CHAPTER VI

CURRENCY RISK MANAGEMENT: FUTURES AND FORWARDS

In an international context, a very important area of risk management is *currency risk*. This risk represents the possibility that a domestic investor's holding of foreign currency will change in purchasing power when converted back to the home currency. Currency risk also arises when a firm has assets or liabilities expressed in a foreign currency. Consider the following example.

Example VI.1: Spec's, the Texas liquor store chain, regularly imports wine from Europe. Suppose Spec's has to pay for those imports EUR 5,000,000 on March 2. Today, February 4, the exchange rate is 1.10 USD/EUR.

Situation: Payment due on March 2: EUR 5,000,000.
$S_{Feb\,4}$ = 1.10 USD/EUR.

Problem: S_t is difficult to forecast $\Rightarrow$ Uncertainty $\Rightarrow$ Risk.
Example: on January 2, $S_{t=Mar\,2}$ > or < 1.10 USD/EUR.

At $S_{Feb\,4}$, Speck's total payment would be: EUR 5M x 1.10 USD/EUR = USD 5.5M.

On March 2 we have two potential scenarios with respect to today's valuation of EUR 5M:
If the $S_{Mar\,2} \downarrow$ (USD appreciates) $\Rightarrow$ Spec's will pay less USD.
If the $S_{Mar\,2} \uparrow$ (USD depreciates) $\Rightarrow$ Spec's will pay more USD.

The second scenario introduces *Currency Risk*. ¶

We have seen that FX movements look like a random walk, then, currency risk is very difficult to avoid for transactions denominated in foreign currency. Then, currency risk becomes relevant when the value of an asset/liability change "a lot" when S_t moves. In finance, we relate "a lot" to the variance or volatility. For currency risk, we will look at the volatility of FX rates: More volatile currencies, higher currency risk.

Example VI.1 (continuation): Consider the following situations:
(A) $S_{Mar\,2}$ can be
(i) 1.09 USD/EUR, for a total payment: EUR 5M * 1.09 USD/EUR = USD 5.45M.
(ii) 1.11 USD/EUR, for a total payment: EUR 5M * 1.11 USD/EUR = USD 5.55M.

(B) $S_{Mar\,2}$ can be
(i) 0.79 USD/EUR, for a total payment: EUR 5M * 0.79 USD/EUR = USD 3.95M.
(ii) 1.49 USD/EUR, for a total payment: EUR 5M * 1.49 USD/EUR = USD 7.45M.

Situation B is riskier (more volatile) for Spec's, since it may result in a higher payment. ¶

How do we measure FX risk in FX markets? We use the distribution of s_t to understand and measure FX risk. Below, Table VI.1 presents the distribution of s_t (with annualized mean & SD) from 1990:Jan - 2017:Dec, using monthly data.

TABLE VI.1
Distribution of Changes in Exchange Rates for Selected Currencies (1990-2017)

Currency	Mean	Standard Deviation	Skewness	Excess Kurtosis	Min	Max	Normal?
GBP/USD	0.0090	0.0951	0.9681	3.4004	-0.0842	0.1386	No
CHF/USD	-0.0097	0.1101	0.2171	1.3365	-0.1226	0.1257	No
EUR/USD	0.0118	0.1030	0.4803	1.2113	-0.0872	0.1139	No
NOK/USD	0.0166	0.1102	0.5253	1.2145	-0.0707	0.1392	No
INR/USD	0.0565	0.0820	3.0932	24.1434	-0.0655	0.2191	No
JPY/USD	-0.0010	0.1056	-0.1936	1.9347	-0.1474	0.1065	No
KRW/USD	0.0295	0.1247	1.7968	15.9320	-0.1657	**0.2723**	No
THB/USD	0.0179	0.1055	2.6493	32.3567	-0.1874	**0.2843**	No
SGD/USD	-0.0095	0.0563	0.5677	2.9251	-0.0557	0.0810	No
CNY/USD	-0.0122	0.0160	-0.4484	7.9325	-0.0337	0.0272	No
SAR/USD	0.0000	0.0030	3.3228	119.9623	-0.0087	0.0109	No
CAD/USD	0.0106	0.0792	0.8378	5.7371	-0.0823	0.1473	No
MXN/USD	0.0818	0.1359	5.0008	51.7441	-0.1282	**0.4531**	No
BRL/USD	0.0861	0.2262	5.1741	52.8000	-0.1735	**0.7049**	No
ZAR/USD	0.0805	0.1416	-0.2684	1.3388	-0.1575	0.1276	No
EGP/USD	0.0408	0.0530	13.9156	216.7728	-0.1662	**1.0017**	No
AUD/AUD	0.0106	0.1144	0.8887	4.3249	-0.0846	0.2023	No
Average	**0.0349**	**0.1180**	**0.7657**	**29.5760**	**-**	**-**	**No**

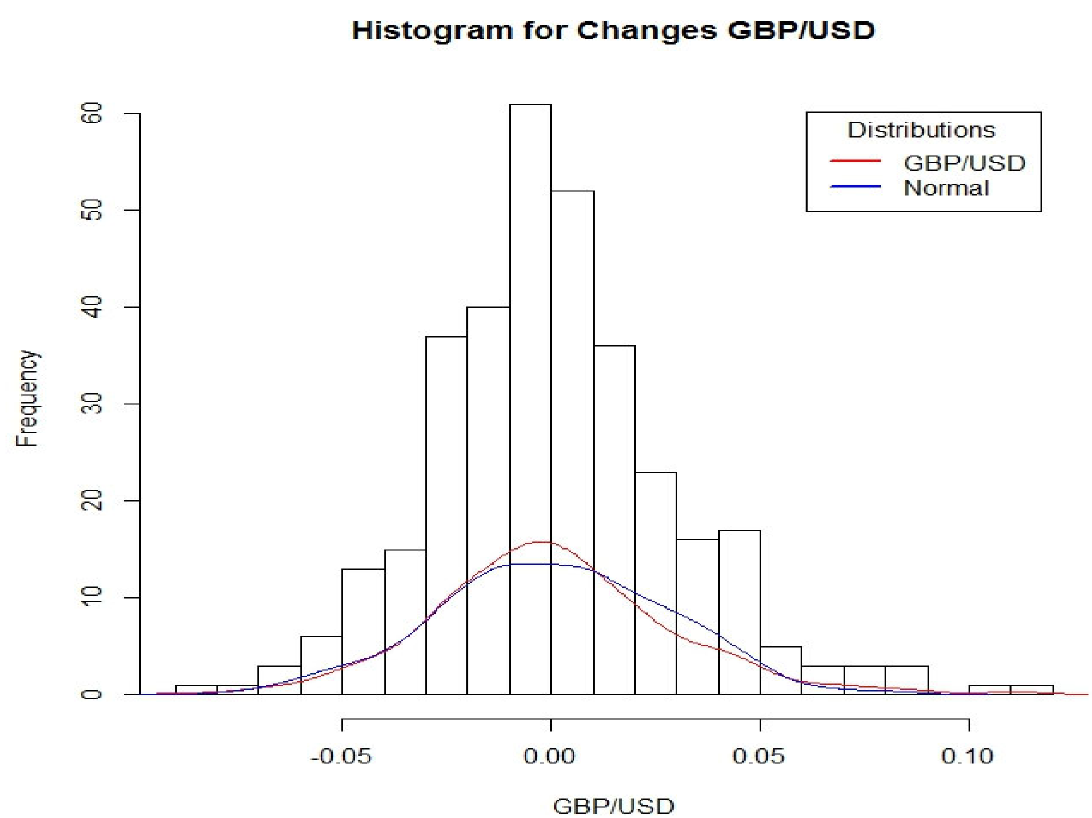
• Observations (typical of financial time series):
- On average, the USD appreciated against international currencies at an annualized mean of **3.50%**. The average annualized SD is 11.80%.
- Against developed currencies: 0.54% annualized change (SD=9.51%)
- *Excess Kurtosis.* It describes the fatness of the tails. Under normality, excess kurtosis equals 0. All the currencies show excess kurtosis, that is, the tails are fatter than the tails of a normal –i.e., probability of a tail event is higher than what the normal distribution implies.

- *Skewness*. If the distribution is symmetric (mean=median, for example, a normal), skewness is 0. Almost all the currencies show positive skewness (mean>median); that is, the fat part of the curve is on the left.
- s_t does not follow a normal distribution, as clearly seen in Graph VI.I.

Graph VI.1 show the histogram and empirical distribution (in red) of changes in the GBP/USD. It is the typical behavior of a developed currency (against the USD).

GRAPH VI.1

Distribution of monthly changes in the GBP/USD exchange rate (1990-2017)



FX volatility is a serious concern for many companies, especially during times of turbulence in FX markets, where extreme behavior can substantially swing the cash flows of firms (excess kurtosis helps to point out series with big swings). The following example illustrates this point: the Thai cement giant, Siam City Cement, had big losses during the 1997 Asian Financial crisis, mainly due to liabilities denominated in foreign currency.

Example VI.2: On July 2, 1997, Thailand devalued its currency, the baht (THB), by 18%. Siam City Cement, Thailand's second largest cement producer, lost THB 5,870 million (USD 146 million), giving a net deficit for the nine-month period of 1997 of THB 5,380 million. Siam City Cement reported a net profit of THB 817 million during the first nine months of 1996. Industry analysts said that the company was affected by foreign exchange losses on USD 590 million foreign debt, reported as of June 30. ¶

These examples show that FX risk is a serious concern for companies and investors in international markets. Managing this risk is very important. Chapter I introduced the instruments of currency risk management. This chapter studies the use of futures and forward contracts to lessen the impact of currency risk on positions denominated in foreign currencies. The next chapter studies currency options as a currency risk management tool.

I. Futures and Forward Currency Contracts

Before we start talking about futures and forwards, we have to answer an important question: why do we care about futures or forward contracts? In order to answer this question, we should recall that the primary goal of risk management is to change the risk-return profile of a cash position (or portfolio) to suit given investment objectives. This involves one of three alternatives: preserving value, limiting opportunity losses, or enhancing returns. Futures and forward contracts are effective in meeting these risk management objectives because they can be used as cost-efficient substitutes or proxies for a cash market position. The determination of the proper equivalency ratio is critical to the use of futures or forwards as a cash market proxy, regardless of whether this ratio will be applied to a hedge, an income enhancement strategy or a speculative position. The difference between futures and forward contracts is the subject of Section I. The determination of the proper equivalency ratio is the subject of Section II.

1.A Futures and Forwards Contracts in Risk Management

This section presents the basic differences between futures and forwards. They are instruments used for buying or selling a stated amount of foreign currency at a stated price per unit at a specified time in the future. When a forward or futures contract is signed there is no up-front payment. Both forward and futures contracts are classified as *derivatives* because their values are derived from the value of the underlying security. Forward and futures contracts play a similar role in the management of currency risk. The empirical evidence shows that both contracts do not show significantly different prices. Although a futures contract is similar to a forward contract, there are many differences between the two.

1.A.1 Forward Contracts

Chapter I introduced forward contracts. A forward contract is a tailor-made contract. Forward contracts are made directly between two parties, and there is no secondary market. In general, at least one of the parties is a bank. Forward contracts are traded over the counter: traders and brokers can be located anywhere and deal with each other over the phone. To reverse a position, one has to make a separate additional forward contract. Reversing a forward contract is not common. Ninety percent of all contracts result in the seller making delivery of the underlying currency.

Forward contracts are quoted in the interbank market for maturities of one, three, six, nine and 12 months. Non-standard maturities are also available. For good clients, banks can offer a maturity extending out to 10 years.

In Example I.8, 30-, 90-, and 180-day forward rate quotations appear directly under the Canadian dollar. The *Wall Street Journal* presents similar forward quotes for the other five major currencies: JPY, GBP, DEM, FRF, and CHF. These quotes are stated as if all months have 30 days. A 180-day maturity represents a six-month maturity. In general, the settlement date of a 180-day forward contract is six calendar months from the spot settlement date for the currency. For example, if today is January 21, 1998, and spot settlement is January 23, the forward settlement date would be April 23, 1998, a period of 92 days from January 21.

1.A.2 Futures Contracts

A futures contract has standardized features and is exchange-traded, that is, traded on organized exchanges rather than over the counter. Foreign exchange futures contracts are for standardized foreign currency amounts, terminated at standardized times, and have minimum allowable price moves (called "ticks") between trades. Foreign exchange futures contracts are traded on the market floor of several exchanges around the world. For example, they are traded on the Chicago Mercantile Exchange (the "Merc"), the Tokyo International Financial Futures Exchange (TIFFE), the Sydney Futures Exchange, the New Zealand Futures Exchange, the MidAmerica Commodities Exchange, the New York Futures Exchange, and the Singapore International Monetary Exchange (SIMEX).

The CME is the biggest and most important market in the world for foreign exchange futures contracts. CME futures contracts have been copied by other organized exchanges around the world. CME futures are quoted in direct quotes -U.S. dollar price of a unit of foreign exchange. CME futures specify a *contract size*, that is, the amount of the underlying foreign currency for future purchase or sale, and the *maturity date* of the contract. Futures contracts have specific *delivery months* during the year in which contracts mature on a specified day of the month. Contracts are traded on the traditional three-month cycle of March, June, September, and December. In addition, a current month contract is also traded. For some currencies, however, the CME offers currency futures with additional expiration dates. For example, for the GBP and the EUR contracts, the CME also offers January, April, July, and October as expiration dates. The month during which a contract expires is called the *spot month*. At the CME, delivery takes place the third Wednesday of the spot month or, if that is not a business day, the next business day. Trading in a contract ends two business days prior to the delivery date (i.e., the third Wednesday of the spot month). CME's trading hours are from 7:20 AM to 2:00 PM (CST).

Futures contracts are netted out through a clearinghouse, so that a clearinghouse stands on the other side of every transaction. This characteristic of futures markets stimulates active secondary markets since a buyer and a seller do not have to evaluate one another's creditworthiness. The presence of a liquid clearing house substantially reduces the credit risk associated with all forward contracts. The clearing house makes a trader only responsible for his/her *net* positions. The clearinghouse is composed of clearing members. Clearing members are brokerage firms that satisfy legal and financial requirements set by the government and the exchange. Individual brokers who are not clearing members must deal through a clearing member to clear a customer's transaction. In the event of default of one side of a futures transaction, the clearing member stands in for the defaulting party, and seeks restitution from that party. Given this structure, it is logical that the clearinghouse requires some collateral from clearing members. This collateral requirement is called margin requirement.

In the U.S., futures trading is regulated by the Commodities Futures Trading Commission (CFTC). The CFTC regulates the activities of all the players: futures commission merchants, clearinghouse members, floor broker and floor traders.

Table VI.2 summarizes the differences between forward and futures contracts.

TABLE VI.2

Comparison of Futures and Forward Contracts

	Futures	**Forward**
Amount	Standardized	Negotiated
Delivery Date	Standardized	Negotiated
Counter-party	Clearinghouse	Bank
Collateral	Margin account	Negotiated
Market	Auction market	Dealer market
Costs	Brokerage and exchange fees	Bid-ask spread
Secondary market	Very liquid	Highly illiquid
Regulation	Government	Self-regulated
Location	Central exchange floor	Worldwide

1.A.2.i Margin requirements

Organized futures markets have margin requirements, to minimize credit risk. There are two types of margin requirements: the *initial margin* and the *maintenance margin*. The idea behind the margin account is that the margin should cover virtually all of the one-day risk. This reduces both one's incentive to default as well as the loss to the clearinghouse in the event of default. If margin is posted in cash, there is an opportunity cost involved in dealing with futures, because the cash could otherwise be held in the form of an interest-bearing asset. In general, however, a part of the initial margin can be posted in the form of interest-bearing assets, such as Treasury bills. This allows market participants to reduce the opportunity costs associated with margin requirements.

A futures contract is *marked-to-market* daily at the *settlement price*. The settlement price is an exchange's official closing price for the session, against which all positions are marked to market. In a liquid contract this may be the last traded price, but for less liquid contracts it may be an average of the last few traded prices, or a theoretical price based on the traded prices of related contracts.

Every favorable (adverse) move in exchange rates creates a cash inflow (outflow) to the margin account. In order to avoid the cost and inconvenience of frequent but small payments, losses are allowed to accumulate to certain levels before a *margin call* (a request for payment) is issued. These small losses are simply deducted from the initial margin until a lower bound is reached. The lower bound is the maintenance margin. Then, additional money should be added to the account to restore the account balance to the initial margin level. This amount of money, usually paid in cash, is called *variation margin*.

Example VI.3: At the CME, the initial margin on a EUR contract is USD 2,565 and the maintenance margin is USD 1,900. As long as the investor's losses do not exceed USD 665 no margin calls will be issued. If the investor's losses accumulate to USD 665, a variation margin of USD 665 will be added to the account. ¶

The CME sets margin requirements according to a formula that takes into account the volatility of each currency. Currencies with lower volatility have lower margin requirements than currencies with higher volatility (see Table VI.3 below).

Margin requirements and the associated cash flows are a major difference between forward and futures contract. In futures contracts traders realize their gains or losses daily, at the end of each trading day. In forward contracts, however, there are no cash flows until the position is closed, that is, the gains or losses are realized at maturity.

Table VI.3 summarizes the contract terms for the major currency contracts traded on the CME as of December 2015.

TABLE VI.3

Most Active Currency Futures Contracts: Specifications

			Margin Requirements	
Security	Size	Minimum price fluctuation	Initial	Maintenance
AUD	AUD 100,000	.0001 (USD 10.00)	USD 1,890	USD 1,400
BRR	BRR 100,000	.0001 (USD 10.00)	USD 5,280	USD 3,000
CAD	CAD 100,000	.0001 (USD 10.00)	USD 1,705	USD 1,550
CHF	CHF 125,000	.0001 (USD 12.50)	USD 4,950	USD 2,475
GBP	GBP 62,500	.0002 (USD 12.50)	USD 2,035	USD 1,850
JPY	JPY 12,500,000	.000001 (USD 12.50)	USD 2,970	USD 2,700
MXN	MXN 500,000	.000025 (USD 12.50)	USD 2,035	USD 1,850
EUR	EUR 125,000	.0001 (USD 12.50)	USD 3,905	USD 3,550

Note: For certain transactions, usually big transactions, the minimum price fluctuation is cut in half. That is, for a big GBP transaction the minimum price fluctuation can be set at .0001, or USD 6.25.

From now on, this chapter will use the word *futures* to denote both futures and forward contracts.

1.A.2.ii Settlement

Prior to expiration, traders have a number of options to either close out or extend their open positions without holding the trade to expiration.

For those traders who take their contract to expiration, settlement will occur. For FX futures, the last trading day is, in general, the second business day prior to the third Wednesday of the contract month.

The contract stops trading and, thus, it needs a reference price for settlement. The price difference between the expiring contract and the next deferred contract, called *calendar spread*, is used to adjust the final price of the expiring contract. The calendar spread is added to a value weighted average price (VWAP) obtained from the last moments (30") of trading of the expiring contract to get the *settlement price*.

CME FX futures can be cash-settled (BRL, RUB) or physically delivered (GBP, EUR, JPY). We will go over a detailed GBP futures example.

Example VI.4: GBP/USD CME futures
On the Monday preceding expiration, the expiring Dec contract is trading at 1.5530, and the next deferred contract, March, is trading at 1.5610.

The *calendar spread* is -0.0080, or 80 ticks.

GBP futures stop trading at 9:16 AM CT on that Monday. In the final 30" of trading, CME Clearing determines the VWAP for the March contract is 1.5620.

Settlement price of the expiring contract: 1.5620 - 0.0080 = 1.5540.

Then, the short side deposits GBP 62,500 pounds per contract with an approved agent bank. The long position deposits USD 97,125 (=1.5540*62,500) per contract in an approved delivery bank. Cash versus currency are exchanged over bank wire. This process is completed by 10:00 AM CT on the third Wednesday of December, two business days after last trading day.

<u>Note</u>: For cash-settled FX futures, the process is simpler. Any profit or loss is calculated by taking the difference between the final settlement price and the previous day's mark-to-market. ¶

Like any other futures contract, an FX trader with an open position may decide to offset or roll forward a position to avoid expiration and delivery.

1.A.2.iii Newspaper Quotes

Financial newspapers publish daily quotes for futures currency contracts. For example, on November 7, 1994, the *Wall Street Journal* published the following quotes:

CURRENCY

	Open	High	Low	Settle	Change	Lifetime High	Lifetime Low	Open Interest
JAPAN YEN (CME) - 12.5 million yen; $ per yen (.00)								
Dec	1.0288	1.0293	1.0233	1.0290	+.0017	1.0490	.9525	62,308
Mar95	1.0338	1.0380	1.0325	1.0377	+.0019	1.0560	.9680	7,722
June	1.0470	1.0470	1.0455	1.0483	+.0021	1.0670	.9915	765
Sept	....	....	....	1.0485	+.0023	1.0775	1.0200	184

Est vol 24,804; vol Thur 32,689; open int 71,055, -1,467.

The title, JAPAN YEN, shows the size of the contract (12.5m JPY) and states that the prices are in USD cents. In each row, the settlement price (*Settle*) is representative of transaction prices around the close. On November 7, 1994, The settlement price of June 1995 increased .0021 cents, which implies that a holder of a purchase contract has made 12.5Mx(.0021/100) = USD 262.5 per contract and that a seller has lost USD 265.6 per contract. For the June 1995 contract, the *High-Low* range is narrower than for the older contracts, since the June 1995 contract has been trading for little more than four months. *Open interest* reflects the number of outstanding contracts. Notice that most of the trading is in the nearest maturity contract. The line below the price information gives an estimate of the volume traded on Friday, November 7, and the previous day, Thursday, November 6. The WSJ also shows the total open interest across the four contracts and the change in open interest relative to November 6.

1.B The Value of a Forward Contract

In many situations, a buyer or a seller of forward and futures contract might want to close their future commitment before the expiration of the contract. Before expiration, the market value of a forward or futures contract is given by the price at which it can be bought or sold in the market.

Example VI.5: Six month ago, in December, Goyco Corporation, a U.S. firm, sold a one-year JPY forward contract at $F_{0,\text{one-year(Dec)}}$ = .0105 USD/JPY. That is, in June, Goyco Corp. is short a six-month forward contract initiated at the rate of .0105 USD/JPY. Suppose that in June a new six-month forward contract is initiated at the rate of .0102 USD/JPY. Given the new market conditions, Goyco Corp. wants to know the market value of the contract sold six months ago.

An investor can close any outstanding contract by taking an opposite position on a similar contract. For example, Goyco Corp. can close its commitment to sell JPY 12.5 million in December (six months from now) by buying a JPY forward contract with a similar expiration date -in this case, six months. The new contract is traded at the current forward price, $F_{t=6\text{-mo(Jun)},6\text{-mo(Dec)}}$. At expiration Goyco Corp. will receive:

$F_{0,\text{one-year}} - F_{t=6\text{-mo},6\text{-mo}}$ = JPY 12.5M x .0105 USD/JPY - JPY 12.5M x .01025 USD/JPY = USD 3,125
= USD 3,750

Today the value of the forward contract would be given by the discounted value of its forward cash flows. Suppose that the U.S. interest rate is 6%. Then, today's value of the forward contract for Goyco Corp. is:

$$\text{Today's value of forward contract} = \frac{F_{0,\text{one-year}} - F_{6\text{-mo},6\text{-mo}}}{[1 + i_{USD} \times (6\text{-mo}/360)]} = \frac{\text{USD } 3{,}125}{[1 + .06\times(180/360)]} = \text{USD } 3{,}034. \P$$

Let r be the annual discount rate. In general, the value of a forward contract sold at time t that was initiated at time t_0 at a rate $F_{t0,T}$ is equal to

$$\frac{F_{t0,T} - F_{t,T}}{[1 + r \times (T-t)/360]}.$$

On the other hand, the value of a forward contract bought at time t that was initiated at time t_0 at a rate $F_{t0,T}$ is equal to

$$\frac{F_{t,T} - F_{t0,T}}{[1 + r \times (T-t)/360]}.$$

Value of Forward Contract at t_0 and T

When the contract is signed (inception) -that is, at time t_0-, the value of a forward contract is equal to zero. This is true for both sides. This is not surprising, since there is no up-front payment when the contract is signed. The zero value of a forward contract at t_0 can be easily seen from the above formula:

$$\frac{F_{t0,T} - F_{t0,T}}{[1 + r * (T-t_0)/360]} = 0.$$

At expiration -that is, at time T-, the value of a futures value is given by the difference between the spot price and the forward price. For example, the value of a forward sold at expiration is: $F_{t0,T} - S_T$.

1.C Using a Forward/Futures Contract

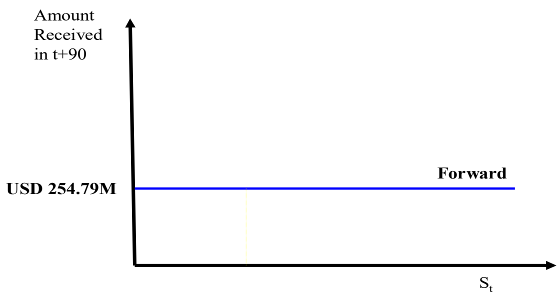
Forward and futures contracts are routinely used to hedge an underlying position or to speculate on the future direction of the exchange rate. In this book we will emphasize hedging. A forward or futures contract can completely eliminate currency risk.

Example VI.6: Iris Oil Inc., a Houston-based energy company, will transfer CAD 300 million to its USD account in 90 days. To avoid currency risk, Iris Oil decides to sell a CAD forward contract. Bank Two offers Iris Oil a 90-day USD/CAD forward contract at $F_{t,90\text{-day}}$ = .8493 USD/CAD.

In 90-days, Iris Oil will receive with certainty:

(CAD 300M) x (.8493 USD/CAD) = USD 254,790,000.

Payoff Diagram for Iris Oil



Note: Now, the exchange rate at time t+90 (S_{t+90}) is irrelevant. ¶

II. Hedging with Futures Currency Contracts

A hedger uses the futures markets to reduce or eliminate the risk of adverse currency fluctuations. Usually, hedging involves taking a position in futures that is the opposite either to a position that one already has in the cash market or to a future cash obligation that one has or will incur. Therefore, the position in the futures market will depend on the position in the cash market.

The *short hedger* sells short in the futures market against a long cash position in the underlying commodity. For example, a typical U.S. short hedger is someone who will receive in the future a payment denominated in a foreign currency. A *long hedger* is long the futures contract and is short a contract denominated in the underlying foreign currency. A short cash position in the underlying currency means that the hedger has a commitment to deliver a given amount of foreign currency. For example, a typical U.S. long hedger is someone who will pay in the future a given amount denominated in a foreign currency.

Hedging with futures is very simple: one takes a position on futures contracts, which is the reverse of the underlying (cash) position. Many argue that the goal of hedging is to construct a *perfect hedge*. A perfect hedge completely eliminates currency risk. In a hedge, risk is eliminated to the extent that the gain (loss) on the futures position exactly compensates the loss (gain) on the underlying (cash) position.

A hedger makes two decisions. First, a hedger has two decide which futures contract to use. Second, a hedger has to determine the *hedge ratio*, that is, the size of the opposite position relative to the size of the underlying position. In the rest of the section, we are going to analyze the determination of the hedge ratio, and second, the contract choice, which in the case of currency markets is easier to make. We will see that determining a hedge ratio that achieves a perfect hedge is, in general, a very difficult task.

2.A Choice of Hedge Ratio: Naïve Approach (Equal Opposite Position)

A simple approach to hedging is to take a position in foreign exchange contracts that is exactly the reverse of the principal being hedged. Under this simple approach, the hedge ratio is equal to one, that is, the size of the hedging position is exactly equal to the size of the underlying position. We will see that, in many situations, this simple approach to hedging is not an optimal approach.

Example VI.7: Long Hedge and Short Hedge.
Situation A: Long hedge.
It is March 1, a U.S. company has to pay JPY 25 million in 180 days. The company decides to hedge currency risk. The company hedges the JPY payables by *buying* JPY September futures for JPY 25 million, that is, two CME contracts.

Situation B: Short hedge.
On September 12, a U.S. investor wants to hedge GBP 1 million invested in British gilts. He fears changes in U.S. interest rates in the next three months and decides to hedge his long GBP position. He sells futures with delivery in December for 1.55 USD/GBP; the spot exchange rate is 1.60 USD/GBP. At the CME one can buy and sell contracts of GBP 62,500 where the futures price is expressed in dollars per pound. That is, the U.S. investor hedges the long GBP position by *selling* 16 CME contracts. ¶

The hedger has a portfolio composed of two positions: the spot position (underlying position) and the futures position (hedging position). We want to explore the effects on the value of the hedger's portfolio of fixing the hedge ratio equal to one.

Let us introduce the following notation:
V_t: value of the portfolio of foreign assets to hedge measured in foreign currency at time (in Example VI.7, Situation B, V_t= GBP 1,000,000).
V_t*: value of the portfolio of foreign assets measured in domestic currency at time t (in Example VI.6, Situation B, $V_t^* = V_t\, S_t$ = USD 1,600,000).

When exchange rates and future rates change, the value of the hedger's portfolio will change. This change in value (profits) will be equal to:

$$\text{profits} = V_tS_t - V_0S_0 + V_0 \times (F_{0,T} - F_{t,T}). \qquad \text{(VI.1)}$$

Let ΔS_t represent changes in S_t and $\Delta F_{t,T}$ represent changes in $F_{t,T}$. Then, if V_t=V_0, the profits are equal to:

$$\text{profits} = V_0 \times (\Delta S_t - \Delta F_{t,T}).$$

If S_t changes by exactly the same amount as $F_{t,T}$ –i.e., ΔS_t=$\Delta F_{t,T}$-, then the hedger's profits will be zero. The hedger has constructed a perfect hedge. The value of the hedger's portfolio is unaffected by changes in exchange rates.

Example VI.8: Calculating the short hedger's profits.
Reconsider Example VI.7, Situation B. Assume that on October 29 the futures and spot exchange rates drop to 1.45 USD/GBP and 1.50 USD/GBP, respectively.

Table VI.4 shows the change in V_t and V_t* and the associated profits from the short hedge.

TABLE VI.4

A. Portfolio Value and Rate of Return

	September 12	October 29	Rates of return (%)
V_t	1,000,000	1,000,000	0.00
V_t*	1,600,000	1,500,000	-6.25
S_t	**1.60**	**1.50**	-6.25
$F_{t,Dec}$	1.55	1.45	+6.25

B. USD Profits from a Short Hedge

Date	Long Position ("Buy")	December Futures ("Sell")
September 12	1,600,000	1,550,000
October 29	1,500,000	1,450,000
Gain	-100,000	+100,000

In USD terms, this loss in the value of the underlying position is USD 100,000, as we see below

$V_t^* - V_0^*$ = (GPB 1,000,000 * **1.50 USD/GBP**) - (GPB 1,000,000 * **1.60 USD/GPB**) = USD -100,000.

On the other hand, the realized gain on the futures contract (hedging position) sale is given by:

$V_0 \times (F_{0,Dec} - F_{t,Dec})$ = GBP 1,000,000 x (1.55 - 1.45) USD/GBP = USD 100,000.

Therefore, the net profit on the hedged portfolio is USD 0, as we see below:

Profit = $V_tS_t - V_0S_0 + V_0 \times (F_{0,Dec} - F_{t,Dec})$ = USD -100,000 + USD 100,000 = USD 0.

Now, suppose the investor, on October 29, liquidates his positions, he receives USD 1,600,000. That is, the investor experiences no loss due to the depreciation of the GBP against the USD. ¶

♦ **Perfect Hedges are Rare**
In Example VI.8 the U.S. investor has constructed a perfect hedge. Two events run in the investor's favor in Example VI.8: the value of the underlying cash position remained the same and the *basis* remained constant –i.e., the spot price and the futures price changed by the same amount (USD .10). As we will see below, maintaining a perfect hedge at any point in time, during the life of the futures contract, is not easy to do. ♦

2.A.1 Changes in V_t

You should note, from formula (VI.1), that the net profits on the hedged position are also a function of V_t. That is, if V_t also changes, the net profit on the hedged position be affected.

Example VI.9: Reconsider Example VI.8. On October 29 we have $F_{t,Dec}$=1.45 USD/GBP and S_t=1.50 USD/GBP. Now, the GBP value of the British gilts rises to 2%. Thus, the USD loss in portfolio value is:

(GBP 1,020,000 * **1.50 USD/GBP**) - (GBP 1,000,000 * **1.60 USD/GBP**) = USD -70,000.

On the other hand, the realized gain on the futures contract sale is still USD 100,000. Therefore, the net profit on the hedged position is USD 30,000. This position is almost perfectly hedged, since the 2% return on the British asset is transformed into a 1.875% return (30,000/1,600,000), despite the drop in value of the GBP.

The small difference between the two numbers is explained by the fact that the investor hedged only the principal (GBP 1 million). The investor did not hedge the price appreciation or the return on the British investment (2%). The 6.25% drop in the GBP value applied to the 2% return exactly equals .125%.

Note: In order to have a perfect hedge, the U.S. investor should hedge the principal and the return on the British gilts, r_{gilts}. If r_{gilts}=.02 is known in advance, a perfect hedge will be achieved by setting the hedge ratio equal to 1.02. ¶

♦ **Efficiency of a Hedge and Returns**

The larger the return (no hedge position), the less efficient the hedge. Let us analyze the result in Example VI.9. Define r_t and r_t*, as the rates of return on V_t and V_t*. The relation between USD and GBP returns on the foreign portfolio is as follows:

$$r_t^* = r_t + s_t\,(1+r_t).$$

Hence, in Example VI.9, -.04375 = .02 - .0625 x (1.02).

The cross-product term (s_t x r_t = 0.125%) explains the difference between the return on the portfolio and the return on the futures position.

Therefore, when the value of the portfolio in local currency fluctuates widely, it is very difficult to maintain a perfect hedge. This is a very common situation, since asset returns are unpredictable. ♦

2.B Choice of a Hedge Ratio: Optimal Hedge Ratio

In Example VI.8, we presented an example where a hedge ratio equal to one delivers a perfect hedge. We should point out two things about that example: (1) V_t remains constant and (2) the spot price and the futures price change by the same amount (.10 USD/GBP). In the previous section, we analyzed the situation where V_t changes. Now, we will analyze the second situation. We will see that the hedge is not fully effective if the difference between the futures price and the price of the underlying asset does not converge smoothly during the life of the futures. Basis risk arises if the difference between the futures price and the spot price deviates from a constant basis per period (in general, per month). That is,

Basis = Futures price - Spot Price = $F_{t,T}$ - S_t.

If the basis remains constant, $r_F = (F_{t,T} - F_{0,T})/S_0$ is equal to the spot exchange rate movement, $s = (S_t - S_0)/S_0$. As seen in Example VI.9, if there is no basis risk, i.e., the basis remains constant, the optimal hedging strategy is to completely hedge the position. In general, if the basis unexpectedly increases (or "weakens"), the short hedger loses. If the basis unexpectedly decreases ("strengthens"), the short hedger gains.

Example VI.10: Reconsider the situation in Example VI.8, Table VI.4, under the following scenarios:
(A) Basis weakens.
Suppose the futures exchange rate drops to **1.50 USD/GBP** (that is, the basis has increased from -5 points to 0 points). The basis has weakened from USD .50 to USD 0. Now, the USD profits from the short hedge are USD -50,000.

Date	S_t	Long Position ("Buy")	$F_{t,Dec}$	December Futures ("Sell")
September 12	**1.60**	1,600,000	1.55	1,550,000
October 29	**1.50**	1,500,000	**1.50**	1,500,000
Gain		-100,000		+50,000

That is, even when V_t remains constant, a hedge ratio equal to one is no longer perfect! In this case, suppose the investor, on October 29, liquidates his positions, he receives USD 1,550,000. That is, the investor experiences a loss of USD 50,000.

(B) Basis strengthens.
Suppose the futures exchange rate drops to **1.40 USD/GBP** (that is, the basis has decreased from -5 points to -10 points). Now, the USD profits from the short hedge are USD 150,000.

Date	S_t	Long Position ("Buy")	$F_{t,Dec}$	December Futures ("Sell")
September 12	**1.60**	1,600,000	1.55	1,550,000
October 29	**1,50**	1,500,000	**1.40**	1,400,000
Gain		-100,000		+150,000

Suppose the investor, on October 29, liquidates his positions, he receives USD 1,650,000. That is, the investor experiences a profit of USD 50,000.

Note: Under scenario A, the investor could have done better by establishing a futures position worth GBP 966,667. This position in futures would have delivered a perfect hedge (value of hedging position = GBP 966,667 * **1.50 USD/GBP** = USD 1,450,000). Similarly, under scenario B, a futures position worth GBP 1,035,714 would have also achieved a perfect hedge. ¶

As Example VI.10 illustrates, when the basis changes, hedgers need to adjust the size of the futures position. A naive portfolio makes the value of the hedger's portfolio dependent on changes in S_t; not an optimal situation.

2.B.1 Basis risk

In Chapter III, we examined the Interest Rate Parity Theorem (IRPT). Futures exchange rates are directly determined by two factors: the spot exchange rate and the interest rate differential between two currencies. For example, for the USD/GBP exchange rate, we have:

$$F_{t,T} = S_t * \frac{(1+ i_{USD} * \frac{T}{360})}{(1+ i_{GBP} * \frac{T}{360})}$$

Note the effective interest rate is a function of time. Let us rewrite the above relation as

$$F_{t,T} = \delta\, S_t, \qquad \text{(VI.2)}$$

Equation (VI.2) shows the relation between movements in the futures price and in the spot price before the expiration date of the futures contract. Equation (VI.2) establishes a linear (proportional) $F_{t,T}$ and S_t. The proportional constant is δ. The coefficient δ is referred to as the futures *delta*. If $\delta > 1$ ($\delta<1$), the futures price will move more (less) than the spot price. Therefore, the hedge should involve a smaller (greater) amount of currency futures than the amount of underlying currency being hedged.

The basis can also be written, for the USD and GBP case, as:

$$\text{Basis} = F_{t,T} - S_t = S_t \; * \frac{(\,i_{USD} - i_{GBP}) * \frac{T}{360}}{(1 + i_{GBP} * \frac{T}{360})}$$

Note that, in general, as i_d and i_f change, the basis will also change. Also, a correlation between currency movements and changes in the interest rate differential will lead to an optimal hedge ratio different from one. This is because a component of the futures return is the change in interest rate differential or basis. Accordingly, the hedge ratio should compensate for this correlation.

Using equation (VI.2) and with a little bit of algebra, we can redefine basis risk as

Basis risk = $F_{t,T} - S_t = (\delta - 1)\, S_t$.

Notice that the futures delta is not constant throughout the life of a futures contract. Thus, as the contract matures $F_{t,T}$ *converges* to S_t. Mathematically, we express this result as:

$$(T \rightarrow 0) \Rightarrow \delta \rightarrow 1.$$

That is, basis risk gets smaller as the contract matures and, thus, the optimal hedge ratio approaches one.

We can also think of the relation between $F_{t,T}$ and S_t in a different way. The correlation between $F_{t,T}$ and S_t. is a function of the futures contract term. Futures prices for contracts near maturity closely follow spot exchange rates because the interest rate differential is a small component of the futures price.

Example VI.11: Consider the futures price of BRR contracts with one, three, and twelve months left until delivery. Let S_t=0.80 USD/BRR, and the interest rates and the calculated values for the futures are as given in TABLE VI.5

TABLE VI.5
Importance of Interest Rate Differentials to Futures Prices

Maturity:	Twelve months	Three months	One month
i_{BRR}	.120	.120	.120
i_{USD}	.040	.040	.040
F(USD/BRR)	0.743	0.784	**0.795**
Basis	-0.057	-0.016	-0.005

One-month calculations (assume a 30-day month):

$$F_{t,T} = S_t + S_t * \frac{(i_{USD} - i_{BRR}) * \frac{T}{360}}{(1 + i_{BRR} * \frac{T}{360})}$$

$$= 0.80 + 0.80 * \frac{-.08 * \frac{30}{360}}{(1 + .12 * \frac{30}{360})} = \mathbf{0.795\ USD/EUR}.$$ ¶

Example VI.11 shows that even though the interest differential is large, its effect on the futures price and the basis is decreasing with maturity. The intuition behind this result is very simple: the spot exchange rate is the driving force behind short-term forward exchange rate movements. This is less true for longer-term forward contracts.

Expected Profits on the Hedge and the Initial Basis

We want to derive the relation between the expected profits on the hedge and the initial basis. We introduce the following additional notation:

n_s: number of units of foreign currency held.(n_s is positive for long positions and negative for short positions).
n_f: number of futures foreign exchange units held (n_f is positive for long positions and negative for short positions). Note that number of contracts is given by n_f/size of the contract.
$\pi_{h,t}$: uncertain profit of the hedger at time t.

The uncertain profit of the hedger at maturity (time T) who holds n_s units of foreign currency and hedges using n_f using futures contracts is:

$$\pi_{h,T} = (S_T - S_0)n_s + (F_{t,T} - F_{0,T})n_f.$$

The expected gain to the hedger over the life of the contract at time t=0 is:

$$E_0(\pi_{h,T}) = (E_0(S_T) - S_0)n_s + (E_0(F_T) - F_{0,T})n_f.$$

If we assume that the current futures price is an unbiased predictor of the futures price at time T, then,

$F_{0,T} = E_0(F_T)$.

At maturity, convergence ensures $E_0(S_T) = E_0(F_T)$. Therefore, the expected gain is:

$E_0(\pi_{h,T}) = (F_{0,T} - S_0) n_s$.

In other words, at time t=0, the expected profit on the hedge is directly proportional to the initial basis.

2.B.2 Derivation of the optimal hedge ratio

To derive the optimal number of futures contracts to hedge a position in a foreign currency, let us rewrite the hedger's profits in terms of profit per unit of the foreign currency position, that is,

$$\pi_{h,T} / n_s = (S_T - S_0) + (F_{t,T} - F_{0,T})\,(n_f/n_s). \qquad \text{(VI.3)}$$

Let h denote the *hedge ratio*, that is, the number of contracts per unit of the underlying position in foreign currency. Now, using Δ to denote changes, we can write equation (VI.3) as

$$\pi_{h,T} / n_s = \Delta S_T + h\, \Delta F_{t,T}. \qquad \text{(VI.4)}$$

A hedger wants to minimize risk. In this case, the hedger wants to minimize the variance of the hedge portfolio profit (σ_h^2), that is, the problem for the hedger is to choose h in such a way that the variability of the hedge portfolio profit is as small as possible. Formally, the hedger problem is:

$$\min_h \; \sigma_h^2 = \sigma_S^2 + h^2\, \sigma_F^2 + 2\, h\, \sigma_{SF}, \qquad \text{(VI.5)}$$

where σ_S^2 is the variance of the spot price change, σ_F^2 is the variance of futures price change and σ_{SF} is the covariance between spot and futures price changes. The value of h that minimizes σ_h^2 is obtained by taking the first derivative of (VI.5) with respect to h and setting it equal to zero:

$$\frac{d\sigma_h^2}{dh} = 2\, h^*\, \sigma_F^2 + 2\, \sigma_{SF} = 0. \qquad \text{(VI.6)}$$

Solving for h^*, the optimal hedge ratio is:

$$h^* = -\,\frac{\sigma_{SF}}{\sigma_F^2}. \qquad \text{(VI.7)}$$

The optimal hedge ratio depends on the covariance between the spot and futures price changes relative to the variance of the futures price change. Note that a covariance over a variance is the estimated slope of a linear –i.e., ordinary least squares (OLS)- regression.

♦ **Remarks on Hedge Ratio Estimates**

(1) The expression for the h^* is the estimated slope coefficient of an OLS regression of the spot price change on the futures price change. Like all slopes, it measures by how many units the dependent variable changes when the independent variable changes by 1 unit.

(2) The OLS estimate of h^* provides an estimate of δ in equation (VI.2):

$$\delta = \frac{(1+ i_d * \frac{T}{360})}{(1+ i_f * \frac{T}{360})} = -\,(1/h^*).$$

It should be clear that, for a perfect hedge, for each unit of an underlying currency you are long (short), you should go short (long) 1/δ units of futures of the same currency. To see this, let POR_t be the value of the portfolio made up by the long and short positions. Also, let ΔS_t and $\Delta F_{t,T}$ represent the change in the spot and futures price, respectively, for a change in the spot rate. Then, the change in the value of the portfolio will be:

$$\Delta POR_t = \Delta S_t + (-1/\delta)\, \Delta F_{t,T} = \Delta S_t - (1/\delta)[\delta \Delta S_t] = 0,$$

provided that the IRPT holds perfectly.

(3) Recall equation (VI.2). When the futures contract is denominated in the same currency as the asset being hedged, we can use IRPT to get the hedge ratio, h^*. As we will see below, however, for situations where the futures contract is denominated in a different currency than the asset being hedged (*cross-hedging*), OLS will provide an estimate of h^*. ♦

Now, consider the third of the previous remarks. It is easy to estimate the hedge ratio using IPRT.

Example VI.12: On December 20, Mr. Krang, a U.S. investor, is long BRR 2,500,000 for six months. Mr. Krang wants to hedge currency risk and therefore for each BRR long, he will sell (1/δ) June EUR futures. Recall that at the CME, the BRR futures contract is for BRR 100,000. Interest rates in the U.S. and Brazil are 4% and 12%, respectively.

Then, assuming 30 days months, δ=.96226, which implies a hedge ratio of 1.03922. Mr. Krang will sell:

$$(-1.03922) * 2{,}500{,}000/100{,}000 = -25.98 \text{ contracts } (\approx 26 \text{ contracts}).$$

Note: The hedge ratio is very close to one. This happens even though the interest rate differential is big, 8%. This is because IRPT defines a very precise relation between $F_{t,T}$ and S_t. There is not a lot of uncertainty about this relation, especially when compared to commodity futures and commodity spot prices, where imbalances between supply and demand, and storage problems usually lead to a significantly higher basis risk. As seen in Table VI.5, basis risk in currency futures tends to be very small. ¶

♦ **Dynamic hedging**

Recall that as $T \rightarrow 0$ (the contract matures), we have $\delta \rightarrow 1$. To have a perfect hedge, at all times, we need to do continuous adjustments to our hedge portfolio.

Example VI.13: In Example VI.12, suppose that in March the June BRR futures delta is δ=.9912. Hence, to obtain a perfect hedge, Mr. Krang now should go short $1/\delta$=1.0089 in June BRR futures for each BrR in the underlying position being hedged.

Therefore, since Mr. Krang is long BRR 2,500,000, he should go short:

(-1.0089) * (2,500,000/105,000) = 25.22 ≈ 25 contracts.

That is, he should go long 1 futures contract. ¶ ♦

2.B.3 OLS estimation of hedge ratios

Consider the following regression equation:

$$\Delta S_t = \mu + \tau\, \Delta F_t + \varepsilon_t, \qquad \text{(VI.8)}$$

where μ and τ are constant parameters and ε_t is the error term. The intercept μ represents an expected return uncorrelated with changes in the futures price. The term $\tau\Delta F_t$ represents the fact that random change in the futures price will be reflected in the spot price according to the slope coefficient, τ. The error term, ε_t, reflects basis risk, which arises from the fact that certain changes in ΔS_t are uncorrelated with ΔF_t. Now, let us go back to equation (VI.4) and substitute in equation (VI.8), that is,

$$\pi_{h,T} / n_s = (\mu + \tau\, \Delta F_T + \varepsilon_T) + h\, \Delta F_T = \mu + (\tau+h)\, \Delta F_T + \varepsilon_T. \qquad \text{(VI.9)}$$

Equation (VI.9) shows that by setting $h = -\tau$ the profit on the hedge portfolio can be made independent of movements in spot and futures prices. For example, if τ=0.5, a one dollar change in the futures price is matched by USD .50 change in the spot price. In this case, h*= -0.5.

A hedge is fully effective only if spot and futures price changes are perfectly correlated. That is, in order to have a perfect hedge we need the error term, ε_t, to be always equal to zero. The degree of efficiency of a hedge is measured by R-squared of the regression (VI.8). Recall that in this case, the R^2 measures how much of the variability of the spot price change is explained by futures price change. Suppose ε_t is always zero, then ΔF_t explains 100% of the variance of ΔS_t. Hence, the hedge is fully effective.

Example VI.14: OLS estimation of h.
Reconsider Example VI.8. We estimate equation (VI.8) using monthly data for USD/GBP spot and futures price changes. We use four years of monthly data for a total of 48 observations. The futures price changes are for the nearby futures contract. The regression results are:

$$\Delta S_t = .001 + .92\, \Delta F_{t,\text{one-month}}, \qquad R^2 = .95.$$

The high R^2 points out the efficiency of the hedge. Changes in futures USD/GBP prices are highly correlated with changes in USD/GBP spot prices.

The hedge ratio is -.92. That is, the number of contracts sold is given by

n_f/size of the contract = h n_s/size of the contract = -.92 x 1,000,000/62,500 = -14.7 ≈ - 15 contracts .

Note: A different interpretation of the R^2: hedging reduces the variance of the cash flows by an estimated 95 percent. ¶

> ♦ **Hedge Ratios and Stationarity**
> The OLS estimate of the hedge ratio is based on past data. The hedge we construct, however, is for a future period, that is, τ estimates are *ex-post*, but hedging decisions are *ex-ante*. Every time we use OLS estimation of hedge ratios we are assuming a stationary relation between ΔF and ΔS. Loosely speaking, under this assumption, the future should be similar to the past. We should be comfortable with this assumption before estimating equation (VI.8). ♦

2.B.4 A different approach: ARCH Models at Work

One limitation of the OLS approach to estimate hedge ratios is that it assumes stationarity of the variance of future price change and the covariance between spot and future price change. The assumption of homoscedasticity, that is, a constant covariance matrix, is a common assumption in time-series. As discussed in chapter V, however, exchange rates and other financial assets are heteroscedastic. That is, variances and covariances of financial assets are time-varying. That is, the covariance matrix changes with time. This finding has implications for hedging since the optimal hedge ratio is a ratio of a covariance relative to a variance. Therefore, it is possible to improve the estimation of the hedge ratio by incorporating a model for time-varying variances, such as the GARCH model.

Example VI.15: Estimation of hedge ratios using GARCH models.
Cannigia Co. wants to hedge GBP 1 million using a forward contract for 6 months. Cannigia Co. uses the following autoregressive model to forecast exchange rates:

$\Delta S_t = S_t - S_{t-1} = a_S + b_S \Delta S_{t-1} + \varepsilon_{St}$,
$\Delta F_t = F_t - F_{t-1} = a_F + b_F \Delta F_{t-1} + \varepsilon_{Ft}$.

The covariance matrix of this bivariate system is given by a 2x2 matrix, Ω_t. The matrix Ω_t is the time-varying covariance matrix of ΔS_t and ΔF_t. The diagonal elements of Ω_t, $\sigma^2_{S,t}$ and $\sigma^2_{F,t}$, represent the variance of ΔS_t and the variance of ΔF_t, respectively. The off-diagonal elements, $\sigma_{SF,t}$, represent the covariance between ΔS_t and ΔF_t.

Each element in the covariance matrix is parameterized as follows:

$\sigma^2_{S,t} = \alpha_{S0} + \alpha_{S1}\, \varepsilon^2_{S,t-1} + \beta_{S1}\, \sigma^2_{S,t-1}$
$\sigma^2_{F,t} = \alpha_{F0} + \alpha_{F1}\, \varepsilon^2_{F,t-1} + \beta_{F1}\, \sigma^2_{F,t-1}$

$\rho = \sigma_{SF,t} / \{\sigma^2_{F,t}\,\sigma^2_{S,t}\}^{1/2}$.

Note that the correlation coefficient, ρ, is constant. That is, ρ does not depend on time t. This version of the multivariate GARCH model is called constant correlations GARCH model.

You work for Cannigia. You estimate the above system and you get the following estimates:
a_S=.004; b_S=.32; a_F=.006; b_F=.15;
α_{S0}=.22; α_{S1}=.25; β_{S1}=.83; α_{F0}=.32; α_{F1}=.09; β_{F1}=.87; ρ=.56.

You are given the following data: spot rates (S_t), 30-day forward rates ($F_{t,one\text{-}month}$) for April, May, June, July, August, and September, and initial estimates for both variances in June.

S_A=1.59; S_M=1.61; S_J=1.65; S_J=1.69; S_A=1.72; S_S=1.73;
F_A=1.60; F_M=1.61; F_J=1.64; F_J=1.65; F_A=1.70; F_S=1.73;
$\sigma^2_{S,June}$=.14; $\sigma^2_{F,June}$=.11.

With these estimates, we have the following equations:

$\Delta S_t = .004 + .32\ \Delta S_{t-1} + \varepsilon_{St}$,
$\Delta F_t = .006 + .15\ \Delta F_{t-1} + \varepsilon_{Ft}$.

$\sigma^2_{S,t} = .22 + .25\ \varepsilon^2_{S,t-1} + .83\ \sigma^2_{S,t-1}$
$\sigma^2_{F,t} = .32 + .09\ \varepsilon^2_{F,t-1} + .87\ \sigma^2_{F,t-1}$
$\sigma_{SF,t} = .56\ \{\sigma^2_{F,t}\,\sigma^2_{S,t}\}^{1/2}$.

At the end of August, you constructed your hedge ratio ($h_{Sep}=\sigma_{SF,Sep}/\sigma^2_{F,Sep}$). Now, at the end of September, you are required to construct your hedge ratio for October, that is, you want h_{Oct}.

Steps to calculate the hedge ratio for July, h_{July}:
(1) Calculate errors: Actual realization - Expected (Forecasted) value
$\varepsilon_{St} = \Delta S_t - \Delta S_t^F = \Delta S_t - (.004 + .32\ \Delta S_{t-1})$
$\varepsilon_{Ft} = \Delta F_t - \Delta F_t^F = \Delta F_t - (.006 + .15\ \Delta F_{t-1})$

$\varepsilon_{SJune} = \Delta S_{June} - \Delta S_{June}^F = .04 - (.004 + .32 \text{ x } .02) = .04 - .104 = .0296$
$\varepsilon_{FJune} = \Delta F_{June} - \Delta F_{June}^F = .03 - (.006 + .15 \text{ x } .01) = .03 - .0075 = .0225$

(2) Construct forecast variance for July (using June's information).
$\sigma^2_{S,t} = .22 + .25\ \varepsilon^2_{S,t-1} + .83\ \sigma^2_{S,t-1}$
$\sigma^2_{F,t} = .32 + .09\ \varepsilon^2_{F,t-1} + .87\ \sigma^2_{F,t-1}$

$\sigma^2_{S,July} = .22 + .25\ \varepsilon^2_{S,June} + .83\ \sigma^2_{S,June} = .22 + .25\ (.0296)^2 + .83\ (.14) = .3364$
$\sigma^2_{F,July} = .32 + .09\ \varepsilon^2_{F,June} + .87\ \sigma^2_{F,June} = .32 + .09\ (.0225)^2 + .87\ (.11) = .4157$

(3) Calculate forecast for covariance and hedge ratio
$\sigma_{SF,t} = .56\ \{\sigma^2_{F,t}\,\sigma^2_{S,t}\}^{1/2}$.
$h_t = -\ \sigma_{SF,t}/\sigma^2_{F,t}$

$\sigma_{SF,July} = .56\ \{\sigma^2_{F,July}\ \sigma^2_{S,July}\}^{1/2} = .56\ \{.3364 \times .4157\}^{1/2} = .2094$
$h_{July} = -\ \sigma_{SF,July}/\sigma^2_{F,July} = -.2094/.4157 = -.5037$

Your estimated hedge ratio for July is h_{July}= -0.5037. That is, the number of contracts you advise Cannigia to be short in July is [-.5037x(1,000,000/62,500)]= -8.06 (short 8 contracts).

To calculate the hedge ratio for the other months recursively repeat the steps 1 to 3. You should get:

	ΔS_t^F	$\varepsilon_{S,t}$	$\sigma^2_{S,t}$	ΔF_t^F	$\varepsilon_{F,t}$	$\sigma^2_{F,t}$	$\sigma_{SF,t}$	h_t
June	.0104	.0296	.1400	.0075	.0225	.1100	.0695	-.6318
July	.0168	.0232	.3364	.0105	-.0005	.4157	.2094	-.5037
August	.0168	.0132	.4994	.0075	.0425	.6817	.3267	-.4793
September	.0136	-.0036	.6345	.0135	.0165	.9132	.4263	-.4668
October	...	...	.7467	...	...	1.1145	.5109	-.4584

Your estimated hedge ratio for October is h_{Oct}=-.4668. That is, the number of contracts you advise Cannigia to be short in October is [-.4584x(1,000,000/62,500)]=-7.33 (short 7 contracts).

Since in September the number of contracts shorted by Cannigia was also 7 (actually, 7.47), at the end of September, you advise Cannigia to keep all contracts open. ¶

2.C Choice of Futures Contracts

In the forward market a party can tailor the amount, date, and the currency to a given exposed position, this is not always possible in the futures market. There are three problems associated with hedging in futures markets:

(1) The contract size is fixed and is unlikely to match the cash position to be hedged.
(2) The expiration dates of futures contracts rarely match those for the currency receivables or payables that the contract is meant to hedge.
(3) The choice of underlying assets in the futures market is limited, and the currency one wishes to hedge may not have a futures contract.

There is very little a hedger can do with respect to (1) in the futures market. Note that in the forward market, in general, contract size is not a problem. With respect to (2) and (3) hedgers can construct imperfect hedges. An imperfect hedge is called a *delta-hedge* when the maturities do not match, and is called *cross-hedge* when the currencies do not match.

2.C.1 Delta-hedging

Suppose a hedger has decided to establish a GBP futures position to hedge a foreign currency cash position. Now, the hedger has to decide on which contract month to use. It might seem logical that the when the expiration of the underlying position corresponds to a delivery month, the contract with that delivery month is selected. Many times, however, a contract with a later delivery month is chosen. This is because futures prices are in some instances very volatile during the delivery month. Other times, hedgers want to minimize basis risk. As we have seen in Table VI.5, near month currency futures contracts track the behavior of the spot exchange rates better and, therefore, they have the higher correlation with spot rates. Thus, near month currency futures are preferable since they minimize the basis variation.

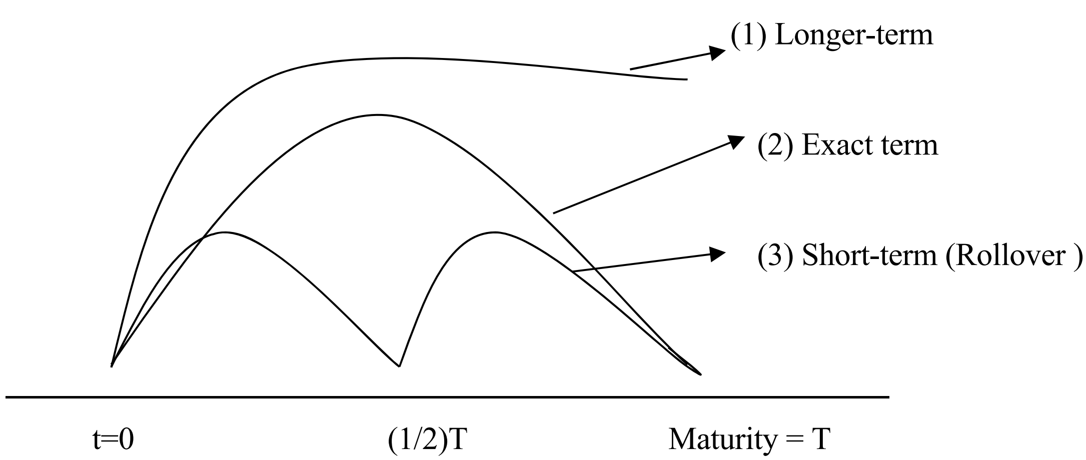
In many situations, basis risk is not the only factor to consider. Liquidity considerations are important. Sometimes, liquidity and basis risk should be treated as trade-off. For short-term currency positions, there is no trade-off: short-term futures contracts minimize basis variation and also have greater trading volume (are more liquid) than long-term contracts. For medium and long-term currency position, however, there is often a trade-off. For example, suppose a hedger needs to establish a position in a contract whose expiration cycle is a year or more in the future. Since the futures price and the spot price converge at delivery, basis risk can be minimized or eliminated by matching spot and futures long-term obligations. Liquidity, however, in this situation is a major consideration. It is common to find that the size of the position you want to establish is too big for the level of open interest now held in your preferred expiration month. It is common to find, for long-term futures, wide bid-ask spreads, which can make the cost of a hedge very expensive. A solution to this illiquidity problem is to establish a position in the nearest contract month. Once the delivery cycle is near, all outstanding futures contracts are closed and, then, *rolled forward* to the next expiration month. Rolling forward, however, often exposes the hedger to basis risk. In addition, transaction costs are greater when futures positions are closed and re-established.

In summary, medium- and long-term hedgers can select from three basic contract terms:

(1) Short-term contracts, which must be rolled over at maturity.
(2) Contracts with a matching maturity.
(3) Longer-term contracts with a maturity extending beyond the hedging period.

Graph VI.2 depicts three such hedging strategies for an expected hedge period of six months.

GRAPH VI.2
Three Hedging Strategies for an Expected Hedge Period of T months



If there is uncertainty regarding the date of a cash obligation, a hedger will not be able to match maturities. In this case, a hedger usually prefers a rolling forward approach to hedge a cash position with near month contracts. Even though a more distant contract might reduce transaction costs, the minimization of basis risk tends to be the main consideration.

Longer hedges can be built using currency swaps, which can be negotiated with long horizons. Frequently, corporations use currency swaps to manage the currency exposure of their assets and liabilities. Portfolio managers, on the other hand, usually take a shorter horizon. In Chapter XIV we will study currency swaps.

2.C.1.i Delta-hedging-Rollover

Rollover occurs when a trader closes out a position in an expiring contract ("the *front month*") and simultaneously reestablishes the same position in a future month. This procedure ("a *roll*") extends the expiration of a position.

A roll is usually carried out shortly before expiration of the initial contract. The gain or loss on the original contract will be settled by taking the difference between the price on the day the roll is executed and the previous day's mark-to-market.

Example VI.16: An FX trader is long two GBP Dec futures trading at USD 1.5530 on December 11 (Friday). The Dec futures expires on the 3rd Wednesday of December, say December 16. This trader closes the long position on December 11 and, simultaneously enters into a March futures at the current market rate, USD 1.5620.

Suppose on December 10, the Dec futures was mark-to-market at USD 1.5525. That is, there is a gain for the long side of USD .0005 per GBP in the long position. Then, on December 11, the long side receives USD 62.50 (=.0005*62500*2) when the Dec futures position is closed. Simultaneously, the FX trader opens a new long position, with two GBP March futures at USD 1.5620. ¶

2.C.2 Cross-Hedging

When a hedger has a cash position on a foreign currency on which a futures contract is traded, it is almost always preferable to hedge with that contract, since futures and spot prices of the same currency have the highest correlation. Futures and forward currency contracts, however, are only actively traded for the major currencies. International portfolios are often invested in assets in Hungary, India, Thailand, Peru, and other countries where futures and forward contracts are either not traded or very illiquid in the domestic currency. In these situations, hedgers try to establish futures positions using closely linked and highly correlated currencies. For example, a U.S. investor could use EUR futures to hedge a currency risk on Hungarian stocks, since the Hungarian forint (HUF) and the EUR are strongly correlated.

Investment managers, with cash positions in many foreign currencies, sometimes worry about the depreciation of only one or two currencies in their portfolio and, therefore, hedge currency risk selectively. Other times they worry about the depreciation of the domestic currency relative to all foreign currencies and, then, hedge all currency risk in their portfolios.

Example VI.17: The strong USD appreciation from February 2012 to March 2020 (close to 40%) was realized against all currencies. This domestic currency appreciation induced a negative currency contribution on all foreign portfolios. ¶

A complete foreign currency hedge can be achieved by hedging the investment in each foreign currency. But this is difficult -and could be very expensive- for many currencies. In a portfolio with assets in many currencies, the residual risk of each currency gets partly diversified away. Optimization techniques can be used to construct a hedge with futures contracts in only a few currencies (JPY, EUR, and GBP). Once a decision has been taken to (cross) hedge with only a few currencies, the manager has to decide the number of contracts needed to hedge her foreign currency exposure. A manager can use OLS estimates of hedge ratios.

Example VI.18: Ruggieri SA, a U.S. firm, has to pay **HUF 10 million** in 90 days. Since there are no futures contracts on the HUF, Ruggieri SA decides to buy two other contracts on currencies that are highly correlated to the HUF: the EUR and the GBP. In order to calculate the appropriate hedge ratios, Ruggieri SA regresses USD/HUF changes against a constant, USD/EUR 3-mo. futures changes, and USD/GBP 3-mo. futures changes. This regression produces the following output:

$$\Delta S_{USD/HUF} = \underset{(0.59)}{0.39} + \underset{(10.61)}{0.84}\, \Delta F_{USD/EUR} + \underset{(6.33)}{0.76}\, \Delta F_{USD/GBP}, \qquad R^2 = 0.81.$$

The high t-statistic (in parentheses) and the high R^2 confirm that the EUR and GBP futures prices are correlated with the HUF spot rate. The exchange rates are .0043 EUR/HUF and 270 HUF/GBP (1/270=.0037 GBP/HUF.

Then, the number of contracts bought by Ruggieri SA is given by:

EUR: (**-10,000,000** * .0043/125,000) * (-0.84) = 2.89 ≈ 3 contracts.
GBP: (**-10,000,000** * .0037/62,500) * (-0.76) = 3.28 ≈ 3 contracts. ¶

The stability of the estimated hedge ratios is of crucial importance in establishing effective hedge strategies especially when cross-hedging is involved. Empirical studies indicate that hedges using futures contracts in the same currency as the asset to be hedged are very effective but that the optimal hedge ratios in cross-hedges that involve different currencies are quite unstable over time.

III. Looking Ahead: Currency Options

We have gone over one basic hedging tool: currency futures. Currency futures set a price for forward delivery of a currency. If hold until they mature, currency futures completely eliminate the uncertainty associated with having assets and liabilities denominated in foreign currency. The next chapter introduces currency options as a hedging tool. Options are more flexible contracts, which can place a cap or a floor on the future value of an asset and liability denominated in foreign currency. Therefore, options reduce currency risk, but do not completely eliminate it.

Interesting readings

Parts of Chapter VI were based on the following books:

International Financial Markets, by J. Orlin Grabbe, published by McGraw-Hill.

International Financial Markets and The Firm, by Piet Sercu and Raman Uppal, published by South Western.

International Investments, by Bruno Solnik, published by Addison Wesley.

Exercises:

1.- You are long GBP 312,500 and you go short a number of forward contracts to offset your long position. The exchange rate is 1.55 USD/GBP. The futures price is 1.61 USD/GBP. One month later the spot price is 1.59 USD/GBP and the futures price is 1.62 USD/GBP. Was the hedge perfect? If not, calculate the net profit of the hedge portfolio.

2.- On January 19, Ms. Sternin, a U.S. investor, wants to hedge a short Bund (German government bond) position valued at EUR 2 million. Ms. Sternin uses a hedge ratio equal to 1 (h=1). She decides to use futures with delivery in September for 1.17 USD/EUR; the spot exchange rate is 1.20 USD/EUR.

i.- Assume that on April 17 the futures and spot exchange rates drop to 1.155 USD/EUR and 1.160 USD/EUR, respectively. Calculate the short hedger's profits/losses. Is h=1 a perfect hedge?
ii.- Now, assume that on April 17 the futures and spot exchange rates drop to 1.155 USD/EUR and 1.153 USD/EUR, respectively. Calculate the short hedger's profits/losses. Is h=1 a perfect hedge?
iii.- Explain the different results obtained under (i) and (ii).

3.- Ms. O'Neil, a U.S. investor has to pay CZK 40,000,000 in 180 days (CZK = Czech coruna). She decides to hedge her position using EUR and GBP futures contracts. The exchange rates are .95 USD/EUR, 1.45 USD/GBP, and 42 CZK/USD. She runs an OLS regression and obtains the following estimates:

$$\Delta S_{USD/CZK} = \underset{(0.20)}{.087} + \underset{(3.13)}{.94}\, \Delta F_{USD/EUR} + \underset{(5.43)}{0.81}\, \Delta F_{USD/GBP}, \qquad R^2 = 0.77.$$

a.- How many EUR and GBP contracts should Ms. O'Neil buy to obtain an optimal hedge?

b.- Suppose three months later, Ms. O'Neil re-estimates the above equation. The exchange rates are .90 USD/EUR, 1.41 USD/GBP, and 49 CZK/USD. Her new estimates are:

$$\Delta S_{USD/CZK} = \underset{(0.78)}{.109} + \underset{(4.78)}{.97}\, \Delta F_{USD/EUR} + \underset{(5.92)}{0.90}\, \Delta F_{USD/GBP}, \qquad R^2 = 0.83.$$

Based on the new estimates, how many contracts should Ms. O'Neil buy or sell?

4.- A U.S. investor holds a portfolio of Japanese stocks worth JPY 200 million. The spot exchange rate is JPY/USD=100 and the three-month forward exchange rate is JPY/USD=105. Our investor fears that the Japanese will depreciate in the next month, but wants to keep the Japanese stocks. What position can the investor take based on three-month forward exchange rate contracts? List all the factors that will make the hedge imperfect.

5.- A Cypriot investor holds a portfolio of Japanese stocks similar to that of our U.S. investor. The current three-month Cypriot Pound (CYP) forward exchange rate is CYP/USD=.5. What position should the Cypriot investor take to hedge the JPY/CYP exchange risk?

6.- A U.S. investor is attracted by the high yield on GBP bonds but is worried about a GBP depreciation. The current market rates are as follows:

	U.S.	U.K.
Bond yield (%)	7	8
Three-month interest rate (%)	6	10
S_t = 1.70 USD/GBP		

A bond dealer has repeatedly suggested that the investor invest in hedged foreign bonds. This strategy can be described as the purchase of foreign currency bonds (here, GBP bonds) with simultaneous hedging in the short-term forward of futures currency markets. The currency hedge is rolled over when the forward or futures contract expires.
(a) What is the current three-month forward exchange rate (USD/GBP)?
(b) Assuming a GBP 2 million investment in British bonds, how would you determine the exact ratio necessary to minimize the currency influence?
(c) When will this strategy be successful (compared to a direct investment in U.S. bonds)?

7.- Futures and forward currency contracts are not easily available for most currencies. Many currencies, however, are closely correlated. A U.S. investor has a portfolio of Hungarian stocks that she wishes to hedge against currency risks. No futures contracts are traded on the Hungarian forint (HUF), so she decides to use euro (EUR) futures contracts traded in Chicago. Here are the data:

Value of the portfolio	HUF 100 million.
Spot exchange rates	HUF/USD = 210.60
	USD/EUR = 1.10.
Futures price (contract of EUR 125,000)	USD/EUR = 1.21.

How many EUR contracts should our U.S. investor trade?

8.- Suppose you want to hedge a long position on Swiss Francs (CHF) for one year. The annual CHF interest rate is 6% and the annual U.S. interest rate is 7.2%. How many contracts do you need to hedge CHF 7 million? Is the CHF a premium currency?