## PURCHASING POWER PARITY

## The Behavior of FX Rates

- Fundamentals that affect FX Rates:
- Inflation rates differentials $\left(\mathrm{I}_{\mathrm{USD}}-\mathrm{I}_{\mathrm{FC}}\right)$
- Interest rate differentials $\left(i_{U S D}-i_{F C}\right)$
- Income growth rates $\left(y_{U S D}-y_{F C}\right)$
- Trade flows
- Other: trade barriers, expectations, taxes, etc.
- Goal 1: Explain $\mathrm{S}_{\mathrm{t}}$ with a theory, say T1. Then, $\mathrm{S}_{\mathrm{t}}{ }^{\mathrm{T} 1}=f($.

Different theories can produce different $f($.)'s.
Evaluation: How well a theory match the observed behavior of $S_{t}$, say the mean and/or variance.

- Goal 2: Eventually, produce a formula to forecast $\mathrm{S}_{\mathrm{t}+\mathrm{T}}=f\left(\mathrm{X}_{\mathrm{t}}\right) \Rightarrow \mathrm{E}\left[\mathrm{S}_{\mathrm{t}+\mathrm{T}}\right]$.
- We want to have a theory that can match the observed $\mathrm{S}_{\mathrm{t}}$. It is not realistic to expect a perfect match, so we ask the question: On average, is $\mathrm{S}_{\mathrm{t}} \approx \mathrm{S}_{\mathrm{t}}^{\mathrm{T1}}$ ? Or, alternatively, is $\mathrm{E}\left[\mathrm{S}_{\mathrm{t}}\right]=\mathrm{E}\left[\mathrm{S}_{\mathrm{t}}^{\mathrm{T1}}\right]$ ?

- Like many macroeconomic series, exchange rates have a trend -in statistics the trends in macroeconomic series are called stochastic trends. It is better to try to match changes, not levels.
- Let's plot changes of MXN/USD exchange rate. Now, the trend is gone.

- Our goal is to explain $\mathrm{e}_{\mathrm{f}, \mathrm{v}}$ the percentage change in $\mathrm{S}_{\mathrm{t}}$. Again, we will try to see if the model we are using, say T1, matches, on average, the observed behavior of $\mathrm{e}_{\mathrm{f}, \mathrm{t}}$. For example, is $\mathrm{E}\left[\mathrm{e}_{\mathrm{f}, \mathrm{t}}\right]=\mathrm{E}\left[\mathrm{e}_{\mathrm{f}, \mathrm{t}}^{\mathrm{T} 1}\right]$ ?
- We will use statistics to formally tests theories.
- Let's look at the distribution of $\mathrm{e}_{\mathrm{f}, \mathrm{t}}$ for the USD/MXN. -in this case, we look at monthly percentage changes from 1986-2011.

- The average ("usual") monthly percentage change is a $0.9 \%$ appreciation of the USD (annualized $-11.31 \%$ change). The SD is $4.61 \%$.
- These numbers are the ones to match with our theories for $\mathrm{S}_{\mathrm{t}}$. A good theory should predict an average annualized change close to $-11 \%$ for $\mathrm{e}_{\mathrm{f}, \mathrm{t}}$.
- Descriptive stats for $\mathrm{s}_{\mathrm{t}}$ for the JPY/USD and the USD/MXN.

|  | JPY/USD | USD/MXN |
| :--- | ---: | ---: |
| Mean | -0.0026 | -0.0090 |
| Standard Error | 0.0014 | 0.0026 |
| Median | -0.0004 | -0.0027 |
| Mode | 0 | 0 |
| Standard Deviation | 0.0318 | 0.0460 |
| Sample Variance | 0.0010 | 0.0021 |
| Kurtosis | 1.6088 | 18.0321 |
| Skewness | -0.2606 | -2.1185 |
| Range | 0.2566 | 0.5833 |
| Minimum | -0.1474 | -0.3333 |
| Maximum | 0.1092 | 0.2500 |
| Sum | -1.2831 | -2.7354 |
| Count | 491 | 305 |

- Developed currencies: less volatile, with smaller means/medians.


## Purchasing Power Parity (PPP)

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PPP is based on the law of one price (LOOP): Goods, once denominated in the same currency, should have the same price.

If they are not, then some form of arbitrage is possible.

Example: LOOP for Oil.
$\mathrm{P}_{\text {oil-USA }}=$ USD 80.
$\mathrm{P}_{\text {oil-SWIT }}=$ CHF 160.

$$
\Rightarrow \mathbf{S}_{\mathbf{t}}^{\text {LOOP }}=\mathrm{USD} 80 / \text { CHF } 160=0.50 \text { USD } / \mathrm{CHF} .
$$

If $S_{t}=0.75 \mathrm{USD} / \mathrm{CHF}$, then a barrel of oil in Switzerland is more expensive -once denominated in USD- than in the US:
$\mathrm{P}_{\text {oil-SwIT }}(\mathrm{USD})=\mathrm{CHF} 160 \times 0.75 \mathrm{USD} / \mathrm{CHF}=\mathrm{USD} 120>\mathrm{P}_{\text {oil-USA }}$

## Example (continuation):

Traders will buy oil in the US (and export it to Switzerland) and sell the US oil in Switzerland. Then, at the end, traders will sell CHF/buy USD. This movement of oil from the U.S. to Switzerland will affect prices: $\mathrm{P}_{\text {oil-USA }} \uparrow ; \mathrm{P}_{\text {oil-SWIT } \downarrow} \downarrow \& \mathrm{~S}_{\mathrm{t}} \downarrow=>\mathbf{S}_{\mathrm{t}}^{\text {LOOP }} \uparrow \quad\left(\mathrm{S}_{\mathrm{t}} \& \mathbf{S}_{\mathrm{t}}^{\text {LOOP }}\right.$ converge $) ~ \llbracket$

LOOP Notes:

- LOOP gives an equilibrium exchange rate.

Equilibrium will be reached when there is no trade in oil (because of pricing mistakes). That is, when
 the LOOP holds for oil.
$\diamond$ LOOP is telling what $S_{t}$ should be (in equilibrium). It is not telling what $S_{t}$ is in the market today.

- Using the LOOP we have generated a model for $\mathrm{S}_{\mathrm{t}}$. We'll call this model, when applied to many goods, the PPP model.

Problem: There are many traded goods in the economy.

Solution: Use baskets of goods.


PPP: The price of a basket of goods should be the same across countries, once denominated in the same currency. That is, USD 1 should buy the same amounts of goods here (in the U.S.) or in Colombia.

- A popular basket: The CPI basket. In the US, the basket typically reported is the CPI-U, which represents the spending patterns of all urban consumers and urban wage earners and clerical workers. ( $87 \%$ of the total U.S. population).
- U.S. basket weights:

- A potential problem with the CPI basket: The composition of the index (the weights and the composition of each category) may be very similar.
- For example, the weight of the food category changes substantially as the income level increases.


Absolute version of PPP: The FX rate between two currencies is simply the ratio of the two countries' general price levels:

$$
\mathrm{S}_{\mathrm{t}}^{\text {PPP }}=\text { Domestic Price level } / \text { Foreign Price level }=\mathrm{P}_{\mathrm{d}} / \mathrm{P}_{\mathrm{f}}
$$

Example: Law of one price for CPIs.
CPI-basket ${ }_{\text {USA }}=\mathrm{P}_{\text {USA }}=$ USD 755.3
CPI-basket $_{\text {swIT }}=\mathrm{P}_{\text {SWIT }}=$ CHF 1241.2

$$
\Rightarrow \mathrm{S}_{\mathrm{t}}^{\mathrm{PPP}}=\mathrm{USD} 755.3 / \mathrm{CHF} 1241.2=0.6085 \mathrm{USD} / \mathrm{CHF} .
$$

If $S_{t} \neq 0.6085 \mathrm{USD} / \mathrm{CHF}$, there will be trade of the goods in the basket between Switzerland and US.

$$
\begin{aligned}
& \text { Suppose } \mathrm{S}_{\mathrm{t}}=0.70 \mathrm{USD} / \mathrm{CHF}>\mathrm{S}_{\mathrm{t}}^{\text {PPP }} \\
& \text { Then, } \quad \begin{aligned}
\mathrm{P}_{\text {SWIT }}(\text { in USD }) & =\text { CHF } 1241.2 * 0.70 \mathrm{USD} / \mathrm{CHF} \\
& =\text { USD } 868.70>\mathrm{P}_{\mathrm{USA}}=\text { USD } 755.3
\end{aligned}
\end{aligned}
$$

## Example (continuation):

$$
\begin{aligned}
\qquad \begin{aligned}
\mathrm{P}_{\text {SWIT }}(\text { in USD }) & =\text { CHF } 1241.2 * 0.70 \text { USD } / \text { CHF } \\
& =\text { USD } 868.70>\mathrm{P}_{\text {USA }}=\text { USD } 755.3
\end{aligned} \\
\text { Potential profit: USD } 868.70-\text { USD } 755.3=\text { USD } 93.40
\end{aligned}
$$

Traders will do the following psendo-arbitrage strategy:

1) Borrow USD
2) Buy the CPI-basket in the US
3) Sell the CPI-basket, purchased in the US, in Switzerland.
4) Sell the CHF/Buy USD
5) Repay the USD loan, keep the profits. $\|$

Note: "Equilibrium forces" at work:

$$
\begin{aligned}
& \text { 2) } \left.P_{\mathrm{USA}} \uparrow \& 3\right) \mathrm{P}_{\mathrm{SWIT}} \downarrow \quad\left(\Rightarrow \mathrm{~S}_{\mathrm{t}}^{\text {PPP }} \uparrow\right) \\
& \text { 4) } \mathrm{S}_{\mathrm{t}} \downarrow \text {. }
\end{aligned}
$$

## - Real v. Nominal Exchange Rates

The absolute version of the PPP theory is expressed in terms of $S_{t}$, the nominal exchange rate.

We can modify the absolute version of the PPP relationship in terms of the real exchange rate, $\mathrm{R}_{\mathrm{t}}$. That is,

$$
R_{t}=S_{t} P_{f} / P_{d}
$$

$\mathrm{R}_{\mathrm{t}}$ allows us to compare prices, translated to DC:
If $\mathbf{R}_{\mathrm{t}}>1$, foreign prices (translated to DC ) are more expensive
If $\mathbf{R}_{t}=1$, prices are equal in both countries -i.e., PPP holds!
If $\mathbf{R}_{\mathrm{t}}<1$, foreign prices are cheaper

Economists associate $\mathbf{R}_{\mathrm{t}}>1$ with a more efficient domestic economy.

Example: Suppose a basket -the Big Mac- cost in Switzerland and in the U.S. is CHF 6.23 and USD 3.58, respectively.
$\mathrm{P}_{\mathrm{f}}=$ CHF 6.23
$\mathrm{P}_{\mathrm{d}}=$ USD 3.58
$\mathrm{S}_{\mathrm{t}}=1.012 \mathrm{USD} / \mathrm{CHF} \quad \Rightarrow \mathrm{P}_{\mathrm{f}}(\mathrm{in}$ USD $)=$ USD 6.3048
$R_{t}=S_{t} P_{\text {SWIT }} / P_{\text {US }}=1.012$ USD $/$ CHF $\times$ CHF $6.23 /$ USD $3.58=1.7611$.

Taking the Big Mac as our basket, the U.S. is more competitive than Switzerland. Swiss prices are $76.11 \%$ higher than U.S. prices, after taking into account the nominal exchange rate.
To bring the economy to equilibrium -no trade in Big Macs-, we expect the USD to appreciate against the CHF.

According to PPP, the USD is undervalued against the CHF. $\Rightarrow$ Trading signal: Buy USD/Sell CHF. $\mathbb{I}$

- The Big Mac ("Burgernomics," popularized by The Economist) has become a popular basket for PPP calculations. Why?

1) It is a standardized, common basket: beef, cheese, onion, lettuce, bread, pickles and special sauce. It is sold in over 120 countries.

2) It is very easy to find out the price.
3) It turns out, it is correlated with more complicated common baskets, like the PWT (Penn World Tables) based baskets.

Using the CPI basket may not work well for absolute PPP. The CPI baskets can be substantially different. In theory, traders can exploit the price differentials in BMs .

- In the previous example, Swiss traders can import US BMs.


From UH (US) to Rapperswill (CH)


- This is not realistic. But, the components of a BM are internationally traded. The LOP suggests that prices of the components should be the same in all markets.

The Economist reports the real exchange rate: $\mathbf{R}_{\mathrm{t}}=\mathrm{S}_{\mathrm{t}} \mathrm{P}_{\text {BigMac, } \mathrm{f}} / \mathrm{P}_{\text {BigMac, } \mathrm{d}}$.

For example, for Norway's crown (NOK): $\mathbf{R}_{\mathrm{t}}=7.02 / 3.58=1.9609$

$$
\Rightarrow(96.09 \% \text { overvaluation })
$$

Example: (The Economist's) Big Mac Index (January 2011)
$\mathbf{R}_{\mathrm{t}}=\mathrm{S}_{\mathrm{t}} \mathrm{P}_{\text {BigMac,f }} / \mathrm{P}_{\text {BigMac,d }} \quad$ (US $=$ domestic) $\quad=>\mathbf{R}_{\mathrm{t}}=1$ under Absolute PPP


Example: (The Economist's) Big Mac Index (January 2016) $=>\mathbf{R}_{\mathrm{t}}$ changes over time!


Example: Big Mac Index - $\mathbf{R}_{\mathrm{t}}$ Changes over time in 2000-2016.

CHF/USD \& BRL/USD : Big Mac R(t)



- Empirical Evidence: Simple informal test:

Test: If Absolute PPP holds $\Rightarrow \mathbf{R}_{\mathrm{t}}=1$.
In the Big Mac example, PPP does not hold for the majority of countries.
Several tests of the absolute version have been performed: Absolute version of PPP, in general, fails (especially, in the short run).

- Absolute PPP: Qualifications
(1) PPP emphasizes only trade and price levels. Political/social factors (instability, wars), financial problems (debt crisis), etc. are ignored.
(2) Implicit assumption: Absence of trade frictions (tariffs, quotas, transactions costs, taxes, etc.).
Q: Realistic? On average, transportation costs add $7 \%$ to the price of U.S. imports of meat and $16 \%$ to the import price of vegetables. Many products are heavily protected, even in the U.S. For example, peanut imports are subject to a tariff as high as $163.8 \%$. Also, in the U.S., tobacco usage and excise taxes add USD 5.85 per pack.
- Absolute PPP: Qualifications

Some everyday goods protected in the U.S.:

- European Roquefort Cheese, cured ham, mineral water (100\%)
- Paper Clips (as high as 126.94\%)
- Canned Tuna (as high as $35 \%$ )
- Synthetic fabrics (32\%)
- Sneakers ( $48 \%$ on certain sneakers)
- Japanese leather (40\%)
- Peanuts (shelled 131.8\%, and unshelled 163.8\%).
- Brooms (quotas and/or tariff of up to $32 \%$ )
- Chinese tires (35\%)
- Trucks (25\%) \& cars ( $2.5 \%$ )

Some Japanese protected goods:

- Rice (778\%)
- Beef ( $38.5 \%$, but can jump to $50 \%$ depending on volume).
- Sugar (328\%)
- Powdered Milk (218\%)
- Absolute PPP: Qualifications
(3) PPP is unlikely to hold if $\mathrm{P}_{\mathrm{f}}$ and $\mathrm{P}_{\mathrm{d}}$ represent different baskets. This is why the Big Mac is a popular choice.
(4) Trade takes time (contracts, information problems, etc.).
(5) Internationally non-traded (NT) goods -i.e. haircuts, home and car repairs, hotels, restaurants, medical services, real estate. The NT good sector is big: $50 \%-60 \%$ of GDP (big weight in CPI basket).

Then, in countries where NT goods are relatively high, the CPI basket will also be relatively expensive. Thus, PPP will find these countries' currencies overvalued relative to currencies in low NT cost countries.

Note: In the short-run, we will not take our cars to Mexico to be repaired, but in the long-run, resources (capital, labor) will move. We can think of the over-/under-valuation as an indicator of movement of resources.

- Absolute PPP: Qualifications

The NT sector also has an effect on the price of traded goods. For example, rent and utilities costs affect the price of a Big Mac. ( $25 \%$ of Big Mac due to NT goods.)

- Empirical Fact

Price levels in richer countries are consistently higher than in poorer ones. This fact is called the Penn effect. Many explanations, the most popular: The Balassa-Samuelson (BS) effect.

- Borders Matter

You may look at the Big Mac Index and think: "No big deal: there is also a big dispersion in prices within the U.S., within Texas, and, even, within Houston!" It is true that prices vary within the U.S.

For example, in 2015, the price of a Big Mac (and Big Mac Meal) in New York was USD 5.23 (USD 7.45), in Texas as USD 4.39 (USD 6.26) and in Mississippi was USD 3.91 (USD 5.69).

But, borders play a role, not just distance!

Engel and Rogers (1996) computed the variance of LOOP deviations for city pairs within the U.S., within Canada, and across the border: Distance between cities within a country matter, but the border effect is significant.

To explain the difference between prices across the border using the estimate distance effects within a country, they estimate the U.S.-Canada border should have a width of 75,000 miles!

This huge estimate has been revised downward in subsequent studies, but a large positive border effect remains.

## - Balassa-Samuelson effect

Labor costs affect all prices. We expect average prices to be cheaper in poor countries than in rich ones because labor costs are lower.

This is the so-called Balassa-Samuelson effect. Rich countries have higher productivity and, thus, higher wages in the traded-goods sector than poor countries do. But, firms compete for workers.

Then, wages in NT goods and services are also higher $\Rightarrow$ Overall prices are lower in poor countries.

- For example, in 2000, a typical McDonald's worker in the U.S. made USD 6.50/hour, while in China made USD 0.42 /hour.

- The Balassa-Samuelson effect implies a positive correlation between PPP exchange rates (overvaluation) and high productivity countries.
- Incorporating the Balassa-Samuelson effect into PPP:

1) Estimate a regression: Big Mac Prices against GDP per capita.

## Burger thy neighbour

Big Mac prices v GDP per person, July 2011


- Incorporating the Balassa-Samuelson effect into PPP:

2) Compute fitted Big Mac Prices (GDP-adjusted Big Mac Prices), along the regression (red) line. Use the difference between GDP-adjusted Big Mac Prices and actual prices (the white/blue dots) to estimate GDP-adjusted PPP over/under-valuation.


- Pricing-to-Market

Krugman (1987) offers an alternative explanation for the strong positive relationship between GDP and price levels: Pricing-to-market -i.e., price discrimination.

Based on price elasticities, producers discriminate: the same exact good is sold to rich countries (lower price elasticity) at higher prices than to poorer countries (higher price elasticity).

Alessandria and Kaboski (2008) report that U.S. exporters, on average, charge the richest country a $48 \%$ higher price than the poorest country.

Again, pricing-to-market struggles to explain why PPP does not hold among developed countries with similar incomes. For example, Baxter and Landry (2012) report that IKEA prices deviate $16 \%$ from the LOOP in Canada, but only $1 \%$ in the U.S.

## Relative PPP

The rate of change in the prices of products should be similar when measured in a common currency (as long as trade frictions are unchanged):
where,

$$
e_{f, T}^{P P P}=\frac{S_{t+T}-S_{t}}{S_{t}}=\frac{\left(1+I_{d}\right)}{\left(1+I_{f}\right)}-1 \quad \text { (Relative PPP) }
$$

$I_{f}=$ foreign inflation rate from $t$ to $t+T$;
$I_{d}=$ domestic inflation rate from $t$ to $t+T$.
Note: $\mathrm{e}_{\mathrm{f}, \mathrm{T}} \mathrm{PPP}$ is an expectation; what we expect to happen in equilibrium.

- Linear approximation: $\quad \mathrm{e}_{\mathrm{f}, \mathrm{T}}{ }^{\mathrm{PPP}} \approx\left(\mathrm{I}_{\mathrm{d}}-\mathrm{I}_{\mathrm{f}}\right) \quad \Rightarrow$ one-to-one relation

Example: From t=0 to $\mathrm{t}=1$, prices increase $10 \%$ in Mexico relative to prices in Switzerland. Then, $\mathrm{S}_{\mathrm{t}}$ should also increase $10 \%$; say, from $\mathrm{S}_{0}=9$ MXN/CHF to $\mathrm{S}_{1}=9.9 \mathrm{MXN} / \mathrm{CHF}$. Suppose $\mathrm{S}_{1}>9.9 \mathrm{MXN} / \mathrm{CHF}$, then according to Relative PPP the CHF is overvalued. $\mathbb{I}$

Example: Forecasting $S_{t}$ (USD/ZAR) using PPP (ZAR=South Africa).
It's 2013. You have the following information:
$\mathrm{CPI}_{\mathrm{US}, 2013}=104.5$,
$\mathrm{CPI}_{\mathrm{SA}, 2013}=100.0$,
$\mathrm{S}_{2011}=.2035$ USD/ZAR.
You are given the 2014 CPI's forecast for the U.S. and SA:
$\mathrm{E}\left[\mathrm{CPI}_{\mathrm{US}, 2014}\right]=110.8$
$\mathrm{E}\left[\mathrm{CPI}_{\mathrm{SA}, 2014}\right]=102.5$.

You want to forecast $\mathrm{S}_{2014}$ using the relative (linearized) version of PPP.
$\left.E[]_{\text {US-2014 }}\right]=(110.8 / 104.5)-1=.06029$
$\mathrm{E}\left[\mathrm{I}_{\mathrm{SA}-2014}\right]=(102.5 / 100)-1=.025$
$\mathrm{E}\left[\mathrm{S}_{2014}\right]=\mathrm{S}_{2013} \mathrm{x}\left(1+\mathrm{e}_{\mathrm{f}, \mathrm{T}}{ }^{\mathrm{PPP}}\right)=\mathrm{S}_{2013} \mathrm{x}\left(1+\mathrm{E}\left[\mathrm{I}_{\mathrm{US}}\right]-\mathrm{E}\left[\mathrm{I}_{\mathrm{SA}}\right]\right)$
$=.2035$ USD/ZAR $\times(1+.06029-.025)=.2107$ USD/ZAR.

Under the linear approximation, we have PPP Line


Look at point A: $\mathrm{e}_{\mathrm{f}, \mathrm{T}}>\mathrm{I}_{\mathrm{d}}-\mathrm{I}_{\mathrm{f}}$,
$\Rightarrow$ Priced in FC, the domestic basket is cheaper
$\Rightarrow$ pseudo-arbitrage against foreign basket $\quad \Rightarrow$ FC depreciates

## - Relative PPP: Implications

(1) Under relative PPP, $\mathbf{R}_{\mathrm{t}}$ remains constant (it can be different from 1!).
(2) Relative PPP does not imply that $S_{t}$ is easy to forecast.
(3) Without relative price changes, an MNC faces no real operating FX risk (as long as the firm avoids fixed contracts denominated in FC).

- Relative PPP: Absolute versus Relative
- Absolute PPP compares price levels.

Under Absolute PPP, prices are equalized across countries: " A mattress costs GBP 200 (= USD 320) in the U.K. and BRL 800 (=USD 320) in Brazil."،

- Relative PPP compares price changes.

Under Relative PPP, exchange rates change by the same amount as the inflation rate differential (original prices can be different): "U.K. inflation was $2 \%$ while Brazilian inflation was $8 \%$. Meanwhile, the BRL depreciated $6 \%$ against the GBP. Then, relative cost comparison remains the same."

- Relative PPP is a weaker condition than Absolute PPP: $\mathbf{R}_{\mathrm{t}}$ can be different from 1.


## - Relative PPP: Testing

Key: On average, what we expect to happen, $\mathrm{e}_{\mathrm{f}, \mathrm{T}} \mathrm{PPP}$, should happen, $\mathrm{e}_{\mathrm{f}, \mathrm{T}}$.
$\Rightarrow$ On average: $\mathrm{e}_{\mathrm{f}, \mathrm{T}} \approx \mathrm{e}_{\mathrm{f}, \mathrm{T}} \mathrm{PPP} \approx \mathrm{I}_{\mathrm{d}}-\mathrm{I}_{\mathrm{f}}$
or $\mathrm{E}\left[\mathrm{e}_{\mathrm{f}, \mathrm{T}}\right]=\mathrm{E}\left[\mathrm{e}_{\mathrm{f}, \mathrm{T}}{ }^{\mathrm{PPP}}\right] \approx \mathrm{E}\left[\mathrm{I}_{\mathrm{d}}-\mathrm{I}_{\mathrm{f}}\right]$

A linear regression is a good framework to test theories. Recall,

$$
\mathrm{e}_{\mathrm{f}, \mathrm{~T}}=\left(\mathrm{S}_{\mathrm{t}+\mathrm{T}}-\mathrm{S}_{\mathrm{t}}\right) / \mathrm{S}_{\mathrm{t}}=\alpha+\beta\left(\mathrm{I}_{\mathrm{d}}-\mathrm{I}_{\mathrm{f}}\right)_{\mathrm{t}+\mathrm{T}}+\varepsilon_{\mathrm{t}+\mathrm{T}},
$$

where $\varepsilon$ : regression error. That is, $\mathrm{E}\left[\varepsilon_{\mathrm{t}+\mathrm{T}}\right]=0$.

Then, $E\left[e_{f, T}\right]=\alpha+\beta E\left[\left(I_{d}-I_{f}\right)_{t+\mathrm{T}}\right]+E\left[\varepsilon_{\mathrm{t}+\mathrm{T}}\right]=\alpha+\beta E\left[e_{f, T}{ }^{\text {PPP }}\right]$

$$
\Rightarrow \mathrm{E}\left[\mathrm{e}_{\mathrm{f}, \mathrm{~T}}\right]=\alpha+\beta \mathrm{E}\left[\mathrm{e}_{\mathrm{f}, \mathrm{~T}}^{\mathrm{PPP}}\right]
$$

$\Rightarrow$ For Relative PPP to hold, on average, we need $\alpha=0 \& \beta=1$.

- Relative PPP: General Evidence

Under Relative PPP: $\mathrm{e}_{\mathrm{f}, \mathrm{T}} \approx \mathrm{I}_{\mathrm{d}}-\mathrm{I}_{\mathrm{f}}$

1. Visual Evidence

Plot $\left(\mathrm{I}_{\mathrm{JPY}}-\mathrm{I}_{\mathrm{USD}}\right)$ against $\mathrm{s}_{\mathrm{t}}(\mathrm{JPY} / \mathrm{USD})$, using monthly data 1971-2015.
Check to see if there is a $45^{\circ}$ line.


No $45^{\circ}$ line $\quad \Rightarrow$ Visual evidence rejects PPP.

- Relative PPP: General Evidence

1. Visual Evidence

Is $\mathrm{R}_{\mathrm{t}}$ close to 1 (actually, constant, under Relative PPP)?


In general, we have some evidence for mean reversion, though slow, for $\mathbf{R}_{\mathrm{t}}$.

## - Relative PPP: General Evidence (continuation)

In general, we have some evidence for mean reversion for $\mathbf{R}_{\mathrm{t}}$ in the long run. Loosely speaking, $\mathbf{R}_{\mathbf{t}}$ moves around some mean number, which we associate with the long-run PPP parity (for the JPY/USD the average $\mathbf{R}_{\mathrm{t}}$ is $0.93)$. But, the deviations from the long-run parity are very persistent-i.e., very slow to adjust.

Economists usually report the number of years that a PPP deviation is expected to decay by $50 \%$ (the balf-life) is in the range of 3 to 5 years for developed currencies. Very slow!

- Descriptive Stats

|  | $\mathbf{I}_{\mathbf{J P Y}}$ | $\mathbf{I}_{\text {USD }}$ | $\mathbf{I}_{\mathbf{J P Y}} \mathbf{I}_{\mathbf{U S D}}$ | $\mathbf{e}_{\mathbf{f , \mathbf { T }}}(\mathbf{\text { JPY/USD} )}$ |
| :--- | :---: | :---: | :---: | :---: |
| Mean | $\mathbf{0 . 0 0 2 1}$ | $\mathbf{0 . 0 0 3 3}$ | $-\mathbf{0 . 0 0 1 2}$ | $\mathbf{- 0 . 0 0 1 5}$ |
| SD | $\mathbf{0 . 0 0 6 3}$ | $\mathbf{0 . 0 0 3 8}$ | $\mathbf{0 . 0 0 6 1}$ | $\mathbf{0 . 0 3 1 6}$ |
| Min | -0.0107 | -0.0191 | -0.0192 | -0.1474 |
| Median | 0.0010 | 0.0030 | -0.0019 | -0.0001 |
| Max | 0.0431 | 0.0177 | 0.0346 | 0.1092 |

2. Statistical Evidence

More formal tests: Regression
$\mathrm{e}_{\mathrm{f}, \mathrm{T}}=\left(\mathrm{S}_{\mathrm{t}+\mathrm{T}}-\mathrm{S}_{\mathrm{t}}\right) / \mathrm{S}_{\mathrm{t}}=\alpha+\beta\left(\mathrm{I}_{\mathrm{d}}-\mathrm{I}_{\mathrm{f}}\right)_{\mathrm{t}+\mathrm{T}}+\varepsilon_{\mathrm{t}+\mathrm{T}}, \quad-\varepsilon:$ regression error, $\mathrm{E}\left[\varepsilon_{\mathrm{t}+\mathrm{T}}\right]=0$.

The null hypothesis is: $\quad \mathrm{H}_{0}$ (Relative PPP true): $\alpha=0$ and $\beta=1$ $\mathrm{H}_{1}$ (Relative PPP not true): $\alpha \neq 0$ and/or $\beta \neq 1$

- Tests: t -test (individual tests on $\alpha$ and $\beta$ ) and F -test (joint test)
(1) t-test $=\left[\right.$ Estimated coeff. - Value of coeff. under $\left.H_{0}\right] /$ S.E.(coeff.) $t-$ test $\sim \mathrm{t}_{\mathrm{v}}(\mathrm{v}=\mathrm{N}-\mathrm{K}=$ degrees of freedom)
(2) F-test $=\left\{\left[\operatorname{RSS}\left(\mathrm{H}_{0}\right)-\operatorname{RSS}\left(\mathrm{H}_{1}\right)\right] / \mathrm{J}\right\} /\left\{\operatorname{RSS}\left(\mathrm{H}_{1}\right) /(\mathrm{N}-\mathrm{K})\right\}$

F-test $\sim \mathrm{F}_{\mathrm{J}, \mathrm{N}-\mathrm{K}} \quad \mathrm{J}=\#$ of restrictions in $\mathrm{H}_{0}, \mathrm{~K}=\#$ parameters in model, $N=\#$ of observations, RSS= Residuals Sum of Squared).

- Rule: If $\mid t$-test $\left|>\left|t_{v, \alpha / 2}\right|\right.$, reject at the $\alpha$ level. If F-test $>\mathrm{F}_{\mathrm{J}, \mathrm{N}-\mathrm{K}, \alpha}$, reject at the $\alpha$ level. Usually, $\alpha=.05$ ( $5 \%$ )

Example: Using monthly Japanese and U.S. data (1/1971-9/2007), we fit the following regression:

$$
e_{f, t}(J P Y / U S D)=\left(S_{t}-S_{t-1}\right) / S_{t-1}=\alpha+\beta\left(I_{J A P}-I_{U S}\right)_{t}+\varepsilon_{t}
$$

$\mathrm{R}^{2}=0.00525$
Standard Error $(\sigma)=.0326$
F-stat $($ slopes $=0$-i.e., $\beta=0)=2.305399(p$-value $=0.130)$
Observations $(N)=439$

|  | Coefficient | Stand Err | t-Stat | P-value |
| :--- | :---: | :--- | :--- | :--- |
| Intercept $(\alpha)$ | 0.00246 | 0.001587 | -1.55214 | 0.121352 |
| $\left(\mathrm{I}_{\mathrm{JAP}}-\mathrm{I}_{\mathrm{US}}\right)(\boldsymbol{\beta})$ | -0.36421 | 0.239873 | -1.51835 | 0.129648 |

We will test the $\mathrm{H}_{0}$ (Relative PPP true): $\alpha=0$ and $\beta=1$
Two tests: (1) t-tests (individual tests)
(2) F-test (joint test)

Example: Using monthly Japanese and U.S. data (1/1971-9/2007), we fit the following regression:

$$
\mathrm{e}_{\mathrm{f}, \mathrm{t}}(\mathrm{JPY} / \mathrm{USD})=\left(\mathrm{S}_{\mathrm{t}}-\mathrm{S}_{\mathrm{t}-1}\right) / \mathrm{S}_{\mathrm{t}-1}=\alpha+\beta\left(\mathrm{I}_{\mathrm{JAP}}-\mathrm{I}_{\mathrm{US}}\right)_{\mathrm{t}}+\varepsilon_{\mathrm{t}}
$$

$\mathrm{R}^{2}=0.00525$
Standard Error $(\sigma)=.0326$
F-stat (slopes $=0$-i.e., $\beta=0$ ) $=2.305399(p-$ value $=0.130)$
F-test $\left(\mathrm{H}_{0}: \alpha=0\right.$ and $\left.\beta=1\right): 16.289$ (p-value: lower than 0.0001$)=>$ reject at $5 \%$ level $\left(\mathrm{F}_{2,467, .05}=3.015\right)$
Observations $=439$

|  | Coefficient | Stand Err | t -Stat | P-value |
| :--- | :--- | :--- | :--- | :--- |
| Intercept $(\alpha)$ | -0.00246 | 0.001587 | -1.55214 | 0.121352 |
| $\left(\mathrm{I}_{\mathrm{JAP}}-\mathrm{I}_{\mathrm{US}}\right)(\beta)$ | -0.36421 | 0.239873 | -1.51835 | 0.129648 |
|  |  |  |  |  |
| Test $\mathrm{H}_{0}$, using t-tests $\left(\mathrm{t}_{437.05}=1.96-\right.$ Note: when $\left.N-K>30, \mathrm{t}_{.05}=1.96\right)$ : |  |  |  |  |
| $\mathrm{t}_{\alpha=0}:(-0.00246-0) / 0.001587=-1.55214(\mathrm{p}$-value $=.12)=>$ cannot reject |  |  |  |  |
| $\mathrm{t}_{\beta=1}:(-0.36421-1) / 0.239873=-5.6872(\mathrm{p}$-value:.00001) $=>$ reject. |  |  |  |  |

## - PPP Evidence:

- Relative PPP tends to be rejected in the short-run (like in the example above). In the long-run, there is a debate about its validity. Researchers find that currencies with high inflation rate differentials tend to depreciate.
- PPP: $\mathrm{R}_{\underline{t}}$ and $\mathrm{S}_{\underline{t}}$

Research shows that $R_{t}$ is much more variable when $S_{t}$ is allowed to float. $R_{t}$ variability tends to be highly correlated with $S_{t}$ variability. This finding comes from Mussa (1986).


After 1973, when floating exchange rates were adopted, $\mathrm{R}_{\mathrm{t}}$ moves like $\mathrm{S}_{\mathrm{t}}$. As a check to the visual evidence: Volatility(changes in $\mathrm{R}_{\mathrm{t}}$ ) $=2.96$ \& Volatility $\left(\right.$ changes in $\left.S_{t}\right)=2.93$. Almost the same!

Implications: Price levels play a minor role in explaining the movements of $\mathrm{R}_{\mathrm{t}}$ (prices are sticky). Engel (1999) reports that prices seem to be sticky also for traded-goods.

Possible explanations:
(a) Contracts:

Prices cannot be continuously adjusted due to contracts. In a stable economy, contracts have longer durations. In high inflation countries (contracts with shorter duration) PPP deviations are not very persistent.
(b) Mark-up adjustments:

Manufacturers and retailers tend to moderate any increase in their prices in order to preserve market share. Changes in $S_{t}$ are only partially transmitted or pass-through to import/export prices.
Average ERPT (exchange rate pass-through) is around $50 \%$ over one quarter and $64 \%$ over the long run for OECD countries (for the U.S., $25 \%$ in the short-run and $40 \%$ over the long run).
(c) Repricing costs (menu costs)

It is expensive to adjust continuously prices; in a restaurant, the repricing cost is re-doing the menu. For example, Goldberg and Hallerstein (2007) estimate that the cost of repricing in the imported beer market is $0.4 \%$ of firm revenue for manufacturers and $0.1 \%$ of firm revenue for retailers.
(d) Aggregation

Q: Is price rigidity a result of aggregation -i.e., the use of price index? Empirical work using micro level data -say, same good (exact UPC!) in Canadian and U.S. grocery stores- show that on average product-level $\mathbf{R}_{\mathrm{t}}$ move closely with $\mathrm{S}_{\mathrm{t}}$. But, micro level prices show idiosyncratic movements, mainly unrelated to $\mathrm{S}_{\mathrm{t}}: 10 \%$ of the deviations from PPP are accounted by $\mathrm{S}_{\mathrm{t}}$.

## - PPP: Puzzle

The fact that no single model of exchange rate determination can accommodate both the high persistent of PPP deviations and the high correlation between $\mathrm{R}_{\mathrm{t}}$ and $\mathrm{S}_{\mathrm{t}}$ has been called the "PPP puzzle."

## - PPP: Summary of Empirical Evidence

$\diamond \mathbf{R}_{\mathrm{t}}$ and $\mathrm{S}_{\mathrm{t}}$ are highly correlated, domestic prices (even for traded-goods) tend to be sticky.
$\diamond$ In the short run, PPP is a very poor model to explain short-term
exchange rate movements.
$\diamond$ PPP deviation are very persistent. It takes a long time (years!) to disappear.
$\diamond$ In the long run, there is some evidence of mean reversion, though very slow, for $R_{t}$. That is, $S_{t}{ }^{\text {PPP }}$ has long-run information:

Currencies that consistently bave high inflation rate differentials -i.e., $\left(I_{d} I_{f}\right)$ positive- tend to depreciate.

- The long-run interpretation for PPP is the one that economist like and use. PPP is seen as a benchmark, a figure towards which the current exchange rate should move.
- Calculating $\mathrm{S}_{\mathrm{t}}^{\mathrm{PPP}}$ (Long-Run FX Rate)

Let's look at the MXN/USD case.
We want to calculate $S_{t}^{\text {PPP }}=P_{d, t} / P_{f, t}$ over time.
(1) Divide $S_{t}{ }^{\text {PPP }}$ by $S_{o}{ }^{\text {PPP }} \quad(t=0$ is our starting point).
(2) After some algebra,

$$
\mathrm{S}_{\mathrm{t}}^{\mathrm{PPP}}=\mathrm{S}_{\mathrm{o}}{ }^{\text {PPP }} \times\left[\mathrm{P}_{\mathrm{d}, \mathrm{t}} / \mathrm{P}_{\mathrm{d}, \mathrm{o}}\right] \times\left[\mathrm{P}_{\mathrm{f}, \mathrm{o}} / \mathrm{P}_{\mathrm{f}, \mathrm{t}}\right]
$$

By assuming $\mathrm{S}_{\mathrm{o}}{ }^{\text {PPP }}=\mathrm{S}_{\mathrm{o}}$, we plot $\mathrm{S}_{\mathrm{t}}{ }^{\text {PPP }}$ over time.

Note: $\mathrm{S}_{\mathrm{o}}{ }^{\text {PPP }}=\mathrm{S}_{\mathrm{o}}$ assumes that at $\mathrm{t}=0$, the economy was in equilibrium. This may not be true: Be careful when selecting a base year.)

Let's look at the MXN/USD case.


- In the short-run, $\mathrm{S}_{\mathrm{t}}{ }^{\text {PPP }}$ is missing the target, $\mathrm{S}_{\mathrm{t}}$.
- But, in the long-run, $\mathrm{S}_{\mathrm{t}}^{\text {PPP }}$ gets the trend right. (As predicted by PPP, the high Mexican inflation rates differentials against the U.S., depreciate the MXN against the USD.)

Another example, let's look at the JPY/USD case.


As predicted by PPP, since U.S. inflation rates have been consistently higher than the Japanese ones, in the long-run, the USD depreciates against the JPY.

- PPP Summary of Applications:
$\bullet$ Equilibrium ("long-run") exchange rates. A CB can use $\mathbf{S}_{\mathbf{t}}{ }^{\text {PPP }}$ to determine intervention bands.
- Explanation of $\mathrm{S}_{\mathrm{t}}$ movements ("currencies with high inflation rate differentials tend to depreciate").
$\diamond$ Indicator of competitiveness or under/over-valuation: $\mathbf{R}_{\mathrm{t}}>1 \Rightarrow \mathrm{FC}$ is overvalued (\& Foreign prices are not competitive).
$\diamond$ International GDP comparisons: Instead of using $S_{t}, S_{t}^{\text {PPP }}$ is used. For example, per capita GDP (World Bank figures, in 2012):

| Country | GDP per capita (in USD) - 2012 |  |
| :--- | :---: | :---: |
|  | Nominal | PPP |
| Luxembourg | 107,476 | 91,388 |
| USA | 49,965 | 49,965 |
| Japan | 46,720 | 35,178 |
| Venezuela | 12,767 | 13,485 |
| Brazil | 11,340 | 11,909 |
| Lebanon | 9,705 | 14,610 |
| China | 6,091 | 9,233 |
| India | 1,489 | 3,876 |
| Ethiopia | 410 | 1,139 |

Example: Nominal vs PPP - Calculations for China
Nominal GDP per capita: CNY 38,068.75;
$\mathrm{S}_{\mathrm{t}}=0.16 \mathrm{USD} / \mathrm{CNY} ;$
$\mathrm{S}_{\mathrm{t}}{ }^{\text {PPP }}=0.2425$ USD/CNY $\quad \Rightarrow$ "U.S. is $51.58 \%$ more expensive"

- Nominal GDP_capita (USD) = CNY 38,068.75 x 0.16 USD/CNY= USD 6,091
- PPP GDP_capita (USD) $=$ CNY 38,068.75 x $\mathbf{0 . 2 4 2 5}$ USD/CNY = USD 9,233. $\mathbb{1}$

