Optimal Patenting and Licensing of Financial Innovations\textsuperscript{1}

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Abstract

Recent court decisions, starting with the State Street decision in 1998, allow business methods to be patentable and now give financial institutions the option to seek patent protection for financial innovations. This new patentability paradigm and the heterogeneity of characteristics associated with financial innovations, poses an immediate decision problem for senior management: what to patent. We present a parsimonious decision framework that answers this question. We show that for innovations with certain characteristics, it is optimal not to patent, even if the option of patenting and licensing is available. Our model emphasizes the role of embedded real options that arise from certain types of financial innovations. The model provides an explanation of observed patenting behavior of financial institutions and the success of a wide class of innovations, including swaps, credit derivatives, and pricing algorithms.

Keywords: Business Methods; Financial Innovations; Patents; Licenses; Real Options

JEL classification codes: G20, L10, O31
1 Introduction

In the last two decades there have been important changes in the administrative and legal environment regarding the patenting of financial innovations and business methods.\(^1\) Historically, financial innovations were generally not eligible for patent protection because of the business method exception to patentability. However, a number of recent court decisions, starting with the celebrated State Street decision, allow business methods to be patentable.\(^2\)

The new patentability paradigm, and the heterogeneity of innovation characteristics, poses an immediate decision problem for senior management of financial institutions: what should they patent? One solution is to patent all new innovations.\(^3\) However, this is unlikely to be always optimal, because prior to the State Street decision there were many successful financial innovations without the availability of patents (Miller (1986), Tufano (1989) and Finnerty and Emery (2002)). Despite a large literature on the many facets of patenting, there is little that offers management a useful framework to answer the basic question of what financial innovations should be patented.

In this paper, we develop a parsimonious framework that helps management identify characteristics of financial innovations that are critical for deciding whether to patent or not, and if the decision is to patent, whether to license the innovation. The parsimony of the model allows us to characterize, in a comprehensive manner, the optimal patenting and licensing policy for a financial institution, taking into account the response of imitators and adopters. Our contribution is to provide a decision framework for management that sets out the drivers of the decision to patent and license financial innovations, and apply this framework to explain observed patenting behavior with respect to a wide range of financial innovations — from the Black-Scholes option pricing formula to the development of the market for credit default swaps.

The benefits of patenting are well known. A patent gives its owner the legal right to exclude others from utilizing the invention covered by the

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\(^1\)First, in 1982, the Federal Court Improvement Act vested almost exclusive patent appellate jurisdiction in a new court – the U.S. Circuit Court of Appeals for the Federal Circuit (CAFC). Second, the running of the U.S. Patent and Trademark Office was changed from a tax revenue funded agency to a profit center that took place during the 1990s.


\(^3\)The increased use of patent protection by financial institutions is highlighted in a recent article “U.S. financial services groups rush to join patent stampede,” Financial Times, December 30, 2006.
patent, although a patent owner can generate revenue by licensing the use of patented inventions. Patents can also be used defensively, as they provide a form of détente in markets where rival firms hold patents and cross licensing agreements can be used to minimize litigation risk. The failure to patent an innovation exposes an institution to the flipside of the benefits associated with patents. Other institutions or individuals may prevent the institution from using a particular innovation; the magnitude of this risk has increased since the advent of patent trolls — firms whose sole assets are patents and who generate revenues via licensing and suing for patent infringement.

In determining whether to patent a financial innovation, an important characteristic is whether the innovation requires participation of other agents or financial institutions for success. Many forms of financial innovation provide a direct service to clients that are visible to competitors, but do not require a secondary market. For example, the innovation could provide clients with a service to improve the performance of their portfolio, or a method that facilitates dynamic portfolio benchmarking. Patent protection appears to be generally useful in these cases: if imitations occur, then the innovator has the option to seek remedy.

However, the situation is quite different if the innovation occurs through the introduction of a new form of security, for example. The risks of underwriting the instrument may be such that the institution wants to form a syndicate in order to spread the risk. Or the payoffs from the innovation are much larger if liquid secondary markets develop, but this requires education of end-users and even market makers; it is therefore in the interest of the innovator to advertise the product.

For example, in the developing credit derivatives market, investment banks produce detailed information about the different forms of derivatives, publish papers about the uses of the derivatives and pricing methodologies in trade journals, and give presentations to potential end-users. In each of these cases, immediate patenting may not be optimal, because of (a version of) the “hold-up” problem (Williamson (1985)). The patent gives the innovator the option to restrict trading in — or development of derivatives from — the basic security; given this, potential partners or market developers will be unwilling to invest capital specializing in the security. For similar reasons, patenting and then licensing (in some form) to third-parties is unlikely to be a solution.

Of course, without the patent the innovating institution has no legal recourse if there is imitation. The trade-off between the advantages of non-patenting with its risks therefore depend on the existence of non-legal imitation barriers. Here, the financial services industry exhibits some special
characteristics. More so than in almost any other industry, specialized hu-
man capital and allied organizational assets are central to effective absorp-
tion of certain financial innovations, such as the development of a new kind
of security or implementation of especially complex formulas (see, Scholes
(1998)). If the institution’s competitors do not have the same in-house ex-
pertise, then the institution can benefit from innovation, at least in the short
run.

By developing a framework that incorporates the considerations men-
tioned above, our paper makes four contributions. First, the model clarifies
that the patenting decision pivots on just a few critical parameters, namely,
the net increase in consumer surplus or benefit from future generations of
the basic innovation, the initial client base of the financial institution, and
the expected increase in this market size through risk-sharing and client
education by imitators. If the innovation has little prospects of further im-
provements in consumer surplus or benefit, through derivative innovations
or refinements and the initial market size is not too small, then patenting
is optimal. At the other extreme, innovations that have a large real options
component, i.e., substantial potential for a series of further innovations that
significantly enhance consumer value, but a low initial market size that can
expand through imitation, will not be patented.

Our analysis highlights the real options embedded in certain types of
financial innovations because of characteristics that are special to the fi-
nancial services industry. While financial innovations share some structural
features with innovations in other industries, where network externalities are
important (see, e.g., Economides (1996)) — such as the computer software
industry — financial innovations have certain unique characteristics that in-
fluence the patentability issue. For example, the innovating institution need
not be the exclusive vendor in order for the innovation to be profitable. 
Rather, the innovator can allow other financial institutions to offer various
versions of the innovation to share risk, increase market depth, liquidity
and price transparency, while using its human capital and expertise-related
advantages to profitably trade with high-value users.

The second contribution is to show that the predictions from the decision-
based model are consistent with observed patenting behavior in the financial
industry. We analyze various types of financial innovations and identify a
class of successful innovations, such as the swap and credit derivative mar-
kets, where the absence of patent protection and imitation were crucial to
their success. These innovations exhibit characteristics that are consistent
with the predictions of our model: larger financial institutions are more
likely to patent (e.g., Lerner (2002)) and non-financial firms file for more
patents per innovation compared to financial firms (Lerner (2006)). More generally, our model contributes to the literature that has developed to explain the considerable inter-industry heterogeneity in patenting policies that can not be explained by variations in R&D alone (see, Pakes and Griliches (1980)).

The third contribution is to the broader patenting and licensing literature. While much of the patenting literature has assumed that patents are always optimal, Horstmann et al. (1985) provide a model where patenting is not always optimal because it reveals the private information of the patent holder. However, in our model patenting can be sub-optimal even without asymmetric information, because the value of the embedded options in a sequence of innovations is amplified with an educated market that follows a non-protected regime. And while the optimal patent policy in software innovations is also complex due to the presence of externalities from imitation (e.g., Bessen and Maskin (2001) and Shelanski (2002)), the expertise-related first-mover advantages in the financial industry are manifestly unique, for the reasons described above. Similarly, our analysis of the effect of expertise-related constraints of licensees on the optimal licensing policy contribute to the literature on analyses of licensing of intangible property (e.g., Katz and Shapiro (1985, 1986a)). A recent literature theoretically and empirically analyzes the real-option aspects of innovations (e.g., Bloom and Van Reneen (2000) and Schwartz (2003)), but this literature does not apply its analysis to the specialized features of financial innovations that we emphasize in this paper.

The fourth contribution is that our analysis extends the first-mover advantage argument for financial innovations that has been advanced in the literature. Tufano (1989) shows that a financial institution introducing a new form of security typically retains a dominant market share for several years after the introduction, even though there is rapid imitation by rival institutions. This insight is central to the first mover argument of Herrera and Schroth (2003). Our framework incorporates the first mover advantage as a special case. While the institution may have a first mover advantage, the economic rents may not be sufficient to justify the innovation in the absence of a patent. However, patenting may not be optimal if the real benefits generated from subsequent innovations, depend on the size and state of the market. The sequential nature of certain types of financial innovations is an issue not addressed by Herrera and Schroth, yet it is often of great practical importance. The possible benefits to the institution are two fold: (a) the rent generated by the initial innovation and (b) the options for further innovation, as it learns more about the market for the initial innovation.
The remaining paper is organized as follows. In Section 2, we analyze characteristics of financial innovations to identify types of innovations where patenting appears optimal and those where it does not. In Section 3, we present the model and its analysis, and relate it to the literature. In section 4 we apply the model. Section 5 summarizes the results and concludes.

2 Innovation Characteristics and Optimality of Patenting

We briefly examine the types of financial innovations that have been patented over the period January 1971 to September 2005. We highlight those types of innovations where patenting appears optimal and those where it does not.

2.1 Patenting is Optimal

Following the approach described in Lerner (2001, 2002), we examine patents in five subclasses of classification 705.\(^4\) The patents in this classification usually describe some method or process that provides the financial institution with a comparative advantage in three broad categories: (a) undertaking some form of back office function; (b) facilitating a service that can be offered to clients or an improvement in the technology of an existing service; and (c) performing a particular task through an improved method. We now argue that patenting these type of innovations is optimal.

We note first that these type of innovations, especially those that fall in category (a) and (c), are not easy to observe from outside, and therefore it is difficult to identify them with the innovating institution. Furthermore, the diffusion of knowledge within the finance industry is rapid; for example, due to mobility of labor and knowledge-sharing in industry conferences and trade magazines. Without patent protection, competitors may quickly reverse engineer the service and offer a competing one.\(^5\) For innovations that fall into category (b), the institution is likely to advertise them because they

\(^4\)The subclasses are as follows. 705/35: finance (e.g. Banking, investment and credit); 705/36: portfolio selection, planning or analysis; 705/37: trading, matching or bidding; 705/38: credit risk processing, loan processing; 705/4: Insurance – calculation of annuity rates, investment of insurance company assets, the management of risk through financial instruments and related topics. See Kumar and Turnbull (2006) for a more detailed discussion.

\(^5\)If an innovation is protected by a trade secret and a third party reverse engineers its innovation, the innovation will not have protection if the third party was not bounded by the trade secret.
enhance the services offered to the client. Rival firms can then attempt to reverse engineer the innovation that facilitated the service. Therefore, we conclude that for the type of financial innovations just described, obtaining a patent may be optimal because it protects the innovator, allowing it to earn rent.

2.2 Patenting is Sub-Optimal

Here we consider the characteristics of innovations for which patent protection is unnecessary or undesirable. To be concrete, we consider the innovation to be some form of financial instrument that will appeal to a wide section of end-users — such as credit swaps. (We will give further examples of such innovations in Section 4.)

In the first round of transactions, the innovator learns how to appropriate price the instrument and how to hedge. In the process, the institution earns rents that reward it for its innovation. It also earns itself a reputation as a market leader. News of the innovation spreads among competitors and imitators start to offer similar products. Consequently, rents dissipate and the innovator ends up earning a fair rate of return. This is the type of first mover advantage argument described by Herrera and Schroth (2003) (HS).

However, for the types of financial innovation that we are considering, the situation is often far more complex than that described by HS. While the innovator may expect the potential market for end-users to be large, it typically has to grow the market. However, to expand the market, it is necessary to increase the liquidity and transparency of the market by educating the end-users about the nature of the innovation and explaining pricing methodologies and settlement procedures. For this to happen, it needs the participation of other market makers. Apart from increasing market depth, the increase in the number of market makers helps in the dissemination of the information about the product to potential end-users. Furthermore, to increase market liquidity, the innovating institution needs the standardization of contracts and the posting of consensus prices visible to end-users.

Increased use of the product by end-users generates a variety of benefits. It helps market makers to lower their costs of hedging its position by taking off-setting positions and generating revenue on the order flow. It also increases the likelihood of derivative innovations involving extensions of the basic product. Indeed, end-users may be the driving forces behind future innovations by suggesting extensions. However, rents from these derivative innovations generally accrue to the innovating market leaders.
‘Cream-skimming’ by the innovating institution in the derivative innovations market, after initially developing the liquidity and transparency of the market by inviting the participation of other institutions, is thus a unique aspect of an important class of financial innovations. Importantly, this aspect of financial innovations poses a dilemma for the innovating institution: whether to patent (and possibly license) the innovation or to forego patent protection. The reason is that obtaining a patent on the basic innovation essentially precludes the participation of other market makers that is crucial for the reasons elucidated above. In the next section, we present a model that analyzes this decision problem in industry equilibrium.

3 A Real Options Model of Financial Innovations

The model has three dates. At the start \((t = 0)\), the financial institution (denoted \(I\)) expends an amount \(c_0\) to develop a new form of a derivative. The derivative can be purchased from the institution and is effective for one-time period. That is, if investors wish to obtain recurring benefits from the derivative, they must re-purchase in every time period. The institution has a client base or initial market of \(m_0\) investors, each of whom obtain a basic value of \(v\) from using the derivative at each date, and this parameter is common knowledge. We assume that \(I\) is the high-quality provider and the leader in the market because of its proprietary intellectual capital and its pool of specialized human capital. Therefore, in addition to the derivative, \(I\) can provide additional services regarding the derivative that add value to the buyers: for example, from expertise in the structuring of the derivative; the resolution of legal and regulatory issues; providing ancillary technology to compute the required cash flows to different stakeholders, and the development of the necessary pricing and hedging methodologies. These additional services are valued at \(\alpha\). Thus, if the derivative premium is \(\phi\), then the net benefit to the investor is, \(\alpha + v - \phi\).

To develop the market beyond the initial client base, the financial institution needs to advertise the derivative to potential end users. But, in this process, imitators learn about the derivative and compete with the institution at \((t = 1)\). These imitators also advertise to their end users. We will assume that there are \(x\) imitators and each imitator has \(z\) end users, who obtain the basic value \(v\) per period from using the derivative. The imitators, however, are not in a position to provide any additional services to the end users. Hence, the net benefit to the customers of investors is, \(v - \phi\).

The provision of specialized services is costly; for example, for financial
instruments, the bulk of the delivery costs are specialized labor wage costs. Because such highly qualified labor is in short-supply, we assume that the unit costs are convex in the market size: thus, if the market size is \( m \), then the unit costs are, \( \frac{1}{2}c m^2 \), for some given parameter \( c > 0 \). The imitators at \( (t = 1) \), on the other hand, act competitively and face a common constant unit cost function, \( \tilde{c}m \), where \( \tilde{c} < v \).

### 3.1 The Patenting Decision

The innovating institution, \( I \), can forestall imitation by patenting the derivative valuation process. If the institution does patent the new derivative, then the market size for the derivative at date \( t = 1 \) is limited by the institution’s own advertising and client reach, namely \( m_0 \). We denote the firm’s decision, \( D \), to obtain a patent by \( D = P \). The decision not to obtain a patent is represented by \( D = N \).

The institution \( I \) expects that the innovation will lead to further innovations. It anticipates that in the process of communicating with end users and observing their value generation from the derivative, it will learn about (a) how to improve the market for the derivative and (b) the potential demand for new forms of innovations related to the derivative. Effectively, the institution has real options for further innovation.

The end user may view the future innovation either as a complement or a substitute. For example, if the initial innovation is a credit default swap and the future innovation is an option on the credit default swap, the end user might invest in both. Alternatively, if viewed as a substitute, the end user switches from investing in the credit default swap to investing in options on the swap. Another example of a substitute would be a collateralized debt obligation and the future innovation being a synthetic collateral debt obligation on a credit index, the later having more transparent pricing. In this paper, we treat the future innovation as a substitute, though the analysis readily extends to the case of a complement.

We model these real options by assuming that at date \( t = 1 \), \( I \) makes an investment \( c_2 \) in a future innovation that will materialize at date \( t = 2 \), with a probability \( s, 0 < s < 1 \). The new innovation increases the buyer valuation to \( \mu > v + \alpha \). Because \( I \) is the originator of this innovation, it has a monopoly over its delivery at date \( t = 2 \), taking as given the total market size for the initial innovation (or derivative) at the end of the previous period (date \( t = 1 \)). The unit cost function for this innovation for \( I \) is \( \frac{1}{2}c' m^2 \). Note that in general the cost of further innovation, \( m_2 \), will depend on the initial patenting decision. If the market has sufficiently developed in size...
and knowledge, then costs may be lower. The size of the market will tend to be larger in the absence of patenting.\footnote{Similarly, we would expect Bayesian updating to occur for the probability of a successful innovation, \( q \), given that at date \( t = 1 \), \( I \) can observe the state of market development. For the present, we ignore the Bayesian updating.}

Under certain situations, \( I \), as a monopolist, may wish to restrict the market it serves. In this case, the residual market can still buy the original derivative in the market place. However, we assume that by date \( t = 2 \), the market for the original derivative is competitive, and all producers face common constant unit costs of production and delivery of, \( c \). We therefore incorporate the idea, well documented by the empirical literature, that the original innovation eventually becomes a commodity over time as the expertise and specialized inputs required for its production and delivery become publicly known and freely available, respectively—see Tufano (1989). Indeed, we also assume that the opportunity to earn rents from this class of derivatives itself expires at the end of date \( t = 2 \), although we can easily allow a competitive market in the product class to remain over the horizon, without materially affecting our results.\footnote{More realistic touches, such as allowing the buyer value from using the original derivative to atrophy over time (because of possible obsolescence) can be easily incorporated at the cost of additional notation, but without materially affecting our results.}

Firms maximize discounted expected profits.\footnote{For simplicity, and notational ease, we have assumed away uncertainty about the size of the market for the innovations and the costs of the second-stage innovation. This is without loss of generality because firms maximize expected profits and we can interpret these quantities in terms of expected market size and costs. Moreover, the results will not be qualitatively affected even if we consider risk-aversion by financial institutions.} The time-interval between adjacent dates is \( \Delta \) and the instantaneous risk-neutral discount rate is \( r \).

### 3.1.1 Analysis

We solve the model through backward induction, starting at date \( t = 2 \). Let, \( m^T_1 \) denote the total number of investors purchasing the initial innovation at the end of date \( t = 1 \). Clearly, \( m^T_1 \) depends on whether \( I \) patented the innovation at date \( t = 0 \) or not. That is,

\[
m^T_1 = \begin{cases} 
m_0 & \text{if } D = P \\
m_1 \equiv m_0 + xz & \text{if } D = N \end{cases}
\]

We first consider the case of no patenting at the initial date: \( D = N \). If \( I \) successfully develops the innovation, then it faces a market where the
buyers’ reservation utility is determined by their ability to purchase the initial innovation at the price $c$. Let $\bar{v} \equiv v + \alpha$. Buyers will only purchase the new innovation at a price $\kappa_2$, if $(\mu - \kappa_2) \geq (\bar{v} - c)$ The constrained profit maximization problem facing $I$, conditional on having a successful innovation at date $t = 2$, is to choose a derivative premium $\kappa_2$ and market size $q_2$ to:

$$\max_{\{\kappa_2, q_2\}} \left\{ \kappa_2 q_2 - \frac{1}{2} c', q_2^2 \right\}, \quad \text{s.t., } (i) \ \kappa_2 \leq \mu - (\bar{v} - c), \ (ii) \ q_2 \leq m_1$$

(2)

In (2), we recognize the upper limit on the price due to the reservation utility of the buyers. As this pricing constraint will be binding in any optimal strategy for $I$, we straightforward compute the optimal price of the new innovation and its market share as:

$$q_2^*(N) = \min \left( m_1, \frac{\mu - (\bar{v} - c)}{c'} \right)$$

$$\kappa_2^* = \mu - (\bar{v} - c)$$

(3)

(where we recall that $N$ denotes the regime where the firm does not obtain a patent). This policy yields the profits,

$$\Pi_2^*(N) = q_2^*(N) \left[ \mu - (\bar{v} - c) - \frac{c' q_2^*(N)}{2} \right]$$

(4)

Note that these profits are positive because, by assumption, $\mu > \bar{v}$. Thus, $I$ will invest in developing the new innovation if and only if

$$c_2(N) \leq \exp(-r\Delta)[s \Pi_2^*(N)]$$

(5)

We turn next to the case where $I$ has taken out a patent at date $t = 0$: that is, $D = P$. In this case, $I$ maintains a monopoly over the market, $m_0$. Of course, $I$ may still wish to segment this market into buyers who receive the second-generation innovation, at a premium of $\kappa_2$, and buyers who receive the original innovation, at a premium of $2$. Buyers of the latest innovation therefore will purchase as long as $\mu - \kappa_2 \geq (\bar{v} - \phi_2)$. Hence, conditional on successfully developing a second-generation innovation, the optimization problem of $I$ is now to,

$$\max_{\{\phi_2, q_2\}} \left\{ [\kappa_2 q_2 - \frac{1}{2} c' q_2^2] + [(\phi_2 - c) \max(0, m_0 - q_2)] \right\}, \quad \text{s.t., } \kappa_2 \leq \mu - (\bar{v} - \phi_2)$$

(6)
The objective function (6) shows how the market gets endogenously segmented between the first and second generation innovations. Analysis of the maximization problem yields the optimal pricing and market segmentation policies: 

\[ q^*_2(P) = \min \left( m_0, \frac{\mu-(\bar{c}-c)}{c} \right); \kappa_2^* = \mu; \text{ and } \phi_2^* = \bar{v}. \]

These policies yield the profits:

\[ \Pi_2^*(P) = q_2^*(P) \left[ \mu - \frac{c_2^*(P)}{2} \right] + (\bar{v} - c) \max(0, m_0 - q_2^*(P)) \]

(7)

(where we recall that \( P \) denotes the regime where the firm decides to patent). Thus, \( I \) will invest in developing the new innovation if and only if

\[ c_2(P) \leq \exp(-r\Delta) |s\Pi_2^*(P)| \]

(8)

We can delineate two sets of conditions for whether it is optimal to patent or not. It then follows from (1), (4) and (7) that,

**Proposition 1** Suppose that \( m_1 > \frac{\mu-(\bar{c}-c)}{c} \). Then, \( \Pi_2^*(N) > \Pi_2^*(P) \) if \( m_0 \) is sufficiently small relative to \( \frac{\mu-(\bar{c}-c)}{c} \).

Proposition 1 confirms the intuition that if imitators bring in a sufficiently large number of buyers into the market at date \( t = 1 \), then the profits from a successful new-generation innovation are higher for \( I \) if it does not patent the initial innovation. This result also indicates that, for a sufficiently large effective market size \( y \equiv xz \), allowing imitation is more likely to be optimal for \( I \) if the second-generation innovation significantly improves buyer value compared to the unit cost, that is, \( (\mu - \bar{v})/c' \) is high and/or if there is a significant cost reduction between the two innovations, that is, \( c/c' \) is high. However, if \( (\mu - \bar{v})/c' \) is low and/or if there is a significant cost increases between the two innovations, that is, \( c/c' \) is low, then we have the reverse case:

**Corollary 1** If \( m_0 > \frac{\mu-(\bar{c}-c)}{c} \), then \( \Pi_2^*(P) > \Pi_2^*(N) \).

Clearly, the quantities \( \frac{\mu-(\bar{c}-c)}{c} \), \( m_0 \), and \( m_1 \) are critical to the optimality of patenting. In Section 3.3 below, we will discuss further the economic interpretation of these quantities.

We turn next to analysis at date \( t = 1 \). We first consider the case of no patenting. Because \( I \) is the market leader, it chooses a premium and market size, with the imitators serving the remaining market at the break-even price of \( \bar{c} \). An end user will buy from \( I \) only if \( \kappa_1 - \bar{c} \leq \alpha \), where \( \kappa_1 \) is the premium charged by \( I \). Hence, \( I \)'s constrained profit maximization problem is to choose a derivative premium \( \kappa_1 \) and market size \( q_1 \):

\[ Max_{\{\kappa_1, q_1\}} \left\{ \kappa_1 q_1 - \frac{1}{2} c q_1^2 \right\}, \]

(9)
\[ s.t., \quad (i) \, \kappa_1 \leq \alpha + \bar{c}, \quad (ii) \, q_1 \leq m_1 \quad (10) \]

The reservation utility constraint in (10) will be binding in the optimal policy. Hence, the solution to (9)-(10) is, \( q_1^*(N) = \text{Min} \left( m_1, \frac{\alpha + \bar{c}}{c} \right) \) and \( \kappa_1^*(N) = \alpha + \bar{c} \). The profits with the optimal policy, \( \Pi_1^*(N) \), are given by

\[ \Pi_1^*(N) = q_1^*(N) \left[ \alpha + \bar{c} - \frac{cq_1^*(N)}{2} \right] \quad (11) \]

If a patent has been taken out at date \( t = 0 \), that is, \( D = P \), then at date \( t = 1 \), \( I \) has a monopoly over the provision of the initial innovation. Thus, \( I \) will charge the premium \( \phi_1 \), subject to the constraint that \( \phi_1 \leq \bar{v} \) and serve its profit maximizing market:

\[ \text{Max}_{\{\phi_1, q_1\}} \left\{ \phi_1 q_1 - \frac{1}{2} cq_1^2 \right\}, \quad \text{s.t.,} \quad (i) \, \phi_1 \leq \bar{v}, \quad (ii) \, q_1 \leq m_0 \quad (12) \]

The optimal policies are therefore, \( q_1^*(P) = \text{Min} \left( m_0, \frac{\bar{v}}{c} \right) \) and \( \phi_1^*(P) = \bar{v} \).

The profits from these strategies are:

\[ \Pi_1^*(P) = q_1^*(P) \left[ \bar{v} - \frac{cq_1^*(P)}{2} \right] \quad (13) \]

Now, we can directly compare \( I \)'s profits at date \( t = 1 \), based on the patent decision at the previous date. Intuitively, this comparison trades-off the higher profit margin and lower market size with patent protection against the lower margin and higher market size without patent protection. Patenting strictly dominates the alternative at date \( t = 1 \) if \( m_0 \) is at least as large as the optimal monopoly market size for \( I \). This is stated formally in the following proposition:

**Proposition 2** If \( m_0 \geq \frac{\bar{v}}{c} \), then \( \Pi_1^*(P) > \Pi_1^*(N) \).

Note that Proposition 2 holds is independent of the effective market size \( y \). That is, if \( I \)'s initial end user base is not too small, then patenting strictly dominates the alternative from the viewpoint of date \( t = 1 \), irrespective of the market extension provided by imitators. On the other hand, the logic of Proposition 2 can be reversed if \( m_0 \) is sufficiently small relative to \( y \). That is,

**Proposition 3** If \( m_0 \) is sufficiently small relative to \( y \), then \( \Pi_1^*(N) > \Pi_1^*(P) \).

Propositions 2 and 3 clarify the essential conflict between patenting and allowing imitation: patenting increases profits on the initial innovation, but
may restrict profits—relative to an open imitation environment—from the second-generation or subsequent innovation.

Indeed, Proposition 3 implies that if the innovating institution’s initial market size, \( m_0 \), is sufficiently small, then it is beneficial not to patent in order to increase the value of the real option of the subsequent innovation. That is, irrespective of \( y \), if \( m_0 \) is sufficiently small, then it is likely that the optimal policy is to forego patenting. Usually, if there is a “break through,” one expects innovations creating large buyer value per unit cost, i.e., a large \((\bar{v}/\bar{c})\), to be more patentable. The following Corollary shows that this is not always the case.

**Corollary 2** There exists some \( 0 < \bar{m}_0 < \frac{\bar{v}}{\bar{c}} \) such that \( \Pi_1^*(N) > \Pi_1^*(P) \) whenever \( m_0 < \bar{m}_0 \).

This result is somewhat counter-intuitive because it implies that, for a given \( m_0 \), patenting is less likely to be optimal if the initial innovation creates large buyer value per unit cost of delivery. Usually, one expects innovations creating greater buyer value (per unit cost) \((\bar{v}/\bar{c})\) to be more patentable. However, this intuition overlooks the fact that high-value initial innovations increase the innovator’s short-run profits without a patent, while also (at least weakly) increasing the innovator’s profits from subsequent innovations. Another way of stating this point is that it may be sub-optimal to patent significant “break throughs,” especially if these break throughs can give rise to further innovations. Some types of financial innovations fall into this category. If patented, so that there are no other suppliers, the market is too small. To be viable, the market needs other suppliers and this can be achieved by not patenting.

We now analyze the determinants of the optimal patenting decision in further detail by comparing the present value of profits, from the view point of date \( t = 0 \). First, with patenting\(^9\)

\[
\Pi_0^*(P) = \exp(-r\Delta) \left[ \Pi_1^*(P) + \left[ \exp(-r\Delta)s\Pi_2^*(P) - c_2(P) \right]^+ \right]
\]

(14)

It will be optimal to patent if

\[
\Pi_0^*(P) > c_0 + c_P
\]

where \( c_P \) are the legal and preparation costs associated with patenting. Without patenting we have

\[
\Pi_0^*(N) = \exp(-r\Delta) \left[ \Pi_1^*(N) + \left[ \exp(-r\Delta)s\Pi_2^*(N) - C_2(N) \right]^+ \right]
\]

(15)

\(^9\)The term \([J]^+ = \max(0,J)\).
We clarify the circumstances under which the institution benefits from patenting versus not patenting in a number of results. The first representation quantifies the intuition that if the initial market size, i.e., \( m_0 \) is large relative to the value increment arising from the second-generation innovation, then it will be optimal for \( I \) to patent at date \( t = 0 \).

**Proposition 4** Suppose that \( m_0 \geq \max \left( \frac{\mu - (\delta - \bar{c})}{\sigma}, \frac{\delta}{c} \right) \). Then, \( \Pi_0^s(P) > \Pi_0^s(N) \).

We note that the condition of Proposition 4 is more likely to be met if, ceteris paribus, the difference \( \mu - \bar{v} \), that is, the buyer value increment between the initial and the subsequent innovation, is not too high. And, for a fixed \( \mu - \bar{v} \), the sufficient condition of Proposition 4 is also more likely to be satisfied if \( c/c' \), that is, the ratio of the delivery costs of the initial and the subsequent innovations is not too large. Put differently, if there is a substantial production cost reduction between the two innovations, then patenting is more likely to be optimal. This is because with a very low production cost at date \( t = 2 \), the advantages of having a large market size due to an open imitation regime are amplified.

The tenor of the foregoing argument suggests that it would be optimal not to patent if there is a substantial buyer value-increment or a substantial production cost reduction between the initial and the subsequent innovation. Our next result clarifies that this intuition is correct provided that the market extension due to an open imitation regime is sufficiently large.

**Proposition 5** Suppose that \( y \) is a large number. Then there exist \( \delta \) and \( \beta \) such that
\[
\Pi_0^s(N) > \Pi_0^s(P) \text{ if } \mu - \bar{v} > \delta \text{ or } c/c' > \beta.
\]

This result and Proposition 2 imply that while it might be optimal to patent the initial innovation, when we consider subsequent innovations, it is optimal not to patent.

### 3.2 The Role of Licensing

So far, we have not allowed the innovating institution \( (I) \) to patent and then share the innovation through licensing. Licensing can potentially resolve the conflict between increasing the market size for the sequential innovation and capturing rents from it. An institution can always patent an innovation and then license the use of the innovation to other institutions. Alternatively, for certain types of innovations that lack the uniqueness to qualify for patent protection, and require the participation of other institutions to help develop a market, the innovator can register the name of the product as a service.
mark\textsuperscript{10} and license its use. We provide a brief discussion of the effect on the patenting decision when \( I \) can ex ante license the sequential innovation to the potential imitators through fixed fees.\textsuperscript{11} Thus, if \( D = P \) (i.e., \( I \) patents in the first period) and the second-generation innovation is subsequently realized, then \( I \) can license it to each of the \( x \) imitators, at a fixed fee. We assume, however, that the licensees cannot match the quality of support and related services offered by \( I \); otherwise, in the absence of any capacity constraints, licensing will not be profitable for \( I \).

The maximum licensing fee that \( I \) can charge, in any equilibrium, is equal to the increase in buyer surplus from the second generation innovation. Hence, profits from licensing will be positively associated with the incremental buyer utility provided by the second-generation innovation. However, the requirement of a substantial buyer-value enhancement in the second-generation innovation is not a sufficient condition for the strategy of patenting and licensing to dominate the no-patenting strategy. This is because if \( I \) does not patent (i.e., \( D = P \)) and induces the imitators to create a larger market, it can attempt to serve that market with the second-generation innovation itself—as we have seen above—without sharing that market with the licensees.

Thus, the conditions under which licensing is profitable also tend to be those under which the strategy of not obtaining a patent (i.e., \( D = N \)) is profitable. In fact, it is easy to show that the strategy of not patenting dominates the strategy of patenting and then licensing whenever the rate of growth of the market induced by the market (i.e., the ratio \( m_1/m_0 \)) is sufficiently large or quality of support provided by the licensees to the end-users is sufficiently low. In the latter case, the licensing fees are low, thereby reducing the licensing revenues. An interesting insight emanating from our analysis, then, is that using a licensing strategy to expand the market size for a sequence of innovations may not be optimal if the market can be expanded

\textsuperscript{10}A service mark is similar to a trademark, except that a trademark promotes products while service marks promote services. See Lanham Trademark Act 15 U.S.C.A.\textsection 1051-1127.

\textsuperscript{11}See Kumar and Turnbull (2006) for a more detailed analysis. We note that while, in theory, licensing can occur through both fixed fees and per unit licensing fees that depend on output (see, e.g., Katz and Shapiro (1985)), the latter appear to be particularly unsuitable in the financial world, in practice, because of prohibitive costs of monitoring output and antitrust laws. Specifically, the use of financial instruments and algorithms often occurs as part of complex bilateral (provider-client) relationships that involve a variety of activities; hence, it may be difficult to write easily verifiable contracts based on output.
through imitators or the licensees can not provide high-quality support and other services that enhance buyer-value.

### 3.3 Critical Determinants of the Patenting and Licensing Decision

In the foregoing analysis, the quantities \( c, c_0, m_0 \), and \( m_1 \) are clearly critical to the optimality of patenting and licensing (subsequent to a patent decision). These quantities have a ready economic interpretation and help relate the model to the literature on the economics of innovation and observed patterns in the patenting of financial innovations.

First, \( \frac{\mu - (\delta - \gamma)}{\gamma} \) is the incremental value margin of the second-generation innovation. The higher is this quantity, the higher is the value — other things held fixed — of the real option of developing and marketing the second generation innovation that is embedded in the initial innovation. However, the profitability of the second-generation innovation also increases with the market size, for a given incremental value margin. Patenting the first-generation innovation restricts the market size for the second-generation innovation to \( m_0 \), but allows the innovator higher profit margins over the innovation cycle. On the other hand, not patenting the initial innovation allows the market size to increase to \( m_1 \) when the second-generation innovation is introduced, but at the cost of lower profit margins.

Therefore, the likelihood of patenting increases with \( m_0 \), ceteris paribus; but, other things being fixed, this likelihood decreases with the incremental value margin and with the ratio \( m_1/m_0 \). But \( m_0 \) is, by definition, the size of the (initial) client-base of the innovating institution; hence, \( m_0 \) is a measure of institution size in our model. Larger institutions are therefore more likely to patent financial innovations, other things held fixed, which is consistent with Lerner (2002). Similarly, innovations that satisfy a well defined immediate need, or more generally satisfy the innovation characteristics for minor innovations set out in Section 2, have low incremental value margins, and therefore are likely to be patented.

By contrast, major new ideas that are potentially attractive to a wide base of users and link a variety of markets have a high incremental value margin; these are also the kind of innovations where the ratio \( m_1/m_0 \) is likely to be very high. Patenting is therefore not likely to be optimal in such cases. And because high incremental value margin innovation cycles are quite common in the financial service industry, financial institutions will tend to file a lower number of patents per innovation than non-financial firms, which is consistent with Lerner (2006).
3.4 Related Literature

Theoretical work on the determinants of financial patenting is relatively rare, and we have discussed related literature in the Introduction. More generally, Gallini (1984) and Gallini and Winter (1985) consider the sharing of innovations in a search-theoretic R & D model, and find that licensing always occurs in equilibrium. In a duopolistic setting, Katz and Shapiro (1985) consider licensing of a cost-reducing innovation that is owned by one of the producers, while Katz and Shapiro (1986) examines licensing by an upstream research lab to a downstream oligopoly of identical producers. And Green and Scotchmer (1995) consider the role of licensing in a model of sequential innovations when different firms contribute to the innovation sequence. Our analysis differs from the existing licensing literature in considering the role of licensing in expanding the market for a sequence of innovations, where the sole innovator is also the high-quality service provider in the industry.

In our model, the innovation sequence improves the basic product (in terms of buyer value), and hence is similar to the “quality ladder” formulation (see, e.g., Scotchmer (2004)). However, in a pure quality ladder, each point in the sequence increases the product quality by a fixed amount, while this is not the case in our model. More importantly, our model allows heterogeneous buyer valuation of the innovation sequence, based on differences in quality of supporting services, unlike the quality ladder model where the quality superiority of the innovations is fixed for the industry (e.g., O’Donoghue et al. (1998)).

There are somewhat superficial similarities between certain aspects of successful financial innovations and innovations in ‘network markets’ — where users purchase products compatible with those brought by other — such as the computer software industry (see, e.g., Besen and Farrell (1994)). In such markets, coexistence of incompatible products is unstable, and dominant technology standards emerge rapidly (Besen and Johnson (1986)). Because expectations about the ultimate size of the network are crucial, market demand can be self-reinforcing for a technology that is expected to be the standard — and hence end up with largest network. Consequently, installing a large user base visibly and early is important for successful innovations in network markets.\textsuperscript{12}

\textsuperscript{12}Innovators therefore use a variety of strategies to maximize their use base early on. Examples of such strategies include penetration pricing (Katz and Shapiro (1986b)); liberal grants of manufacturing licenses to potential rivals and commitments for joint development of derivative innovations (Bensen and Farrell (1994)); actively attracting producers of complementary products, such as applications for software platforms; and, strategic ‘pre-announcements’ of products to disrupt the installation of user base for rivals (Farrell and
However, profit-generation in successful innovations in network markets and financial innovations differ in at least one important aspect. Successful innovations in network markets generate profits by becoming dominant standards and increasing the size of the user base. In such 'winner-take-all' markets, axiomatically, there is little scope for the simultaneous existence of different versions of the basic technology provided by different vendors. By contrast, successful financial innovations are often characterized by various derivative innovations (of the basic innovations) being simultaneously offered by different financial institutions. The innovating institution typically earns profits not by grabbing the entire market, but by expropriating the most profitable trading segments through a first-mover advantage based on expertise.

4 Applications of the Model

The model developed in the last section can be expressed in the form

\[ \Pi_0^t(D) = PV_0(D) + PV_0[option(D)] \]  

(16)

where, for a given patenting policy denoted by \( D \in \{ P, N \} \), the term \( PV_0(D) \) represents the present value of the initial innovation and \( PV_0[option(D)] \) the present value of options associated with subsequent innovations that depend on the initial innovation. In practice, it is difficult to actually compute the option value because of obvious data constraints. Notwithstanding this constraint, the expression (16) provides a useful framework for senior management to identify the salient features that will determine the merits of patenting and licensing a financial innovation.

We now apply this model to two quite different examples. The first is a financial instrument that was successfully introduced without a patent. The second example, a pricing algorithm, falls into the category of innovations that "facilitates a service that is offered to clients." In Section Two, we argued that this type of innovation would likely be protected by a patent. There are conditions when patenting is not optimal.

4.1 Case Study One: Swap Innovations

Our first example is that of an interest rate swap. This type of financial contract allows one end user to trade fixed rate coupon payments over a defined horizon for a series of floating rate payments over the same period.
with another end user. The first interest rate swap contract was introduced in the 1980’s.\(^\text{13}\)

If the innovating institution had patented the idea of an interest rate swap, preventing other institutions from competing\(^\text{14}\), then it must estimate the present value of the cash flows from marketing swaps to end users: \(PV_0(P)\). Central to the analysis is an estimate of the size of the market \(m_0\), the growth in the market\(^\text{15}\), the cost of servicing each transaction \(c\) and the value added \((\tilde{v})\). The cost of the innovation \((c_0)\), depends on the costs associated with designing the contract, addressing legal issues associated with the exchange of cash flows in the presence of counterparty risk, addressing regulatory issues, designing the back office, designing hedging strategies and having the staff to run a swaps desk.

If the institution does not patent, then information about the swap contract and the potential profits will be disseminated, attracting other institutions \((x)\) to enter the market, each able to reach a client base \((z)\). The operating cost per contract are \((\tilde{c})\) and the value added \((v)\). The institution \(I\) is assumed to have an advantage in execution, at least over some initial period and the value added to end users is \((\tilde{v} = \alpha + v, \alpha > 0)\). The size of the potential market has now expanded to \((m_1 = m_0 + y)\). The institution must estimate the present value of the cash flows in this more competitive environment. This is denoted by \(PV_0(N)\).

In the Herrera and Schroth (2002) analysis, it is argued that it is possible for \(PV_0(N) > PV_0(P)\), implying that \(I\) may not require patent protection to recoup the costs of innovation. For financial innovation involving financial instruments, the situation is usually more complicated. It is possible for \(PV_0(N) < PV_0(P)\), yet it is still optimal for \(I\) not to seek patent protection. The difference arises from the present value of subsequent innovations motivated by the initial innovation.

Here future innovations may take many forms. One example is instead of exchanging fixed for floating payments, exchange floating for floating payments referenced to two different interest rates\(^\text{16}\). Another example would be to trade options on swaps. The success of future innovations depends

\(^{13}\)Note that this example occurs before the State Street decision. However, the same discussion applies to credit default swaps that were introduced the late 1990s.

\(^{14}\)The institution could have allowed other institutions to offer swaps under licensing agreements. This would however have hindered the development of the market. The pricing of the licensing agreement would also be an issue - see the discussion in Section 3.2.

\(^{15}\)In the model developed in the last section, we did not address this issue in order to avoid complication.

\(^{16}\)For example, exchange LIBOR payments for Federal Fund payments.
on the acceptance of the initial innovation. End users must be aware of the benefits of using swaps. The size of the market affects the liquidity of the market. If \( I \) had patented the innovation, it reaches a market of size \( m_0 \). This may affect the costs of introducing new forms of swaps, as it needs to educate end users about the merits of swaps, the liquidity of the swap market may be quite limited, restricting its development. It also must address the legal and regulatory issues that arise from the new forms of innovation.

If \( I \) had not patented the innovation, the size of the swap market will be enhanced and with more end users there will be more knowledge about the product. Institutions in the swap market will also learn from each other about the pricing and hedging of swaps\(^\text{17}\). Standardization will occur with the development of an ISDA contract, which will enhance liquidity. Consequently, the costs of introducing a new form of swap should be lower compared to the patent case: \( c_2(P) > c_2(N) \). The institution \( I \) must estimate the present value of the option to undertake further innovation: \( PV_0[\text{option}(D)] \). Therefore \( I \) is now in a position to calculate the net present value of initial innovation plus the option for further innovation.

Note that in our model we assumed that \( I \) is the innovator for subsequent innovations. This is not necessary. Usually there are a small number of leading institutions that act as market leaders, each being able to capture some rent over some finite period. The analysis can incorporate this possibility. The analysis readily extends to cover the case of multiple innovations. Experienced managers will have views about possible subsequent innovations and the importance of imitators in facilitating the development of new markets. The model provides a framework for senior management to identify the significance of different development paths, when reaching a decision about whether to patent or not.

The analysis for credit default swaps is similar, so we omit the details.

4.2 Pricing Algorithms

This example is quite different in nature from the previous example. The algorithm could be for pricing of options using simulation\(^\text{18}\) or it could be a risk model. For example, the RiskMetrics algorithm first developed by J. P. Morgan for risk management or the risk model developed by Lehman

\(^{17}\)In investment banking there is high mobility of labor, so knowledge is readily diffused. There are also industry publications and conferences resulting in the dissemination of knowledge.

\(^{18}\)Patent 6,381,586, granted to International Business Machines, prices options using importance and stratified sampling Monte Carlo simulation.
Brother for analyzing the risk characteristics of fixed income portfolios -see Naldi, Chu and Wang (2002). To analyze this type of innovation, the first step is to identify the objectives of the innovation project. If the pricing algorithm is developed to be part of a package of algorithms, it is hoped that it increases the value added, \( (\alpha) \), to end users. If the pricing algorithm is developed to speed up pricing for, say, risk management, it helps to lower the cost per unit \( (c) \). In both cases the option for further development may be non-existent. Patenting in either case provides a barrier preventing competitors copying the innovation. The economic analysis is straight forward in theory, if not in practice.

For the case of the algorithm being a risk model, we first consider the J. P. Morgan case. Here the initial motivation for the innovation is dictated by the need to meet Basel I regulatory requirements. It could either purchase the necessary software or develop in-house. The advantage of a leading institution developing in-house is the flexibility it allows to incorporate new structures into a risk management system\(^{19}\). Viewed in isolation, J. P. Morgan would have benefitted from obtaining a patent, as other institutions would be forced to bear the full costs of development: \( PV_0(N) < PV_0(P) \). However, they did not apply for a patent (this was after the State Street Decision), instead they followed a policy of full disclosure and became a market leader.

The option for further innovation in this case is to capitalize on the development of the software by transferring it over to a separate risk management entity. This stand alone entity generates revenue by providing risk management consulting to other institutions and corporations. By making the development open and becoming an industry standard enhances the value of the option. In this case \( PV_0[\text{option}(N)] > PV_0[\text{option}(P)] \).

For the case of the algorithm being a risk model for fixed income securities, it is developed as an aid to clients who manage their fixed income portfolios relative to one of the institution’s bond indices. The benefit to the institution is that it lowers the costs to their clients in managing their portfolios and it is hoped that they will continue to use the institution for trading. To be acceptable to clients, the model must be transparent, so details are public knowledge. A patent may prevent competitors from developing similar models. Absent a patent, the institution still has an advantage. It is one of the premier fixed income trading houses, and has a large data bank of bond and fixed income index data, which acts a barrier to entry. The data bank facilitates the calibration of the risk model. It also allows the

\(^{19}\) Software vendors can be quite tardy in responding to clients requests.
institutions to provide additional services to clients, such as testing the efficacy of different trading strategies. The options for further innovation might entail extending the risk model to the many different indices that are used in practice.\(^{20}\)

To undertake a formal analysis, the institution must quantify the benefits of the innovation, the value of subsequent innovations and the effects of patenting on these values. While this is extremely difficult, the analysis represented by expression (16) at least provides a framework.

5 Summary and Conclusions

The State Street decision, recognizing that business methods can be patented, presents financial institutions with an option to obtain patent protection for different types of financial innovations. The possibility of patenting also brings with it the option of licensing. This new patentability paradigm for financial innovations, poses an immediate decision problem for senior management of financial institutions. What innovations should they patent, or patent and license, and are there certain innovations where it is optimal not to patent? This is an important issue because financial innovations differ from innovations in other areas due to certain market and regulation related aspects that are unique to the financial industry. In particular, public exposure of innovations, ease of imitation, importance of educating end-users, and leveraging on the participation of other market makers to reduce the costs of adoption and increase liquidity, are some of the features that are especially important in the industry.

We provide a parsimonious decision framework for management that sets out the drivers of the decision to patent and license financial innovations, and apply this framework to explain observed patenting behavior with respect to a wide range of financial innovations — from the pricing algorithms to the development of the new financial instruments. Our analysis highlights the real options for subsequent innovations and market expansion that are embedded in certain types of financial innovations, as the primary determinant of whether patent protection is warranted; furthermore, we also examine the role of licensing in financial innovations. Interestingly, our model illuminates characteristics of financial innovations that have been successful because they were not protected by patents and imitation was allowed.

To our knowledge, our paper is among the first analyses to systematically examine the determinants of long-term profits from financial innova-

\(^{20}\)Many of these indices are designed to meet particular needs of a client.
tion. This analysis is of independent interest because it extends and refines the first mover advantage argument for financial innovations in the literature (Tufano (1998) and Herrera and Schroth (2003)); and, it is also contributes to the broader patenting and licensing literature by showing that patenting can be sub-optimal even in the absence of asymmetric information (Horstmann et al. (1985)).
Appendix

Proof of Proposition 1 Put $\theta \equiv \frac{\mu - (\bar{v} - \bar{c})}{\bar{c}}$. We want to find a set of additional conditions under which $L \equiv \Pi_2^*(N) - \Pi_1^*(P) > 0$. Let, $\beta \equiv \left[ (\bar{v} - \bar{c}) - \frac{c'm_0}{2} \right] m_0$. Then, under the conditions annunciated in the Proposition, we can write, $L = \frac{c'}{2} \left[ (\theta - m_0)^2 - m_0^2 - 2\beta/c' \right]$. The critical value of $\theta$ for which $L = 0$ is, $\theta = m_0 = ( +/- ) (m_0^2 + 2\beta/c')^{1/2}$. Now,

$$m_0^2 + 2(\beta/c') = m_0^2 + 2 \left[ (\bar{v} - \bar{c}) - \frac{c'm_0}{2} \right] m_0 = \frac{2}{c'} (\bar{v} - \bar{c}) m_0$$

Hence if,

$$\theta - m_0 > \left[ \frac{2}{c'} (\bar{v} - \bar{c}) m_0 \right]^{1/2},$$

then $L > 0$. Thus, if $m_0$ is sufficiently small then (17) will be satisfied.

Proof of Corollary 1 In this parametric range, we have internal solutions to the profit maximizing output condition in both cases: $P$ or $N$. This implies that

$$\Pi_2^*(P) = \frac{1}{2c'} [\mu - (\bar{v} - \bar{c})]^2 + (\bar{v} - \bar{c})m_0$$
$$\Pi_2^*(N) = \frac{1}{2c'} [\mu - (\bar{v} - \bar{c})]^2$$

(18)

Given that $\bar{v} \geq c$, then the result follows.

Proof of Proposition 2 Given the conditions of the Proposition, then as $v > \bar{c}$, if follows that $m_1 \geq (\alpha + \bar{c})/\bar{c}$. This implies an interior solution, so that $\Pi_1^*(N) = (\alpha + \bar{c})^2/\bar{c}$. Hence

$$\Pi_1^*(P) - \Pi_1^*(N) \geq [(\bar{v} + \alpha)^2 - (\alpha + \bar{c})^2]/\bar{c} > 0.$$

Proof of Proposition 3 For $m_0 = \varepsilon$, where $\varepsilon$ is a small number, it follows from (13) that $\Pi_1^*(P) \leq \bar{v} \varepsilon$. Meanwhile, for $y \geq \frac{\alpha + \varepsilon}{\bar{c}}$, it follows from (11) that, $\Pi_1^*(N) = \frac{(\alpha + \varepsilon)^2}{2c}$. Hence, $\Pi_1^*(N) - \Pi_1^*(P)$ if we choose $\varepsilon \leq \frac{(\alpha + c)^2}{2c' \bar{c}}$.

Proof of Corollary 2 Follows immediately from Proposition 3 if we set $\tilde{m}_0 = \frac{(\alpha + \varepsilon)^2}{2c' \bar{c}}$.

Proof of Proposition 4 The proof follows from Corollary 1 and Proposition 2.

Proof of Proposition 5 The proof follows from Proposition 3 and Corollary 2.
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