PURCHASING POWER PARITY

The Behavior of FX Rates

- Fundamentals that affect FX Rates: — Formal Theories

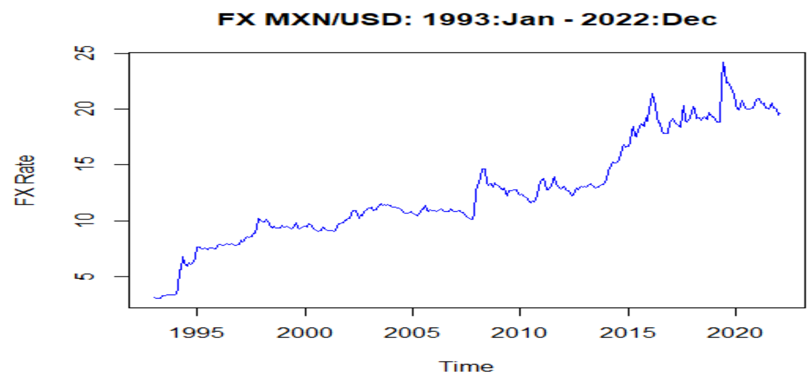
Fundamentals	Formal Theories
- Inflation rates differentials $(I_{USD} - I_{FC})$	PPP
- Interest rate differentials $(i_{USD} - i_{FC})$	IFE
- Income growth rates $(y_{USD} - y_{FC})$	Monetary Approach
- Trade flows	Balance of Trade
- Other: trade barriers, expectations, taxes, etc.	

- ***Goal 1***: Explain S_t with a theory, say T1. Then, $S_t^{T1} = f(.)$
  Different theories can produce different $f(.)$'s.
  Evaluation: How well a theory match the observed behavior of S_t.

- ***Goal 2***: Eventually, produce a formula to forecast $S_{t+T} = f(X_t) \Rightarrow E[S_{t+T}]$.

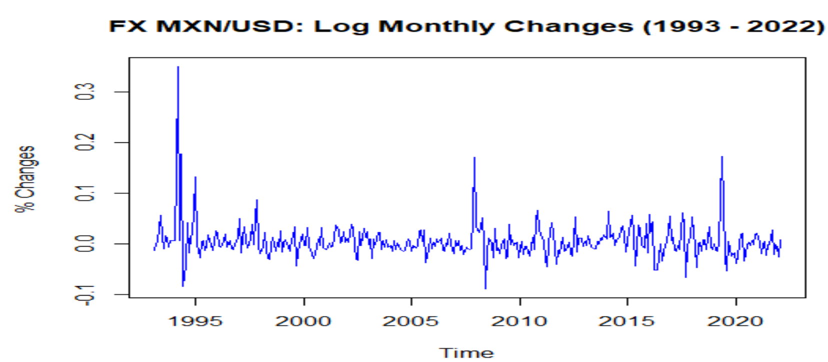
• We want a theory that matches observed S_t. But, not realistic to expect a perfect match.

Q: On average, is $S_t \approx S_t^{T1}$? Or, alternatively, is $E[S_t] = E[S_t^{T1}]$?



• Like many macroeconomic series, exchange rates have a trend –in statistics the trends in macroeconomic series are called *stochastic trends*. It is better to try to match changes, not levels.

• Now, the trend is gone. Our goal is to explain $e_{f,t}$, the percentage change in S_t. (Notation: Many times $s_t = e_{f,t}$).



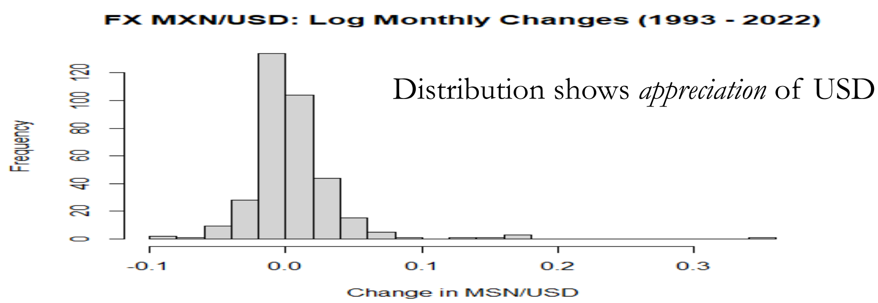
• The data will show us if the model we are using, say T1, matches, on average, the observed behavior of $e_{f,t}$.

Q for the data: Is $E[e_{f,t}] = E[e_{f,t}^{T1}]$?

• We will use statistics to formally tests theories.

• Data:

Distribution of MXN/USD monthly % changes, $e_{f,t}$ (1993:Feb – 2022: Dec)



Descriptive Stats:

$E[e_{f,t}]$ = Average monthly % change = **0.52%** (**6.3%**, annualized)

$SD[e_{f,t}$ = **3.51%** (**12.2%** annualized).

• A good theory should predict, on average, an annualized change of **6.3%** for $e_{f,t}$. A better theory should also predict a **12%** annualized volatility.

• Descriptive stats for $e_{f,t}$ for monthly JPY/USD and the MXN/USD.

	JPY/USD	*USD/MXN*
Mean	-0.0014	**0.0052**
Standard Error	0.0011	0.0019
Median	0.0002	0.0004
Standard Deviation	**0.0262**	**0.0351**
Sample Variance	0.0007	0.0021
Kurtosis	**4.0886**	**33.3631**
Skewness	**-0.4276**	**3.9122**
Minimum	-0.1052	-0.0887
Maximum	0.0807	0.3500
Count	577	350

• Developed currencies tend to be less volatile, with smaller means/medians. They are not normal distributed, but closer to "normal."

Purchasing Power Parity (PPP)

Purchasing Power Parity (PPP)

PPP is based on the law of one price (LOOP): Goods, once denominated in the same currency, should have the same price.

If they are not, then some form of arbitrage is possible.

Example: LOOP for Oil.

$P_{oil\text{-}USA}$ = **USD 60**

$P_{oil\text{-}SWIT}$ = **CHF 120**

$\Rightarrow S_t^{LOOP}$ = **USD 60** / **CHF 120** = **0.50 USD/CHF**.

If S_t = **0.75 USD/CHF** ⇒ Oil in Switzerland is more expensive (in USD) than in the US:

$P_{oil\text{-}SWIT}$ (USD) = **CHF 120** * **0.75 USD/CHF** = **USD 90** > $P_{oil\text{-}USA}$

Example (continuation):

S_t = **0.75 USD/CHF** > S_t^{LOOP} (LOOP is not holding)

Trading strategy:

(1) Buy oil in the US at $P_{oil\text{-}USA}$ = **USD 60**.

(2) Export oil to Switzerland

(3) Sell US oil in Switzerland at $P_{oil\text{-}SWIT}$ = **CHF 120**.

(4) Sell CHF/buy USD at then S_t.

Strategy, exporting US of oil to Switzerland, will affect prices:

1) $P_{oil\text{-}USA}\uparrow$
2) $P_{oil\text{-}SWIT}\downarrow$ ⇒ $S_t^{LOOP}\uparrow$ (= $P_{oil\text{-}USA}\uparrow / P_{oil\text{-}SWIT}\downarrow$)
3) $S_t\downarrow$ $S_t \Leftrightarrow S_t^{LOOP}$ (convergence). ¶

Example (continuation):

LOOP Notes :

◊ LOOP gives an *equilibrium* exchange rate.
Equilibrium is achieved when there is no trade in oil.
(because of pricing mistakes): LOOP holds for oil!

◊ LOOP is telling what S_t *should be* (in equilibrium). Not what S_t *is* in the market today.

◊ Using the LOOP we have generated a model for S_t. When applied to many goods, we have the *PPP model*.

Problem with LOOP: There are many traded goods in the economy.

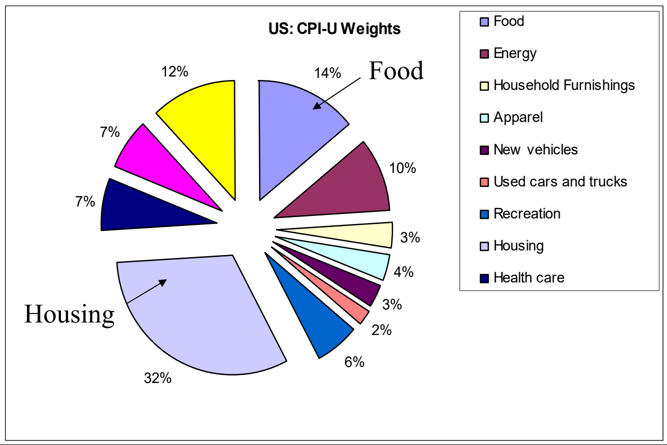
Solution: Use **baskets** of goods.

PPP: The price of a basket of goods should be the same across countries, once denominated in the same currency. That is, USD 1 should buy the same amounts of goods in the U.S. or in Colombia.

• A popular basket: The CPI basket.

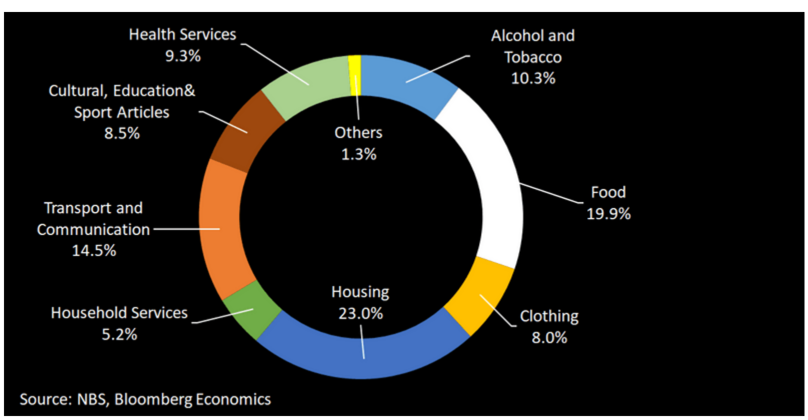
• In the U.S., the basket typically reported is the **CPI-U**. It represents the spending patterns of *all urban consumers and urban wage earners and clerical workers*. (**87%** of U.S. population).

• U.S. basket weights:



• Weights are different in different countries.

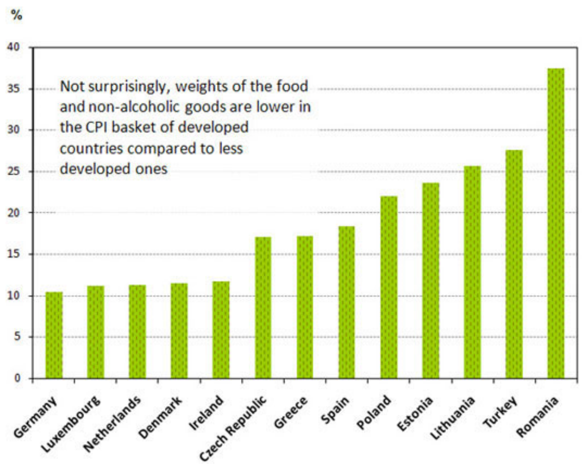
• China's basket weights:



• Relative to the U.S. weights, heavier weight given to Food & Clothing (Apparel, in the U.S.) and lower to Housing and Household Services (Energy, in the U.S.).

• The different weights is a problem when comparing CPI baskets: The composition of the index may vary widely across countries.

• For example, in Europe, the weight of the food category changes substantially as the income level increases.



Absolute version of PPP: The FX rate between two currencies is the ratio of the two countries' general price levels:

$$\mathbf{S_t^{PPP}} = \text{Domestic Price level / Foreign Price level} = \mathbf{P_d / P_f}$$

Example: LOOP for CPIs.

CPI-basket$_{USA}$ = P_{USA} = **USD 5,577**

CPI-basket$_{SWIT}$ = P_{SWIT} = **CHF 6,708**

$\Rightarrow \mathbf{S_t^{PPP}}$ = **USD 5,577/CHF 6,708 = 0. 8314 USD/CHF**.

If $S_t \neq$ **0. 8314 USD/CHF**, there will be trade of the goods in the baskets.

Suppose S_t = **1.09 USD/CHF** > $\mathbf{S_t^{PPP}}$.

Then,

P_{SWIT} (in USD) = **CHF 6,708** * **1.09 USD/CHF**

= **USD 7,311.72** > P_{USA} = **USD 5,577**

Example (continuation): (disequilibrium: S_t= 1.09 USD/CHF > S_t^{PPP})

P_{SWIT} (in USD) = **CHF 6,708** * 1.09 USD/CHF

= **USD 7,311.72** > P_{USA} = **USD 5,577**

Potential profit: **USD 7,311.72** – **USD 5,577** = **USD 1,734.72**

Traders will do the following *pseudo-arbitrage* strategy:

1) Borrow USD
2) Buy the CPI-basket in the U.S.
3) Sell the CPI-basket, purchased in the U.S., in Switzerland.
4) Sell the CHF/Buy USD
5) Repay the USD loan, keep the profits.

Note: "Equilibrium forces" at work:

2) $P_{USA}\uparrow$

3) $P_{SWIT}\downarrow$ (⇒ $S_t^{PPP}\uparrow = P_{USA}\uparrow / P_{SWIT}\downarrow$)

4) $S_t\downarrow$ $S_t \Leftrightarrow S_t^{PPP}$ (converge) ¶

• Real v. Nominal Exchange Rates

The absolute version of the PPP theory is expressed in terms of S_t, the *nominal exchange rate*.

We can write the absolute version of the PPP relationship in terms of the *real exchange rate*, R_t. That is,

$$R_t = S_t\, P_f / P_d = 1$$

R_t allows us to compare prices, translated to DC:

If $R_t > 1$, foreign prices (translated to DC) are more expensive

If $R_t = 1$, prices are equal in both countries –i.e., PPP holds!

If $R_t < 1$, foreign prices are cheaper

Economists associate $R_t > 1$ with a more efficient domestic economy.

Example: We have Big Mac ("the basket") prices in Switzerland & the US:

P_f = **CHF 6.70**

P_d = **USD 5.36**

S_t = **1. 0836 USD/CHF** $\Rightarrow$ P_f (in USD) = USD 7.26 > P_d

$R_t = S_t\, P_{SWIT}/P_{US}$ = **1. 0836 USD/CHF** * **CHF 6.70/USD 5.36** = **1.3545**

Taking the Big Mac as our basket, the U.S. is more competitive than Switzerland. Swiss prices are **35.45%** higher than U.S. prices, after taking into account the nominal exchange rate.

To bring the economy to equilibrium –no trade in Big Macs-, we expect the USD to appreciate against the CHF.

According to PPP, the USD is *undervalued* against the CHF.

$\Rightarrow$ Trading Signal: Buy USD/Sell CHF. ¶

• The Big Mac ("Burgernomics," popularized by *The Economist*) has become a popular basket for PPP calculations. Why?

1) Standardized, common basket: beef, cheese, onion, lettuce, bread, pickles and special sauce. (CPI baskets, not standardized). Sold in 120+ countries.

Big Mac (Sydney)  Big Mac (Tokyo) 

2) Very easy to find out the price.

3) It turns out, it is correlated with more complicated common baskets.

• In theory, traders can exploit the price differentials in BMs.

The Economist's Big Mac Index

• In the previous example, Swiss traders can import US BMs.

From UH (US) to Rapperswill (CH)

• Not realistic. But, the components of a BM are internationally traded. LOOP suggests that prices of components should be similar in all markets.

The Economist reports the real exchange rate: $\mathbf{R_t} = S_t\, P_{BigMac,f}/P_{BigMac,d}$.

For example, in **Dec 2022**, for the British pound (GBP):

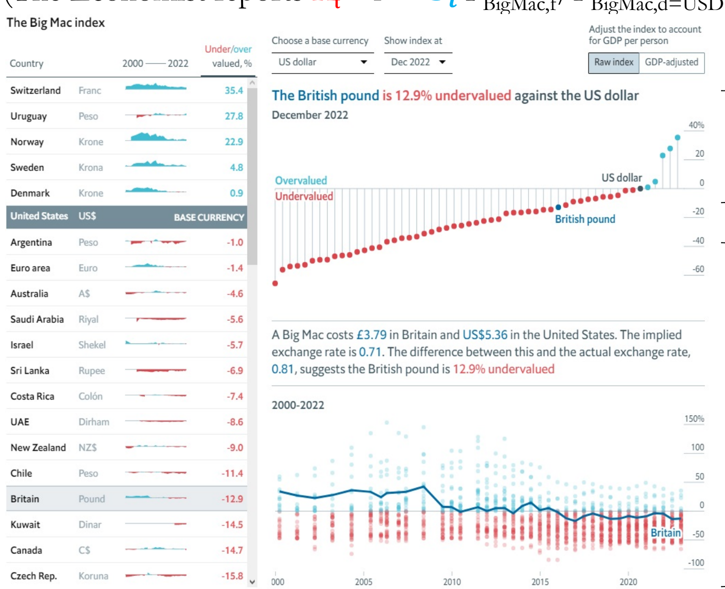
$$\mathbf{R_t} = [\mathbf{1.2318\ USD/GBP} * \text{GBP } 3.79] / \text{USD } 5.36 = \mathbf{0.87099}$$

$\Rightarrow$ (**12.90%** overvaluation)

Example: (The Economist's) Big Mac Index in **Dec 2022**.

$\mathbf{S_t^{PPP}} = P_{BigMac,d} / P_{BigMac,f}$

(The Economist reports $\mathbf{R_t} - 1 = S_t\, P_{BigMac,f}/P_{BigMac,d=USD} - 1$).



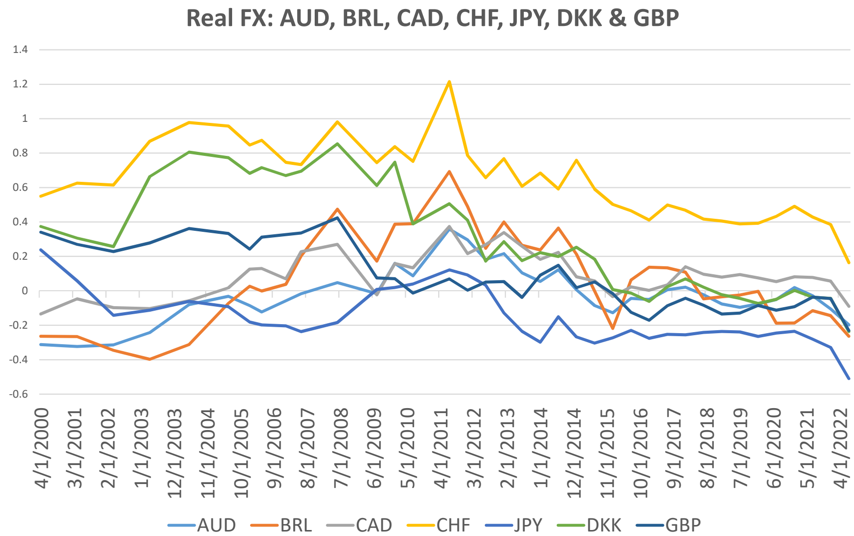
$\mathbf{R_t} > 1$

$\mathbf{R_t} < 1$

Example: (The Economist's) Big Mac Index (January 2011)

$R_t = S_t P_{BigMac,f} / P_{BigMac,d}$ (US=domestic) $\Rightarrow$ R_t=1 under Absolute PPP

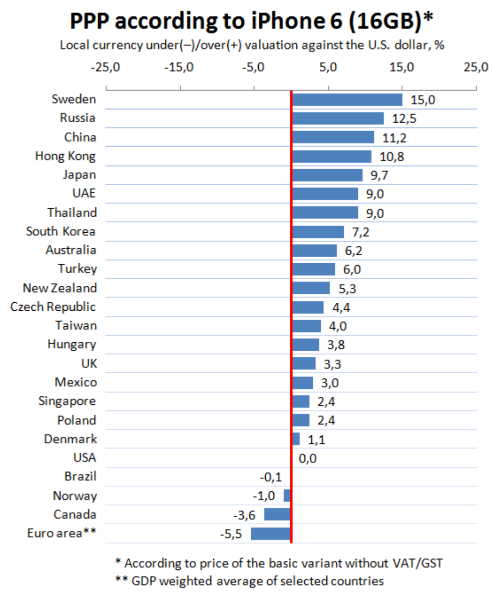
Example: Big Mac Index - (R_t - 1). Changes over time in **2000 - 2022**.



R_t does move over time. R_t departures from 1, can be very persistent.

Example: Iphone 6 (March 2015, taken from seekingalpha.com).

$\mathbf{R}_t = S_t\, P_{IPhone,f}/P_{IPhone,d}$ (d=US) $\Rightarrow$ $\mathbf{R}_t$=1 under Absolute PPP



• Empirical Evidence: Simple informal test:

Test: If Absolute PPP holds $\Rightarrow$ $\mathbf{R}_t$ = 1.

In the Big Mac example, PPP does not hold for the majority of countries.

$\Rightarrow$ Absolute PPP, in general, fails (especially, in the short-run).

• Absolute PPP: Qualifications

(1) *PPP emphasizes only trade and price levels*. Political/social factors, financial problems, etc. are ignored.

(2) Implicit assumption: *Absence of trade frictions* (tariffs, quotas, taxes, etc.).

Q: Realistic?

- On average, transportation costs add **7%** to the price of U.S. imports of meat and **16%** to the import price of vegetables.

- Many products are heavily protected, even in the U.S. For example, peanut imports are subject to a tariff as high as **163.8%.**

- Absolute PPP: Qualifications

Some everyday goods protected in the U.S.:
- Peanuts (shelled **131.8%**, and unshelled **163.8%**).
- Paper Clips (as high as **126.94%**)
- European Roquefort Cheese, cured ham, mineral water (**100%**)
- Japanese leather (**40%**)
- Sneakers (**48%** on certain sneakers)
- Chinese tires (**35%**)
- Canned Tuna (as high as **35%**)
- Synthetic fabrics (**32%**)
- Steel (**25%**)
- Indian wood furniture (**25%**)
- Italian footwear & eyeglasses (**25%**)
- Brooms (quotas and/or tariff of up to **32%**)
- Trucks (**25%**) & cars (**2.5%**)

- Absolute PPP: Qualifications

Some Japanese protected goods:
- Rice (**778%**)
- Sugar (**328%**)
- Powdered Milk (**218%**)
- Beef (38.5%, but can jump to 50% depending on volume).

Some European protected goods:
- Knitted Clothes (**100%**)
- Fresh Cheese (48.3%)
- Bovine Meat, boneless (41%)
- Fresh or dried grapefruit (25%)
- Atlantic Salmon (25%)

• Absolute PPP: Qualifications

(3) PPP is unlikely to hold if P_f and P_d represent *different baskets*. This is why the Big Mac is a popular choice.

(4) *Trade takes time* (contracts, information problems, etc.).

(5) *Internationally non-traded/non-tradable (NT) goods* –i.e. haircuts, home and car repairs, medical services, real estate. The NT good sector is big: **50%-60%** of consumption (big weight in CPI basket).

Then, in countries where NT goods are relatively expensive, the CPI basket will be relatively expensive. Thus, PPP will find these countries' currencies *overvalued* relative to currencies in low NT cost countries.

Note: In the short-run, cars will not be taken to Mexico to be repaired, but in the long-run, resources (capital, labor) will move.

⇒ Over-/under-valuation: An indicator of movement of resources.

• Absolute PPP: Qualifications

The NT sector also has an effect on the price of traded goods. For example, rent and utilities costs affect the price of a Big Mac: 25% of Big Mac due to NT goods.

• Empirical Fact

Price levels in richer countries are consistently higher than in poorer ones. This fact is called the *Penn effect*. Many explanations, the most popular: The *Balassa-Samuelson (BS) effect*.

• Borders Matter

You may look at the Big Mac Index and think: "No big deal: there is also a big dispersion in prices within the U.S., within Texas, and, even, within Houston!"

True. Prices vary within the U.S. For example, in **2015**, the price of a Big Mac (and Big Mac Meal) in New York was USD 5.23 (USD 7.45), in Texas as USD 4.39 (USD 6.26).

But, borders play a role, not just distance!

Engel and Rogers (1996) computed the variance of LOOP deviations for **city pairs** within the **U.S.**, within **Canada**, and **across the border**.

Conclusion: Distance between cities within a country matter, but the **border effect** is **significant**.

To explain the difference between prices across the border using the estimate distance effects within a country, they estimate the U.S.-Canada border should have a width of **75,000 miles**!

This huge estimate has been revised downward, but a large positive border effect remains.

• Balassa-Samuelson Effect

Labor costs affect all prices. We expect average prices to be cheaper in poor countries than in rich ones because **labor costs** are **lower**.

This is the *Balassa-Samuelson effect*: Rich countries have higher productivity and, thus, higher wages in the traded-goods sector than poor countries do. But, firms compete for workers.

Then, wages in NT goods and services are also higher

$\Rightarrow$ Overall prices are lower in poor countries.

- For example, in **2000**, a typical McDonald's worker in the U.S. made **USD 6.50/hour**, while in China made **USD 0.42/hour**.

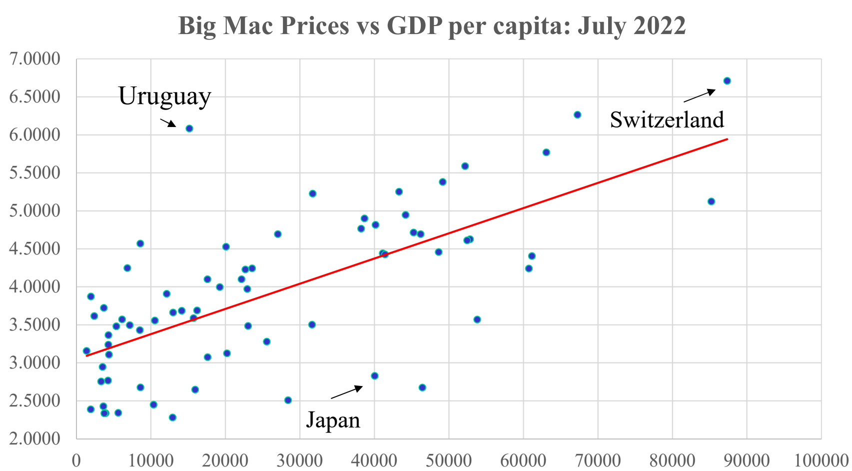
In **2021**, the same numbers for a cashier are **USD 10/hour** and **USD 1.76**.

- Balassa-Samuelson effect: A positive correlation between *PPP exchange rates (overvaluation)* and high productivity countries.

Incorporating the Balassa-Samuelson effect into PPP:

1) Estimate a regression: Big Mac Prices against GDP per capita.

P_{BM} (in USD)$_t = \alpha + \beta$ GDP_per_capita$_t + \varepsilon_t$



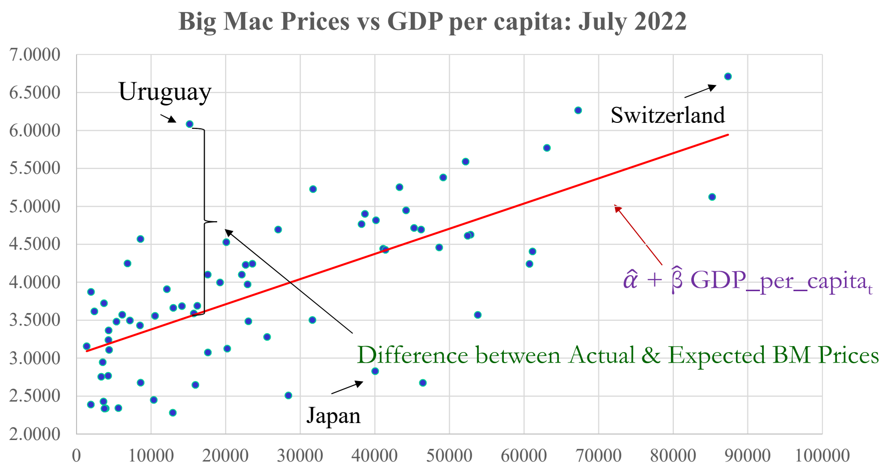
Points on Red line: *Fitted (Expected) Big Mac Prices*, given a *GDP per person*.

$\hat{P}_{BM,GDP\text{-}adj} = \hat{\alpha} + \hat{\beta}$ GDP_per_capita$_t$

Incorporating the Balassa-Samuelson effect into PPP:

2) Compute fitted values:

$\hat{P}_{BM,GDP\text{-}adj} = \hat{\alpha} + \hat{\beta}$ GDP_per_capita$_t$



GDP-adjusted over/under valuation: (BM Price/$\hat{P}_{BM,GDP\text{-}adjusted}$) – 1.

Incorporating the Balassa-Samuelson effect into PPP: Computations

Using data from The Economist for July 2022, we estimate the red line:

$\hat{P}_{BM,GDP\text{-}adj}$ = 3.045895 + 0.0000332 * GDP_per_capita$_t$

Now, we can compute the "Expected BM prices, given the GDP of a given country." Let's compute the above value for Uruguay. Uruguay's GDP per capita in July 2022 was **USD 15,169.153**. Then,

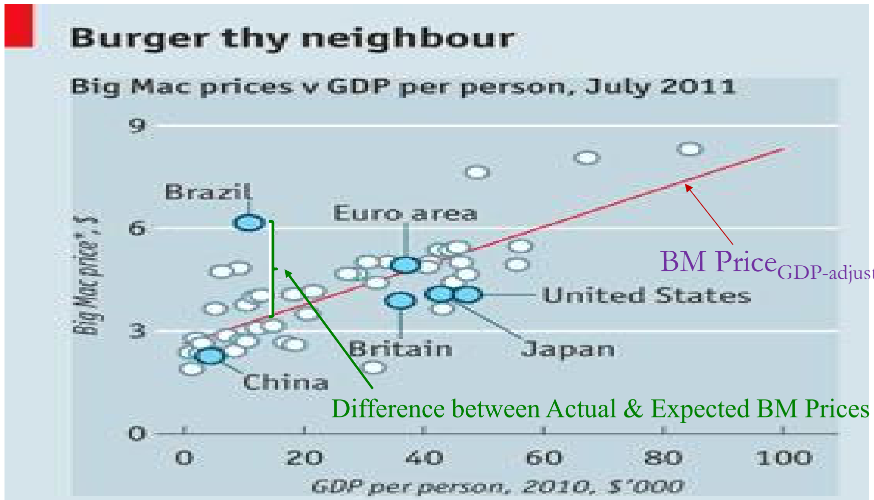
$\hat{P}_{BM,GDP\text{-}adj}$ (Uruguay) = 3.045895 + 0.0000332 * **15,169.153** = **3.549511**

That is, the expected BM in Uruguay in July 2022, given its GDP per capita, was **USD 3.55**. Since the observed local BM price was UYU 255, which translates to **USD 6.08** (= UYU 255 * **41.91 USD/UYU**), then the *GDP-adjusted over/under valuation* was:

6.08 / **3.549511** – 1 = 71.29% (71.29% overvalued)

Incorporating the Balassa-Samuelson effect into PPP: July 2011

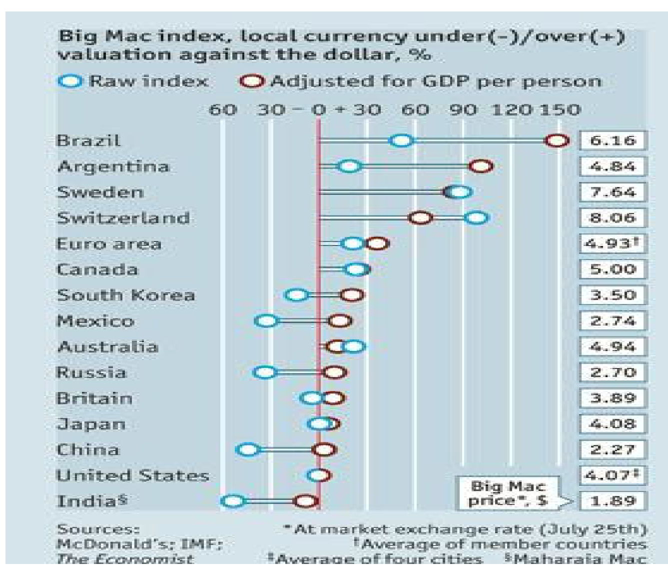
Same computation for **July 2011**.



Points on Red line: GDP-*adjusted Big Mac Prices* (BM Price$_{\text{GDP-adjusted}}$).

Incorporating the Balassa-Samuelson effect into PPP:

The GDP adjustment can make a difference.



Example: Raw vs GDP-Adjusted Big Mac Index in **Dec 2022**.

The Big Mac index

Country		2000 —— 2022	Under/over valued, %
Switzerland	Franc		35.4
Uruguay	Peso		27.8
Norway	Krone		22.9
Sweden	Krona		4.8
Denmark	Krone		0.9
United States	US$		BASE CURRENCY
Argentina	Peso		-1.0
Euro area	Euro		-1.4
Australia	A$		-4.6
Saudi Arabia	Riyal		-5.6
Israel	Shekel		-5.7
Sri Lanka	Rupee		-6.9
Costa Rica	Colón		-7.4
UAE	Dirham		-8.6
New Zealand	NZ$		-9.0
Chile	Peso		-11.4
Britain	Pound		-12.9
Kuwait	Dinar		-14.5
Canada	C$		-14.7
Czech Rep.	Koruna		-15.8

The Big Mac index

Country		2011 —— 2022	Under/over valued, %
Uruguay	Peso		52.7
Switzerland	Franc		37.3
Norway	Krone		23.6
Argentina	Peso		21.2
Sri Lanka	Rupee		16.6
Sweden	Krona		12.1
Costa Rica	Colón		11.0
Euro area	Euro		8.4
Saudi Arabia	Riyal		8.2
Chile	Peso		5.1
Denmark	Krone		3.3
Brazil	Real		1.1
United States	US$		BASE CURRENCY
Israel	Shekel		-0.9
Nicaragua	Córdoba		-1.2
UAE	Dirham		-2.3
New Zealand	NZ$		-2.8
Australia	A$		-3.1
Colombia	Peso		-4.3
Bahrain	Dinar		-4.9

• Pricing-to-Market

Krugman (1987): Positive relationship between GDP and price levels is caused by *Pricing-to-market* –i.e., price discrimination.

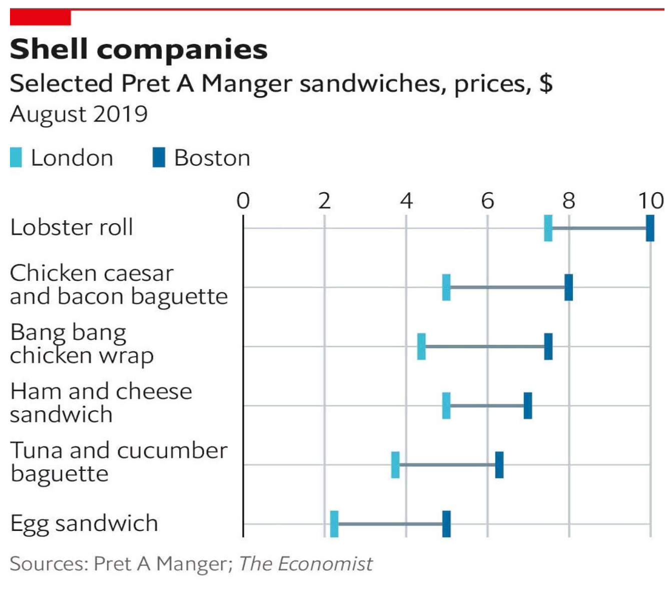
Producers discriminate: Same good is sold to rich countries at higher prices than to poorer countries.

Alessandria and Kaboski (2008): U.S. exporters, on average, charge the richest country a **48%** higher price than the poorest country.

But pricing-to-market struggles to explain why PPP does not hold among developed countries with similar incomes.

For example, Baxter and Landry (2012) report that IKEA prices deviate **16%** from the LOOP in Canada, but only **1%** in the U.S.

Example: Pricing to Market? Pret A Manger (August 2019, from The Economist). Comparing: $S_t\, P_{PM,London}$ & $P_{PM,Boston}$.



Main PPP criticism

Absolute PPP does not incorporate transaction costs and frictions. Relative PPP allows for fixed transaction costs/frictions (say, a fixed USD amount).

Relative PPP

The rate of change in the prices of products should be similar when measured in a common currency (as long as trade frictions are unchanged):

$$e_{f,t,T}^{PPP} = \frac{S_{t+T}^{PPP} - S_t}{S_t} = \frac{(1 + I_d)}{(1 + I_f)} - 1 \qquad \text{(Relative PPP)}$$

where,

I_f = foreign inflation rate from t to t+T.

I_d = domestic inflation rate from t to t+T.

Note: $e_{f,t,T}^{PPP}$ is an expectation; what we expect to happen in equilibrium from t to t+T.

- Linear approximation: $e_{f,t,T}^{PPP} \approx (I_d - I_f)$ $\Rightarrow$ one-to-one relation

Relative PPP

• Linear approximation: $e^{PPP}_{f,t,T} \approx (I_d - I_f)$ ⇒ one-to-one relation

Example: From t=0 to t=1, prices increase **10%** in Mexico relative to prices in Switzerland. Then, S_t should also increase 10%.

If $S_{t=0}$ = 9 MXN/CHF ⇒ $\mathbf{S^{PPP}_{t=1}}$ = E[$S_{t=1}$] = **9.9 MXN/CHF**.

Suppose at t=1, S_t increases 13.33%. Then,

$S_{t=1}$= **10.2 MXN/CHF** > $\mathbf{S^{PPP}_{t=1}}$ **= 9.9 MXN/CHF**

⇒ According to Relative PPP, the CHF is overvalued. ¶

Notation: E[$S_{t=1}$] = Expected value of $S_{t=1}$ (model-based), a predicted value.

Example: Forecasting S_t (USD/ZAR) using PPP (ZAR=South Africa).

It's Dec 2022. You have the following information:

$CPI_{US,2022}$ = 104.5,

$CPI_{SA,2022}$ = 100.0,

$S_{t=2022}$ = **.2035 USD/ZAR**.

You are given the 2023 CPI's forecast for the U.S. and SA:

E[$CPI_{US,2023}$] = 110.8

E[$CPI_{SA,2023}$] = 102.5.

You want to forecast S_{2023} using the relative (linearized) version of PPP.

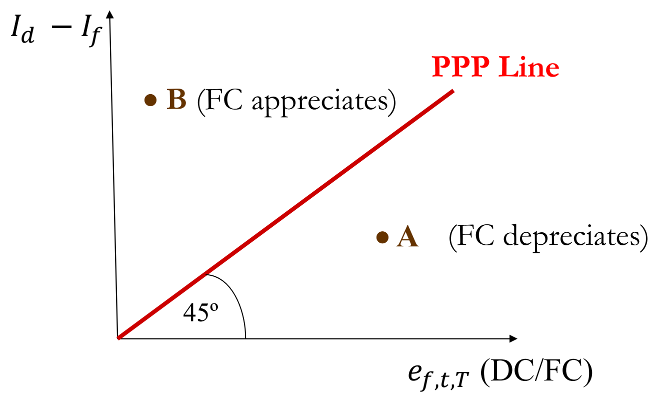
E[$I_{US,2023}$] = (110.8/104.5) - 1 = **.06029**

E[$I_{SA,2023}$] = (102.5/100) - 1 = **.025**

E[S_{2023}] = S_{2022} * (1 + $e^{PPP}_{f,t=2022,T=2023}$) = S_{2022} * (1 + E[I_{US}] - E[I_{SA}])

= **.2035 USD/ZAR** * (1 + **.06029** - **.025**) = .2107 USD/ZAR..

• Under the linear approximation, $e_{f,t,T}^{PPP} \approx (I_d - I_f)$, we have a PPP Line



Look at point **A**: $e_{f,t,T} > (I_d - I_f)$,

⇒ Priced in FC, the domestic basket is cheaper

⇒ pseudo-arbitrage (trade) against foreign basket ⇒ FC depreciates

• Relative PPP: Implications

(1) Under relative PPP, $\mathbf{R_t}$ remains constant (it can be different from 1!).

(2) Without relative price changes, an MNC faces no real operating FX risk (as long as the firm avoids fixed contracts denominated in FC).

• Relative PPP: Absolute versus Relative

- Absolute PPP compares price levels.

Under Absolute PPP, prices are equalized across countries:

"*A mattress costs* **GBP 200** *(=* **USD 320***) in the U.K. and* **BRL 800** *(=* **USD 320***) in Brazil.*"

- Relative PPP compares price changes.

Under Relative PPP, exchange rates change by the same amount as the inflation rate differential (original prices can be different):

"*U.K. inflation was 2% while Brazilian inflation was 8%. Meanwhile, the BRL depreciated 6% against the GBP. Then, relative cost comparison remains the same.*"

• Relative PPP is weaker than Absolute PPP: **R_t** can be different from 1.

• Relative PPP: Testing

Key: On average, what we expect to happen, $e^{\text{PPP}}_{f,\text{t},\text{T}}$, should happen, $e_{f,t,T}$.

⇒ On average: $e_{f,t,T} \approx e^{\text{PPP}}_{f,\text{t},\text{T}} \approx (I_d - I_f)$

or $\quad \text{E}[e_{f,t,T}] = \text{E}[\, e^{\text{PPP}}_{f,\text{t},\text{T}} \,] \approx \text{E}[(I_d - I_f)]$

A linear regression is a good framework to test theories. Recall,

$$e_{f,t,T} = \frac{S_{t+T} - S_t}{S_t} = \alpha + \beta\, (I_d - I_f)_{t+T} + \varepsilon_{t+T},$$

where ε_t: regression error. That is, $\text{E}[\varepsilon_{t+T}] = 0$.

Then, $\quad \text{E}[e_{f,t,T}] = \alpha + \beta\, \text{E}[(I_d - I_f)_{t+T}] + \text{E}[\varepsilon_{t+T}] = \alpha + \beta\, \text{E}[\, e^{\text{PPP}}_{f,\text{t},\text{T}}]$

$\Rightarrow \text{E}[e_{f,t,T}] = \alpha + \beta\, \text{E}[\, e^{\text{PPP}}_{f,\text{t},\text{T}}]$

⇒ For Relative PPP to hold, on average, we need **α=0** & **β=1**.

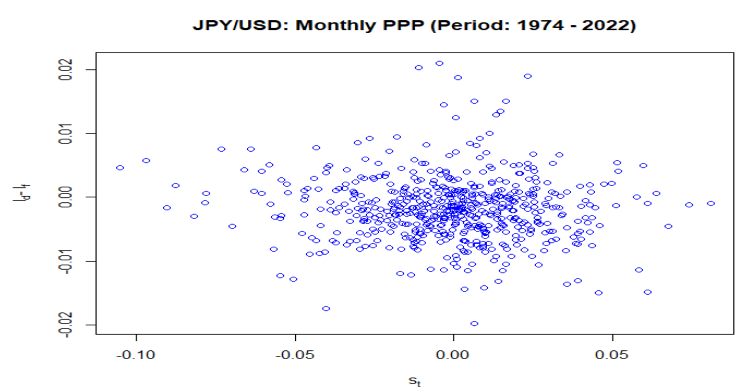
• Relative PPP: General Evidence

Under Relative PPP: $e_{f,t,T} \approx (I_d - I_f)$

1. Visual Evidence

Plot $(I_d - I_f)$ against $e_{f,t}$(JPY/USD), using monthly data **1975 - 2022**.

Test: Is there a 45° line?



No 45° line ⇒ Visual evidence rejects PPP.

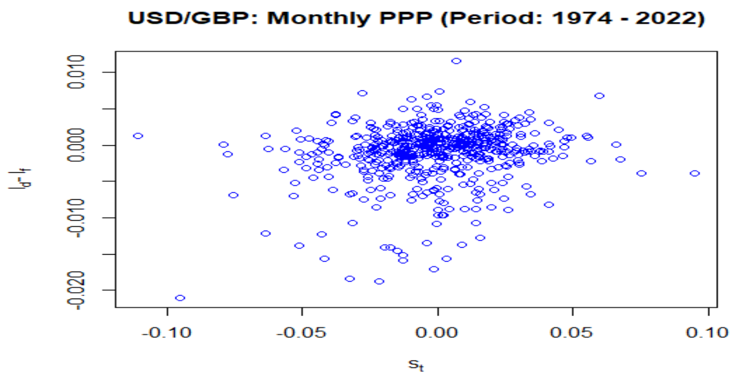
• Relative PPP: General Evidence

Under Relative PPP: $e_{f,t,T} \approx (I_d - I_f)$

1. Visual Evidence

Plot ($I_{USD} - I_{GBP}$) against $e_{f,t}$ (USD/GBP), using monthly data **1975 - 2022**.

Test: Is there a 45° line?



• Relative PPP: General Evidence

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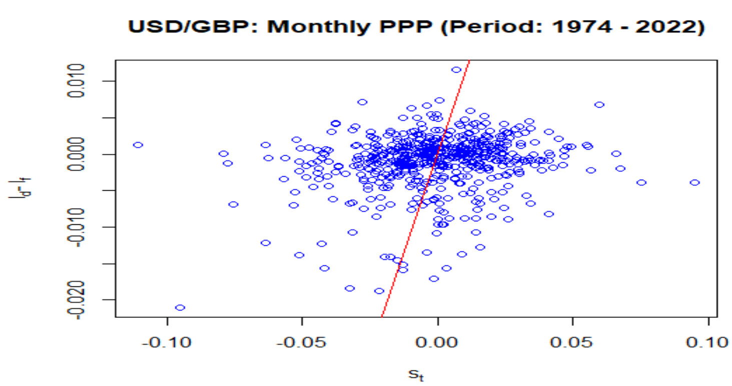
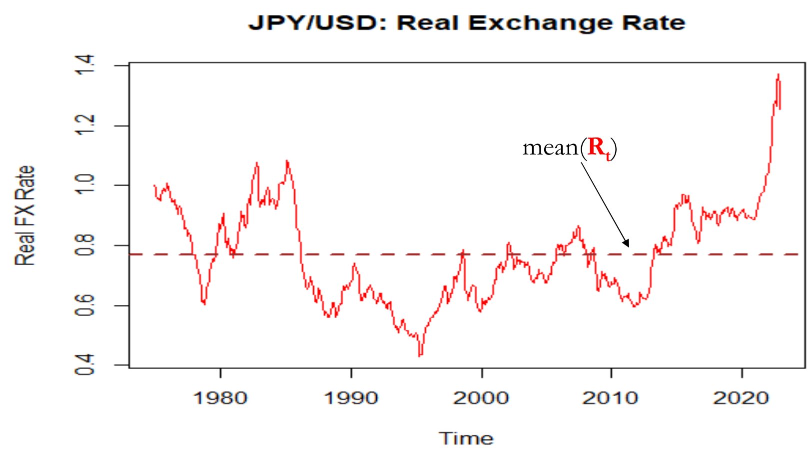


No 45° line ⇒ Visual evidence rejects PPP.

• Relative PPP: General Evidence

1. Visual Evidence

Test: Is R_t ≈ Constant? (Under Absolute PPP ≈ 1)



Some evidence for mean reversion, though slow, for R_t (average = 0.77).

• Relative PPP: General Evidence (continuation)

In the long run, R_t moves around some mean number (long-run PPP parity?). But, the deviations from long-run parity are very *persistent*.

Economists report the number of years that a PPP deviation is expected to decay by 50%, the *half-life*. The half-life is in the range of **3 to 5 years** for developed currencies. Very slow!

• Descriptive Stats (**1975:Jan – 2022:Dec**)

Long-run, on average.

	I_{JPY}	I_{USD}	$I_{JPY} - I_{USD}$	$e_{f,t,T}$ (JPY/USD)
Mean	**0.00125**	**0.00303**	**-0.00179**	**-0.00139**
SD	**0.00485**	**0.00322**	**0.00502**	**0.02622**
Min	-0.01095	-0.01786	-0.01981	-0.08065
Median	0.00102	0.00266	-0.00184	0.00022
Max	0.02558	0.01420	0.02104	0.08066

Big difference in volatility.

2. Statistical Evidence

Formal test: Regression

$$e_{f,t,T} = \alpha + \beta\,(I_d - I_f)_{t+T} + \varepsilon_{t+T}, \qquad (\varepsilon_t\text{: error term, E}[\varepsilon_t] = 0).$$

The null hypothesis is: H_0 (Relative PPP true): **α=0** and **β=1**

H_1 (Relative PPP not true): **$\alpha\neq0$** and/or **$\beta\neq1$**

• **Tests:** *t-test* (individual tests on α and β) & *F-test* (joint test)

(1) Individual test: *t-test*

$$t\text{-test} = t_\theta = [\hat{\theta} - \theta_0]/\text{S.E.}(\hat{\theta})$$

where θ represents α or β ⇒ (θ_0 = α or β evaluated under H_0).

Statistical distribution: $t_\theta \sim t_v$ ($v = N - K$ = degrees of freedom)

K = # parameters in model, & N = # of observations.

Rule: If $|t\text{-test}| > |t_{v,\alpha/2}|$, reject H_0 at the α level.

When $v = N - K > 30$, $t_{30+,.025} \approx$ **1.96** ⇒ 2-sided C.I. α = .05 (5 %)

2. Statistical Evidence

(2) Joint Test: *F-test*

$$F = \frac{[\text{RSS}(H_0) - \text{RSS}(H_1)]/J}{\text{RSS}(H_1)/(N - K)}$$

Statistical distribution: $F \sim F_{J,N-K}$

J = # of restrictions in H_0 (under PPP, J=2: **α=0** & **β=1**)

K = # parameters in model (under PPP model, K=2: α & β)

N = # of observations

RSS = Residuals Sum of Squared, $\hat{\varepsilon}_t = e_t = e_{f,t,T} - [\hat{\alpha} + \hat{\beta}\,(I_{d,t} - I_{f,t})]$.

$$\text{RSS}(H_0) = \textstyle\sum_{t=1}^{N} [s_t - (I_{d,t} - I_{f,t})]^2$$

$$\text{RSS}(H_1) = \textstyle\sum_{t=1}^{N} (\hat{\varepsilon}_t)^2$$

Rule: If $F > F_{J,N-K,\alpha}$, reject at the α level. Usually, α = .05 (5 %)

When $N > 300$, $\mathbf{F_{J=2,300+,\alpha=.05}} \approx$ **3**.

Example: Using monthly Japanese and U.S. data (**1975:Jan - 2022:Dec**), we fit the following regression (Observations = 576):

$$e_{f,t,T}(\text{JPY/USD}) = (S_t - S_{t-1})/S_{t-1} = \alpha + \beta\,(I_{JAP} - I_{US})_t + \varepsilon_t.$$

$R^2 = 0.005621$
Standard Error (σ) = .02617
F-stat (slopes=0 –i.e., $\beta=0$) = 3.244 (*p-value* = 0.07219)
Observations (*N*) = 576

	Coefficient	Stand Err	t-Stat	P-value
Intercept ($\hat{\alpha}$)	-0.00209	0.001157	-1.804	0.0717
($I_{JAP} - I_{US}$) ($\hat{\beta}$)	-0.39148	0.217343	-1.801	0.0722

We will test the H_0 (Relative PPP true): **$\alpha=0$ & $\beta=1$**

Two tests: (1) *t-tests* (individual tests)
(2) *F-test* (joint test)

Example: Using monthly Japanese and U.S. data (**1975:Jan - 2022:Dec**), we fit the following regression (Observations = 576):

$$e_{f,t,T}(\text{JPY/USD}) = (S_t - S_{t-1})/S_{t-1} = \alpha + \beta\,(I_{JAP} - I_{US})_t + \varepsilon_t.$$

$R^2 = 0.005621$
Standard Error (σ) = .02617
F-stat (slopes=0 –i.e., $\beta=0$) = 3.244 (*p-value* = 0.07219)
F-test (H_0: **$\alpha=0$ & $\beta=1$**): **19.185** (*p-value*: < 0.00001) ⇒ reject H_0 at 5% level ($F_{2,550,.05}$= **3.012**)

	Coefficient	Stand Err	t-Stat	P-value
Intercept ($\hat{\alpha}$)	-0.00209	0.001157	-1.804	0.0717
($I_{JAP} - I_{US}$) ($\hat{\beta}$)	-0.39148	0.217343	-1.801	0.0722

Test H_0, using t-tests ($t_{574,.05}$ = **1.96** – <u>Note</u>: when *N-K* > 30, $t_{.05}$ = **1.96**):

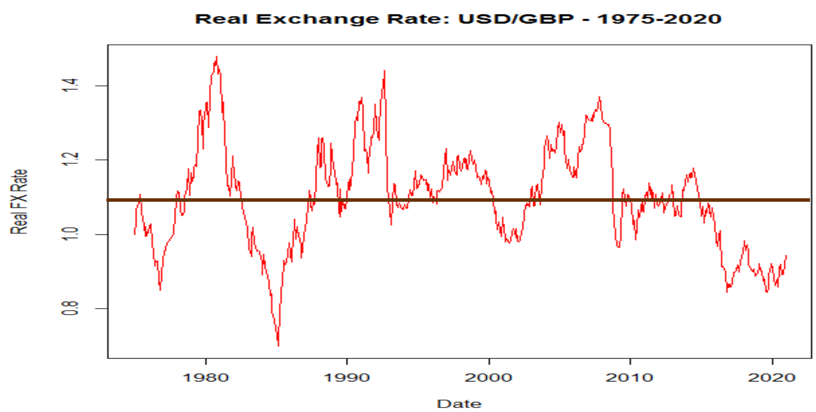
$t_{\alpha=0}$: (-0.00209 – 0)/0.001157 = -1.804 (*p-value* = .07) ⇒ cannot reject H_0.

$t_{\beta=1}$: (-0.39148 – 1)/0.217343 = -6.402 (*p-value*: < .00001) ⇒ reject H_0. ¶

• PPP Evidence:

◊ Relative PPP tends to be rejected in the short-run. In the long-run, there is debate about its validity: Currencies with high inflation rate differentials tend to depreciate.

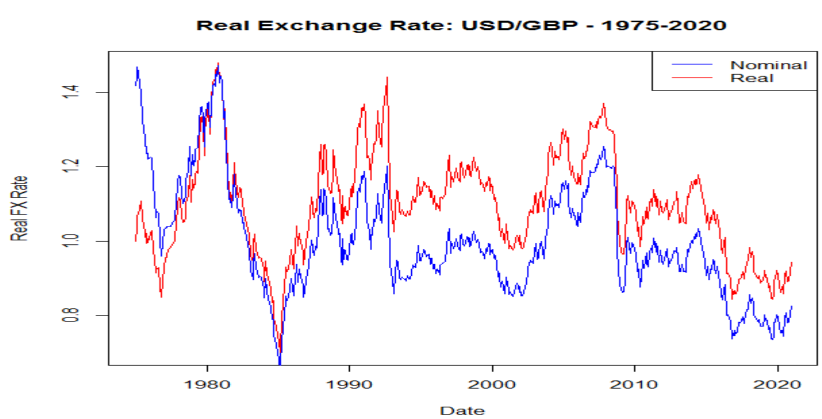
◊ Some evidence for a mean reverting $\mathbf{R_t}$ (average $\mathbf{R_t}$ = 1.10). But deviations can last for years!



• PPP: R_t and S_t

Mussa (1986): $\mathbf{R_t}$ is more variable under a free float.
$\mathbf{R_t}$ variability is highly correlated with S_t variability.

Check Second Moments: Volatility (changes in $\mathbf{R_t}$) = 2.706% & Volatility (changes in S_t) = 2.622 (correlation = .983). Almost the same!



Implications: Price levels play a minor role in explaining the movements of R_t (prices are *sticky*).

Possible explanations:

(a) Contracts:

Prices cannot be continuously adjusted due to contracts.

(b) Mark-up adjustments:

Manufacturers and retailers moderate increases in their prices in order to keep market share. Changes in S_t are only partially transmitted or *pass-through* to import/export prices.

Average ERPT (exchange rate pass-through) is around **50%** over one quarter and **64%** over the long run for **OECD countries** (for the **U.S., 25%** in the short-run and **40%** over the long run).

(c) Repricing costs (*menu costs*)

Expensive to adjust continuously prices –a restaurant, re-printing the *menu*.

(d) Aggregation

Q: Is price rigidity a result of aggregation –i.e., the use of price index? Empirical work using **micro level data** –say, same good (exact UPC!) in Canadian and U.S. grocery stores– show that on average product-level R_t moves with S_t. But, evidence is not as solid.

• PPP: Puzzle

The fact that no single model of exchange rate determination can accommodate both the high persistent of PPP deviations and the high correlation between R_t and S_t has been called the "*PPP puzzle.*"

• PPP: Summary of Empirical Evidence

◊ R_t and S_t are highly correlated, P_d tends to be sticky.

◊ In the short run, PPP is a poor model to explain short-term S_t movements.

◊ PPP deviations are very persistent. They take years to disappear.

◊ In the long run, there is some evidence of mean reversion, though slow, for R_t. That is, S_t^{PPP} has long-run information:

Currencies that consistently have high inflation rate differentials tend to depreciate.

• The long-run interpretation is the one that economists like and use: S_t^{PPP} is seen as a benchmark.

• Calculating S_t^{PPP} (Long-Run FX Rate)

We want to calculate $S_t^{PPP} = P_{d,t} / P_{f,t}$ over time.

(1) Divide S_t^{PPP} by $S_{t=0}^{PPP}$ (t = 0 is our starting point).

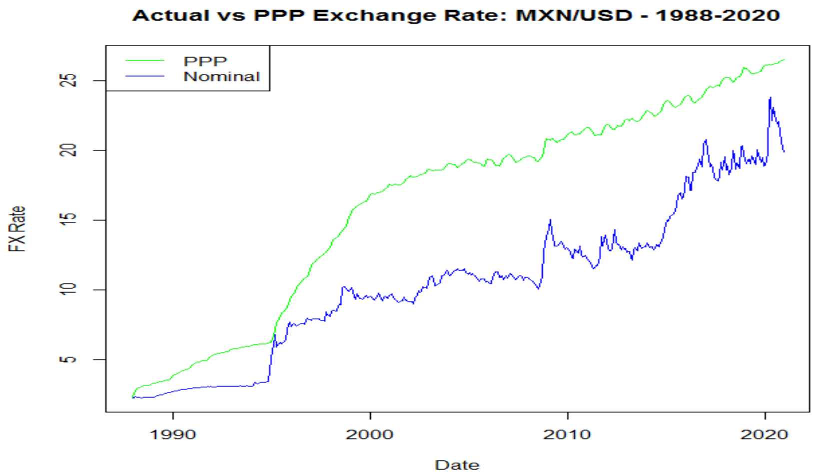
(2) After some algebra,

$$S_t^{PPP} = S_{t=0}^{PPP} * [P_{d,t} / P_{d,0}] * [P_{f,0}/P_{f,t}]$$

By assuming $S_{t=0}^{PPP} = S_0$, we plot S_t^{PPP} over time.

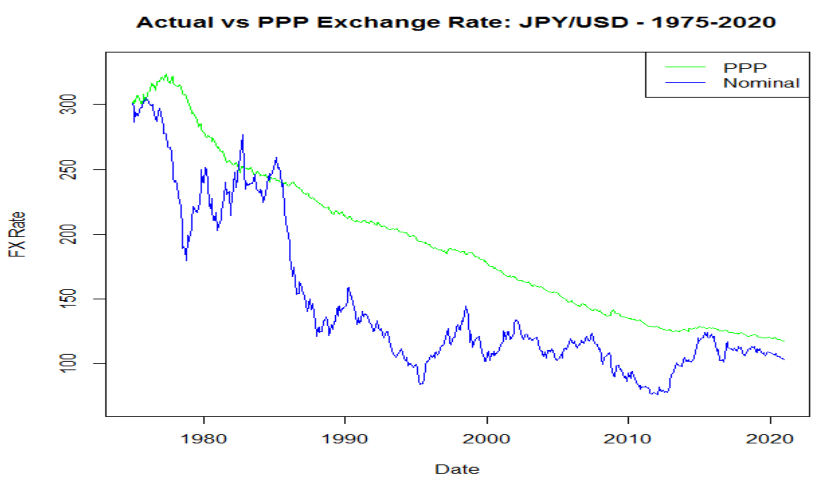
Note: $S_{t=0}^{PPP} = S_0$ assumes that at t=0, the economy was in *equilibrium*. This may not be true: Be careful when selecting a base year.

Let's look at the MXN/USD case.



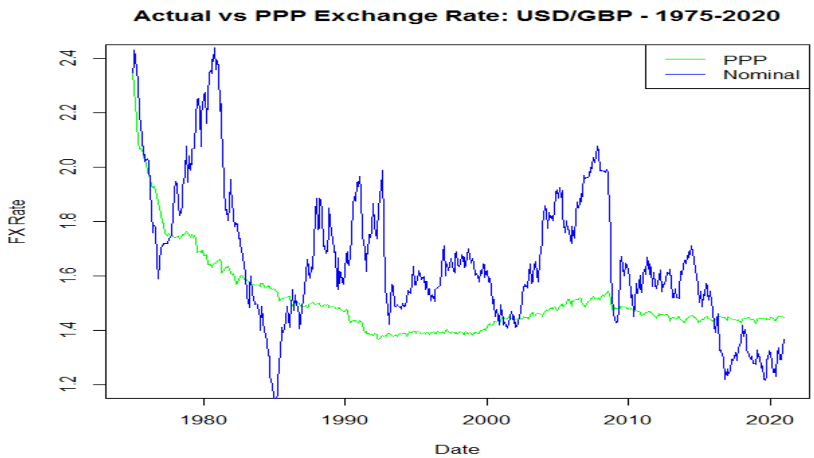
- In the short-run, $\mathbf{S_t^{PPP}}$ misses the target, S_t.
- But, in the long-run, $\mathbf{S_t^{PPP}}$ gets trend right, reflecting a consistent higher inflation in Mexico.

Another example, the JPY/USD case.



As predicted by PPP, since I_{US} has been consistently higher than I_{JAP}, in the long-run, the USD depreciates against the JPY.

Another example, the USD/GBP case.



As predicted by PPP, I_{US} was consistently lower than I_{UK} until the mid-90s, the USD appreciated against the GBP. Since then, it has been moving around a constant value.

• PPP Summary of Applications:

◊ Equilibrium ("*long-run*") exchange rates.

◊ Explanation of S_t movements.

◊ Indicator of competitiveness or under/over-valuation.

◊ International GDP comparisons: Instead of using S_t, $\mathbf{S_t^{PPP}}$ is used to translate local currencies to USD. For example, Chinese per capita GDP (World Bank figures, in 2017):

Nominal GDP per capita: **CNY 59,670.52**;
S_t = **0.14792 USD/CNY**;
- Nominal GDP_cap (USD)= **CNY 59,670.52** * **0.1479 USD/CNY**=**USD 8,827**

$\mathbf{S_t^{PPP}}$ = **0.2817 USD/CNY** ⇒ "U.S. is 90% more expensive"
- PPP GDP_cap (USD)= **CNY 59,670.52** * **0.2817 USD/CNY** = **USD 16,807**.

Country	GDP per capita (in USD) - 2017	
	Nominal	**PPP**
Luxembourg	104,103	103,745
USA	59,532	59,532
Japan	38,428	43,279
Italy	31,953	39,427
Czech Republic	20,368	36,504
Costa Rica	11,631	17,044
Brazil	9,821	15,484
China	**8,827**	**16,807**
Lebanon	8,524	14,676
Algeria	4,123	15,275
India	1,937	7,056
Ethiopia	767	1,899
Mozambique	416	1,247

Note: PPP GDP/Nominal GDP = **USD 16,807**/ **USD 8,827** = 1.9040
⇒ "U.S. is 90% more expensive." ¶