CHAPTER III

EXCHANGE RATES, INTEREST RATES, PRICES AND EXPECTATIONS

This chapter presents simple models of exchange rate determination. These models apply arbitrage arguments in different contexts to obtain equilibrium relations that determine exchange rates. In this chapter, we define arbitrage as the activity that takes advantages of pricing mistakes in financial instruments in one or more markets, facing *no risk* and using *no own capital*.

The no own capital requirement is usually met by buying and selling (or borrowing and lending) the same or equivalent assets or commodities. The no risk requirement is usually met by doing the buying and selling (or borrowing and lending) simultaneously. Obviously, arbitrageurs will engage in this activity only if it is profitable, which means there should be a pricing mistake. Financial markets are said to be in equilibrium if no arbitrage opportunities exist.

The equilibrium relations derived in this chapter are called parity relations. Because of the underlying arbitrage argument, parity relations establish situations where economic agents are indifferent between two financial alternatives. Thus, parity relations provide an "equilibrium" value or a "benchmark." These benchmarks are very useful. For example, based on a parity benchmark, investors or policy makers can analyze if a foreign currency is "overvalued" or "undervalued."

I. Interest Rate Parity Theorem (IRPT)

The IRPT is a fundamental law of international finance. Open the pages of the *Wall Street Journal* and you will see that Argentine bonds yield 10% and Japanese bonds yield 1%. Why wouldn't capital flow to Argentina from Japan until this differential disappeared? Assuming that there are no government restrictions to the international flow of capital or transaction costs, the barrier that prevents Japanese capital to fly to Argentina is currency risk. Once yens are exchanged for pesos, there is no guarantee that the peso will not depreciate against the yen.

There is, however, one way to guarantee a conversion rate between the peso and the yen: a trader can use a forward foreign currency contract. Forward foreign currency contracts eliminate currency risk. A forward foreign currency contract allows a trader to compare domestic returns with foreign returns translated into the domestic currency, without facing currency risk. Arbitrage will ensure that both known returns, expressed in the same currency, are equal.

That is, world interest rates are linked together through the currency markets. The IRPT embodies this relation:

If the interest rate on a foreign currency is different from that of the domestic currency, the forward exchange rate will have to trade away from the spot exchange rate by a sufficient amount to make profitable arbitrage impossible.

1.A Covered interest arbitrage

Covered interest arbitrage is the activity that forces the IRPT to hold. Assume that there are no barriers to the free movement of capital across international borders –i.e., there is *perfect capital mobility*. Consider the following notation:

i_d = domestic nominal risk-free interest rate for T days.
i_f = foreign nominal risk-free interest rate for T days.
S_t = time t spot rate (direct quote: units of domestic currency per unit of foreign currency).
$F_{t,T}$ = forward rate for delivery at date T, at time t.

Now, consider the following strategy:

(1) At time 0, we borrow from a foreign bank one unit of a foreign currency for T days. At time T, we should pay the foreign bank ($1+i_f$ x T/360) units of the foreign currency.

(2) At time 0, we exchange the unit of foreign currency for domestic currency, that is, we get S units of domestic currency.

(3) At time 0, we deposit S_t units of domestic currency in a domestic bank for T days. At time T, we should receive from the domestic bank $S_t(1+i_d$ x T/360) units of domestic currency.

(4) At time 0, we also enter into a T-day forward contract to buy foreign (sell domestic currency) at a pre-specified exchange rate ($F_{t,T}$).

At time T, we exchange the $S_t(1+i_d)$ units of domestic currency for foreign currency, using the pre-specified exchange rate in the forward contract. That is, we get $S_t(1+i_d$ x T/360)/$F_{t,T}$ units of foreign currency.

This strategy will not be profitable if at time T, what we receive in units of foreign currency is equal to what we have to pay in units of foreign currency. Since arbitrageurs will be searching for an opportunity to make a risk-free profit, arbitrage will ensure that

$$\frac{S_t * (1 + i_d * \frac{T}{360})}{F_{t,T}} = (1 + i_f * \frac{T}{360})$$

Solving for $F_{t,T}$, we obtain the following expression for the IRPT:

$$F_{t,T} = S_t * \frac{(1+ i_d * \frac{T}{360})}{(1+ i_f * \frac{T}{360})} \qquad \text{(III.1)}$$

The IRP theory, also called *covered IRPT*, as presented above was first clearly exposed by John Maynard Keynes (1923).

Notes:
◊ Steps **(2)** and **(4)** simultaneously done produce a FX swap transaction! In this case, we buy the FC forward at $F_{t,T}$ and go sell the FC at S_t. We can think of ($F_{t,T}$ - S_t) as a profit from the FX swap.
◊ We get the same IRPT equation if we start the covered strategy by **(1)** borrowing DC at i_d; **(2)** exchanging DC for FC at S_t; **(3)** depositing the FC at i_f; and **(4)** selling the FC forward at $F_{t,T}$.

If the forward rate is not set according to (III.1), arbitrage will occur. If a bank trader quotes a forward rate that violates (III.1), other traders, immediately, will take advantage of the arbitrage opportunity. How can a bank make sure that other banks do not profit from its forward quotes? The answer is very easy: use (III.1) to price forward foreign currency contracts.

Example III.1: The IRPT at work.
A Japanese company wants to calculate the one-year forward JPY/USD rate. With spot yen selling at 150 JPY/USD and the JPY annual interest rate equal to 7% and the USD annual interest rate equal to 9%, the one-year forward rate should be:

$$F_{t,1-year} = S_t * \frac{(1+ i_d\,)}{(1+ i_f\,)}) = 150\,\frac{\text{JPY}}{\text{USD}} * \frac{(1+.07)}{(1+.09)} = 147.25 \text{ JPY/USD.}$$

Now, suppose instead that the IRPT is violated. For example, Bertoni Bank is quoting the forward rate for delivery in one-year at time t at $F_{t,\text{one-year}}$=140 JPY/USD. Arbitrageurs will use covered interest arbitrage to take advantage of this situation.

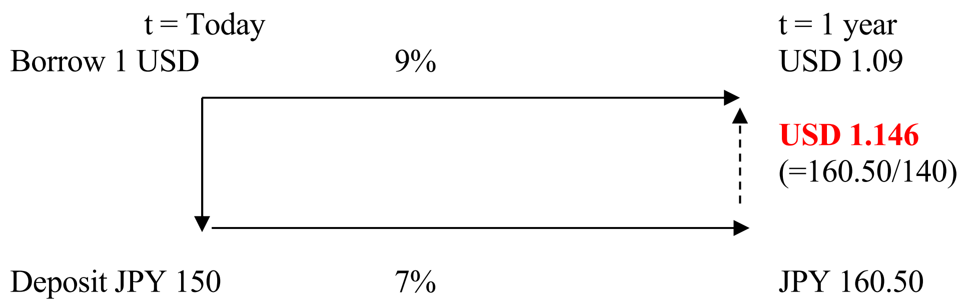
The forward rate, $F_{t,\text{one-year}}$=140 JPY/USD, is less than what the arbitrage-free valuation should be. That is, the forward JPY is currently overvalued. Therefore, an arbitrageur would like to take advantage of this overvaluation of the forward JPY.

A covered interest arbitrage strategy works as follows:
(1) Borrow one USD from a U.S. bank for one year.
(2) Exchange the USD for JPY 150
(3) Deposit the JPY 150 in a Japanese bank for one year.
(4) Sell JPY (Buy USD) forward to Bertoni Bank at the forward rate 140 JPY/USD.

For example, a U.S. arbitrageur simultaneously does the following steps:
1) She borrows USD 1 for a year at 9% (and she will pay back USD 1.09 at the end of the year).
2) She takes this USD 1 and buys JPY 150.
3) She lends the JPY 150 for a year at 7% (and she receives JPY 160.50 at the end of the year).
4) Simultaneously, she buys a one year forward contract at the exchange rate of 140 JPY/USD.

Her cash flows at the end of the year:
- She pays USD 1.09 to close loan in Step 1.
- She exchanges JPY 160.5 for USD using the 140 JPY/USD forward rate, getting USD 1.146 (=160.5/140).

Graphically,



After one year, the U.S. arbitrageur will realize a risk-free profit of **USD. 056** per USD borrowed. Arbitrageurs will take advantage of this situation. Bertoni Bank will soon realize its forward quote is not correct, because it will receive an unusually large number of "sell JPY forward" orders. Arbitrage of this type will ensure that $F_{t,\text{one-year}}$=147.25 JPY/USD. ¶

We can manipulate (III.1) to obtain a simpler expression for the IRPT. By dividing both sides of (III.1) by S_t, we obtain:

$$\frac{F_{t,T}}{S_t} = \frac{(1+ i_d * \frac{T}{360})}{(1+ i_f * \frac{T}{360})}$$

Now, we subtract 1 from both sides, giving us:

$$\frac{F_{t,T}}{S_t} - 1 = \frac{F_{t,T} - S_t}{S_t} = \frac{\left(1+ i_d * \frac{T}{360}\right)}{\left(1+ i_f * \frac{T}{360}\right)} - 1 = \frac{(i_d - i_f) * \frac{T}{360}}{\left(1+ i_f * \frac{T}{360}\right)}.$$

The above expression can be approximated by

$$\frac{F_{t,T} - S_t}{S_t} \approx (i_d - i_f) * \frac{T}{360} \qquad \text{(III.2)}$$

Example III.2: Interest differentials and the linear approximation.
Go back to Example I.11. The USD/GBP spot rate is S_t=1.62. The 180-day USD/GBP forward rate is $F_{t,180}$=1.6167. That is, the USD is expected to appreciate with respect to the GBP in the next months. The forward price of the GBP appears to be decreasing at a rate of about .4% a year (-0.204% in 180

days). This suggests that the short-term risk-free annual interest rate is about .4% lower in the U.S. than in the U.K. ¶

The approximation in (III.2) is quite accurate when i_d and i_f are small, usually lower than 10%. The above equation gives us a linear approximation to formula (III.1):

$$F_{t,T} \approx S_t * [1 + (i_d - i_f) * \frac{T}{360}]$$

The above formulae assume discrete compounding. We can also use the following continuous formulation:

$$F_{t,T} = S_t \exp[(i_d - i_f) * T/360].$$

♦ **IRPT: Remark**
IRPT is a mathematical relation. You can think of the forward rate as an identity linking interest rate differentials and currency rates. The economic intuition of this mathematical relation is simple: the forward rate is the rate that eliminates an arbitrage profit. ♦

1.A.1 IRPT: Assumptions

Behind the covered arbitrage strategy -steps **(1)** to **(4)**-, we have implicitly assumed:
(1) Funding is available. That is, step **(1)** can be executed.
(2) Free capital mobility. No barriers to international capital flow –i.e., step **(2)** and later **(4)** can be implemented.
(3) No default/country risk. That is, steps **(3)** and **(4)** are safe.
(4) Absence of significant frictions. Typical examples: transaction costs & taxes. Small transactions costs are OK, as long as they do not impede arbitrage.

We are also implicitly assuming that the forward contract for the desired maturity T is available. This may not be true. In general, the forward market is liquid for short maturities (up to 1 year). For many currencies, say from emerging market, the forward market may be liquid for much shorter maturities (up to 30 days).

1.B The Forward Premium and the IRPT

Recall the definition of forward premium, *p*:

$$p = \frac{F_{t,T} - S_t}{S_t} * \frac{360}{T}$$

Using IRPT derived above, we get that the forward premium is a function of the interest rate differential.

$$p = \frac{F_{t,T} - S_t}{S_t} * \frac{360}{T} = \frac{(i_d - i_f)}{\left(1 + i_f * \frac{T}{360}\right)}.$$

If the domestic interest rate is higher (lower) than the foreign interest rates, the forward premium is positive and the foreign currency is called a *premium currency*.

We have seen that the difference between the forward and the spot exchange rates is called forward points (sometimes this difference is also called the *swap rate*). Using the IRPT, T-days forward points are calculated as:

$$F_{t,T} \; - \; S_t = S_t * [\frac{\left(1 + i_d * \frac{T}{360}\right)}{\left(1 + i_f * \frac{T}{360}\right)} - 1].$$

That is, the forward points are a function of the interest rate differential. If the domestic interest rate is higher (lower) than the foreign interest rates, the forward points will be added (subtracted) to the spot rate.

Example III.3: Using the information from Example III.1 we can calculate the one-year forward points as follows:

$$150 \, \frac{\text{JPY}}{\text{USD}} * [\frac{(1 + .07)}{(1 + .09)} - 1] = \text{- 2.7523 JPY/USD.} \P$$

Consider (III.2). That is,

$$\frac{F_{t,T} - S_t}{S_t} \approx (i_d \; - i_f) * \frac{T}{360}$$

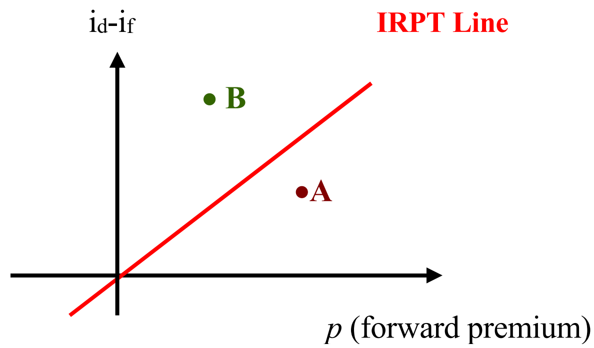
Suppose, now, that we consider a 360-day forward contract (i.e., T=360), then we can approximate the above equation as:

$$p \approx i_d \; - i_f$$

That is, covered arbitrage forces the forward premium to be approximately equal to the interest rate differential. In equilibrium, the forward premium exactly compensates the interest rate differential. Under this equilibrium condition, there are no arbitrage opportunities and no capital flows moving from one country to another due to covered arbitrage strategies.

Exhibit III.1 presents the relation between the forward premium and interest rate differentials in equilibrium.

Exhibit III.1
IRPT Line



If p > i_d -i_f, then domestic capital will fly to the foreign economy. That is, what an investor loses on the lower interest rate from the foreign investment is more than compensated by the high forward premium. Therefore, a point like **A** in the above graph represents a situation where there are capital outflows from the domestic economy. Note that covered arbitrage strategy will affect i_d, i_f, S_t, and $F_{t,T}$. The (i_d-i_f) will clearly increase: domestic interest rates will tend to increase (higher demand for domestic loans); while foreign interest rates will tend to decrease (higher bank deposits in the foreign country). On the other side, the forward premium will clearly decrease: the exchange rate will tend to increase (higher demand for the foreign currency); while the forward rate will decrease (higher foreign currency forward sales). Thus, arbitrageurs will force the equilibrium back to the IRPT line.

On the other hand, if p < i_d -i_f, then foreign capital will fly to the domestic economy. That is, what an investor makes on the high interest rate from the domestic investment is more than what the investor gets by investing in the covered foreign investment. That is, a point like **B** in the above graph represents a situation where the domestic economy experiences capital inflows. Similar to the previous case, covered interest strategies will move the economy from **B** to the IRPT line.

Example III.4: Suppose you are given the following data -taken from Example III.1:
S_t = 150 JPY/USD
$i_{JPY,1\text{-}yr}$ = 7%
$i_{USD,1\text{-}yr}$ = 9%
$F_{t,1\text{-}yr}$ = 140 JPY/USD.

With this information, we calculate p and the interest rate differential:

p = (140 - 150)/150 = -.06667 (p < 0, a discount)
i_{JPY} - i_{USD} = .07 - .09 = -.02.

Since p < i_{JPY} - i_{USD}, we expect foreign capital to fly to Japan (the domestic country) to buy Japanese assets (we are in a point like B, in Exhibit III.1). For instance, U.S. investors will buy Japanese government bonds or bank deposits, which is consistent with the second part of Example III.1. ¶

1.C IRPT with Bid-Ask Spreads

As illustrated in Example I.6, exchange rates are prices quoted with bid-ask spreads. Let $S_{bid,t}$ and $S_{ask,t}$ be the bid and asked domestic spot rates. Let $F_{bid,t,T}$ and $F_{ask,t,T}$ be the bid and asked domestic forward rates for delivery at date T. In addition, interest rates are also quoted with bid-ask spreads. Let $i_{bid,d}$, $i_{bid,f}$, and $i_{ask,d}$, $i_{ask,f}$ be the bid and asked relevant interest rates on Eurodeposits denominated in the domestic and the foreign currency. Now, consider a trader in the interbank market. The trader will have to buy or borrow at the other party's asked price while she will sell or lend at the bid price. If the trader wishes to do arbitrage, there are two roads to take: borrow domestic currency or borrow foreign currency.

1.C.1 Bid's Bound: Borrow Domestic Currency

Consider the following covered arbitrage strategy:
1. Borrow one unit of domestic currency for T days at $i_{ask,d}$.
2. Exchange the domestic currency for foreign currency at $S_{ask,t}$.
3. Deposit the foreign currency for T days at $i_{bid,f}$.
4. Sell the foreign currency forward at $F_{bid,t,T}$.

That is, the trader can borrow 1 unit of domestic currency at time t=0, and repay $1+i_{ask,d}$ at time T. Using the borrowed domestic currency, she can buy spot foreign currency at $S_{ask,t}$ and sell the currency forward for T days at $F_{bid,t,T}$, while depositing the foreign currency at the foreign interest rate, $i_{bid,f}$. This strategy would yield, in terms of domestic currency:

$(1/S_{ask,t})\ (1 + i_{bid,f} * T/360) * F_{bid,t,T}$.

For this strategy to yield no profit, it must be the case that it produces an amount less than or equal to $(1+i_{ask,d})$ units of domestic currency that must be repaid on the domestic loan. That is,

$(1/S_{ask,t})\ (1 + i_{bid,f} * T/360)\ F_{bid,t,T} \leq (1 + i_{ask,d} * T/360)$.

Solving for $F_{bid,t,T}$,

$$F_{bid,t,T} \leq S_{ask,t} * \frac{\left(1+ i_{ask,d} * \frac{T}{360}\right)}{\left(1+ i_{bid,f} * \frac{T}{360}\right)} = \mathbf{U_{bid}}.$$

1.C.2 Ask's Bound: Borrow Foreign Currency

Now, consider the following covered arbitrage strategy:
1. Borrow one unit of foreign currency for T days.
2. Exchange the foreign currency for domestic currency.
3. Deposit the domestic currency for T days.

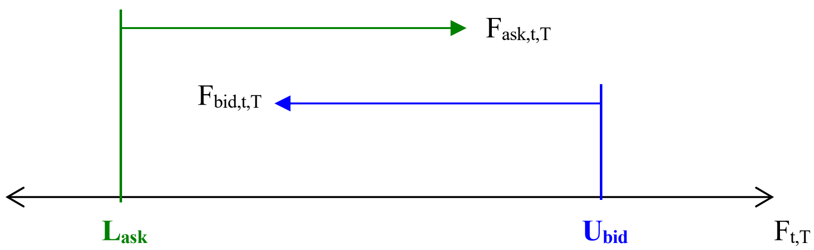
4. Buy the foreign currency forward.

That is, the trader can borrow 1 unit of foreign currency at time t=0, and repay $1+i_{ask,f}$. Following a similar procedure as the one detailed above, we get:

$$F_{ask,t,T} \geq S_{bid,t} * \frac{\left(1+i_{bid,d} * \frac{T}{360}\right)}{\left(1+i_{ask,f} * \frac{T}{360}\right)} = \mathbf{L_{ask}}.$$

<u>Note</u>: the above inequalities provide bounds (together with the condition $F_{ask,t,T} > F_{bid,t,T}$) for the bid and ask forward rates.

Exhibit III.2
Trading bounds for the Forward bid and the Forward ask



Example III.5: Suppose we have the following information: S_t=1.6540-.0080 USD/GBP, i_{USD}=7¼-½, i_{GBP}=8 1/8–3/8, and $F_{t,one\text{-}year}$=1.6400-.0050 USD/GBP. Given these prices, we should check if there is an arbitrage possibility.

If a trader borrows one USD, she will repay USD 1.07500. If she buys GBP, deposit them at the GBP rate, and sells GBP forward, she will obtain

(1/1.6620) * (1 + .08125) * 1.64 = USD 1.06694.

Therefore, there is no arbitrage opportunity. For each USD the trader borrows, she would lose USD .00806.

On the other hand, if the trader borrows one GBP, she will repay GBP 1.08375. If she buys USD, deposit them at the USD rate, and buy GBP forward, she will obtain

1.6540 * (1 + .07250) * (1/1.6450) = GBP 1.07837.

Again, there is no arbitrage opportunity. That is, the bid-ask forward quote is consistent with no arbitrage. This is due to the fact that the forward quote is within the IRPT bounds. To check this point, we calculate the bounds for the forward rate, U_{bid} and L_{ask}.

$$\mathbf{U_{bid}} = S_{ask,t} * \frac{(1+i_{ask,d})}{(1+i_{bid,f})} = 1.6620 \text{ USD/GBP} * \frac{(1.075)}{(1.08125)} = 1.6524 \text{ USD/GBP}$$

$$\Rightarrow \mathbf{U_{bid}} \geq F_{bid,t,T} = 1.6400 \text{ USD/GBP}.$$

L_{ask} $= S_{bid,t} * \frac{(1 + i_{bid,d})}{(1+ i_{ask,f})}$ = 1.6540 USD/GBP * $\frac{(1.0275)}{(1.08375)}$ = 1.6368 USD/GBP

$\Rightarrow$ **L_{ask}** $\leq F_{ask,t,T}$ = 1.6450 USD/GBP. ¶

To check your understanding of the IRPT with bid-ask spreads, do exercises III.4 and III.5.

1.D Synthetic Forward Rates

A *synthetic* asset is a combination of different assets that exactly replicates the cash flows of the original asset. We have already used this concept to construct a covered arbitrage opportunity. We have already constructed synthetic forward rates by combining the spot rate and the domestic and foreign interest rate. If the synthetic forward rate is cheaper than the market forward rate, then there is an arbitrage opportunity. Note, however, that sometimes it is possible to observe a synthetic forward rate less or more expensive than the forward rate, but there are no arbitrage opportunities, because of transaction costs. In this case, a trader would use the less expensive forward rate.

Now, it is possible that for some currencies there is no active market for forward exchange rates. For many currencies, this is the usual case, especially for long-term forward contracts. In general, the majority of the governments around the world issue long terms bonds. A trader can use the yields on long term bonds to obtain a forward rate quote. This trader can replicate the forward contract using a spot currency contract combined with borrowing and lending government bonds. This replication is done using equation (III.1).

Example III.6: Replicating a 10-year forward bid quote.
A trader at Bertoni Bank is unable to obtain a USD/JOD 10-year forward bid quote (JOD = Jordanian Dinar). She decides to replicate a USD/JOD forward contract using 10-year government bond yields and the spot exchange rate. The yield for 10-year government bonds at the bid is 6% in the U.S. and at the ask 8% in Jordan. The ask USD/JOD spot quote is 1.60 USD/JOD. She shorts the domestic (USD) bond, converts the USD into JOD and buys the Jordanian (JOD) bond. Ignoring transaction costs, she creates a 10-year forward bid quote:

$F_{bid,t,10\text{-}year} = S_{bid,t} * [\frac{(1 + i_{bid,d,10-year})}{(1+ i_{ask,f,10-year})}]^{10}$

= 1.60 USD/JOD * $[1.06/1.08]^{10}$ = 1.3272 USD/JOD. ¶

Synthetic forward contracts are very useful for exotic currencies. When countries impose borrowing or lending restrictions, it will be difficult for traders to construct synthetic forward contracts.

1.E IRPT: Evidence

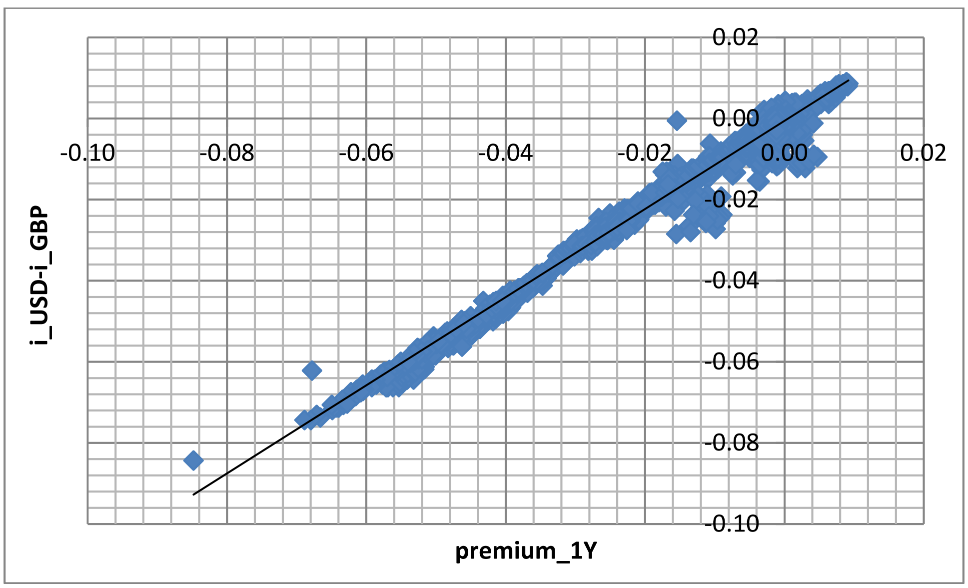
Testing IRPT is very simple. Recall the relation between the forward premium and the interest rate differential, $p \approx i_d - i_f$. Then, we can plot the forward premium and the interest rate differential for several currencies in a graph similar to Exhibit III.1. The visual test would accept the IRPT if we observe a 45% degree line in the plot. A more formal test of the IRPT can be designed by using the following regression:

$$p = \alpha \ + \ \beta\,(i_d \ - i_f\,) * \frac{T}{360} + \xi,$$

where ξ represents a regression error term. Under the IRPT, the null joint hypothesis is α=0 and β=1. An F-test can be used to test this joint hypothesis.

Starting from Frenkel and Levich (1975), there is a lot of evidence that supports IRPT. As an example, in Figures III.1 we plot the daily interest rate differential against the annualized forward premium. They plot very much along the 45° line. Moreover, the correlation coefficient between these two series is 0.995, highly correlated series!

FIGURE III.1
USD/GBP premia and interest rates differentials (1990-2015)



Using intra-daily data (10' intervals), Taylor (1989) also find strong support for IRPT. At the tick-by-tick data, Akram, Rice and Sarno (2008, 2009) show that there are short-lived (from 30 seconds up to 4 minutes) departures from IRP, with a potential profit range of 0.0002-0.0006 per unit. There are, however, small deviations from IRP. What is the meaning of these small deviations? Are arbitrageurs not taking advantages of these departures from IRP? The answer to the last question is no. There are several variables that explain departures from IRP.

The first reason behind departures from IRP is the time lag that exists between the observation of an arbitrage opportunity and the actual execution of the covered arbitrage strategy. Once an arbitrageur decides to take advantage of the IRPT not holding, the deviation from IRP has disappeared. That is, the prices we use to test the IRPT --*p* and (i_d -i_f)—are, many times, misleading. Arbitrageurs were not able to use those quoted prices.

The second reason, and the most obvious, for observing deviations from the IRPT is transaction costs. Arbitrageurs cannot take advantage of violations of the IRPT that are smaller than the transaction costs they need to pay to carry out a covered arbitrage strategy. That is, the existence of transaction costs would allow deviations from IRP equal or smaller than these transaction costs.

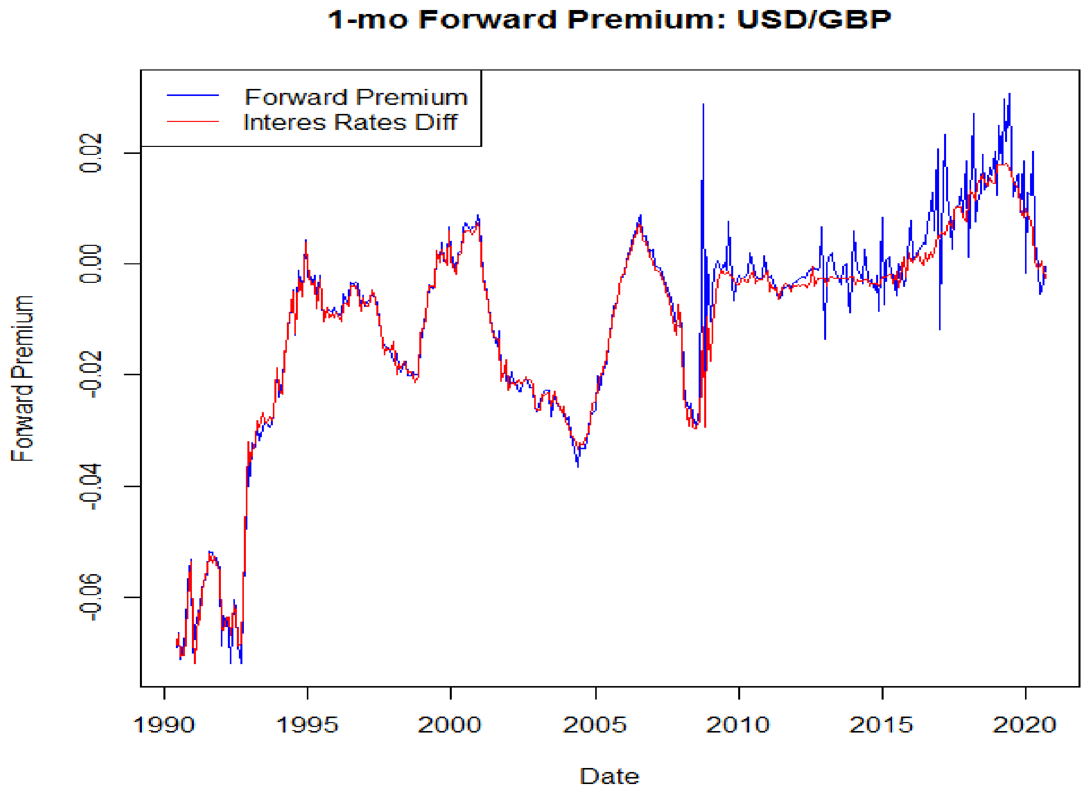
There are situations, however, where we observe significant and more persistent deviations from the IRPT line. These situations are usually attributed to monetary policy, credit risk, funding conditions, risk aversion of investors, lack of capital mobility, default risk, country risk, and market microstructure effects. Let's focus on country risk. The forward contract locks in the rate at which foreign currency should be converted into domestic currency. There is, however, no guarantee that the funds will be allowed to leave the country. A political or economic crisis in the foreign market might trigger capital controls. If governments can effectively control the flows of capital into and from the country, then one or more of the steps of the covered arbitrage strategy, step (4) and/or (1), cannot take place. Moreover, the threat of capital controls or default on foreign debt can be enough deterrent for arbitrageurs not to act. In general, any potential impediment to the free flow of capital in and out from a country will make deviation from the IRPT very likely.

For example, during the 2008-2009 financial crisis there were several violations of IRPT (in Figure III.1, the point well over the line, on top, (-.0154,-.0005) is from May 2009). These violations are attributed to funding constraints –i.e., difficulties to do step (1): borrow. See Baba and Parker (2009) and Griffoli and Ranaldo (2011).

Since the financial crisis 2008-2009, there is evidence that the IRPT is not holding, as seen in Figure III.2. A potential reason, interest rates are no longer "risk-free" rates.

FIGURE III.2

USD/GBP 1-month forward premium and interest rate differential over time (1990-2020)



Another variable to consider is differential taxation. Taxes tend to be different in different countries. Thus, the same arbitrage opportunity in one country will result in a different return to residents of a different country. Note that in this section we have considered pre-tax returns. Differential taxes can substantially affect a covered arbitrage strategy.

II. Purchasing Power Parity (PPP)

Suppose the price of an ounce of silver in California is significantly higher –say, USD 20- to the price of an ounce of silver in Arizona. We should expect traders to buy silver in Arizona and sell it in California. This arbitrage activity will continue until silver in Arizona and in California sell for about the same price, allowing for transaction costs. Similar arbitrage activity will appear if the price of computers or wheat is significantly different, allowing for transaction costs, in different countries. Arbitrage in goods and services provides a link between prices and exchange rates. This relationship is known as the *purchasing power parity* (PPP).

2.A Absolute PPP and the Law of one price (LOOP)

Our first version of purchasing power parity is absolute PPP, which was developed by the Swedish economist Gustav Casell in 1918. Casell's PPP is based on the law of one price (LOOP): goods expressed in the same currency should have the same price.

Example III.7: Law of one price for Oil.
If the price of one barrel of oil is USD 80 in the U.S. and the exchange rate is 0.80 USD/CHF, then, the price of oil in Switzerland should be CHF 100. Conversely, given the price of oil in the U.S. and in Switzerland, we should be able to calculate the equilibrium exchange rate USD/CHF, $\mathbf{S_t^{LOOP}}$. In this case,

$\mathbf{S_t^{LOOP}}$ = $P_{oil,US}$ /$P_{oil,SWIT}$ = USD 80/CHF 100 = 0.80 USD/CHF.

Suppose the exchange rate is $\boldsymbol{S_t}$ = **1.00 USD/CHF**, instead. Then, the price of oil will be more expensive in Switzerland (USD 100). U.S. oil imports will flood the Swiss market, forcing the exchange rate, S_t, and/or the price of oil, $P_{oil,SWIT}$, to adjust to its appropriate PPP level. ¶

In the absence of substantial trade barriers and other transaction costs, the law of one price should hold, otherwise, arbitrage opportunities will arise. The LOOP, however, should only apply to international traded goods. It is unthinkable to use the LOOP to price land or haircuts. Land may be much cheaper in Australia than in the U.S., but this will not induce U.S. residents to import land from Australia.

The *absolute version* of the PPP theory postulates that the equilibrium exchange rate between two currencies is simply the ratio of the two countries' general price levels:

$\mathbf{S_t^{PPP}}$ = Domestic Price level / Foreign Price level = P_d / P_f (Absolute PPP).

Thus, absolute PPP applies the LOOP to a basket of goods: the basket of goods used to calculate price indices. These consumption baskets are thought to represent the consumption of a typical (or average) consumer in a given country. That is, aggregate price levels determine exchange rates.

For absolute PPP to work, we need arbitrage based on aggregate price levels. For example, suppose aggregate prices in the U.S. increase and the exchange rate remains constant. Traders will take advantage of this disequilibrium situation: U.S. exports will decrease and U.S. imports will increase. A new equilibrium will be reached when the USD depreciates to compensate for the increase in the U.S. aggregate price level. We can think of PPP as providing an exchange rate at which there is no "arbitrage" of the consumption basket. Thus, according to absolute PPP, the ratio of aggregate price levels delivers an equilibrium (*fair valuation*) exchange rate. This equilibrium ratio is also called PPP parity.

The *Consumer Price Index* (CPI) basket is often used as a representative basket. The CPI is reported monthly. It indicates the change in the prices paid by urban consumers for a representative basket of goods and services –the CPI basket.

Example III.8: Suppose that the cost of the consumption basket for an average consumer in Switzerland is **CHF 1,677** and in the U.S. is **USD 1,394.25**. Then, according to PPP, the equilibrium exchange rate is:

$\mathbf{S_t^{PPP}}$ = P_{USA}/ P_{SWIT} = **USD 1,394.25**/ **CHF 1,677** = **0.8314 USD/CHF**.

Suppose that the actual exchange rate is $\boldsymbol{S_t}$ **= 1.0836 USD/CHF**. Then, according to PPP, we consider the CHF to be *overvalued* (with respect to "PPP fair valuation") by:

(**1.0836** /**0.8314** - 1) = **0.3033** (or **30.33%**). ¶

The problem with using CPI or a broad aggregate price index is that the components of the aggregate basket are usually different across countries. The law of one price does not apply to different baskets. Thus, economists have devoted significant efforts to construct similar baskets of goods to estimate PPP implied exchange rates. One popular price index is based on the PWT (Penn World Tables) data set. The PWT presents price measures for different countries that are based on a common market basket of approximately 150 detailed categories of goods.

Another common market basket was popularized by The British magazine *The Economist.* The Economist uses the Big Mac as a basket of common goods: beef, cheese, onion, lettuce, bread, pickles and special sauce. The Big Mac is sold in 120 countries around the world and it represents a standardized basket of goods. Most of the ingredients that are used in the Big Mac are traded in international markets. The Economist uses the prices of Big Macs around the world to derive PPP implied exchange rates (relative to the USD) and, then, to derive an indicator of undervaluation or overvaluation of a currency. This indicator is usually called the *Big Mac Index*. Exhibit III.2 shows the Big Mac Index calculated on January 2012 for different countries.

Using 2000 data, Pakko and Pollard (2003), two economists from the Federal Reserve Bank of St. Louis, found a .73 correlation between the PPP measured derives from the PWT and the Big Mac Index..

Example III.9: *The Economist* reports the price of a Big Mac in the Euro-area and in the U.S. as **EUR 3.721** and USD 4.93, respectively. That is,

P_f = **EUR 3.72** (USD 4.00)

P_d = **USD 4.93**

S_t = **1.0753 USD/EUR**.

$\mathbf{S_t^{PPP}}$ = P_{USA}/ P_{EUR} = **USD 4.93**/ **EUR 3.72** = **1.3253 USD/EUR** > $\boldsymbol{S_t}$ = **1.0753 USD/EUR**.

Taking the Big Mac as our basket, the EUR is *undervalued* by 18.9% (=**1.0753**/**1.3253**). ¶

In theory, traders can exploit the price differentials in Big Macs. In Example III.9, Euro-area traders can export Big Macs to the U.S. But, this scenario is not realistic; a Big Mac sandwich shipped from the Europe to the U.S. would probably not be very appealing. But, since the components of a Big Mac are traded on world markets, the LOOP suggests that prices of the components should be similar in all markets.

2.A.1 Real v. Nominal Exchange Rates

The absolute version of the PPP theory is expressed in terms of S_t, the *nominal exchange rate*. This is "nominal" because it is expressed in terms of money rather than in units of a *real* good or consumption basket. We can modify the absolute version of the PPP relationship in terms of another exchange rate, the *real exchange rate*, R_t. That is,

$$R_t = S_t \; P_f / P_d.$$

The real exchange rate allows us to compare foreign prices, translated into domestic terms with domestic prices. If absolute PPP holds, then R_t should be equal to one. If R_t is different than one, one country is more competitive than the other is. This is not an equilibrium situation --or at least, a long-run equilibrium situation. If prices and exchange rates are flexible, absolute PPP will force an adjustment via inflation or/and the nominal exchange rate, until R_t is equal to one.

Example III.10: Suppose that the cost of the consumer basket represented by the Consumer Price Index (CPI) in Switzerland and in the U.S. is **CHF 1677.0** and **USD 1,394.25**., respectively. Also, suppose that S_t = **1.0836 USD/CHF**. Then,

$R_t = S_t \, P_{SWIT} / P_{US}$ = **1.0836 USD/CHF** * **CHF 1,677.0** / **USD 1,394.25**. = **1.3036**.

We can conclude that Switzerland is less competitive than the U.S. since its prices are **30.36%** higher than U.S. prices, after taking into account the nominal exchange rate.

Swiss residents will buy more U.S. goods, than U.S. residents buy Swiss goods. This is not an equilibrium situation (under absolute PPP, $R_t = 1$). One way to get back to the equilibrium level is to have the CHF depreciate against the USD, over time. ¶

A currency can experience a *real exchange rate appreciation*, when a country's inflation is much higher than that of a foreign trading partner and the exchange rate, S_t, does not move exactly to compensate for the difference in inflation rates. That is, the real exchange rate can appreciate or depreciate without movements of the nominal exchange rate. For instance, in 1999, the Argentine peso (ARS) experienced a real depreciation against the USD, since the inflation rate in Argentina was -1.8%, the inflation rate in the U.S. was 2.5%, while the ARS/USD exchange rate remained fixed at 1 ARS/USD. The ARS had a real depreciation against the USD. Therefore, U.S. goods for Argentine residents became relatively more expensive, while Argentine goods for U.S. residents became relatively more attractive.

♦ **The Real Exchange Rate as an Indicator of a Currency Crisis**
Recent studies have identified certain variables that signal that a country is vulnerable to a currency crisis. The signal that appears to be the most important is the real exchange rate. A study by the IMF has estimated that when the inflation of a country is much higher than that of its trading partners, and the exchange rate remains fixed, the probability of a currency crisis increases to 67%. That is, when a currency is significantly overvalued, in real terms, it indicates a high chance of a crisis. ♦

2.A.2 Absolute PPP: Does It Hold?

The Economist's Big Mac Index reports the real exchange rate for many countries:

$$R_t = S_t \; P_{BigMac,d} \,/\, P_{BigMac,f}$$

A test of Absolute PPP is very simple: Check if $R_t = 1$. If big deviations are observed, a shadow is cast over the validity of this simple theory, Exhibit III.2 shows the real exchange rate, as reported by The Economist, for the major currencies in December 2022.

Exhibit III.2
Big Mac Index

The Big Mac index

Country		2000 —— 2022	Under/over valued, %
Switzerland	Franc		35.4
Uruguay	Peso		27.8
Norway	Krone		22.9
Sweden	Krona		4.8
Denmark	Krone		0.9
United States	US$		BASE CURRENCY
Argentina	Peso		-1.0
Euro area	Euro		-1.4
Australia	A$		-4.6
Saudi Arabia	Riyal		-5.6
Israel	Shekel		-5.7
Sri Lanka	Rupee		-6.9
Costa Rica	Colón		-7.4
UAE	Dirham		-8.6
New Zealand	NZ$		-9.0
Chile	Peso		-11.4
Britain	Pound		-12.9
Kuwait	Dinar		-14.5
Canada	C$		-14.7
Czech Rep.	Koruna		-15.8

Country		2000 —— 2022	Under/over valued, %
Bahrain	Dinar		-15.9
Lebanon	Pound		-16.5
Singapore	S$		-16.6
Brazil	Real		-17.2
Nicaragua	Córdoba		-21.2
Mexico	Peso		-21.8
Colombia	Peso		-22.4
Poland	Zloty		-23.6
Honduras	Lempira		-24.5
Turkey	Lira		-25.6
South Korea	Won		-26.0
Thailand	Baht		-27.2
Qatar	Riyal		-28.3
Hungary	Forint		-29.9
Oman	Rial		-31.2
Peru	Sol		-33.2
China	Yuan		-34.0
Jordan	Dinar		-34.3
Guatemala	Quetzal		-35.8
Pakistan	Rupee		-36.8

We observe big departures from Absolute PPP. With some exceptions, the Big-Mac tends to be more expensive in developed countries (U.S.A., Euro area, Australia) than in less developed countries (Colombia, Guatemala, Pakistan).

2.A.3 Trade Frictions and Other Factors affecting PPP

The big deviations from absolute PPP are usually attributed to different reasons:

(1) PPP emphasizes only trade and price levels. Since the PPP approach focuses solely on trade as a determinant of the supply and demand of foreign exchange, other economic, political and financial factors are ignored. For example, in Exhibit III.2 we see that countries with high country risk (Ukraine, Russia, Egypt) are at the bottom of the table.

(2) Implicit assumption: Absence of trade frictions (tariffs, quotas, transactions costs, etc.). The existence of unequal taxes or tariffs in goods imported or exported across countries will cause a separation of the true price levels between countries. Export restrictions cause the currency of the country with high *export* tariffs to be undervalued (on PPP basis) relative to the currency of a country with lower export tariffs. Conversely, a country with higher *import* restrictions will be overvalued on PPP basis. In practice, many products are heavily protected, even in the U.S. For example, peanut imports are subject to a U.S. tariff between 131.8% (for shelled peanuts) and 163.8% (for unshelled peanuts).

Transportation costs are assumed negligible by PPP. But, for certain goods, transportation costs can be important. For example, having the Big Mac Index in mind, Hummels (2001) estimates that transportation costs add 7% to the price of U.S. imports of meat and 16% to the import price of vegetables.

(3) Perfect competition. Imperfect competition, usually related to (2) can create price discrimination. Companies take advantage of price elasticities in different countries to price-to-market. For example, U.S. pharmaceuticals sell the same drug in the U.S. and in Canada at different prices. Baxter and Landry (2012) report that IKEA prices deviate 16% from the LOOP in Canada, but only 1% in the U.S.

(4) Instantaneous adjustments. Another implicit PPP assumption, related to another trade friction. Not realistic. Trade takes time and it also takes time to adjust contracts.

(5) PPP assumes P_f and P_d represent the same basket. PPP is unlikely to hold if the prices of individual goods comprising the consumption basket are not the same across countries. This is why the Big Mac is a popular basket to calculate real exchange rates: it is standardized around the world with an easy to get price.

(6) Internationally non-traded (NT) goods. Not all goods in an economy are traded in international markets. There are many goods and services that are non-traded (NT): real estate, home and car repair, restaurants, retail trade, hotel rooms, haircut, doctor visits, etc. Thus, PPP cannot be used for these goods. This is not a sector of the economy with a small weight in the PPP basket. The NT goods sector accounts for 50% to 60% of the GDP in developed economies and, therefore, it has a big weight in the CPI basket. Thus, a direct comparison of price indexes may not be very informative. For example, in countries where NT goods are relatively high, the CPI basket will also be relatively expensive. Thus, a standard application of PPP will tend to find these countries' currencies *overvalued* relative to currencies in countries with low costs of NT goods.

(7) The NT sector also has an effect on the price of traded goods. The NT sector affects not only the composition of the basket, but it has also an impact on the price of internationally traded goods. For example, rent, distribution and utilities costs affect the price of a Big Mac. (It is estimated that 25% of Big Mac's cost is due to NT goods.)

2.A.4 Balassa-Samuelson Effect

Labor costs affect all prices. We expect average prices to be cheaper in poor countries than in rich ones because labor costs are lower. This is the basis of the so-called "*Balassa-Samuelson effect,*" due to Balassa (1964) and Samuelson (1964). Rich countries have higher productivity and, thus, higher wages in the traded-goods sector than poor countries do. But, since firms compete for workers, wages in NT goods and services are also higher. Then, overall prices are lower in poor countries. This Balassa-Samuelson effect implies a positive correlation between high productivity countries and PPP currency overvaluation.

There is a different explanation for the fact that average prices tend to be cheaper in poor countries. It is based on the work of three economists, Bhagwati, Kravis and Lipsey. They emphasize the differences in endowment of labor and capital, not lower levels of productivity. Poor countries have more labor relative to capital, so marginal productivity of labor is greater in rich countries than in poor countries. NT goods tend to be labor-intensive; therefore, because labor is less expensive in poor countries and is used mostly for NT goods, NT goods are cheaper in poor countries. Wages are high in rich countries, so NT goods are relatively more expensive.

Practitioners tend to incorporate the Balassa-Samuelson effect into PPP calculations in a straightforward manner. Suppose we want to adjust Big Mac PPP-implied exchange rates. Then:

1) Estimate a regression using Big Mac Prices (in USD, $P_{BM,t}$) as the dependent variable against GDP per capita (GDP_p). That is, run the following regression:

$$P_{BM,t} = \alpha + \beta \text{ GDP\_p}_t + \varepsilon_t$$

2) Compute fitted values (GDP-adjusted Big Mac Prices). That is,

$$\hat{P}_{BM,GDP\text{-}adjusted} = \hat{\alpha} + \hat{\beta} \text{ GDP\_per\_capita}_t$$

Based on the GDP-adjusted Big Mac Prices, re-estimate the PPP implied over/under-valuation:

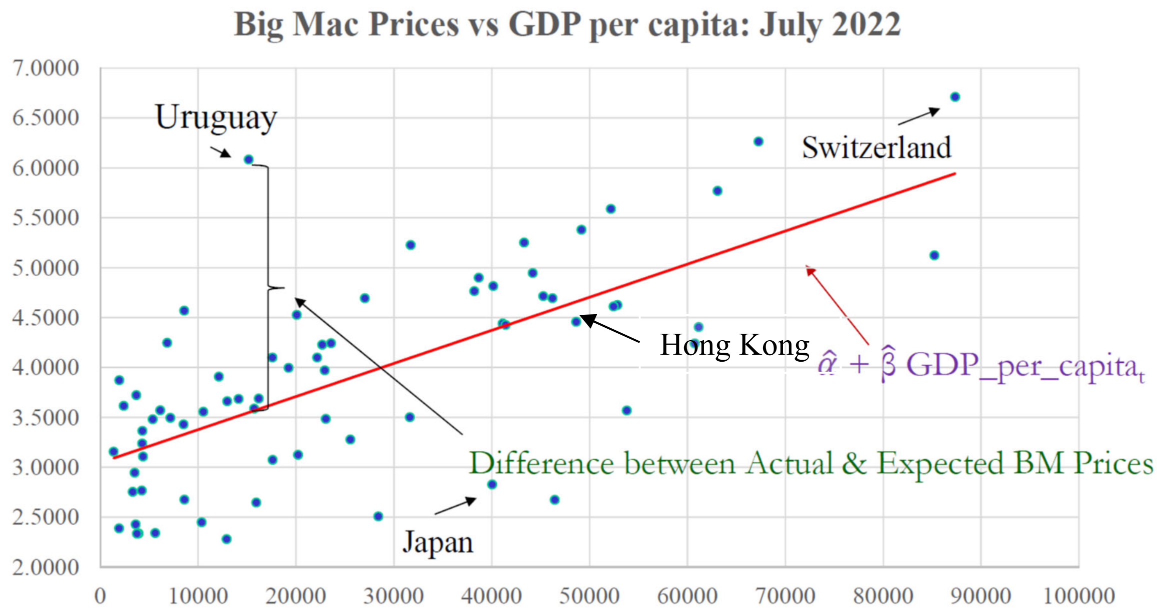
GDP-adjusted over/under valuation: (BM Price/$\hat{P}_{BM,GDP\text{-}adjusted}$) – 1

The regression line tells us what the "expected price" in a country is, once we take into consideration its GDP level.

Using data from The Economist for July 2022, we estimated the above regression:

$$\hat{P}_{BM,GDP\text{-}adjusted} = 3.045895 + 0.0000332 * \text{GDP\_per\_capita}_t$$

Below, we show the regression line using data from in July 2022:



Now, using the computed red line above, we calculate the "Expected BM prices, given the GDP of a given country." For example, we compute the above value for Uruguay. Uruguay's GDP per capita in July 2022 was **USD 15,169.153**. Then,

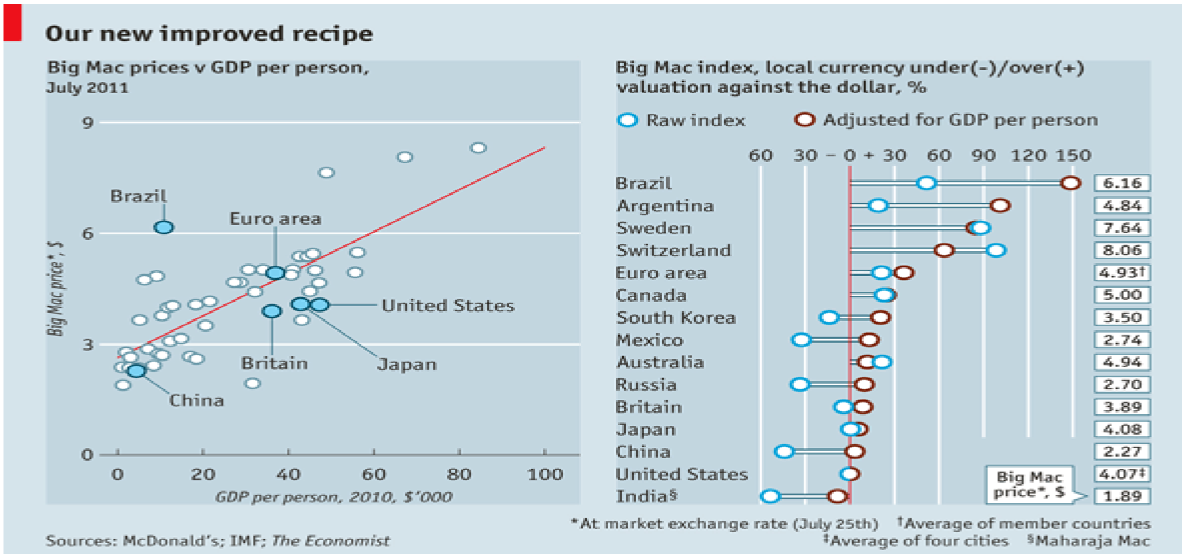
$\hat{P}_{\text{BM,GDP-adj}}$ (Uruguay) = 3.045895 + 0.0000332 * **15,169.153** = **3.549511**

That is, the expected USD Big Mac price, in Uruguay, given its GDP per capita, was **USD 3.55.** Since the observed local BM price was UYU 255, which translates to **USD 6.08** (= UYU 255 * 41.91 USD/UYU), then the *GDP-adjusted over/under valuation* was:

6.08 / **3.549511** – 1 = **71.29%** (**71.29%** overvalued)

On the other hand, we have Japan; which according to the adjusted index, its currency is undervalued by **35%**. These adjustments to PPP implied exchange rates can be significant; China goes from undervalued by 38.4% on unadjusted basis to undervalued by **4.8%** on GDP-adjusted basis. In Exhibit III.3, we report a comparison of both PPP exchange rates for Juliy 2011.

Exhibit III.3
Adjusted Big Mac Index



PPP: Borders matter

You may look at the Big Mac Index and think: "No big deal: there is also a big dispersion in prices within the U.S., within Texas, and, even, within Houston!" It is true that prices vary within the U.S. For example, in 2015, the price of a Big Mac (and Big Mac Meal) in New York was USD 5.23 (USD 7.45), in Texas as USD 4.39 (USD 6.26) and in Mississippi was USD 3.91 (USD 5.69).

Engel and Rogers (1996) computed the variance of LOOP deviations for city pairs within the U.S., within Canada, and across the border. They found that distance between cities within a country matter, but the border effect is very significant. To explain the difference between prices across the border using the estimate distance effects within a country, they estimate the U.S.-Canada border should have a width of 75,000 miles!

This huge estimate of the implied border width between the U.S. and Canada has been revised downward in subsequent studies, but a large positive border effect remains.

2.B Relative Purchasing Power Parity

As noted above, one important criticism of absolute PPP is the assumption of absence of transportation costs, tariffs, or other obstruction to the free flow of trade. Because of these trade frictions, prices can differ from country to country. The *relative version* of the PPP theory takes into account trade frictions, which will be assumed constant. Thus, relative PPP is a weaker version of PPP.

Under the assumption that trade frictions are constant, the difference between the two country's price indices is constant. Therefore, the rate of change in the prices of products should be similar when measured in a common currency --as long as trade frictions are unchanged. The following formula reflects the relationship between relative inflation rates and changes in exchange rate according to the relative version of PPP:

$$s_{t,T}^{PPP} = \frac{S_{t+T}^{PPP} - S_t}{S_t} = \frac{(1 + I_d)}{(1 + I_f)} - 1 \qquad \text{(Relative PPP),}$$

where
$s_{t,T}^{PPP}$ = change in S_t from t and t+T.
I_f = $(P_{f,,t+T}/P_{f,t})$ -1 = foreign inflation rate from t to t+T;
I_d = $(P_{d,,t+T}/P_{d,t})$ -1 = domestic inflation rate from t to t+T.

We can use a linear approximation to the above formula; similar to the approximation we use for the IRPT formula. This linear approximation works very well for small inflation rates. Under this approximate formula, the percent change in exchange rates is equal to the inflation rate differential between the two countries. That is,

$$s_{t,T}^{PPP} \approx I_d - I_f.$$

Since this relationship is not expected to hold at every time interval, it is usually rewritten in terms of conditional expectations (averages):

$$E_t[s_{t,T}^{PPP}] \approx E_t[I_d] - E_t[I_f].$$

We expect to observe a one-to-one relation between (I_d-I_f) and s_t. For example, if prices are expected to double in the U.S. relative to those in Switzerland, the exchange rate of the CHF with respect to the USD should be expected to double (say, from .75 to 1.50).

Example III.11: Forecasting with PPP the USD/South African rand exchange rate (USD/ZAR).
You have the following information:
$CPI_{US,2021}$=104.5
$CPI_{SA,2021}$ = 100.0
S_{2021} = 0.2035 USD/ZAR.

You are given the 2022 forecast for the CPI in the U.S. and South Africa:
$CPI_{US,2022}$ = 110.8
$CPI_{SA,2022}$ = 102.5.

You want to forecast S_{2022} using the relative (linearized) version of PPP. Then,
$E[I_{US,2022}]$ = (110.8/104.5) – 1 = .06029
$E[I_{SA,2022}]$ = (102.5/100) – 1 = .025

S^F_{2022} = **S_{2021}** * $[1 + I^F_{US,\,2022} - I^F_{SA,\,2022}]$
= **.2035 USD/ZAR** * [1 + .06029 -.025] = .2107 USD/ZAR.

You forecast an appreciation of the ZAR against the USD. ¶

As long as there are no changes in transportation costs, obstructions to trade, or the ratio of traded goods to non-traded goods, the change in the exchange rate should be roughly proportional to the change in the ratio of the two countries' general price levels. That is, under the relative version of PPP, the real exchange rate, R_t, remains constant.

Relative PPP is also used to classify a currency as *overvalued* or *undervalued*. The term overvalued or undervalued insinuates that exchange rates are not supposed to be what the free-market rates are. For example, suppose that, over time, domestic inflation is higher than foreign inflation. According to PPP, we should expect a depreciation of the domestic currency. If the domestic currency depreciates less than what PPP suggests, it is said that the domestic currency is overvalued. Similarly, if the domestic currency depreciates by more than what PPP suggests, the domestic currency is undervalued.

2.C PPP: Implications

Relative PPP does not imply that the exchange rate, S_t, is easy to forecast. As seen in Example III.11, the quality of the forecast of S_t depends on the quality of the forecasts of price levels in both countries. For example, a country with high and unpredictable inflation, like Russia, will show a high and unpredictable exchange rate.

Without relative price changes, a multinational corporation faces no real operating exchange risk. As long as the firm avoids fixed contracts denominated in foreign currency, its foreign cash flows will change with the foreign rate of inflation. Therefore, once the corporation translates the cash flows to the domestic currency, the domestic cash flows will be unchanged. That is, the distinction between the nominal exchange rate, S_t, and the real exchange rate, R_t, is very important. They have different implications for exchange rate risk.

2.D PPP: Absolute vs. Relative

Absolute PPP compares price levels, while Relative PPP compares price changes (or movements). Under Absolute PPP prices are equalized across countries, but under Relative PPP exchange rates move by the same amount as the inflation rate differential (original prices can be different).

Relative PPP is a weaker condition than the absolute one: $\mathbf{R_t}$ can be different from 1.

The following sentences contrast both theories:

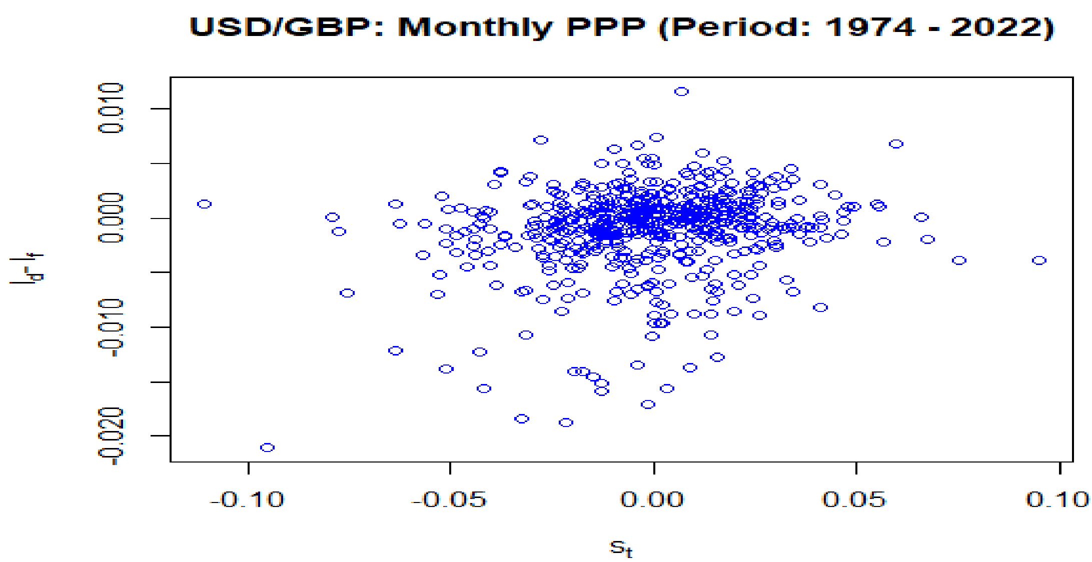
Absolute PPP: "*A mattress costs GBP 200 (= USD 320) in the U.K. and BRL 800 (=USD 320) in Brazil –i.e., same cost in both countries*." (S_t =1.6 USD/GBP & S_t =0.4 USD/BRL.)

Relative PPP: "*U.K. inflation was 2% while Brazilian inflation was 8%. Meanwhile, the BRL depreciated 6% against the GBP. Then, relative cost comparison remains the same*."

2.D PPP: Evidence

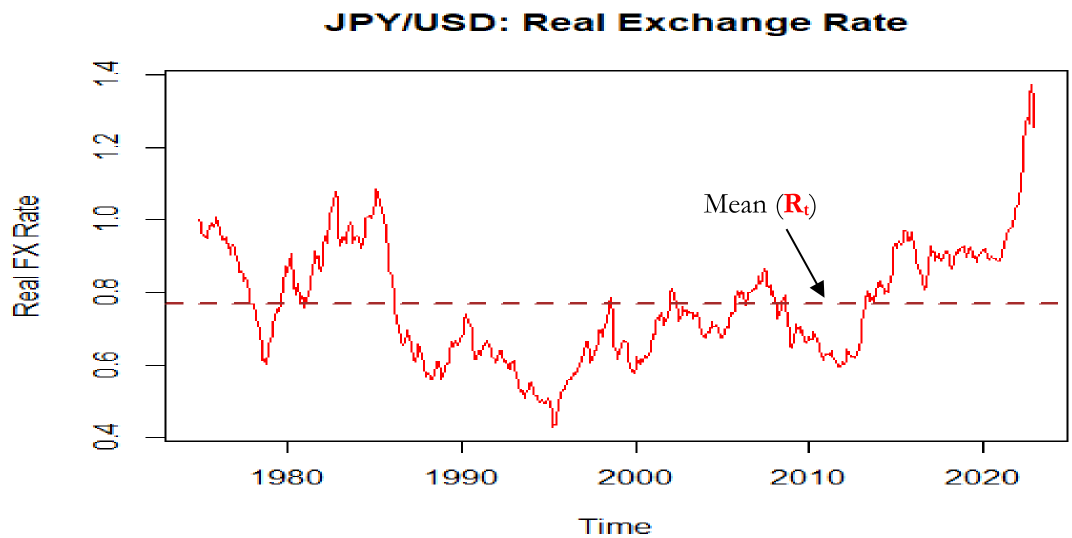
There is overwhelming visual evidence against relative PPP in the short-run. Figure III.3 plots the inflation rate differentials between the U.S. and the U.K. ($I_{USD} - I_{GBP}$) against s_t(USD/GBP), using monthly data 1974 - 2022. There is no one-to-one relation between ($I_{USD} - I_{GBP}$) and s_t, –no 45° line in Figure III.3– as the linearized version of relative PPP implies. This figure is typical for developed currencies. There is no one-to-one relation between (I_d-I_f) and s_t. In the short run, financial prices, like exchange rates, adjust very quickly to disequilibrium situations.

FIGURE III.3
USD/GBP Exchange Rate Changes and Inflation Rate Differentials (1974-2022)



Economists also use the real exchange rate, **R_t**, to test PPP. Under absolute PPP, **R_t** equals 1, but under relative PPP, **R_t** should be constant, not 1. Then, the question is: Do we observe a constant **R_t**? The short answer is no, but, tn general, we have some evidence for *mean reversion* for **R_t** in the long run. See Figure III.4, where the monthly JPY/USD real exchange rate is plotted for the period 1974 - 2022. Loosely speaking, **R_t** moves around some mean number, which we associate with a *long-run PPP parity* (for the JPY/USD the average **R_t** is 0.77). But, the deviations from the long-run PPP parity are very *persistent* –i.e., very slow to adjust. Note that the deviations from long-run PPP parity are big (up to 60%) and happen in every decade.

FIGURE III.4
JPY/USD: Real exchange rate (1971 - 2022)



Economists usually report the number of years that a PPP deviation is expected to decay by 50% (the *half-life)* is in the range of 3 to 5 years for developed currencies. Very slow!

Long-term contracts and implicit price agreements make many prices in the economy sticky, in the short- and medium-run. Thus, since prices, trade, and commodity arbitrage respond sluggishly, PPP is not expected to be a good model. In the long-run, however, there is evidence supporting a role for inflation rate differentials. Over time, countries with persistent positive inflation rate differentials tend to see a depreciation of their domestic currencies. Similarly, over time, countries with persistent negative inflation rate differentials tend to see an appreciation of their domestic currencies. Over the years, relative price levels matter.

The experience with other currencies is similar to the experience with the JPY/USD exchange rate displayed above.

2.D.1 PPP: Formal Statistical Evidence

Let's look at the usual descriptive statistics for $(I_d - I_f)_{t+T}$ and $s_{t,T}$. For monthly JPY/USD from 1975:Jan to 2022:Dec, they have similar means, but quite different standard deviations (look at the very different minimum and maximum stats). A simple t-test for equality of means (t-test=0.175) cannot reject the null hypothesis of equal means, which is expected given the large SDs, especially for $e_{f,t}$.

	I_{JPY}	I_{USD}	$I_{JPY} - I_{USD}$	$s_{t,T}$ (JPY/USD)
Mean	**0.00125**	**0.00303**	**-0.00179**	**-0.00139**
SD	**0.00485**	**0.00322**	**0.00502**	**0.02622**
Min	-0.01095	-0.01786	-0.01981	-0.08065

Median	0.00102	0.00266	-0.00184	0.00022
Max	0.02558	0.01420	0.02104	0.08066

But, the average relation over the whole sample is not that informative, especially with such a big SD. We are more interested in the contemporaneous relation between $s_{t,T}$ and $(I_d - I_f)_t$. That is, what happens to $s_{t,T}$ when $(I_d - I_f)_t$ jumps?

To test the contemporaneous relation we have a more formal test, a regression:

$$s_{t,T} = \boldsymbol{\alpha} + \boldsymbol{\beta}\,(I_d - I_f)_{t+T} + \varepsilon_{t+T},$$

where ε_t is the regression error, with mean 0 –i.e., $E[\varepsilon_t] = 0$. To do this regression we need to collect data on exchange rates and inflation rates for the two countries involved. We will estimate two parameters, α and β. We will use the following notation: *K* refers as the number of parameters in a regression and *N* refers to the number of observations used in a regression.

Under relative PPP, we have the following null hypothesis:

H_0 (Relative PPP holds): $\alpha = 0$ and $\beta = 1$

H_1 (Relative PPP does not hold): $\alpha \neq 0$ and/or $\beta \neq 1$

The statistical tests are t-tests, for the individual estimated coefficients α and β, and F-tests, for a joint test on the estimated coefficients α and β:

1) Individual test:

t-test = [Estimated coeff. – Value of coeff. under H_0]/S.E.(coeff.)

The t-test follows a t_v distribution, where v=N-K refers to the degrees of freedom. The decision rule is simple: if |t-test| > $t_{v,\alpha/2}$, reject H_0 at the α level. Usually, α = .05 (5 %).

(2) Joint test:

$$F = \frac{[\mathbf{RSS(H_0)} - \mathbf{RSS(H_1)}]/J}{\mathbf{RSS(H_1)}/(N-K)}$$

The F-test follows an $F_{J,N-K}$ distribution, where J is equal to the number of restrictions imposed by H_0, and SSR refers to the sum of squared residuals of the regression. The decision rule is simple: if F-test > $F_{J,N-K,\alpha}$, reject H_0 at the α level.

Example III.12: Using monthly Japanese and U.S. data from the graph (1/1975 - 12/2022), we fit the following regression:

s_t (JPY/USD) = $(S_t - S_{t-1})/S_{t-1} = \boldsymbol{\alpha} + \boldsymbol{\beta}\,(I_{JAP} - I_{US})_t + \varepsilon_t$.

$R^2 = 0.005621$
Standard Error (σ) = .02617
F-stat (slopes=0 –i.e., **β=0**) = 3.244 (*p-value* = 0.07219)
F-test (H_0: **α=0** and **β=1**) = **19.185** (*p-value*: lower than 0.0001) => reject at 5% level ($F_{2,477,.05}$= **3.015**)

Observations = 552

	Coefficients	Stand Error	t Stat	P-value
Intercept ($\hat{\alpha}$)	**-0.00209**	**0.001157**	-1.804	0.0717
$I_{JAP} - I_{US}$ ($\hat{\beta}$)	-0.39148	0.217343	-1.801	0.0722

Let's test H_0, using t-tests ($t_{477,.05}$=1.96 –when N-K > 30, $t_{.05}$ = **1.96**):
$t_{\alpha=0}$ (t-test for **α = 0**): (**-0.00209** – 0)/**0.001157** = **-1.804** (*p-value* = .07) ⇒ cannot reject H_0 at the 5% level
$t_{\beta=1}$ (t-test for **β = 1**): (-0.39148 – 1)/0.217343 = **-6.402** (*p-value*: < .00001) => reject H_0 at the 5% level

Regression Notes:
◊ If we look at the R^2, the variability of monthly (I_{JAP} – I_{US}) explain very little, 0.01%, of the variability of monthly s_t.
◊ We can modify the regression to incorporate the Balassa-Samuelson effect, by incorporating GDP differentials. Say,

$$s_t \text{ (JPY/USD)} = \alpha + \beta\,(I_{JAP} - I_{US})_t + \delta\,(\text{GDP\_cap}_{JAP} - \text{GDP\_cap}_{US})_t + \varepsilon_t. \P$$

Example III.12 formally rejects relative PPP. Formal tests of PPP arrive to similar conclusions for other currencies: Relative PPP tends to be rejected in the short-run.. In the long-run, there is a debate about its validity. As mentioned above there is some evidence of (slow) mean reversion. In the long-run, inflation differential matter: Currencies with high inflation rate differentials tend to depreciate.

Taylor (2002), using real exchange rates for 20 countries for over 100 years, finds strong evidence for PPP. However, deviation from PPP parity can be substantial in the short-run. In a survey of the PPP literature, Rogoff (1996) describes a consensus among PPP researchers that half the deviation from the PPP parity disappears between 3 to 5 years. It can take 5 to 10 years for the real exchange rate to revert back to its equilibrium level.

Officer, in a paper published in the IMF Staff Papers, in 1976, points out that PPP emphasizes monetary demand and supply disturbances. For instance, other factors being constant or negligible, a tight domestic money supply policy decreases the rate of inflation and, therefore, leads to a higher value for the domestic currency. In the short-run, other factors are not constant and changes in price levels are not solely determined by monetary factors. In the short-run, the existence of contracts makes prices sticky. In the long run, however, monetary factors are the main determinant of the inflation rate, therefore, PPP tends to hold in the long run.

♦ PPP and High Inflation

Officer's considerations help PPP to provide a good description of exchange rates movements in high inflation countries, even in the short-run. Under high inflation, all other factors that influence prices become relatively negligible. In high inflation countries, contracts are written to adapt to the high inflation conditions. Economic agents are very sensitive to price changes and, thus, prices adjust very rapidly in response to monetary disturbances. ♦

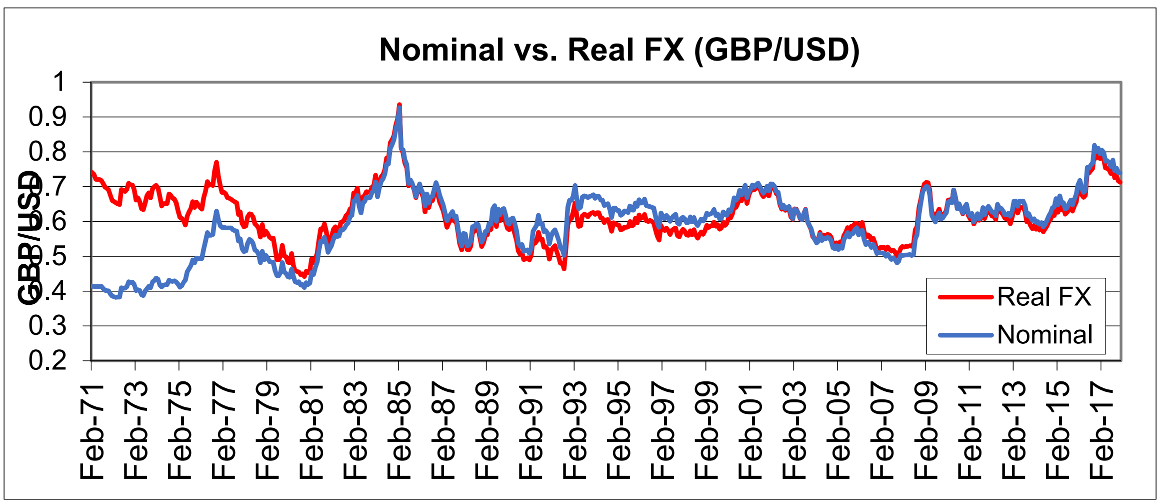
2.D.2 PPP: R_t, S_t and Sticky Prices

Research shows that $\mathbf{R_t}$ is much more variable when S_t is allowed to float. $\mathbf{R_t}$'s variability tends to be highly correlated with S_t's variability. This finding comes from Mussa (1986).

Figure III.5 shows the finding of Mussa (1986) for the USD/GBP exchange rate: After 1973, when floating exchange rates were adopted, $\mathbf{R_t}$ moves like S_t. As a check to the visual evidence: the monthly volatility of changes in $\mathbf{R_t}$ is 2.96 and the monthly volatility of changes in S_t is 2.93, with a correlation coefficient of .979. Almost the same!

FIGURE III.5

Sticky Prices: Nominal vs Real FX Rates (1971-2017)



From the above USD/GBP graph, which is representative of the usual behavior of $\mathbf{R_t}$ and S_t, we infer that price levels play a minor role in explaining the movements of $\mathbf{R_t}$ (&S_t). Prices are *sticky/rigid* –i.e., they take a while to adjust to shocks/disequilibria.

A potential justification for the implied price rigidity: NT goods. Price levels include traded and NT goods; traded-goods should obey the LOOP. But, Engel (1999) and others report that prices are sticky also for traded-goods (measured by disaggregated producer price indexes). A strange result for many of us that observe gas prices change frequently!

Several possible explanations have been advanced for this empirical fact:

(a) Contracts

Prices cannot be continuously adjusted due to contracts. In a stable economy, with low inflation, contracts may be longer. We find that economies with high inflation (contracts with very short duration) PPP deviations are not very persistent.

(b) Mark-up adjustments
There is a tendency of manufacturers and retailers to moderate any increase in their prices in order to preserve their market share. For example, changes in S_t are only partially transmitted or *pass-through* to import/export prices. The average ERPT (exchange rate pass-through) is around 50% over one quarter and 64% over the long run for OECD countries (for the U.S., 25% in the short-run and 40% over the long run). The average ERPT seems to be declining since the 1990s. Income matters: ERPT tends to be bigger in low income countries (2-3 times bigger) than in high countries.

(c) Repricing costs (*menu costs*)
It is expensive to adjust continuously prices; in a restaurant, the repricing cost is re-doing the menu. For example, Goldberg and Hallerstein (2007) estimate that the cost of repricing in the imported beer market is 0.4% of firm revenue for manufacturers and 0.1% of firm revenue for retailers.

(d) Aggregation
Q: Is price rigidity a result of aggregation –i.e., the use of price index? Empirical work using detailed micro level data –say, same good (exact UPC barcode!) in Canadian and U.S. grocery stores– show that on average product-level $\mathbf{R_t}$ –i.e., constructed using the same traded goods– move closely with S_t. But, individual micro level prices show a lot of idiosyncratic movements, mainly unrelated to S_t: Only 10% of the deviations from PPP are accounted by S_t.

• PPP: Puzzle
The fact that no single model of exchange rate determination can accommodate both the high persistent of PPP deviations and the high correlation between R_t and S_t has been called the "*PPP puzzle.*" See Rogoff (1996).

2.D.3 PPP: Summary of Empirical Evidence

We have presented several facts related to PPP:
- ◊ R_t and S_t are highly correlated, domestic prices (even for traded-goods) tend to be sticky.
- ◊ In the short run, PPP is a very poor model to explain short-term exchange rate movements.
- ◊ PPP deviation are very persistent. It takes a long time (years!) to disappear.
- ◊ In the long run, there is some evidence of mean reversion, though very slow, for R_t. That is, $\mathbf{S_t^{PPP}}$ has long-run information: Currencies that consistently have high inflation rate differentials –i.e., (I_d-I_f) positive- tend to depreciate.

The long-run interpretation for PPP is the one that economist like and use. PPP is seen as a benchmark, a figure towards which the current exchange rate should move.

2.D.3.a Calculating S_t^{PPP} (Long-Run FX Rate)

We want to calculate $\mathbf{S_t^{PPP}}$ = $P_{d,t}$ / $P_{f,t}$ over time. Steps:

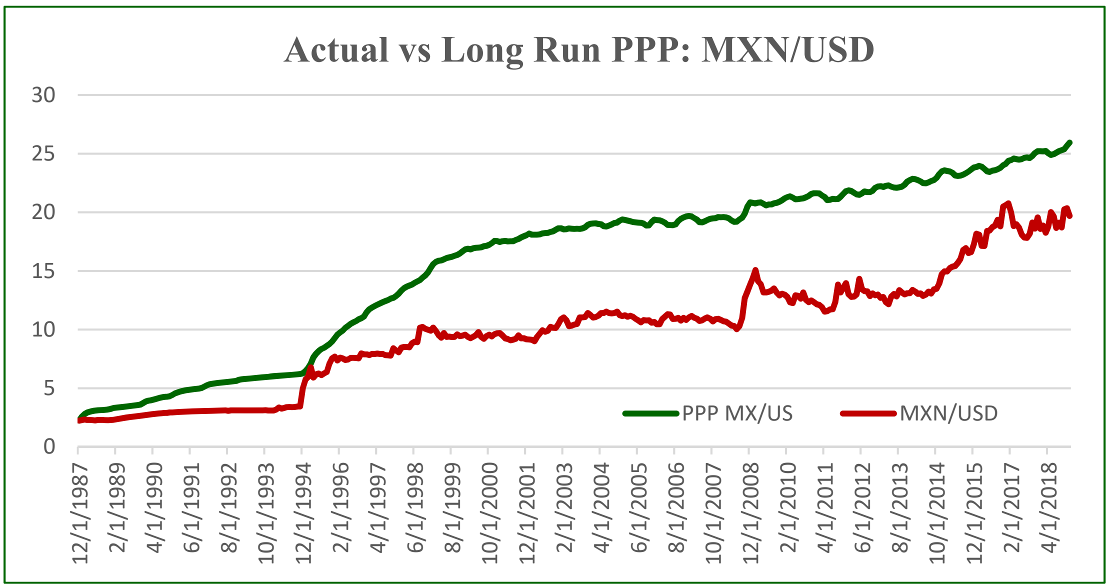
(i) Divide S_t^{PPP} by S_o^{PPP} (where t=0 is our starting point or base year).
(ii) After some algebra,

$$S_t^{PPP} = S_o^{PPP} * [P_{d,t} / P_{d,o}] * [P_{f,o}/P_{f,t}]$$

By assuming $S_{t=0}^{PPP}$ = S_o, we can plot S_t^{PPP} over time. (Note: $S_{t=0}^{PPP}$ = S_o assumes that at time 0, the economy was in *equilibrium*. This may not be true. That is, be careful when selecting a base year.)

Let's look at the MXN/USD case during the 1987-2018 period. During the sample, Mexican inflation rates were consistently higher than U.S. inflation rates. Relative PPP predicts a consistent appreciation of the USD against the MXM. Figure III.6 plots S_t^{PPP} for the MXN/USD.

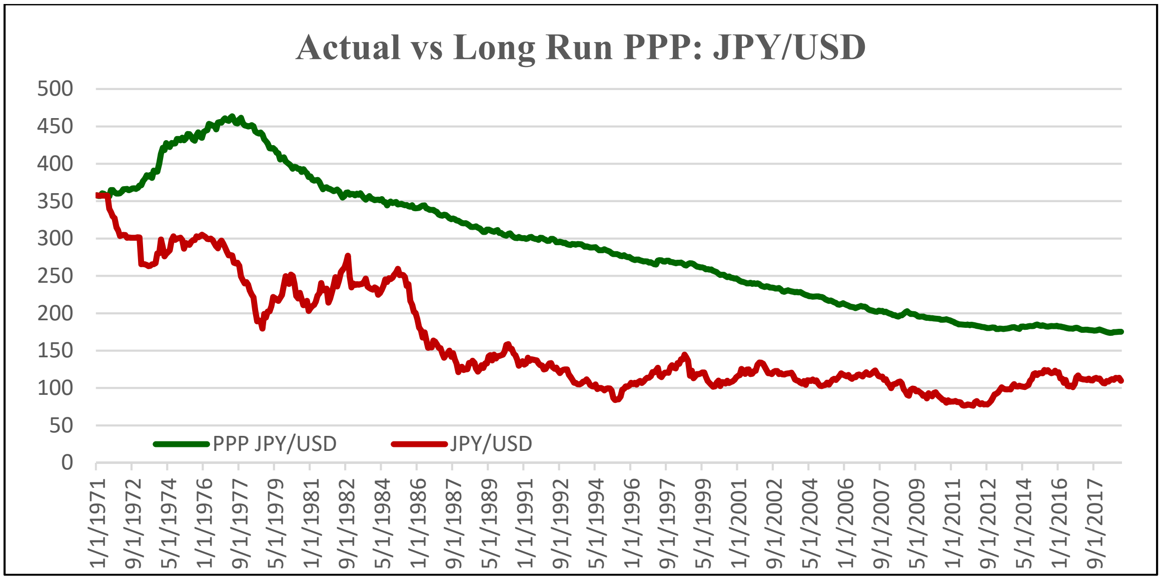
FIGURE III.6
MXN/USD exchange rate: Does PPP hold in the long run?



In the short-run, Relative PPP is missing the target, S_t. But, in the long-run, PPP gets the trend right. That is, inflation rate differentials matter: as predicted by PPP, the high Mexican inflation rates differentials against the U.S depreciate the MXN against the USD.

Similar behavior is observed for the JPY/USD, see Figure III.7, using data from 1971-2018. The inflation rates in the U.S. have been consistently higher than in Japan, then, according to Relative PPP, the USD should depreciate against the JPY. PPP gets the long term trend right, but misses S_t in the short-run.

FIGURE III.7
JPY/USD exchange rate: Does PPP hold in the long run?



Note that in both graphs, S_t^{PPP} is smoother than S_t, which is why a very poor model to explain the short-term MXN/USD movements. Both exchange rates, however, share the same long-run trend.

2.F PPP: Two practical applications

Given the basic economic intuition behind PPP and the empirical evidence that gives PPP some long-run support, many analysts use PPP exchange rates to compare economic fundamentals across countries. In addition, PPP exchange rates are more stable than actual exchange rates and, thus, big swings in actual exchange rates do not affect PPP valuations of economic fundamentals very much. For example, GDP is usually reported in both actual and PPP figures.

Example III.13: In 2011, using market prices –actual exchange rates- the U.S. GDP was USD 15.06 trillion, which amounted to a 23.1% share of the world's GDP (27.5% share in 1996), while China had a GDP equal to USD 6.99 trillion, for a 9.3% share of the world's GDP (3.1% share in 1996). If PPP exchange rates were used, the U.S. GDP was the same, USD 15.06 trillion for a 20% share of the world's GDP and the Chinese GDP was USD 11.3 trillion, for a 14.4% share. ¶

Many central banks follow a very simple rule to establish a crawling peg. They adjust the domestic currency, with the goal of maintaining a stable real exchange rate. In this way, the exchange rate becomes the inflationary anchor as the nominal depreciation (appreciation) rate matches the growth in domestic prices, thus reducing expectations and loss of competitiveness.

Example III.14: The Bolivian Central Bank followed a crawling peg from 1985 to 1994, through a system of mini-devaluations of the peso boliviano (BOB) against the USD to achieve a stable real exchange rate. The following table shows the changes from 1992 to 1994 in exchange rates (BOB/USD), Bolivian inflation and U.S. inflation:

	1992	1993	1994
S_t (BOB/USD)	4.10	4.48	4.70
s_t (%)	9.33	9.27	4.91
I_{BOL}	10.46	9.31	8.52
I_{US} (%)	1.73	0.85	2.44

The depreciation of the BOB closely followed the inflation rate differential. From June 1994 on, the Bolivian Central Bank has devalued its domestic currency to maintain a stable exchange rate against a basket of currencies. The basket of currencies represents a weighted average of the currencies in Bolivia's six largest trading partners. ¶

III. International Fisher Effect (IFE)

Along with the PPP theory, another major theory is the International Fisher Effect (IFE) theory. It uses nominal interest rate differentials rather than inflation rate differentials to explain why exchange rates change over time, but it is closely related to the PPP theory because nominal interest rates are highly correlated with inflation rates. Recall that PPP emphasizes trade as the determinant of supply and demand for foreign exchange. IFE, on the other hand, emphasizes financial transactions.

3.A Arbitrage in Perfect Financial Markets

Assume that there are perfect international capital markets. That is, there are no restrictions to the free flow of capital across national borders. Also, assume that investors consider a foreign asset a perfect substitute of a similar domestic asset. Then, under the IFE, the expected return to investors who invest in money markets in their home country should be equal to the return to investors who invest in foreign money markets once adjusted for currency fluctuations. For example, using equation (I.1) and ignoring transactions costs, taxes and uncertainty, the "effective" T-day return on a foreign bank deposit is given by

$$r_f \text{ (in DC)} = \left(1 + i_f * \frac{T}{360}\right) (1 + s_{t,T}) - 1.$$

where,
i_f = foreign interest rate for T days;
i_d = domestic interest rate for T days.

On the other hand, the effective T-day return on a home bank deposit is:

$$r_d \text{ (in DC)} = i_d * T/360.$$

Setting r_d (in DC) = r_f (in DC and solving for $s_{t,T}$ (= $s_{t,T}^{IFE}$) we get:

$$s_{t,T}^{IFE} = \frac{\left(1 + i_d * \frac{T}{360}\right)}{\left(1 + i_f * \frac{T}{360}\right)} \qquad \text{(III.3)}$$

$s_{t,T}^{IFE}$ represents the expected change, at time t, in the exchange rate over the next T days. If IFE, as expressed in (III.3) does not hold, capital will flow to the country where the effective T-day return is higher. According to the IFE capital flows will force the equalization of effective rates of return across currencies.

Using a linear approximation, we have that the change in exchange rates is proportional to the change in the ratio of the two countries' interest rates:

$$s_{t,T}^{IFE} \approx (i_d - i_f) * T/360.$$

This linear approximation says that if $i_d > i_f$ investors will sell foreign currency and buy domestic currency as long as the foreign currency is not expected to appreciate by the amount equal to the interest rate differential -i.e., $\approx i_d = i_f$. You should be careful with this pseudo-arbitrage strategy. This strategy ignores currency (depreciation) risk.

IFE gives us an expectation for a future exchange rate, $S_{t,T}^{IFE}$. If we believe in IFE, we can use this expectation as a forecast.

Example III.15: Forecasting exchange rates using IFE.
You work for Euroland Inc., a German manufacturer. You have the following information: $i_{USD,2022:I}$=1%, $i_{EUR,2022:I}$=0.5%, and **$S_{2022:I}$=1.1659 USD/EUR**. You want to forecast $S_{2003:II}$ using IFE.

$$E[S_{2022:II}] = S_{t,T=2022:II}^{IFE} = S_{2022:I} * \frac{(1 + i_{USD,2022:I} * \frac{T}{360})}{(1 + i_{EUR,2022:I} * \frac{T}{360})}$$

$$= \textbf{1.1659 USD/EUR} * \frac{(1 + 0.01 * \frac{184}{360})}{(1 + 0.005 * \frac{184}{360})} = \textbf{1.168872 USD/EUR}$$

That is, for the second semester of 2022, IFE expects an appreciation of the EUR against the USD. This appreciation of the EUR compensates EUR deposits for the higher interest rates in the U.S.¶

3.B PPP and IFE

IFE is related to the domestic Fisher effect, postulated by the economist Irving Fisher in 1930. The Fisher effect states that the nominal interest rate, i, is approximately equal to the real interest rate, Θ, plus expected inflation, E[I], over the life of the interest rate. That is,

$i = \Theta + E[I]$.

If as postulated by Fisher the real interest rate, Θ, is stable over time, then changes in interest rates are driven by changes in inflationary expectations.

Implicitly, PPP states that real interest rates are equal across countries. Thus, differences in inflationary expectations between currencies drive their interest rate differentials:

$$i_d - i_f = (\Theta + E[I_d]) - (\Theta + E[I_f]) = E[I_d] - E[I_f].$$

3.C IFE: Implications

If IFE holds, and without hedging, the expected cost of borrowing funds is identical across currencies. Similarly, the expected return of lending is identical across currencies. *Carry trades*, where the low interest rate currency is borrowed to invest in the high interest rate currency, are not profitable. When the expected change in exchange rates is incorporated into the calculations, all currencies have the same expected nominal interest rate when expressed in the same numeraire.

If an investor expects a consistent departure from IFE, a profitable carry trade can be designed. Suppose that the high currency interest rate consistently changes, against a currency with a low interest rate, by less than the IFE predicts. Then, borrowing the low interest currency and investing in the high interest rate currency is a profitable strategy.

Example III.16: During the 1990s, the Mexican peso depreciated by 5% a year, on average, against the USD. The short-term interest rate differential between the MXN and the USD ranged from 7% to 16%. The realized deviations from IFE were substantial (2% to 11%).

Many U.S. investment firms and U.S. mutual funds, like Fidelity Short-Term World Income Fund, did carry trades expecting to take advantage of $E_t[s_{t,T}] \neq (i_{MXN} - i_{USD})$. Steps:
1) Borrow funds in the U.S. at i_{USD}
2) Convert to MXN at S_t
3) Invested MXN in Mexican government securities at i_{MXN}.
4) Wait until T. Convert back to USD at S_{t+T}, which expected to be: $E[S_{t+T}] = S_t*(1+ E_t[\mathbf{s_{t,T}}])$.

Some simple calculations:
Expected foreign exchange loss 5% ($E[s_{t,T}]$ = -5%)
Assume $(i_{USD} - i_{MXN})$ = -7%. (For example: i_{USD}= 6%, i_{MXN}=13%, (T=1 year).)
The $E[s_{t,T}]$= -5% > $s^{IFE}_{t,T}$ = -7% $\Rightarrow$ "on average" strategy (1)-(4) should work.

Annual expected return (MXN investment):

$$r_f \text{ (in USD)} = (1 + i_{MXN} * T/360) * (1 + E_t[\mathbf{s_{t,T}}]) - 1 = (1 + .13) * (1 - .05) - 1 = 0.074$$

Annual cost for USD borrowing:

$$r_d \text{ (in USD)} = i_{USD} * T/360 = .06$$

Expected USD Profit = .014 per year.

Note: This strategy worked well for a couple of years, when $E_t[s_{t,T}]$ = 5%. But, it failed big in December 1994, when the MXN lost 40% of its value and the accumulated gains were wiped out in a matter of days. ¶

The IFE pseudo-arbitrage strategy differs from covered arbitrage in the final step. Step (4) involves no coverage. It's an uncovered strategy. IFE is also called *Uncovered Interest Rate Parity* (UIRP).

3.D IFE: Evidence

Testing IFE is more complicated than PPP, since IFE involves an expectation (an *unobservable*). In general, we test IFE assuming that on average what we expect occurs. That is, the observed average $s_{t,T}$ equals the expected change at time t, or $E_t[s_{t,T}]$.

IFE has been extensively tested, in general, assuming that on average what we expect occurs ("rational expectations" assumptions). Similarly to the PPP formal tests, a formal test of IFE can be done with a regression based on the linearized version of equation (III.3). That is:

$$s_{t,T} = \alpha + ß\,[(i_d - i_f)_t * T/360] + \varepsilon_t.$$

The test is based on the following null joint hypothesis

H_0 (IFE true): $\alpha = 0$ and $ß = 1$.

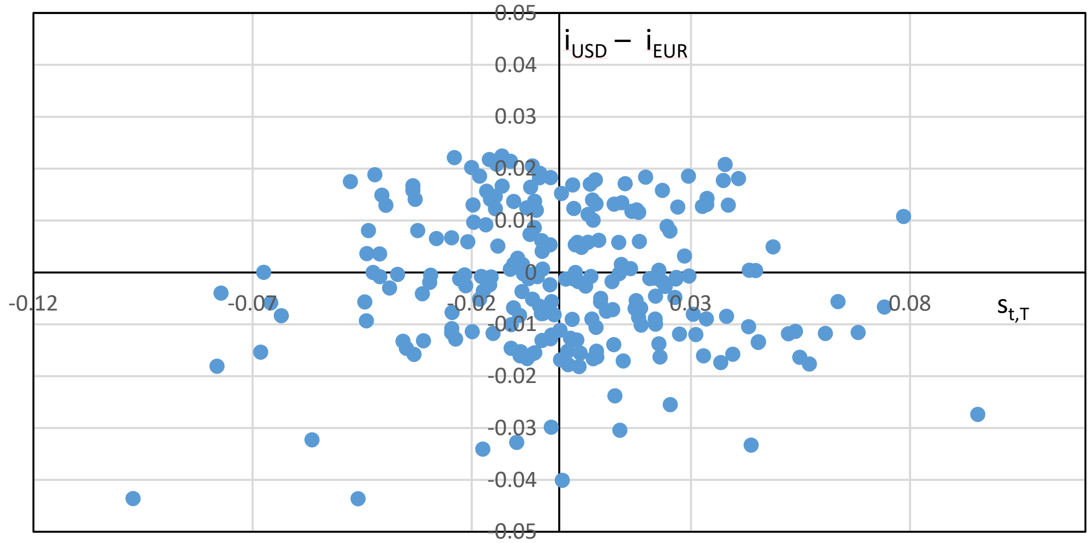
H_1 (IFE not true): $\alpha \neq 0$ and/or $\beta \neq 1$

An F-test can be used to test this null hypothesis. The null hypothesis has been soundly rejected by the data. In general the rejection arises because ß is not statistically different from zero.

Example III.17: Short-run tests of IFE for the USD/EUR

We collected monthly interest rates differentials ($i_{USD} - i_{EUR}$) and e_f (USD/EUR) from 01/99 to 12/17.

IFE: USD/EUR



We immediately see that there is no clear 45 degree line. The visual evidence rejects IFE.

For a formal test of IFE, we estimate the following regression:

$$s_{t,T} = \alpha + \beta\,(i_d - i_f)_t + \varepsilon_t$$

$R^2 = 0.00641$
Standard Error = 0.02907
F-statistic (slopes=0) = 0.1459 (p-value=0.70305)
F-test (α=0 and β=1) = 68.63369 (p-value= lower than 0.0001) => rejects H_0 at the 5% level ($F_{2,193,.05}$=3.05)
Observations = 228

	Coefficients	*Stand Error*	*t-Stat*	*P-value*
Intercept (α)	0.000588	0.001935	0.30400	0.76141
($i_{USD} - i_{EUR}$) (β)	-0.05477	0.14350	-0.38169	0.70305

Let's test H_0, using t-tests ($t_{104,.05}$ = 1.96) :
$t_{\alpha=0}$ (t-test for $\alpha = 0$): (0.000588 – 0)/0.001935 = 0.304 $\Rightarrow$ cannot reject at the 5% level.
$t_{\beta=1}$ (t-test for $\beta = 1$): (-0.05477 – 1)/0.14350 = -8.045 $\Rightarrow$ reject at the 5% level.

Formally, IFE is rejected in the short-run (both the joint test and the t-test reject H_0). Also, note that β is negative, not positive as IFE expects.

Note: During the 1999-2017 period, the average monthly ($i_{USD} - i_{EUR}$) was -0.00164/12 = -.00015. That is, s_t^{IFE} = **-0.015%** per month (IFE expects a 0.015% monthly depreciation of the EUR). But, the actual average monthly s_t was .0007 (s_t =**0.07%** per month; statistically speaking not different from zero), which is different from s_t^{IFE}.

If we use the regression to derive an expectation, the regression expects **$E_t[s_t]$** = .000588-.005477*(-.00164) = **0.0006**, which is statistically speaking not different from zero. That is, we expect a very close to zero monthly change in the EUR against the USD. This zero change is still different from **s_t^{IFE} = -0.15%,** which is statistically significantly different from 0.

Recall that consistent deviations from IFE point out that carry trades are profitable: During the 1999-2017 period, USD-EUR carry trades should have been profitable. ¶

Similar to PPP, there is no short-run evidence. As pointed out above, consistent IFE departures make carry trades profitable: Burnside (2008) show that the average excess return of an equally weighted carry trade strategy, based on up to 20 currencies and executed monthly over the period 1976–2007, was about 5% per year. Lower than excess returns for equity markets, but with a Sharpe ratio twice as big as the S&P500! (Annualized volatility of the carry trade returns was much less than that for stocks).

IFE, however, has some empirical support in the long run: interest rate differentials have some power to predict exchange rates movements. As predicted by the IFE, we find over extended periods of time (5, 10 years) that currencies with relatively high interest rates tend to depreciate and currencies with relatively low interest rates tend to appreciate. Chinn and Meredith (2004) find that estimates of the β are usually not significantly different from 1, at 5 and 10 year horizons.

A different test of IFE is provided by dropping the rational expectations assumption. Froot and Frankel (1989) rely upon survey-based measures of exchange rates to calculate expected depreciation. They find that for reserve currencies (against the U.S. dollar) it is much more difficult to reject the null hypothesis that β=1. But, for other currencies and using a more recent extended sample period, the evidence for IFE is not very strong.

Frankel and Poonawala (2006) find that support on IFE depends to some extent on the exchange rate system: highly managed exchange rate regimes are associated with currencies that show greater deviations from IFE.

Some practitioners use rule of thumbs based on long-run IFE. For example, a 1% change in the nominal 10-year bond yield differential -between USD bonds and EUR bonds- is used to forecast a change in the USD/EUR exchange rate of 10%.

We should note that in Section 1.B we mentioned that PPP is not supported by the data, especially in the short-run. Since IFE is based on some form of purchasing power parity, it should not be surprising that IFE is also rejected by the data.

IV. Expectations Hypothesis of Exchange Rates

The expectations hypothesis of exchange rates states that the expected spot rate T periods from now (S_{t+T}) is equal to today's forward rate for delivery T periods from now ($F_{t,T}$):

$$E_t[S_{t+T}] = F_{t,T}. \qquad \text{(III.4)}$$

Under this equation forward rates are *unbiased* predictors of future spot rates. That is, the average difference between the forward rate and the future spot rate will be a small number, close to zero, over long periods of time.

Equation (III.4) has a strong intuitive appeal. If markets are perfect, speculators will trade forward contracts at prices equal to the expected future rate. Now, suppose $E_t[S_{t+T}] \geq F_{t,T}$. According to the expectation hypothesis, a profit opportunity arises. For instance, assume speculators believe that $E_t[S_{t+T}] > F_{t,T}$, then speculators will buy foreign currency forward and in T days they will sell the foreign currency at a higher price. Note, however, that this cannot be an equilibrium situation. Speculators will be buying foreign currency forward and no investor will be selling foreign currency forward! Obviously, this divergence between $E_t[S_{t+T}]$and $F_{t,T}$ cannot last.

Example III.18: Suppose a South African investor does not behave according to the expectations hypothesis. He expects that in 180 days the ZAR/USD the spot rate will be 5.3400 (S_{t+180} = 5.3400 ZAR/USD). Today the 180 day forward rate is 5.1764 ($F_{t,180}$ = 5.1764 ZAR/USD). For this investor, a potential profit exists. The strategy for the non-expectations hypothesis investor is to buy USD forward at ZAR 5.1764 and, in 180 days, sell the USD for ZAR 5.3400.

Now, if everybody expects the exchange rate in 180 days to be 5.3400 ZAR/USD, a disequilibrium situation will result. Everybody will be buying USD forward and nobody will be selling USD forward. ¶

As example III.18 illustrates, according to the expectations hypothesis, expectations on average should adapt to the forward rate.

Let's manipulate the expectations hypothesis equation (III.4). We will subtract from (III.4) S_t and then divide by S_t:

$$(E_t[S_{t+T}] - S_t)/S_t = (F_{t,T} - S_t)/S_t.$$

Using the IRPT, the left side of the above equation is approximately equal to (i_d - i_f). That is, we can rewrite the above equation as

$$(E_t[S_{t+T}] - S_t)/S_t \approx (i_d - i_f) * T/360. \qquad \text{(III.5)}$$

Equation (III.5) is another way of stating the expectations hypothesis. We have seen this equation before, it is the IFE. But, it was derived from a different intuition.

Note, that equation (III.5) is equal to IRPT, when $E[S_{t+T}]=F_{t,T}$. For this reason, equation (III.5) is referred as the *uncovered interest rate parity* (UIRP). The IRPT is a relation derived from arbitrage considerations. The IRPT involves no risk. UIRP, however, involves an expectation about future spot rates, it does not involve a set price for future spot rates. Therefore, UIRP involves risk. Risk considerations might create a differential between the forward rate and the expected future spot rate.

4.A Expectations Hypothesis: Implications

Under the expectations hypothesis, the expected cash flows associated with hedging or not hedging currency risk are the same. A hedger converts her foreign currency assets and liabilities at the forward rate. A non-hedger expects to convert her foreign currency assets and liabilities at the expected future spot rate. Therefore, under the expectations hypothesis, both the hedger and non-hedger have the same expected cash flow expressed in the domestic currency.

4.B Expectations Hypothesis: Evidence

In general, expectations are unobservable. However, some companies and organizations survey "experts" and compile FX expectations (*Bloomberg*, in the U.S., *Japan Center for International Finance*, in Japan, *Banxico*, in Mexico, etc.). EH is not tested based on these surveys, but on the implications of the EH.

Once we test the implications, testing the EH theory is simple. We have to answer a key question: are forward rates good predictors of future spot rates? The expectations hypothesis can be tested based on equation (III.4) and using a simple regression:

$$(S_{t+T} - F_t)/S_t = a + b\, Z_t + \varepsilon_t,$$

where Z_t represents any economic variable that might have power to explain exchange rates, for example, $(i_d\text{-}i_f)$. The expectations hypothesis implies that a=b=0. That is, there are no variables capable of forecasting the prediction error. Tests of this form have found that b is negative and significant when $Z_t=(i_d\text{-}i_f)$. The R^2, however, is very low.

The expectations hypothesis can also be tested based on the UIRPT formulation of equation (III.5), using the following regression:

$$(S_{t+T} - S_t)/S_t = a + b\, (i_d - i_f) + \varepsilon_t.$$

Under the expectations hypothesis, the null hypothesis to test is

H_0: a=0 and b=1.

As illustrated by Example III.17, it is common to find that b<0. That is, when $(i_d\text{-}i_f)$=2%, the exchange rate depreciates by (b * .02) --instead of appreciating by 2% as predicted by UIRP.

In summary, tests of the expectations hypothesis find that forward rates have little power for forecasting spot rates. That is, the forward rate is a biased estimator of the future spot rate.

4.C Explanations for the Forward Bias

Given that the forward rate is not a good predictor of futures spot rates, many economists have attempted to provide rational explanations for this counterintuitive result.

4.C.1 Risk Premium

A possible explanation for the failure of the expectations hypothesis is the existence of a risk premium. Recall that the risk premium of a given security is defined as the return on this security, over and above the risk-free return. A foreign exchange risk premium induces risk-averse agents to take a risk in the foreign exchange market. Thus, the existence of a divergence between $E_t[S_{t+T}]$ and $F_{t,T}$ can be justified by risk-aversion.

Now, let us formalize the idea of a risk premium in the foreign exchange market. After some simple algebra, we find that the expected excess return on the foreign exchange market is given by:

$$(E_t[S_{t+T}] - F_{t,T})/S_t = RP_{t,t+T},$$

where $RP_{t,t+T}$ represents the foreign exchange risk premium.

Example III.19: Understanding the meaning of the foreign exchange risk premium.
Suppose you have the following data:
S_t= 1.58 USD/GBP,
$E_t[S_{t+6\text{-}mo}]$ = 1.65 USD/GBP
$F_{t,6\text{-}mo}$ = **1.62 USD/GBP**.

The expected change in the exchange rate is equal to:

$$E[s_{t+6\text{-}mo}] = (E_t[S_{t+6\text{-}mo}] - S_t)/S_t = (1.60 - \mathbf{1.58})/\mathbf{1.58} = \mathbf{0.0127}.$$

The 6-mo foreign exchange forward premium on the GBP is:

$$p_{6\text{-}mo} = (\mathbf{F_{t,6\text{-}mo}} - S_t)/S_t = (\mathbf{1.62} - 1.58)/1.58 = \mathbf{0.0253}.$$

According to this example, in the next 6-month period, the GBP is expected to appreciate against the USD by **1.27%**, while the forward premium suggests a GBP appreciation of **2.53%.**

$$\Rightarrow \; E[s_{t+6\text{-}mo}] < p_{6\text{-}mo} \qquad (\approx\; (\boldsymbol{i_{d=USD}} - i_{f=GBP})/2)$$

The discrepancy arises from the presence of a foreign exchange risk premium, $RP_{t,t+6\text{-}mo}$, which makes the forward rate a biased predictor of the exchange rate six months from now.

Given the positive risk premium on the GBP, the expected (USD) return from holding a GBP deposit will be more than the USD return from holding a USD deposit. This non-zero return differential might be an equilibrium result consistent with rational investor behavior. The higher return from holding a GBP deposit is necessary to induce investors to hold the riskier GBP denominated investments. ¶

A risk premium in foreign exchange markets implies that the expectation hypothesis should be written differently:

$$E_t[S_{t+T}] = F_{t,T} + S_t\, RP_{t,t+T}.$$

As long as the risk premium, $RP_{t,t+T}$ is consistently different from zero, foreign exchange markets will display a forward bias.

The empirical evidence for a risk premium in foreign exchange markets is weak. Several researchers have assumed that the forward rate is an unbiased predictor of future spot rates. Then, they have tried to explain the risk premium using the fundamental variables used in the finance literature to explain risk premia in financial assets, such as volatility. No significant relation has been found between the foreign exchange risk premium and fundamental variables.

♦ **Risk Premium and Diversifiable Risk**
Note that the existence of a divergence between $E_t[S_{t+T}]$ and $F_{t,T}$ can be justified by the existence of a risk premium. Many economists claim, however, that a risk premium is justified if exchange rate risk is not diversifiable. If a risk is diversifiable, then there is no need to expect a compensation for holding it. ♦

4.C.2 Errors in Forming Expectations

In an uncertain environment, economic agents are expected to make forecasting mistakes. Rational agents, however, will eventually learn and, thus, errors will not consistently persist. Nevertheless, some economists have argued that investors make consistent errors in forecasting exchange rates. One explanation for these consistent mistakes relies on the assumption that it takes time for investors to learn about new market conditions. For example, suppose there is a new chairman on the Bank of Japan. It might take years for economic agents to learn the Bank of Japan's new monetary policy. That is, there is "slow learning."

Karen Lewis, in a paper published in the Journal of Monetary Economics in 1989, showed that even when slow learning of money supply rules is taken into account the forward bias observed in the early 1980s did not disappear.

4.C.3 The "Peso Problem"

A peso problem is a very specific form of a small sample problem that affects statistical inference. According to this view, for long periods of time investors assign a small but positive probability to an extreme change in the asset price (such as a devaluation or a stock market crash), which may never materialize in a limited sample period. The frequency of the extreme events in the sample studied does not equal the ex ante anticipated probability. The forward rate, however, will reflect the ex-ante probability distribution. Since the event may never materialize, markets will observe a persistent forward bias.

The small sample problem is called *peso problem*, in reference to the discrete changes in the Mexican peso in 1976. Before 1976, the Mexican peso had been successfully pegged to the

USD for 23 years. Mexican interest rates were substantially higher than U.S. interest rates, creating a MXP/USD forward rate higher from the MXP/USD spot rate. Therefore, the MXP/USD showed a persistent premium. The peso problem, however, is not a new problem, nor it is constrained to developing economies. It applies to any situation in which there can be a discrete jump in prices or shift in policy regimes.

Example III.20: Peso problem: Now and then.
The Mexican peso used to show a real and continuous appreciation until the Mexican government finally devalued the peso (generally after an election). Before the devaluation, since markets were expecting a devaluation, the peso used to have a strong forward bias.

During the period 1890-1908 the USD/GBP showed a peso problem. That is, during that period financial markets expected the USD to depreciate against the GBP, but this never happened –i.e., expectations were persistently biased. Different events created this bias. One of them was the 1896 Presidential Election, in which the U.S. adherence to the gold standard was in question. ¶

V. Looking Ahead

The exchange rate models based on arbitrage that we have studied do not enjoy a strong support from the data, especially in the short-run. Note that we have not explicitly mentioned supply and demand factors when the parity relations were developed. In Chapter I, however, we emphasized that exchange rates are just prices. In the next chapter, we are going to explicitly model supply and demand for the foreign currency to gain more insight into exchange rates.

Interesting readings:

International Financial Markets, by J. Orlin Grabbe, published by McGraw-Hill.

International Financial Markets and The Firm, by Piet Sercu and Raman Uppal, published by South Western.

International Investments, by Bruno Solnik, published by Addison Wesley.

Cassel, Gustav (1918), "Abnormal deviations in international exchanges," The Economic Journal.

Hummels, David (2001). "Toward a Geography of Trade Costs." Unpublished manuscript, Purdue University, September.

Parsley, D. and Wei, S., 2007. A Prism into the PPP Puzzles: The Micro-foundations of Big Mac Real Exchange Rates, The Economic Journal, October, 117, 1336-1356.

Rogoff, Kenneth (1996), "The purchasing power parity puzzle", Journal of Economic Literature, June 1996, 647-68.

Taylor, Alan M. (2002), "A century of purchasing power parity", Review of Economics and Statistics, 84: 139-150

Exercises:

1.- The spot USD/ZAR is equal to .19630 (ZAR = South African Rand). The one-year interest rates on the Eurocurrency market are 5% in ZAR and 7% in USD. What is the one-year forward exchange rate (USD/ZAR)? The one-month rates are 5.5% in ZAR and 6% in USD. What is the one-month forward exchange rate? (Remember to transform the annual rate to a one-month rate.)

2.- Suppose you are given the following data for the Israeli shekel (ISS) and the USD:
S_t = 3.40 ISS/USD
$i_{ISS,1\text{-}yr}$ = 8%
$i_{USD,1\text{-}yr}$ = 5.5%
$F_{t,1\text{-}yr}$ = 4.30 ISS/USD.

(i) Determine if the ISS is a discount or premium currency.
(ii) Determine if Israel will experience capital inflows or capital outflows.
(iii) Is it possible to construct a covered interest rate strategy to profit from the above prices.

3.- Ms. Sternin, a U.S. investor, has USD 500,000 to invest. The one-year interest rate offered in the U.S. is 5.5%, while the one-year interest rate offered in Japan is 2.1%. The spot rate is .009 USD/JPY, that is USD .009 per Japanese yen. Ms. Sternin is offered a one-year forward contract at .008 USD/JPY.

(i) Determine the arbitrage-free one-year forward contract exchange rate.
(ii) Can Ms. Sternin make a risk-free profit? If yes, describe a covered arbitrage strategy and determine Ms. Sternin's profits.

4.- The bid-ask rates are as follows:
(i) CHF/USD 1.5100-40.
(ii) One year Euro-CHF 4¼-5/8 %, which means that the bank is ready to borrow CHF for a year at 4.25% or to lend CHF for a year at 4.625%.
(iii) One year Euro-USD 6 3/4-7/8 %.

Provide a quotation for the one-year CHF/USD forward exchange rate.

5.- The French bank Le Meridian quotes the following exchange rates: EUR/USD=0.9250-0055. The Euro one year interest rates for the EUR, i_{EUR}, and for the USD, i_{USD}, are 6 1/4-1/2 and 7 1/8-1/4. You work for a U.S. bank. What is a proper bid-ask quotation for the one-year USD/EUR forward exchange rate?

6.- Using the data of Example III.1 design an arbitrage strategy if F = 150 JPY/USD.

7.- Based on the empirical findings that reject the expectations hypothesis theory, construct a trading strategy that takes advantage of the failure of the expectations hypothesis to hold.

8.- The U.S. and Poland both produce ricotta cheese. A pound of ricotta cheese sells in the U.S. for USD 5.50. An equivalent pound of ricotta cheese sells in Poland for PLN 20 (PLN: Polish Zloty).

(a) According to purchasing power parity (PPP), what should be the USD/PLN exchange rate?

(b) Suppose S_t=.3010 USD/PLN Is the USD overvalued or undervalued? That is, what kind of signal have you generated? Calculate the real exchange rate, R_t.

(c) Suppose the price of a pound of ricotta cheese in the U.S. is expected to rise to USD 6 over the next year, while the price of an equivalent Polish pound of ricotta cheese remains constant. According to PPP, what should be the expected USD/PLN exchange rate one-year from now?

9.- You work for HK Bank, a Hong Kong company. You have the following information for the first semester of 2001: $i_{USD,2000:I}$=5.80%, $i_{HK,2001:I}$=7.10%, and $S_{2001:I}$=8.0523 HK/USD. Forecast $S_{2001:II}$ using IFE. Do you expect a depreciation of the HKD? HKD has a currency board. Does your $S_{2001:II}$ forecast surprise you?