CHAPTER XIII

INTERNATIONAL BOND MARKETS: PRICING AND HEDGING

This chapter applies some of the concepts introduced in Chapter XI and in Appendix XI. First, the chapter introduces the techniques used to price and select new issues in an international context. Then, the chapter focuses on *interest risk management*. To this end, we introduce the specifics of the most popular government bond futures contract, delivery, and pricing. This chapter concludes with a couple of examples on interest rate risk management.

I. Pricing and Selection of a New Eurobond Issue

Pricing a fixed-income public issue is one of the critical functions of an issuing house. The major issuing houses with long experience in bond and derivative markets use their skills to (1) read a market, (2) structure and price an issue, and (3) anticipate market changes and interpret their impact on pricing. It is not rare, however, to find pricing mistakes in new bond issues. Tight competition has induced many lead managers to underprice in the dubious interest of market share and rarely in the interest of their issuer or investor clientele. Issues sometimes are too complex and their distribution is poor. Issues are also vulnerable to weak market conditions. Clearly, the issuer and the investors have a vital interest in correct pricing, but it falls primarily on the issuing house to form a view which it then has to back up by committing substantial amounts to underwriting the issue.

♦ **International Bond Pricing: Same Domestic Techniques**
The techniques used for pricing issues in international markets are similar to the ones used domestically. The wide range of instruments and currencies available in international bond markets, however, makes pricing international bonds more complicated. ♦

We will concentrate on the dominant international bond market: the Eurobond market. The process of pricing a Eurobond involves the collection and evaluation of information, and the evaluation of market conditions. Now, we turn our attention to these two points.

1.A Information

1.A.1 Borrowing Requirements

The borrower's funding requirement determines the amount to be raised over a certain period and it is left to the issuing house to submit, from time to time, proposals for issues. Usually, the borrower will select for each issue the currency of exposure, amount, maturity range, call options, and target cost of funds. Then the issuing house determines the most cost-effective procedure, including opportunities for issuing in other currencies, the availability of hedging instruments and, in the case of corporate borrowers, the merits of a convertible issue or an issue with equity warrants attached.

1.A.2 Preliminary Analysis of the Issue

Before any pricing analysis is done, the issuing house examines the market's assessment of the borrower's outstanding issues. This is an excellent guide to pricing a new issue. A preliminary analysis also includes the trading history, quality of market makers, liquidity, the perceptions of market participants, and yields on similar issues. Benchmark issues are now readily available in most Euro-currency sectors. Therefore, the analysis of yields is done in terms of the "spread" over the yield of the benchmark issue.

1.A.3 Market Conditions

A detailed study of market conditions is essential. It places the issue in relation to what is going on in other markets. This study takes into account not only Eurobond markets but domestic bond markets and foreign exchange markets.

Example XIII.1: In August 1998, the government of Philippines appointed JP Morgan and Warburg Dillon Read to lead-manage a Eurobond EUR issue. The fixed-rate transaction was to be issued in tandem with a dollar floating-rate note to be lead-managed by Goldman Sachs. The two issues were postponed in September after Russia's debt default triggered a crisis of confidence in emerging market debt. In January 1999, the country successfully re-entered the international capital market with a USD 1 billion global bond. The Eurobond EUR was placed three months later. ¶

The study covers derivative markets such as swaps and even expectations in stock markets. Other relevant information may include credit ratings of Euro-issues by the borrower and by the sovereign of its country of origin, economic information such as inflation, growth of GDP, balance of payments, etc.

Sometimes, market conditions not only affect the pricing of an issue, but also its size. When markets are very receptive to some issuers or to some markets, issues can be oversubscribed. It is common, that companies increase the size of the issue when the demand for an issue is very strong. Of course, the opposite also occurs: when demand for an issue is very weak, issuers can cut the size of the bond issue.

Example XIII.2: In March 2003, Petrobras, Brazil's state owned oil company, sold USD 400 million 5-year eurobonds. Petrobras was initially planning to sell USD 200mm in bonds, but due to strong demand ended up placing USD 400mm. Market sources said demand surpassed $1bn.

1.A.4 Perception of the Issuer

Issuing houses collect objective and subjective information about the borrower's outstanding issues. The input of the bond traders and the bond sales force is crucial. Traders have continuous contact with institutional and retail investors and will gather information about their interest in the structure and pricing of particular new issues.

In the case of a first-time issuer, a study of the market's perception of the issuer may cover:

(1) the perception of the borrower by its competitors.
(2) the perception of the issuer within its domestic financial markets in relation to other borrowers.
(3) the perception of the borrower, if any, in the Euromarkets.

In the case of an existing issuer, the perception is already reflected in the performance of outstanding issues. An issue maybe trading poorly because of bad design (for example, small size), and not because the issuer has a negative perception. Perceptions can be changed at the cost of a more expensive issue and a determination to support outstanding issues.

1.B Evaluation

The next step is to evaluate the information. In new markets, pricing occasionally looks like informed guesswork, as in the case of the early EUR issues. In established markets, however, proposals are noted more for their convergence than differences in pricing.

Example XIII.3: On April 23, 1998, Olivetti, the Italian electronics company, issued the largest corporate bond denominated in euros. The EUR 600 million five-year issue followed a EUR 500 million convertible bond issued by Parmalat, the Italian food company, earlier in 1998. Lehman Brothers, sole lead manager, said it was difficult to price the issue owing to the absence of any sizable corporate benchmark in euros. Nevertheless, the unrated bond, which was priced to yield 128 basis points over the five-year EUR OAT, was very well received. Source: *Financial Times*. ¶

The evaluation process does not fall on the issuing house alone. Experienced issuers are able to evaluate their perception and market conditions. Now, we will illustrate the process described above.

1.C Case Studies

1.C.1 Pricing a New Straight Bond: Merotex

Merotex is a leading construction firm, based in Goritzia, Italy. It ranks 18th among the top 20 Italian companies. Merotex has regularly issued in the DEM Eurobond market where it has consistently obtained the best terms for corporate borrowers. It has not recently issued in ITL in either the domestic or other Eurobond markets.

Merotex has recently bought two U.S. construction companies in South Florida. These acquisitions were financed by bank loans for a total of USD 250 million, which Merotex wants to refinance with medium-term debt. A flow of USD denominated receivables and a low USD interest rate structure point to a Euro-USD issue. Merotex would like to issue a simple straight bond with no early call options.

General market conditions seem favorable for a USD Eurobond issue. The Euro-USD bond market is presently very good; U.S. economic conditions are above expectations; and the USD is currently a very strong currency. In addition, the primary market has quickly absorbed a recent 10-year Euro-USD issue by Fina.

Merotex has a good track record in the international bond markets. Merotex, however, has no outstanding Euro-USD issues. In order to have an idea of the magnitude of the spread the market may require, we analyze issues by similar borrowers from Italy and overseas.

Comenti is a multinational Italian construction firm, which has launched several Eurodollar issues, one of which has approximately six years of remaining life. The issue is currently trading at 40 basis points (bps) over 6-year U.S. Treasuries. Comenti enjoys an excellent reputation in Euromarkets.

Fix Constructions (FC) is Merotex's major U.S. competitor in Florida. FC has launched a 10-year Eurodollar issue five years ago, with a call option two years from now. The issue is currently trading at a yield equivalent to a spread of 65 bps over 5-year U.S. Treasuries. FC is well regarded but, lately, its performance has been just average and Merotex is invading FC's traditional markets.

Some large Italian companies have also issued Euro-USD bonds and those outstanding with 5-year maturity trade within a range of 40-70 bps.

1.C.1.i Evaluation

The FC issue is trading at a relative high spread. This may be accounted for by numerous factors:

(1) In the primary market.

i.- poor lead-manager and/or weak syndicate.
ii.- an issue size not large enough to sustain liquidity.
iii.- FC is a diversified company and therefore more difficult to evaluate.
iv.- poor timing.

(2) In the secondary market.

i.- a poor trading history (leading some market markets to withdraw).
ii.- deterioration of the image or credit of the business.
iii.- the call provision.

Merotex's track record in the international bond market is limited but very good. Merotex's DEM bond issues have been well received in the market. German houses are familiar with Merotex, therefore, Merotex would like to include one German house in the management group.

The lead manager outlines the structure of a Euro-USD issue for Merotex as follows:

Amount: For a first-time issue in the Euro-USD market, the issue size should be sufficient to promote liquidity, but not so much as to make the placement process difficult. This suggests an issue size of USD 200 million with a possibility of an increase to USD 300 if the initial placement is well received.

Maturity: The clear choice is five years. Market conditions would permit a longer maturity, but for a first-time issuer a shorter maturity is preferable.

Yield spread: An aggressive spread would be 40 bps over 5-year U.S. Treasuries. This could be justified by the strong reputation, size and ranking of the company, and by good market conditions. For a first-time issue, a small premium is suitable. The spread is set at 45 bps.

The lead manager is able to formulate a pricing scheme:

U.S. Treasury:	6.915% s.a. (semiannual)
Merotex spread:	0.45% s.a.
Merotex yield:	7.365% s.a., or 7.501% p.a. (annual)

Therefore, investors will be offered a 5-year Merotex Eurodollar bond at a price to yield 7.50% annual. The price is expressed in terms of the issue price -usually 100 percent or par- and a coupon in this instance of 7½% p.a.

The issuing house structures its proposal to Merotex with a selling concession of ¾%. That is, the issuing house buys the issue at a price of 99¼. In addition, the issuing house charges ¼% for managing the issue and ¾% for making an underwriting commitment. That is, Merotex has to pay an additional 1% to the issuing house.

In competitive bidding, issuing houses may forgo some of the fees so as to lower the all-in cost of the issue. In this example, we will assume that the issuing house sells the issue at 99.24 --i.e., it forgoes part of the selling concession. However, at a price of 99.24 percent, the coupon required to yield 7½% is lower. That is, the issuing house calculates the coupon rate using the present value formula introduced in the Appendix XII. Assuming YTM=r=7½, T=5, P=99.24, and FV=100, the only unknown is C. Solving for C, the issuing house obtains 7.3113%. Rounding up, the coupon rate is set at 7 (5/16). (Recall that (5/16)=0.3125)

In this case, the lead-manager decides, under competitive pressure, to forgo the selling concession and to lower the coupon from 7½ to 7 (5/16). The issue is said to be priced "at the selling concession." By joining the syndicate, other members of the management group rely entirely on the management fee and underwriting fee for their remuneration on the issue.

The issue's commission structure will still include a selling concession of ¾%. The buyers of the issue are indifferent, because the yield of the issue is still 7½%. The only party to benefit is the issuer, who benefits from lower coupon payments.

1.C.1.ii Expenses

Expenses on the issue include the following items:

1.- Paying Agency: Merotex issues 100,000 bonds in USD 1,000 denominations and 10,000 bonds in USD 10,000 denominations.

Total number of bonds:	110,000.
Coupon charge p.a.:	USD .07 per coupon payment or USD 7,700 payable in arrears.
Redemption charge:	USD .70 per bond or USD 77,000 flat on redemption.
Authentication:	USD 4,000 flat on delivery of bonds.
Administration:	USD 2,000 (p.a) payable in arrears.

2.- Listing: USD 20,000 flat payable in advance.

3.- Trustee: USD 8,000 (p.a) payable in advance.

4.- Printing, fees of lawyers, traveling, and other reimbursable out-of-pocket expenses: USD 80,000.

1.C.1.iii Pro Forma of the Issue

Borrower:	Merotex C.A.
Guarantor:	None
Amount:	USD 200 million
Maturity:	5 years
Coupon:	7 (5/16)
Issue price:	100%
Amortization:	Bullet repayment on final maturity date
Issuer's call option:	None
Listing:	London
Denominations:	USD 1,000 and USD 10,000
Form:	Bearer securities
Status:	Direct, unconditional and unsecured obligations ranking equal with all senior unsecured debt of the issuer.
Negative pledge:	Undertaking not to enhance the status or security of any outstanding senior unsecured debt of the issuer without granting the benefit of such enhancement to the bonds under the issue.
Events of default:	Standard including cross-default
Rating:	Applied for and expected to be AA
Tax status:	All payments of interest and principal to be made free of deduction or any withholding. In the event that such withholding or deduction is required under the laws of the jurisdiction of the issuer, the issuer will pay additional amounts, which fully compensate the holders as if no deduction or withholding has been levied.
Commissions:	1¾% flat
Yield:	7.3125% (at issue price), 7.50% (at 99.24% or after deduction of full selling concession)

1.C.1.iv Cost of Funds

To calculate the cost of funds, we write the cash flows associated with the issue in Table XIII.A.

TABLE XIII.A
Cash Flows of Merotex C.A. (in USD million)

Year	0	1	2	3	4	5
Principal	200.0000	-	-	-	-	-200.0000
Interest	-	-14.6250	-14.6250	-14.6250	-14.6250	-14.6250
Commissions	-3.5000	-	-	-	-	-
Paying Agency	-	-.0077	-.0077	-.0077	-.0077	-.0847
Authentication and Admin.	-0.0040	-0.0020	-0.0020	-0.0020	-0.0020	-0.0020
Listing	-0.0200	-	-	-	-	-
Trustee	-0.0080	-0.0080	-0.0080	-0.0080	-0.0080	-0.0080 -
Reimbursable expenses	-0.0800	-	-	-	-	-
Cash Flow	196.388	-14.6427	-14.6427	-14.6427	-14.6427	-214.7117

1.C.1.iv.1 Cost of Funds Inclusive of Commissions and Reimbursable Expenses

This figure takes account of the coupon, the commissions of 1¾% flat on the issue amount, and the reimbursable managers' expenses. From Table XIII.A we gather all the necessary information. The commissions amount to USD 3.5 million and the reimbursable expenses to USD 80,000, or a total of USD 3.58 million. The issuer then receives the net proceeds of:

USD 200,000,000 - USD 3,580,000 = USD 196,420,000 (or 98.21% of the issue amount.)

The all-in cost can be calculated as the IRR of a 5-year project that has a positive cash flow of USD 196.42 million in year zero. Every year the project has negative cash flows of USD 14.625 million. In the final year, the project has an additional negative cash flow of USD 200 million. The IRR is equal to 7.7580. That is, Merotex obtains an all-in cost of 7.7580 percent annually.

1.C.1.iv.2 Cost of Funds Inclusive of all Expenses

The different additional costs impact at different times and they should be included in the calculation of the all-in cost of funds. From Table XIII.A we obtain all the necessary inputs to calculate the IRR of the project. Now, the all-in cost is equal to 7.7778 percent annually.

1.C.2 Pricing a New Eurobond with Equity Warrants: Voeller Oel und Mineral Forschung

1.C.2.i Eurobond with Equity Warrants: General Consideratios

By 1990, Eurobond issues with equity warrants attached (*equity warrant issues*) became the second largest category of instruments in the Eurobond market. Motivated by a great bull market, Japanese borrowers issued 95% of all equity warrants issues, denominated mainly in USD, CHF, and DEM. Equity warrant issues have two different components: a standard fixed-rate Eurobond issue, and a detachable equity warrant. The equity warrant is a standard call option on the stock of the issuer. Thus, during its life, the warrant value is made up of the intrinsic value and a time value. Warrants tend to have longer maturities than standard call options. On Euro-USD bonds, which offer maturities of up to 10 years, the exercise periods generally range between 4 and 7 years. Bond issues in CHF, EUR, or GBP, which offer longer maturities, permit longer warrant exercise periods.

The bond is priced as normal in line with current market yields and the warrant is priced separately as a function of the desired warrant premium and equity content. The two components are packaged with a unique price of either (1) 100 percent, in which case the bond price will be at a discount to par and the coupon at a below-market level (*discount bond*); or (2) in excess of 100 percent where the bond is issued at a normal market price, that is, close to 100 percent (*full coupon bond*). Once the issue is launched, the equity warrant is typically detached from the package, leading to a secondary market in three distinct securities. These are the *bond and the warrant* (cum-warrant); *the bond alone* (ex-warrant); and *the warrant alone*.

Equity warrants, like convertibles, are an instrument for raising equity capital at a premium to the share price ruling at the time of launch. Unlike convertibles, however, the amount of equity raised through the exercise of the warrants is not tied to the nominal amount of the bond issue. The ratio of equity raised to the issue size is called the *equity ratio*, or *warrantability*, and can range from 100% to 200% of the nominal amount of the bond issue.

On launch, equity warrants attached to Eurobond issues are priced out-of-the money. The intrinsic value of the warrant at launch is nil and the warrant has time value alone. The Black-Scholes formula, introduced in Appendix VI, is used to approximate the value of the warrant. Warrant traders also evaluate the value of the warrant on the basis of practical standards and the "feel" of the market. For example, traders make adjustments based on (1) the prices of other warrants in the market, (2) market perception of the company and (3) expectations of the performance of the stock market over the life of the warrant.

1.C.2.ii Voeller Oel und Mineral Forschung

1.C.2.ii.1 Information

Voeller Oel und Mineral Forschung (VOMF) is a leading German company engaged in the exploration and exploitation of uranium deposits. Recently, uranium prices have been raising. The German economy is coming out of a prolonged recession, inflation is very low, and the stock market is expected to do very well in the near future. VOMF's share price has been boosted by its recent exploration agreements with Russia. VOMF is looking to refinance short-term debt amounting to USD 100 million. VOMF hires an investment bank to study strategies and to propose a refinancing plan.

The investment bank suggests an equity-linked financing for the following reasons:

i.- The German stock market is expected to do well in the future.
ii.- VOMF has a strong reputation in international markets.
iii.- The EUR (DEM) has a record as a strong currency.
iv.- Shareholders might vote for an increase in capital in the next assembly.

VOMF tells the investment bank that it is not urgent to increase equity. VOMF is also adverse to the high cost of issuing ordinary equity. The investment bank proposes a straight bond with equity warrants attached.

The investment bank has the following data available:

US Treasury yields:	3-yr 6.530% (s.a.); 5-yr 6.915% (s.a.); 7-yr 7.135% (s.a.)
EUR Bund interest rate:	1-year 4.52% (p.a.)
VOMF Euro-USD bond yield:	U.S. Treasuries + 9 bps
Current VOMF's share price (P_0):	EUR 120
Historic dividend yield:	5.50%
Historic stock price volatility:	3-yr 19.90%; 5-yr 21.40%; 7-yr 25.50%.
Outstanding warrants	
Outstanding life:	3½ years
Current price (W_0):	USD 10-10.80 (EUR 15.625-17)
Exercise price (X):	EUR 145
Current exchange rate:	.64 USD/EUR (1.5625 EUR/USD).

1.C.2.ii.2 Evaluation

Usual market practice requires the equity content to range from 100-200% of the issue amount. In this case, since present market conditions are good, the investment bank suggests an equity content of 150%. The investment bank also sets the *exercise ratio* equal to 1, which is the same exercise ratio set for the outstanding warrants. An exercise ratio equal to one means that one warrant entitles the holder to purchase one share.

Using the Black-Scholes formula, setting the strike price at EUR 150, the investment bank calculates a theoretical price of a three-year warrant of EUR 12.23.

Reminder: Black-Scholes formula

Recall the Balck-Scholes formula: $C = P\ N(d1) - X\ e^{-rfxT}\ N(d2)$, where
$d1 = [\ln(P/X) + (r_f + .5\ \sigma^2)T]/[\sigma T^{1/2}]$,
$d2 = [\ln(P/X) + (r_f - .5\ \sigma^2)T]/[\sigma T^{1/2}]$,
and N(.) represents the cumulative normal distribution function.

We have seen this formula in Chapter VII.
Inputs:
P = current price = EUR 120
X = strike price = EUR 150
r_f = risk free rate (annual)= .0452
σ = volatility (annual) = .1990
T = time to maturity = 3 years.

Replacing in the Black-Scholes formula, we get

$C = EUR\ 120\ N(-.08165) - EUR\ 150\ e^{-.0452x3}\ N(-.42633) = 12.226.$

VOMF's outstanding warrants with 3½ years until expiration have been trading recently in the range USD 10-10.80, or EUR 15.625-17 at today's exchange rate of .64 USD/EUR. The most recent quote was EUR 16. The exercise price is EUR 145, which produces a global premium (GP) of

$GP = (X + W_0)/P_0 = (145 + 16)/120 = 1.3417.$ (or 34.17%)

The investment bank proposes 3-year equity warrants with a global premium set close to 35%. To minimize competition with the outstanding warrants, the strike price for the new warrants is set a bit higher at EUR 150. Now, the investment bank derives a price of:

Warrant price for new issue = GP x P_0 - X = 1.35 * EUR 120 - EUR 150 = EUR 12,

which is in line with the theoretical price derived above. At this price, the implied volatility is equal to 19.63%

The investment bank also assumes a conversion exchange rate based on the current exchange rate of .64 USD/EUR. Given an equity content of 150% of the issue, the following generic terms are obtained:

Amount of equity raised:	USD 100M * 1.5= USD150M = EUR 234,375,000
Number of shares created on exercise:	EUR 234,375,000/EUR 150 = 1,562,500
Exercise ratio:	1
Number of warrants:	1,562,500
Number of bonds:	100,000
Number of warrants per bond:	1,562,500/100,000 = 15.625
Value of the warrants attached to each bond of USD 1,000:	15.625 * 12 = EUR 187.50 = USD 120 (12% of the nominal amount of each bond)

Now, the investment bank sets the terms for the bond. The issue amount is USD 100 million. Market conditions indicate a 7-year bond and denominations of USD 1,000. The 7-year U.S. Treasury yield is 7.135% semiannual. We add the spread of 9 bps to derive a desired yield to investors of 7.225% s.a. or 7.3555% p.a. Total commissions are 2%. However, due to competitive pressures, the investment bank decides to forgo part of the selling concession of 1%. The bonds are offered at 98.78 percent. Thus, VOMF's coupon is reduced to 7 1/8% p.a. The investment bank also chooses a full-coupon bond, which trades better in the secondary market than a discount bond.

1.C.2.ii.3 The Pro forma of VOMF's New Issue

1. The bond

Borrower:	Voeller Oel und Mineral Forschung
Guarantor:	None

Amount:	USD 100 million
Maturity:	7 years
Coupon:	7 1/8%
Issue price:	100%
Amortization:	Bullet repayment on final maturity date
Issuer's call option:	None
Listing:	London
Denominations:	USD 1,000
Form:	Bearer securities
Number of bonds:	100,000
Commissions:	2%
Yield:	7.125% (at issue price), 7.3555% (at 98.78% or after deduction of full selling concession)

2. The warrants

Price of warrant:	EUR 12
Exercise price:	EUR 150
Period of exercise:	3 years
Exercise premium:	12%
Global premium:	35%
Implied volatility:	19.63%
Issue price (bond and warrants):	112%
Cost of funds (based on total issue price less commissions of 2%):	5.372% p.a. or 5.302% s.a.

The cost in semiannual terms is 183 bps below the yield on 7-year U.S. Treasuries. If, in addition, VOMF accepts a swap into floating-rate funds at say, 80 bps over U.S. Treasuries, the benefit would be worth 263 bps in semiannual bond terms or, in money market terms 259 bps (below LIBOR).

1.D Choosing a Particular Type of Bond for a New Issue

Given the wide variety of instruments in the Eurobond market, it is natural to ask the following question: how does a firm select a particular type of bond for a new issue? The selection process is quite simple. First, a firm compares the cost of funds of different instruments under different scenarios. Then, based on its risk tolerance, a firm decides on the best instrument. In this section, we will show how a Portuguese firm, Bioneth Engineering, selects a Eurobond issue with currency options attached.

1.D.1 Eurobonds with Currency Options Attached

Attached to a Eurobond issue, a currency option is securitized as a tailor-made listed warrant. This presents several advantages over standard currency options:

i.- It gets around the prohibition in some countries that prevents retail and institutional investors from buying currency options per se.

ii.- Currency options can be purchased (1) in smaller denominations or contract sizes and (2) with longer exercise periods.

For the issuer, adding a securitized option reduces the cost of the borrowing. The proceeds of the sale of the currency option warrants are applied (as in the case of any warrant) as a once-and-for-all income to reduce the IRR of the financing. The issuer, however, is creating an exposure for itself. The issuer may choose to hedge this exposure.

1.D.2 Case Study: Bioneth Engineering

Bioneth Engineering is a leading bio-technology firm based in Oporto, Portugal. Bioneth's British subsidiary has GBP 100 million of short-term debt. Bioneth has decided to refinance the GBP short-term debt with a straight 7-year 8% Euro-GBP bond. Market competitive pressures have recently driven down the commissions paid for issuing straight bonds to 1¾%. An investment bank approaches Bioneth and offers to issue a similar straight bond, but with tradeable three-year warrants attached giving entitlement to an American EUR-call/GBP-put option, with the following terms:

1. Terms of the bond.

Amount:	GBP 100 million.
Maturity:	7 years.
Issue price:	100%
Denominations:	GBP 1,000
Interest:	8% p.a. payable annually in arreas.
Early redemption:	None.
Redemption price:	100%
Issuance commissions:	1¾%
Listing:	London

2. Terms of the warrants.

Exercise price:	1.50 EUR/GBP (.6667 GBP/EUR)
Exercise period:	At any time within a period commencing 2 weeks after settlement date and terminating on the third anniversary of the issue.
Current exchange rate:	1.60 EUR/GBP (.6250 GBP/EUR)
Structure:	Each bond of GBP 1,000 has a detachable warrant giving the holder the right to receive the difference between (1) the GBP equivalent of EUR 1,600 at the then prevailing spot rate and (2) the GBP equivalent of EUR 1,600 at a rate of 1.50 EUR/GBP.
Warrant price:	EUR 0.04935 per GBP. At the current exchange rate, this is equivalent to GBP 0.0308 per GBP, or GBP 30.80 per bond (or 3.08% per bond).
Issue price:	100% + 3.08% = 103.08%
Premium of X/S:	.6667/.6250 = 1.0667 or 6.67%
Cost of funds (including commissions):	8.34%

The warrants are a three-year securitized American out-of-the money EUR call option which would be listed on selected exchanges separately from the bond. To get a theoretical price, the investment bank uses the following additional inputs: EUR risk-free rate 12.0%, GBP risk-free rate 5.5%, and the annual EUR/GBP volatility during the past three years was 23%

In the secondary market, the warrant price should track the time value and intrinsic value, if any, of the warrant. The warrant holders are more likely to trade their warrants before expiration than exercise them, which would imply surrendering their time value. At expiration, the holders will exercise their rights only if the warrant is in-the-money. For example, if at expiration, the exchange rate remains above 1.50 EUR/GBP, investors will not exercise their rights. On the other hand, if at expiration the GBP depreciates to 1.40 EUR/GBP (.7143 GBP/EUR), investors will exercise and receive, per bond:

EUR 1,600 * .7143 GBP/EUR - EUR 1,600 * .6667 GBP/EUR = GBP 76.19.

If Bioneth decides to use the bond with currency options attached, Bioneth will be exposed to currency risk. For example, suppose that at expiration date, the GBP depreciates to 1.40 EUR/GBP. Then, Bioneth will have an additional cash flow of

GBP 76.19 * 100,000 = GBP 7,619,000.

To hedge this exposure, the investment bank offers Bioneth an identical currency option at a cost of EUR 0.04 per GBP or GBP 25 per GBP 1000 bond (2.5% of the nominal amount). Suppose the investment bank considers likely an exchange rate of 1.40 EUR/GBP in three years.

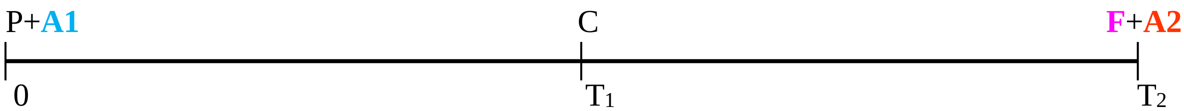
Before deciding on an instrument, Bioneth compares the cash flows under different alternative scenarios:

Dates	Straight bond	Currency Option Bond (unhedged and not exercised)	Currency Option Bond (unhedged and exercised)	Currency Option Bond (hedged)
0	98.250	101.330	101.330	98.830
1	8.000	8.000	8.000	8.000
2	8.000	8.000	8.000	8.000
3	8.000	8.000	15.619	8.000
4	8.000	8.000	8.000	8.000
5	8.000	8.000	8.000	8.000
6	8.000	8.000	8.000	8.000
7	108.000	108.000	108.000	108.000
IRR:	8.340%	7.747%	8.904%	8.227%

For the currency option bond, unhedged and exercised, we have assumed that $S_{t=3\text{-year}}$= 1.40 EUR/GBP. Note that the cost of the hedged alternatives is lower than the cost of a straight bond. Based on the IRR of each alternative, Bioneth decides to issue a bond with currency options attached. Bioneth also decides to hedge the whole issue.

II. Forward Price of a Coupon Bond

To price a forward bond, consider a covered arbitrage strategy:
(1) Borrow short-term for T_2 days at an interest rate r_2.
(2) Buy a coupon bond at the quoted price P plus accrued interest **A1** (coupon C paid at the end of T_1).
(3) Sell the bond at the forward price F for delivery at the end of T_2 days (receiving accrued interest **A2**).



(Recall: Cash price = P + Accrued interest)

The cash flows associated with the arbitrage stratey are as follows:
(1) The borrower pays interest on P+**A1** for T_2 days.
(2) The borrower receives interest on C for (T_2-T_1) days.
(3) The borrower receives **F**+**A2** after T_2 days.

Note that since the interest rate for T_2-T_1 days is unknown at time 0, we compute the (T_2-T_1)-day *implied forward rate*, *f*, at time T_1. Then, we used *f* to compute the cash receive for depositing C in a bank at time T_1 for T_2-T_1 days. We compute ***f*** with the following formula:

$$[1 + f\,(T_2 - T_1)/360] = \frac{(1 + r_2\, T_2/360)}{(1 + r_1\, T_1/360)},$$

where we use money market day-count basis -i.e., Actual/360- to calculate accrued interest for the short term instruments (bank deposits and bank loans).

Arbitrage will force that the borrower gets less cash at time T_2, than what he paid out. That is,

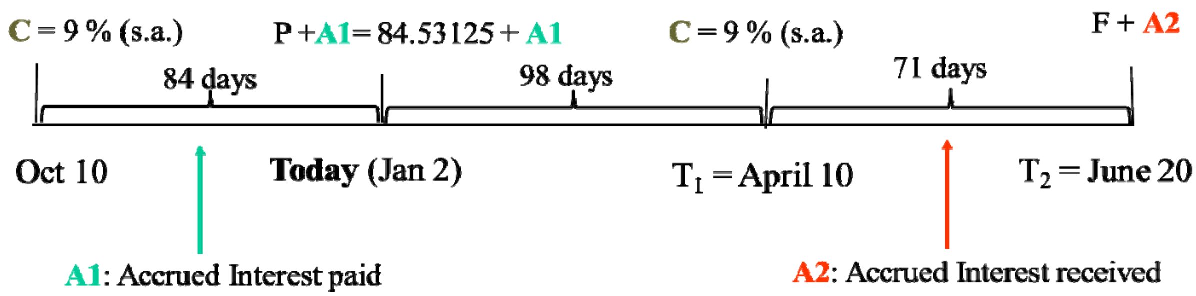
$$(P+A1)*(1 + r_2\, T_2/360) \geq C\,[1 + f\,(T_2 - T_1)/360] + (F+A2).$$

Therefore, F must be equal to

$$F = (P+A1)*(1 + r_2\, T_2/360) - C\,\frac{(1 + r_2\, T_2/360)}{(1 + r_1\, T_1/360)} - A2.$$

Example XIII.4: Calculation of F.
Today is January 2, 1997. The 9% French bond is trading at a price of 109-13 -that is, 109(13/32). The bond has coupon payables on April 10 and October 10. Short-term interest rates as of January 2, 1997 are 7¼% for four months or less and 7½ for more than four months. Recall that for French bonds government bonds accrued interest is calculated on actual/actual basis. The forward price of the bond (**F**) calculated to June 20 is calculated as follows:

T_2 = 169 days (January 2 to June 20), T_1 = 98 (January 2 to April 10).
$C = (1.09)^{1/2} - 1 = .044$. (European actuarial semiannual coupon.)



a.- **A1**

There are 84 days of accrued interest as of January 2, which calculated on actual/actual basis is:

(.044) * (84/182) = .020322.

⇒ **A1** = .020322 * 100 = 2.0322.

b.- **A2**

There are 71 days of accrued interest (April 10 to June 20) which calculated on an actual/actual basis, is

(.044) * (71/183) = .0171.

⇒ **A2** = 1.7083.

c.- F

Convert the bond price 109'13 to its decimal equivalent of 109.4063%. Now, apply Forward formula:

$$\mathbf{F} = (109.4063 + 2.0322) * (1 + .0750 * (169/360) - \frac{4.40 * (1 + .0750 * (169/360))}{(1 + .0725 * (98/360))} - 1.7083 = 109.1839. ¶$$

III. Bond Futures

Bond portfolio managers and bond traders use futures contracts to manage the risks on their portfolios. We will present three types of bond futures: notional, cheapest-to-deliver, and index-based.

3.A Notional Bond Futures

A notional bond future is a fictional bond of fixed principal, coupon, and maturity. For instance, a futures contract could be based on a GBP 100,000 government bond with a 7 percent coupon, paid semiannually, and maturing in ten years. This example describes the notional bond behind the Long Gilt bond futures contract traded at the London International Financial Futures and Options Exchange (LIFFE). It is very probable that such actual cash bond does not exist. A notional bond, however, could be priced using standard bond pricing techniques. A futures contract written on a notional bond could be used to hedge traded cash bonds.

Example XIII.5: Managers of U.K. government securities (gilts) portfolios can manage their interest rate risk exposure using Long Gilts Futures. Suppose managers are concerned about rising interest rates. They can hedge their bond portfolios by going short Long Gilts futures. If interest rates rise, the price of Long Gilts futures will fall, and any profit on the futures position will offset capital loss on the portfolio. A short cash bond position can be hedged by going long Long Gilts futures. The precision of this hedge will depend on the change in value of the Long Gilts futures contract compared with the change in value of the position in government securities. That is, the precision of the hedge will depend on the behavior of the basis. If the basis changes, the hedge will not be perfect. ¶

Notional government bond futures trade in many exchanges around the world. Among them, we find the Chicago Mercantile Exchange (CME), the Tokyo Stock Exchange (TSE), LIFFE, the MATIF in Paris, the Deutsche Terminborse in Frankfurt, the FTA in Amsterdam, and the MEFF in Barcelona. In this section we will present four of the most widely used notional bond futures: the U.S. Treasury Bond futures, the 10-year Japanese government bond futures, the Bund futures, and the French government bond futures.

3.A.1 The U.S. Treasury Bond Contract

In August 1977, the CBOT introduced the U.S. Treasury bond futures contract, or T-bond futures contract. Prices and yields on the CBOT T-bond futures contract are based on a (fictional) 20-year 8% U.S. Treasury bond. The CBOT allows many different bonds to be delivered in satisfaction of a short position in the contract. Specifically, any Treasury bond with at least 15 years to maturity or to first call date, whichever comes first, qualifies for delivery. The CBOT T-bond contract calls for the short side (i.e., the seller) to deliver USD 100,000 face value of any one of the qualifying bonds. Different contracts trade for delivery in March, June, September, and December. The CBOT T-bond futures contract became a huge success. Soon after its introduction, it became the second most traded interest rate futures, behind the CME eurodollar futures. Given this success, in 1982, the CBOT introduced a similar contract, the 10-year U.S. Treasury note futures contract, which is based on notional 10-year Treasury bond. The CBOT soon introduced a 5-year T-note futures and a 2-year T-notes futures. The 10-year T-note futures is the most traded of the group, followed by the 2-year T-note futures, the 5-year T-note futures, and the 30-year T-bond futures. In 2000, the CBOT change the coupon on the notional bond from 8% to 6%.

At the CBOT, the short position decides when, in the delivery month, delivery actually will take place. This is called the *timing option*. The seller of the bond futures contract may initiate delivery on any business day of the delivery month, or on the last two business days of the previous month. The delivery process takes three days. On the third day, delivery day, the buyer (long side) pays the seller for the bonds, and in return receives book-entry T-bonds, transferred via the Federal Reserve wire system.

While cash bond deliveries may take place until the end of the month, the deliverable futures contract itself stops trading on the eighth to last day of the delivery month.

The futures price is quoted in terms of par being 100. Quotes are in 32nds of 1%. The minimum price fluctuation (tick size) for the Treasury bond futures contract is a 32nd of 1%. The dollar value of a 32nd for a USD 100,000 par value -the par value for the underlying Treasury bond- is USD 31.25.

Example XIII.6: A price of 91'21 denotes 91(21/32) percent of par. If the price changes from 91'21 to 91'22, the long side of the T-bond futures contract makes (and the short side loses):

(1/32)x(.01)x(USD 100,000) = USD 31.25.

If the price goes from 91'21 to 91'16, the long (short) side loses (makes):

(5/32) * (.01) * (USD 100,000) = USD 156.30. ¶

3.A.2 The 10-year Japanese Government Bond Futures

The Japanese government bond (JGB) market is the second largest sovereign debt market in total market capitalization after the U.S. Treasury bond market. The JGB market comprises: (1) 2- to 4-year medium-term bonds, (2) 5-year (zero-coupon) bonds, (3) 10-year long-term bonds, and (4) 20-year super-long-term bond. The 10-year JGB is the largest segment of the JGB market. The total amount of outstanding 10-year issues represents almost 80% of all JPY-denominated government debt issues. Significant amounts of the 10-year JGBs are traded in London and New York.

The Tokyo Stock Exchange (TSE) offers a futures contract similar to the CBOT-T bond futures but based instead on 10-year JGBs. This contract was launched on October 19, 1985. Today, it is the most actively traded debt contract at the TSE. Prices and yields on the 10-year JGB futures contract are based on a (fictional) 10-year 6% JGB, but the TSE allows many different bonds to be delivered in satisfaction of a short position in the contract. Specifically, any exchange listed JGB with at least 7 years to maturity or more but less than eleven years qualifies for delivery. The TSE JGB futures contract calls for the short side (i.e., the seller) to deliver JPY 100,000,000 face value of any one of the qualifying bonds. Different contracts trade for delivery in March, June, September, and December. The delivery date is on the 20th of each contract month. This delivery arrangement contrasts with the CBOT T-bond contract where bonds can be delivered at any time during the contract month. The futures price is quoted in terms of par being 100. The tick size is 1/100 point per 100 points, or JPY 10,000 per contract.

3.A.3 The Bund Futures

In September 1988 the London International Financial Futures Exchange (LIFFE) began trading futures contracts based on the German government bond (Bundesrepublikanleihe or Bund). Because it is not possible to sell Bunds short in the cash market, the contract was very attractive to firms wishing to hedge their cash Bund position by selling futures. The 1988 German Government Bond Future contract was modified in 1999 to accommodate the contract to new European currency, the Euro.

The Bund contract is based on a deliverable 10-year notional bond with a 6% coupon. The contract size is EUR 100,000 nominal with maturities in March, June, September, and December. LIFFE allows many different bonds to be delivered in satisfaction of a short position in the contract. Specifically, any Bund with at least 8½ years to maturity or more but less than ten years qualifies for delivery, provided that any such Bund has a minimum amount in issue of EUR 2 billion as listed by LIFFE. There is only one delivery date for each contract: the 10th day of the delivery month. The futures price is quoted in terms of par being 100. The tick size is 1/100 point per 100 points, or EUR 10 per contract.

Note: The original 1988 Bund contract was based on a deliverable 10-year notional DEM bond with a 6% coupon. The contract size was DEM 250,000 nominal.

3.A.4 The French Government Bond Futures

In February 1986 the Marché à Terme d'instruments Financiers (MATIF) began trading its first product: the notional futures contract based on the French government bond. This notional contract, in FRF, has been replaced by a Euro Notional contract.

The Euro contract is based on a deliverable 10-year notional bond with a 5.50% coupon. The contract size is EUR 100,000 nominal with maturities in March, June, September, and December. Bonds are selected by the seller from an official list of 8½ to 10½ year deliverable government bonds. The bonds in the official MATIF list should be redeemable at maturity, with a minimum outstanding amount of EUR 6,000 million. There is only one delivery date for each contract: the 3rd Wednesday of the contract month. The futures price is quoted in terms of par being 100. The tick size is 1/100 point per 100 points, or EUR 10 per contract. The initial margin is EUR 2,200.

3.B Cheapest-to-Deliver Bond (CDB) Futures

At the closing of a delivery month, the short side of an open contract has to deliver the underlying bond against cash settlement. The underlying bonds in the most popular government bond futures contracts are notional bonds, which are not deliverable. The bonds exchanged at delivery are those bearing characteristics as close as possible to those of the notional bond. These bonds are designated by the exchange and grouped in the *deliverable bond pool*, also called *delivery basket*. The seller of the bond futures contract (the short side) chooses, from the bond pool, which bond to deliver. The seller of the futures contract will select for delivery the bond that costs the least amount of money. That is, the seller will select the *cheapest-to-deliver* bond. Because bond futures traders know this, the futures price will move in conjunction with the cash bond issue that is cheapest to deliver.

Example XIII.7: At LIFFE, there is an Italian Government Bond (BTP) CDB futures contract. It is based on a notional 10-year government bond with a 6% coupon. The contract size is EUR 100,000. Deliverable issues are any Buoni del Tesoro Poliennnali (BTP) with a maturity between 8½ and 10½, provided any such BTP has a minimum issue of EUR 2 billion. ¶

3.B.1 Equilibrium Price for a Deliverable Government Bond Futures

During the delivery month, traders will compare the value of a cash bond (deliverable), P, plus accrued interest (A1), to its equivalent in the futures market. When the long side takes delivery, she pays a futures invoice price (**I**), representing the cash price as a percentage of face value, plus accrued interest on the cash bond (A1).

The invoice price is the futures price (**Z**), times an exchange *conversion factor* (cf) associated with the bond selected for delivery, say Bond *i*:

I = **Z** * **cf**$_i$.

Then, the net short gain from delivering bond *i* is:

Z * **cf**$_i$. – P_i (in equilibrium: P_i = **Z** * **cf**$_i$)

Example XIII.8: A JGB 4% coupon bonds maturing on 9 years is priced at P = 69.10, with a **.862465** conversion factor can be delivered to cover a CDB JGB futures. An expiring futures JGB is trading at **Z** = **80.133**. Assume the underlying note has a face amount of JPY 100,000,000 and accrued interest (A1) on futures delivery day of JPY 1,000,000. The futures invoice total is calculated as:

I = JPY 100,000,000 * **.862465** * .80133 + JPY 1,000,000 = JPY 70,111,907.85

The total should approximately equal its invoice price in the cash market. That is,

Cash invoice total = P + A1 = JPY 100,000,000 * .6910 + JPY 1,000,000 = 70,100,000.

Note: The short side will compare many deliverable bonds and look for the deliverable bond that maximizes **Z** * **cf**$_i$. – P_i. ¶

The deliverable bond pool groups together bonds with different nominal interest rates and maturities. The conversion factor allows the price of the deliverable bond to be adjusted to that of the futures contract. At the CBOT, the exchange conversion factor (cf) is calculated by setting the yield to maturity (or to first call, if callable) on the bond to be delivered equal to 6% (with semiannual compounding), and dividing the resulting price by 100. The maturity of the bond is rounded down to the nearest quarter, as calculated from the first day of the delivery month. This allows the CBOT to produce comprehensible tables. If after rounding the bond does not last for an exact number of half years (i.e., there is an extra 3 months), the first coupon is assumed to be paid after 3 months and accrued interest is subtracted.

♦ **Conversion Factors in Other Markets**

The same CBOT principles apply to other CDBs. For example, at the TSE, the 10-year JGB futures' conversion factor (cf) is calculated by setting the yield to maturity on the bond to be delivered equal to 6% (with semiannual compounding), and dividing the resulting price by 100. ♦

Example XIII.9: Calculation of cf.

(A) *T-bond futures with no Accrued interest.*

It is March 1995. Suppose the physical bond that will be delivered is the U.S. Treasury 10% coupon bond maturing on May 15, 2015, which has 20 years and two months to maturity. Rounding down, we obtain a maturity of 20 years. We assume that the first payments are made after 6 months. Setting the YTM on the bond equal to 6% results in a (decimal) price of

$$\Sigma_{i=1 \text{ to } 40}\, 5/(1+.03)^i + 100/(1+.03)^{40} = 146.2295.$$

Dividing by 100 gives a conversion factor of **cf** = **1.462295**. If the futures price were **Z** = **90**, the invoice price would be

I = **90** * **1.462295** = 131.60659.

This price applies to a principal amount of USD 100,000, so the actual cash payment is USD 131,606.59.

(B) *JGB futures with no Accrued interest*.
In June 1996, the JGB 4% coupon bond maturing on June 2005 has exactly 9 years to maturity. Suppose that this is the physical JGB that will be delivered. The first payments are assumed to be made after 6 months. Setting the YTM on the bond equal to 6% results in a (decimal) price of

$$\Sigma_{i=1 \text{ to } 18}\ 2/(1+.03)^i + 100/(1+.03)^{18} = 86.2465.$$

Dividing by 100 gives a conversion factor of **cf** = **0.862465**. If the futures price were **Z** = **80.133**, the invoice price would be

$$\mathbf{I} = \mathbf{80.133} * \mathbf{.862465} = 69.111908.$$

This price applies to a principal amount of JPY 100,000,000, so the actual cash payment is JPY 69,111,908. In exchange for this cash payment, a buyer of the J-bond futures will receive the 4% JGB maturing on June 2005.

(C) *T-bond futures with Accrued interest*.
Consider a U.S. Treasury 10% coupon bond with 18 years and 4 months to maturity, which is the physical bond to be delivered against the T-bond futures. For the purposes of calculating the conversion factor, the bond is assumed to have exactly 18 years and 3 months to maturity. Discounting all the payments back to a point in time 3 months from today gives a value of

$$5 + \Sigma_{i=1 \text{ to } 36}\ 5/(1+.03)^i + 100/(1+.03)^{36} = 148.665.$$

The interest rate for a 3-month period is $[(1+.03)^{.5} - 1]$=1.49%. Therefore, discounting back to the present gives the bond's value as

$$148.665/1.0149 = 146.48398.$$

Subtracting the accrued interest of, approximately, 2.5, gives a price of 143.98398. The conversion factor is **1.4398398**. If the futures price were **Z** = **92.125**, the invoice price would be

$$I = \mathbf{92.125} * \mathbf{1.4398398} = 132.64524.$$

This price applies to a principal amount of USD 100,000, so a buyer of the T-bond futures has to make a USD 132,645.24 cash payment. In exchange, a buyer receives the 10% U.S. Treasury with 18 years and 4 months to maturity. ¶

Intuition behind the Conversion Factor

To understand the system of conversion, you should recall that, other factors being held constant, the lower the coupon rate, the lower the bond's price (see Appendix XI). Since the seller of the government bond futures contract can deliver any government bond in the deliverable bond pool, a method of conversion is needed to offset the economic incentive to deliver the lowest coupon bond. For example, the CBOT's system adjusts the future price, which is based on an 6% coupon, to a price that corresponds to the coupon of the issue being delivered.

Example XIII.10: First, consider the bond of Example XIII.9, Part A. The price of the 10% T-bond with 20 years and two months to maturity, with a YTM of 6%, is 146.2295. Now, suppose there is another deliverable T-bond with coupon rate is 6%. The price of this bond will be 100.00. If the YTM is 6%, the second T-bond seems cheaper. However, once adjusted by the conversion factor, both bonds have the same futures equivalent price (cash price * cf).

Note that the only difference between the two bonds is that the first bond has a higher coupon rate. Owing the 10% coupon bond is like owing 1.462295 6% bonds. Since the price is based on an 6% coupon bond, the futures price is multiplied by a conversion factor of 1.462295 to compute the amount paid (delivery price) by the long to the short if the short delivers the 10% coupon bond. ¶

The conversion factor has some impact on futures prices. Other factors being held constant, the lower the coupon rate, the lower the bond's price. Now, suppose the equilibrium YTM is 10%. By lowering the yield to 6%, the conversion factor will favor delivery of lower-coupon, longer-maturity bonds. Conversely, higher-coupon, shorter-maturities securities may be cheaper to deliver when yields are below 6%. These biases are greater the farther the YTM moves from the 6% standard.

Carry considerations are the primary reason why the lowest-priced bond will not necessarily prove to be the cheapest to deliver. The cash bond with the lowest price often has the lowest coupon, and the relatively low income its coupon produces may not offset the cost of financing its purchase.

3.B.2 The Components of Basis

Recall that the *basis* represents the difference in value between the price for today's settlement of a cash bond (P_i) and the price of its corresponding contract for future delivery (Z * cf). That is, for bond *i:*

$$\text{Basis}_i = P_i - \mathbf{Z} * \mathbf{cf_i}.$$

Calculation of the basis must start by establishing which deliverable cash bonds will be used to measure the corresponding bond futures contract price. If basis were comprised only of carry, the adjusted futures price would be approximately equal to the price of the underlying CDB bond, net of any carrying costs, throughout the futures trading cycle.

While carry considerations are the most important component of the basis, there is another component to it. The design of the standardized futures contract also influences its price. The fact that the seller has unique delivery choices gives the seller a comparative advantage. For example, suppose a dealer can realize a profit by selling his relatively expensive cash bond and replacing it with the deliverable bond that has become cheaper. This right to select the date and the deliverable bond is called *implied delivery option*. Both forward and futures incorporate a discount for carry, but the futures price also discounts for the implied delivery option.

The cash and adjusted futures prices seldom converge fully, even on the last day of trading, because the holder of a short position enjoys another option, the *wild card option*. The wild card arises, in the case of the CBOT, because the futures market closes at 2:00 PM (Chicago time), which locks in that day's futures settlement price (I is fixed), while the short side has until 8:00 PM to declare an intention to deliver. The wild card option is a six-hour put option. The strike price of the put is the futures invoice price fixed by the 2:00 PM futures settlement. In the event the cash bond price has fallen below the invoice price by 8:00 PM, the short side of the bond futures contract can exercise this put and receive the higher invoice price.

The delivery option and the wild card option have positive value. Thus, in the market equilibrium the futures price should be bid down by the value of these options: Z = P/cf - option values.

In general, P > Z * cf.

That is, the basis should be positive even during the delivery month.

Example XIII.11: It is October 2, 2000. You purchase a Long Gilt Dec futures and you want to calculate the value of the deliverable options with respect to December 31, 2000. The price of the Dec Long Gilt futures contract is 113.27. The Sep 27 2013 8% Long gilt has a 1.08356 conversion factor and it is trading at 123.05. The short rate is 5.50%. With this information you calculate the value of the delivery options. First, you calculate the carry component, which is equal to the interest income received minus the financing cost:

Carry = A2 – (P + A1) * r_2 T_2/360 = 1.98895 – (123.05 + 0.110497) * .055 * 90/360 = 0.29545

Second, you recognize that the basis has two components, carry and delivery options value. Thus,

Option value = 0.31516 – 0.29545 = 0.019665.

That is, the option value represents 6.24% of the basis. ¶

3.B.3 Identification of the CDB

Traders cannot buy a cash bond and make immediate delivery during the period prior to the delivery month. Thus, traders would compare; not the current cash bond price to its futures equivalent, but instead the forward price (F) of the bond to its futures equivalent.

The forward bond price takes into account the impact of the short-term borrowing cost, and the coupon interest that will be earned on the cash bond.

Define the *basis after carry* (BAC) as: BAC = F - Z * cf.

The BAC indicates the cost, in terms of forward dollars, of buying a cash bond and delivering it against the futures contract. A trader want to minimize this cost in terms of each futures contract he is short, so we can characterize the CDB as that bond with the smallest BAC.

Example XIII.12: Tables XIII.B and XIII.C give the relevant calculations for determining which T-bond is cheapest to deliver against the Dec 1998 CBOT bond futures contract. The calculations are made on September 17 with respect to Dec 31, the first available delivery day, assuming a 5.31% short (repo) rate.

TABLE XIII.B

Conversion Value of Selected Deliverable T-Bonds, September 17, 1998

Maturity (or 1st call)	Coupon (%)	cf	June Future (Z)	Invoice Price (I) (Z * cf)	Basis (P-Z * cf)
Feb. 15, 2015	11¼	1.2879	131'07	168.99663	0.03462
Aug. 15, 2015	10 5/8	1.2361	131'07	162.19950	0.05050
Nov. 15, 2015	9 7/8	1.1701	131'07	153.53906	0.92969

TABLE XIII.C

Forward Value of Deliverable T-Bond, Calculated from April 16 to June 1, with a Short Rate of 8%

Maturity (or 1st call)	Price (P)	Accrued Interest (9/17)	Coupon	Accrued Interest (12/1)	Forward Price	BAC
Feb. 15, 2015	169'01	1.00883	0	3.27105	169.45268	0.46605
Aug. 15, 2015	162'07	0.95278	0	3.03159	162.66706	0.46757
Nov. 15, 2015	154'15	3.54280	4.9375	1.25483	154.04177	0.50271

From the above calculations, we see that the 11¼ maturing February 15, 2015, has the smallest BAC and hence is cheapest to deliver.

Let's go through most of the relevant calculations for one of these bonds.

Take the first bond listed, with a coupon of 11¼%. This bond matures in 2015. Hence as of December 1, 1998, there will be a little over sixteen years and two months to call. To get the exchange conversion factor, we round down to the nearest quarter, and therefore calculate the price of the bond from November 15, 1998, which produces a 1.2879 conversion factor.

The June future is at 131'07 or 131.21875. The invoice price is therefore

$$131.21875 * 1.2879 = 168.99663.$$

The price in the cash market, meanwhile, is 169'01 or 169.03125. As of September 17, accrued interest from August 15 is calculated as

$$A1 = (.1125/2) * (33/184) = 1.00883.$$

On December 1, there are 107 days of accrued interest from August 15:

$$A2 = (.1125/2) * 107/184 = 3.2710598$$

Note that there is no coupon payment from September 17 to December 1. Thus, applying the equation for the forward bond price, we get

$$F = (169.03125 + 1.00883)x(1 + .0531 * 107/360) - 3.2710598 = 169.45268.$$

Finally, BAC = 169.45268 - 168.99663 = 0.45605.

The calculations are similar for the other bonds. ¶

All bond traders at the major derivatives exchanges have hand-held computers that rank bonds according to cheapness and display the cash price, basis, carry value, and implied repo rate associated with each bond.

3.C Index-Based Futures

Several bond futures contracts belong to this group. A futures may be based on an index constructed from an average of cash bond prices. Or it may be based on an average yield, YTM_A. A typical example is given by the 3-year and 10-year Australian government bond contracts at the Sydney Futures Exchange that are quoted in terms of an index price IP defined as:

$$IP = 100 - YTM_A,$$

where YTM_A is the average yield to maturity of a pre-determined group of bonds.

A quotation of 92 thus corresponds to a YTM_A of 8%. Once YTM_A is determined, there is a one-to-one relationship between the YTM_A and the price Z of a notional bond expressed as a percentage of face value. Index-based bond futures are settled in cash.

At the Sydney Futures Exchange, the YTM_A is determined from a sample of twelve randomly selected bond dealers. At 11:30 AM of the last trading day of the contract month, the twelve dealers provide the yields at which they would buy and sell each of the series of Commonwealth Treasury bonds designated by the Board of the exchange for that contract month. After discarding the two highest buying and the two lowest selling quotations for each designated bond, the arithmetic mean of the remaining yields are calculated for cash settlement price.

IV. Hedging With Bond Futures

The basic techniques for hedging using bond futures use the concept of *basis point value* (*bpv*). The basis point value of a bond is the change in the bond's price for a one basis point movement in yield. That is,

bpv = Change in the bond's (USD) price for a 1 bp change in YTM (r).

The *bpv* measures the interest risk or price sensitivity to changes in interest rates of underlying asset, in our case a bond. It is also called the *delta* or *DV01 (dollar duration,* when change in YTM is infinitesimal, dr).

Example XIII.13: Calculation of bpv for Bond X.

Bond X: FV = USD 100; T = 3; freq = annual; coupon = YTM = 3%

$\Rightarrow P_X$ = **USD 100**.

If YTM = 2.99% $\Rightarrow P_{X'}$ = **USD 100.0283**.

$\Rightarrow bpv_X = P_{X'} - P_X$ = **100.0283** – **100** = USD 0.0283

Interpretation: YTM increases by .01% $\Rightarrow P_X$ decreases to USD 99.9717. ¶

Hedging bonds using *bpv*'s is based on a simple idea. We want to create a hedging position that moves together with the underlying bond position. Suppose we have two bonds: Bond A with a with a $\mathbf{bpv_A} = \mathbf{2}$ and Bond B with a $\mathbf{bpv_B} = \mathbf{1}$ Assume that the yields on the bond will move in a *parallel fashion* (the spread between the yields is constant). Then, for 1 bp rise in YTMs

$\Rightarrow$ Price of the A will fall by **2**.

$\Rightarrow$ Price of the B will fall by 1.

Suppose our u*nderlying position* is *Long* USD 1 in Bond A. Then, to build a matching *Hedging position* we need to be *short* USD 2 of the bond B. Defining the hedge ratio of bond B (h_B) as the number of units of bond B needed to create a hedged position against another bond, A, we have:

$$h_B = -(\mathbf{bpv_A}/\mathbf{bpv_B}) = -2.$$

Although a government bond futures tracks the movement of an underlying CDB, it will not show the same price sensitivity of the CDB. For example, a Bund futures contract exhibits the price sensitivity of an issue with a 6% coupon because it is priced on that standard. Conversion factors must be used to calculate the price sensitivity correlation between a cash security and its corresponding futures contract. That is, for CDB futures, we can calculate the *bpv* of the futures by first calculating the bpv of the CDB, and then dividing by the exchange conversion factor:

$$bpv_{\text{future}} = bpv_{\text{CDB}} / \text{cf}.$$

Similarly, the hedged ratio of a bond future (the number of futures contracts needed to establish a hedged position) is

$$\text{hedge ratio future} = h_{futures} = -(bpv_{\text{bond}}/bpv_{\text{future}}).$$

From the above equations, we can calculate the hedge ratio for a CDB future, $h_{CDB\text{-}futures}$:

$$h_{CDB\text{-}futures} = -(bpv_{\text{bond}}/bpv_{\text{CDB}}) * \text{cf}.$$

These equations assume that yield spreads remain constant. This might not be a correct assumption. If we change this assumption and estimate a coefficient k such that

$$\text{change in yield B} = k * \text{change in yield A}$$

then, $h_B = -k * (\mathbf{bpv_A}/\mathbf{bpv_B}) =$

Example XIII.14: It is January 10, 1997. Suppose the bpvs of the following deliverable Long Gilts futures are as follows:

Maturity (or 1st call)	Coupon (%)	P	bpv	cf
December 2008	9	109(13/16)	.0821	1.00000
January 2009	8	103(1/16)	.0872	0.90247
March 2010	6¼	86(25/32)	.0793	0.77248

Notice what this means. If yields drop 10 basis points, the price of the 8% January 2009 bond will rise by .872%, from 103.0625 to 103.9345. Alternatively, a basis point value of .0872 means that GBP 50,000 face value of these bonds will change in market value by GBP 43.60 for a one basis point move.

Assume the March 2010 bond is the CDB. Suppose a manager is trying to hedge a GBP-Eurobond with a bpv=.127. Thus, from the above formulas, the number of Long Gilts futures contracts needed to hedge GBP 50,000 in face value of this bond is on January 10, 1997:

hedge ratio = – (.127/.0793) * 0.77248 = -1.2371.

Therefore, to hedge GBP 2,500,000 of the given Eurobond, one would go short 62 (-1.2371x50) Long Gilts futures.

Now, suppose that in June 1998 the January 2009 Long Gilts becomes cheapest to deliver, the new hedge ratio is

hedge ratio = – (.127/.0872) * 0.90247 = -1.3144.

Now, to hedge GBP 2,500,000 of the given Eurobond, one would go short 65.72 (-1.3144x50) Long Gilts futures. That is, one would need to sell approximately four more futures contracts. ¶

4.A Duration-Based Hedging Strategies

Recall the definition of modified duration, D* (or *ModD*):

$$\frac{1}{P}\frac{dP}{dr} = -\frac{1}{P}\Sigma_t\, t\,\frac{C_t}{(1+r)^{t+1}} = -D\,\frac{1}{(1+r)} = -D^*.$$

where D is (Macaulay) Duration, which is also referred as *MacD*. Duration measures the weighted average number of years an investor must maintain a position in the bon until the PV of the bond's cash flows equals the amount paid for the bond. Modified Duration, D*, measures the average cash-weighted term to maturity of a bond. It is also a measure of the percentage change in bond value that would result from a small change in yield, say one basis point.

Example XIII.15: Calculation of D and D* for Bond X.

Data for Bond X: FV = USD 100; *T* = 3; freq = annual; coupon = YTM = 3%. Then, P_X = **USD 100**.

D = [2.9126 + 2 * 2/28278 + 3 * 2.7454 + 3 * 91.5141]/ **100** = 29135/**100** = 2.9135

D* = 2.9135/1.03 = **2.826**

Interpretation of D*: A 1% decrease in YTM increases Bond X's expected price by 2.826%.

Note: *bpv* = DV01 = D* * P = **2.826** * **USD 100** * .0001 = USD .0283. ¶

In more general terms, the expected percentage bond's price change when YTM changes is given by:

$$dP/P = \text{Expected \% Price change} = D^* * dr.$$

Example XIII.16: Expected Percentage Price Change for Bond X

Suppose we expect a decrease in YTM of 15 bps –i.e., YTM is expected to drop to 2.85%. Then, given that D* = **2.826**, & dr = 15 bps,

$$dP/P = \text{Expected \% Price change} = -\mathbf{2.826} * .0015 = -0.00424.$$

That is, we expect Bond X's price to increase by 0.424%. ¶

The above equalities are a linear (first-order) approximation to the change of a bond's price, when the change in YTM is small. In general, convexity (second-order approximation) also matters.

Bond portfolio managers use duration to speculate (when YTM are expected to go up, a portfolio manager decreases duration) or to hedge. Consider a situation where a position in an interest rate dependent asset, such as a Eurobond portfolio is being hedged using an interest rate futures contract. Define:

F: Contract price for the interest rate futures contract
D_F: Duration of asset underlying futures contract
S: Value of the asset (Eurobond portfolio) being hedged
D_s: Duration of asset (Eurobond portfolio) being hedged

Assume that the change in interest rates, dr, is the same for all maturities (we only allow for parallel shifts in the yield curve). Then, the number of contracts required to hedge against an uncertain change in interest rates, dr, is:

$$N = S\, D_s\, (1+r_F)/[F\, D_F\, (1+r_s)] = (S\, D_s^*)/(F\, D_F^*).$$

This is the *duration-based hedge ratio*. It is also called the *price sensitivity hedge ratio*.

Example XIII.17: It is January 19 and a bank manager with EUR 20 million in Eurobonds is concerned that interest rates are expected to be volatile over the next three months. The bank manager decides to use the June Euro government futures contract traded at MATIF to hedge the value of the portfolio. The current price is 91.25. Recall that the size of the MATIF's government bond futures contract is EUR 100,000. Therefore, the futures contract price is EUR 91,250.

The average duration of the Eurobond portfolio over the next three months will be 7.80 years and the yield on the Eurobond portfolio is 7.92%. The cheapest-to-deliver bond in the French government futures contract is expected to be a 10-year 7% per cent annum coupon bond. The yield on this bond is currently 6.80% per year, and the duration is 7.20 years at maturity.

The bank manager requires a short position to hedge the Eurobond position. If interest rates go up (down), a gain (loss) will be made on the short position and a loss (gain) will be made on the Eurobond portfolio.

The number of bond futures contracts is given by:

N = [20,000,000 * 7.80 * (1+ .0680)]/[91,250 * 7.20 * (1 + .0792)] = 234.98

The portfolio manager should short 235 futures contracts. ¶

4.A.1 Application: Asset Allocation

Portfolio management involves decisions concerning what types of asset should be bought (see Chapter XVII). A fund manager might decide to invest 40 percent of the fund in bonds and 60 percent in stocks. Once the asset allocation decision is made, substantial changes to the allocation are usually avoided because transaction costs could be very high. Managers will use futures and options to change the asset allocation indirectly.

For example, suppose a portfolio manager has a fund with X euros invested in stocks and Y euros invested in bonds. Suppose the portfolio manager wants to change her bond allocation from Y to Y^1. The bond portfolio has a modified duration of D_Y*. Instead of selling (buying) stocks to buy (sell) bonds, the portfolio manager can make the change by buying (selling) T-bond futures contracts. Suppose she wants to have income D_Y*Y^1 from her bond position if interest rates fall by 1 percent. She plans to generate that amount with income from the current bond portfolio, D_Y*Y, and income from a T-bond futures position, $N D_F$*F, that is,

$$D_B^* Y^1 = D_B^* Y + N D_F^* F.$$

Solving for N, we get

$$N = D_B^* (Y^1 - Y)/(D_F^* F).$$

Note that if the euro investment in bonds is to be reduced ($Y^1 < Y$), T-bonds futures contracts are sold, and if the euro investment in bonds is to be increased ($Y^1 > Y$), T-bonds futures contracts are bought. Presumably, the reduction (increase) in bond investment is transferred to stocks through buying (selling) stock index futures contracts.

Example XIII.18: Ms. O'Neal, a portfolio manager, has EUR 90,000,000 in a stock portfolio whose composition matches the French stock market index, the CAC-40, and EUR 60,000,000 in a bond portfolio with a modified duration of 7.8 years. Ms. O'Neal forecasts that French stocks are going to do extremely well over the next six months. She wants to take advantage of this situation and eliminate her interest rate exposure. Transaction costs, however, are very high (especially because at the end of the six-month period she wants to go back to her 60-40 portfolio allocation). Assume that the CDB at the MATIF has a duration of 6.5 years and that the price of the six-month futures bond contract is 127.14. Also assume that the six-month CAC-40 futures is priced at 2300.

First, with respect to eliminating the interest rate exposure, the number of T-bond futures to sell is

$$N = 7.8\ (0 - 60{,}000{,}000)/(6.5 * 1.2714 * 100{,}000) = -566.30 \text{ contracts.}$$

That is, selling 566 MATIF government bonds futures contracts is equivalent to completely wipe out interest rate risk (i.e., completely liquidating the bond investment position).

Second, to take a long position of EUR 60,000,000 in stocks, Ms O'Neil will buy CAC-40 futures contracts at the MATIF. The size of each CAC-40 futures contract is equal to EUR 200 per index point. That is, the number of contracts to buy is

N = 60,000,000/(2,300 * 200) = 130.434, or 130 contracts. ¶

V. Looking Ahead

In this Chapter we have introduced T-Bond futures as an interest risk management tool. CDB futures are very well suited to hedge medium- and long-term bonds. In the next chapter we will introduce swaps, which are a more flexible instrument that can be used to hedge interest rate risk for any interest rate sensitive instrument. A swap represents an exchange of cash flows at periodic intervals. Thus, to value a swap we should use the same techniques that we used in this chapter to value a bond: present value models.

Related readings

International Investments, by Bruno Solnik, published by Addison Wesley.

International Financial Markets, by J. Orlin Grabbe, published by McGraw-Hill.

Practical Use of Treasury Futures, Chicago Board of Trade.

For an outstanding reference on pricing derivatives (futures and options), swaps, and hedging, see **Options, Futures and Other Derivative Securities**, second edition, by John C. Hull. This book will be a very useful reading for the next part of the course.

Exercises:

1. A U.K. gilt is trading at a price of 95'29, and has a 7.8% coupon payable on May 15 and November 15. Short-term interest rates as of September 15 are 6.5% for one month, 6.75% for two months, and 7.0% for three months. What is the forward price of the gilt calculated to December 15?

2. Consider an 8% coupon bond that pays coupons on March 15 and September 15, and which matures on March 15, 2001. If the yield is 9.403% what would be the price on January 3, 1995 for
(a) a U.S. corporate bond? (Recall the day count is 30/360.)
(b) Australian government bond? (Recall the day count is actual/actual.)

3. During the subscription period of a Euro-EUR bond issue, a member of the managing group sells short 100 Bund futures at the LIFFE at an average price of 99.30. Later the futures position is closed out at an average price of 99.48. What was the net gain or loss on the futures position (in EUR)?

4. It is September 31, 2000. You purchase a Bund Mar futures. You want to calculate the value BAC and the value of the deliverable options with respect to March 31, 2001. The price of Mar Bund contract is 106.23. The Feb 15 2010 7% Bund has a 1.05498 conversion factor and it is trading at 113.05. The short rate is 5.75%.

5. Check all the numbers (do the calculations) presented for the third bond in Example XIII.12.

6. In January 1996, the long-term French government bond 7½% coupon bonds maturing on September, 2006, have exactly 10 years and 9 months to maturity. Calculate the MATIF's conversion factor for this bond. Assume that the futures price is 95.750. Calculate the actual cash payment to the short side delivering this bond.

7. A market maker with a long position of GBP 460,250 in Eurobonds at a current price of 101 goes short eight Long Gilt futures contracts at 89'16. Rising interest rates caused the price of the eurobonds to fall to 97½, while the price of Long Gilt futures falls to 79'44. What is the change in the total value of the market maker's position? (The notional U.K. Gilts futures contract is based on a GBP 50,000 bond with a 9% coupon with 10 years to maturity. The tick size is 1/64).

8. Calculate the hedge ratio for a short bond with a bpv=.27, which is hedged with a CBOT T-bond futures with a conversion factor of 1.057 and a bpv_{CDB}=.089.

9. It is February 10. You are the manager of a Eurobond portfolio worth JGB 1,200 million. The average duration of the portfolio is 9.2 years with an annual YTM of 7.3%. The September 10-year JGB futures price is currently 104.047, with an annual yield of 7.9%, and the expected CDB has a duration of 7.8 years. How should you hedge against changes in interest rates over the next 6 months?

10. Siete Swiss, one of the largest watch manufacturers in the world, had an excellent year in 1996. Its profits increased by 36%, with worldwide revenue of USD 2.5 billion. Siete is looking to refinance medium-term debt amounting to GBP 200 million. An investment bank suggests issuing a straight bond with equity warrants attached. The investment bank has the following data available:

GBP gilts yields:	3-yr 7.150 (s.a.); 5-yr 7.275 (s.a.) 7-yr 7.303 (s.a.)
CHF government bond yield:	1-year 4.232% (p.a.)
Siete Euro-GBP bond yield:	GBP gilts + 60 bps
Current Siete's share price (P_0):	CHF 60
Historic dividend yield:	4.20%
Historic stock price volatility:	3-yr 16.30%; 5-yr 18.00%; 7-yr 20.15%.
Outstanding warrants	
Outstanding life:	4½ years
Current price (W_0):	GBP 6.82 (CHF 13.64)
Exercise price (X):	CHF 70
Current exchange rate:	.50 GBP/CHF (2.00 CHF/GBP).

Given the current tight market conditions, the investment bank suggests:

1. For the warrants: an equity content of 100%, an *exercise ratio* equal to 1.50, and a 5-year warrant.

2. For the bond: a 7-year full-coupon bond, denominations of GBP 1,000, and an issue price of 100%.

Total commissions are 2½%. Due to competitive pressures, the investment bank decides to forgo ¾% of the selling concession.

The investment bank also assumes a conversion exchange rate based on the current exchange rate.

Following usual market practices:

i. Write down the following generic terms for the issue:

Amount of equity raised:
Number of shares created on exercise:
Number of warrants per bond:
Value of the warrants attached to each
bond of GBP 1,000:

ii. Calculate the information required below to complete the pro forma of Siete's issue:

1. The bond
Amount:
Maturity:
Coupon:
Issue price:
Yield:

2. The warrants
Price of warrant:
Exercise price:
Period of exercise:

Exercise premium:
Global premium:
Issue price (bond and warrants):
Cost of funds (based on total issue price less commissions):

11. Nairong Co. is a Chinese software development firm. More than 50% of Nairong's revenue is linked to the JPY. Nairong has decided to refinance JPY 5,000 million of short-term debt with a 4-year 6% JPY Asianbond issue. Nairong is considering two alternatives: (1) a straight JPY bond and (2) a bond with currency options attached. The latter is a straight JPY bond but with tradeable two-year warrants attached giving entitlement to an American USD put option, with the following terms:

1. Terms of the bond.

Amount:	JPY 5,000 million.
Maturity:	4 years.
Issue price:	100%
Denominations:	JPY 1 million
Interest:	6% p.a. payable annually in arreas.
Early redemption:	None.
Redemption price:	100%
Issuance commissions:	2¼%
Listing:	Singapore

2. Terms of the warrants.

Exercise price:	.0095 USD/JPY
Exercise period:	At any time within a period commencing 2 weeks after settlement date and terminating on the second anniversary of the issue.
Current exchange rate:	.012 USD/JPY
Structure:	Each bond of JPY 1,000,000 has a detachable warrant giving the holder the right to receive the difference between (1) the JPY equivalent of USD 12,000 at a rate of .0095 USD/JPY and (2) the JPY equivalent of USD 12,000 million at the then prevailing spot rate.
Warrant price:	
Issue price:	
Premium of exercise price relative to current spot price:	
Cost of funds (including commissions):	

You work for Nairong Co. Determine the price of the warrant, which is solely based on the Black - Scholes formula. To get a theoretical price, Nairong Co. gives you the following additional inputs: U.S. risk-free rate 6.50%, Japan's risk-free rate 4.40% and the annual USD/JPY volatility during the past two years was 15% (you take this estimate as a measure of future annual volatility). You should also determine the issue price, the premium of exercise price, and the cost of funds.

Assume that at expiration date, the JPY depreciates to .009 USD/JPY.

The investment bank offers Nairong an identical currency option at a cost of USD 0.063 per JPY 100. Determine the best financing alternative for Nairong.