

Optimal Capital Structure and Investment with Real Options and Endogenous Debt Costs*

Praveen Kumar

University of Houston

Vijay Yerramilli

University of Houston

Abstract

We examine the joint optimization of financial leverage and irreversible capacity investment in a real options framework with risky debt and endogenous interest costs. Higher capacity, *ceteris paribus*, increases operating leverage and default probability, but lowers *ex post* adjustment costs and generates larger tax shields. A key insight is that financial leverage and capacity are substitutes in the debt market equilibrium. We develop novel predictions about the effects of capital adjustment costs, operating costs, and uncertainty on optimal financial leverage and capacity that may potentially help explain ambiguous empirical results in the literature regarding the determinants of capital structure and investment. (*JEL* G31, G32, G33, D24)

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Firms are levered because of both their financial and operational choices. It has long been recognized that both debt repayment costs and fixed operating costs, which are determined by the firm's operating capacity, are important in leveraging the exposure of assets to economic risks (e.g., [Lev 1974](#); [Novy-Marx 2011](#)). Hence, the trade-off between financial and operating leverage is of substantial interest because many firms simultaneously face both financial frictions and technological frictions, such as capital irreversibility and adjustment costs. Although a number of studies in the broader corporate finance literature endogenize both capital structure and investment, we still have a limited understanding of the trade-offs involved in the choice of financial and operating leverage. In this paper, we examine the joint optimization of financial leverage and irreversible investment in a real options framework with risky debt financing, where shareholders may strategically default on their debt obligations, and the cost of debt is determined endogenously.

We examine a three-period model. In the first period, shareholders choose irreversible capacity investment and financial leverage to maximize equity value when future profits are uncertain. Similar to other models of operating leverage in the literature (e.g., [Carlson, Fisher, and Giammarino 2004](#)), capacity investment is positively related to the firm's operating leverage because of the presence of fixed operating costs that increase with capacity. In the second period, conditional on the realization of a profit shock and the prior decisions, the firm may choose to default, or to continue operation at the installed capacity, or to expand capacity by incurring capital adjustment costs and additional financing costs. Debt generates interest tax shields but exposes the firm to the risk of costly default because of the presence of deadweight bankruptcy costs, and the cost of debt is endogenously determined by a competitive loan market.

We characterize the firm's optimal financial leverage and capacity investment policies in a perfect Bayesian equilibrium in which shareholders follow their dynamically consistent production and default strategies in the second period. Consistent with the literature on

incomplete contracting (e.g., [Grossman and Hart 1986](#); [Hart and Moore 1998](#)), we assume that shareholders cannot credibly pre-commit to state-contingent production and default policies. Hence, along the equilibrium path, shareholders optimally choose to default when the levered cash flow is nonpositive even though the unlevered cash flow may be positive. There is thus a conflict of interest between shareholders and lenders because the former do not internalize the efficiency costs on the latter when they default; that is, debt in our model is exposed both to the cash flow risk and the agency risk. The loan market in the first period prices debt with rational expectations on the shareholders' dynamically consistent policies in the second period.

Standard corporate finance analysis of optimal financial leverage emphasizes the trade-off between higher tax shields and bankruptcy costs. Meanwhile, the real options analysis of optimal irreversible investment highlights the trade-off between preserving the value of the expansion option and minimizing capital adjustment costs (see [Caballero 1991](#); [Abel and Eberly 1997](#)). Our framework helps clarify the trade-offs among tax shields, bankruptcy costs, capital adjustment costs, and real options on the joint choice of capital structure and investment when the firm's risky debt is priced in a competitive loan market.

To facilitate intuition on these trade-offs, it is helpful to consider perturbations in financial leverage and capacity investment that leave the cost of debt unchanged in equilibrium.¹ When the debt market is in equilibrium, an increase in financial leverage, while holding the capacity investment and cost of debt fixed, violates the lenders' participation constraint because an increase in financial leverage raises both the level of debt and the probability of default. By a similar logic, an increase in capacity investment, while holding financial leverage and cost of debt fixed, also violates the lenders' participation constraint although the

¹Such perturbations effectively define the marginal rate of substitution of financial leverage for capacity investment in the debt market equilibrium. We note that allowing the capacity investment size to be flexible (e.g., to not be restricted to a finite number of levels) is important to define this notion of local substitutability.

effect is more complicated because, apart from its effect on default probability, the capacity investment also has *real* effects on the unlevered profit. Overall, to maintain the debt market equilibrium, an upward perturbation in financial leverage needs to be accompanied by a downward one in the capacity investment, and vice versa. Formally, the marginal rate of substitution of financial leverage for capacity investment in the debt market equilibrium is negative. Therefore, the firm’s financial leverage and capacity investment may optimally respond to variations in technological and financial parameters in offsetting ways.

We find that the levered firm with endogenous borrowing and cost of debt “overinvests” relative to the benchmark optimal investment of the unlevered firm, which is consistent with the intuition that shareholders do not internalize the deadweight costs of bankruptcy. Furthermore, we develop novel refutable predictions on the effects of technological and financial parameters on the optimal choice of financial and operating leverage, which we now describe.

We show that higher adjustment costs *ceteris paribus* have a positive effect on the optimal capital investment of the levered firm because the firm economizes on future adjustment (or option exercise) costs, which is consistent with the findings of real options models for unlevered firms. However, because of the local substitutability between financial and operating leverage, we show that higher capital adjustment costs also lower the firm’s financial leverage in equilibrium. Although there is a long literature on the effects of adjustment costs on capital investment, to the best of our knowledge, our paper is the first to highlight the impact of capital adjustment costs on financial leverage. The effect of capital adjustment costs and other technological parameters on financial leverage, which we highlight below, may help explain the interindustry and intraindustry variation in leverage ratios that cannot be explained by tax rates and financial distress costs ([Lemmon, Roberts, and Zender 2008](#)).

The effects of cash flow uncertainty on capital structure and investment attract much attention in the finance and economics literatures. [Caballero \(1991\)](#) shows that the effects of uncertainty on investment are ambiguous when capital irreversibility and adjustment costs

are jointly present, as is the case in our model although he does not consider capital structure. There is a negative “real option” effect on investment because higher uncertainty raises the option value from waiting, but there is also an opposing positive “adjustment cost” effect because higher uncertainty increases the likelihood of incurring capital adjustment costs in the future. Meanwhile, static trade-off models of capital structure predict a negative relation between financial leverage and uncertainty (Kraus and Litzenberger 1976).

We find that if capital adjustment costs are low, then higher uncertainty raises the option value from waiting and lowers the firm’s optimal capacity investment even in the case of levered firms. But because of the substitutability between capital structure and investment, financial leverage is *positively* related to uncertainty in this region. However, when the adjustment costs and uncertainty are sufficiently high, then the adjustment cost effect dominates the real option effect, so that capital investment increases and financial leverage decreases as uncertainty increases. Thus, our analysis clarifies that a negative relation of uncertainty and financial leverage will not generally obtain when capital structure, investment, and debt costs are jointly endogenous. To our knowledge, the change in sign of the effects of uncertainty on capital structure depending on the size of capital adjustment costs is a novel result from our analysis. These theoretical results also appear to be consistent with the mixed empirical results on the relation of financial leverage and uncertainty in the literature (see Parsons and Titman 2010).²

Capital adjustment costs also play an important role in determining how financial leverage and capacity investment respond to changes in fixed operating costs. Because of adjustment costs, the firm’s per-unit fixed operating costs are higher when it expands capacity ex post. Therefore, in situations in which the real option to expand capacity is very valuable — that is, when capital adjustment costs are low and cash-flow uncertainty is high — the

²In particular, although studies such as Bradley, Jarrell, and Kim (1984) and Booth et al. (2001) find a negative relation of leverage and uncertainty, Toy et al. (1974) and Long and Malitz (1985) find the opposite relation, and Titman and Wessels (1988) find no significant relation.

unlevered firm's optimal capacity *increases* with operating costs as it attempts to economize on higher future operating costs. However, the impact of higher fixed operating costs on the levered firm's policies are more subtle because fixed operating costs also affect the levered firm's default probability and expected bankruptcy costs. Overall, we find that the capital investment increases and financial leverage decreases as fixed operating costs increase only when the real option to expand capacity is sufficiently valuable and the fixed operating costs are sufficiently large.

In sum, the joint optimization of capital structure and investment with realistic production and financial contracting features — specifically, endogenous capacity size, capital irreversibility and adjustment costs, and risky debt with endogenous interest costs — distinguishes our analysis from the existing corporate finance literature. For example, papers that consider the dynamic trade-offs between capital structure and investment (e.g., [Hackbarth and Mauer 2012](#)) or real investment frictions (e.g., [Tsyplakov 2008](#); [Tserlukevich 2008](#); [Sundaresan, Wang, and Yang 2015](#); [Hackbarth and Sun 2015](#)) take investment size as given and/or assume exogenous cost of debt. On the other hand, real options models that allow flexible capacity and capital adjustment costs (e.g., [Caballero 1991](#)) do not consider capital structure. [Ozdogli \(2012\)](#) studies a dynamic model with optimal financial leverage and investment irreversibility but restricts attention to riskless debt, which is secured by a fraction of the firm's capital. In contrast, the trade-offs between financial and operating leverage with respect to endogenous cost of risky debt are fundamental to our analysis.

Our paper also complements the growing literature that examines the effects of agency conflicts due to asymmetric information in models of real options. For example, [Grenadier and Wang \(2005\)](#) and [Gryglewicz and Hartman-Glaser \(2017\)](#) consider the effects of management-shareholder agency conflicts in the presence of asymmetric information on the exercise of real options and investment timing; [Grenadier and Malenko \(2011\)](#) examine how investment timing may signal the private information of agents; and [Cong \(2014\)](#) studies optimal security

design in auctions of assets with embedded real options. By contrast, in our model the agency conflicts are due to the inability to write state-contingent production contracts and not due to asymmetric information.

1. The Basic Model

1.1 Production technology and cash-flow uncertainty

We model a firm in a risk-neutral economy that owns a single project. Shareholders of the firm choose the capacity and capital structure for the project *ex ante*, and decide on default policy, production policy, and capacity expansion policy *ex post* (that is, after the resolution of uncertainty) to maximize the equity value of the firm. We first describe the timing conventions for the “real” side of the project. Subsequently, we describe the timing conventions for external financing and default.

The three dates in the model are denoted by $t = 0, 1, 2$. The firm chooses a capacity investment of Q at $t = 0$, which requires an irreversible capital investment of AQ , where $A > 0$ denotes the unit capacity installation cost. For notational convenience, and without any loss of generality, we set $A = 1$. Consistent with the literature on irreversible capital investment (see [Dixit and Pindyck 1994](#)), we assume that once installed, the capacity Q cannot be drawn down. At the beginning of $t = 1$, the firm observes the realization of a profit shock $\tilde{\delta}$, following which it chooses its production level q . The cash flow from the project is realized at the final date $t = 2$, and the project and the firm are terminated immediately thereafter.

The unlevered cash flows from the project depend on the profit shock, the production level, fixed operating costs that increase with capacity, and the firm’s corporate income tax rate. For a given production level q and initial capacity Q , the firm’s pre-tax profit is given by $2\tilde{\delta}\sqrt{q} - cQ$, where c denotes the fixed per-unit capacity operating cost incurred at

$t = 1$. Consistent with the existing literature on operating leverage (e.g., [Carlson, Fisher, and Giammarino 2004](#); [Sagi and Seasholes 2007](#)), we assume that these are sunk costs that the firm has to incur independent of its output decision at $t = 1$. For example, the firm may have to sign contracts for capacity maintenance services, administrative and legal services, specialized labor services, and so forth; and being fixed, these contracts cannot be made contingent on the actual output. Thus, for capacity Q , the fixed operating cost of cQ is treated as a sunk cost. As we will make explicit below, the choice of the initial capacity Q thus also determines the firm's initial operating leverage, which is defined as the ratio of the fixed operating costs cQ to firm value at $t = 0$, in line with the definition employed by [Carlson, Fisher, and Giammarino \(2004\)](#).

The profit shock $\tilde{\delta}$ is distributed uniformly on $[\alpha - Z, \alpha + Z]$, so that $\mathbb{E}[\tilde{\delta}] = \alpha$ and $Var[\tilde{\delta}] = \frac{Z^2}{3}$. Hence, Z represents the cash-flow uncertainty because an increase in Z is a mean-preserving spread on $\tilde{\delta}$. For technical convenience and to ensure that the project has sufficient downside risk, we will assume that $Z \geq \alpha$. The distribution of $\tilde{\delta}$ and the parameters α and Z are common knowledge at $t = 0$.

All uncertainty regarding the project's profit is resolved at the beginning of $t = 1$ when the firm observes the realization of $\tilde{\delta}$ and chooses its production, q . The chosen production level may exceed the initial capacity Q because the firm has the option to expand capacity by incurring a total cost of $\lambda(1 + c)(q - Q)$, where λ is a measure of the capital adjustment costs. We assume that $\lambda > 1$, which means that waiting for the resolution of uncertainty for the possible exercise of the capacity expansion option is costly to the firm, because both the capacity installation cost and the fixed operating cost increase by a factor of λ for additional capacity installed at $t = 1$. This assumption is motivated by several real-life observations. First, it is well recognized in the investment and economic growth literature that because of indivisibilities, "cost lumpiness" exists and generates economies of scale for initial capacity installation ([Weitzman 1970](#)) that are typically not available in subsequent

expansions. Second, if capacity takes time to build and generate revenues (Kydland and Prescott 1982), then λ reflects the opportunity cost to the firm from production delays. Third, with putty-clay technology (see Johansen 1959; Gilchrist and Williams 2000), which is typically associated with capital irreversibility, the initial capacity embodies the technology available and is found most suitable at $t = 0$. With the evolution of technologies, expansion of the initial capacity at subsequent dates will be generally costlier because of unavailability of parts and/or because the firm may incur costs to retrain new labor for a previous technology; that is, it will be costlier to obtain capacity maintenance and skilled labor services, leading to higher unit fixed operating costs (λc). Indeed, dynamic cost minimization of capacity expansion is a major challenge that firms generally attempt to address in production planning (Luss 1982).

1.2 Capital, leverage, and bankruptcy costs

At $t = 0$, the firm has no financial resources and finances its capacity investment through a mixture of debt and external equity issued to investors in competitive capital markets; we abstract away from capital issuance costs to keep the model simple.³ Given the timing conventions of the project, all payoffs to investors are made at $t = 2$. We let $\phi \in [0, 1]$ denote the fraction of the initial investment financed using debt, with the remaining $1 - \phi$ fraction being financed with external equity. That is, the firm raises debt $D = \phi Q$ at $t = 0$. Let $F \equiv D(1 + r)$ denote the promised debt repayment at $t = 2$, where r denotes the firm's cost of debt, and is determined endogenously in the model. Without loss of generality, we normalize the risk-free rate to zero so that r can be construed as the risk premium due to the firm's default risk on its two-period debt at $t = 0$.

The main advantage of debt is that it generates interest tax shields for the firm if the

³Our analysis and results will be isomorphic to a model in which the firm starts with positive internal equity (or assets) and issues *nonrecourse* debt (Hart and Moore 1998) whereby lenders cannot access the firm's nonproject assets in default.

firm does not default on its debt. Let τ denote the firm’s effective income tax rate. Then, conditional on D and r , the interest tax shield is τDr if the firm does not default on its debt. The disadvantage of debt is that it can generate shareholder-debtholder conflicts of interest and subject the firm to deadweight bankruptcy (or financial distress) costs, which we will specify momentarily. We model these conflicts by assuming that the shareholders’ production, expansion and default policies at $t = 1$, which depend on the realization of the profitability shock $\tilde{\delta}$, are not contractible ex ante, consistent with the incomplete contracting literature.⁴

Formally, let $\hat{q}_{UL}(Q, \delta)$ denote the optimal production policy at $t = 1$ for an “unlevered” firm (i.e., when $\phi = 0$), and let $\hat{\pi}_{UL}(Q, \delta)$ be the corresponding optimal unlevered cash flow. If the levered firm follows the same production policy and does not default on its debt, then (given limited liability) shareholders will obtain a levered cash flow of $\max[0, \hat{\pi}_{UL}(Q, \delta) - \phi QR_\tau]$, where $R_\tau \equiv 1 + r(1 - \tau)$ denotes the after-tax (gross) cost of debt. Hence, shareholders in a levered firm will prefer to default on the debt if

$$\hat{\pi}_{UL}(Q, \delta) \leq \phi QR_\tau. \tag{1}$$

Condition (1) indicates a “debt overhang” effect on the shareholders’ production decision (Myers 1977). Whereas an unlevered firm will choose a positive $\hat{q}_{UL}(Q, \delta)$ as long as $\hat{\pi}_{UL}(Q, \delta) > 0$, a levered firm instead prefers to default on the debt if condition (1) holds. We note that the assumption that shareholders declare bankruptcy when equity falls to zero is standard in the literature on endogenous bankruptcy (see Leland and Toft 1996).⁵

⁴An economically appealing reason for this assumption is that the realized profit shock δ is not verifiable so that profit-contingent production and default policies are not legally enforceable (Hart and Moore 1998). An alternative view is that investment or default policies that are *not* dynamically consistent — that is, are not in the best interests of equityholders ex post — can be negated or renegotiated by a change of equity ownership and control of the firm (see Grossman and Hart 1986; Kumar and Langberg 2009).

⁵In practice, shareholders may also incur a utility cost of managing the firm, and hence, may strictly prefer to “walk away” from the firm if condition (1) is satisfied despite the fact that no new capital is required to operate the firm with $q \leq Q$.

If default occurs when there is value in the project ($\hat{\pi}_{UL}(Q, \delta) > 0$), then lenders take ownership of the project and implement the optimal production policy of an unlevered firm. However, the default process subjects the firm to deadweight bankruptcy costs – both direct costs (Gruber and Warner 1977) and indirect economic distress costs (Andrade and Kaplan 1998) – that are (eventually) borne by the lenders. Similar to the approach used in models of endogenous bankruptcy (e.g., Leland and Toft 1996), we model these bankruptcy costs through a proportional reduction in the firm’s unlevered (or asset) value, which represents the underlying economic value of the firm at the time of default. Formally, the lenders suffer a loss of $\gamma \hat{\pi}_{UL}(Q, \delta)$, where $0 < \gamma < 1$ is a measure of the deadweight bankruptcy costs. Overall, the value of the firm for lenders following a default is $(1 - \gamma) \hat{\pi}_{UL}(Q, \delta)$.

As is typical in the bankruptcy process, the deadweight economic loss from shareholder default (or bankruptcy) is borne by the lenders following default. There is, hence, a conflict of interest between shareholders and lenders because the former do not internalize the efficiency costs for the latter when they default and transfer control to the lenders. However, these agency costs for lenders should be priced in the cost of debt ex ante, which we consider next.

1.3 Lending market equilibrium

In a competitive, risk-neutral debt market the equilibrium cost of debt r is determined such that the lenders’ expected payoff is $D = \phi Q$ because the opportunity cost for lenders is the risk-free return, which is normalized to zero. Let $I_D \in \{0, 1\}$ be an indicator variable for default (where $I_D = 1$ denotes default), and let $p(I_D)$ denote the endogenous probability of default. Hence, if the debt market is competitive, then the following condition must hold in equilibrium:

$$(1 - \gamma) \mathbb{E}_\delta [\hat{\pi}_{UL}(Q, \delta) | I_D = 1] + [1 - p(I_D)] \cdot \phi Q (1 + r) = \phi Q. \quad (2)$$

Henceforth, we refer to Equation (2) as the competitive debt market condition. In effect, Equation (2) determines the project’s cost of debt r as a function of the initial capacity Q and financial leverage ϕ . In particular, it is apparent from (2) that the cost of debt r increases with the distress costs γ , holding other things fixed.

1.4 Equilibrium definition

A subgame-perfect Bayesian equilibrium in this set-up satisfies the following conditions:

1. At $t = 1$, the firm chooses its production and default policies to maximize the shareholders’ levered cash flow from the project.
2. Debt market determines the firm’s cost of debt in accordance with condition (2), after observing the firm’s choices at $t = 0$ and rationally anticipating the firm’s policies at $t = 1$.
3. The firm chooses its optimal initial capacity and financial leverage at $t = 0$ to maximize equity value (which is equivalent to the net levered project value).

It facilitates intuition for our results if we first analyze the benchmark case when the firm does not take on any debt at $t = 0$ (i.e., is “unlevered”).

2. Optimal Policies for the Unlevered Firm

We derive the unlevered firm’s optimal policies in the usual manner through backward recursion, first characterizing the optimal production policy at $t = 1$, and then solving for the optimal initial capacity Q that maximizes the net project value at $t = 0$. For notational ease, we will let $\psi \equiv \lambda(1 + c)$ denote the marginal cost of new capacity at $t = 1$.

Given that the realized δ is common knowledge at $t = 1$, it follows that any capacity expansion is financially feasible only if there is no risk for the new investors. For risk-free

debt, the interest cost is zero. Because tax shields apply only to interest payments, the Modigliani and Miller (1958) assumptions apply in our model at $t = 1$ and the firm is therefore indifferent between debt and equity financing at this date.

The unlevered firm's production choice problem at $t = 1$ is thus to maximize the after-tax unlevered cash flow and is given by :

$$\max_{q \geq 0} \pi_{UL}(q|Q, \delta) \equiv (2\delta\sqrt{q} - cQ - \max\{0, \psi(q - Q)\}) \cdot (1 - \tau), \quad (3)$$

subject to the requirement that $\pi_{UL}(q, Q, \delta) \geq 0$. The optimal production for the unlevered firm $\hat{q}_{UL}(Q, \delta)$ (defined in the previous Section) thus solves problem (3), with $\hat{\pi}_{UL}(Q, \delta)$ being the optimal unlevered cash flow at $t = 1$.

Intuitively, the firm will exercise its capacity expansion option only if the profit shock δ is sufficiently high. The following result makes this precise.

Lemma 1 *An unlevered firm expands project capacity at $t = 1$ (i.e., $\hat{q}_{UL}(Q, \delta) > Q$) iff $\delta > \psi\sqrt{Q}$. Otherwise, it operates the project at full capacity (i.e., $\hat{q}_{UL}(Q, \delta) = Q$) if $0 < \delta \leq \psi\sqrt{Q}$, and chooses $\hat{q}_{UL}(Q, \delta) = 0$ if $\delta \leq 0$.*

The marginal value of an additional unit of production when the firm is operating at full capacity is $\delta(1 - \tau)/\sqrt{Q}$, whereas the marginal cost of adding capacity is $\psi(1 - \tau)$. Thus, the firm exercises the expansion option only if the realized profitability $\delta > \psi\sqrt{Q}$. The expressions for $\hat{q}_{UL}(Q, \delta)$ and $\hat{\pi}_{UL}(Q, \delta)$ are fully characterized in the proof of Lemma 1.

Lemma 1 shows that the firm's production and expansion policy at $t = 1$ depends not only on the realized δ but also on its choice of initial capacity Q . Thus, an important feature of our model is that the value of the firm's assets-in-place or AIP (i.e., Q) and real options are not independent. Rather, the value of the real options depend on the value of the AIP, which is also determined endogenously. The unlevered firm's optimal initial capacity, Q_{UL}^* ,

is derived from the following optimization problem:

$$\max_{Q \geq 0} V_{UL}(Q) = \mathbb{E}_\delta [\hat{\pi}_{UL}(Q, \delta)] - Q. \quad (4)$$

Note that the chosen capacity also effectively determines the optimal operating leverage for the firm through the ratio $\Omega_{UL}(Q) \equiv \frac{cQ}{V_{UL}(Q)}$. In particular, the comparative statics of the model parameter with respect to the optimal capacity Q_{UL}^* have the same sign as the comparative statics with respect to the optimal operating leverage $\Omega_{UL}(Q_{UL}^*)$.⁶

Lemma 1 indicates that the firm exercises the capacity expansion option only if $\delta > \psi\sqrt{Q}$. Therefore, since δ cannot exceed $\alpha + Z$, capacity expansion occurs with positive probability in equilibrium iff $\psi\sqrt{Q_{UL}^*} < \alpha + Z$. Intuitively, this condition is more likely to be met when the cash-flow uncertainty Z is high and/or the capital adjustment cost factor λ is low. We verify this intuition. The appendix shows that there exist thresholds $\bar{\lambda}_{UL}$ and \bar{Z}_{UL} such that an unlevered firm exercises the capacity expansion option with positive probability at $t = 1$ iff $\lambda < \bar{\lambda}_{UL}$ and $Z > \bar{Z}_{UL}$ (“expansion region”). And under the assumption that $Z \geq \alpha$ (which ensures that uncertainty is not too low), a sufficient condition for the firm to be in the expansion region is that $\lambda < \frac{\bar{\lambda}_{UL}}{2}$. Given the focus of our study, henceforth, we restrict attention to the expansion region so that the real option to expand capacity is economically meaningful.

We now examine the effect of the salient model parameters on the optimal initial capacity of the unlevered firm.

Proposition 1 *The optimal initial capacity Q_{UL}^* (and hence, operating leverage) for the unlevered firm has the following properties in equilibrium:*

⁶Note that $\frac{\partial \Omega_{UL}(Q)}{\partial Q} \propto cV_{UL}(Q) - cQ \frac{\partial V_{UL}(Q)}{\partial Q}$. But, by the Envelope theorem, at the optimum capacity, the second term is zero so that $\frac{\partial \Omega_{UL}(Q_{UL}^*)}{\partial Q} \propto cV_{UL}(Q_{UL}^*) > 0$. Hence, for any model parameter x , $\frac{\partial \Omega_{UL}(Q_{UL}^*)}{\partial x} = \frac{\partial \Omega_{UL}(Q_{UL}^*)}{\partial Q} \frac{\partial Q_{UL}^*}{\partial x} \propto \frac{\partial Q_{UL}^*}{\partial x}$.

1. Q_{UL}^* increases as the adjustment cost factor λ increases. Moreover, Q_{UL}^* is increasing with the fixed unit costs c if λ is sufficiently low and Z is sufficiently high.
2. If $\lambda < \frac{\bar{\lambda}_{UL}}{2}$, then Q_{UL}^* decreases as profit uncertainty Z increases, and $Q_{UL}^* = 0$ for sufficiently high Z . However, if $\bar{\lambda}_{UL} > \lambda \geq \frac{\bar{\lambda}_{UL}}{2}$, then Q_{UL}^* is increasing with Z .
3. Q_{UL}^* is decreasing in the tax rate τ .

The optimal capacity trades off two conflicting forces with respect to variations in the real (or production related) costs λ and c . Because capital investment is irreversible and profits are uncertain, lower initial capacity allows the firm to make a more efficient choice of production at $t = 1$ following the observation of the true δ (the “real option” effect). However, a low Q also raises the expected capital adjustment costs ex post (the “adjustment cost” effect). Thus, Q_{UL}^* is ceteris paribus positively related to the adjustment cost factor λ in the expansion region ($\lambda < \bar{\lambda}_{UL}$ and $Z > \bar{Z}_{UL}$), because a higher λ strengthens the adjustment cost effect. In a similar vein, the marginal cost of new capacity, $\lambda(1 + c)$, is increasing with the fixed unit cost c . Hence, if there is a high likelihood of exercising the expansion option — which is the case when λ is sufficiently low and Z is sufficiently high — then the firm economizes on the exercise costs of the expansion option by increasing the initial capacity as c increases.⁷

Meanwhile, the effect of cash-flow uncertainty Z on Q_{UL}^* is more ambiguous, because an increase in Z strengthens both the real option effect and the adjustment cost effect. If adjustment costs are low, the real option effect dominates the adjustment cost effect, and we obtain a negative effect of uncertainty Z on investment Q_{UL}^* . Conversely, the adjustment cost effect dominates the real option effect when adjustment costs are sufficiently high, and we obtain a positive relationship between Q_{UL}^* and Z .

⁷We show in the appendix that the conditions on λ and Z that are sufficient to generate the positive relation of Q_{UL}^* to c are stronger than the conditions for the firm to be in the expansion region.

The ambiguous effect of cash-flow uncertainty on investment arises because of the joint presence of capital irreversibility and capital adjustment costs (λ), as has been noted before in the literature (Caballero 1991). To see why, suppose we had capital irreversibility without the expansion option. Then, as is well known, a mean-preserving increase in cash-flow uncertainty raises the option value of waiting and delays investment (Dixit and Pindyck 1994). Next, consider the other polar case without capital irreversibility, that is, the firm can contract or expand capacity through a symmetric adjustment cost function. In this case, a mean-preserving increase in price uncertainty leads to *higher* investment (Caballero 1991) because, without irreversibility, the investment decision does not depend on the prior capacity and is based only on the comparison of investment costs with the current and future marginal investment returns. And since higher price uncertainty raises the expected return on investment, uncertainty is positively related to investment.

For the unlevered firm, a higher tax rate unambiguously reduces the marginal value productivity of capacity since the optimal unlevered cash flow $\hat{\pi}_{UL}(Q, \delta)$ is decreasing in τ (cf. (3)). Hence, Q_{UL}^* is negatively related to τ . To anticipate the analysis below, note that the situation would be more complex for a levered firm because higher tax rates ceteris paribus also increase the tax shield effect of debt and influence the endogenous default threshold.

Finally, we note that by choosing Q_{UL}^* , the firm effectively determines the allocation of its equity value between assets-in-place and growth options. In the polar case in which $Q_{UL}^* = 0$ (which will be realized if $\lambda < \frac{\bar{\lambda}_{UL}}{2}$ and Z is sufficiently high), equity value is entirely embedded in the expansion option. In contrast, as the exercise costs of the expansion option increase, a greater proportion of value optimally resides in the assets-in-place.

3. Optimal Policies for the Levered Firm

Using the same backward recursion procedure from the previous section, we first analyze the firm's capacity expansion and strategic default policies at $t = 1$, and then derive the optimal capacity investment and capital structure policies in the initial stage.

3.1 Optimal capacity option exercise and default

At $t = 1$ the firm chooses its optimal production and default policies conditional on the realized profitability shock δ , taking as given the initial capacity Q , leverage $0 < \phi \leq 1$, and the equilibrium borrowing cost r determined in the prior period. That is, while $\omega \equiv (Q, \phi, r)$ are endogenous at $t = 0$, they are given (or state variables) at $t = 1$. Conditional on not being in default ($I_D = 0$), the firm's optimal production and default policies at $t = 1$ are derived from:

$$\max_{q \geq 0, I_D \in \{0,1\}} \pi_L(q|\omega, \delta) \equiv (1 - I_D) \cdot [\pi_{UL}(q|Q, \delta) - \phi QR_\tau], \quad (5)$$

where $\pi_{UL}(q|Q, \delta)$ is the unlevered cash flow defined in Equation (3) and R_τ has been defined earlier as the after-tax (gross) cost of debt. (The levered cash flow from the project is zero if the firm is in default). Since $\omega \equiv (Q, \phi, r)$ are predetermined variables at $t = 1$, it follows from problems (3) and (5) that if the firm does not default, then its production decision is formally equivalent to the unlevered firm's decision. Moreover, the firm will not default if the realized δ is sufficiently high because the optimal unlevered cash flow $\hat{\pi}_{UL}(Q, \delta)$ increases with δ (see Lemma 1). That is, there exists some default threshold profit state $\bar{\delta}(\omega)$ such that the firm will default iff $\delta \leq \bar{\delta}(\omega)$.

We now characterize the firm's equilibrium default strategy at $t = 1$, conditional on ω . We will then use the shareholders' dynamically consistent default strategy to analyze the

firm's initial period choice of capacity and financial leverage (Q, ϕ) , taking as given the effect of these choices on the cost of debt in the competitive lending market (given by (2)). Like in the case of the unlevered firm, we will restrict attention to situations in which the firm exercises the real option with positive probability (the "expansion" region). Furthermore, an equilibrium in which the firm defaults for high values of δ at which the expansion option is in-the-money (i.e., the region $\delta \in [\psi\sqrt{Q}, \alpha + Z]$) is not economically appealing. Hence, we will restrict attention to situations in which the default threshold is below the firm's expansion threshold, that is, $\bar{\delta}(\omega) < \psi\sqrt{Q}$. We will show that the conjectures, $\bar{\delta}(\omega) < \psi\sqrt{Q} < \alpha + Z$, are satisfied in equilibrium for an open set of parameters.

Our next results provides an analytic characterization of the default threshold, $\bar{\delta}(\omega)$.

Lemma 2 *For a given $\omega = (Q, \phi, r)$, there exists a threshold*

$$\bar{\delta}(\omega) = \frac{\sqrt{Q}[\phi R_\tau + c(1 - \tau)]}{2(1 - \tau)} \quad (6)$$

such that:

1. *If $\delta \leq \bar{\delta}(\omega)$, then the levered firm defaults on its debt and shareholders receive zero cash flow, whereas lenders receive $(1 - \gamma)\hat{\pi}_{UL}(Q, \delta)$.*
2. *If $\delta > \bar{\delta}(\omega)$, then the firm follows the same production policy as an unlevered firm, that is $\hat{q}_L(\omega, \delta) = \hat{q}_{UL}(Q, \delta)$ (cf. Lemma 1); shareholders receive a levered cash flow of $\hat{\pi}_L(\omega, \delta) = \hat{\pi}_{UL}(Q, \delta) - \phi Q R_\tau$, whereas lenders receive the promised debt repayment $\phi Q(1 + r)$.*

Since the debt repayment, $F = \phi Q(1 + r)$, is positively related to Q , ϕ and r , it is not surprising that the default threshold $\bar{\delta}(\omega)$ also increases with these variables. Moreover, it

follows from this result that the survival (or nondefault) probability conditional on ω is

$$1 - p(I_D) = \Pr(\delta > \bar{\delta}(\omega)) = \frac{(\alpha + Z) - \bar{\delta}(\omega)}{2Z}. \quad (7)$$

We note that under the maintained assumptions, namely, $\alpha \leq Z$ and $\bar{\delta}(\omega) < (\alpha + Z)$, the default probability $p(I_D) \in (0, 1)$ in equilibrium.

A few important aspects of the default decision are worth emphasizing because these have important implications for the choice of the firm's financial and operating leverage at $t = 0$. In the interval $\delta \in (0, \bar{\delta}(\omega))$, the default decision by shareholders is strategic rather than liquidity driven, and $\bar{\delta}(\omega)$ itself depends not only on the initial capacity and leverage, but also on other model parameters such as the fixed unit costs c and the tax rate τ . On the other hand, the adjustment cost λ does not affect the default threshold because shareholders do not exercise the capacity expansion option when they decide to default.

3.2 Optimal capital structure and capacity investment

Using Lemma 2, we can rewrite the competitive debt market condition (2) as

$$\Gamma(\omega) \equiv (1 - \gamma)\mathbb{E}_\delta [\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}(\omega)] + [1 - p(I_D)]\phi Q(1 + r) - \phi Q = 0, \quad (8)$$

Now, let $V_L(\omega)$ denote the equity value of the firm (or the net levered value of the project) with an initial capacity of Q , financed with debt $D = \phi Q$. Then, $V_L(Q, \phi) = \mathbb{E}_\delta [\hat{\pi}_L(\omega, \delta)] - (1 - \phi)Q$ and substituting for $\hat{\pi}_L(\omega, \delta)$ from Lemma 2 yields

$$V_L(\omega) = \mathbb{E}_\delta [\hat{\pi}_{UL}(Q, \delta) | \delta \geq \bar{\delta}(\omega)] - \phi Q(1 - p(I_D))R_\tau - (1 - \phi)Q, \quad (9)$$

It facilitates intuition to utilize the competitive debt market condition (8) to rewrite $V_L(Q, \phi)$ in adjusted present value (APV) terms as

$$V_L(\omega) = V_{UL}(Q) + (1 - p(I_D)) \cdot \phi Q r \tau - \gamma \mathbb{E}_\delta [\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}(\omega)] \quad (10)$$

In Equation (10), $V_{UL}(Q)$ denotes the value of the corresponding unlevered firm, the second term denotes the expected tax shield created by the debt, and the final term denotes the expected costs of default arising from deadweight bankruptcy costs. Note that $\gamma \mathbb{E}_\delta [\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}(\omega)] > 0$ because the lenders will not operate the firm following default if $\hat{\pi}_{UL}(Q, \delta) \leq 0$.

Thus, the firm's optimal capacity, Q_L^* , and financial leverage, ϕ^* , are determined by the solution to the following optimization problem:

$$\max_{Q \geq 0, \phi \in [0, 1]} V_L(\omega), \quad (11)$$

subject to the competitive debt market condition (8), which also determines the equilibrium cost of debt r^* . In sum, the joint optimization of capital investment and financial leverage takes into account the real effects of capacity, expected gains to shareholders from tax deductibility of interest on debt, and competitively determined cost of debt. In particular, the cost of debt depends on the default risk and lenders' expected post-default payoffs based on the dynamically consistent default policy of shareholders. Finally, note that the levered firm's operating leverage is $\Omega_L(\omega) \equiv \frac{cQ_L}{V_L(\omega)}$; that is, for given financial leverage and cost of debt, the operating leverage is determined by the initial capacity Q_L .

To build intuition on the trade-offs involved in the joint optimization of (Q, ϕ) , it is helpful to consider the following necessary optimality conditions for $Q_L^* > 0$ and $\phi^* \in (0, 1)$,

respectively,

$$\frac{\partial V_{UL}}{\partial Q} + (\phi\tau r) \left[(1 - p(I_D)) - Q \frac{\partial p(I_D)}{\partial Q} \right] - \gamma \frac{\partial \mathbb{E}_\delta [\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}]}{\partial Q} = 0, \quad (12)$$

$$(Qr\tau) \left[(1 - p(I_D)) - \phi \frac{\partial p(I_D)}{\partial \phi} \right] - \gamma \frac{\partial \mathbb{E}_\delta [\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}(\omega)]}{\partial \phi} = 0. \quad (13)$$

Note that $\frac{\partial V_{UL}}{\partial Q} = 0$ is the necessary optimality condition for the unlevered firm. Hence, Equation (12) indicates that in addition to the marginal net benefits of Q considered by the unlevered firm, the levered firm also takes into account the marginal effects of capacity on interest tax shields and on the expected distress cost for debt holders, which are captured by the second and third terms, respectively. Since equity holders only directly benefit from the tax shields if the firm stays solvent, the second term appropriately considers both the marginal increase in tax shields conditional on being solvent (which is positive) and the higher probability of the loss of these tax shields due to default (which is negative). The final term, which represents the marginal effect of Q on the lenders' expected bankruptcy cost, is positive in our model because $E_\delta[\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}] > 0$; this also ensures that the value function $V_L(\omega)$ is strictly concave in Q .⁸

Meanwhile Equation (13) indicates that the optimal financial leverage takes into account the net expected benefit to shareholders of tax shields from debt, conditional on remaining solvent. At the margin, raising ϕ increases the tax shield but also increases the likelihood of losing these shields due to default, and these conflicting effects are jointly captured by the first term in Equation (13). Furthermore, the firm considers the marginal effect of ϕ on the expected distress costs of creditors, namely, $\gamma \mathbb{E}_\delta[\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}]$, which is given by the second term. Variations in Q affect the expected distress costs by impacting both $\hat{\pi}_{UL}(Q, \delta)$ and the default threshold $\bar{\delta}$ (see (6)), whereas changes in ϕ impact this variable only through

⁸Intuitively, if raising Q always increases lenders' expected payoff in default, then higher capacity investment could lower the endogenous cost of debt, and the optimal capacity may not be well defined.

the default threshold.

Since these optimality conditions (12) and (13) must hold simultaneously, we expect the tax rate τ and the bankruptcy costs γ to influence Q both directly and indirectly, through the effect of these parameters on ϕ . In particular, the levered firm may increase the optimal capacity compared with the unlevered firm, other things held fixed, to exploit interest tax shields if this benefit exceeds the marginal impact of higher debt on expected financial distress costs. Similarly, we expect the “real” parameters — capital adjustment costs λ and fixed costs c — to affect ϕ either directly (for example, the impact of c on the default policy) or indirectly through the effect of these parameters on Q . We now turn to a more formal analysis of optimal financial leverage and capacity investment policies.

3.3 Characterization and comparative statics

Since the firm can choose to be unlevered (i.e., $\phi^* = 0$), the sufficient conditions for $\phi^* > 0$ and the relation of the optimal levered capacity Q_L^* to the corresponding quantity for the unlevered firm (Q_{UL}^*) are of substantial interest. The next result clarifies these issues.⁹

Proposition 2 *If $\tau > 0$, then it is optimal for the firm to be financially levered, that is, $\phi^* > 0$. And holding other things fixed, the optimal capacity investment for the levered firm is greater than that of the corresponding unlevered firm, that is $Q_L^* > Q_{UL}^*$.*

The result that it is optimal to be levered (i.e., $\phi^* > 0$) is consistent with the standard (or text book) theories of capital structure: at very low levels of ϕ , the marginal tax shield benefits of higher leverage exceed the debt costs from higher default risk (cf. (13)). But we find that the optimal capacity for the levered firm also exceeds that of the unlevered

⁹Note that the equilibrium policies are derived through backward recursion and asymmetric information does not exist in our model. Furthermore, the characterization and comparative statics results derived below hold for any cost of debt that satisfies the competitive debt market condition (8). Hence, these results will hold for all possible equilibria in the model.

firm because of the tax shield effect. As we mentioned before, the tax shields in our model are positively related to both financial leverage and capacity (since $D = \phi Q$). It turns out that at the optimal unlevered capacity, Q_{UL}^* , the incremental tax shield benefit of higher Q dominates the marginal increase in expected bankruptcy costs.

Proposition 2 characterizes necessary properties of any optimum of the levered firm. Prior to any further analysis, we also need to identify sufficient conditions to ensure that (1) the local second-order conditions for the optimality conditions (12) and (13) are satisfied; (2) the firm is in the expansion region, that is, $\psi\sqrt{Q_L^*} < (\alpha + Z)$; and (3) the default threshold is below the firm's expansion threshold in equilibrium, that is, $\bar{\delta}(\omega) < \psi\sqrt{Q_L^*}$. These conditions are derived in Lemma A1 in the appendix, and henceforth, we take these to be our maintained assumptions.

By intuition similar to that used in the case of the unlevered firm, we show that there exist thresholds $\bar{\lambda}_L$ and \bar{Z}_L such that the levered firm is in the expansion region iff $\lambda < \bar{\lambda}_L$ and $Z > \bar{Z}_L$. Next, consider the requirement that the default threshold be below the expansion threshold in equilibrium. Intuitively, this requires that the endogenously determined financial leverage, ϕ^* , and the cost of debt, r^* , be not too high. Now, ϕ^* depends on the tax benefits of debt, which are proportional to τr^* ; and r^* is likely to be low when the nonstochastic component of the project's profitability, α , is not too low in relation to the uncertainty, Z , so that the probability of default is not too high. Overall, we show in Lemma A1 that if α is not too low relative to Z , if τ is not too high, and if the adjustment costs (λ) are not too low, then $r^* < 1$ and the requirement that $\bar{\delta}(\omega) < \psi\sqrt{Q_L^*}$ is met in equilibrium. Moreover, these conditions are also sufficient to guarantee that the local second-order conditions are satisfied.

Before we describe the effects of the salient model parameters on the optimal financial leverage (ϕ^*) and operating leverage (Q_L^*), it is useful to highlight both the substitutability and complementarity considerations in the choice of operating and financial leverage. To see

the substitutability, note that when the debt market is in equilibrium (i.e., condition 8 is satisfied), raising ϕ , while holding (Q, r) fixed, violates the lenders' participation constraint, because an increase in ϕ raises both the level of debt (ϕQ) and the probability of default ($p(I_D)$). By a similar logic, an increase in Q , while holding (ϕ, r) fixed, also violates the lenders' participation constraint, although the effect is more complicated because, apart from its effect on default probability, Q also directly affects the expected costs of financial distress (i.e., $\mathbb{E}_\delta [\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}(\omega)]$) through its *real* effects on the unlevered profit, $\hat{\pi}_{UL}$ (see Lemma 1). Hence, to maintain the debt market equilibrium, a upward perturbation in ϕ needs to be accompanied with a downward one in Q , and vice versa.¹⁰

Thus, there is a substitutability in financial and operating leverage choice from the viewpoint of maintaining a given cost of debt. In particular, raising financial leverage and lowering operational leverage to maintain the cost of debt involves the trading off lower operating costs and changes in tax shields with higher expected capital adjustment costs. And because of the substitutability in financial and operating leverage in the debt market, these variables may sometimes optimally respond to variations in technological and financial parameters in offsetting ways.¹¹

Of course, apart from their effects on the cost of debt, perturbations in (Q, ϕ) also affect the net levered value $V_L(\omega)$, which are captured in the optimality conditions (12) and (13). There may, therefore, also be complementarity between the response of (Q_L^*, ϕ^*) to certain parameters, such as deadweight bankruptcy costs that, intuitively, lower the benefits of both Q and ϕ , at the margin.

¹⁰Formally, the marginal rate of substitution of financial leverage for capacity investment in the debt market equilibrium is given by the ratio $-\Gamma_Q(\omega) / \Gamma_\phi(\omega)$, computed by differentiating (8) with respect to Q and ϕ and setting it to zero to maintain the equilibrium. We show in Lemma A2 in the appendix that this is negative in equilibrium.

¹¹While the assumption of uniform distribution on the profit shocks is convenient for our analysis, our main qualitative results, in particular the substitutability in financial and operating leverage due to endogenous debt costs and the opposing effects of the real parameters on these choice variables, are not based on this assumption and should hold in general stochastic specifications of the profit shocks.

Given the focus of our paper, the effects of the technological (“real”) parameters — λ , Z , and c — on the joint choice of financial and operating leverage are of substantial interest. We showed in Proposition 1 that an unlevered firm chooses its optimal initial capacity by trading off the adjustment cost effect and the real option effect. Similar considerations apply in case of the levered firm as well, except that the levered firm also has to trade off the tax benefits of debt and the expected distress costs. Overall, we expect the real parameters to broadly have similar effects on Q_L^* as they have on Q_{UL}^* , but based on the substitutability argument sketched above, expect the opposing effect on ϕ^* . We now formalize these arguments.

Proposition 3 *The optimal capacity investment for the levered firm (Q_L^*) and optimal financial leverage (ϕ^*) vary with the adjustment cost factor (λ) and fixed operating cost (c) as follows:*

1. Q_L^* increases as λ increases, whereas ϕ^* decreases as λ increases.
2. There exist thresholds — λ_c , Z_c and \bar{c} — such that if $\lambda < \lambda_c$, $Z > Z_c$ and $c > \bar{c}$, then Q_L^* increases, whereas ϕ^* decreases as c increases.

We showed in Proposition 1 that a higher adjustment cost (λ), which ceteris paribus strengthens the adjustment cost effect and weakens the value of the real option to expand capacity, has a positive effect on the initial capacity of the unlevered firm. By similar considerations, Q_L^* increases as λ increases. But holding ϕ fixed, an increase in Q_L^* leads to an increase in the amount of debt and the probability of default, which violates the lenders’ participation constraint. Hence, by the substitutability argument sketched above, the levered firm also optimally lowers its financial leverage (ϕ^*) as λ increases. Figure 1 graphs the relation of (Q_L^*, ϕ^*) with λ for a profile of parameters that satisfy our maintained assumptions. In particular, the marginal corporate income tax rate is $\tau = 0.28$, which is the mean simulated marginal tax rate from data on earnings before taxes reported by [Graham \(2000\)](#).

As indicated by Proposition 1, the unlevered firm optimally increases its capacity investment as c increases if the capacity option is sufficiently valuable; that is, when λ is sufficiently low and Z is sufficiently high. However, the analysis is more complex for the levered firm because c also affects the equilibrium default probability ($p(I_D)$), and hence, the expected bankruptcy cost ($\mathbb{E}_\delta[\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}]$). We show that the marginal effect of Q on expected bankruptcy cost (i.e., $\frac{\partial \mathbb{E}_\delta[\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}]}{\partial Q}$) is low when the fixed operating cost, c , is sufficiently high. Hence, if c is sufficiently high, then Q_L^* also increases with c when the capacity expansion option is sufficiently valuable — that is, the adjustment costs are sufficiently low and the cash-flow uncertainty is sufficiently high. But if these conditions apply, then optimal financial leverage is negatively related to c because of the substitutability between Q_L^* and ϕ^* in the debt market equilibrium.

We turn next to the relation of optimal financial leverage and capacity investment to the cash flow uncertainty, Z .

Proposition 4 *The effect of cash flow uncertainty (Z) on the optimal capacity investment (Q_L^*) and optimal financial leverage (ϕ^*) varies with capital adjustment cost (λ) as follows:*

1. *If λ is sufficiently low, then Q_L^* decreases and ϕ^* increases as Z increases.*
2. *If λ and Z are sufficiently high, then Q_L^* increases and ϕ^* decreases as Z increases.*

We showed in Proposition 1 that an increase in Z strengthens both the real option effect and the adjustment cost effect, and hence, the effect of Z on the initial capacity of the unlevered firm (Q_{UL}^*) varies with the adjustment cost factor λ , which determines the relative magnitudes of the two effects. By a similar logic, the optimal initial capacity of the levered firm, Q_L^* , also decreases as Z increases for low values of λ for which the real options effect dominates the adjustment cost effect (formally, if $\lambda \leq \hat{\lambda}_L$, where we characterize the threshold $\hat{\lambda}_L > 1$ in the proof of Proposition 4). Then, because of the substitutability between Q_L^* and ϕ^* in the debt market equilibrium, ϕ^* increases with Z in this region.

The analysis is more complicated when λ is high. However, we show that the adjustment cost effect dominates the real option effect if λ and Z are sufficiently high; formally if $\lambda > \tilde{\lambda}(Z)$, where we characterize the threshold $\tilde{\lambda}(Z) > \hat{\lambda}_L$ in the proof of Proposition 4 and show that $\tilde{\lambda}(Z)$ is decreasing in Z . Therefore, Q_L^* increases and ϕ^* decreases as Z increases when λ and Z are sufficiently high.

Proposition 4 is of substantial interest because the usual textbook presumption is that profit uncertainty should have a negative impact on financial leverage. (We will discuss the empirical implications of this and other results in Section 4.). Figure 2 graphs the relation of (Q_L^*, ϕ^*) with Z , where the top and bottom panels correspond to the low and high adjustment cost regimes, respectively. The parameter specifications are chosen to satisfy our maintained assumptions and the additional requirements of the sufficiency conditions in the high adjustment cost regime.

Next, we examine the effects of the “financial” parameters — the corporate tax rate τ and the deadweight bankruptcy cost γ — on the joint choice of financial and operating leverage. We showed in Proposition 1 that an increase in the tax rate τ has an unambiguous negative effect on the initial capacity of an unlevered firm. However, the effect of a higher tax rate on a levered firm is more complex because a higher tax rate also increases both the value of tax shields and the probability of default, and the relative magnitudes of these effects depend on the endogenously chosen Q_L^* and ϕ^* .¹² Thus, it is hard to analytically characterize the effect of τ on the levered firm’s policies. In the next result, we identify sufficient conditions for the tax rate τ to be positively related to financial leverage and negatively related to the capacity investment.

Proposition 5 *The optimal capacity investment for the levered firm (Q_L^*) and optimal financial leverage (ϕ^*) vary with the tax rate (τ) and bankruptcy cost (γ) as follows:*

¹²It follows from Equations (6) and (7) that the default probability $p(I_D)$ is increasing in τ .

1. *There exist thresholds, $\tilde{\tau} \leq \bar{\tau}$ and $\lambda_\tau < \bar{\lambda}_L$, such that if $\tau < \tilde{\tau}$ and $\lambda \geq \lambda_\tau$, then Q_L^* decreases and ϕ^* increases as τ increases.*
2. *ϕ^* decreases as γ increases. Moreover, there exists a cost threshold c_γ , such that Q_L^* also decreases as γ increases if $c < c_\gamma$.*

The marginal benefit of ϕ is increasing in τ (i.e. $V_{\phi\tau}^L > 0$) if the effect of τ on the tax shields dominates its effect on expected bankruptcy costs. Intuitively, this is more likely when ϕ^* is relatively low, which in turn, is more likely when τ is not too high. Next, recall that the marginal benefit of Q to the unlevered firm unambiguously decreases with τ (i.e., $V_{Q\tau}^{UL} < 0$). Hence, intuitively, the marginal benefit of Q to the levered firm is also likely to decrease with τ (i.e., $V_{Q\tau}^L < 0$) if Q_L^* is sufficiently high and close to Q_{UL}^* , and ϕ^* is small. It follows that these conditions are more likely to be met when the adjustment cost λ is not too high, because we have shown that Q_L^* is increasing and ϕ^* is decreasing in λ (see Proposition 3). Overall, an upper bound on τ and a lower bound on λ are sufficient to prove that τ is positively related to ϕ^* and negatively related to Q_L^* . Otherwise, the effects of τ on ϕ^* and Q_L^* are hard to characterize analytically.

Higher bankruptcy costs ceteris paribus raise the equilibrium cost of debt. Hence, we expect ϕ^* to be negatively related to γ , which is consistent with the usual intuition. But having lowered ϕ in response to higher γ , will the firm optimally raise or lower its capacity Q_L^* ? The answer is theoretically ambiguous because, conditional on reducing financial leverage, raising capacity in response to increased bankruptcy costs has two opposing effects: a higher Q increases expected profits in nondefault states but it also raises expected bankruptcy costs that are positively related to γ , other things held fixed. We show that the latter effect dominates the former if the fixed operating cost c is not too high, and hence, Q_L^* also decreases as γ increases if c is sufficiently low.

4. Discussion of Empirical Implications

The comparative statics analysis in Section 3.3 generates novel and refutable empirical predictions on the determinants of financial leverage and capital investment, which emerge from the endogeneity of these choice variables with endogenous costs of debt. We now discuss the empirical implications of the comparative statics results and relate these to the literature.

Our theoretical analysis applies to firms with valuable real options (“real options-intensive” firms), that is, firms with low capital adjustment costs that are exposed to high cash-flow uncertainty. These parametric restrictions can be checked empirically. For example, a large literature estimates capital adjustment costs using a variety of microlevel data sources (e.g., [Cooper and Haltiwanger 2006](#)). Similarly, forward-looking measures of cash flow uncertainty can be obtained from options on commodity futures ([Doshi, Kumar, and Yerramilli 2017](#)). Meanwhile, following [Myers \(1977\)](#), the empirical corporate finance literature often uses high market-to-book equity ratios as an indicator of valuable growth options. At the industry level, the literature considers valuable real options to reside in R&D-intensive and commodity-based industries, such as oil & gas and mining ([Jagle 1999](#); [Grullon, Lyandres, and Zhdanov 2012](#)).

Proposition 3 implies that capital adjustment costs will have a negative effect on the financial leverage of real options-intensive firms. In a similar vein, Proposition 3 also implies that financial leverage will be negatively related to fixed unit operating costs for firms with real high real option intensity and fixed operating costs. We note that various strands of literatures in finance, economics, and engineering use a variety of methods to estimate operating costs.¹³ To our knowledge, these predictions on the impact of technological or production-based parameters on optimal capital structure have not been tested in the liter-

¹³Although the applied finance literature uses financial accounting ratios, a large literature in industrial and process engineering has developed procedures for estimating various types of operating costs (e.g., see [Humphreys and Wellman 1996](#); [Ulrich and Vasudevan 2004](#)).

ature, and may help explain both the inter-industry and intra-industry variation in leverage ratios that cannot be explained by tax rates and financial distress costs (Lemmon, Roberts, and Zender 2008).

Next, Proposition 4 predicts a positive relation of financial leverage and cash-flow risk when capital adjustment costs are low, but predicts a negative relation when the adjustment costs are high. Our numerical simulations verify these theoretical predictions. To our knowledge, the change in sign of the effects of uncertainty on financial leverage depending on the size of capital adjustment costs is a novel aspect of our analysis. As we mentioned before, although the textbook trade-off model of capital structure predicts a negative relation of financial leverage to cash-flow risk (Kraus and Litzenberger 1976), the empirical evidence on this relation is quite mixed (Parsons and Titman 2010).

In the textbook model of capital structure, corporate taxes are *ceteris paribus* positively related to financial leverage because of the debt tax shield effect. However, in our model taxes also impact the value of the real option and the firm's endogenous default risk, so that the effect of taxes on financial leverage is not so straightforward. Overall, we are able to derive a positive relation between financial leverage and taxes in Proposition 5 only for low marginal tax rates. Notably, the empirical capital literature also presents mixed results on the relation of corporate taxes to financial leverage.¹⁴

Turning to the empirical implications with respect to optimal capital investment, the positive effects of fixed operating costs for firms with relatively low adjustment costs, high cash-flow risk, and high fixed operating costs (cf. Proposition 3) is a novel prediction that, to our knowledge, has not been tested in the literature.¹⁵ Similarly, the negative impact of deadweight bankruptcy costs on capital investment for firms with relatively low operating

¹⁴For example, Graham (2000) estimates large debt tax savings and suggests that firms could raise value by increasing leverage. However, MacKinlay (2012) finds that, on average, corporate taxes do not significantly affect firms' preference for issuing debt.

¹⁵Abel and Eberly (2002) examine the effects of the fixed components of capital adjustment costs on investment, but do not examine the effects of fixed operating costs *per se*.

costs appears not to have been tested in the empirical literature.

Finally, the joint determination of financial leverage and production capacity complicates the effects of corporate income taxes on investment. In particular, we are able to derive an unambiguous negative relation of taxes and investment only under restrictive conditions on capital adjustment costs. The importance of capital adjustment costs for the relation of capital investment to corporate income taxes is a novel aspect of our analysis. Moreover, the generally ambiguous theoretical relation appears to be consistent with the data.¹⁶

5. Conclusion

In this paper, we have analyzed the joint optimization of financial leverage and irreversible capacity investment, while incorporating realistic aspects of both production technologies and financial contracting that include capital adjustment and fixed operating costs, deadweight bankruptcy costs, debt tax shields, strategic default by equity holders, and endogenous interest costs. Because of the endogenous cost of debt, there is local substitutability in the choice financial and operational leverage as the firm trades off reduced operational costs and changes in tax shields from lowering capacity investment and raising financial leverage against higher expected capital adjustment costs, while maintaining the costs of debt. However, complementarity effects may also exist in response to variations in variables, such as deadweight bankruptcy costs, that lower the benefits of both financial leverage and capacity investment, at the margin.

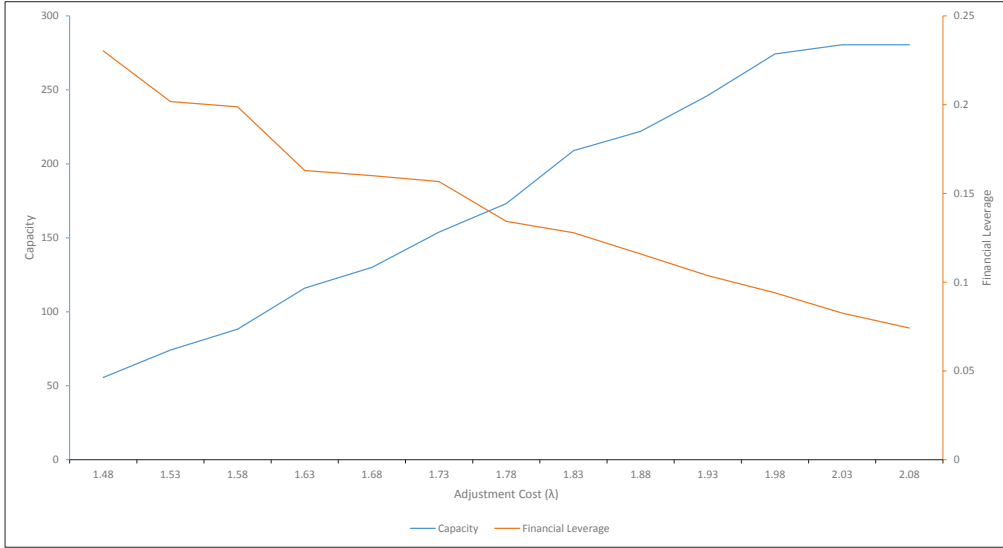
Building on these substitutability and complementarity relationships, we derive novel empirical predictions about the offsetting effects of technological parameters, such as capital adjustment costs, fixed operating costs, and cash-flow uncertainty, and corporate income

¹⁶There is a large empirical literature that studies the relation of corporate taxes and aggregate investment, but there are relatively only a few studies that use firm-level data. This literature generally finds small investment elasticities of corporate income taxes (Gruber and Rauh 2007), with an insignificant relation of corporate income taxes and investment in structures (Cummins and Hassett 1992).

taxes on optimal capital structure and capacity investment. Overall, the joint effects of technological and financial contracting variables on the choice variables at hand are also consistent with, and potentially help explain, some of ambiguous empirical results in the existing literature on the determinants of capital structure and investment.

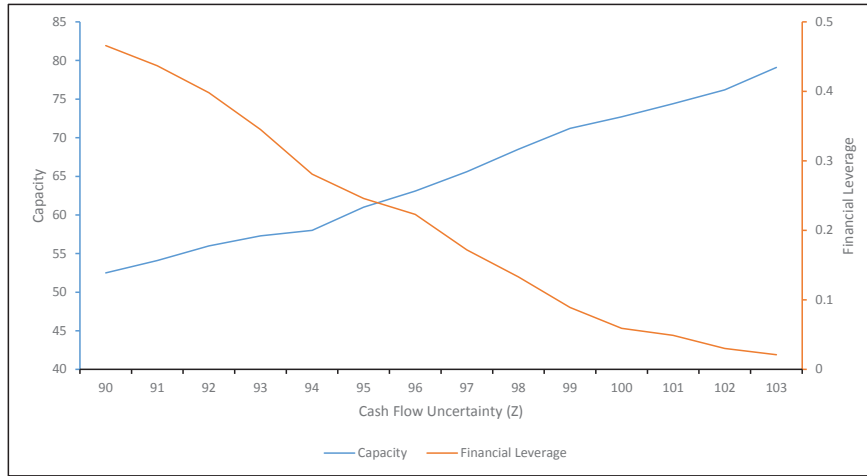
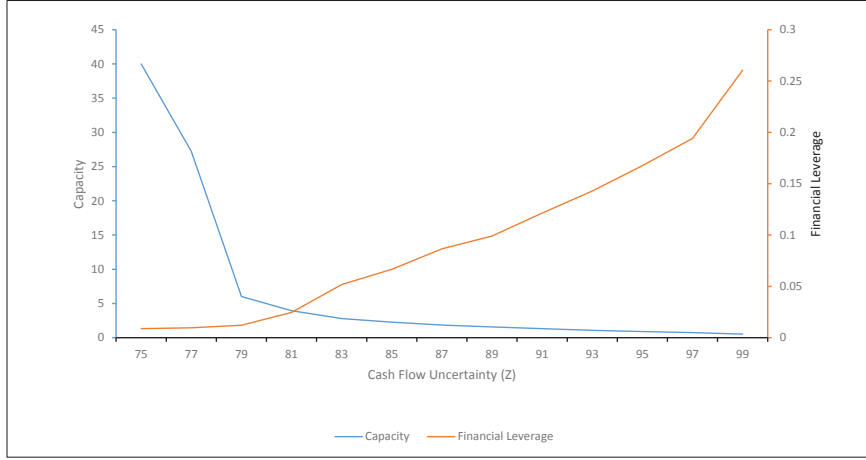
For tractability, we restricted attention to a two-period model in which the firm issues risky debt only once. A dynamic extension would allow the examination of sequential issues of risky debt with endogenous debt costs and is an important topic for future research. Nevertheless, it appears that the local substitutability of financial leverage and capacity investment size in the debt market equilibrium highlighted in our analysis will exist in dynamic extensions with attendant implications for the generally opposing effects of real (or operating cash flow related) parameters on the optimal capital structure and investment paths. We leave these issues for future research.

Figure 1: Variation of Q_L^* and ϕ^* with adjustment cost, λ



This figure plots the variation in optimal capacity investment, Q_L^* , and financial leverage, ϕ^* , with respect to the capital adjustment cost, λ . The vertical axis on the left-hand side denotes Q_L^* , whereas the vertical axis on the right-hand side denotes ϕ^* . The numerical analysis is based on the following parameter values: $\alpha = 70$, $Z = 75$, $\tau = 0.28$, $\gamma = 0.1$, and $c = 0.2$.

Figure 2: Variation of Q_L^* and ϕ^* with cash flow uncertainty, Z



This figure plots the variation in optimal capacity investment, Q_L^* , and financial leverage, ϕ^* , with respect to cash flow uncertainty, Z . The top panel plots the relationships when the capital adjustment cost, λ , is *LOW*, using the following parameter values: $\alpha = 70$, $\tau = 0.3$, $\gamma = 0.1$, $c = 0.2$, and $\lambda = 1.45$. The bottom panel plots the relationships when λ is *HIGH*, using the following parameter values: $\alpha = 70$, $\tau = 0.24$, $\gamma = 0.1$, $c = 0.8$, and $\lambda = 1.9$. In both panels, the vertical axis on the left-hand side denotes Q_L^* , whereas the vertical axis on the right-hand side denotes ϕ^* .

Appendix: Proofs

Proof of Lemma 1: Recall that $\psi \equiv \lambda(1+c)$ and note that the marginal value of output q is

$$\frac{d\pi_{UL}(q|Q, \delta)}{dq} = \begin{cases} \delta q^{-0.5} (1-\tau) & \text{if } q \leq Q \\ (\delta q^{-0.5} - \psi) (1-\tau) & \text{if } q > Q. \end{cases} \quad (\text{A1})$$

Clearly, $\hat{q}_{UL}(Q, \delta) = 0$ and $\hat{\pi}_{UL}(Q, \delta) = 0$ if $\delta < 0$. For $\delta \geq 0$, the optimal policy can be characterized as:

$$\hat{q}_{UL}(Q, \delta) = \begin{cases} Q & \text{if } \delta \leq \psi\sqrt{Q} \\ \left(\frac{\delta}{\psi}\right)^2 & \text{if } \delta > \psi\sqrt{Q}. \end{cases} \quad (\text{A2})$$

Consequently,

$$\hat{\pi}_{UL}(Q, \delta) = \begin{cases} (2\delta\sqrt{Q} - cQ) (1-\tau) & \text{if } \delta \leq \psi\sqrt{Q} \\ \left(\frac{\delta^2}{\psi} + Q(\psi - c)\right) (1-\tau) & \text{if } \delta > \psi\sqrt{Q}. \end{cases} \quad (\text{A3})$$

As $\alpha+Z$ is the upper bound for $\tilde{\delta}$, capacity expansion at $t = 1$ occurs with positive probability only if $\psi\sqrt{Q} < \alpha + Z$. ■

Proof of Proposition 1: The firm chooses $Q \geq 0$ to maximize $\mathbb{E}_\delta [\hat{\pi}_{UL}(Q, \delta)] - Q$.

Step 1: Solving for Q_{UL}^ .* Per Lemma 1, the expression for $\mathbb{E}_\delta [\hat{\pi}_{UL}(Q, \delta)]$ depends on $\psi\sqrt{Q} < \alpha + Z$, which determines whether the firm expands capacity at $t = 1$ with positive probability. Therefore, we consider the following cases separately to characterize Q_{UL}^* .

We characterize Q_{UL}^* under the conjecture that $\psi\sqrt{Q_{UL}^*} < \alpha + Z$, so that the firm expands project capacity with positive probability at $t = 1$, and verify that the conjecture holds in equilibrium.

If so, then based on the results in Lemma 1, and the fact that δ is uniformly distributed on $[\alpha - Z, \alpha + Z]$, we obtain the following expression for $\mathbb{E}_\delta [\hat{\pi}_{UL}(Q, \delta)]$:

$$\begin{aligned} \mathbb{E}_\delta [\hat{\pi}_{UL}(Q, \delta)] &= \frac{(1-\tau)}{2Z} \left[\int_0^{\psi\sqrt{Q}} 2\delta\sqrt{Q}d\delta + \int_{\psi\sqrt{Q}}^{\alpha+Z} \left(\frac{\delta^2}{\psi} + \psi Q\right) d\delta - 2ZcQ \right] \\ &= \frac{(1-\tau)}{2Z} \cdot \left[\frac{(\alpha+Z)^3}{3\psi} - \frac{\psi^2 Q^{1.5}}{3} + \psi Q(\alpha+Z) - 2ZcQ \right]. \end{aligned} \quad (\text{A4})$$

The firm's problem is to $\max_Q V_{UL}(Q) \equiv \mathbb{E}_\delta [\hat{\pi}_{UL}(Q, \delta)] - Q$. For convenience, we use the notation $V_Q^{UL} = \frac{\partial V_{UL}}{\partial Q}$. Therefore, if $Q_{UL}^* > 0$, it must satisfy the following first-order

condition:

$$V_Q^{UL} = \frac{(1-\tau)}{2Z} \cdot \left[\psi(\alpha + Z) - \frac{\psi^2 \sqrt{Q}}{2} - 2Zc \right] - 1 = 0. \quad (\text{A5})$$

(The second-order condition is clearly satisfied.) It follows that Q_{UL}^* is given by:

$$\psi \sqrt{Q_{UL}^*} = \max \left\{ 0, 2(\alpha + Z) - \frac{4Z(c(1-\tau) + 1)}{\psi(1-\tau)} \right\}. \quad (\text{A6})$$

We still need to verify the conjecture that $\psi \sqrt{Q_{UL}^*} < \alpha + Z$. The conjecture is clearly satisfied if $Q_{UL}^* = 0$. On the other hand, if $Q_{UL}^* > 0$, then per Equation (A6), $\psi \sqrt{Q_{UL}^*} < \alpha + Z$ is equivalent to the condition

$$\alpha + Z < Z \left(\frac{4(c(1-\tau) + 1)}{\psi(1-\tau)} \right). \quad (\text{A7})$$

Substituting $\psi = \lambda(1+c)$, and rearranging terms, it is evident that this condition is satisfied if and only if

$$\lambda < \bar{\lambda}_{UL} \equiv \frac{4(c(1-\tau) + 1)}{(1+c)(1-\tau)}, \quad (\text{A8})$$

and

$$Z > \bar{Z}_{UL} \equiv \frac{\alpha\lambda(1+c)(1-\tau)}{(4-\lambda)(1+c)(1-\tau) + 4\tau}. \quad (\text{A9})$$

Henceforth, we refer to the parametric region characterized by conditions $\lambda < \bar{\lambda}_{UL}$ and $Z > \bar{Z}_{UL}$ as the “expansion” region for the unlevered firm. It is easily verified that \bar{Z}_{UL} is increasing in λ and decreasing in τ . Also, recall that $Z \geq \alpha$ by assumption. Therefore, a sufficient condition for the firm being in the expansion region is that $\frac{4(c(1-\tau)+1)}{\psi(1-\tau)} > 2$, which is equivalent to the requirement that $\lambda < \frac{\bar{\lambda}_{UL}}{2}$. Finally, it is evident from Equation (A6) that $Q_{UL}^* = 0$ only if $\alpha + Z \leq \frac{2Z(c(1-\tau)+1)}{\psi(1-\tau)}$, which is possible only if

$$Z \geq \hat{Z}_{UL} \equiv \frac{\alpha\lambda(1+c)(1-\tau)}{(2-\lambda)(1+c)(1-\tau) + 2\tau}. \quad (\text{A10})$$

Note that $\hat{Z}_{UL} > \bar{Z}_{UL}$.

Step 2: Characterizing the comparative statics of $Q_{UL}^ > 0$ with respect to salient model parameters in the expansion region.*

Suppose that the firm is in the expansion region and $Z < \hat{Z}_{UL}$ so that $Q_{UL}^* > 0$.

a. Comparative statics with respect to λ : It follows by implicit differentiation of Equation

(A5) that

$$\frac{dQ_{UL}^*}{d\lambda} \propto \alpha + Z - \psi\sqrt{Q_{UL}^*},$$

which is positive, by definition, in the expansion region. Hence, Q_{UL}^* is strictly increasing in λ in this region.

b. Comparative statics with respect to c : Implicit differentiation of (A5) with respect to c shows that

$$\frac{dQ_{UL}^*}{dc} \propto \lambda \left[(\alpha + Z) - \lambda(1+c)\sqrt{Q_{UL}^*} \right] - 2Z, \quad (\text{A11})$$

which is ambiguous in sign since the term in the parentheses is positive in the expansion region. However, substituting the expression for $\psi\sqrt{Q_{UL}^*} > 0$ (cf. (A6)) in (A11) yields:

$$\begin{aligned} \frac{dQ_{UL}^*}{dc} &\propto \lambda \left[\frac{4Z(c(1-\tau)+1)}{\lambda(1+c)(1-\tau)} - (\alpha + Z) \right] - 2Z, \\ &= Z(\bar{\lambda}_{UL} - \lambda - 2) - \lambda\alpha, \end{aligned}$$

where the last equality is obtained by substituting for $\bar{\lambda}_{UL}$ (cf. (A8)). Therefore, $\frac{dQ_{UL}^*}{dc} > 0$ if the following two conditions are satisfied:

$$2 + \lambda < \bar{\lambda}_{UL} \text{ and } Z > \frac{\lambda\alpha}{(\bar{\lambda}_{UL} - \lambda - 2)}.$$

c. Comparative statics with respect to Z : It is clear from Equation (A6) that

$$\frac{dQ_{UL}^*}{dZ} \propto 2 - \frac{4(c(1-\tau)+1)}{\lambda(1+c)(1-\tau)} = 2 - \frac{\bar{\lambda}_{UL}}{\lambda}.$$

Hence $\frac{dQ_{UL}^*}{dZ} < 0$ iff $\lambda < \frac{\bar{\lambda}_{UL}}{2}$ (which is a sufficient condition for the firm to be in the expansion region).

d. Comparative statics with respect to τ : It is evident from Equation (A6) that Q_{UL}^* is decreasing in τ . ■

Proof of Lemma 2: The default threshold, $\bar{\delta}(\omega)$, is the level of δ at which $\hat{\pi}_{UL}(Q, \delta) = \phi QR_\tau$, so that the firm is just about indifferent between defaulting and not defaulting. If $\bar{\delta}(\omega) < \psi\sqrt{Q}$ in equilibrium (which we verify in Lemma A1 below), then using Equation

(A3)) and the definition of $\bar{\delta}(\omega)$, it follows that $(2\bar{\delta}\sqrt{Q} - cQ)(1 - \tau) = \phi QR_\tau$, and thus

$$\bar{\delta}(\omega) = \frac{\sqrt{Q}[\phi R_\tau + c(1 - \tau)]}{2(1 - \tau)}. \quad (\text{A12})$$

It is clear that $\bar{\delta}(\omega)$ is increasing in $\omega = (Q, \phi, r)$. ■

Proof of Proposition 2: The optimization problem is:

$$\max_{\phi \in [0,1], Q \geq 0} V^L(\omega) = \mathbb{E}_\delta[\hat{\pi}_{UL}(Q, \delta) | \delta \geq \bar{\delta}(\omega)] - \phi Q(1 - p(I_D))R_\tau - (1 - \phi)Q, \quad (\text{A13})$$

subject to

$$\Gamma(\omega) \equiv (1 - \gamma)\mathbb{E}_\delta[\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}(\omega)] + \phi Q[(1 - p(I_D))R - 1] = 0. \quad (\text{A14})$$

As shown in the text, substituting the constraint in the objective function yields the unconstrained optimization problem:

$$\max_{\phi \in [0,1], Q \geq 0} V_L(\omega) = V_{UL}(Q) + (1 - p(I_D)) \cdot \phi QR_\tau - [\gamma \mathbb{E}_\delta[\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}(\omega)]]. \quad (\text{A15})$$

If $\bar{\delta} \leq \psi\sqrt{Q}$, then as we showed in the proof of Lemma 2 that, for $\phi > 0$, $\bar{\delta}(\omega)$ is given by Equation (A12), so that

$$1 - p(I_D) = \frac{1}{2Z} \int_{\bar{\delta}}^{\alpha+Z} d\delta = \frac{\alpha + Z - \bar{\delta}}{2Z}. \quad (\text{A16})$$

Note that $1 - p(I_D) > 0$ since $\bar{\delta} < (\alpha + Z)$ in equilibrium. Moreover,

$$\begin{aligned} \mathbb{E}_\delta[\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}] &= \frac{(1 - \tau)}{2Z} \left[\int_0^{\bar{\delta}} (2\delta\sqrt{Q} - cQ) d\delta \right] \\ &= \frac{Q^{3/2} [(\phi R_\tau)^2 - c^2(1 - \tau)^2]}{8Z(1 - \tau)}, \end{aligned} \quad (\text{A17})$$

where the last equation is obtained by substituting for $\bar{\delta}$ using Equation (A12) and simplifying. Note that $\mathbb{E}_\delta[\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}] > 0$ since the default threshold, $\bar{\delta} > 0$ if $Q > 0$. Hence, $\phi R_\tau > c(1 - \tau)$ in any equilibrium with $\phi > 0$ and $Q > 0$.

We characterize the optimum for $Q_L^* > 0$ and $0 < \phi^* < 1$. Subsequently, we provide suf-

ficient conditions on the parameters to validate these assumptions, which will also identify conditions for $Q_L^* = 0$ and $\phi^* \in \{0, 1\}$. Note that conditional on (ϕ^*, Q_L^*) the competitive interest rate r^* is determined by the debt market condition (8). (To ease the notational burden, we will usually suppress the “*” superscript unless necessary for clarity.) For convenience, we use the notation $V_Q^{UL} = \frac{\partial V_{UL}}{\partial Q}$, $V_y^L = \frac{\partial V_L}{\partial y}$, $y \in \{\phi, Q\}$, and $V_{xy}^L \equiv \frac{\partial^2 V_L}{\partial xy}$, $x, y \in \{\phi, Q\}$.

Step 1: Characterizing Q_L^ .* The necessary first-order condition for $Q_L^* > 0$ is

$$V_Q^L = V_Q^{UL} + \phi\tau r \left[(1 - p(I_D)) - Q \frac{\partial p(I_D)}{\partial Q} \right] - \gamma \frac{\partial \mathbb{E}_\delta [\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}]}{\partial Q} = 0. \quad (\text{A18})$$

The components of Equation (A18) may be computed as follows:

$$\begin{aligned} V_Q^{UL} &= \frac{(1 - \tau)}{2Z} \cdot \left[\psi(\alpha + Z) - \frac{\psi^2 \sqrt{Q}}{2} - 2Z \left(c + \frac{1}{1 - \tau} \right) \right], \\ \frac{\partial \mathbb{E}_\delta [\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}]}{\partial Q} &= \frac{3Q^{1/2} [(\phi R_\tau)^2 - c^2(1 - \tau)^2]}{16Z(1 - \tau)}, \\ \text{and } \frac{\partial p(I_D)}{\partial Q} &= \frac{Q^{-1/2} [\phi R_\tau + c(1 - \tau)]}{8Z(1 - \tau)}. \end{aligned}$$

Substituting these expressions into Equation (A18) and simplifying, we can rewrite Equation (A18) as

$$V_Q^L = -K_1(\phi, r) \cdot \sqrt{Q} + K_2(\phi, r) = 0, \quad (\text{A19})$$

where

$$K_1(\phi, r) \equiv \frac{(1 - \tau)\psi^2}{4Z} + \frac{3\phi\tau r [\phi R_\tau + c(1 - \tau)]}{8Z(1 - \tau)} + \frac{3\gamma [(\phi R_\tau)^2 - c^2(1 - \tau)^2]}{16Z(1 - \tau)} \quad (\text{A20})$$

and

$$K_2(\phi, r) \equiv \frac{(1 - \tau)}{2Z} \cdot [\psi(\alpha + Z) - 2Zc] - 1 + \left(\frac{\alpha + Z}{2Z} \right) \phi\tau r. \quad (\text{A21})$$

Step 2: Characterizing ϕ^ .* Denoting $V_\phi^L = \frac{\partial V_L}{\partial \phi}$, the necessary first-order condition for $0 < \phi^* < 1$ is

$$V_\phi^L = (1 - p(I_D)) Q r \tau - (\phi Q r \tau) \frac{\partial p(I_D)}{\partial \phi} - \gamma \frac{\partial \mathbb{E}_\delta [\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}(\omega)]}{\partial \phi} = 0. \quad (\text{A22})$$

Next, the components of Equation (A22) may be computed as follows:

$$\begin{aligned}\frac{\partial \mathbb{E}_\delta[\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}]}{\partial \phi} &= \frac{Q^{3/2} \phi R_\tau^2}{4Z(1-\tau)}, \\ \text{and } \frac{\partial p(I_D)}{\partial \phi} &= \frac{\sqrt{Q} R_\tau}{4Z(1-\tau)}.\end{aligned}$$

Substituting the above expressions in Equation (A22) and simplifying, we can rewrite Equation (A22) as

$$V_\phi^L = -M_1(Q, r) \cdot \phi + M_2(Q, r) = 0, \quad (\text{A23})$$

where

$$M_1(Q, r) \equiv \frac{Q^{3/2} R_\tau \cdot (2\tau r + \gamma R_\tau)}{4Z(1-\tau)} \quad (\text{A24})$$

and

$$M_2(Q, r) \equiv \frac{Qr\tau \cdot [2(\alpha + Z) - c\sqrt{Q}]}{4Z}. \quad (\text{A25})$$

Step 3: Proving that $\phi^ > 0$ if $\tau > 0$.*

We prove this by contradiction. Suppose $\phi^* = 0$, so that $r^* = 0$ and $\bar{\delta} = 0$. Then, it is clear from the first-order condition (A18) that $Q_L^* = Q_{UL}^*$, and hence, $V^L(\omega) = V_{UL}(Q_{UL}^*)$. Then consider a perturbation with $\phi' = \varepsilon$ and $Q = Q_{UL}^*$, where ε is a small number. We denote by r' the interest rate that satisfies the competitive debt market condition (A14) for $\phi' = \varepsilon$ and $Q = Q_{UL}^*$, and denote $\omega' = (\varepsilon, Q_{UL}^*, r')$. Hence, we have from Equation (A15) that

$$V_L(\omega') - V^L(\omega) = (1 - p(I_D)) \cdot \varepsilon Q r' \tau - \gamma \mathbb{E}_\delta[\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}(\omega')]. \quad (\text{A26})$$

But it follows from the competitive debt market condition that

$$\gamma \mathbb{E}_\delta[\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}(\omega)] = \mathbb{E}_\delta[\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}(\omega)] + \varepsilon Q [(1 - p(I_D))R' - 1]. \quad (\text{A27})$$

Substituting (A27) in (A26) gives

$$V_L(\omega') - V^L(\omega) = Q\varepsilon [p(I_D) - (1 - p(I_D))r'(1 - \tau)] - \frac{Q^{3/2} [(\varepsilon R'_\tau)^2 - c^2(1 - \tau)^2]}{8Z(1 - \tau)}. \quad (\text{A28})$$

Note that

$$\lim_{\varepsilon \downarrow 0} [p(I_D) - (1 - p(I_D))r'(1 - \tau)] = 0.$$

Furthermore, $\lim_{\varepsilon \downarrow 0} p(I_D) = 0$ and $\lim_{\varepsilon \downarrow 0} (\varepsilon R'_\tau)^2 = 0$. Using these facts, we conclude that

$$\lim_{\varepsilon \downarrow 0} (V_L(\omega') - V^L(\omega)) = \frac{Q^{3/2} c^2 (1 - \tau)^2}{8Z(1 - \tau)} > 0. \quad (\text{A29})$$

Finally note from (A28), the functional expression of $p(I_D)$, and the competitive debt condition (that implicitly determines r') that $V_L(\omega') - V^L(\omega)$ is continuous in ε . Hence, it follows from (A29) that there exists some $\bar{\varepsilon} > 0$ such that $V_L(\omega') > V^L(\omega)$ if $0 < \varepsilon < \bar{\varepsilon}$. But this contradicts that $\phi^* = 0$ is an equilibrium outcome. Hence, we conclude that $\phi^* > 0$ if $\tau > 0$.

Step 4: Proving that $Q_L^ > Q_{UL}^*$.*

This is equivalent to showing that $[V_Q^L]_{Q=Q_{UL}^*} > 0$. Note that we can rewrite (A22) as

$$\begin{aligned} (1 - p(I_D)) r \tau &= (\phi r \tau) \frac{\partial p(I_D)}{\partial \phi} + \frac{\gamma}{Q} \frac{\partial \mathbb{E}_\delta [\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}(\omega)]}{\partial \phi} \\ &= \frac{\phi Q^{1/2} R_\tau (r \tau + \gamma R_\tau)}{4Z(1 - \tau)}. \end{aligned} \quad (\text{A30})$$

Substituting the above expression for $(1 - p(I_D)) r \tau$ in (A18), and simplifying (after a lot of algebra) yields

$$V_Q^L = V_Q^{UL} + \frac{\phi r \tau Q^{1/2} (\phi R_\tau - c(1 - \tau))}{8Z(1 - \tau)} + \frac{\gamma Q^{1/2} [(\phi R_\tau)^2 + 3c^2(1 - \tau)^2]}{16Z(1 - \tau)}. \quad (\text{A31})$$

Because $[V_Q^{UL}]_{Q=Q_{UL}^*} = 0$ and the remaining V_Q^L on the right-hand side of Equation (A31) are positive (because $\phi R_\tau - c(1 - \tau) > 0$ in equilibrium), it follows that $[V_Q^L]_{Q=Q_{UL}^*} > 0$, which implies that $Q_L^* > Q_{UL}^*$. ■

Lemma A1 *There exist thresholds $-\bar{\lambda}_L$, $\lambda_{low} < \bar{\lambda}_L$, \bar{Z}_L , $\bar{\tau}$ and $\bar{\rho} < 1$ such that if $\lambda \in (\lambda_{low}, \bar{\lambda}_L)$, $Z > \bar{Z}_L$, $\tau < \bar{\tau}$, and $\frac{\alpha}{Z} > \bar{\rho}$, then the following equilibrium conjectures are met: (1) $\psi \sqrt{Q_L^*} < \alpha + Z$; (2) $r^* < 1$; (3) $\bar{\delta} < \psi \sqrt{Q_L^*}$; and (4) the second-order conditions are satisfied.*

Proof of Lemma A1: We will deal with each conjecture sequentially below.

Part (1): Verifying the conjecture that $\psi \sqrt{Q_L^*} < \alpha + Z$.

Proving this conjecture is equivalent to showing that $[V_Q^L]_{\sqrt{Q} = \frac{\alpha + Z}{\psi}} < 0$. We will derive

sufficient conditions under which this is true. Now,

$$[V_Q^L]_{\sqrt{Q}=\frac{\alpha+Z}{\psi}} = -K_1 \cdot \frac{(\alpha+Z)}{\psi} + K_2, \quad (\text{A32})$$

where we have omitted the arguments of K_1 and K_2 for convenience. It is clear from Equation (A20) that $K_1(\phi, r) > \frac{(1-\tau)\psi^2}{4Z}$, and from Equation (A21) that

$$K_2 < \frac{(1-\tau)}{2Z} \cdot [\psi(\alpha+Z) - 2Zc] - 1 + \left(\frac{\alpha+Z}{2Z}\right),$$

because $\phi \leq 1$, $\tau < 1$ and $r < 1$ (as we prove in step (2) below). Substituting these in (A32), it follows that

$$\begin{aligned} [V_Q^L]_{\sqrt{Q}=\frac{\alpha+Z}{\psi}} &< \frac{\psi(\alpha+Z)(1-\tau) - 4Z(1+c(1-\tau)) + 2(\alpha+Z)}{4Z} \\ &= \frac{Z \cdot [\psi(1-\tau) - (2+4c(1-\tau))] + \psi\alpha(1-\tau) + 2\alpha}{4Z}. \end{aligned} \quad (\text{A33})$$

Hence, a sufficient condition for $[V_Q^L]_{\sqrt{Q}=\frac{\alpha+Z}{\psi}} < 0$ is that the expression on the right-hand side of (A33) is negative, which is true if

$$\lambda < \bar{\lambda}_L \equiv \frac{2+4c(1-\tau)}{(1+c)(1-\tau)} \quad (\text{A34})$$

and

$$Z > \bar{Z}_L \equiv \alpha \cdot \left[\frac{2+\lambda(1+c)(1-\tau)}{(2+4c(1-\tau) - \lambda(1+c)(1-\tau))} \right]. \quad (\text{A35})$$

In sum, when parameter values are such that $r < 1$, then there exist thresholds $\bar{\lambda}_L$ and \bar{Z}_L such that the conjecture $\psi\sqrt{Q_L^*} < \alpha+Z$ is valid if $\lambda < \bar{\lambda}_L$ and $Z > \bar{Z}_L$. Comparing Equations (A8) and (A34), it is evident that $\frac{\bar{\lambda}_{UL}}{2} \leq \bar{\lambda}_L < \bar{\lambda}_{UL}$.

Part (2): Verifying the conjecture that $r < 1$ in equilibrium.

Since $(1-\gamma)\mathbb{E}_\delta[\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}(\omega)] \geq 0$, it follows from the competitive debt market condition (8) that $(1-p(I_D)) \cdot (1+r) < 1$. Hence, $r < 1$ implies that $1-p(I_D) > 0.5$, and vice versa. Therefore, to derive sufficient conditions under which $r < 1$, we begin with the conjecture that $r < 1$ and then derive the parametric restrictions that are sufficient to guarantee that $1-p(I_D) > 0.5$. Since $1-p(I_D) = \frac{\alpha+Z-\bar{\delta}}{2Z}$, the requirement that $1-p(I_D) >$

0.5 is equivalent to $\alpha > \bar{\delta}$; that is, that

$$2\alpha(1-\tau) > [\phi R_\tau + c(1-\tau)] \cdot \sqrt{Q_L^*}. \quad (\text{A36})$$

Since $\phi \leq 1$ and $R_\tau < 2-\tau$ if $r < 1$, a sufficient condition that guarantees that $1-p(I_D) > 0.5$ is

$$\begin{aligned} 2\alpha(1-\tau) &> [1 + (1+c)(1-\tau)] \cdot \sqrt{Q_L^*} \\ &= [1 + (1+c)(1-\tau)] \cdot \left(\sqrt{Q_{UL}^*} + \varepsilon \right), \end{aligned} \quad (\text{A37})$$

where we define $\varepsilon \equiv \sqrt{Q^*} - \sqrt{Q_{UL}^*}$. We know from Proposition 2 that as $\tau \rightarrow 0$, $\phi \rightarrow 0$ and $Q_L^* \rightarrow Q_{UL}^*$, so that $\varepsilon \rightarrow 0$; that is, ε is small for low values of τ .

Let $\rho \equiv \frac{\alpha}{Z} \leq 1$ so that $\alpha = \rho Z$. We know from Proposition 1 that the unlevered firm is in the expansion region if $\lambda > \bar{\lambda}_{UL}$ and $Z < \bar{Z}_{UL}$, and in this region, $\psi\sqrt{Q_{UL}^*} < \alpha + Z = (1+\rho)Z$. Therefore, if ε is sufficiently small, then a sufficient condition for condition (A37) to hold is that

$$\begin{aligned} 2\rho\psi(1-\tau) &> [1 + (1+c)(1-\tau)] \cdot (1+\rho), \\ \text{that is } \frac{2\rho}{1+\rho} &> \frac{1}{\lambda} \left[1 + \frac{1}{(1+c)(1-\tau)} \right]. \end{aligned} \quad (\text{A38})$$

Since $\frac{2\rho}{1+\rho} \leq 1$ and is increasing in ρ , it follows that condition (A38) is met if ρ is sufficiently high, τ is sufficiently low, and $\lambda > \left[1 + \frac{1}{(1+c)(1-\tau)} \right]$. But τ being low also ensures that ε is low. Overall, we have shown that there exist thresholds $\bar{\rho} < 1$, $\bar{\tau}_1 > 0$, and $\lambda_{low} = \left[1 + \frac{1}{(1+c)(1-\tau)} \right]$ such that the conjecture $r^* < 1$ is met if $\frac{\alpha}{Z} > \bar{\rho}$, $\tau < \bar{\tau}_1$, and $\lambda > \lambda_{low}$. It is easily verified that $\lambda_{low} < \bar{\lambda}_L$.

Part (3): Verifying the conjecture that $\bar{\delta} < \psi\sqrt{Q_L^*}$.

Substituting for $\bar{\delta}$ from Equation (A12), this is equivalent to the requirement that $\frac{\phi R_\tau}{1-\tau} < 2\psi - c$, that is,

$$\phi \left(\frac{1}{1-\tau} + r \right) < 2\lambda(1+c) - c. \quad (\text{A39})$$

But since $\phi \leq 1$, (A39) will hold if $r \leq 1$ and $\tau = 0$. Hence, by continuity there exists some $\bar{\tau}_2 > 0$ such that $\bar{\delta} < \psi\sqrt{Q_L^*}$ if $0 < \tau < \bar{\tau}_2$.

Part (4): Verifying the conjecture that the second-order conditions are satisfied.

The second-order conditions for the first-order optimality conditions $V_y^L = \frac{\partial V_L}{\partial y} = 0$, $y \in$

$\{\phi, Q\}$ are expressed in the usual way through the Hessian matrix:

$$\mathcal{F} = \begin{bmatrix} V_{QQ}^L & V_{Q\phi}^L \\ V_{Q\phi}^L & V_{\phi\phi}^L \end{bmatrix}. \quad (\text{A40})$$

The second-order conditions require that $V_{QQ}^L < 0$, $V_{\phi\phi}^L < 0$, and $|\mathcal{F}| = V_{QQ}^L V_{\phi\phi}^L - (V_{Q\phi}^L)^2 > 0$.

Note that, from (A19), $V_{QQ}^L < 0$ if $K_1(\phi, r) > 0$, and a sufficient condition for this to be true is that $\phi R_\tau > c(1 - \tau)$, which holds in any equilibrium. Next, $M_1(Q_L, r) > 0$ (cf. (A24)) implies that $V_{\phi\phi}^L < 0$.

Finally, we characterize the conditions under which $|\mathcal{F}| > 0$, which requires us to first characterize $V_{Q\phi}^L$. Note that $V_{Q\phi}^L = V_{\phi Q}^L$. Hence, computing the partial derivative of Equation (A23) with respect to Q yields that $V_{Q\phi}^L = -\phi \cdot \frac{\partial M_1}{\partial Q} + \frac{\partial M_2}{\partial Q}$. But

$$\frac{\partial M_1}{\partial Q} = \frac{3Q^{1/2} R_\tau \cdot (2\tau r + \gamma R_\tau)}{8Z(1 - \tau)} = \frac{3M_1}{2Q} = \frac{3M_2}{2Q\phi},$$

where the last equation is obtained by noting that $M_2 = M_1\phi$ in equilibrium. Hence,

$$\begin{aligned} V_{Q\phi}^L &= -\frac{3M_2}{2Q} + \frac{\partial M_2}{\partial Q} \\ &= -\frac{3r\tau \cdot [2(\alpha + Z) - c\sqrt{Q}]}{8Z} + \frac{r\tau \cdot [4(\alpha + Z) - 3c\sqrt{Q}]}{8Z} \\ &= -\frac{(\alpha + Z)r\tau}{4Z} < 0. \end{aligned} \quad (\text{A41})$$

Therefore the second-order condition, $|\mathcal{F}| > 0$, is equivalent to

$$M_1 K_1 > 2\sqrt{Q_L^*} \cdot \left[\frac{(\alpha + Z)r\tau}{4Z} \right]^2. \quad (\text{A42})$$

Note now from (A24) that $[M_1(Q_L, r)]_{\tau=0} = \frac{Q^{3/2}\gamma R^2}{4Z} > 0$, while it can readily be checked from (A20) that $[K_1(\phi, r)]_{\tau=0} = \frac{4\psi^2 - 3\gamma c^2}{16Z} > 0$ (because $\gamma < 1$ and $\psi^2 > c^2$). Hence, condition (A42) is satisfied if $\tau = 0$. From continuity, it follows that there exists $\bar{\tau}_3 > 0$ such that (A42) holds for $\tau \in (0, \bar{\tau}_3)$.

Combining parts (1) through (4) above, it follows that there exist thresholds λ_{low} , $\bar{\lambda}_L$, \bar{Z}_L , and $\bar{\tau} \equiv \min\{\bar{\tau}_1, \bar{\tau}_2, \bar{\tau}_3\}$, such that all the four equilibrium conjectures are satisfied if $\lambda \in (\lambda_{low}, \bar{\lambda}_L)$, $\tau < \bar{\tau}$ and $Z > \bar{Z}_L$. ■

Lemma A2 *The marginal rate of substitution of financial leverage for capacity investment in the debt market equilibrium is negative.*

Proof of Lemma A2: When the debt market is in equilibrium (i.e., $\Gamma(\omega) = 0$), consider perturbations of (Q, ϕ) that maintain the debt market in equilibrium. Using $\Gamma_x(\omega)$ to denote the $\frac{\partial \Gamma(\omega)}{\partial x}$, the marginal rate of substitution of Q for ϕ in the debt market equilibrium is given by

$$\left[\frac{d\phi}{dQ} \right]_{\Gamma=0} = -\frac{\Gamma_Q(\omega)}{\Gamma_\phi(\omega)}. \quad (\text{A43})$$

For notational convenience, denote $\mathbb{E}_\delta^- \equiv \mathbb{E}_\delta [\hat{\pi}_{UL}(Q, \delta) | \delta \leq \bar{\delta}(\omega)]$. Then

$$\Gamma_\phi(\omega) = (1 - \gamma) \frac{\partial \mathbb{E}_\delta^-}{\partial \phi} + QR[1 - p(I_D)] - Q - R\phi Q \frac{\partial p(I_D)}{\partial \phi}.$$

But $(1 - \gamma) \frac{\partial \mathbb{E}_\delta^-}{\partial \phi} \leq \frac{\partial \mathbb{E}_\delta^-}{\partial \phi}$, and it follows from (2) that $QR[1 - p(I_D)] \leq Q$. Hence,

$$\Gamma_\phi(\omega) \leq \frac{\partial \mathbb{E}_\delta^-}{\partial \phi} - R\phi Q \frac{\partial p(I_D)}{\partial \phi} = \frac{Q^{3/2} \phi R_\tau (R_\tau - R)}{4Z(1 - \tau)} < 0. \quad (\text{A44})$$

Next,

$$\begin{aligned} \Gamma_Q(\omega) &= (1 - \gamma) \frac{\partial \mathbb{E}_\delta^-}{\partial Q} + \phi R[1 - p(I_D)] - \phi - R\phi Q \frac{\partial p(I_D)}{\partial Q} \\ &= (1 - \gamma) \frac{\partial \mathbb{E}_\delta^-}{\partial Q} - \frac{(1 - \gamma) \mathbb{E}_\delta^-}{Q} - R\phi Q \frac{\partial p(I_D)}{\partial Q}, \end{aligned}$$

where the second equality follows from (2). Substituting for $\frac{\partial \mathbb{E}_\delta^-}{\partial Q}$, \mathbb{E}_δ^- and $\frac{\partial p(I_D)}{\partial Q}$ from above, and simplifying, it follows that:

$$\begin{aligned} \Gamma_Q(\omega) &= \frac{Q^{1/2} [\phi R_\tau + c(1 - \tau)] \cdot [(1 - \gamma)[\phi R_\tau - c(1 - \tau)] - 2\phi R}{16Z(1 - \tau)} \\ &< 0, \end{aligned} \quad (\text{A45})$$

where the inequality follows because $(1 - \gamma) \phi R_\tau < 2\phi R$.

Since $\Gamma_Q < 0$ and $\Gamma_\phi < 0$, it follows from (A43) that $\frac{d\phi}{dQ} |_{\Gamma=0} < 0$. ■

Proof of Proposition 3: From Cramer's rule, the comparative statics with respect to any parameter x are:

$$\begin{aligned}\frac{dQ_L^*}{\partial x} &= \frac{\begin{vmatrix} -V_{Qx}^L & V_{Q\phi}^L \\ -V_{\phi x}^L & V_{\phi\phi}^L \end{vmatrix}}{|\mathcal{F}|} \propto -V_{Qx}^L V_{\phi\phi}^L + V_{Q\phi}^L V_{\phi x}^L \\ \frac{d\phi^*}{\partial x} &= \frac{\begin{vmatrix} V_{QQ}^L & -V_{Qx}^L \\ V_{Q\phi}^L & -V_{\phi x}^L \end{vmatrix}}{|\mathcal{F}|} \propto -V_{QQ}^L V_{\phi x}^L + V_{Q\phi}^L V_{Qx}^L,\end{aligned}\tag{A46}$$

because $|\mathcal{F}| > 0$ (see Lemma A1). These comparative statics reflect perturbations in (Q_L^*, ϕ^*) in response to small variations in the model parameters that continue to satisfy the competitive debt market condition $\Gamma(\omega) = 0$.

Part (1): Comparative statics with respect to λ . It follows from Equation (A19) that

$$V_{Q\lambda}^L = -\frac{\partial K_1}{\partial \lambda} Q + \frac{\partial K_2}{\partial \lambda} = \frac{(1-\tau)(1+c)(\alpha+Z-\psi\sqrt{Q_L^*})}{2Z} > 0,\tag{A47}$$

because $\psi\sqrt{Q_L^*} < \alpha+Z$ in equilibrium. On the other hand, it is clear from Equation (A23) that $V_{\phi\lambda}^L = 0$.

Combining with Equation (A46), it follows that $\frac{dQ_L^*}{d\lambda} \propto -V_{Q\lambda}^L V_{\phi\phi}^L > 0$, because $V_{Q\lambda}^L > 0$ (cf. (A47)) and $V_{\phi\phi}^L = -M_1 < 0$.

Similarly, $\frac{d\phi^*}{d\lambda} \propto V_{Q\lambda}^L V_{Q\phi}^L < 0$ because $V_{Q\lambda}^L > 0$ and $V_{Q\phi}^L < 0$ (cf. (A41)).

Part (2): Comparative statics with respect to c . We have

$$\frac{dQ_L^*}{dc} \propto -V_{Qc}^L V_{\phi\phi}^L + V_{Q\phi}^L V_{\phi c}^L,\tag{A48}$$

$$\frac{d\phi^*}{dc} \propto -V_{QQ}^L V_{\phi c}^L + V_{Q\phi}^L V_{Qc}^L.\tag{A49}$$

It follows from Equations (A19), (A20), and (A21) that $V_{Qc}^L = -\sqrt{Q} \frac{\partial K_1}{\partial c} + \frac{\partial K_2}{\partial c}$, where

$$\begin{aligned}\frac{\partial K_1}{\partial c} &= \frac{(1-\tau)\lambda^2(1+c)}{2Z} + \frac{\phi\tau r}{8Z} - \frac{3\gamma c(1-\tau)}{8Z}, \\ \text{and } \frac{\partial K_2}{\partial c} &= \frac{(1-\tau)}{2Z} \cdot [\lambda(\alpha+Z) - 2Z].\end{aligned}$$

Hence,

$$V_{Qc}^L = \frac{\lambda(1-\tau)(\alpha + Z - \psi\sqrt{Q})}{2Z} + \frac{\sqrt{Q}(3\gamma c(1-\tau) - \phi\tau r)}{8Z} - (1-\tau). \quad (\text{A50})$$

Similarly, it follows from Equations (A23), (A24), and (A25) that

$$V_{\phi c}^L = \frac{\partial M_2}{\partial c} = -\frac{r\tau Q^{3/2}}{4Z} < 0. \quad (\text{A51})$$

Therefore, if $V_{Qc}^L > 0$, then it is clear from (A48) and (A49), respectively, that $\frac{dQ_L^*}{dc} > 0$ and $\frac{d\phi^*}{dc} < 0$.

We now evaluate the conditions for $V_{Qc}^L > 0$. Since $\phi\tau r \leq 1$ in equilibrium, the second term in (A50) is positive if $c > \frac{1}{3\gamma(1-\tau)}$. Meanwhile, the sum of the remaining two terms is nonnegative if

$$\sqrt{Q_L^*} \leq \frac{\lambda(\alpha + Z) - 2Z}{\lambda^2(1+c)} \equiv Y. \quad (\text{A52})$$

By the same logic in the proof of Lemma A1, proving that $\sqrt{Q_L^*} < Y$ is equivalent to proving that $[V_Q^L]_{\sqrt{Q}=Y} = -K_1 \cdot Y + K_2 < 0$. We will derive *sufficient* conditions under which this is true. It is clear from Equation (A20) that $K_1 > \frac{(1-\tau)\psi^2}{4Z}$, and from Equation (A21) that

$$K_2 < \frac{(1-\tau)}{2Z} \cdot [\psi(\alpha + Z) - 2Zc] - 1 + \left(\frac{\alpha + Z}{2Z}\right),$$

because $\phi\tau r < 1$. Hence,

$$\begin{aligned} [V_Q^L]_{\sqrt{Q}=Y} &< -\frac{(1-\tau)\psi^2}{4Z} \cdot Y + \frac{(1-\tau)}{2Z} \cdot [\psi(\alpha + Z) - 2Zc] - 1 + \left(\frac{\alpha + Z}{2Z}\right) \\ &= \frac{\alpha \cdot [(1-\tau)\psi + 2] + Z \cdot [(1-\tau)(\psi - 2c) - 2\tau]}{4Z}. \end{aligned} \quad (\text{A53})$$

Hence, it is sufficient to show that the expression on the right-hand side of (A53), which is possible if $(1-\tau)(\psi - 2c) - 2\tau < 0$, that is, if

$$\lambda < \frac{2\tau + 2c(1-\tau)}{(1-\tau)(1+c)} \equiv \lambda_c \quad (\text{A54})$$

and

$$Z > \alpha \cdot \left[\frac{\lambda(1-\tau)(1+c) + 2}{2\tau + 2c(1-\tau) - \lambda(1-\tau)(1+c)} \right] \equiv Z_c. \quad (\text{A55})$$

In Lemma A1, we imposed the restriction that $\lambda > \lambda_{low}$. Hence, we need to verify that $\lambda_c > \lambda_{low}$, which is equivalent to $c > \frac{2-3\tau}{1-\tau}$. Moreover, it is easily checked by comparing Equations (A34) and (A54) that $\lambda_c < \bar{\lambda}_L$, and by comparing Equations (A35) and (A55) that $Z_c > \bar{Z}_L$. Overall, we have shown that there exist thresholds – λ_c , Z_c , and $\bar{c} = \max\left\{\frac{1}{3\gamma(1-\tau)}, \frac{2-3\tau}{1-\tau}\right\}$ – such that $V_{Qc}^L > 0$ and hence, $\frac{dQ_L^*}{dc} > 0$ and $\frac{d\phi^*}{dc} < 0$, if $\lambda < \lambda_c$, $Z > Z_c$, and $c > \bar{c}$. ■

Proof of Proposition 4: Using (A46), we obtain

$$\frac{dQ^*}{dZ} \propto -V_{QZ}^L V_{\phi\phi}^L + V_{\phi Z}^L V_{QQ}^L, \quad (\text{A56})$$

$$\frac{d\phi^*}{dZ} \propto -V_{QQ}^L V_{\phi Z}^L + V_{QZ}^L V_{Q\phi}^L. \quad (\text{A57})$$

It follows from Equations (A19), (A20), and (A21) that $V_{QZ}^L = -\sqrt{Q}\frac{\partial K_1}{\partial Z} + \frac{\partial K_2}{\partial Z}$, where

$$\frac{\partial K_1}{\partial Z} = -\frac{K_1}{Z} \text{ and } \frac{\partial K_2}{\partial Z} = -\frac{(1-\tau)\psi\alpha}{2Z^2} - \frac{\alpha\phi\tau r}{2Z^2}.$$

Note that $\frac{K_1\sqrt{Q}}{Z} = \frac{K_2}{Z}$ per Equation (A19). Hence,

$$V_{QZ}^L = \frac{K_2}{Z} + \frac{\partial K_2}{\partial Z} = \frac{[(1-\tau)(\lambda(1+c) - 2c) + \phi\tau r - 2]}{2Z}. \quad (\text{A58})$$

It is evident from Equation (A58) that V_{QZ}^L increases with λ and that $[V_{QZ}^L]_{\lambda=1} < 0$. Hence, there exists a $\hat{\lambda}_L > 1$ such that $V_{QZ}^L \leq 0 \iff \lambda \leq \hat{\lambda}_L$.

Similarly, it follows from Equations (A23), (A24), and (A25) that $V_{\phi Z}^L = -\phi\frac{\partial M_1}{\partial Z} + \frac{\partial M_2}{\partial Z}$, where

$$\frac{\partial M_1}{\partial Z} = -\frac{M_1}{Z} \text{ and } \frac{\partial M_2}{\partial Z} = -\frac{Qr\tau \cdot (2\alpha - c\sqrt{Q})}{4Z^2}.$$

Note that $M_1\phi = M_2$ per Equation (A23). Hence,

$$V_{\phi Z}^L = \frac{M_2}{Z} + \frac{\partial M_2}{\partial Z} = \frac{Qr\tau}{2Z} > 0. \quad (\text{A59})$$

Consider the following two cases separately:

Case 1: Suppose $\lambda \leq \hat{\lambda}_L$ (so that $V_{QZ}^L \leq 0$). Then, since $V_{QQ}^L < 0$, $V_{\phi\phi}^L < 0$, and $V_{Q\phi}^L < 0$, it is clear from (A56) and (A57) that $\frac{dQ_L^*}{dZ} < 0$ and $\frac{d\phi^*}{dZ} > 0$.

Case 2: Suppose $\lambda > \hat{\lambda}_L$ (so that $V_{QZ}^L > 0$).

a. First consider $\frac{dQ_L^*}{dZ}$. After substituting for the terms in (A56), we obtain

$$\begin{aligned}\frac{dQ_L^*}{dZ} &\propto M_1 V_{QZ}^L - \frac{Qr\tau}{2Z} \frac{(\alpha + Z)r\tau}{2Z} \\ &> \left(\frac{(\alpha + Z)r\tau}{2Z} \right)^2 \frac{\sqrt{Q}}{K_1} \cdot \left(2V_{QZ}^L - \frac{K_1\sqrt{Q}}{\alpha + Z} \right), \\ &= \left(\frac{(\alpha + Z)r\tau}{2Z} \right)^2 \frac{\sqrt{Q}}{K_1} \cdot \left(2V_{QZ}^L - \frac{K_2}{\alpha + Z} \right),\end{aligned}\tag{A60}$$

where the first inequality follows because $M_1 > \frac{2\sqrt{Q}}{K_1} \cdot \left[\frac{(\alpha+Z)r\tau}{2Z} \right]^2$ by the second-order condition (A42) (and because $V_{QZ}^L > 0$ in the case under consideration), and the second equality follows since $K_1\sqrt{Q} = K_2$ from the first-order condition for Q . But from (A21)

$$\frac{K_2}{\alpha + Z} = \frac{\psi(1 - \tau) + \phi\tau r}{2Z} - \frac{[1 + c(1 - \tau)]}{\alpha + Z}.\tag{A61}$$

Hence, using Equations (A58) and (A61), and substituting $\phi\tau r \geq 0$, it follows that

$$2V_{QZ}^L - \frac{K_2}{\alpha + Z} \geq \frac{\lambda(1 + c)(1 - \tau)}{2Z} - \frac{2(2\alpha + Z) \cdot [1 + c(1 - \tau)]}{2Z(\alpha + Z)}.\tag{A62}$$

It follows from (A60) and (A62) that $\frac{dQ_L^*}{dZ} > 0$ if

$$\lambda > \left(2 + \frac{2\alpha}{\alpha + Z} \right) \cdot \left(\frac{1 + c(1 - \tau)}{(1 + c)(1 - \tau)} \right) \equiv \tilde{\lambda}_L(Z).\tag{A63}$$

It is easily checked that $\tilde{\lambda}_L(Z) > \hat{\lambda}_L$ and that $\tilde{\lambda}_L(Z)$ is decreasing in Z . Hence, we conclude that $\frac{dQ_L^*}{dZ} > 0$ if λ and Z are sufficiently high.

b. Next, consider $\frac{d\phi^*}{dZ}$. After substituting for the terms in (A57), we obtain

$$\begin{aligned}\frac{d\phi^*}{dZ} &\propto -V_{QQ}^L V_{\phi Z}^L + V_{QZ}^L V_{Q\phi}^L \\ &= \frac{\sqrt{Q}r\tau}{2Z} \left(\frac{K_1}{2} - \frac{(\alpha + Z)}{\sqrt{Q}} V_{QZ}^L \right) \\ &= \frac{r\tau}{4Z} (K_2 - 2(\alpha + Z)V_{QZ}^L),\end{aligned}\tag{A64}$$

where the last equality in (A64) again follows from the fact that $K_1\sqrt{Q} = K_2$ from the first-order condition for Q . But if $\lambda > \tilde{\lambda}_L(Z)$, then $K_2 < 2(\alpha + Z)V_{QZ}^L$. Thus, (A64) implies that $\frac{d\phi^*}{dZ} < 0$ if $\lambda > \tilde{\lambda}_L(Z)$, that is, if λ and Z are sufficiently high. ■

Proof of Proposition 5: Part (1): Comparative statics with respect to τ . We have

$$\frac{dQ^*}{d\tau} \propto -V_{Q\tau}^L V_{\phi\phi}^L + V_{Q\phi}^L V_{\phi\tau}^L, \quad (\text{A65})$$

$$\frac{d\phi^*}{d\tau} \propto -V_{QQ}^L V_{\phi\tau}^L + V_{Q\phi}^L V_{Q\tau}^L. \quad (\text{A66})$$

Step 1: Characterizing $V_{\phi\tau}^L$. It follows from Equations (A23), (A24), and (A25) that $V_{\phi\tau}^L = -\phi^* \frac{\partial M_1}{\partial \tau} + \frac{\partial M_2}{\partial \tau}$, where

$$\begin{aligned} \frac{\partial M_1}{\partial \tau} &= \frac{Q^{3/2} \cdot [(1-\tau)(2r(1-\gamma)R_\tau - 2r^2\tau) + 2r\tau R_\tau + \gamma R_\tau^2]}{4Z(1-\tau)^2} \\ &= \frac{rQ^{3/2} \cdot [(1-\gamma)R_\tau - r\tau]}{2Z(1-\tau)} + \frac{M_1}{(1-\tau)}, \end{aligned}$$

and

$$\frac{\partial M_2}{\partial \tau} = \frac{Qr \cdot [2(\alpha + Z) - c\sqrt{Q}]}{4Z} = \frac{M_2}{\tau} = \frac{M_1\phi^*}{\tau},$$

where the last equation follows by noting that $M_1\phi^* = M_2$ (from Equation (A23)). Substituting from the above expressions yields that

$$V_{\phi\tau}^L = \phi^* \left[\frac{(1-2\tau)M_1}{\tau(1-\tau)} - \frac{rQ^{3/2} \cdot ((1-\gamma)R_\tau - r\tau)}{2Z(1-\tau)} \right]. \quad (\text{A67})$$

After substituting for M_1 and simplifying, it is easily verified that $V_{\phi\tau}^L \geq 0$ iff

$$(1-2\tau)R_\tau \cdot (2\tau r + \gamma R_\tau) \geq 2\tau r(1-\tau) \cdot ((1-\gamma)R_\tau - r\tau). \quad (\text{A68})$$

Note that condition (A68) is satisfied at $\tau = 0$, hence, by continuity, $V_{\phi\tau}^L > 0$ when $\tau \in (0, \hat{\tau})$ for some $\hat{\tau} \ll 0.5$. Since $\tau \in (0, \bar{\tau})$ under our maintained assumptions, define $\tilde{\tau} = \min[\hat{\tau}, \bar{\tau}]$. Hence, we have shown that $V_{\phi\tau}^L > 0$ for $\tau \in (0, \tilde{\tau})$.

Step 2: Characterizing $V_{Q\tau}^L$. Next, it follows from Equations (A19), (A20), and (A21) that $V_{Q\tau}^L = -\sqrt{Q} \frac{\partial K_1}{\partial \tau} + \frac{\partial K_2}{\partial \tau}$, where

$$\begin{aligned} \frac{\partial K_1}{\partial \tau} &= -\frac{\psi^2}{4Z} + \frac{2\phi rc + 3\gamma c^2}{16Z} + \frac{\phi^2 [2r(R_\tau - r\tau(1-\tau)) + 3\gamma R_\tau(R_\tau - 2r(1-\tau))]}{16Z(1-\tau)^2} \\ &> -\frac{\psi^2}{4Z}, \end{aligned}$$

because the next two terms are positive, and

$$\frac{\partial K_2}{\partial \tau} = -\frac{\psi(\alpha + Z)}{2Z} + c + \left(\frac{\alpha + Z}{2Z}\right) \phi^* r^* \leq -\frac{\psi(\alpha + Z)}{2Z} + c + \phi^* r^*,$$

where the inequality follows because $\alpha \leq Z$. Therefore,

$$V_{Q\tau}^L < \frac{\psi^2 \sqrt{Q_L^*}}{4Z} - \frac{\psi(\alpha + Z)}{2Z} + c + \phi^* r^* < (1 + c) \left(1 - \frac{\lambda(\alpha + Z)}{4Z}\right), \quad (\text{A69})$$

where the second inequality follows after substituting $\psi = \lambda(1 + c)$ and noting that $\psi \sqrt{Q_L^*} < \alpha + Z$ and $\phi^* r^* < 1$, in equilibrium. It is evident from (A69) that $V_{Q\tau}^L < 0$ if $\lambda > \frac{4Z}{\alpha + Z}$. It follows from (A65) and (A66) that there exist thresholds $\tilde{\tau}$ and $\lambda_\tau = \max(\frac{4Z}{\alpha + Z}, \lambda_{low})$ such that $\frac{dQ^*}{d\tau} < 0$ and $\frac{d\phi^*}{d\tau} > 0$ if $\tau \in (0, \tilde{\tau})$ and $\lambda \in (\lambda_\tau, \bar{\lambda}_L)$.

Part (2): Comparative statics with respect to γ . Note that

$$\frac{dQ^*}{d\gamma} \propto -V_{Q\gamma}^L V_{\phi\phi}^L + V_{\phi\gamma}^L V_{Q\phi}^L, \quad (\text{A70})$$

$$\frac{d\phi^*}{d\gamma} \propto -V_{QQ}^L V_{\phi\gamma}^L + V_{Q\gamma}^L V_{Q\phi}^L. \quad (\text{A71})$$

It is clear from Equations (A19), (A20), and (A21) that

$$V_{Q\gamma}^L = -\sqrt{Q} \frac{\partial K_1}{\partial \gamma} = -\frac{3Q^{1/2} [(\phi R_\tau)^2 - c^2 (1 - \tau)^2]}{16Z(1 - \tau)} < 0. \quad (\text{A72})$$

Similarly, it follows from Equations (A23), (A24), and (A25) that

$$V_{\phi\gamma}^L = -\phi \frac{\partial M_1}{\partial \gamma} = -\frac{Q^{3/2} \phi R_\tau^2}{4Z(1 - \tau)} < 0. \quad (\text{A73})$$

Step 1: Characterizing $\frac{d\phi^}{d\gamma}$.* Note that $V_{QQ}^L = -\frac{K_1}{2\sqrt{Q}} = -\frac{K_2}{2Q}$, because $K_1 = \frac{K_2}{\sqrt{Q}}$ in equilibrium. Multiplying this with Equation (A73) yields

$$-V_{QQ}^L V_{\phi\gamma}^L = -\frac{K_2 \sqrt{Q} \phi R_\tau^2}{8Z(1 - \tau)} = -\frac{[(1 - \tau) \psi^2 \sqrt{Q_{UL}^*} + 2(\alpha + Z) \phi r \tau] \sqrt{Q} \phi R_\tau^2}{32Z^2(1 - \tau)}, \quad (\text{A74})$$

where the second equation follows by substituting for K_2 .

Meanwhile, multiplying Equations (A41) and (A72) yields

$$V_{Q\gamma}^L V_{Q\phi}^L = \frac{3\sqrt{Q}[(\phi R_\tau)^2 - c^2(1-\tau)^2](\alpha + Z)r\tau}{64Z^2(1-\tau)}. \quad (\text{A75})$$

It follows from (A71), (A74) and (A75) that $\frac{d\phi^*}{d\gamma} < 0$ iff

$$\left[2(1-\tau)\psi^2\sqrt{Q_{UL}^*} + (\alpha + Z)\phi r\tau \right] \phi R_\tau^2 > -3c^2(1-\tau)^2(\alpha + Z)r\tau,$$

which always holds. Hence, we have shown that $\frac{d\phi^*}{d\gamma} < 0$.

Step 2: Characterizing $\frac{dQ^}{d\gamma}$.* Note that $V_{\phi\phi}^L = -M_1 = \frac{M_2}{\phi}$. Hence, by logic similar to that in step (1), it follows that $\frac{dQ^*}{d\gamma} < 0$ iff

$$|V_{Q\gamma}^L| \cdot M_2 > \phi V_{\phi\gamma}^L V_{Q\phi}^L \iff |V_{Q\gamma}^L| \cdot M_2 > \frac{Q^{3/2}\phi^2 R_\tau^2 (\alpha + Z)r\tau}{16Z^2(1-\tau)}. \quad (\text{A76})$$

After substituting for $|V_{Q\gamma}^L|$ and M_2 , the above condition simplifies to

$$3[(\phi R_\tau)^2 - c^2(1-\tau)^2] \cdot [2(\alpha + Z) - c\sqrt{Q}] > 4\phi^2 R_\tau^2 (\alpha + Z). \quad (\text{A77})$$

It is easily verified that condition (A77) is met at $c = 0$ if $\phi^* > 0$, which we have shown to be true if $\tau > 0$. Hence, by continuity of the L.H.S. of (A77) in c , there exists some $c_\gamma > 0$ such that condition (A77) is met if $c \in (0, c_\gamma)$. ■

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