as possible, which corresponds to $q(0,1) = 1$ and $q(1,1) = \hat{q} = 1 - \frac{m}{\delta\Delta A}$ so that IC(1) binds. However, feasibility also requires that condition (c), $(\gamma - 1)B \leq q(1,1)A$, is met. Substituting for $q(1,1)$ and rearranging yields the feasibility condition $m \leq \delta\Delta(A - (\gamma - 1)B)$. Sufficiency of this condition follows by noting that, when this condition is met, the equilibrium characterized in part (1) of Proposition 3 is feasible.

Case (2): $\ell_b(0) = 1,\ \ell_b(1) = \gamma$. In this case, $V_b(0) = V_n(0)$ and $V_b(1) = \gamma B + \frac{pm}{\Delta} + \delta V(1)$. Note that $\Lambda = \phi[V_n(0) - V_n(1)] + (1-\phi)[V_b(0) - V_b(0)]$. Rearranging terms, it is easily shown that condition (a) is equivalent to

$$q(0,1)A - \phi q(1,1)A - (1-\phi)(\gamma - 1)B = m/\delta\Delta. \tag{48}$$

Condition (b) requires that $V_b(0) = V_n(0)$ dominates lending γ and receiving an expected payoff of $\gamma B + pm/\Delta + \delta V(1)$, which is equivalent to the condition $q(0,1)A \geq (\gamma - 1)B$. Finally, condition (c) requires that $V_b(1)$ dominates lending 1 and receiving an expected payoff of $V_n(1)$, which is equivalent to $q(1,1)A \leq (\gamma - 1)B$.

Clearly, for condition (a) to be satisfied, it is necessary that $A \geq (1-\phi)(\gamma - 1)B + m/\delta\Delta$ (Condition "F1"). Also, condition (b) requires that $A \geq (\gamma - 1)B$ (Condition "F2"). This proves the necessity of these two feasibility conditions.

To prove sufficiency, we show that if feasibility conditions F1 and F2 are met, then we can find monitoring probabilities $q(0,1)$ and $q(1,1)$ that are consistent with conditions (a), (b), and (c). Consider $q(0,1) = q_0 \equiv \min\{1, [\frac{(\gamma-1)B}{A} + \frac{m}{\delta\Delta A}]\}$ and $q(1,1) = q_0 - \frac{m}{\delta\Delta A} - \frac{(1-\phi)(\gamma-1)B}{A}$, which have been constructed such that condition (a) is satisfied. Moreover, it is clear that $0 < q_0 \leq 1$. It only remains to be shown that $q(1,1) \in [0,1]$ and that conditions (b) and (c) are satisfied. Consider the following two subcases:

(i) Suppose $A \geq (\gamma-1)B + m/\delta\Delta$ so that $q_0 = (\gamma-1)B/A + m/\delta\Delta A < 1$. Then it must be that $q(1,1) = \phi(\gamma-1)B/A < 1$, where the inequality follows from condition F2 and because $\phi < 1$. Condition (c) is met because $q(1,1)A = \phi(\gamma-1)B < (\gamma-1)B$. Also, condition (b) is satisfied because $q(0,1)A = (\gamma-1)B + m/\delta\Delta > (\gamma-1)B$.

(ii) Suppose $A < (\gamma-1)B + m/\delta\Delta$ so that $q_0 = 1$. Then $q(1,1) = 1 - \frac{m}{\delta\Delta A} - \frac{(1-\phi)(\gamma-1)B}{A} > 0$ by condition F1. Also in this case, $q(1,1) \leq \frac{\phi(\gamma-1)B}{A}$, which implies that condition (c) is satisfied. Finally, condition (b) is also satisfied because $q(0,1)A = A \geq (\gamma-1)B$ by condition F2.

Case (3): $\ell_b(0) = \ell_b(1) = \gamma$. In this case, $V_n(d)$ is the same as in Case (1) above, but $V_b(d) = \gamma B + \frac{pm}{\Delta} + \delta V(1)$. Then, $\Lambda = V(0) - V(1) = \phi[V_n(0 - V_n(1)] = \phi[q(0,1) - q(1,1)] \cdot A$. Hence, IC(1) can bind only if $q(0,1) > q(1,1)$.

If the high-reputation bank switches to lending 1 in a boom, it receives $V_n(0)$. Condition (b) requires that this be no more than $V_b(0)$; comparing with the expression for $V_b(0)$ and rearranging, this is equivalent to $q(0,1)A \leq (\gamma-1)B$. Similar logic shows that condition (c) is equivalent to $q(1,1)A \leq (\gamma-1)B$. Because $q(0,1) > q(1,1)$, this is implied by condition (b).

For condition (a) to be satisfied, it is necessary that $\phi A \geq \frac{m}{\delta\Delta}$, that is, $m \leq \phi\delta\Delta A$. In addition, it is also necessary that $q(0,1)\phi A \geq \frac{m}{\delta\Delta}$; combining this with the requirement that

$(\gamma-1)B \geq q(0,1)A$ (condition (b)) yields the necessity of the condition $\phi(\gamma-1)B \geq \frac{m}{\delta\Delta}$, or equivalently, $m \leq \phi\delta\Delta(\gamma-1)B$. These two necessary conditions can be combined into a single necessary condition, $m \leq \phi\delta\Delta\min\{A, (\gamma-1)B\}$.

To prove sufficiency, suppose $m \leq \phi\delta\Delta\min\{A, (\gamma-1)B\}$. Then, consider the monitoring probabilities $q(0,1) = \min\{1, \frac{(\gamma-1)B}{A}\}$ and $q(1,1) = \min\{1, \frac{(\gamma-1)B}{A}\} - \frac{m}{\phi\delta\Delta A}$, which have been constructed so that condition (a) is satisfied. The feasibility condition guarantees that $q(1,1) \geq 0$, which also implies that $q(0,1) > 0$ so that these probabilities are well defined ($q(1,1) < q(0,1) \leq 1$ by construction). Next, both conditions (b) and (c) are satisfied because $q(0,1)A = \min\{A, (\gamma-1)B\} \leq (\gamma-1)B$.

Case (4): $\ell_b(0) = \gamma$, $\ell_b(1) = 1$. Then $V_n(d)$ is as in Case (1) above, $V_b(1) = V_n(1)$, and $V_b(0) = \gamma B + pm/\Delta + \delta V(1)$. By a similar logic as in case (2), condition (a) becomes

$$\phi[q(0,1) - q(1,1)] \cdot A + (1-\phi)[(\gamma-1)B - q(1,1)A] = \frac{m}{\delta\Delta}. \tag{49}$$

Turning to condition (b), if the high-reputation bank deviates and lends 1, it earns $V_n(0)$; thus, condition (b) becomes $V_b(0) \geq V_n(0)$, which is equivalent to $(\gamma-1)B \geq q(0,1)A$.

Condition (c) requires that the low-reputation bank prefers $V_b(1) = V_n(1)$ over lending γ and earning $\gamma B + pm/\Delta + \delta V(1)$. Thus, condition (c) becomes $(\gamma-1)B \leq q(1,1)A$.

Combining conditions (a) and (c), we have $\phi[q(0,1) - q(1,1)] \cdot A > m/\delta\Delta$, which in turn implies $q(0,1) > q(1,1)$. Thus, it must be that $q(0,1)A > q(1,1)A \geq (\gamma-1)B$, where the last inequality follows from condition (c). But this contradicts condition (b). Hence, it follows that Case (4) cannot exist. ∎

Proof of Lemma 6: (i) By the usual method, we can rule out the possibility that $\Lambda > \frac{m}{\delta\Delta}$, because then $q(0) = q(1) = 1 \Rightarrow \Lambda = 0$. Therefore, $\Lambda = \frac{m}{\delta\Delta}$. Then it follows that

$\bar{S}(0) = \delta\Lambda(1-p)$ and $\bar{S}(1) = -\delta p\Lambda$. Solving for $\bar{R}(0)$ and $\bar{R}(1)$ yields

$$\bar{R}(0) = C + \frac{1}{(p + \Delta q(0))} \cdot \left(1 - C + \frac{\delta\Lambda(1-p)}{(1+\beta)}\right)$$

and

$$\bar{R}(1) = C + \frac{1}{(p + \Delta q(1))} \cdot \left(1 - C - \frac{\delta p\Lambda}{1+\beta}\right).$$

The borrower's surplus at the high-reputation bank's break-even rate is

$$\begin{aligned}
\bar{U}(0) &\equiv (p + \Delta q(0))\left(X - \bar{R}(0)\right) \\
&= (p + \Delta q(0))(X - C) + C - 1 - \frac{\delta\Lambda(1-p)}{(1+\beta)}.
\end{aligned}$$

Similarly, the borrower's surplus at the low-reputation bank's break even rate is

$$\begin{aligned}
\bar{U}(1) &\equiv (p + \Delta q(1))\left(X - \bar{R}(1)\right) \\
&= (p + \Delta q(1))(X - C) + C - 1 + \frac{\delta p\Lambda}{1+\beta}.
\end{aligned}$$

Note that

$$\begin{aligned}
\bar{U}(0) - \bar{U}(1) &= \Delta(q(0) - q(1))(X - C) - \frac{\delta\Lambda}{(1+\beta)} \\
&= \frac{A[q(0) - q(1)] - \delta\Lambda}{(1+\beta)}.
\end{aligned} \tag{50}$$

Finally, as in the base model,

$$\begin{aligned}
V(d, none) &= Aq(d) + B - mq(d) + \delta[V(1) + (p + \Delta q(d))\Lambda] \\
&= Aq(d) + B + \delta[V(1) + p\Lambda].
\end{aligned}$$

(ii) We will prove by contradiction that $q(0) > q(1)$. Suppose not, i.e., let's suppose that

$q(0) < q(1)$.

Then, $\bar{U}(1) > \bar{U}(0)$; the low-reputation bank's break-even offer is more attractive to the borrower than the high-reputation bank's break-even offer. So when a low-reputation incumbent meets a high-reputation rival, the incumbent can win the loan by offering a rate R_{10} at which the borrower's surplus is $\bar{U}(0)$; i.e., $(p + \Delta q(1))(X - R_{10}) = \bar{U}(0)$. The low-reputation incumbent's current surplus under this offer is

$$S_{10} = (1 + \beta)\left[(p + \Delta q(1))(R_{10} - C) + C - 1\right].$$

Expanding $R_{10} - C = X - C - (R_{10} - C)$, and substituting $(p + \Delta q(1))(X - R_{10}) = \bar{U}(0)$, we obtain $S_{10} = \delta\Lambda(1 - p) + [q(1) - q(0)]A$. The low-reputation incumbent's continuation value in this case is

$$\begin{aligned} V_{10} &= S_{10} - mq(1) + \delta\left[V(1) + (p + \Delta q(1))\Lambda\right] \\ &= \delta\Lambda + [q(1) - q(0)]A + \delta V(1) \end{aligned}$$

Therefore, we have the following expression for $V(1)$:

$$\begin{aligned} V(1) &= (1 - \lambda)V(1, none) + \lambda\varphi V_{10} + \lambda(1 - \varphi)\delta V(1) \\ &= (1 - \lambda)V(1, none) + \lambda\varphi(\delta\Lambda + [q(1) - q(0)]A) + \lambda\delta V(1) \end{aligned}$$

On the other hand, a high-reputation incumbent's continuation value when faced with a rival is $\delta V(0)$, regardless of the rival's reputation, and whether or not the incumbent gets to make the loan. Hence,

$$V(0) = (1 - \lambda)V(0, none) + \lambda\delta V(0)$$

Subtracting $V(1)$ from $V(0)$ yields

$$\Lambda = -A(1-\lambda)[q(1)-q(0)] + \lambda\delta\Lambda - \lambda\varphi(\delta\Lambda + [q(1)-q(0)]A)$$

Rearranging the above equation yields

$$\begin{aligned}\Lambda(1-\lambda\delta) &= -A(1-\lambda)[q(1)-q(0)] - \lambda\varphi(\delta\Lambda + [q(1)-q(0)]A) \\ &< 0 \text{ because } q(1) \geq q(0).\end{aligned}$$

However, that contradicts the requirement that $\Lambda = \frac{m}{\delta\Delta} > 0$. Therefore, it must be that $q(0) > q(1)$. ■

Proof of the Lemma 7: *Step 1: We prove that it is not feasible to have a monitoring equilibrium in which $\bar{U}(0) \leq \bar{U}(1)$. with $A[q(0)-q(1)] \leq \delta\Lambda$.*

We prove this by contradiction. Suppose there exists an equilibrium with $\bar{U}(0) \leq \bar{U}(1)$. From Equation (50), this is equivalent to the condition, $A[q(0)-q(1)] \leq \delta\Lambda$. The expressions for $V(0)$ and $V(1)$ are the same as those in the proof of Lemma 6. Hence, we have the following expression for $\Lambda = V(0) - V(1)$:

$$\Lambda = A(1-\lambda)[q(0)-q(1)] + \lambda\delta\Lambda - \lambda\varphi(\delta\Lambda - [q(0)-q(1)]A).$$

Rearranging terms, we obtain

$$\Lambda[1-\lambda\delta(1-\varphi)] = A[q(0)-q(1)] \cdot [1-\lambda(1-\varphi)]$$

Hence, it must be that

$$A[q(0)-q(1)] = \Lambda\left(\frac{1-\lambda\delta(1-\varphi)}{1-\lambda(1-\varphi)}\right) > \Lambda,$$

where the inequality follows because $\delta < 1$. But this contradicts the assumption that $A[q(0) - q(1)] \leq \delta\Lambda$. Hence, it is not feasible to have a reputational monitoring equilibrium in which $\bar{U}(0) \leq \bar{U}(1)$.

Step 2: We characterize how competition is resolved in any monitoring equilibrium.

(i) It is clear that if the incumbent and the rival have the same reputation d, then each bank will bid the break-even rate $\bar{R}(d)$ at which it is indifferent between making the loan and withdrawing. Hence, the incumbent's continuing value is $\delta V(d)$.

(ii) Consider the case when a high-reputation bank competes with a low-reputation bank. Given that $\bar{U}(0) > \bar{U}(1)$ in any feasible monitoring equilibrium, the high-reputation bank will win the loan by offering a rate R_{01} at which the borrower is indifferent between its offer and the low-reputation bank's offer; i.e., $(p + \Delta q(0))(X - R_{01}) = \bar{U}(1)$. The high-reputation bank's surplus if the offer is accepted is $S_{01} \equiv (p + \Delta q(0))(R_{01} - C) + C - 1$. After expanding $R_{01} - C = X - C - (X - R_{01})$, and substituting $(p + \Delta q(0))(X - R_{01}) = \bar{U}(1)$ from the borrower's indifference condition, we obtain that

$$\begin{aligned} S_{01} &= (1+\beta) \cdot \left[(p + \Delta q(0))(X - C) - \bar{U}(1) + C - 1\right] \\ &= A[q(0) - q(1)] - \delta p \Lambda. \end{aligned}$$

The high-reputation bank's continuation value if it makes the loan is

$$\begin{aligned} V_{01} &= S_{01} - mq(0) + \delta[V(1) + (p + \Delta q(0))\Lambda] \\ &= A[q(0) - q(1)] + \delta V(1). \end{aligned}$$

From Step 1 of the proof, we have $\bar{U}(0) > \bar{U}(1)$, which is equivalent to the condition that $A[q(0) - q(1)] > \delta\Lambda$. It follows that $V_{01} > \delta V(0)$. On the other hand, the low reputation bank's continuation value is $\delta V(1)$. ■

Proof of the Proposition 4: First, we establish the feasibility condition for a monitoring equilibrium. As shown in Lemma 7, we must have $A[q(0) - q(1)] > \delta\Lambda$ in any monitoring equilibrium. Combining this with our assumptions regarding the manner in which ties are resolved, we have the following expression for $V(0)$:

$$\begin{aligned} V(0) &= (1-\lambda)V(0, none) + \lambda\varphi\delta V(0) + \lambda(1-\varphi)V_{01} \\ &= (1-\lambda)V(0, none) + \lambda\delta V(0) + \lambda(1-\varphi)([q(0) - q(1)]A - \delta\Lambda) \end{aligned}$$

The expression for $V(1)$ is as follows:

$$V(1) = (1-\lambda)V(1, none) + \lambda\delta V(1)$$

Differencing the above equations yields

$$\Lambda = A(1-\lambda)[q(0) - q(1)] + \lambda\delta\Lambda + \lambda(1-\varphi)([q(0) - q(1)]A - \delta\Lambda)$$

Rearranging terms yields

$$\Lambda(1 - \lambda\delta\varphi) = A[q(0) - q(1)] \cdot (1 - \lambda\varphi)$$

Substituting $\Lambda = \frac{m}{\delta\Delta}$ and solving for $q(1)$ yields

$$q(1) = q(0) - \frac{m(1-\lambda\delta\varphi)}{\delta\Delta A(1-\lambda\varphi)}.$$

As $q(0) \leq 1$, the requirement that $q(1) > 0$ can be met only if condition (32) is satisfied.

Sufficiency follows by noting that when condition (32) is satisfied, then the monitoring equilibrium with $q(0) = 1$ and $q(1) = 1 - \frac{m(1-\lambda\delta\varphi)}{\delta\Delta A(1-\lambda\varphi)}$ is feasible. Note that, in this equilibrium, $[q(0) - q(1)]A = \frac{\Lambda(1-\lambda\delta\varphi)}{(1-\lambda\varphi)} > \Lambda$, which ensures that $\bar{U}(0) > \bar{U}(1)$, as required by Lemma 7.

1. In a full-monitoring equilibrium, $q(0) = 1$ and $q(1) = \hat{q} = 1 - \frac{m(1-\lambda\delta\varphi)}{\delta\Delta A(1-\lambda\varphi)}$. Then,

$$\begin{aligned} V(0, none) &= A + B + \delta(V(1) + p\Lambda) \\ &= v + \delta V(0) - \frac{m(1-p)}{\Delta}, \end{aligned}$$

where the second equation is obtained by substituting $v = A + B$, $V(1) = V(0) - \Lambda$ and $\Lambda = \frac{m}{\delta\Delta}$. Substituting the expression for $V(0, none)$, $\Lambda = \frac{m}{\delta\Delta}$, and $1 - \hat{q} = \frac{m(1-\lambda\delta\varphi)}{\delta\Delta A(1-\lambda\varphi)}$ into the expression for $V(0)$ in step 1, and simplifying, yields:

$$V(0) = (1-\lambda)\left[v + \delta V(0) - \frac{m(1-p)}{\Delta}\right] + \lambda\delta V(0) + \frac{m\lambda(1-\varphi)(1-\delta)}{\delta\Delta(1-\lambda\varphi)}$$

Solving the above equation for $V(0)$ yields the expression for $V^*_{compete}$ in Equation (31). Next, by the IC condition, it must be that $V(1) = V^*_{compete} - \frac{m}{\delta\Delta}$.

2. It is easy to show that the expression on the right-hand side of Condition (32) is decreasing in λ and φ. Similarly, it is easy to show that $V^*_{compete}$ is decreasing in φ, as claimed.

Thus, we need only examine how $V^*_{compete}$ changes with λ. Note that,

$$\frac{dV^*_{compete}}{d\lambda} = -V^* + \frac{m}{\delta\Delta} \cdot \frac{1-\varphi}{(1-\lambda\varphi)^2}.$$

This derivative is positive if and only if

$$V^* < \frac{m}{\delta\Delta} \cdot \frac{1-\varphi}{(1-\lambda\varphi)^2} \tag{51}$$

We also have

$$\frac{m}{\delta\Delta} \leq A\frac{1-\lambda\varphi}{1-\delta\lambda\varphi} < A, \tag{52}$$

where the first inequality follows from Condition (32) and the second inequality follows because $\delta < 1$ and λ and $\varphi \leq 1$.

Substituting into Condition (51) for V^*, rearranging, and using $v \equiv A + B$, we have that this condition is equivalent to

$$\begin{aligned} v - \frac{m(1-p)}{\Delta} &< \frac{m}{\delta\Delta} \cdot \frac{(1-\delta)(1-\varphi)}{(1-\lambda\varphi)^2} \Leftrightarrow \\ A + B &< \frac{m}{\delta\Delta}\left\{\delta(1-p) + (1-\delta)\frac{1-\varphi}{(1-\lambda\varphi)^2}\right\}. \end{aligned}$$

Combining this with Condition (52), it follows that $dV^*_{compete}/d\lambda$ is positive only if

$$A + B < A(1-\delta\lambda\varphi)^{-1}\left\{\delta(1-p)(1-\lambda\varphi) + (1-\delta)\frac{1-\varphi}{1-\lambda\varphi}\right\}.$$

Now, because $0 \leq 1 - \varphi \leq 1 - \lambda\varphi$, the RHS of this expression is less than or equal to

$$\begin{aligned} & A(1-\delta\lambda\varphi)^{-1}\{\delta(1-p)(1-\lambda\varphi) + (1-\delta)\} \\ = \; & A(1-\delta\lambda\varphi)^{-1}\{1 - \delta\lambda\varphi - p\delta(1-\lambda\varphi)\} \\ = \; & A \cdot \frac{1-\delta\lambda\varphi - p\delta(1-\lambda\varphi)}{1-\delta\lambda\varphi} = A \cdot \left[1 - \frac{p\delta(1-\lambda\varphi)}{1-\delta\lambda\varphi}\right] < A, \end{aligned}$$

where the last inequality follows from $p\delta < 1$ and $1 - \lambda\varphi \leq 1 - \delta\lambda\varphi$. But then it follows that if $V^*_{compete}$ increases with λ, we must have $A + B < A$, which is a contradiction. Thus, whenever a monitoring equilibrium is feasible, $V^*_{compete}$ decreases with λ. ■

Appendix B

This Appendix contains several extensions of the basic model.

B.1. Characterizing the Reputation Equilibrium with $N = 2$

We now characterize equilibria where $N = 2$, i.e., where market participants condition their beliefs about the bank's monitoring based on the number of defaults d incurred by the bank in the previous 2 periods. Because the bank can experience two outcomes every period (default or no default), its past performance profile, x, can take on $2^2 = 4$ possible combinations. Denoting default and no default by 1 and 0, respectively, the four possible combinations are: 00, 01, 10 and 11, where the *left-most digit denotes the outcome in the most recent period.* Observe that in the binary system, these performance profiles correspond to $x = 0$, 1, 2 and 3, respectively. The number of defaults, d, is obtained by summing the two digits in the performance profile; i.e.,

$$d(x) = \begin{cases} 2 & \text{if} \quad x = 3 \\ 1 & \text{if} \quad x = 1, 2 \\ 0 & \text{if} \quad x = 0 \end{cases} \tag{53}$$

Observe that while both the performance profiles $x = 1$ and $x = 2$ have the same reputation today (because $d(1) = d(2) = 1$), the default is more recent in the $x = 2$ profile compared with the $x = 1$ profile. As we show below, this affects the transition in the bank's reputation over the next period.

The bank's next period performance profile and reputation will depend on whether or not its current period loan defaults. Given the current performance profile x, let $x^{-}(x)$ and $x^{+}(x)$ denote its performance profile next period following a default and no default, respectively. It is easily verified that

$$x^-(x) = \begin{cases} 2 & \text{if} \quad x = 0, 1 \\ 3 & \text{if} \quad x = 2, 3 \end{cases} \tag{54}$$

and

$$x^+(x) = \begin{cases} 0 & \text{if} \quad x = 0, 1 \\ 1 & \text{if} \quad x = 2, 3 \end{cases} \tag{55}$$

Let $V(x)$ denote the expected discounted value of the bank's profits in equilibrium, given the performance profile x. By the same intuition as in the $N = 1$ case, monitoring is incentive compatible for a bank with the performance profile x only if $V(x^+) - V(x^-) \geq \frac{m}{\delta\Delta}$. Using the performance profile transitions in equations (54) and (55), we obtain the following incentive compatibility conditions:

$$V(0) - V(2) \geq \frac{m}{\delta\Delta}, \tag{56a}$$

$$V(1) - V(3) \geq \frac{m}{\delta\Delta} \tag{56b}$$

By the same logic as in Section 2, the Bellman equation can be written as

$$V(x) = q(d(x)) \cdot (A - m) + B + \delta(p + \Delta q(d(x))) \cdot (V(x^+) - V(x^-)) + \delta V(x^-) \tag{57}$$

Lemma 8 *In any monitoring equilibrium, the incentive compatibility conditions (56a) and (56b) bind with equality. Moreover, $q(0) > q(1) > q(2)$; the probability that a bank monitors in the current period is strictly decreasing in the number of defaults it has caused in the previous two periods.*

Although the proof of Lemma 8 is more involved than that of Lemma 1, the underlying intuition is very similar. For the incentive compatibility condition (56a) to hold, it is necessary that a bank with no past defaults monitor more intensively than a bank that has experienced one default in the past two period. Similarly, for condition (56b) to hold, it is necessary that a bank with only one past default monitor more intensively than one with two past defaults. These conditions can be met only if the two incentive compatibility conditions bind with equality.

As in Section 2, we now solve for a full monitoring equilibrium in which the bank fully monitors the loan in the highest reputation state ($q(0) = 1$), but monitors with lower probability in the lower reputation states such that the probability of monitoring is strictly decreasing in the number of past defaults. Specifically, let $q(d)$ be of the form $q(d) = 1 - d\theta$,where $\theta > 0$ is a constant that needs to be characterized. For such an equilibrium to exist, there must exist a $0 < \theta < \frac{1}{N}$ to ensure that the bank monitors with positive probability in all states.

Our next result describes the conditions under which the full monitoring equilibrium is feasible, and characterizes θ and the value function $V(x)$ for $x \in \{0, 1, 2, 3\}$.

Proposition 5 *The full monitoring equilibrium described above is feasible if, and only if,*

$$m \leq \frac{\delta}{2}(1 + \delta)\Delta^2(1 + \beta)(X - C). \tag{58}$$

If condition (58) is satisfied, then the equilibrium is characterized by

$$\theta = \frac{m}{\delta(1 + \delta)\Delta^2(1 + \beta)(X - C)} \tag{59}$$

and the value function given by: $V(0) = V^*$, $V(1) = V^* - \frac{m}{\delta(1+\delta)\Delta}$, $V(2) = V^* - \frac{m}{\delta\Delta}$, *and* $V(3) = V^* - (\frac{2+\delta}{1+\delta})\frac{m}{\delta\Delta}$.

Using the Bellman Equation (57), and the fact that the incentive compatibility conditions bind with equality (Lemma 8), it is easy to show that $V(0)-V(2) = (1+\delta)\theta A$. But incentive compatibility requires that $V(0)-V(2) = \frac{m}{\delta\Delta}$. Equating these two expressions and solving for θ yields the expression in equation (59). For the full monitoring equilibrium to be feasible, it must be that $\theta < \frac{1}{2}$, because otherwise $q(2) = 1-2\theta \leq 0$. Setting $\theta < \frac{1}{2}$ yields the feasibility condition in (58).

Note that condition (58) is more likely to be met when the monitoring cost m is low, when the impact of monitoring Δ is high, when the value of liquidity β is high, and when the bank's discount factor δ is high. Also note that, because $\frac{1+\delta}{2} < 1$, condition (58) is more stringent than the equivalent condition for the $N = 1$ case. Substituting $d = 0$ and $V(0) - V(2) = \frac{m}{\delta\Delta}$ in equation (57), and solving the equation for $V(0)$ yields $V(0) = V^*$, where V^* is as defined in equation (9). The expressions for $V(1)$, $V(2)$ and $V(3)$ are obtained using the incentive compatibility conditions and the Bellman equation.

Thus, we have shown that using two-period reputation histories do not make full-monitoring equilibria more likely to be feasible, and do not increase the maximum value achieved by the bank.

B.2. Equilibrium with Grim-Trigger Strategies

In this subsection, we illustrate how the reputation equilibrium characterized in Proposition 1 differs from alternative equilibria supported by grim-trigger strategies, which feature coordinated randomized punishments for defaults.

Consider an equilibrium with the following randomized punishment if the bank experiences a default: with some probability $\pi > 0$, the bank receives the "unmonitored" loan price of (i.e., loan price based on the belief that the bank hasn't monitored the loan) for the rest of its life. Otherwise, the market prices the bank's loan under the belief that it has monitored with some positive probability q. The parameters π and q are endogenous. Note that, because setting monitoring at zero forever corresponds to an infinite repetition of the one-shot game's Nash equilibrium (no monitoring), it follows that the punishment strategy is in fact subgame perfect.

Let $V(q, \pi)$ denote the "continuation value" (i.e., the expected value of current and future profits) of the bank, given π and the market's conjecture of the bank's monitoring effort q. Let V_{punish} denote the bank's value if it receives the randomized punishment. Recall that the bank's current period surplus under the belief that it hasn't monitored the loan is $B = (1+\beta)(p(X-C)+C-1-u)$. Hence, $V_{punish} = (1-\delta)^{-1}B$. If the bank monitors with intensity q, then the probability that it will receive the randomized punishment next period is $\pi(1-p-q\Delta)$. Therefore, $V(q,\pi)$ must satisfy the following equation:

$$\begin{aligned} V(q,\pi) &= (A-m)q + B + \delta[1-\pi(1-p-q\Delta)] \cdot V(q,\pi) \\ &\quad +\delta\pi(1-p-q\Delta)\cdot(1-\delta)^{-1}B, \end{aligned} \tag{60}$$

where A and B are defined in equations (5) and (6).

As the current period surplus, $Aq+B$, is sunk at the time the bank chooses its monitoring q, monitoring is incentive-compatible only if

$$\pi[V(q,\pi) - (1-\delta)^{-1}B] \geq \frac{m}{\delta\Delta}. \tag{61}$$

Our focus is focus on a full monitoring equilibrium in which the bank fully monitors the loan (i.e., $q = 1$) till it receives the randomized punishment described above. We have the following result.

Proposition 6 *A full monitoring equilibrium in which the bank always monitors till it receives the randomized punishment (i.e. $q = 1$) is feasible if and only if*

$$m \leq \frac{\delta\Delta^2(1+\beta)(X-C)}{1-\delta p} \tag{62}$$

The highest possible continuation value that can be sustained under a full monitoring equilibrium is

$$V^* = \frac{1}{(1-\delta)}\left(v - \frac{m(1-p)}{\Delta}\right). \tag{63}$$

Note that, because $1-\delta p < 1$, condition (62) is less stringent than the equivalent feasibility condition (10) for the one-period reputation equilibrium characterized in Proposition 1. Furthermore, if condition (62) is satisfied, then the bank's continuation value V^* is that same as that characterized in equation (9) above. This establishes that, compared with reputational equilibria, coordinated randomized punishments make it easier to attain a full-monitoring equilibrium. The comparative statics on the feasibility condition that were mentioned in the text follow immediately from condition (62).

B.3. Stochastic Loan Booms when Booms are Common

In this section, we examine equilibria with stochastic lending booms in which IC(γ) binds, that is, the bank has incentives to monitor even when it lends γ units. Note that if IC(γ) binds, then IC(1) must hold strictly. So an immediate implication is that, regardless of its reputation, the bank monitors with probability 1 when it lends 1 unit.

Proposition 7 *Equilibria in which monitoring is incentive-compatible for the bank even when it lends γ units (i.e., IC(γ) is binding) may be feasible as follows:*

1. *An equilibrium in which the bank lends 1 unit in both boom and normal states, regardless of its reputation is never feasible.*

2. *An equilibrium in which the bank lends γ units in a boom and 1 unit in a normal economy, regardless of its reputation is feasible if, and only if,*

$$m \leq (1 - \phi)\,\delta\Delta A, \tag{64}$$

and

$$\frac{(\gamma - 1)}{\gamma} \cdot (A + B - m) \geq \frac{m}{(1 - \phi)\delta\Delta}. \tag{65}$$

In this equilibrium, a high-reputation bank monitors with probability 1 in both boom and normal states, whereas a low-reputation bank monitors with probability 1 in the *normal state and with probability* $q(1,\gamma) = 1 - \frac{m}{(1-\phi)\delta\Delta A}$ *in the boom state.*

3. *An equilibrium in which a high-reputation bank lends* γ *units in a boom and 1 unit in a normal economy, whereas a low-reputation bank lends 1 unit in both boom and normal states is feasible if, and only, if the following condition is satisfied in addition to conditions (64) and (65):*

$$A \geq (\gamma - 1)(B - m). \tag{66}$$

   In this equilibrium, a low-reputation bank monitors with probability 1 in both boom and normal states, whereas a high-reputation bank monitors with probability 1 in the normal state and with probability $q(0,\gamma) = \frac{1}{\gamma} - A^{-1} \cdot \left[\frac{(\gamma-1)(B-m)}{\gamma} - \frac{m}{(1-\phi)\delta\Delta}\right]$ *in the boom state.*

4. *An equilibrium in which a low-reputation bank lends* γ *units in a boom and 1 unit in a normal economy, whereas a high-reputation bank lends 1 unit in both boom and normal states is never feasible.*

6.1 Proofs of Results in Appendix B

Proof of Lemma 8: The proof utilizes the following expressions that are obtained by using the Bellman equation (57) in conjunction with the transition equations (54) and (55):

$$V(0) = q(0) \cdot (A - m) + B + \delta(p + \Delta q(0)) \cdot (V(0) - V(2)) + \delta V(2), \tag{67a}$$

$$V(1) = q(1) \cdot (A - m) + B + \delta(p + \Delta q(1)) \cdot (V(0) - V(2)) + \delta V(2), \tag{67b}$$

$$V(2) = q(1) \cdot (A - m) + B + \delta(p + \Delta q(1)) \cdot (V(1) - V(3)) + \delta V(3), \tag{67c}$$

$$\text{and } V(3) = q(2) \cdot (A - m) + B + \delta(p + \Delta q(2)) \cdot (V(1) - V(3)) + \delta V(3), \tag{67d}$$

(1) Proving that $V(0) - V(2) = \frac{m}{\delta\Delta}$.

We will prove this by contradiction. Suppose $V(0) - V(2) > \frac{m}{\delta\Delta}$. Then, banks with types

$x = 0$ and $x = 1$ will strictly prefer to monitor, so that $q(0) = q(1) = 1$, which in turn implies that $V(0) - V(1) = 0$. But if $V(0) = V(1)$, then it must be that $V(1) - V(2) = V(0) - V(2) > \frac{m}{\delta\Delta}$.

Next, subtracting equation (67c) from equation (67b), and using the fact that $V(0) - V(1) = 0$ yields $V(1) - V(2) = \delta[1 - p - \Delta q(1)] \cdot [V(2) - V(3)]$. As $V(1) - V(2) > \frac{m}{\delta\Delta} > 0$ and $1 - p - \Delta q(1) > 0$, it follows that $V(2) - V(3) > 0$. Combining $V(1) - V(2) > \frac{m}{\delta\Delta}$ and $V(2) - V(3) > 0$ yields that $V(1) - V(3) > \frac{m}{\delta\Delta}$.

Next, if $V(1) - V(3) > \frac{m}{\delta\Delta}$, then it follows that banks with types $x = 2$ and $x = 3$ will strictly prefer to monitor, so that $q(1) = q(2) = 1$. However, $q(1) = q(2)$ implies that $V(2) - V(3) = 0$, which contradicts our earlier finding that $V(2) - V(3) > 0$. Therefore, it must be that $V(0) - V(2) = \frac{m}{\delta\Delta}$. By a similar logic, it can be argued that $V(1) - V(3) = \frac{m}{\delta\Delta}$.

(2) Proving that $q(0) > q(1) > q(2)$.

After substituting $V(0) - V(2) = V(1) - V(3) = \frac{m}{\delta\Delta}$, it is easy to see that $V(0) - V(1) = (q(0) - q(1))A$, and $V(2) - V(3) = (q(1) - q(2))A$. Therefore, it is sufficient to show that $V(0) - V(1) > 0$ and $V(2) - V(3) > 0$.

Note that $V(0) - V(2) = V(1) - V(3)$ implies that $V(0) - V(1) = V(2) - V(3)$. Therefore, it is sufficient to show that $V(2) - V(3) > 0$.

Subtracting equation (67c) from equation (67b), and substituting $V(0) - V(2) = V(1) - V(3) = \frac{m}{\delta\Delta}$, yields $V(1) - V(2) = \delta \cdot (V(2) - V(3))$. Therefore,

$$\begin{aligned} V(1) - V(3) &= V(1) - V(2) + V(2) - V(3) \\ &= (1 + \delta) \cdot (V(2) - V(3)) \end{aligned} \tag{68}$$

which proves that $V(2) - V(3) > 0$ because $V(1) - V(3) = \frac{m}{\delta\Delta} > 0$. ■

Proof of Proposition 5: In this proof, we make use of equations (67a) through (67d) that we used in the proof of Lemma 8, after substituting $V(0) - V(2) = V(1) - V(3) = \frac{m}{\delta\Delta}$.

Step I: Solving for θ.

Substituting $q(0) = 1$, $q(1) = 1 - \theta$, $q(2) = 1 - 2\theta$, and $V(0) - V(2) = V(1) - V(3) = \frac{m}{\delta\Delta}$ in equations (67a) through (67d) that we used in the proof of Lemma 8, it follows that

$$V(0) - V(1) = \theta A, \tag{69}$$

$$V(2) - V(3) = \theta A, \tag{70}$$

and

$$V(0) - V(2) = \theta A + \delta(V(2) - V(3)) = (1 + \delta)\theta A, \tag{71}$$

where the second equation above is obtained using equations (69) and (70).

But $V(0) - V(2) = \frac{m}{\delta\Delta}$ by the IC constraint (56a). Setting $(1 + \delta)\theta A = \frac{m}{\delta\Delta}$, and solving for θ yields the expression for θ in the proposition. For the equilibrium to be well defined, it must be that $\theta \leq \frac{1}{N} = \frac{1}{2}$, which is equivalent to condition (58).

Step II: Solving the value function $V(x)$ for $x \in \{0, 1, 2, 3\}$.

We begin by solving for $V(0)$. Substituting $q(0) = 1$, $V(0) - V(2) = \frac{m}{\delta\Delta}$, and $\delta V(2) = \delta V(0) - \frac{m}{\Delta}$ in equation (67a) yields $V(0) = A + B + \delta V(0) - \frac{m(1-p)}{\Delta}$. Substituting for A and B from equations (5) and (6), and solving for $V(0)$ yields $V(0) = V^*$. Once we have solved for $V(0)$, it is fairly straightforward to obtain $V(1)$, $V(2)$ and $V(3)$ using equations (69), (56a), and (56b), respectively. ■

Proof of the Proposition 6: Let V^* denote the highest possible continuation value that can be achieved under the full monitoring equilibrium. As $V(q, \pi)$ is decreasing in π, it is efficient to choose the lowest π at which the IC constraint (61) binds, i.e., $\pi^* = \frac{m}{\delta\Delta(V^* - (1-\delta)^{-1}B)}$. Substituting $q = 1$ and the expression for π^* in the Bellman equation (60) yields $V^* = A - m + B + \delta V^* - \frac{m(1-p-\Delta)}{\Delta}$. Solving for V^*, and substituting $v = A + B$, yields the expression for V^* in Proposition 6. For the equilibrium to be feasible, it is necessary that

$\pi^* \leq 1$, or equivalently, that $m \leq \delta\Delta(V^* - (1-\delta)^{-1}B)$. Substituting for V^* and simplifying yields the feasibility condition (62). ■

Proof of Proposition 7: Suppose IC(γ) binds, so that $\Lambda = \gamma m/\delta\Delta$. Then IC(1) must hold strictly, which implies that $q\,(d,1) = 1$ and $V_n\,(d) = V_n$ for all d. Moreover, substituting $\Lambda = \gamma m/\delta\Delta$ in the expression for $V_n\,(d)$, it follows that

$$V_n = A + B + (\gamma - 1)m + p\gamma m/\Delta + \delta V(1). \tag{72}$$

Because we know what the bank does in the normal state, to describe an equilibrium, we only to characterize the bank's loan volume and monitoring intensity (for high loan volume) when the economy is in a boom: $\ell_b(0)$, $\ell_b(1)$, $q(0,\gamma)$, and $q(1,\gamma)$. (If the bank chooses $\ell_b(d) = 1$, then it monitors with full intensity and receives V_n.) There are four cases to consider, based on how the bank chooses its lending volume $l \in \{1, \gamma\}$ in a boom, in the high and low reputation states. In each case, we must check three necessary conditions: (a) IC(γ) must bind, i.e., $\Lambda = \gamma m/\delta\Delta$; (b) in a boom, a high-reputation bank must not want to switch its loan volume choice, $\ell_b\,(0)$; and (c) in a boom, a low-reputation bank must not want to switch its loan volume choice, $\ell_b\,(1)$.

Case (1): $\ell_b(0) = \ell_b(1) = 1$. This case can be ruled out immediately, since it requires that $V_b(d) = V_n$ for both $d = 0$ and $d = 1$. But then, $V(0) = V(1)$ and $\Lambda = 0$, contradicting the assumption that IC(γ) is binding and IC(1) holds strictly.

Case (2): $\ell_b(0) = \ell_b(1) = \gamma$. In this case,substituting $\Lambda = \gamma m/\delta\Delta$ in the expression for $V_b\,(d)$ yields,

$$V_b(d) = \gamma[q(d,\gamma) \cdot A + B] + p\gamma m/\Delta + \delta V(1). \tag{73}$$

It follows from equations (72) and (73) that $\Lambda = V\,(0) - V\,(1) = (1-\phi)\gamma A \cdot [q(0,\gamma) - q(1,\gamma)]$. Therefore, for condition (a) to be met, it must be that $q(0,\gamma) - q(1,\gamma) = \frac{m}{(1-\phi)\delta\Delta A}$. Rearranging this equation, and noting that $q\,(0,\gamma)$, yields $q\,(1,\gamma) \leq 1 - \frac{m}{(1-\phi)\delta\Delta A}$. However, for the equilibrium to be well-defined, it is necessary that $q\left(1,\gamma\right) \geq 0$. Hence, it must be that $m \leq (1-\phi)\delta\Delta A$, which proves the necessity of condition (64).

Next, condition (b) requires that $V_b(0) \geq V_n$, so that a high-reputation bank prefers to lend γ over 1 unit in a boom. Similarly, condition (c) requires that $V_b\left(1\right) \geq V_n$. As $q\left(0,\gamma\right) > q\left(1,\gamma\right)$, it is sufficient to check that condition (c) holds. After some algebraic manipulation, the condition $V_b\left(1\right) \geq V_n$ is equivalent to

$$\left(\gamma q(1,\gamma)-1\right)A+(\gamma-1)(B-m) \geq 0. \tag{74}$$

It follows that condition (c) is most likely to hold when $q(1,\gamma)$ is as high as possible, subject to condition (a). This corresponds to $q(0,\gamma)=1$ and $q(1,\gamma)=1-\frac{m}{(1-\phi)\delta\Delta A}$. Substituting into Equation (74) and rearranging, it follows that condition (c) will hold only if $\left(\gamma-1\right)\left(A+B-m\right) \geq \frac{\gamma m}{(1-\phi)\delta\Delta}$, which proves the necessity of condition (65).

Finally, suppose conditions (64) and (65) hold. Then, it is easily verified that the equilibrium characterized by $q(0,\gamma)=1$, $q(1,\gamma)=1-\frac{m}{(1-\phi)\delta\Delta A}$, and $\ell_b(0)=\ell_b(1)=\gamma$ satisfies conditions (a) through (c) above. This proves the sufficiency of conditions (64) and (65).

Case (3): $\ell_b(0)=\gamma,\ \ell_b(1)=1$. In this case, $V_b(0)$ is given by Equation (73) with $d=0$, whereas $V_b(1)=V_n$. Therefore, in this case,

$$\begin{aligned}\Lambda &= (1-\phi)[V_b(0)-V_b(1)] = (1-\phi)\left[V_b(0)-V_n\right] \\ &= (1-\phi)\cdot\left[\left(\gamma q(0,\gamma)-1\right)A+(\gamma-1)(B-m)\right],\end{aligned}$$

where the second equation follows by substituting for $V_b\left(0\right)$ and V_n. Hence, condition (a) is equivalent to

$$\left(\gamma q(0,\gamma)-1\right)A+(\gamma-1)(B-m)=\frac{\gamma m}{(1-\phi)\delta\Delta}. \tag{75}$$

Because $q(0,\gamma) \leq 1$, this requires that $(\gamma-1)(A+B-m) \geq \gamma m/(1-\phi)\delta\Delta$, which proves the necessity of condition (65).

Note that condition (a) will hold only if $V_b\left(0\right) > V_n$, which also guarantees condition (b) that a high-reputation bank prefers lending γ over 1 unit in a boom.

That leaves condition (c) that $V_n \geq V_b\left(1\right)$; a low-reputation bank must prefer lending 1 unit over γ units in the boom state. Let $q\left(1, \gamma\right)$ denote the (off-equilibrium) belief about a low-reputation bank's monitoring if it switched to lending γ units in the boom state. Then, the requirement $V_n \geq V_b\left(1\right)$ is equivalent to

$$(\gamma q\left(1, \gamma\right) - 1)A + (\gamma - 1)(B - m) \leq 0. \tag{76}$$

Clearly, condition (c) is more likely to be met for low values of $q\left(1, \gamma\right)$. As $q\left(1, \gamma\right) \geq 0$, condition (c) can be met only if $A \geq \left(\gamma - 1\right)\left(B - m\right)$, which explains the necessity of condition (66). Combining this with equation (75), it must be that $\gamma q(0, \gamma)A \geq \gamma m/(1 - \phi)\delta\Delta$. As $q\left(0, \gamma\right) \leq 1$, it is necessary that $A \geq m/(1 - \phi)\delta\Delta$ or $m \leq (1 - \phi)\delta\Delta A$, which proves the necessity of condition (64).

Finally, suppose conditions (64), (65), and (66) are satisfied. Then, it is easily verified that the equilibrium characterized by $\ell_b(0) = \gamma$, $\ell_b(1) = 1$, and $q\left(0, \gamma\right) = \frac{1}{\gamma} - A^{-1} \cdot \left[\frac{(\gamma-1)(B-m)}{\gamma} - \frac{m}{(1-\phi)\delta\Delta}\right]$ satisfies conditions (a) through (c) above. This proves the sufficiency of the feasibility conditions.

Case (4): $\ell_b(0) = 1$, $\ell_b(1) = \gamma$. In this case, we have $V_b(0) = V_n$, and $V_b(1)$ is given by Equation (73) with $d = 1$. Then, using similar analysis as in Case (3), condition (a) is equivalent to the requirement $(1 - \phi)[V_n - V_b(1)] = \gamma m/\delta\Delta$. Hence, condition (a) requires that $V_n > V_b\left(1\right)$. However, this contradicts condition (c), which requires $V_b(1) \geq V_n$. Thus, Case (4) is impossible. ■