

# Analyst Coverage and the Cross Sectional Relation Between Returns and Volatility

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## **Analyst Coverage and the Cross Sectional Relation Between Returns and Volatility**

The January effect conceals a significant negative relation between returns and idiosyncratic volatility at horizons up to two years. Controlling for January and the influence of penny stocks, we find that this relation, first documented by Ang, Hodrick, Xing and Zhang (2006), is robust to various measures of idiosyncratic volatility even after skipping a month to account for bid-ask reversals. The relation is not attributable to small firms or stocks with lottery-like payoffs. We model and empirically test the hypothesis that low returns to high volatility stocks are corrections of optimistic mispricing that arises because news shocks generate disagreement among traders. Our empirical tests, which use low analyst coverage as a proxy for disagreement, are consistent with this explanation of both AHXZ's result and the negative relation between returns and turnover volatility documented by Chordia, Subrahmanyam and Anshuman (2001). Among other findings, we show that the negative relations between returns and idiosyncratic volatility, and returns and turnover volatility, exist only among low coverage stocks.

## Introduction

Traditional asset pricing models predict there should be no return premium to securities' idiosyncratic volatility because investors eliminate such risk by holding optimally diversified portfolios. However, Merton (1987) considers a setting of limited diversification where investors' holdings are restricted to subsets of stocks that are "known" to them. Idiosyncratic volatility then contributes to portfolio risk and is priced, with higher volatility stocks earning higher average returns. From either of these perspectives, the result documented by Ang, Hodrick, Xing and Zhang (2006, 2009) is puzzling. They show empirically that high idiosyncratic volatility stocks earn low average returns (henceforth referred to as the AHXZ result).

Several explanations have been proposed for this finding. Han and Kumar (2008) and Bali, Cakici and Whitelaw (2011) argue that investors prefer securities with lottery-like payoffs, implying that such stocks have low equilibrium expected returns. Jiang, Xu and Yao (2009) argue that high idiosyncratic return volatility predicts low earnings, which are accompanied by low returns. Others argue that the AHXZ result is spurious by documenting its sensitivity to sampling choices [e.g., Bali and Cakici (2008)] and to accounting for short-term reversals [e.g., Fu (2009), Huang, Liu, Rhee and Zhang (2010), and Han and Lesmond (2010)]. AHXZ examine returns beginning in the month immediately following the computation of idiosyncratic volatility. Since stock returns are positively correlated with idiosyncratic volatility in the ranking month, the beginning price from which the next month's return is computed is more likely at the ask than the bid. The ensuing reversal of these price concessions to liquidity providers [see Kaul and Nimalendran (1990)] then leads to a negative relation between idiosyncratic volatility and returns in the month immediately following the ranking.

In this paper, we reconsider the AHXZ result by examining separately the returns in the month immediately following the ranking and returns up to two years later. This follows the approach of Jegadeesh and Titman (1993) in studying the profits to momentum strategies. We also examine the impact of January and penny stocks. We confirm the findings that led others to conclude the AHXZ result is sensitive to sampling choices.

However, we also show that even after skipping the first month, the AHXZ result is quite robust up to two years after the ranking once we control for January and the influence of penny stocks. High volatility stocks are prime candidates for tax-loss selling [see Roll (1983), D’Mello, Ferris and Hwang (2003) and Grinblatt and Moskowitz (2004)]. The impact of this selling pressure is especially pronounced for penny stocks, which are relatively illiquid. Their positive January returns conceal what is otherwise a strong and persistent negative relation between future returns and idiosyncratic volatility. After accounting for the January effect, the AHXZ result is consistently significant in raw and risk adjusted returns and when using idiosyncratic volatility measured from (i) daily returns of the past month, (ii) daily returns of the past year and (iii) monthly returns of the past five years. These findings indicate quite strongly that the AHXZ result is real and that it requires an economic explanation.

Among our findings, we show that the AHXZ result is not driven by small firms (though small firms do have high idiosyncratic volatility) or because high idiosyncratic volatility predicts low earnings (it does, but not for the stocks that drive AHXZ’s result). We also show that it is not explained by measures of whether stocks have lottery-like payoffs. Several high daily returns in one month do predict low returns in the following month, but not in later months. This suggests the high daily returns identify stocks that experience buying pressure, and the low return in the subsequent month is a reversal of the price concession to liquidity providers. We then consider an information based explanation that predicts which firms should dominate in generating the AHXZ result in the sample. We test its implications and find they fit both the AHXZ result and the equally puzzling findings of Chordia, Subrahmanyam and Anshuman (2001) that returns are low for stocks with high volatility of share turnover.

Our explanation is based on the substantial accumulation of evidence that the differential higher cost of short versus long positions leads to upward biased prices when traders disagree [see Chen, Hong and Stein (2002), Diether, Malloy and Scherbina (2002), Jones and Lamont (2002), Gopalan (2003), Lamont (2004), Nagel (2005), Boehm, Danielsen and Sorescu (2006), Sadka and Scherbina (2007), and Boehm, Danielsen, Kumar and Sorescu (2009)]. This idea is attributed to Miller (1977) who argues verbally that costly short

sales prevent pessimists from trading as aggressively as optimists, which leads to upward biased prices. Several rigorous models have this feature also [see, e.g., Harrison and Kreps (1978), Chen, Hong and Stein (2002), Hong and Stein (2003) and Gopalan(2003)]. As disagreement eventually is resolved, the bias dissipates and prices fall. This explains a principal finding in the papers cited above that future returns are low for stocks with high dispersion among analysts' earnings forecasts.

We examine whether this phenomenon explains AHXZ's result. It might seem immediate that this can be done by considering whether analyst dispersion or idiosyncratic return volatility (IVOL) is the stronger predictor of returns in a test that casts them as substitutes.<sup>1</sup> There are two problems with this. First, disagreement and IVOL play distinct roles in the economics of Miller's hypothesis. If Miller's hypothesis is true, they are not substitutes but complements as described below. Second, dispersion among analysts' forecasts can only be computed if there are two or more analysts. This omits a large portion of the sample, precisely the firms for which a paucity of analyst coverage leaves traders most prone to disagreement.

In order for Miller's hypothesis to affect returns, disagreement (or costs) must *change* through time because a constant bias in prices will not affect returns. For disagreement to change, beliefs must change. IVOL is a measure of changes in beliefs because IVOL is computed from market clearing prices, which reflect the impact of information arrivals on traders' beliefs. If Miller's hypothesis explains AHXZ's result, then only the subset of news arrivals that change beliefs, *and also* generate disagreement, will impart an upward bias into prices. High measures of IVOL and disagreement are therefore complements in predicting a bias in prices. Alternatively, information arrivals that do not generate disagreement will produce a high IVOL ranking, but they will not impart an upward bias into prices. A proper test of whether Miller's hypothesis explains AHXZ's result requires *interacting* IVOL and a measure of disagreement.

We present a simple dynamic model that captures these ideas. We assume that traders are strategic, short positions are costly, and traders can disagree about the interpretation of news. Consensus beliefs are correct despite disagreements, however. When news arrives,

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<sup>1</sup>AHXZ conduct such a test as a robustness check.

it changes consensus beliefs. This shock to beliefs is reflected in equilibrium prices, and it is the source of high return volatility but not bias.

If the news also creates disagreement, equilibrium prices are biased (too high) in relation to fundamental value because pessimists optimally trade less aggressively than optimists. We show that the anticipation of news causes prices to rise even *prior* to the actual news arrival. This is because traders bet on the appearance of a future bias in prices when they expect it. The true value of the security is eventually revealed, disagreement dissipates, and the bias in prices disappears. The temporal pattern of equilibrium returns therefore mirrors that of optimistic mispricing—i.e., a runup prior to a period of significant news arrival(s) followed by low returns thereafter. Alternatively, if news does not generate disagreement, this mispricing pattern does not arise even though return volatility is high.

We test this explanation of AHXZ’s result in four ways, using low analyst coverage as a proxy for whether traders disagree about the interpretation of significant news. Our first test examines whether the relation between returns and idiosyncratic volatility is different depending on whether stocks have low coverage or not. We use the three measures of IVOL described above, ex-post return horizons of one month to two years, and raw and risk adjusted returns. Most of the results indicate that the low returns following high IVOL rankings are attributable to low coverage stocks only. The exception is returns in the month immediately following the ranking, which are affected by short-term reversals. As expected, those returns are negative regardless of coverage.

Disagreement generates trading in our model as it does in many other models [e.g., Harris and Raviv (1993)], so shocks to turnover indicate the arrival of information about which traders disagree. In our second test, we examine the relation between returns and the volatility of turnover. We find that the low returns to high turnover volatility, first documented by Chordia, et.al. (2001), are also attributable to stocks with low coverage. This finding is very strong and uniform across returns horizons, suggesting that disagreement plays a role in this puzzle as well.

Our third test considers whether return dynamics are consistent with optimistic mispricing—a runup followed by low returns for high IVOL stocks. This pattern is indeed significant and is driven by stocks with low coverage. It is robust across the various IVOL

and turnover volatility measures, and it is incremental to the commonly observed reversal pattern at intermediate horizons [e.g., DeBondt and Thaler (1985)]. Finally, we examine returns around earnings announcements because the concreteness of earnings should *resolve* disagreement. We find that average earnings announcement returns are significantly negative for high IVOL low coverage stocks, and insignificant for low IVOL stocks and for high IVOL stocks with high coverage. Similar results hold in both tests when the volatility of turnover is used instead of IVOL.

The results of all four tests support the hypothesis that mispricing associated with disagreement explains low returns to stocks that sustain significant shocks to prices or turnover. In fact, the strength and uniformity of the turnover results suggest that turnover volatility coupled with low coverage is actually a better indicator of information arrivals that generate disagreement than is IVOL and low coverage.

Finally, we attempt to characterize how coverage affects disagreement, and to identify the type of information about which traders disagree. We compare stocks with long and short histories of low coverage and find that those with long histories drive the result. Excess returns are not significantly negative for high volatility stocks that are new to the low coverage group—in many cases, their excess returns are positive though not significant. What matters for mispricing is the level of coverage over a long period of time and not merely coverage at the time of an information arrival. This suggests that whether news arrivals create disagreement is related to how the general availability of analysis (financial models, commentary, and forecasts) helps traders to interpret news, and not the specifics of the forecast revisions or recommendation changes that analysts make in response to particular news items.

We then examine accounting operating performance (ROA) for five years prior and two years after the year in which stocks are ranked as having high IVOL or turnover volatility. Among high volatility stocks, the time paths of ROA are quite different between those with a history of low coverage and those without. For low coverage stocks, ROA increases strongly in the three years before ranking and decreases in the two years after—i.e., operating performance improves prior to the ranking year then reverses afterward. In contrast, for high coverage stocks, ROA decreases in the years both prior *and* subsequent

to the ranking year. These patterns suggest that whether improvements in operating performance will continue is a common issue about which disagreement exists.

Our paper makes several contributions. First, we show that the AHXZ result is robust even after accounting for short term reversals, and that it is not attributable to illiquid or small stocks, or stocks with lottery-like payoffs. Second, we model an informational explanation for the AHXZ result that links it to mispricing that arises from disagreement among traders. Third, we demonstrate that the returns patterns that characterize *both* the AHXZ and Chordia, et.al. (2001) results are consistent with the mispricing explanation, as are the returns associated with earnings announcements. These findings are robust to the definition of IVOL and to the choice of returns horizon, and they provide a unified explanation for two separate puzzles (AHXZ and Chordia, et.al.). Finally, we show that the patterns in coverage and ROA suggest that the mispricing underlying the AHXZ and Chordia, et.al. results arises because traders disagree about the persistence of recent performance improvements for stocks with a history of low coverage.

The next section of the paper presents the model. Section 2 describes the sampling procedure and the idiosyncratic volatility measures. Section 3 examines the robustness of the AHXZ result. Section 4 presents tests of the mispricing hypothesis. Section 5 concludes.

## 1. Model

Idiosyncratic components of price changes arise from arrivals of firm specific news or idiosyncratic liquidity shocks. We focus on news because, as discussed in the introduction, others have addressed the role of liquidity shocks and their reversals in explaining the AHXZ result. In our model, equilibrium prices aggregate traders' beliefs, so news that has a large impact on consensus beliefs generates a price shock that leads to high measured return volatility. We examine the impact on security returns when news also generates disagreement among traders.

Dynamics are an important part of our story. As noted in the introduction, if disagreement does not change through time, evidence of mispricing will not appear in returns. We model a firm's *transition* from a period of no information flow to a period in which



significant information arrives, then back again after uncertainty is resolved. This informational approach to explaining the AHXZ result is quite different from treating IVOL as an intrinsic attribute or characteristic, and arguing that investors prefer high IVOL stocks to low IVOL stocks. Instead, our approach is consistent with Sonmez-Seryal (2008), who examines changes in IVOL rankings to gauge how much of the AHXZ result is driven by stocks that change versus persist in their IVOL quintile rankings. She shows that returns are very high when stocks rise in IVOL quintile ranking, and returns are low when stocks fall in ranking. The relation between returns and IVOL is also positive for stocks that persist in their quartile ranking. She concludes that the AHXZ result is therefore driven by the subset of high IVOL stocks that *fall* from their high ranking—i.e., the *transitions* are what matter. We offer an explanation as to why they matter.

To keep things simple, we model the transitions associated with a single information arrival. This allows the price sequence and whether return volatility is high or low to be endogenous. In addition, traders in our model are strategic, and they account for their impact on prices when formulating trading strategies. Gopalan (2003) presents a static model of perfect competition with assumptions that are similar to ours in order to model Miller’s (1977) hypothesis. A static model suffices for his purpose, but dynamics are required to capture the transitions that we hypothesize drive the AHXZ and Chordia, et.al. results.

### 1.a Timing and Beliefs

Assume that  $2N$  traders participate in a market for a single security whose per-capita supply is  $X > 0$ . Traders have time additive mean-variance preferences over profit, with common risk aversion parameter  $\alpha$ . Trading occurs at dates 1 and 2, and the security pays off  $\tilde{v}$  at date 3.

Traders are identical at date 1. They all believe  $\tilde{v}$  will be drawn at date 3 from a distribution whose expectation is  $v_o$ . With probability  $1 - q$ , no information arrives at date 2, traders continue to hold this belief, and at date 3  $\tilde{v}$  is in fact drawn from a distribution having expectation  $v_o$  and variance  $\sigma_v^2$ .

However, with probability  $q$ , information arrives at date 2 that generates disagreement

among the traders about the security’s expected payoff.  $N$  traders adopt the optimistic belief that  $\tilde{v} \sim (\bar{v}_H, \sigma_v^2)$ , and the other  $N$  traders adopt the pessimistic belief that  $\tilde{v} \sim (\bar{v}_L, \sigma_v^2)$  where  $\bar{v}_H > \bar{v}_L$ . One group’s interpretation of the information will turn out to be correct, meaning that  $\tilde{v}$  will be drawn at date 3 from one of these distributions. We assume the objective probability that  $\tilde{v}$  is drawn from either distribution is  $1/2$ . This means that, conditional on an information arrival at date 2, the expectation of the objective distribution from which  $\tilde{v}$  will be drawn at date 3 is  $\bar{\bar{v}} \equiv \frac{1}{2}\bar{v}_H + \frac{1}{2}\bar{v}_L$ . However, traders in both groups behave as though their subjective beliefs are correct. Figure 1 illustrates the sequence of possible events.<sup>2</sup>

We assume that traders are aware ex-ante of the level of disagreement that will exist if information arrives and creates disagreement at date 2—i.e.,  $d \equiv \bar{v}_H - \bar{v}_L$  is common knowledge. This implies that if information arrives, traders learn both their own revised beliefs and the revised beliefs of the other group. This assumption prevents us from having to model inferences from prices that traders would otherwise draw about the beliefs of others, and the strategic response of each trader to knowing that others attempt to forecast his beliefs from prices. Although the difference  $d$  is fixed, the scale of  $\bar{v}_H$  and  $\bar{v}_L$  can be viewed by traders ex-ante as random.

Traders hold common date-1 beliefs, which are consistent with Bayes rule. In particular, traders’ date-1 expectation is consistent with the three possible conditional expectations they will adopt at date 2, and the probability of each:

$$v_o = qE_1 \left[ \frac{1}{2}\bar{v}_H + \frac{1}{2}\bar{v}_L \right] + (1 - q)v_o = \frac{1}{2}E_1 [\bar{v}_H] + \frac{1}{2}E_1 [\bar{v}_L].$$

This ensures that the prior  $v_o$  reflects what traders know at date 1 about how the future will unfold, so a bias in prices does not arise because traders are misinformed at date 1.

A crucial assumption is that short sales are costly. This is comprised of the direct fees paid to a broker, the difficulty in locating shares to borrow, the opportunity cost associated with constraints on selling shares posted as collateral, and the extra effort

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<sup>2</sup>An *even* split in the population between optimists and pessimists is convenient for exposition, but not necessary. What is important is that the split has the same proportions as the probability of  $\tilde{v}$  being drawn from the “H” and “L” distributions. This ensures that consensus beliefs are correct, and that any bias in prices is the result of strategic choices and not traders merely being misinformed on average.

involved in monitoring a short versus a long position [see Lamont (2004)]. For simplicity, we assume there is a constant marginal cost  $c_s$  per share per period to maintain a short position. The profit that trader  $j$  realizes from holding  $x_{tj}$  shares between dates  $t$  and  $t + 1$  is therefore

$$\tilde{\pi}_{tj} \equiv (\tilde{p}_{t+1} - p_t)x_{tj} + c_s I_{tj} x_{tj}, \quad \text{where} \quad I_{tj} = \begin{cases} 1 & \text{if } x_{tj} < 0 \\ 0 & \text{otherwise,} \end{cases}$$

$p_t$  is the market price per share at dates  $t = 1$  and  $t = 2$ , and  $p_3 \equiv v$ .

At date 2, trader  $j$  selects a demand schedule,  $x_{2j}(p_2)$ , that maximizes his utility conditional on his date-2 beliefs about the distribution of  $\tilde{v}$  and the date-2 demand schedules chosen by other traders:

$$J_{2j} = \max_{x_{2j}(\cdot)} \{E_{2j} [\tilde{\pi}_{2j}] - \alpha \text{Var}_{2j} [\tilde{\pi}_{2j}]\}.$$

The solution conditional on no information arrival at date 2 is denoted by  $\hat{x}_{2j}^*(\cdot)$ , and the solution conditional on an information arrival is denoted by  $x_{2j}^*(\cdot)$ —throughout the discussion, a “hat” means conditional on no information arrival at date 2.

Likewise, at date 1, trader  $j$  selects a demand schedule  $x_{1j}(p_1)$ , that maximizes his utility conditional on his date-1 beliefs about prices,  $\tilde{v}_1$ , and the date-1 *and* date-2 demand schedules of the other traders:

$$J_{1j} = \max_{x_{1j}(\cdot)} \left\{ E_{1j} [\tilde{\pi}_{1j}] - \alpha \text{Var}_{1j} [\tilde{\pi}_{1,j}] + E_{1j} [\tilde{J}_{2j}] \right\}.$$

We solve for the optimal schedules  $\{x_{1j}^*(\cdot), \hat{x}_{2j}^*(\cdot), x_{2j}^*(\cdot) : j = 1, \dots, 2N\}$  by backward induction. If a solution exists in which all traders’ beliefs about the strategies of others are correct, such a solution constitutes a subgame perfect Nash equilibrium. Expressions for the equilibrium prices  $\{p_1^*, \hat{p}_2^*, p_2^*\}$  can be obtained from the market clearing conditions that equate per-capita demand and per-capita supply:

$$\frac{1}{2N} \sum_{j=1}^{2N} \hat{x}_{2j}^*(\hat{p}_2^*) = X \quad \text{and} \quad \frac{1}{2N} \sum_{j=1}^{2N} x_{tj}^*(p_t^*) = X \quad \text{for } t = 1, 2.$$

## 1.b Equilibrium Prices

Whenever traders have identical beliefs, they all hold long positions equal to their share of per-capita supply, and short sale costs have no impact on holdings or prices. In order for the cost of short sales to affect holdings and prices, the difference between optimistic and pessimistic beliefs must be large enough that the cost actually deters short positions the pessimists would otherwise enter. This occurs over parameter regions in which pessimists hold zero shares (but would short if it were costless) and short positions (that are smaller in magnitude than they would be if shorting were costless). To simplify the exposition, we ignore the former region and consider levels of disagreement that are sufficient to generate non-zero short sales.

We show in the Appendix that there exists a  $\underline{d} > 0$  such that if  $d > \underline{d}$ , there is a subgame perfect Nash equilibrium in symmetric linear strategies.<sup>3</sup> In each subgame, the optimal strategies of traders whose beliefs are the same are identical affine functions of their expectation of the price change over the next period. Conditional on an information arrival at date 2, optimists hold long positions and pessimists hold short positions. This equilibrium is unique in the class of symmetric linear equilibria. We now describe the dynamics of prices in this equilibrium to flesh out the connections between IVOL, disagreement and returns.

If no information arrives at date 2, traders have identical beliefs about the date-3 payoff—i.e., that  $\tilde{v} \sim (v_o, \sigma_v^2)$ . Their optimal demand schedules are of the form  $\hat{x}_{2j}^*(\hat{p}_2) = \hat{\beta}(v_o - \hat{p}_2)$  for all  $j$ , and the market clearing price is

$$\hat{p}_2^* = v_o - \frac{X}{\hat{\beta}}.$$

The first term is traders' consensus belief about the security's expected payoff, and the second term is a discount to compensate traders with a positive return for bearing risk.<sup>4</sup> Since  $v_o$  is the expected value of the distribution from which  $\tilde{v}$  will be drawn if information does not arrive, the risk adjusted price is an unbiased estimate of the security's payoff. At equilibrium, all traders hold their per-capita share of supply:  $\hat{x}_{2j}^*(\hat{p}_2^*) = X$ .

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<sup>3</sup>An explicit expression for  $\underline{d}$  is given in Equation (A.27) in the Appendix.

<sup>4</sup>An explicit expression for  $\hat{\beta}$  is obtained by combining Equations (A.14) and (A.23) in the Appendix.

If information does arrive at date 2, optimists' strategies are  $x_{2H}^*(p_2) = \beta(\bar{v}_H - p_2)$ , and pessimists' strategies are  $x_{2L}^*(p_2) = \beta(\bar{v}_L - p_2 + c_s)$ . It turns out that the  $\beta$  coefficients are the same for optimists and pessimists, and are equal to  $\hat{\beta}$  in the case when information does not arrive. The positive  $c_s$  term in the pessimists' demand schedule reduces the aggressiveness with which they short because shorting is costly. The market clearing price is

$$p_2^* = \bar{v} + \frac{c_s}{2} - \frac{X}{\beta}.$$

The first term is traders' consensus belief about the security's expected payoff after information arrives, and the third term is a discount due to risk. Since  $\tilde{v}$  is equally likely to be drawn from a distribution with expectation  $\bar{v}_H$  or  $\bar{v}_L$ , the expectation of its objective distribution is  $\bar{v}$ , so the consensus component of the price is unbiased as an estimate of the security's payoff. However, the risk adjusted price is biased upward because pessimists hold back when shorting is costly. This bias is reflected in the middle term,  $c_s/2$ . It rewards pessimists, who are short, with an expected drop in price to compensate for bearing the cost.

The bias is analogous to the upward bias Miller (1977) argues will arise when beliefs are divergent and short sales are costly. This does not necessarily hold in a carefully structured equilibrium model. For example, prices are not biased in Diamond and Verrecchia's (1987) analysis of costly short sales with divergent beliefs. Their model is based on Glosten and Milgrom (1985) wherein prices are set by the unbiased beliefs of a market maker who does not bear costs if his inventory is short. In our model, prices are determined by market clearing, so beliefs move prices through the orders that traders submit. Even though beliefs are unbiased on average across traders, market clearing prices are biased upward because pessimists trade less aggressively on their beliefs than optimists.

The next point is less obvious and follows from the dynamics of traders' strategic choices—the bias is incorporated into prices *before* the information arrival because traders anticipate divergent beliefs in the future. When traders anticipate that future prices might be high because pessimists hold back, they bet now that prices will rise. This shifts current demand and prices upward, incorporating a bias into the *current* price. The implication is that the upward bias due to costly short sales is imparted into prices even before the

arrival of the shock to beliefs that generates disagreement.

At date 1, traders have identical beliefs and their demand schedules are of the form  $x_{1j}^*(p_1) = \gamma(E_1[p_2] - p_1)$  for all  $j$ .<sup>5</sup> The market clearing price is

$$p_1^* = v_o + q\frac{c_s}{2} - \frac{X}{\beta} - \frac{X}{\gamma}.$$

The first three terms constitute traders' date-1 consensus expectation of the date-2 price. The  $q\frac{c_s}{2}$  term is a bias in the date-1 price associated with traders' *anticipation* that information will arrive with probability  $q$  and generate a bias of  $\frac{c_s}{2}$  in *next* period's price. The more strongly traders anticipate an information arrival at date 2, the more the date-1 price incorporates the future bias. The last term is a discount that compensates traders with a positive expected return for bearing risk between dates 1 and 2. At date 1, traders all hold their share of per-capita supply at equilibrium:  $x_{1j}^*(p_1^*) = X$  for all  $j$ .<sup>6</sup>

This price sequence clarifies the possible connections between disagreement, short sale costs and IVOL in generating the IVOL puzzle. First, IVOL is not a measure of disagreement because prices do not bounce between traders' divergent beliefs. Prices *aggregate* beliefs, so IVOL measures shocks to *consensus* (i.e.,  $\bar{v} - v_o$ ). Second, disagreement generates an upward bias in prices. However, the bias appears *prior* to information arrivals to the extent that information arrivals are anticipated. The bias reverses as the disagreement dissipates when  $\tilde{v}$  is drawn.

If we define as date zero a time at which traders believe  $q = 0$  then, by an argument similar to that used to derive  $p_1^*$ , the date-zero price will have the form  $p_o^* = v_o + \text{risk premium}$ . Conditional on an information arrival at date 2, the sequence of equi-

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<sup>5</sup>An explicit expression for  $\gamma$  is obtained from substituting Equation (A.26) into Equation (A.21.1a) in the Appendix.

<sup>6</sup>The discount  $\frac{X}{\gamma}$  is compensation for the risk that information will arrive, which also depends positively on  $q$ . This will be small for idiosyncratic news, but may seem important here only because we are working with a single security model in which opportunities for diversification are suppressed. If we think of  $\alpha$  as representing traders' aversion to the incremental risk associated with this security in a diversified portfolio, then  $\alpha$  will be small and indeed  $X/\gamma$  vanishes (and  $q\frac{c_s}{2}$  does not) as  $\alpha \rightarrow 0$  (by Equation (A.26) in the Appendix). Alternatively, if traders do hold undiversified portfolios as in Merton (1987), then the risk premium could be large enough to compete with the bias in affecting the price.

librium risk adjusted price *changes* is

$$R_{o,1} = q \frac{c_s}{2} \quad (1)$$

$$R_{1,2} = \{\bar{v} - v_o\} + (1 - q) \frac{c_s}{2} \quad (2)$$

$$R_{2,3} = \{\tilde{v} - \bar{v}\} - \frac{c_s}{2}. \quad (3)$$

These equations illustrate how disagreement affects returns around significant information arrivals. News causes  $\{\bar{v} - v_o\}$  to contribute significant variation to prices, which leads to a high IVOL ranking. The expected value of the terms in curly brackets is equal to zero because consensus is unbiased. However, expected returns are positive both before, and coincident with, the news arrival. The expected subsequent return is negative as disagreement dissipates and the bias disappears.

Equations (1) - (3) relate to the case in which information arrivals that shock consensus also generate disagreement. If they do not (i.e.,  $d = 0$ ), there is an equilibrium in which traders have identical beliefs in all subgames, traders each hold their share of per-capita supply, and there is no bias in prices in either period whether information arrives or not. Conditional on an information arrival at date 2 that does not generate disagreement, Equations (1) - (3) are replaced by similar equations but without the terms involving  $c_s$ , because traders do not short. When information arrives, prices change because consensus shifts ( $\bar{v} - v_o$  will still contribute significant variation to prices). Such a shock will lead to a high IVOL ranking, which is *not* accompanied by a prior price runup and subsequent price drop. Thus, a pattern of mispricing will not be apparent on average around high IVOL rankings if information arrivals do not also generate disagreement.

Disagreement in our model drives trading volume as well. If information does not arrive at date 2 (or if it does, but there is no disagreement), all traders continue to hold their per-capita share of supply and there is no trading. However, if information arrives that generates disagreement, trading occurs between optimists and pessimists. A large value for a statistic that measures shocks to trading volume therefore indicates both the arrival of news *and* the presence of disagreement. Further conditioning on low analyst coverage is an even tighter screen for the presence of disagreement. If our model is correct, a pattern of optimistic mispricing should be at least as pronounced if a measure of volume

volatility is used in place of IVOL as a ranking variable in the empirical tests.

These observations suggest several empirically refutable hypotheses. First, the negative returns following a high IVOL ranking are reversals of biases that build prior to the ranking, so there should be an association between the size of the prior runup and the subsequent price drop. In other words, the price pattern should resemble optimistic mispricing around a high IVOL ranking that is corrected ex-post. Second, and most important, information arrival *and* disagreement are both necessary to generate the AHXZ result in our model. Information arrivals that are not accompanied by disagreement will not generate a bias. Consequently, only the subset of high IVOL stocks for which there is significant disagreement should exhibit a pattern consistent with optimistic mispricing. Third, the pattern of mispricing should be at least as strong when a measure of shocks to turnover is used in place of IVOL, which measures shocks to prices.

In our empirical tests, we use low analyst coverage to identify whether shocks to beliefs are likely to generate disagreement. When coverage is low, traders have less guidance to process the value relevance of news. In contrast, if many analysts follow a firm, investors have a common and large set of professional opinions to anchor their beliefs and to help with interpreting the meaning of significant information. Using analyst coverage as a proxy for disagreement has the advantage of allowing all firms to be included in the sample. Using dispersion in analysts forecasts instead requires that two analysts follow a firm in order for the firm to be included in the sample. This limitation is severe. For example, the average sample size in our Table 2 regressions is 5220 stocks per month. This falls to 2494 under the requirement of two or more analysts.

## 2. Data and Methods

The data consist of monthly prices, returns and other characteristics of the NYSE, AMEX and Nasdaq companies covered by CRSP from 1963 through 2006. Price, return and volume data are obtained from CRSP. Financial information is obtained from Compustat. Data on analyst coverage are obtained from the Summary History data set compiled by the Institutional Brokerage Estimation System (I/B/E/S). Stocks are classified each month as having low coverage ( $LCOV = 1$ ) if three or fewer analysts are listed as pro-



viding one-year earnings forecasts.<sup>7</sup> For expositional ease, we refer to stocks outside the low coverage group as high coverage stocks. Although I/B/E/S coverage begins in 1976, we follow Diether, Malloy and Scherbina (2002) in limiting our sample period to begin in January 1983. Until 1983, the I/B/E/S coverage is sparse and unreliable.

Following AHXZ, we measure the idiosyncratic volatility of each stock as the standard deviation of residuals from a time series regression of stock returns on the Fama-French (1993) factors:

$$R_{it} = \alpha_i + \beta_{i,MKT}MKT_t + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \epsilon_{it}. \quad (4)$$

We construct three idiosyncratic volatility measures that differ both in data frequency and in the length of the time series used to estimate the regression. The first is the original AHXZ idiosyncratic volatility measure (*IVOL20D*). It is estimated from regressions using one prior month of daily returns and factor data, including firm months with at least 20 observations. We also construct two other measures to identify firms by their volatility over longer periods of time. *IVOL200D* is estimated using the prior 12 months of daily returns and factor data, requiring at least 200 non-missing observations in the past year. The third measure *IVOL60M* uses the prior 60 months of monthly returns and factor data, requiring at least 24 months of non-missing observations [see also Fama and MacBeth (1973), Lehmann (1990), Malkiel and Xu (2006) and Spiegel and Wang (2005)]. Considering a variety of volatility measures enables us to provide a clearer picture of the robustness of the results than would be possible using a single measure alone.

Following Chordia et al. (2001), we measure the volatility of trading volume as the standard deviation of share turnover (*STURN*). Each month, turnover is calculated as trading volume divided by shares outstanding as reported by CRSP. *STURN* is calculated over 36 months ending in the second-to-last month prior to portfolio formation.<sup>8</sup> Nasdaq volume includes inter-dealer trades and NYSE/AMEX volume does not, so we divide

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<sup>7</sup>Partitioning between three and four divides the overall sample nearly in half. Low coverage stocks represent 45% of the sample of NYSE/AMEX stocks and 67% of the sample of Nasdaq stocks. The average number of analysts covering stocks in our sample is five.

<sup>8</sup>Note that this results in a month being skipped by construction, so the results reported for turnover volatility are not subject to the critique of short term reversals associated with bid-ask bounce discussed in the Introduction.

volume by two in computing *STURN* for Nasdaq stocks. Trading volume data is not available prior to November 1982 for Nasdaq stocks.

We follow the Fama-MacBeth (1973) style regression approach taken in George and Hwang (2004) and Grinblatt and Moskowitz (2004) to measure and compare the returns to portfolios formed by different investment strategies. This approach has the advantage of using all the stocks in the sample. The regression coefficient estimates isolate the returns to portfolios exhibiting particular characteristics by hedging (zeroing out) the impact of other variables that are included as controls [see Fama (1976)].

We examine returns over future horizons of different lengths. This involves computing returns in a given month to portfolios that were formed in each of several past months. Consider the strategy of forming portfolios every month and holding the portfolios for the next  $T$  months. In a given month  $t$ , the return to pursuing this strategy is the equal-weighted average of the returns to  $T$  portfolios, each formed in one of the  $T$  past months  $t - j$  (for  $j = 1$  to  $j = T$ ). The contribution of the portfolio formed in month  $t - j$  to the strategy's month- $t$  return can be identified by the coefficient estimates of a cross sectional regression of month- $t$  returns on portfolio selection criteria in month  $t - j$ .

The main regression specification we work with is as follows.

$$\begin{aligned}
R_{it} = & b_{0jt} + b_{1jt}LVOL_{i,t-j} + b_{2jt}HVOL_{i,t-j} + b_{3jt}LCOV_{i,t-j} * LVOL_{i,t-j} \\
& + b_{4jt}LCOV_{i,t-j} * HVOL_{i,t-j} + b_{5jt}BM_{i,t-1} + b_{6jt}Size_{i,t-1} + b_{7jt}R_{i,t-1} \quad (5) \\
& + b_{8jt}52WKHW_{i,t-j} + b_{9jt}52WKHW_{i,t-j} + e_{ijt},
\end{aligned}$$

where  $R_{it}$  is the return to stock  $i$  in month  $t$ ,  $HVOL_{i,t-j}$  ( $LVOL_{i,t-j}$ ) equals one if stock  $i$  is among the top (bottom) 20% of stocks in month  $t - j$  when ranked by idiosyncratic volatility, and  $LCOV_{i,t-j}$  takes the value of one if stock  $i$  has no more than three analysts covering it in month  $t - j$  as reported in the I/B/E/S Summary History file. Equity market capitalization and book-to-market in month  $t - 1$  are used as control variables to capture the size and book-to-market effects on returns. We include the prior month's return as a control to capture predictability due to bid-ask bounce, though as we observe later, this does not fully capture short term reversals. These variables are included as deviations from cross sectional means to facilitate the interpretation of the intercept. We also include

winner and loser dummies based on the 52-week high momentum measure in George and Hwang (2004), which they show dominates momentum measures based on past returns.

In light of the control variables, the estimate of  $b_{0jt}$  is the return in month  $t$  to a “neutral” portfolio, formed in month  $t - j$ , that has neither low nor high idiosyncratic volatility (i.e. the portfolio that includes stocks in the middle three idiosyncratic volatility quintiles) and that hedges (zeros out) the effects of deviations from average prior month return, average size and average book-to-market, and also the effects of momentum in predicting returns. The sum of the estimates  $b_{0jt} + b_{1jt}$  is the month- $t$  return to the low volatility portfolio of high coverage stocks that was formed in month  $t - j$  and that has hedged out all other effects. Similarly, the sum  $b_{0jt} + b_{1jt} + b_{3jt}$  is the month- $t$  return to a portfolio of low volatility stocks with low coverage that was formed in month  $t - j$  that has hedged out all other effects. The individual coefficients are, therefore, excess returns that isolate specific characteristics. For example,  $b_{3jt}$  is the excess return in month  $t$  associated specifically with low coverage in a low volatility portfolio formed in month  $t - j$ . The remaining coefficients have similar interpretations.

For a given month  $t$ , the coefficients in Equation (5) are estimated in  $T$  separate cross sectional regressions—one regression for each  $j = 1, \dots, T$ . The portfolio returns associated with various strategies and holding periods are calculated as averages of the appropriate coefficient estimates. For example, consider the strategy of investing in low volatility (and low coverage) portfolios. The excess return in month- $t$  (and the incremental component attributable to low coverage) to such a strategy with a  $K$ -month holding period beginning  $p$  months after portfolio formation is  $S_{1t} = \frac{1}{K} \sum_{j=p}^{p+K} b_{1jt}$  (and  $S_{3t} = \frac{1}{K} \sum_{j=p}^{p+K} b_{3jt}$ ). All the coefficients in  $S_t$  are estimated using the same month- $t$  returns as the dependent variable, so the sums  $S_t$  and  $S_s$  are non-overlapping returns for  $t \neq s$ .

The numbers reported in the tables are time series means of these non-overlapping returns (e.g.,  $\bar{S}_1$  and  $\bar{S}_3$ ), and the corresponding  $t$ -statistics are computed from the temporal distribution of returns. We estimate regressions out to  $T = 24$  and report results for horizons  $\{p = 0, K = 1\}$ ,  $\{p = 1, K = 11\}$  and  $\{p = 12, K = 12\}$ , and sometimes others.

## 2.1 Summary Statistics

Table 1 reports summary statistics for the variables used in our tests. The numbers reported are time series averages of the cross sectional mean, median, maximum, and minimum of each variable, and the correlations among the variables.  $RET(-1, -12)$  is the past 12-month return, and  $IVOL20D$ ,  $IVOL200D$  and  $IVOL60M$  are the idiosyncratic volatility measures defined earlier. The low coverage dummy  $LCOV$  is a key variable in our tests. It has a mean of 0.55—on average, 55% of the sample stocks have three or less analysts covering them. Not surprisingly, the correlation between  $LCOV$  and market capitalization is negative, and the correlations between  $LCOV$  and measures of idiosyncratic volatility are moderate and positive—low coverage stocks tend to be smaller and have higher idiosyncratic volatility. The magnitude of the correlations is between 0.18 and 0.25. There is little correlation between  $LCOV$  and  $STURN$  at -0.05—low coverage stocks tend only slightly to have low volatilities of share turnover.

## 3. Average Returns to High and Low Volatility Portfolios

This section shows that the AHXZ result is robust across all three idiosyncratic volatility measures, across different holding periods ranging from the first month through the second year after portfolio formation, and to the inclusion of controls for firm size and Bali, Cakici and Whitelaw’s (2011) “MAX” variable.

### 3.1 Idiosyncratic Return Volatility

Some previous studies examining the robustness of the AHXZ result [e.g., Bali and Cakici (2008) and Huang et al. (2010)] focus on the one-month idiosyncratic volatility measure ( $IVOL20D$ ) used by AHXZ and do not examine returns beyond the first year after portfolio formation. These studies find the AHXZ result is strong among value weighted portfolios but nonexistent when portfolios are equally weighted. To be sure our explanation addresses the source of their findings, we begin with equally weighted portfolios by estimating Equation (5) with  $HVOL$  and  $LVOL$  dummies as the only independent variables:

$$R_{it} = b_{0jt} + b_{1jt}LVOL_{i,t-j} + b_{2jt}HVOL_{i,t-j} + e_{ijt}. \quad (6)$$

The results are reported in the top panel of Table 2 for *IVOL20D*. The table contains eight columns. Columns 1 and 2 report average portfolio returns one month after portfolio formation  $\{p = 0, K = 1\}$  with and without January, respectively. Columns 3 and 4 report the average monthly portfolio returns during the second month  $\{p = 1, K = 1\}$ , and columns 5 and 6 the third month  $\{p = 2, K = 1\}$ . Columns 7 and 8 report the first year after portfolio formation, excepting the first month  $\{p = 1, K = 11\}$ .

As discussed earlier, the numbers reported as the coefficients of *LVOL* and *HVOL* are time series means (and  $t$  statistics in parentheses) of returns such as  $S_{1t} = \frac{1}{K} \sum_{j=p+1}^{p+K} b_{1jt}$  and  $S_{2t} = \frac{1}{K} \sum_{j=p+1}^{p+K} b_{2jt}$ . However, in columns 2, 4, 6 and 8, January returns are excluded from the calculations. These are important to examine, especially for high volatility portfolios, because high volatility stocks tend to be small in market capitalization and their high volatility makes them more likely than other stocks to be big winners and big losers. Big losers are prime candidates for tax-loss selling at year end. The relative illiquidity of these stocks magnifies the tax loss selling effect on January returns [see Roll (1983), D’Mello, Ferris and Hwang (2003), and Grinblatt and Moskowitz (2004)].

The coefficients reported for *LVOL* and *HVOL* are the excess returns to equally weighted low and high volatility portfolios relative to a benchmark equally weighted portfolio of stocks in the middle three volatility quintiles. We confirm the Bali and Cakici (2008) findings in our sample. High idiosyncratic volatility stocks do not have low returns in the month following portfolio formation when equally weighted portfolios that include January returns are considered (Column 1). Low *IVOL* stocks have significantly lower returns than middle three quintile stocks, and the returns to high *IVOL* stocks are not significantly different from the returns to middle quintile stocks.

The results are opposite when January returns are excluded, however. Column 2 shows that the equally weighted portfolio of top *IVOL* quintile stocks earns negative excess returns that are quite significant. The excess return is -0.87% ( $t = -5.08$ ) in the first month after portfolio formation. This indicates why the original AHXZ result fails for equally weighted portfolios—high idiosyncratic volatility stocks tend to have large positive January returns, which conceal the AHXZ result. This also explains why the AHXZ result is stronger in value weighted portfolios. Tax loss selling is more prevalent among small firms, which tend

also to have high IVOL. Weighting by value minimizes the impact of the positive January returns to small firms on the returns to the high IVOL portfolio.

Recall, however, that Huang et.al. (2010) and Han and Lesmond (2011) show that the first month's return contains a reversal due to price concessions to liquidity providers in the ranking month. Skipping the first month to avoid the short term reversal shows that the January returns conceal the AHXZ result at longer horizons as well.

There is no significant return difference between either high or low IVOL stocks and stocks in the middle three quintiles when January returns are included (columns 3, 5 and 7). However, the results in columns 4, 6 and 8 show that the AHXZ result is robust and persistent in non-January months following portfolio formation. For example, the  $\{p = 1, T = 11\}$  horizon in columns 7 and 8 shows that excluding January changes an insignificant premium to high volatility of 0.26% ( $t = 1.35$ ) per month into a significant discount of -0.35% ( $t = -2.18$ ) per month.<sup>9</sup> Despite the influence of microstructure biases on the month one returns, the persistence of returns out to two years (seen later in Table 4) indicates strongly that the AHXZ result is much more than just a short term liquidity reversal.

These results are based on a sample similar to those in AHXZ, Bali and Cakici (2008) and Huang et al. (2010) that includes "penny stocks," whose relative illiquidity introduces noise and possibly bias into volatility rankings and measured returns even at longer horizons [see Amihud (2002)]. In Table 3, we repeat the analysis after excluding stocks with share prices smaller than \$5 at the end of the month of portfolio formation. This has a noticeable effect on the results, which are even stronger and consistent with AHXZ.

The excess returns to the equally weighted portfolio of high volatility stocks are significant and negative in the first month and the first year after portfolio formation even when January is included. This is because the January effect is especially strong for penny stocks. The exclusion of penny stocks leads to a uniformly significant pattern of negative returns to high IVOL stocks, and uniformly insignificant returns to low IVOL stocks (relative to stocks in the middle three quintiles). The average excess returns to high volatility

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<sup>9</sup>Similar results hold under both the medium and long term idiosyncratic volatility measures, though they are not tabulated to save space.

portfolios are -1.24% ( $t = -8.00$ ) in the first month and -0.40% per month ( $t = -2.65$ ) in the next eleven months after portfolio formation even with January included. When January returns are excluded, the excess returns become even more negative. The excess returns in the first month and first year are -1.39% ( $t = -8.62$ ) and -0.70% ( $t = -4.83$ ) per month, respectively.<sup>10</sup> Here again, the impact of short term reversals can be seen in the first month where the coefficient -1.39 is about double the average of -0.70 over the next eleven months.

The influence of January returns and (illiquid) penny stocks creates the appearance that the AHXZ result is non-existent. Their influence might also distort inferences concerning the *relative* importance of IVOL versus other variables thought to explain the AHXZ result. A prominent example is the “MAX” variable of Bali, Cakici and Whitelaw (2011), which is designed to capture the degree to which investors view a stock as having lottery-like payoffs. It is defined as the average of the five highest daily returns during the prior month. Bali, et.al. document negative one-month returns for high MAX stocks and argue that this drives the AHXZ result because, after controlling for MAX, the return discount to high IVOL stocks becomes a premium. The bottom panels of Tables 2 and 3 examine the impact of January and penny stocks on their conclusions.

Column 1 of the bottom panel in Table 2 confirms their finding. High MAX (low MAX) stocks earn a large and significant return discount (premium) in the first post-ranking month. After controlling for MAX, the relation between returns and IVOL is positive. This is also true when January is excluded from month one returns. However, in months two and three, and in the eleven months after the first month, the results are different. When January is included, both the high and low MAX dummies are insignificant, as are the high and low IVOL dummies. When January is excluded, the high IVOL dummies are all negative and significant despite having controlled for MAX. The coefficients of the MAX dummies are inconsistent in sign and significance across horizons. High MAX is significant and negative at month two, insignificant at month three and insignificant in the

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<sup>10</sup>The untabulated results for medium and long term volatility measures are very similar, although somewhat smaller in magnitude. For example, the excess return to the high *IVOL200D* portfolios one month and one year after portfolio formation are -0.57% ( $t=-2.89$ ) and -0.29% ( $t=-1.55$ ) per month, respectively. The corresponding figures when excluding January are -0.81% ( $t=-3.99$ ) and -0.67% ( $t=-3.70$ ) per month.

eleven months after month one. Low MAX is insignificant at month two, significant and *negative* at month three and insignificant in the eleven months after month one.

The bottom panel of Table 3 reports the same analysis after excluding penny stocks. As in table 2, the month one returns are low for high MAX stocks whether January is included or not. However, in month two, neither of the MAX dummies are significant. In month three, and the eleven months following month one, the high MAX dummy is not significant while the low MAX dummy is significant and *negative*.

In contrast, the results for the IVOL dummies are quite consistent across horizons. Regardless of whether January is included or not, the coefficients of the high IVOL dummy are uniformly negative and strongly significant. For example, in the eleven months following month one, with January included, the coefficient of the low IVOL dummy is insignificant and that of the high IVOL dummy is  $-0.41$  ( $t = -5.00$ ). Excluding January, the estimates are  $0.18$  ( $t = 2.12$ ) for the low IVOL dummy and  $-0.63$  ( $t = -8.93$ ) for the high IVOL dummy. Interestingly, this evidence of a significant negative relation between returns and IVOL is *stronger* after having controlled for MAX than the evidence in the top panel where MAX is not included.

The relation between returns and IVOL is the more robust of the two effects. It is more persistent, and it survives controlling for biases in returns due to tax-loss selling and accounting for the illiquidity of penny stocks. If the MAX variable really does capture investors' willingness to pay premium prices for lottery-like stocks, investors' perception of which stocks possess this attribute is very fleeting—the price premium dissipates by the end of month one. The speed with which this disappears suggests that the MAX effect is a liquidity reversal. This interpretation could also explain the stronger IVOL results after controlling for MAX. If MAX accounts for liquidity effects that are not accounted for by skipping a month, then adding MAX as a control improves the specification so the true (negative) relation between returns and IVOL is estimated with greater precision.

Table 4 reports results for *risk adjusted* returns for all three measures of IVOL and over return horizons out two years from portfolio formation. Penny stocks are excluded from this and all later tables. The figures reported for *LVOL* and *HVOL* are intercepts (and  $t$ -statistics) from time series regressions in which  $S_{1t}, S_{2t}$ , etc., which are non-overlapping



returns, are regressed on contemporaneous Fama-French (1993) factors.

Risk adjusting strengthens the AHXZ result in both high and low volatility portfolios. In sixteen of eighteen cases, the excess returns to high idiosyncratic volatility portfolios are significantly negative both with and without January. Low volatility portfolio returns are insignificant in ten cases and significantly positive in eight cases. The magnitude and significance of the relation for both high and low idiosyncratic volatility groups is again stronger when January is excluded. Outside January, the high volatility portfolios have risk adjusted excess returns that are all significantly negative, irrespective of the holding period and the volatility measure. The excess returns are strongest for *IVOL20D*. The weakest returns correspond to *IVOL60M*, which are still strong. The first month, first year and the second year risk adjusted excess returns to the high *IVOL60M* portfolios are -0.33% ( $t = -3.07$ ), -0.50% ( $t = -5.12$ ) and -0.47% ( $t = -4.76$ ) per month, respectively.

Table 5 examines the degree to which the AHXZ result is attributable to small versus large firms. Regressions similar to those in Table 4 are estimated with the addition of a dummy variable, *SMALL*, defined as one for stocks whose market capitalization is below the cross-sectional monthly median and zero otherwise. This dummy is included by itself, and also interacted with the *HVOL* and *LVOL* dummies. The coefficient of *HVOL* (*LVOL*) is the excess return to high (low) volatility large firms relative to medium volatility large firms. The corresponding excess return to small firms is the sum of the coefficient of *HVOL* (*LVOL*) and the interaction between *SMALL* and *HVOL* (*LVOL*). Raw returns are examined in this table.

Two results are noteworthy. First, the coefficients of the *HVOL* dummy are significantly negative in all cases except two (they are significantly negative in *all* cases in risk adjusted returns that are omitted to save space). This indicates that the AHXZ result is strong among large firms. Second, the AHXZ result is actually weaker among small than large firms. There are many cases in which the *SMALL\*HVOL* coefficient is significant, and in all those cases it is *positive*—i.e., the relation between returns and high idiosyncratic volatility is less negative among small firms than among large firms. The AHXZ result is therefore not attributable to small firms. Although high idiosyncratic volatility stocks tend to be small firms, those responsible for the negative *relation* between returns and

idiosyncratic volatility are not small.

Summarizing, our results indicate that the influence of January returns are responsible for the seeming lack of robustness of the AHXZ result in equally weighted portfolios, particularly when the sample includes penny stocks. Once we exclude January returns and penny stocks, equally weighted portfolios of high idiosyncratic volatility stocks have low raw and risk adjusted returns in the first month, the first year, and even the second year after portfolio formation. Low idiosyncratic volatility portfolios have either insignificant or high returns. This holds for the short term idiosyncratic volatility measure used in AHXZ and also for the medium and long term idiosyncratic volatility measures, and it is not attributable to small firms or firms that have lottery-like payoffs. Next, we document that the relation between returns and the volatility of share turnover is even stronger than the relation between returns and idiosyncratic return volatility.

### 3.2 Volatility of Turnover

Table 6 reports an analysis in which the *HVOL* and *LVOL* dummies are defined with respect to the volatility of share turnover (highest and lowest quintiles of *STURN* defined above) rather than idiosyncratic return volatility. The results for raw returns confirm Chordia et. al.'s (2001) finding that there is a strong negative relation between turnover volatility and subsequent returns. The high turnover volatility portfolio has significant excess returns ranging from -0.63% to -0.44% per month. Only the year-two return with January included is insignificant at -0.23% per month. All but one of the excess returns to the low turnover volatility portfolios are insignificant in raw returns.

After risk adjustment, high (low) turnover volatility portfolios have uniformly significantly negative (positive) excess returns for all holding periods with and without January. For example, a zero investment strategy of taking a long position in the low volatility portfolio and a short position in the high volatility portfolio nets 0.70% (0.25% + 0.45%) per month in the first month after portfolio formation, and 0.69% (0.28% + 0.41%) per month in the following eleven months. Excluding January, the profit is 0.83% in the first month and 0.81% per month in the following eleven months. Similar monthly profits persist for holding periods extending out two years.

If exploitable, this is one of the most profitable investment strategies documented in the literature. Note that penny stocks have already been excluded, we use 20% cutoffs rather than more extreme 10% cutoffs for ranking by *STURN*, and a month is skipped between ranking and computing returns. So if there is a short term reversal in ranking by *STURN*, it is not included here. These results are consistent with the observation in Section 1 that low ex-post returns should be stronger using the volatility of trading volume than return volatility if the AHXZ result arises from disagreement among traders.

## 4. Tests of the Mispricing Hypothesis

The results so far describe two robust negative relations—one between returns and idiosyncratic return volatility, and another between returns and the volatility of share turnover. In this section we examine whether these relations are consistent with the mispricing predictions discussed in Section 1.

### 4.1 Analyst Coverage and the Return-Volatility Relations

Our first test examines whether the two negative relations are stronger among low coverage firms than high coverage firms. The results are consistent with this hypothesis, and the low returns to high IVOL low coverage stocks persist for years. Outside the low coverage subsample, the relation between returns and idiosyncratic return volatility (and turnover volatility) is mostly positive and sometimes significant. Results for risk adjusted returns are reported in Table 7. The results for raw returns are similar and not tabulated to save space. These regressions include the control variables defined above in Equation (5), but the coefficient estimates for the control variables have been omitted to save space.

The first three panels of the table report results for the three measures of idiosyncratic return volatility. In columns 3 - 8, which skip the first month, all the significant negative relations between returns and IVOL are attributable to low coverage stocks. When January is excluded, all the coefficients of interactions between low coverage and high IVOL are significantly negative, and the interactions between low coverage and low IVOL are significantly positive. When January is included, the results are weaker, but still significant in many cases.

In contrast, excess returns are generally positive, and in two cases significantly positive, for stocks with high IVOL and *high* analyst coverage in columns 3 - 8. For example, the results for the second year after portfolio formation using *IVOL200D* show that outside January high IVOL stocks with high coverage earn a positive excess return of 0.65% per month, but high IVOL stocks with low coverage earn a significant 1.12% less, or -0.47% per month. The results in columns 3 - 8 indicate that the strong and significant negative relation between IVOL and future returns after the first month is attributable to stocks that have low analyst coverage. Among high coverage stocks, there is an insignificant or positive relation between returns and idiosyncratic return volatility.

The results in columns 1 and 2 for returns in the month immediately following the ranking by IVOL are different. The return-volatility relation is negative for *both* high and low coverage stocks, and it is especially strong using the short horizon measure *IVOL20D*. For example, the difference between the coefficients of *HVOL* and *LVOL* with January included is -1.20%, which relates to high coverage stocks. For low coverage stocks, the difference is between  $(HVOL + LCOV * HVOL)$  and  $(LVOL + LCOV * LVOL)$ , which is  $(-1.09 + 0.37)$  minus  $(0.11 - 0.04)$  or -0.79%. Comparing this to the results from columns 3 - 8 suggests there is a source of bias associated with a high IVOL ranking that is both unrelated to disagreement and that dissipates quickly. This is consistent with the findings of Fu (2008), Huang, Liu, Rhee and Zhang (2010) and Han and Lesmond (2010) that liquidity based reversals exist in the first post ranking month for high IVOL stocks. These reversals overstate the strength of the AHXZ result at the one-month horizon regardless of analyst coverage.

The results in the last panel of Table 7 are based on the volatility of share turnover. There is a significant negative coefficient on the interaction between low coverage and high volatility for all holding periods including columns 1 and 2, and the low coverage low volatility coefficients are positive and in most cases significant. These results are quite strong and they are consistent with the hypothesis that mispricing is attributable to low coverage stocks. They are also consistent with liquidity reversals driving the IVOL results reported above for month one. Recall that the computation of *STURN* ends one month before portfolio formation, so a month is skipped between ranking and the returns analyzed

in these regressions. This eliminates the short term reversal, and the uniformly low returns to high volatility stocks in month one. The negative relation between returns and *STURN* is attributable to low coverage stocks only.

## 4.2 Analyst Coverage and the Mispricing Reflected in Past Returns

The model predicts that the negative ex-post returns to high volatility stocks are corrections of mispricing that arises when disagreement is high. If this is true, returns *prior* to and including the ranking month are greater than was justified by fundamentals, and the post-ranking correction of mispricing should be related to the increase in prices. This implies that the negative excess returns documented for high *IVOL* and high *STURN* stocks should be larger in magnitude if returns leading up to the ranking are high. This should hold after controlling for the general short term continuations and long term reversals in returns [see Jegadeesh and Titman (1993) and DeBondt and Thaler (1985)].

Table 8 reports regressions of returns on high and low volatility dummies, indicator variables for whether the prior three year return ranks in the top or bottom third of the cross section, and interactions between the high volatility dummy and the high and low prior three year returns indicators. Each panel reports results for a different measure of idiosyncratic return volatility or the volatility of turnover. Within each panel, results are reported for the entire sample on the left, and separately for the high coverage subsample ( $LCOV = 0$ ) on the right.

The results are quite consistent across volatility measures. Consider the full sample results on the left side of the table. The coefficients of the past return variables do pick up significant general continuation and reversal patterns in returns. Past loser stocks continue losing over the next year, and past winners reverse in year two following the ranking year irrespective of volatility rankings. The mispricing prediction is supported as well. Either a negative coefficient of the interaction between *HVOL* and the high past return dummy, a positive coefficient of the interaction between *HVOL* and the low past return dummy, or both, indicate that pre- and post-ranking returns are more negatively related for high volatility stocks than stocks outside the high volatility group. Across all volatility measures and return horizons, all but one of the former coefficients are significantly negative. The

latter coefficients are not significant, however. The mispricing prediction is supported by stocks whose past returns are in the highest third of the cross-section.

The magnitudes of the coefficients of the interactions between high past returns and high volatility are striking. In all cases, the *incremental* reversal associated with high volatility is even larger than the general reversal associated with a high past return. For example, in Panel A using the short-run volatility measure, the general reversal in year two is -0.17% per month for past winners, but -0.46% per month (-0.17% plus -0.29%) for high volatility winners. This follows a first year excess return of insignificant 0.00% for past winners and a significant -0.26% per month (0.00% plus -0.26%) for high volatility past winners.

The high coverage sample results on the right side of the table are different. The relation is nearly non-existent even for past returns in the highest third of the cross section. All but one of the interactions between high past returns and high volatility are insignificant in the high coverage group, and none are significant when January is excluded. Taken together, these results show there is a significant “extra” return reversal among high volatility stocks with the biggest price runups, and it is attributable to the stocks with low analyst coverage.

### **4.3 Analyst Coverage and Earnings Announcement Returns**

In this subsection, we examine whether earnings announcement returns corroborate the mispricing interpretation of the regression tests in the prior tables. The regressions show that low returns are earned by high volatility stocks with low coverage. If this reflects an upward bias in prices associated with disagreement, then we expect earnings announcement returns for these stocks to be significantly negative on average, because realized earnings resolve some of the disagreement. They should also be more negative than the announcement returns for stocks that appear not to be mispriced in the earlier tests—i.e., all low volatility stocks, and high volatility stocks outside the low coverage group. This is indeed the pattern that appears below, both when volatility is measured using idiosyncratic returns and turnover.

We follow the approach of Chopra, Lakonishok and Ritter (1992), La Porta (1996) and

La Porta, Lakonishok, Shleifer and Vishny (1997) for examining corrections of mispricing. Each June, we sort stocks independently by volatility (of either idiosyncratic returns or turnover) and analyst coverage. As before, those with three or fewer analysts are defined as low coverage stocks, the rest as high coverage stocks. High, medium and low volatility groups consist of stocks in the top, middle three, and bottom volatility quintiles, respectively. For each stock, we record the cumulative announcement return over a 3-day window (-1, 0, +1) around the next four quarterly earnings announcements. We calculate “size adjusted” returns by subtracting the return of the firm with median book-to-market among stocks in the same size decile as the announcer. For each stock, the size adjusted “annual” return is the sum of the four quarterly size adjusted returns. The numbers reported in Table 9 are temporal averages of cross sectional means (one for each year) computed within each group. The  $p$ -values reported correspond to  $t$  tests conducted using the time series of yearly cross sectional means and differences in cross sectional means.

The results in Table 9 corroborate the interpretation of the evidence in Tables 7 and 8 as consistent with the model. The announcement returns for high idiosyncratic volatility stocks with low analyst coverage range from -0.87% ( $p$  value 0.02) using *IVOL20D* to -1.22% ( $p$  value 0.00) using *IVOL60M*. When investors receive information about these firms’ fundamentals via earnings announcements, returns are negative *on average*. These are significant and in most cases more negative than the insignificant return to high idiosyncratic volatility stocks with high analyst coverage. Note also that the earnings announcement returns are not significantly different between low and high idiosyncratic volatility stocks with *high* coverage. This coincides with the earlier results in Table 7 where there is no significant difference between the returns of high and low idiosyncratic volatility stocks having high analyst coverage. Both sets of results suggest that a bias exists in the pricing of only the stocks in the low coverage high IVOL group.

The results based on turnover volatility (reported at the bottom of Table 9) are similar to those above, except that the magnitude is much larger. The average announcement return to high turnover volatility stocks with low analyst coverage is -2.27% ( $p$  value 0.00). When coverage is low, the announcement returns of the high turnover volatility stocks are much more negative than those of the low turnover volatility stocks—the difference is a

striking -3.12% ( $p$  value 0.00). However, when coverage is high, the announcement returns of high turnover volatility stocks are not significantly different from zero and not different from low turnover volatility stocks. These results further support the hypothesis that the AHXZ result and the turnover volatility puzzle of Chordia et.al. (2001) are attributable to the mispricing of low coverage stocks.

#### 4.4 Persistence in Low Coverage and Operating Performance

We now turn from testing whether the mispricing hypothesis explains the AHXZ and Chordia et.al. (2001) results to characterizing the mispricing. Specifically, we attempt to shed light on how disagreement is related to low coverage, and whether mispricing relates to information that analysts are likely to have an advantage at interpreting. We examine two issues. First, since analysts' reports are forward looking, they provide context for interpreting future news. So if a stock migrates from high to low coverage, we would not expect mispricing to materialize immediately. Instead, whether a stock has a history of low coverage should be the important factor for explaining cross-sectional variation in mispricing.

Second, security analysts specialize in interpreting the impact of news for the purpose of forecasting earnings. This specialty is distinct from that of "strategists" or "technicians" whose forecasts are based more on macroeconomic trends, perceptions of sentiment, trading activity and liquidity than on individual companies' earnings. If our story that analyst coverage resolves disagreement about news is correct, it should relate most clearly to disagreement about future earnings. We showed earlier that earnings announcements seem to resolve past disagreement, which is concentrated among high IVOL low coverage stocks. Now we examine whether the pattern in earnings itself is different for high IVOL low coverage stocks than stocks in the other groups.

We first examine firms' coverage histories to determine whether recent or persistent low coverage is associated with the mispricing of high IVOL stocks. Table 10 reports estimates of regressions similar to those in Table 7, but the regressions incorporate a variable to distinguish between all low coverage high volatility stocks, and those with persistent low coverage. The variable  $PCOV$  is defined as unity if a stock was covered by three or



less analysts three years prior to the ranking month, and zero otherwise. The regression coefficient of  $LCOV * HVOL$  is the risk adjusted return to high volatility stocks that only recently became low coverage, in excess of the return to all high volatility stocks. The coefficient of  $PCOV * LCOV * HVOL$  is the additional risk adjusted return associated with a history of low coverage.

Two observations from Table 10 are noteworthy. First, the additional return to persistent low coverage is negative across volatility measures and return horizons. It is significantly negative in many cases across the four panels of the table, and uniformly significantly negative when volatility is measured using turnover. Second, the returns to high IVOL stocks that have low coverage but not persistent low coverage are mostly positive and insignificant. This means the negative returns to low coverage high volatility stocks documented earlier are driven by those that have *persistent* low coverage. High volatility stocks that are new to the low coverage group have indistinguishable or higher risk adjusted returns than stocks with high volatility that are outside the low coverage group. Viewed through the lens of our model, these findings suggest that significant news arrivals generate disagreement for stocks that have had low coverage for an extended period of time, but not stocks that are recently dropped from coverage. The disciplining effect of analyst coverage on beliefs and prices fades slowly when firms are dropped from coverage.

Second, we examine earnings patterns via firms' return on assets (ROA) from five years prior to two years after portfolio formation. Table 11 reports average ROA for firms by annual volatility ranking (as in Table 9) and by whether coverage is high or low. Since historical low coverage drives the overpricing, this table groups firms into cells by current volatility and historical coverage. A firm with low coverage for the past five years will be included in low coverage cells throughout. However, consider a firm with a high volatility ranking that had low coverage in only the past year. Its ROA is included in the average computed for the low coverage high volatility groups in years -1 through +2, but its ROA is included in the average for high coverage high volatility stocks in years -5 and -2 when it had high coverage. For future years, firms are grouped using year-zero coverage, so the year-two numbers show the average ROA for firms ranked in a particular volatility/coverage category at year zero.

The table indicates three important differences between low and high coverage stocks with high volatility. First, the ROA of *high* coverage high volatility stocks is similar to that of their medium and low volatility counterparts. In contrast, *low* coverage high volatility stocks' ROA is strikingly low compared to stocks in all other groups. For example, the difference in ROA between high and low volatility high coverage stocks using the 200-day measure of volatility is -1.10% (3.82% versus 4.92%) in year -1. For low coverage stocks the difference is -5.46% (1.59% versus 7.04%). Comparisons in years -5 and -3 are similar. Second, the trend in ROA for high coverage high volatility stocks is negative from year -3 to 0 and 0 to +2. The trend in ROA for low coverage high volatility stocks is strongly positive until year zero, and then it reverses. The *p* values indicate that both the trend and the reversal are highly significant for all volatility measures, including the volatility of share turnover.

The path of earnings for high volatility stocks with low coverage is quite distinct from that of their high coverage counterparts, and that of stocks outside the high volatility group. Low coverage high volatility stocks have historically low, but strongly up-trending, earnings that deteriorate subsequent to their rankings as high volatility stocks. In contrast, the earnings pattern for high volatility stocks outside the low coverage groups is a continued mild down trend from year -3 to +2.

The distinction between these patterns suggests that disagreement among traders concerns the persistence of improvements in operating performance of low coverage firms. This disagreement leads to optimistic mispricing of improvements in operating performance that are not sustained to the extent expected by optimistic traders. A correction in prices occurs when earnings are subsequently announced, thus generating the AHXZ and Chordia, et.al. (2001) results.

The third important difference relates to the sheer magnitude of the price runup in the three years prior to ranking, which is reported in the bottom panels in Table 11. Within each volatility group, low coverage stocks outperform high coverage stocks. The difference is most dramatic in the high volatility group, where returns to low coverage stocks are between two and three times as large as those of the high coverage stocks. For example, among stocks ranked high using the 200-day measure of idiosyncratic volatility, returns

are 172% over the past three years for low coverage stocks versus 51% for high coverage stocks. This also is consistent with optimistic mispricing of the positive earnings trend experienced by low coverage stocks.

We do not have a formal hypothesis (or test) that explains the *sign* of the trend in past operating performance by coverage—i.e., the fact that a high volatility ranking for low coverage stocks is associated with an *improvement* in past operating performance, versus a deterioration for stocks outside the low coverage group. However, the improvement for low coverage stocks occurs from a very low baseline relative to other stocks. If analysts prefer not to cover firms whose operating performance is very poor (perhaps because of a distaste for making bleak forecasts), then very poor past performance itself contributes to low coverage [see McNichols and O’Brien (1997)]. Among firms that exist at a point in time, those with very poor historical performance will be covered by few analysts and will likely have experienced a performance improvement. These feedback and survival effects could explain the directional association between coverage that is low and performance that is improving among stocks that survived until the ranking month.

Finally, accounting for *historical* coverage is important to documenting the distinct patterns in ROA in Table 11. If stocks were grouped by current coverage, so the low coverage portfolios also included firms just recently dropped from coverage, the evidence of growth in ROA would have been undetected. This is because firms that are new to the low coverage group have deteriorating operating performance and low past returns. These offset the high past returns and the uptrend in ROA of the historically low coverage firms. This also is consistent with a feedback and survival explanation of the link between analyst coverage and past operating performance.

## 5. Conclusion

We reexamine weaknesses others have found in the evidence of a negative relation between returns and idiosyncratic volatility (IVOL) first documented by Ang, Hodrick, Xing and Zhang (2006) (AHXZ). We confirm the weaknesses exist, then show they are attributable to the January effect and penny stocks (< \$5 per share). Controlling for these effects, we find that the AHXZ result is robust to variations in the data frequency, the

length of the time series used to construct idiosyncratic volatility, controls for firm size, and the degree to which security returns are lottery-like. Moreover, the significant lower returns to high IVOL portfolios last at least into the second year after portfolio formation.

We argue that the AHXZ result arises from mispricing that is consistent with Miller’s (1977) hypothesis, which we capture in a stylized dynamic model of strategic trading with costly short sales. In our model, significant information arrivals generate disagreement among traders when analyst coverage is low, and costly short sales lead pessimistic beliefs to be underrepresented in prices. Strategic traders anticipate this, which biases prices upward *prior* to information arrivals. Since information arrivals cause return volatility, this leads to a price runup prior to a high volatility ranking and then a correction as disagreement dissipates, but only for low coverage stocks.

Our empirical results are consistent with this explanation of the AHXZ result. First, low average returns to high IVOL stocks occur almost exclusively among firms with low analyst coverage. Outside the low coverage group, the return premium to high IVOL is insignificant or positive. Second, returns to high IVOL stocks are lower, the greater are their returns in the prior three years. This relation also is attributable to low coverage stocks, suggesting the low returns are corrections of prior optimistic mispricing of low coverage stocks.

These conclusions are reinforced by an analysis of returns around earnings announcements. If low coverage high IVOL stocks are mispriced too high, their returns should be negative *on average* when earnings are announced because the concreteness of earnings news should reduce disagreement among traders. We find that earnings announcement returns are significantly negative for stocks with high IVOL when coverage is low, and only in this case. This indicates that investors systematically revise their valuations downward with news on earnings, consistent with these stocks having been mispriced too high beforehand.

Our model also makes a prediction that is unrelated to return volatility—trading volume is driven by disagreement. This prediction is not unique to our model, but it provides another way to test it. Since disagreement coupled with costly short sales drives optimistic mispricing, shocks to trading volume should predict mispricing as well or better

than shocks to returns. When we run our tests by substituting share turnover volatility for return volatility, the results should be similar or stronger. This is exactly what we find. We confirm the finding of Chordia, Subrahmanyam and Anshuman (2001) that high turnover volatility predicts low returns, and we show this relation is attributable to stocks with low coverage. Results of the other tests described above involving returns of the prior three years and those involving earnings announcement returns are also stronger when turnover volatility is used in place of idiosyncratic return volatility.

Finally, we attempt to further characterize why mispricing arises and what type of information it relates to. We distinguish between firms that have a history (3 years) of low coverage from those that are new to the low coverage group. We find that the AHXZ and Chordia et. al. results are driven by firms with a history of low coverage. We then examine accounting operating performance (return on assets) for stocks in various volatility and coverage groups. The patterns in ROA are quite different for high volatility stocks inside versus outside the low coverage group.

Low coverage high volatility stocks, on average, have weak ROA compared to stocks in the other groups, but the trend in past ROA is strong and positive. This trend reverses in the two years following the high volatility ranking. ROA for high volatility stocks outside the low coverage group is also weak, but the trend is downward in the past and the future. Since these stocks experience no predictable returns, the market prices the downward trend in operating performance without bias. By contrast, the market seems to overestimate the persistence of the upward trend in operating performance for low coverage stocks, which is followed by a reversal. Coupling this with the model suggests that traders disagree about the persistence of improving operating performance when analyst coverage is low. Since short sales are costly, pessimists do not trade as aggressively as optimists, resulting in optimistic mispricing of those stocks while performance is improving.

Our findings shed light on hypotheses advanced in other recent papers. Han and Kumar (2008) find that the AHXZ result is concentrated among stocks that are dominated by retail investors as measured by the proportion of trades smaller than \$5,000. This is consistent with some of our results because stocks dominated by retail investors typically have low coverage. However, their explanation is that retail investors *prefer* to hold and

actively trade in high idiosyncratic volatility stocks. They hypothesize that the utility gained from active speculation leads investors to be willing to suffer low returns to holding these stocks. In a similar vein, Bali, Cakici and Whitelaw (2011) argue that investors prefer securities with lottery-like payoffs. These explanations do not fit with the insignificant or positive return premiums we document for high volatility stocks with high analyst coverage. Their stories are also not consistent with negative excess returns that are larger after bigger price runups, or downward average revaluations at earnings announcements for high volatility firms with low coverage. Moreover, we examine explicitly Bali, et.al.'s MAX variable, and we find their conclusion that IVOL is subsumed by MAX is driven by January and penny stocks. Significance of their MAX variable is limited to the first month after portfolio formation, suggesting that it captures short term reversals relating to bid-ask bounce [Fu (2009), Huang et. al (2010) and Han and Lesmond (2011)] rather than investors' preferences.

Jiang, Xu and Yao (2009) argue that high IVOL predicts low returns because high IVOL predicts poor future earnings. For the sample as a whole, we also find that high IVOL predicts poor earnings. This is not why high IVOL predicts low stock returns, however. High IVOL stocks outside the low coverage group do not have low returns even though they have negative earnings growth. In fact, among high IVOL stocks, the deterioration in earnings between years 0 and 2 is greater for those with high coverage than those with low coverage; yet excess returns are not negative for the high coverage firms. Our results instead favor the interpretation that high IVOL predicts low stock returns because disagreement generates optimistic mispricing among low coverage firms that is later corrected.

## APPENDIX

### Date 2

Whether or not information arrives at date 2, trader  $j$  solves

$$J_{2j} = \max_{x_{2j}(\cdot)} \mathbb{E}_{2j} [(\tilde{v} - p_2)x_{2j}(p_2) + c_s I_{2j} x_{2j}(p_2) - \psi_2 x_{2j}(p_2)^2]. \quad (\text{A.1})$$

The  $\psi_2$  parameter captures the utility cost associated with risk aversion and equals the product of the risk aversion coefficient and the variance of profit for trader  $j$  between dates 2 and 3. Since the variance of profit in each subtree is endogenous, we first solve the model for unspecified  $\psi$  parameters, then we close the model at the end by solving for the equilibrium  $\psi$  parameters in terms of the underlying model parameters.

Pointwise optimization of (A.1) yields a family of first-order conditions

$$(\mathbb{E}_{2j}[\tilde{v}] - p_2) - \frac{\partial p_2}{\partial x_{2j}} x_{2j} + c_s I_{2j} - 2\psi_2 x_{2j} = 0$$

that the trader's optimal choice must satisfy at each  $p_2$ . It will be apparent later that the second order condition for a maximum,  $\frac{\partial p_2}{\partial x_{2j}} + 2\psi_2 > 0$ , is satisfied in equilibrium. Rearranging the FOC yields an expression for trader  $j$ 's optimal demand schedule at date 2:

$$x_{2j}^*(p_2) = \frac{\mathbb{E}_{2j}[\tilde{v}] - p_2 + c_s I_{2j}}{\frac{\partial p_2}{\partial x_{2j}} + 2\psi_2}. \quad (\text{A.2})$$

### Date 2 with Information Arrival

We now establish the existence of a symmetric Nash equilibrium conditional on an information arrival at date 2. (In what follows, time subscripts are dropped where this creates no ambiguity.) Suppose a given pessimist  $j$  conjectures the other traders follow symmetric linear strategies, where the strategies can differ by “type” (pessimist vs. optimist). Specifically, trader  $j$  conjectures:

$$x_k = \begin{cases} \beta_L(\bar{v}_L - p - c_s) & \text{for all } k = L \text{ and } k \neq j \\ \beta_H(\bar{v}_H - p) & \text{for all } k = H. \end{cases} \quad (\text{A.3})$$

Under this conjecture, pessimist  $j$  perceives the market clearing condition to be

$$x_j + (N - 1)x_L + Nx_H = 2NX.$$

Substituting from (A.3) and solving for  $p$ ,

$$p = \frac{(N-1)\beta_L(\bar{v}_L + c_s) + N\beta_H\bar{v}_H}{(N-1)\beta_L + N\beta_H} + \frac{x_j + 2NX}{(N-1)\beta_L + N\beta_H}.$$

Therefore, trader  $j$  perceives

$$\frac{\partial p}{\partial x_j} = \frac{1}{(N-1)\beta_L + N\beta_H} \quad (\text{A.4.1})$$

if he is a pessimist (i.e.,  $j = L$ ). Similar reasoning implies

$$\frac{\partial p}{\partial x_j} = \frac{1}{N\beta_L + (N-1)\beta_H} \quad (\text{A.4.2})$$

if  $j = H$ . Combining (A.4.1) and (A.4.2) with (A.2) implies that if trader  $j$  conjectures the others follow the strategies in (A.3), then trader  $j$ 's optimal strategy is

$$x_{2j}^* = \begin{cases} \frac{\bar{v}_L - p + c_s I_L}{\frac{1}{(N-1)\beta_L + N\beta_H} + 2\psi} & \text{if } j = L \\ \frac{\bar{v}_H - p + c_s I_H}{\frac{1}{N\beta_L + (N-1)\beta_H} + 2\psi} & \text{if } j = H. \end{cases} \quad (\text{A.5})$$

This is the same form as the conjectured strategies in Eq. (A.3) provided that  $I_L = 1$  and  $I_H = 0$ . Thus, if trader  $j$  conjectures that others follow the strategies in (A.3), it is optimal for trader  $j$  to follow the same strategy if the following conditions are satisfied:

$$\frac{1}{\beta_L} = \frac{1}{(N-1)\beta_L + N\beta_H} + 2\psi \quad \text{and} \quad \frac{1}{\beta_H} = \frac{1}{N\beta_L + (N-1)\beta_H} + 2\psi \quad (\text{A.6.1})$$

$$\beta_L > 0 \quad \text{and} \quad \beta_H > 0 \quad (\text{A.6.2})$$

$$\beta_L(\bar{v}_L - p^* + c_s) < 0 \quad \text{and} \quad \beta_H(\bar{v}_H - p^*) > 0. \quad (\text{A.6.3})$$

Eq. (A.6.1) says that pessimists share a common strategy coefficient, and optimist share a common strategy coefficient. Eq. (A.6.2) ensures the second-order condition is satisfied for both trader types. Eq. (A.6.3) says that pessimists hold short positions ( $I_L = 1$ ) and optimists hold long positions ( $I_H = 0$ ) at the price,  $p^*$ , that clears the market. Therefore, a symmetric equilibrium exists with optimists taking long positions and pessimists short positions if (A.6.1) - (A.6.3) are satisfied.

First, we find a solution to the pair of equations in (A.6.1) that satisfies (A.6.2). Rewrite the equations in (A.6.1) as

$$N\beta_L + N\beta_H = 2\beta_L + 2\psi\beta_L [(N-1)\beta_L + N\beta_H] \quad (\text{A.7})$$

$$N\beta_L + N\beta_H = 2\beta_H + 2\psi\beta_H [(N-1)\beta_H + N\beta_L].$$



Equating these

$$\begin{aligned}
2\beta_H \{1 + \psi [(N - 1)\beta_H + N\beta_L]\} &= 2\beta_L \{1 + \psi [(N - 1)\beta_L + N\beta_H]\} \\
\beta_H - \beta_L &= \psi \{\beta_L [(N - 1)\beta_L + N\beta_H] - \beta_H [(N - 1)\beta_H + N\beta_L]\} \\
\beta_H - \beta_L &= \psi(N - 1)(\beta_L + \beta_H)(\beta_L - \beta_H).
\end{aligned}$$

Either  $\beta_H - \beta_L = 0$  or  $\beta_H + \beta_L = \frac{-1}{\psi(N-1)}$ . The latter possibility is not consistent with (A.6.2). Consider then the possibility that  $\beta_H = \beta_L = \beta$ . Using either of the equations in (A.7) to solve for  $\beta$ :

$$\begin{aligned}
2N\beta &= 2\beta + 2\psi\beta[(2N - 1)\beta] \\
\beta &= \frac{N - 1}{\psi(2N - 1)} > 0
\end{aligned} \tag{A.8}$$

Thus, (A.6.1) has a unique solution that satisfies (A.6.2).

To verify (A.6.3), we need an expression for the market-clearing price. In the proposed equilibrium, the market-clearing condition and price are

$$\begin{aligned}
N\beta(\bar{v}_L - p^* + c_s) + N\beta(\bar{v}_H - p^*) &= 2NX \\
p^* &= \frac{1}{2}(\bar{v}_L + \bar{v}_H) + \frac{c_s}{2} - \frac{X}{\beta}.
\end{aligned} \tag{A.9}$$

The equilibrium price equals the consensus valuation at date 2 plus an upward bias equal to one half the short-sale cost, minus a risk premium. Using (A.9), the condition in (A.6.3) that pessimists have short positions in equilibrium can be written as

$$\beta \left[ \bar{v}_L - \frac{1}{2}(\bar{v}_L + \bar{v}_H + c_s) + \frac{X}{\beta} + c_s \right] < 0,$$

or equivalently as

$$\bar{v}_H - \bar{v}_L > c_s + 2\frac{X}{\beta}. \tag{A.10}$$

The difference between optimists' and pessimists' beliefs about value must be greater than the short sale cost plus two times the price discount due to risk in order for pessimists to hold short positions in equilibrium. The condition in (A.6.3) that optimists hold long positions in equilibrium can be written as

$$\beta \left[ \bar{v}_H - \frac{1}{2}(\bar{v}_L + \bar{v}_H + c_s) + \frac{X}{\beta} \right] > 0,$$

or equivalently as

$$\bar{v}_H - \bar{v}_L > c_s - 2\frac{X}{\beta} \quad (\text{A.11})$$

which is implied by (A.10).

Therefore, if an information arrival at date 2 generates divergence in beliefs that is large enough to satisfy (A.10), then there is a symmetric Nash equilibrium in linear strategies at date 2 where optimists hold long positions and pessimists hold short positions. In that equilibrium, the market clearing price has the form:

$$\text{price} = \text{consensus beliefs} + \text{bias} - \text{risk premium}.$$

To solve for strategies at date 1 below, we need expressions for equilibrium (i.e., optimized) expected utility at date 2 for each trader type. Using Eq. (A.1), expected optimist utility is

$$\begin{aligned} J_{2H}(p_2^*) &\equiv J_{2H}|_{p_2^*} \\ &= (\bar{v}_H - p_2^*)x_{2H}^*(p_2^*) - \psi_2 x_{2H}^*(p_2^*)^2 \\ &= \beta(1 - \psi\beta)(\bar{v}_H - p_2^*)^2, \end{aligned} \quad (\text{A.12.1})$$

and expected pessimist utility is

$$\begin{aligned} J_{2L}(p_2^*) &\equiv J_{2L}|_{p_2^*} \\ &= (\bar{v}_L - p_2^* + c_s)\beta(\bar{v}_L - p_2^* + c_s) - \psi_2\beta^2(\bar{v}_L - p_2^* + c_s)^2 \\ &= \beta(1 - \psi\beta)(\bar{v}_L - p_2^* + c_s)^2, \end{aligned} \quad (\text{A.12.2})$$

where  $\beta$  is defined in (A.8) and  $p_2^*$  is defined in (A.9). The key observation is that neither  $J_{2H}(p_2^*)$  nor  $J_{2L}(p_2^*)$  depend on choices or prices at date 1, so  $J_{2H}(p_2^*)$  and  $J_{2L}(p_2^*)$  are irrelevant to the optimization of date-1 holdings.

#### Date 2 without Information Arrival

If no information arrives at date 2, all traders continue to believe  $E[\tilde{v}] = v_o$ . If trader  $j$  conjectures the other  $2N - 1$  traders follow a strategy of the form

$$\hat{x}_k = \hat{\beta}(v_o - \hat{p}) \quad \text{for all } k \neq j,$$

then his perception of market clearing is

$$\begin{aligned}\hat{x}_j + (2N - 1)\hat{\beta}(v_o - \hat{p}) &= 2NX \\ \hat{p} &= v_o + \frac{1}{(2N - 1)\hat{\beta}}(\hat{x}_j - 2NX)\end{aligned}$$

and therefore

$$\frac{\partial \hat{p}}{\partial \hat{x}_j} = \frac{1}{(2N - 1)\hat{\beta}}.$$

By (A.2), trader  $j$ 's optimal strategy is then

$$\hat{x}_j^* = \frac{v_o - \hat{p} + c_s I_j}{\frac{1}{(2N-1)\hat{\beta}} + 2\psi},$$

which is the same form as the conjecture above provided that  $I_j = 0$  (i.e., trader  $j$  holds a long position). A symmetric equilibrium with all traders holding long positions will exist if the following conditions are satisfied:

$$\frac{1}{\hat{\beta}} = \frac{1}{(2N - 1)\hat{\beta}} + 2\psi \tag{A.13.1}$$

$$\hat{\beta} > 0 \tag{A.13.2}$$

$$\hat{\beta}(v_o - \hat{p}^*) > 0. \tag{A.13.3}$$

The interpretations of these equations are analogous to those of (A.6.1) - (A.6.3). Solving (A.13.1) implies

$$\hat{\beta} = \frac{N - 1}{\psi(2N - 1)} > 0 \tag{A.14}$$

which satisfies (A.13.2). The market-clearing price derives from the market clearing condition

$$\begin{aligned}2N\hat{x}_j^*(\hat{p}^*) &= 2NX \\ \hat{p}^* &= v_o - \frac{X}{\hat{\beta}},\end{aligned} \tag{A.15}$$

so (A.13.3) is satisfied because

$$\hat{\beta}(v_o - \hat{p}^*) = \hat{\beta} \left( v_o - v_o + \frac{X}{\hat{\beta}} \right) = X > 0.$$

Thus, if information does not arrive, there is a symmetric Nash equilibrium in linear strategies at date 2 where all traders hold (identical) long positions. In this equilibrium, trader  $j$ 's expected utility is

$$\begin{aligned}
J_{2o}(\hat{p}_2^*) &\equiv J_{2o}|_{\hat{p}_2^*} \\
&= (v_o - \hat{p}_2^*) \hat{x}_2^*(\hat{p}_2^*) - \psi \hat{x}_2^*(\hat{p}_2^*)^2 \\
&= \hat{\beta} (v_o - \hat{p}_2^*)^2 - \psi \hat{\beta}^2 (v_o - \hat{p}_2^*)^2 \\
&= \hat{\beta} (1 - \psi \hat{\beta}) (v_o - \hat{p}_2^*)^2,
\end{aligned}$$

where  $\hat{\beta}$  and  $\hat{p}_2^*$  are given in (A.14) and (A.15). The key observation is that  $J_{2o}(\hat{p}_2^*)$  does not depend on choices or prices at date 1, so  $J_{2o}(\hat{p}_2^*)$  is irrelevant to the optimization of date-1 holdings.

### Date 1

At date 1, trader  $j$  seeks to maximize expected long-run utility, given a probability  $q$  that information will arrive next period and generate divergent beliefs, and a probability  $1 - q$  that information will not arrive:

$$\begin{aligned}
\max_{x_{1j}(\cdot)} \quad & q \mathbb{E}_1 \left[ (p_2^* - p_1) x_{1j}(p_1) + \tilde{J}_{2j}(p_2^*) \right] + (1 - q) \mathbb{E}_1 \left[ (\hat{p}_2^* - p_1) x_{1j}(p_1) + \tilde{J}_{2o}(\hat{p}_2^*) \right] \\
& + c_s I_{1j} x_{1j}(p_1) - \psi_1 x_{1j}(p_1)^2.
\end{aligned}$$

To simplify notation, the term involving the short-position indicator,  $I_{1j}$ , is suppressed. It eventually drops out (just as in the analysis of date 2 when no information arrives) because traders are identical at date 1.<sup>11</sup> Trader  $j$ 's problem is therefore

$$\begin{aligned}
\max_{x_{1j}(\cdot)} \quad & \left( q \mathbb{E}_1 [p_2^*] + (1 - q) \mathbb{E}_1 [\hat{p}_2^*] - p_1 \right) x_{1j}(p_1) - \psi_1 x_{1j}(p_1)^2 \\
& + q \mathbb{E}_1 \left[ \tilde{J}_{2j}(p_2^*) \right] + (1 - q) \mathbb{E}_1 \left[ \tilde{J}_{2o}(\hat{p}_2^*) \right]. \tag{A.16}
\end{aligned}$$

Now,  $\mathbb{E}_1 \left[ \tilde{J}_{2o}(\hat{p}_2^*) \right] = J_{2o}(\hat{p}_2^*)$  because no new information arrives between dates 1 and 2 in the ‘‘hat’’ case. For the case where information does arrive, there is equal likelihood that trader  $j$  will adopt optimistic and pessimistic beliefs, so

$$\mathbb{E}_1 \left[ \tilde{J}_{2j}(p_2^*) \right] = \frac{1}{2} \mathbb{E}_1 \left[ \tilde{J}_{2H}(p_2^*) \right] + \frac{1}{2} \mathbb{E}_1 \left[ \tilde{J}_{2L}(p_2^*) \right].$$

---

<sup>11</sup>When traders are identical and there is a symmetric equilibrium, all traders hold their share of per-capita supply of the security, which results in long positions because the security is in positive net supply.

We established earlier that  $J_{2o}(\hat{p}_2^*)$ ,  $J_{2H}(p_2^*)$  and  $J_{2L}(p_2^*)$  are all independent of date-1 holdings and prices, so these terms in (A.16) are irrelevant to the date-1 optimization of (A.16). Consequently, the family of first-order conditions that characterize  $x_{1j}(\cdot)$  is

$$qE_1 [p_2^*] + (1 - q)E_1 [\hat{p}_2^*] - p_1 - \frac{\partial p_1}{\partial x_{1j}} x_{1j} - 2\psi_1 x_{1j} = 0$$

for each  $p_1$ , noting that both  $p_2^*$  and  $\hat{p}_2^*$  are independent of date-1 choices. It will be apparent later that the second order condition for a maximum,  $\frac{\partial p_1}{\partial x_{1j}} + 2\psi_1 > 0$ , is satisfied in equilibrium. Rearranging the FOC yields an expression for trader  $j$ 's optimal demand schedule at date 1:

$$x_{1j}^*(p_1) = \frac{qE_1 [p_2^*] + (1 - q)E_1 [\hat{p}_2^*] - p_1}{\frac{\partial p_1}{\partial x_{1j}} + 2\psi_1}. \quad (\text{A.17})$$

Suppose trader  $j$  conjectures that other traders follow the strategy

$$x_{1k} = \gamma \left( v_o + q \frac{c_s}{2} - \frac{X}{\beta} - p_1 \right) \quad \text{for all } k \neq j. \quad (\text{A.18})$$

Then trader  $j$ 's perception of market clearing is that

$$\begin{aligned} x_{1j} + (2N - 1)\gamma \left( v_o + q \frac{c_s}{2} - \frac{X}{\beta} - p_1 \right) &= 2NX \\ p_1 &= v_o + q \frac{c_s}{2} - \frac{X}{\beta} + \frac{x_{1j} - 2NX}{(2N - 1)\gamma}. \end{aligned}$$

and so

$$\frac{\partial p_1}{\partial x_{1j}} = \frac{1}{(2N - 1)\gamma}. \quad (\text{A.19})$$

Substituting (A.19) into (A.17) and using the fact that  $j$  has a rational prior about how his beliefs will change if information arrives (i.e.,  $v_o = E_1 \left[ \frac{1}{2}\bar{v}_L + \frac{1}{2}\bar{v}_H \right]$ ) implies that trader  $j$ 's demand schedule is

$$x_{1j}^*(p_1) = \frac{q \left( v_o + \frac{c_s}{2} - \frac{X}{\beta} \right) + (1 - q) \left( v_o - \frac{X}{\beta} \right) - p_1}{\frac{1}{(2N-1)\gamma} + 2\psi_1} = \frac{v_o + q \frac{c_s}{2} - \frac{X}{\beta} - p_1}{\frac{1}{(2N-1)\gamma} + 2\psi_1}. \quad (\text{A.20})$$

where we have used the fact that  $\hat{\beta} = \beta$  in the two date-2 equilibria above. Eq. (A.20) is the same form as the conjecture in (A.18). A symmetric equilibrium with all traders

holding long positions will therefore exist if the following conditions are satisfied:

$$\frac{1}{\gamma} = \frac{1}{(2N-1)\gamma} + 2\psi_1 \quad (\text{A.21.1})$$

$$\gamma > 0 \quad (\text{A.21.2})$$

$$\gamma \left( v_o + q \frac{c_s}{2} - \frac{X}{\beta} - p_1^* \right) > 0. \quad (\text{A.21.3})$$

The interpretation of these is the same as (A.13.1) - (A.13.3). Solving (A.21.1) implies

$$\gamma = \frac{N-1}{\psi_1(2N-1)} > 0 \quad (\text{A.21.1a})$$

which satisfies (A.21.2). The market-clearing price satisfies

$$\begin{aligned} 2Nx_{1j}^*(p_1^*) &= 2NX \\ p_1^* &= v_o + q \frac{c_s}{2} - \frac{X}{\beta} - \frac{X}{\gamma}, \end{aligned} \quad (\text{A.22})$$

so (A.21.3) is satisfied because

$$\gamma \left( v_o + q \frac{c_s}{2} - \frac{X}{\beta} - p_1^* \right) = X > 0.$$

Therefore, there is a symmetric Nash equilibrium in linear strategies at date 1 where all traders hold identical long positions. In that equilibrium, the market clearing price (A.22) has the form:

$$\text{price} = \text{consensus beliefs} + (q \times \text{date-2 bias}) - \text{risk premium}.$$

The bias here is exactly the bias in the date-2 price conditional on an information arrival, scaled by the probability of an information arrival.

### **Solving for $\psi_1$ , $\psi_2$ and $d$**

Since traders maximize mean-variance preferences, the quadratic cost modeled above using  $\psi$  parameters arises from risk aversion. The common risk aversion parameter is  $\alpha$ , so the equivalence between the quadratic costs and the variance component of mean-variance preferences is as follows:

$$\begin{aligned} \psi_t (x_{tj}^*)^2 &= \alpha \text{Var}_{tj} [\tilde{\pi}_{jt}] = \alpha \text{Var}_{tj} [\tilde{p}_{t+1} - p_t] (x_{tj}^*)^2 \\ \psi_t &= \alpha \text{Var}_{tj} [\tilde{p}_{t+1} - p_t] \end{aligned}$$

so, for each sub-tree of the game,

$$\psi_t = \begin{cases} \alpha \text{Var} [\tilde{v} - p_2^*] & \text{if info arrives at } t = 2 \\ \alpha \text{Var} [\tilde{v} - \hat{p}_2^*] & \text{if info does not arrive at } t = 2 \\ \alpha \text{Var} [\tilde{p}_2 - p_1^*] & \text{if } t = 1. \end{cases}$$

where  $\tilde{p}_2$  equals  $p_2^*$  with probability  $q$  and  $\hat{p}_2^*$  with probability  $1 - q$ . We derive explicit expressions for the equilibrium values of the  $\psi$  parameters next.

Case 1: If information arrives at  $t = 2$  then

$$p_2^* = \bar{v} + \frac{c_s}{2} - \frac{X}{\beta}$$

$$\tilde{v} - p_2^* = \tilde{v} - \bar{v} - \frac{c_s}{2} + \frac{X}{\beta}.$$

At  $t = 2$ , agent  $j$  knows  $\bar{v}$  because he knows his own beliefs  $\bar{v}_j$  and those of the other group. Therefore,

$$\text{Var}_{2j} [\tilde{v} - p_2^*] = \text{Var}_2 [\tilde{v}] = \sigma_v^2.$$

Case 2: If information does not arrive at  $t = 2$  then

$$\hat{p}_2^* = v_o - \frac{X}{\beta}$$

$$\tilde{v} - \hat{p}_2^* = \tilde{v} - v_o + \frac{X}{\beta}$$

therefore

$$\text{Var}_{2j} [\tilde{v} - \hat{p}_2^*] = \text{Var}_2 [\tilde{v}] = \sigma_v^2.$$

Cases 1 and 2 together imply that  $\psi_2$  is the same in both date-2 subtrees:

$$\psi_2 = \alpha \sigma_v^2. \tag{A.23}$$

Case 3: Recall from above that the definition of  $\tilde{p}_2$  is

$$\tilde{p}_2 = \begin{cases} p_2^* = \bar{v} + \frac{c_s}{2} - \frac{X}{\beta} & \text{with probability } q \\ \hat{p}_2^* = v_o - \frac{X}{\beta} & \text{with probability } 1 - q. \end{cases}$$

Subtracting Eq. (A.22) from these expressions we have

$$\tilde{p}_2 - p_1^* = \begin{cases} \bar{v} - v_o + (1 - q) \frac{c_s}{2} + \frac{X}{\gamma} & \text{with probability } q \\ -q \frac{c_s}{2} + \frac{X}{\gamma} & \text{with probability } 1 - q. \end{cases} \tag{A.24}$$

The form of this is

$$\tilde{Y} = \begin{cases} \tilde{Z} & \text{with probability } q \\ K & \text{with probability } 1 - q, \end{cases}$$

where  $K$  is a constant, and  $\tilde{Z}$  is a random variable conditional on the top state occurring.

The variance of a random variable of this form is

$$\text{Var} [\tilde{Y}] = q\text{Var} [\tilde{Z}] + q(1 - q) \left( E [\tilde{Z}] - K \right)^2. \quad (\text{A.25})$$

Substituting from (A.24) into (A.25) yields

$$\text{Var}_1 [\tilde{p}_2 - p_1^*] = q\text{Var}_1 [\bar{v}] + q(1 - q) \left( \frac{c_s}{2} \right)^2,$$

so

$$\psi_1 = \alpha q \left\{ \text{Var}_1 [\bar{v}] + (1 - q) \frac{c_s^2}{4} \right\}. \quad (\text{A.26})$$

The differences between the expressions (A.23) and (A.26) arise because the uncertainty resolved between dates 1 and 2 relates to whether or not information arrives that shifts the mean of the distribution of  $\tilde{v}$  and by how much, whereas the uncertainty resolved between dates 2 and 3 is the realization of  $\tilde{v}$ .

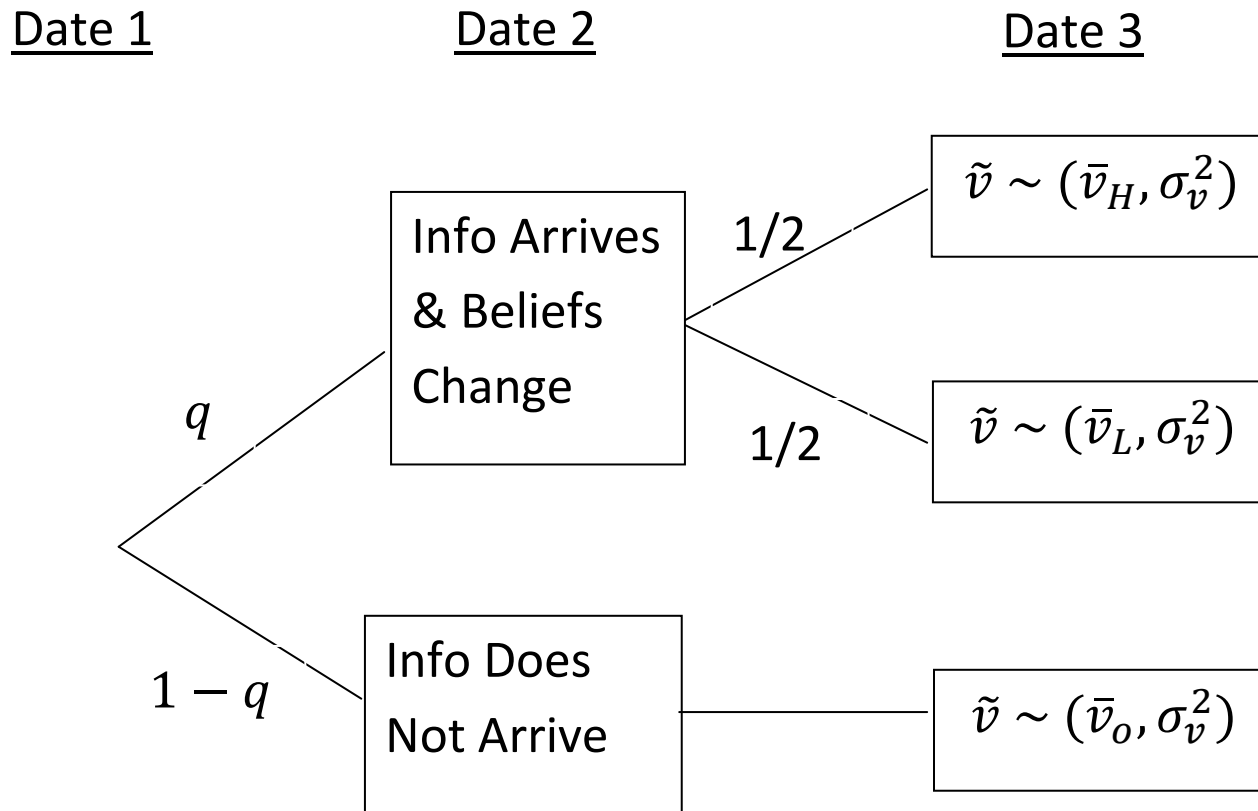
Finally, combining (A.10) and (A.23) yields an expression, in terms of exogenous variables, for the extent of divergence in beliefs required to support an equilibrium where pessimists hold short positions and optimists long positions:

$$d = \bar{v}_H - \bar{v}_L > c_s + 2\alpha\sigma_v^2 X \left( \frac{2N - 1}{N - 1} \right) \equiv \underline{d}. \quad (\text{A.27})$$



# Figure 1

## Sequence of Events



**Table 1: Summary Statistics**

Panel A reports time-series averages of equally-weighted monthly cross-sectional mean, median, maximum and minimum of each variables used in the paper. Panel B reports time-series averages of equally-weighted monthly cross-sectional correlations. Using monthly data from January 1963 to December 2006, we construct indicator variables for each of the measures described in the text. *Market CAP* is market equity capitalization, *Ret(-1,-12)* is the one year return prior to month *t*, *STURN* is the standard deviation of turnover calculated over the past 36 months ending in month -1, *IVOL20D (IVOL200D)* is idiosyncratic volatility calculated from daily returns in the past month (year), *IVOL60M* is idiosyncratic volatility calculated from monthly returns over the past five years. *LCOV* is a dummy that takes the value 1 if the stock is covered by three or fewer analysts, and takes the value zero otherwise.

Panel A

	Mean	Median	Min	Max
Market Cap (Millions)	1460.53	1818.88	2.80	123198.14
Ret(-1,-12)	0.214	0.13	-0.72	8.05
Ret(-1,-36)	0.616	0.29	-0.84	25.45
STURN	0.036	0.02	0.01	0.13
IVOL200D	0.024	0.02	0.01	0.11
IVOL20D	0.021	0.02	0.00	0.15
IVOL60M	0.095	0.09	0.03	0.49
LCOV	0.551	0.96	0.00	1.00

Panel B

	Market Cap	Ret(-1,-12)	Ret(-1,-36)	STURN	IVOL200D	IVOL20D	IVOL60M	LCOV
Market Cap	1.000							
Ret(-1,-12)	-0.002	1.000						
Ret(-1,-36)	0.004	0.452	1.000					
STURN	-0.093	0.087	0.194	1.000				
IVOL200D	-0.213	0.162	0.114	0.423	1.000			
IVOL20D	-0.161	0.076	0.070	0.275	0.639	1.000		
IVOL60M	-0.202	0.177	0.254	0.500	0.744	0.524	1.000	
LCOV	-0.210	0.070	0.006	-0.045	0.253	0.184	0.198	1.000

**Table 2: Raw Returns of High and Low Idiosyncratic Portfolios (Including Penny Stocks)**

Each month between January 1963 and December 2006, 24 ( $j=1, \dots, 24$ ) cross-sectional regressions of the following forms are estimated:

$$R_{it} = b_{0t} + b_{1jt}LVOL_{i,t-j} + b_{2jt}HVOL_{i,t-j} + e_{ijt} \quad \text{and} \quad R_{it} = b_{0t} + b_{1jt}LVOL_{i,t-j} + b_{2jt}HVOL_{i,t-j} + b_{3jt}LMAX5_{i,t-j} + b_{4jt}HMAX5_{i,t-j} + e_{ijt}$$

where  $R_{it}$  is the return to stock  $i$  in month  $t$ ,  $LVOL_{i,t-j}$  ( $HVOL_{i,t-j}$ ) is the low (high) idiosyncratic volatility dummy that takes the value of 1 if the idiosyncratic volatility for stock  $i$  is ranked in the top (bottom) 20% in month  $t-j$ , and zero otherwise.  $LMAX5_{i,t-j}$  ( $HMAX5_{i,t-j}$ ) is the low (high) idiosyncratic volatility dummy that takes the value of 1 if the  $MAX5$  (the average of the five highest daily return in the month) for stock  $i$  is ranked in the top (bottom) 20% in month  $t-j$ , and zero otherwise. The coefficient estimates of a given independent variable are for  $j=1$  for columns labeled ( $p=0, K=1$ ), and averaged over  $j=2$  to 12 for columns labeled ( $p=1, K=11$ ), and  $j=13$  to 24 for columns labeled ( $p=12, K=12$ ). The numbers reported in the table are the time-series averages of these averages. They are in percent per month. The accompanying  $t$ -statistics are calculated from the time series. This sample includes penny socks (price < \$5).  $NOBS$  is the average number of stocks used in the monthly cross-sectional regressions.

Raw Returns, IVOL20D (NOBS=5220)								
	Column 1 (p=0,K=1)	Column 2 (p=0,K=1) Jan. excluded	Column 3 (p=1,K=1)	Column 4 (p=1,K=1) Jan. excluded	Column 5 (p=2,K=1)	Column 6 (p=2,K=1) Jan. excluded	Column 7 (p=1,K=11)	Column 8 (p=1,K=11) Jan. excluded
Intercept	1.38 (5.60)	0.99 (4.04)	1.31 (5.35)	0.92 (3.78)	1.30 (5.29)	0.90 (3.72)	1.26 (5.04)	0.85 (3.46)
LVOL	-0.27 (-2.11)	-0.06 (-0.49)	-0.11 (-0.88)	0.10 (0.82)	-0.11 (-0.91)	0.09 (0.74)	-0.09 (-0.70)	0.13 (1.06)
HVOL	-0.21 (-1.01)	-0.87 (-5.08)	-0.08 (-0.39)	-0.72 (-4.21)	0.00 (0.01)	-0.62 (-3.71)	0.26 (1.35)	-0.35 (-2.18)
Raw Returns, IVOL20D (NOBS=5220)								
Intercept	1.46 (5.94)	1.08 (4.42)	1.32 (5.38)	0.93 (3.85)	1.29 (5.30)	0.91 (3.77)	1.22 (5.06)	0.84 (3.54)
LVOL	-0.49 (-4.53)	-0.24 (-2.33)	-0.19 (-1.79)	0.05 (0.49)	-0.03 (-0.27)	0.19 (1.97)	-0.06 (-0.63)	0.16 (1.79)
HVOL	0.72 (4.25)	0.09 (0.72)	0.01 (0.08)	-0.54 (-4.30)	-0.17 (-1.10)	-0.68 (-5.44)	0.17 (1.25)	-0.33 (-2.95)
LMAX5	0.16 (2.06)	0.09 (1.16)	0.10 (1.35)	0.06 (0.76)	-0.11 (-1.51)	-0.16 (-2.04)	-0.01 (-0.13)	-0.05 (-0.72)
HMAX5	-1.29 (-10.33)	-1.33 (-10.11)	-0.14 (-1.20)	-0.25 (-2.17)	0.21 (1.93)	0.06 (0.54)	0.05 (0.56)	-0.08 (-0.89)

**Table 3: Raw Returns of High and Low Idiosyncratic Portfolios**

Each month between January 1963 and December 2006, 24 ( $j=1, \dots, 24$ ) cross-sectional regressions of the following forms are estimated:

$$R_{it} = b_{0t} + b_{1jt}LVOL_{i,t-j} + b_{2jt}HVOL_{i,t-j} + e_{ijt} \quad \text{and} \quad R_{it} = b_{0t} + b_{1jt}LVOL_{i,t-j} + b_{2jt}HVOL_{i,t-j} + b_{3jt}LMAX5_{i,t-j} + b_{4jt}HMAX5_{i,t-j} + e_{ijt}$$

where  $R_{it}$  is the return to stock  $i$  in month  $t$ ,  $LVOL_{i,t-j}$  ( $HVOL_{i,t-j}$ ) is the low (high) idiosyncratic volatility dummy that takes the value of 1 if the idiosyncratic volatility for stock  $i$  is ranked in the top (bottom) 20% in month  $t-j$ , and zero otherwise.  $LMAX5_{i,t-j}$  ( $HMAX5_{i,t-j}$ ) is the low (high) idiosyncratic volatility dummy that takes the value of 1 if the  $MAX5$  (the average of the five highest daily return in the month) for stock  $i$  is ranked in the top (bottom) 20% in month  $t-j$ , and zero otherwise. The coefficient estimates of a given independent variable are for  $j=1$  for columns labeled ( $p=0, K=1$ ), and averaged over  $j=2$  to 12 for columns labeled ( $p=1, K=11$ ), and  $j=13$  to 24 for columns labeled ( $p=12, K=12$ ). The numbers reported in the table are the time-series averages of these averages. They are in percent per month. The accompanying  $t$ -statistics are calculated from the time series. Penny socks (price < \$5) are excluded.  $NOBS$  is the average number of stocks used in the monthly cross-sectional regressions.

Raw Returns, IVOL20D (NOBS=3997)								
	Column 1 (p=0, K=1)	Column 2 (p=0, K=1) Jan. excluded	Column 3 (p=1, K=1)	Column 4 (p=1, K=1) Jan. excluded	Column 5 (p=2, K=1)	Column 6 (p=2, K=1) Jan. excluded	Column 7 (p=1, K=11)	Column 8 (p=1, K=11) Jan. excluded
Intercept	1.33 (5.58)	1.04 (4.29)	1.26 (5.28)	0.95 (3.97)	1.24 (5.22)	0.94 (3.91)	1.20 (4.89)	0.86 (3.54)
LVOL	-0.22 (-1.82)	-0.08 (-0.68)	-0.09 (-0.75)	0.06 (0.49)	-0.09 (-0.78)	0.05 (0.42)	-0.06 (-0.49)	0.10 (0.88)
HVOL	-1.24 (-8.00)	-1.39 (-8.62)	-0.70 (-4.50)	-0.87 (-5.44)	-0.61 (-4.08)	-0.84 (-5.58)	-0.40 (-2.65)	-0.70 (-4.83)
Raw Returns, IVOL20D (NOBS=3997)								
Intercept	1.38 (5.79)	1.08 (4.50)	1.26 (5.35)	0.96 (4.04)	1.25 (5.28)	0.95 (3.99)	1.16 (4.98)	0.86 (3.68)
LVOL	-0.37 (-3.92)	-0.21 (-2.18)	-0.06 (-0.64)	0.10 (1.12)	0.03 (0.34)	0.18 (1.95)	0.02 (0.22)	0.18 (2.12)
HVOL	-0.68 (-7.03)	-0.86 (-8.99)	-0.70 (-7.35)	-0.84 (-8.70)	-0.73 (-7.58)	-0.89 (-9.57)	-0.41 (-5.00)	-0.63 (-8.93)
LMAX5	0.13 (2.04)	0.09 (1.30)	-0.06 (-0.92)	-0.08 (-1.17)	-0.18 (-2.83)	-0.19 (-2.90)	-0.10 (-1.94)	-0.12 (-2.16)
HMAX5	-0.77 (-5.40)	-0.74 (-4.85)	-0.01 (-0.07)	-0.06 (-0.45)	0.15 (1.21)	0.05 (0.40)	-0.01 (-0.10)	-0.10 (-0.88)

**Table 4: Risk Adjusted Returns of High and Low Idiosyncratic Volatility Portfolios**

Each month between January 1963 and December 2006, 24 ( $j=1, \dots, 24$ ) cross-sectional regressions of the following form are estimated:

$$R_{it} = b_{0t} + b_{1jt} LVOL_{i,t-j} + b_{2jt} HVOL_{i,t-j} + e_{ijt}$$

where  $R_{it}$  is the return to stock  $i$  in month  $t$ ,  $LVOL_{i,t-j}$  ( $HVOL_{i,t-j}$ ) is the low (high) idiosyncratic volatility dummy that takes the value of 1 if the idiosyncratic volatility for stock  $i$  is ranked in the top (bottom) 20% in month  $t-j$ , and zero otherwise. The coefficient estimates of a given independent variable are for  $j=1$  for columns labeled (p=0,K=1), and averaged over  $j=2$  to 12 for columns labeled (p=1,K=11), and  $j=13$  to 24 for columns labeled (p=12,K=12). To obtain risk-adjusted returns, we further run times-series regressions of these averages (one for each average) on the contemporaneous Fama-French factor realizations to hedge out the factor exposure. The numbers reported for risk-adjusted returns are intercepts from these time-series regressions. They are in percent per month and their  $t$ -statistics are in parentheses. Penny socks (price < \$5) are excluded. *NOBS* is the average number of stocks used in the monthly cross-sectional regressions.

Risk Adjusted Returns, IVOL20D (NOBS=3997)						
	Column 1 (p=0,K=1)	Column 2 (p=0,K=1) Jan. excluded	Column 3 (p=1,K=11)	Column 4 (p=1,K=11) Jan. excluded	Column 5 (p=12,K=12)	Column 6 (p=12,K=12) Jan. excluded
Intercept	0.08 (1.92)	0.04 (0.87)	-0.06 (-1.33)	-0.13 (-2.74)	-0.07 (-1.24)	-0.15 (-3.08)
LVOL	-0.01 (-0.28)	0.02 (0.33)	0.14 (2.72)	0.20 (3.81)	0.10 (1.76)	0.17 (3.12)
HVOL	-1.22 (-10.95)	-1.31 (-11.57)	-0.46 (-4.47)	-0.66 (-6.90)	-0.14 (-1.30)	-0.38 (-4.15)
Risk Adjusted Returns, IVOL200D (NOBS=3664)						
Intercept	-0.01 (-0.13)	-0.04 (-0.91)	-0.05 (-1.04)	-0.11 (-2.15)	-0.04 (-0.81)	-0.12 (-2.39)
LVOL	0.12 (2.04)	0.16 (2.82)	0.13 (2.22)	0.20 (3.31)	0.10 (1.60)	0.18 (2.92)
HVOL	-0.59 (-4.14)	-0.75 (-5.32)	-0.32 (-2.31)	-0.60 (-4.70)	-0.22 (-1.53)	-0.54 (-4.44)
Risk Adjusted Returns, IVOL60M (NOBS=2519)						
Intercept	0.02 (0.47)	0.02 (0.33)	0.01 (0.23)	0.00 (-0.06)	0.00 (0.05)	-0.03 (-0.54)
LVOL	0.06 (1.08)	0.08 (1.43)	0.05 (0.95)	0.08 (1.41)	0.03 (0.59)	0.07 (1.22)
HVOL	-0.21 (-1.93)	-0.33 (-3.07)	-0.31 (-2.93)	-0.50 (-5.12)	-0.25 (-2.37)	-0.47 (-4.76)

**Table 5: Small Firms and Raw Returns of High and Low Idiosyncratic Volatility Portfolios**

Each month between January 1963 and December 2006, 24 ( $j=1, \dots, 24$ ) cross-sectional regressions of the following form are estimated:

$$R_{it} = b_{0j} + b_{1j}SMALL_{i,t-j} + b_{2j}LVOL_{i,t-j} + b_{3j}HVOL_{i,t-j} + b_{4j}SMALL_{i,t-j} * LVOL_{i,t-j} + b_{5j}SMALL_{i,t-j} * HVOL_{i,t-j} + e_{ijt}$$

where  $R_{it}$  is the return to stock  $i$  in month  $t$ ,  $LVOL_{i,t-j}$  ( $HVOL_{i,t-j}$ ) is the low (high) idiosyncratic volatility dummy that takes the value of 1 if the idiosyncratic volatility for stock  $i$  is ranked in the top (bottom) 20% in month  $t-j$ , and zero otherwise.  $SMALL_{i,t-j}$  is a dummy that takes the value of 1 if firm  $i$ 's market capitalization is below the median of the sample in month  $t-j$ , and is zero otherwise. The coefficient estimates of a given independent variable are for  $j=1$  for columns labeled (p=0,K=1), and averaged over  $j=2$  to 12 for columns labeled (p=1,K=11), and  $j=13$  to 24 for columns labeled (p=12,K=12). The numbers reported in the table are the time-series averages of these averages. They are in percent per month. The accompanying  $t$ -statistics are calculated from the time series. Penny socks (price < \$5) are excluded.  $NOBS$  is the average number of stocks used in the monthly cross-sectional regressions.

Raw Returns, IVOL20D (NOBS=3990)						
	Column 1 (p=0,K=1)	Column 2 (p=0,K=1) Jan. excluded	Column 3 (p=1,K=11)	Column 4 (p=1,K=11) Jan. excluded	Column 5 (p=12,K=12)	Column 6 (p=12,K=12) Jan. excluded
Intercept	1.22 (5.03)	1.00 (4.05)	1.08 (4.53)	0.86 (3.54)	1.02 (4.13)	0.75 (3.03)
SMALL	0.31 (3.19)	0.09 (0.93)	0.22 (2.32)	-0.02 (-0.18)	0.18 (1.91)	-0.05 (-0.58)
LVOL	-0.12 (-0.98)	-0.04 (-0.28)	-0.01 (-0.12)	0.09 (0.73)	-0.05 (-0.43)	0.07 (0.61)
HVOL	-1.34 (-7.20)	-1.41 (-7.16)	-0.58 (-3.42)	-0.82 (-4.86)	-0.23 (-1.43)	-0.55 (-3.64)
SMALL*LVOL	-0.27 (-3.96)	-0.16 (-2.31)	-0.09 (-1.62)	0.00 (0.07)	-0.01 (-0.20)	0.09 (1.55)
SMALL*HVOL	0.00 (-0.03)	-0.04 (-0.31)	0.20 (2.26)	0.24 (2.53)	0.19 (2.30)	0.26 (3.05)
Raw Returns, IVOL200D (NOBS=3534)						
Intercept	1.17 (4.62)	0.95 (3.67)	1.13 (4.50)	0.89 (3.50)	1.05 (4.05)	0.77 (2.98)
SMALL	0.25 (2.49)	0.03 (0.36)	0.22 (2.30)	0.01 (0.11)	0.20 (2.13)	-0.01 (-0.14)
LVOL	-0.06 (-0.40)	0.06 (0.38)	-0.07 (-0.53)	0.06 (0.43)	-0.06 (-0.48)	0.08 (0.60)
HVOL	-0.82 (-3.29)	-1.00 (-3.78)	-0.57 (-2.62)	-0.86 (-3.95)	-0.45 (-2.12)	-0.83 (-4.08)
SMALL*LVOL	-0.16 (-2.00)	-0.05 (-0.68)	-0.05 (-0.76)	0.04 (0.51)	-0.07 (-0.94)	0.02 (0.28)
SMALL*HVOL	0.24 (1.46)	0.25 (1.49)	0.33 (2.73)	0.34 (2.67)	0.26 (2.13)	0.30 (2.34)

**Table 5 (Continued)**

Raw Returns, IVOL60M (NOBS=2420)						
Intercept	1.26 (5.53)	1.06 (4.58)	1.23 (5.47)	1.03 (4.52)	1.14 (5.01)	0.91 (3.98)
SMALL	0.24 (2.57)	0.01 (0.12)	0.24 (2.60)	0.01 (0.16)	0.20 (2.26)	-0.02 (-0.21)
LVOL	-0.13 (-1.11)	-0.05 (-0.42)	-0.14 (-1.26)	-0.05 (-0.46)	-0.14 (-1.23)	-0.03 (-0.27)
HVOL	-0.28 (-1.43)	-0.42 (-2.05)	-0.43 (-2.41)	-0.62 (-3.40)	-0.35 (-2.07)	-0.63 (-3.79)
SMALL*LVOL	-0.03 (-0.38)	0.04 (0.53)	-0.07 (-1.01)	0.00 (-0.06)	-0.05 (-0.71)	0.03 (0.37)
SMALL*HVOL	-0.02 (-0.20)	-0.02 (-0.20)	0.15 (1.50)	0.11 (1.05)	0.14 (1.28)	0.17 (1.45)

**Table 6: Raw and Risk Adjusted Returns of High and Low Turnover Volatility Portfolios**

Each month between January 1963 and December 2006, 24 ( $j=1, \dots, 24$ ) cross-sectional regressions of the following form are estimated:

$$R_{it} = b_{0t} + b_{1jt} LVOL_{i,t-j} + b_{2jt} HVOL_{i,t-j} + e_{ijt}$$

where  $R_{it}$  is the return to stock  $i$  in month  $t$ ,  $LVOL_{i,t-j}$  ( $HVOL_{i,t-j}$ ) is the low (high) turnover volatility dummy that takes the value of 1 if the volatility of share turnover for stock  $i$  is ranked in the top (bottom) 20% in month  $t-j$ , and zero otherwise. Turnover volatility is measured as the standard deviation of the share turnover using data for the past 36 months, ending in month  $t-2$ . The coefficient estimates of a given independent variable are for  $j=1$  for columns labeled (p=0,K=1), and averaged over  $j=2$  to 12 for columns labeled (p=1,K=11), and  $j=13$  to 24 for columns labeled (p=12,K=12). To obtain risk-adjusted returns, we further run times-series regressions of these averages (one for each average) on the contemporaneous Fama-French factor realizations to hedge out the factor exposure. The numbers reported for risk-adjusted returns are intercepts from these time-series regressions. They are in percent per month and their  $t$ -statistics are in parentheses. Penny socks (price < \$5) are excluded. *NOBS* is the average number of stocks used in the monthly cross-sectional regressions.

Raw Return, STURN (NOBS=3084)						
	Column 1 (p=0,K=1)	Column 2 (p=0,K=1) Jan. excluded	Column 3 (p=1,K=11)	Column 4 (p=1,K=11) Jan. excluded	Column 5 (p=12,K=12)	Column 6 (p=12,K=12) Jan. excluded
Intercept	1.24 (4.79)	1.00 (3.80)	1.17 (4.37)	0.88 (3.27)	1.18 (4.41)	0.83 (3.14)
LVOL	0.10 (0.80)	0.19 (1.48)	0.14 (1.10)	0.26 (2.08)	0.04 (0.33)	0.21 (1.73)
HVOL	-0.46 (-2.99)	-0.58 (-3.58)	-0.44 (-2.83)	-0.63 (-4.22)	-0.23 (-1.61)	-0.48 (-3.59)
Risk Adjusted Return STURN (NOBS=3084)						
Intercept	-0.01 (-0.23)	-0.03 (-0.63)	-0.07 (-1.21)	-0.04 (-0.74)	-0.05 (-0.84)	-0.01 (-0.23)
LVOL	0.25 (4.01)	0.29 (4.60)	0.28 (4.30)	0.31 (4.85)	0.19 (2.90)	0.25 (4.01)
HVOL	-0.45 (-5.09)	-0.54 (-5.98)	-0.41 (-4.14)	-0.50 (-5.72)	-0.27 (-2.74)	-0.45 (-5.09)



**Table 7: Analyst Coverage and Risk-Adjusted Returns of High and Low Idiosyncratic Volatility Portfolios**

Each month between January 1983 and December 2006, 24 ( $j=1, \dots, 24$ ) cross-sectional regressions of the following form are estimated:

$$R_{it} = b_{0jt} + b_{1jt}LVOL_{i,t-j} + b_{2jt}HVOL_{i,t-j} + b_{3jt}LCOV_{i,t-j} * LVOL_{i,t-j} + b_{4jt}LCOV_{i,t-j} * HVOL_{i,t-j} \\ + b_{5jt}BM_{i,t-1} + b_{6jt}Size_{i,t-1} + b_{7jt}Ret_{i,t-1} + b_{8jt}52WKHW_{i,t-j} + b_{9jt}52WKHL_{i,t-j} + e_{ijt}$$

where  $R_{it}$  is the return to stock  $i$  in month  $t$ ,  $LVOL_{i,t-j}$  ( $HVOL_{i,t-j}$ ) is the low (high) idiosyncratic volatility dummy that takes the value of 1 if the idiosyncratic volatility for stock  $i$  is ranked in the top (bottom) 20% in month  $t-j$ , and zero otherwise.  $LCOV_{i,t-j}$  is a dummy that takes the value of 1 if the number of analyst coverage for stock  $i$  is three or less in month  $t-j$ , and is zero otherwise. The control variables are defined in the text, and their coefficients are omitted to save space. The coefficient estimates of a given independent variable are for  $j=1$  for columns labeled ( $p=0, K=1$ ), and averaged over  $j=2$  to 12 for columns labeled ( $p=1, K=11$ ), and  $j=13$  to 24 for columns labeled ( $p=12, K=12$ ). To obtain risk-adjusted returns, we further run times-series regressions of these averages (one for each average) on the contemporaneous Fama-French factor realizations to hedge out the factor exposure. The numbers reported for risk-adjusted returns are intercepts from these time-series regressions. They are in percent per month and their  $t$ -statistics are in parentheses. Penny socks (price < \$5) are excluded. *NOBS* is the average number of stocks used in the monthly cross-sectional regressions.

Risk Adjusted Return, IVOL20D (NOBS=3549)								
	Column 1 (p=0,K=1)	Column 2 (p=0,K=1) Jan. excluded	Column 3 (p=1,K=1)	Column 4 (p=1,K=1) Jan. excluded	Column 5 (p=1,K=11)	Column 6 (p=1,K=11) Jan. excluded	Column 7 (p=12,K=12)	Column 8 (p=12,K=12) Jan. excluded
Intercept	-0.11 (-1.43)	-0.16 (-2.15)	-0.02 (-0.25)	-0.08 (-1.04)	-0.06 (-0.75)	-0.13 (-1.75)	-0.02 (-0.28)	-0.11 (-1.59)
LVOL	0.11 (1.67)	0.12 (1.76)	0.10 (1.52)	0.13 (2.00)	0.04 (0.61)	0.04 (0.65)	-0.03 (-0.48)	-0.05 (-0.82)
HVOL	-1.09 (-5.88)	-0.92 (-4.88)	-0.30 (-1.59)	-0.35 (-1.81)	0.01 (0.06)	-0.02 (-0.17)	0.22 (1.91)	0.16 (1.36)
LCOV*LVOL	-0.04 (-0.71)	-0.01 (-0.09)	-0.01 (-0.20)	0.01 (0.15)	0.09 (1.69)	0.14 (2.67)	0.17 (2.58)	0.25 (4.07)
LCOV*HVOL	0.37* (1.81)	0.18 (0.88)	-0.21 (-1.01)	-0.22 (-1.05)	-0.21 (-1.60)	-0.31 (-2.30)	-0.25 (-1.84)	-0.33 (-2.33)
Risk Adjusted Return , IVOL200D (NOBS=3525)								
Intercept	-0.15 (-1.95)	-0.21 (-2.63)	-0.04 (-0.50)	-0.10 (-1.29)	-0.05 (-0.70)	-0.12 (-1.63)	-0.02 (-0.27)	-0.10 (-1.46)
LVOL	0.16 (2.02)	0.19 (2.46)	0.08 (1.05)	0.12 (1.50)	0.03 (0.39)	0.03 (0.34)	0.00 (0.01)	-0.03 (-0.46)
HVOL	0.02 (0.04)	0.14 (0.34)	0.38 (0.87)	0.40 (0.85)	0.34 (1.23)	0.25 (0.89)	0.66 (2.30)	0.65 (2.19)
LCOV*LVOL	-0.02 (-0.29)	0.00 (-0.07)	0.02 (0.28)	0.03 (0.56)	0.10 (1.64)	0.14 (2.45)	0.16 (2.42)	0.24 (3.72)
LCOV*HVOL	-0.13 (-0.33)	-0.35 (-0.85)	-0.34 (-0.81)	-0.50 (-1.10)	-0.47 (-1.79)	-0.58 (-2.17)	-0.91 (-3.25)	-1.12 (-3.88)

**Table 7 (cont.)**

	Risk Adjusted Return IVOL60M (NOBS=2596)							
	Column 1 (p=0,K=1)	Column 2 (p=0,K=1) Jan. excluded	Column 3 (p=1,K=1)	Column 4 (p=1,K=1) Jan. excluded	Column 5 (p=1,K=11)	Column 6 (p=1,K=11) Jan. excluded	Column 7 (p=12,K=12)	Column 8 (p=12,K=12) Jan. excluded
Intercept	-0.09 (-1.05)	-0.11 (-1.37)	0.02 (0.30)	0.00 (-0.02)	0.02 (0.30)	-0.01 (-0.10)	0.05 (0.72)	0.02 (0.21)
LVOL	0.16 (1.98)	0.19 (2.31)	0.11 (1.33)	0.15 (1.75)	0.04 (0.58)	0.06 (0.70)	-0.02 (-0.25)	-0.04 (-0.44)
HVOL	0.21 (0.89)	0.26 (1.09)	0.07 (0.29)	0.06 (0.26)	-0.08 (-0.39)	-0.06 (-0.31)	0.11 (0.53)	0.08 (0.37)
LCOV*LVOL	0.09 (1.48)	0.11 (1.68)	0.07 (1.10)	0.08 (1.21)	0.12 (2.06)	0.15 (2.39)	0.16 (2.41)	0.21 (3.20)
LCOV*HVOL	-0.26 (-1.01)	-0.42 (-1.62)	-0.36 (-1.53)	-0.48 (-1.99)	-0.30 (-1.53)	-0.47 (-2.33)	-0.50 (-2.42)	-0.64 (-3.01)
	Risk Adjusted Return STURN (NOBS=3089)							
Intercept	-0.11 (-1.29)	-0.15 (-1.81)	-0.01 (-0.12)	-0.06 (-0.75)	-0.04 (-0.46)	-0.10 (-1.28)	0.02 (0.29)	-0.05 (-0.68)
LVOL	0.12 (1.35)	0.16 (1.87)	0.02 (0.22)	0.06 (0.67)	0.04 (0.47)	0.06 (0.76)	0.08 (1.04)	0.08 (0.96)
HVOL	-0.11 (-1.02)	-0.13 (-1.23)	-0.01 (-0.13)	-0.07 (-0.66)	-0.09 (-0.86)	-0.12 (-1.21)	-0.04 (-0.33)	-0.13 (-1.07)
LCOV*LVOL	0.15 (1.85)	0.16 (2.05)	0.28 (3.88)	0.32 (4.35)	0.29 (3.87)	0.34 (4.77)	0.18 (1.97)	0.28 (3.29)
LCOV*HVOL	-0.32 (-2.26)	-0.40 (-2.73)	-0.42 (-3.06)	-0.46 (-3.20)	-0.39 (-3.09)	-0.51 (-4.10)	-0.37 (-3.16)	-0.44 (-3.65)

**Table 8: Past Returns and High and Low Volatility Portfolios**

Each month between January 1963 (1983 for the high coverage sample) and December 2006, 24 ( $j=1, \dots, 24$ ) cross-sectional regressions of the following form are estimated:

$$R_{it} = b_{0jt} + b_{1jt} \text{Low } 3Y \text{ Ret}_{i,t-j} + b_{2jt} \text{High } 3Y \text{ Ret}_{i,t-j} + b_{3jt} \text{Low } 3Y \text{ Ret}_{i,t-j} * HVOL_{i,t-j} + b_{4jt} \text{High } 3Y \text{ Ret}_{i,t-j} * HVOL_{i,t-j} + b_{5jt} LVOL_{i,t-j} + b_{6jt} HVOL_{i,t-j} + b_{7jt} BM_{i,t-1} + b_{8jt} Size_{i,t-1} + b_{9jt} Ret_{i,t-1} + b_{10jt} 52WKHW_{i,t-j} + b_{11jt} 52WKHL_{i,t-j} + e_{ijt}$$

where  $R_{it}$  is the return to stock  $i$  in month  $t$ ,  $LVOL_{i,t-j}$  ( $HVOL_{i,t-j}$ ) is the low (high) idiosyncratic volatility dummy that takes the value of 1 if the idiosyncratic volatility for stock  $i$  is ranked in the top (bottom) 20% in month  $t-j$ , and zero otherwise.  $High\ 3Y\ Ret_{i,t-j}$  ( $Low\ 3Y\ Ret_{i,t-j}$ ) is a dummy that takes the value of 1 if the past three year return for stock  $i$  is ranked in the top (bottom) 30% in month  $t-j$ , and is zero otherwise. The control variables are defined in the text, and their coefficients are omitted to save space. The coefficient estimates of a given independent variable are averaged over  $j=2$  to 12 for columns labeled (p=1,K=11), and  $j=13$  to 24 for columns labeled (p=12,K=12). To obtain risk-adjusted returns, we further run times-series regressions of these averages (one for each average) on the contemporaneous Fama-French factor realizations to hedge out the factor exposure. The numbers reported for risk-adjusted returns are intercepts from these time-series regressions. They are in percent per month and their  $t$ -statistics are in parentheses. Penny socks (price < \$5) are excluded. *NOBS* is the average number of stocks used in the monthly cross-sectional regressions.

**Panel A: 20-Day Idiosyncratic Return Volatility Measure**

	Risk Adjusted Returns IVOL20D All Firms (NOBS=2330)				Risk Adjusted Returns IVOL20D High Coverage Sample (NOBS=1174)			
	Column 1 (p=1,K=11)	Column 2 (p=1,K=11) Jan. excluded	Column 3 (p=12,K=12)	Column 4 (p=12,K=12) Jan. excluded	Column 1 (p=1,K=11)	Column 2 (p=1,K=11) Jan. excluded	Column 3 (p=12,K=12)	Column 4 (p=12,K=12) Jan. excluded
Intercept	0.05 (1.04)	0.04 (0.69)	0.08 (1.60)	0.05 (1.12)	-0.02 (-0.18)	0.02 (0.20)	0.02 (0.25)	0.06 (0.70)
Low 3 yr Ret	-0.09 (-1.26)	-0.21 (-2.97)	-0.04 (-0.61)	-0.12 (-1.81)	-0.07 (-0.48)	-0.24 (-1.75)	-0.17 (-1.18)	-0.30 (-2.12)
High 3 yr Ret	0.00 (0.05)	0.02 (0.43)	-0.17 (-3.44)	-0.17 (-3.32)	-0.03 (-0.40)	-0.01 (-0.13)	-0.10 (-1.27)	-0.13 (-1.54)
Low 3yr Ret*HVOL	0.03 (0.29)	0.04 (0.42)	-0.06 (-0.58)	-0.07 (-0.59)	0.32 (1.42)	0.23 (0.99)	-0.19 (-0.69)	-0.11 (-0.40)
High 3yr Ret*HVOL	-0.26 (-2.97)	-0.23 (-2.63)	-0.29 (-3.24)	-0.34 (-3.68)	-0.02 (-0.11)	0.09 (0.47)	-0.25 (-1.13)	-0.22 (-0.96)
LVOL	-0.01 (-0.18)	0.01 (0.28)	-0.02 (-0.50)	0.00 (0.04)	0.07 (1.24)	0.11 (1.86)	0.03 (0.57)	0.08 (1.40)
HVOL	-0.15 (-1.97)	-0.24 (-3.04)	0.08 (1.10)	0.02 (0.29)	-0.03 (-0.26)	-0.08 (-0.68)	0.23 (1.55)	0.13 (0.87)

Panel B: 200-Day Idiosyncratic Return Volatility Measure

	Risk Adjusted Returns IVOL200D All Firms (NOBS=2340)				Risk Adjusted Returns IVOL200D High Coverage Sample (NOBS=1305)			
	Column 1 (p=1,K=11)	Column 2 (p=1,K=11) Jan. excluded	Column 3 (p=12,K=12)	Column 4 (p=12,K=12) Jan. excluded	Column 1 (p=1,K=11)	Column 2 (p=1,K=11) Jan. excluded	Column 3 (p=12,K=12)	Column 4 (p=12,K=12) Jan. excluded
Intercept	0.06 (1.21)	0.04 (0.82)	0.09* (1.85)	0.07 (1.45)	-0.08 (-0.68)	-0.04 (-0.30)	0.01 (0.11)	0.06 (0.50)
Low 3 yr Ret	-0.12 (-1.66)	-0.23 (-3.31)	-0.05 (-0.71)	-0.13 (-1.94)	-0.14 (-0.86)	-0.31 (-1.94)	-0.31 (-1.86)	-0.43 (-2.56)
High 3 yr Ret	0.01 (0.25)	0.04 (0.77)	-0.18 (-3.64)	-0.18 (-3.48)	0.03 (0.28)	0.05 (0.47)	-0.14 (-1.47)	-0.19 (-1.84)
Low 3yr Ret*HVOL	0.15 (0.94)	0.11 (0.69)	-0.28 (-1.53)	-0.27 (-1.44)	0.72 (0.87)	0.58 (0.74)	-0.29 (-0.30)	-0.29 (-0.28)
High 3yr Ret*HVOL	-0.49 (-3.37)	-0.57 (-3.92)	-0.37 (-2.57)	-0.42 (-2.87)	-1.35 (-1.83)	-1.13 (-1.60)	0.42 (0.42)	0.48 (0.46)
LVOL	-0.02 (-0.40)	-0.01 (-0.13)	-0.04 (-0.69)	-0.03 (-0.61)	0.14 (1.45)	0.18 (1.69)	0.03 (0.28)	0.08 (0.83)
HVOL	-0.06 (-0.43)	-0.17 (-1.20)	-0.08 (-0.55)	-0.21 (-1.46)	0.57 (1.11)	0.51 (1.13)	0.14 (0.18)	0.12 (0.15)

Panel C: 60-Month Idiosyncratic Return Volatility Measure

	Risk Adjusted Returns IVOL60M All Firms (NOBS=1989)				Risk Adjusted Returns IVOL60M High Coverage Sample (NOBS=1023)			
	Column 1 (p=1,K=11)	Column 2 (p=1,K=11) Jan. excluded	Column 3 (p=12,K=12)	Column 4 (p=12,K=12) Jan. excluded	Column 1 (p=1,K=11)	Column 2 (p=1,K=11) Jan. excluded	Column 3 (p=12,K=12)	Column 4 (p=12,K=12) Jan. excluded
Intercept	0.04 (0.71)	0.04 (0.66)	0.08 (1.50)	0.07 (1.38)	-0.02 (-0.24)	0.01 (0.10)	0.00 (0.03)	0.05 (0.50)
Low 3 yr Ret	-0.10 (-1.41)	-0.23 (-3.16)	-0.06 (-0.89)	-0.14 (-2.10)	-0.06 (-0.39)	-0.24 (-1.56)	-0.14 (-0.83)	-0.26 (-1.65)
High 3 yr Ret	0.07 (1.30)	0.10 (1.84)	-0.10 (-2.16)	-0.10 (-1.94)	0.02 (0.25)	0.04 (0.48)	-0.06 (-0.80)	-0.09 (-1.11)
Low 3yr Ret*HVOL	-0.02 (-0.13)	0.03 (0.23)	0.06 (0.38)	0.14 (0.90)	0.12 (0.25)	0.50 (1.06)	0.55 (1.01)	0.59 (1.08)
High 3yr Ret*HVOL	-0.39 (-4.09)	-0.36 (-3.70)	-0.33 (-3.46)	-0.32 (-3.31)	-0.24 (-0.62)	-0.01 (-0.03)	0.31 (0.87)	0.22 (0.62)
LVOL	0.00 (0.02)	0.00 (0.02)	-0.04 (-0.69)	-0.04 (-0.79)	0.05 (0.56)	0.10 (1.13)	0.04 (0.45)	0.08 (-0.93)
HVOL	-0.05 (-0.48)	-0.18 (-1.65)	-0.07 (-0.69)	-0.20 (-1.85)	0.04 (0.14)	-0.13 (-0.44)	-0.06 (-0.18)	-0.03 (-0.09)

Panel D: Volatility of Share Turnover

	Risk Adjusted Returns STURN All Firms (NOBS=2030)				Risk Adjusted Returns STURN High Coverage Sample (NOBS=1068)			
	Column 1 (p=1,K=11)	Column 2 (p=1,K=11) Jan. excluded	Column 3 (p=12,K=12)	Column 4 (p=12,K=12) Jan. excluded	Column 1 (p=1,K=11)	Column 2 (p=1,K=11) Jan. excluded	Column 3 (p=12,K=12)	Column 4 (p=12,K=12) Jan. excluded
Intercept	0.02 (0.39)	0.01 (0.18)	0.07 (1.28)	0.05 (0.99)	0.02 (0.24)	0.08 (0.81)	0.01 (0.13)	0.07 (0.76)
Low 3 yr Ret	-0.08 (-0.92)	-0.22 (-2.62)	-0.02 (-0.23)	-0.10 (-1.26)	0.04 (0.26)	-0.08 (-0.45)	-0.11 (-0.65)	-0.17 (-1.01)
High 3 yr Ret	0.08 (1.55)	0.11 (2.11)	-0.13 (-2.81)	-0.13 (-2.71)	0.01 (0.13)	0.03 (0.37)	-0.10 (-1.36)	-0.12 (-1.45)
Low 3yr Ret*HVOL	0.05 (0.49)	0.09 (0.80)	0.00 (-0.04)	0.02 (0.15)	0.04 (0.15)	-0.05 (-0.21)	0.11 (0.47)	0.01 (0.04)
High 3yr Ret*HVOL	-0.34 (-4.49)	-0.28 (-3.60)	-0.16 (-2.21)	-0.14 (-1.89)	-0.27 (-2.08)	-0.16 (-1.25)	-0.08 (-0.66)	-0.11 (-0.91)
LVOL	0.13 (2.41)	0.15 (2.90)	0.08 (1.49)	0.11 (2.12)	0.04 (0.53)	0.07 (0.84)	0.10 (1.24)	0.15 (1.73)
HVOL	-0.13 (-1.67)	-0.23 (-3.06)	-0.12 (-1.55)	-0.23 (-3.04)	0.05 (0.45)	-0.08 (-0.66)	0.06 (0.54)	-0.04 (-0.35)

**Table 9: Earnings Announcement Returns for Portfolios Sorted on Volatility and Analyst Coverage**

Every June from 1983 to 2006, we sort firms independently into two groups by analyst coverage (three or less analysts is low coverage, greater than three is high coverage) and three groups by idiosyncratic volatility (top 20%, middle 60% and bottom 20%), and form portfolios based on these groupings. For each firm, we then compute the average abnormal return over the four quarterly announcement returns following portfolio formation and annualize this number by multiplying by four. Following La Porta et al (1997), we benchmark each earnings announcement return by the firm with median book-to-market in the same size decile as the announcer. The numbers in the table are the equally weighted average annualized earnings announcement abnormal (net of benchmark) returns in percent. The column labeled H-L is the difference between the returns to high and low leverage groups, and p-values relate to a test of the null hypothesis that the difference between the mean abnormal returns of high and low leverage groups is zero. Penny socks (price < \$5) are excluded.

Cumulative Abnormal Returns						Number of Stocks			
<b>IVOL20D</b>						<b>IVOL20D</b>			
Coverage	L	M	H	H-L	p-value	Coverage	L	M	H
L	0.34	-0.05	-0.87	-1.21	0.00	L	264	855	380
p-value	0.12	0.67	0.02			H	273	811	159
H	-0.14	0.51	-0.87	-0.73	0.18	<b>IVOL200D</b>			
p-value	0.43	0.00	0.10			Coverage	L	M	H
<b>IVOL200D</b>						<b>IVOL200D</b>			
Coverage	L	M	H	H-L	p-value	Coverage	L	M	H
L	0.38	0.03	-1.03	-1.41	0.00	L	259	855	385
p-value	0.09	0.84	0.00			H	296	799	147
H	0.04	0.47	-0.60	0.19	0.38	<b>IVOL60M</b>			
p-value	0.87	0.01	0.39			Coverage	L	M	H
<b>IVOL60M</b>						<b>IVOL60M</b>			
Coverage	L	M	H	H-L	p-value	Coverage	L	M	H
L	0.05	0.51	-1.22	-1.27	0.00	L	189	649	265
p-value	0.85	0.00	0.00			H	228	593	143
H	-0.26	0.49	-0.16	0.72	0.81	<b>STURN</b>			
p-value	0.21	0.00	0.66			Coverage	L	M	H
<b>STURN</b>						<b>STURN</b>			
Coverage	L	M	H	H-L	p-value	Coverage	L	M	H
L	0.85	0.13	-2.27	-3.12	0.00	L	333	737	213
p-value	0.00	0.29	0.00			H	139	765	283
H	-0.05	0.45	-0.47	-0.42	0.26				
p-value	0.85	0.00	0.13						

**Table 10: Persistent Low Coverage and High and Low Volatility Portfolios**

Each month between January 1983 and December 2006, 24 ( $j=1, \dots, 24$ ) cross-sectional regressions of the following form are estimated:

$$R_{it} = b_{0jt} + b_{1jt}LVOL_{i,t-j} + b_{2jt}HVOL_{i,t-j} + b_{3jt}LCOV_{i,t-j} * LVOL_{i,t-j} + b_{4jt}LCOV_{i,t-j} * HVOL_{i,t-j} + b_{5jt}PLCOV_{i,t-j} * LCOV_{i,t-j} * HVOL_{i,t-j} + b_{6jt}BM_{i,t-1} + b_{7jt}Size_{i,t-1} + b_{8jt}Ret_{i,t-1} + b_{9jt}52WKHW_{i,t-j} + b_{10jt}52WKHL_{i,t-j} + e_{ijt}$$

where  $R_{it}$  is the return to stock  $i$  in month  $t$ .  $LVOL_{i,t-j}$  ( $HVOL_{i,t-j}$ ) is the low (high) idiosyncratic volatility dummy that takes the value of 1 if the idiosyncratic volatility for stock  $i$  is ranked in the top (bottom) 20% in month  $t-j$ , and zero otherwise.  $LCOV_{i,t-j}$  is a dummy that takes the value of 1 if the number of analysts covering stock  $i$  is three or less in month  $t-j$ .  $PLCOV_{i,t-j}$  is a dummy that takes the value of 1 if the number of analysts covering for stock  $i$  is 3 or less in month  $t-j-36$ . The coefficient estimates of a given independent variable are for  $j=1$  for columns labeled (p=0,K=1), and averaged over  $j=2$  to 12 for columns labeled (p=1,K=11), and  $j=13$  to 24 for columns labeled (p=12,K=12). To obtain risk-adjusted returns, we further run times-series regressions of these averages (one for each average) on the contemporaneous Fama-French factor realizations to hedge out the factor exposure. The numbers reported for risk-adjusted returns are intercepts from these time-series regressions. They are in percent per month and their  $t$ -statistics are in parentheses. Penny stocks (price < \$5) are excluded.  $NOBS$  is the average number of stocks used in the monthly cross-sectional regressions.

Risk Adjusted Returns, IVOL20D (NOBS=3134)								
	Column 1 (p=0,K=1)	Column 2 (p=0,K=1) Jan. excluded	Column 3 (p=1,K=1)	Column 4 (p=1,K=1) Jan. excluded	Column 5 (p=1,K=11)	Column 6 (p=1,K=11) Jan. excluded	Column 7 (p=12,K=12)	Column 8 (p=12,K=12) Jan. excluded
LVOL	0.09 (1.26)	0.11 (1.57)	0.06 (0.85)	0.11 (1.58)	-0.01 (-0.14)	0.02 (0.26)	-0.07 (-1.26)	-0.07 (-1.11)
HVOL	-0.89 (-4.04)	-0.72 (-3.16)	-0.08 (-0.32)	-0.11 (-0.44)	0.06 (0.52)	0.06 (0.50)	0.18 (1.40)	0.15 (1.11)
LCOV*LVOL	-0.07 (-1.08)	-0.06 (-0.87)	-0.05 (-0.79)	-0.06 (-0.91)	0.06 (1.03)	0.08 (1.43)	0.13 (2.00)	0.18 (2.87)
LCOV*HVOL	0.44 (1.01)	0.30 (0.68)	-0.20 (-0.36)	-0.24 (-0.42)	0.16 (0.73)	0.08 (0.39)	-0.17 (-0.87)	-0.25 (-1.19)
PLCOV*LCOV*HVOL	-0.31 (-0.79)	-0.34 (-0.83)	-0.07 (-0.13)	-0.06 (-0.10)	-0.44 (-2.27)	-0.50 (-2.49)	0.02 (0.09)	0.00 (0.01)
Risk Adjusted Returns, IVOL200D (NOBS=3144)								
LVOL	0.07 (0.90)	0.13 (1.60)	0.00 (-0.03)	0.06 (0.75)	-0.05 (-0.69)	-0.03 (-0.35)	-0.07 (-0.96)	-0.07 (-0.92)
HVOL	-0.06 (-0.11)	0.19 (0.33)	0.18 (0.33)	0.31 (0.53)	0.24 (0.70)	0.28 (0.76)	0.60 (1.63)	0.52 (1.41)
LCOV*LVOL	-0.03 (-0.52)	-0.05 (-0.66)	0.01 (0.13)	-0.01 (-0.18)	0.07 (1.23)	0.09 (1.45)	0.13 (2.00)	0.18 (2.68)
LCOV*HVOL	1.31 (1.61)	0.85 (1.00)	0.56 (0.71)	0.27 (0.33)	-0.17 (-0.34)	-0.41 (-0.76)	-0.70 (-1.27)	-0.82 (-1.42)
PLCOV*LCOV*HVOL	-1.34 (-2.11)	-1.22 (-1.86)	-0.62 (-1.08)	-0.60 (-1.01)	-0.21 (-0.57)	-0.24 (-0.63)	-0.17 (-0.39)	-0.22 (-0.48)



**Table 10 (Continued)**

Risk Adjusted Returns, IVOL60M (NOBS=2693)								
	Column 1 (p=0,K=1)	Column 2 (p=0,K=1) Jan. excluded	Column 3 (p=1,K=1)	Column 4 (p=1,K=1) Jan. excluded	Column 5 (p=1,K=11)	Column 6 (p=1,K=11) Jan. excluded	Column 7 (p=12,K=12)	Column 8 (p=12,K=12) Jan. excluded
LVOL	0.08 (0.94)	0.12 (1.42)	0.02 (0.24)	0.08 (0.89)	-0.04 (-0.54)	-0.02 (-0.22)	-0.07 (-0.87)	-0.07 (-0.80)
HVOL	0.26 (1.05)	0.28 (1.11)	0.13 (0.54)	0.11 (0.47)	-0.02 (-0.09)	-0.01 (-0.05)	0.25 (1.12)	0.19 (0.83)
LCOV*LVOL	0.08 (1.13)	0.09 (1.27)	0.06 (0.84)	0.06 (0.79)	0.10 (1.66)	0.12 (1.85)	0.12 (1.73)	0.17 (2.37)
LCOV*HVOL	1.52 (2.70)	1.21 (2.09)	0.52 (1.00)	0.37 (0.68)	0.47 (1.11)	0.29 (0.65)	0.09 (0.21)	-0.12 (-0.28)
PLCOV*LCOV*HVOL	-1.88 (-3.41)	-1.70 (-2.97)	-0.91 (-1.90)	-0.88 (-1.73)	-0.84 (-2.24)	-0.83 (-2.08)	-0.71 (-1.82)	-0.61 (-1.52)
Risk Adjusted Returns, STURN (NOBS=2708)								
LVOL	0.08 (0.96)	0.16 (1.71)	-0.03 (-0.33)	0.04 (0.40)	0.02 (0.23)	0.08 (0.89)	0.12 (1.43)	0.16 (1.84)
HVOL	-0.09 (-0.69)	-0.15 (-1.13)	0.05 (0.35)	-0.04 (-0.33)	-0.01 (-0.09)	-0.08 (-0.67)	0.01 (0.07)	-0.12 (-0.88)
LCOV*LVOL	0.11 (1.45)	0.10 (1.28)	0.25 (3.48)	0.25 (3.35)	0.23 (3.18)	0.25 (3.44)	0.11 (1.18)	0.16 (1.87)
LCOV*HVOL	0.43 (1.71)	0.50 (1.91)	0.08 (0.31)	0.10 (0.37)	0.02 (0.09)	-0.07 (-0.31)	0.18 (0.79)	0.12 (0.52)
PLCOV*LCOV*HVOL	-0.95 (-3.81)	-1.10 (-4.42)	-0.72 (-2.62)	-0.75 (-2.68)	-0.61 (-3.07)	-0.65 (-3.15)	-0.64 (-2.70)	-0.66 (-2.65)





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