FX Derivatives

2. FX Options

Options: Brief Review

• Terminology

Major types of option contracts:

- *calls* gives the holder the right to buy the underlying asset
- *puts* gives the holder the right to sell the underlying asset.

The complete definition of an option must specify:

- *Exercise* or *strike price* (X): price at which the right is "exercised."
- *Expiration date* (T): date when the right expires.
- When the option can be exercised: anytime (*American*)
  at expiration (*European*).

The right to buy/sell an asset has a price: the *premium* (X), paid upfront.

• More terminology:

- An option is in-the-money (ITM) if, today, we would exercise it.
  - For a call: $X < S_t$ (better to buy at a cheaper price than S_t)
  - For a put: $S_t < X$ (better to sell at a higher price than S_t)

- An option is at-the-money (ATM) if, today, we would be indifferent to exercise it.
  - For a call: $X = S_t$ (same to buy at X or S_t)
  - For a put: $S_t = X$ (same to sell at X or S_t)

In practice, you never exercise an ATM option, since there are some small brokerage costs associated with exercising an option.

- An option is out-of-the-money (OTM) if, today, we would not exercise it.
  - For a call: $X > S_t$ (better to buy at a cheaper price than X)
  - For a put: $S_t > X$ (better to sell at a higher price than X)

• The Black-Scholes Formula

• Options are priced using variations of the Black-Scholes formula:

$$C = \text{call premium} = e^{-i_f T} S_t \, N(\text{d1}) - X e^{-i_d T} N(\text{d2})$$

• Fischer Black and Myron Scholes (1973) changed the financial world by introducing their Option Pricing Model. At the time, both were at the University of Chicago.

• The model, or formula, allows an investor to determine the fair value of a financial option. Almost all financial securities have some characteristics of financial options, the model can be widely applied.

- The Black-Scholes formula is derived from a set of assumptions:
  - Risk-neutrality
  - Perfect markets (no transactions costs, divisibility, etc.)
  - Log-normal distribution with constant moments
  - Constant risk-free rate
  - Continuous pricing
  - Costless to short assets
- According to the formula, FX premiums are affected by six factors:

Variable	Euro Call	Euro Put	Amer. Call	Amer. Put
S_t	+	-	+	-
X	-	+	-	+
T	?	?	+	+
σ	+	+	+	+
i_d	+	-	+	-
i_f	-	+	-	+

- The Black–Scholes does not fit the data. In general:
  - It overvalues deep OTM calls and undervalue deep ITM calls.
  - It misprices options that involve high-dividend stocks.

- The Black-Scholes formula is taken as a useful approximation.

- Limitations of the Black-Scholes Model
  - Log-normal distribution: Not realistic (& cause of next 2 limitations).
  - Underestimation of extreme moves: *left tail risk* (can be hedged)
  - Constant moments: *volatility risk* (can be hedged)
  - Trading is not cost-less: *liquidity risk* (difficult to hedge)
  - No continuous trading: *gap risk* (can be hedged)

Trading in FX Options

- **Markets for foreign currency options**

(1) Interbank (OTC) market centered in London, New York, and Tokyo.

OTC options are tailor-made as to amount, maturity, and exercise price.

(2) Exchange-based markets centered in Philadelphia (PHLX, now NASDAQ), NY (ISE, now Eurex) and Chicago (CME Group).

- PHLX options are on spot amounts of 10,000 units of FC (MXN 100K, SEK 100K, JPY 1M).
- PHLX maturities: 1, 3, 6, and 12 months.
- PHLX expiration dates: March, June, Sept, Dec, plus 2 spot months.
- Exercise price of an option at the PHLX or CME is stated as the price in USD cents of a unit of foreign currency.

OPTIONS

PHILADELPHIA EXCHANGE

	Calls		Puts	
	Vol.	Last	Vol.	Last
Euro				**135.54**
10,000 Euro-cents per unit.				
132 Feb	...	0.01	3	0.38
132 Mar	3	0.74	90	0.15
134 Feb	3	1.90	...	...
134 Mar	...	0.01	25	1.70
136 Mar	8	1.85	12	2.83
138 Feb	75	0.43	...	0.01
142 Mar	1	0.08	1	7.81
Swedish Krona				**15.37**
100,000 Swedish Krona -cents per unit.				

• **Note on the value of Options**
For the same maturity (T), we should have:

value of ITM options > value of ATM options > value of OTM options

ITM options are more expensive, the more in-the-money they are.

Example: Suppose S_t = 1.3554 USD/EUR. We have two ITM Dec puts
X_{put}= 1.36 USD/EUR
X_{put}= 1.42 USD/EUR.

premium (X=1.36) = USD 0.0170
premium (X=1.42) = USD 0.0781. ¶

Using FX Options

• Iris Oil Inc., a Houston-based energy company, will transfer CAD 300 million to its USD account in 90 days. To avoid FX risk, Iris Oil decides to use a USD/CAD option contract.

Data:
S_t = .8451 USD/CAD
Available Options for the following 90-day options

X	Calls	Puts
.82 USD/CAD	----	0.21
.84 USD/CAD	1.58	0.68
.88 USD/CAD	0.23	----

Iris Oil decides to use the .84 USD/CAD put ⇒ Cost of USD 2.04M.

• Iris Oil decides to use the .84 USD/CAD put ⇒ Cost of USD 2.04M.

At T = t+90, there will be two situations: Option is ITM (exercised) or OTM (not exercised):

	If S_{t+90} < .84 USD/CAD	If S_{t+90} > .84 USD/CAD
Option CF:	(.84 – S_{t+90}) CAD 300M	0
Plus	S_{t+90} CAD 300M	S_{t+90} CAD 300M
Total	USD 252M	S_{t+90} CAD 300M

Net CF in 90 days:

USD 252M - USD 2.04 = USD 249.96M for all S_{t+90} < .84 USD/CAD

S_{t+90} CAD 300M – USD 2.04M for all S_{t+90} > .84 USD/CAD

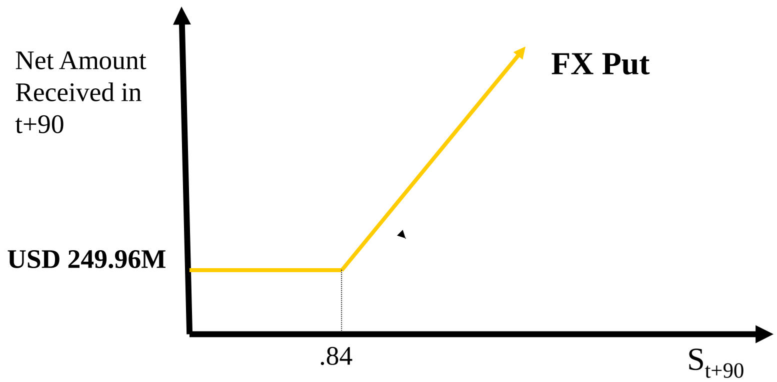
Worst case scenario (floor) : USD 249.96M (when put is exercised.)

Remark: The final CFs depend on S_{t+90}!

The payoff diagram shows that the FX option limits FX risk, Iris Oil has established a floor: USD 249.96M.

But, FX options, unlike Futures/forwards, have an upside:

⇒ At time t, the final outcome is unknown. There is still (some) uncertainty!



• With options, there is a choice of strike prices (premiums). A feature not available in forward/futures.

• Suppose, Iris Oil also considers the .82 put => Cost of USD .63M.

At T = t+90, there will be two situations: Option is ITM (exercised) or OTM (not exercised):

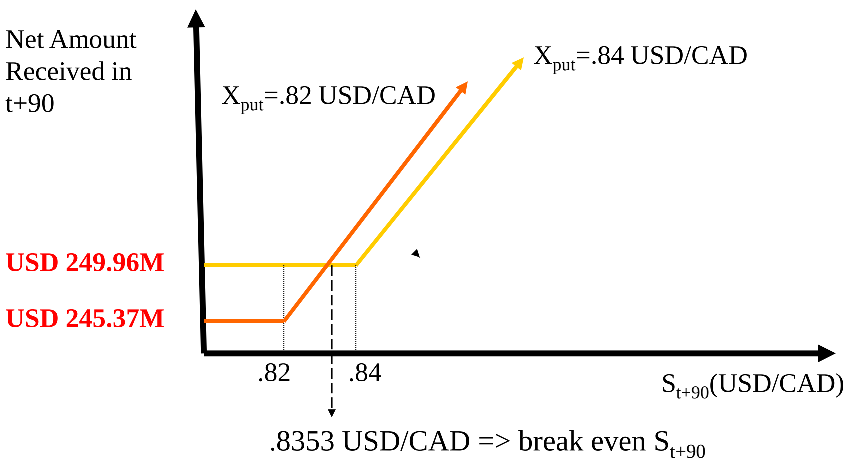
	If S_{t+90} < .82 USD/CAD	If S_{t+90} > .82 USD/CAD
Option CF:	(.82 – S_{t+90}) CAD 300M	0
Plus	S_{t+90} CAD 300M	S_{t+90} CAD 300M
Total	USD 246M	S_{t+90} CAD 300M

Net CF in 90 days:
USD 246M - USD .63 = USD 245.37M for all S_{t+90} < .82 USD/CAD
S_{t+90} CAD 300M – USD .63M for all S_{t+90} > .82 USD/CAD

Worst case scenario (floor) : USD 245.37M (when put is exercised).

• Both FX options limit Iris Oil FX risk:
- X_{put}=.84 USD/CAD ⇒ floor: USD 249.96M (cost: USD 2.04 M)
- X_{put}=.82 USD/CAD ⇒ floor: USD 245.37M (cost: USD .63M)

Note: Higher premium, higher floor (better coverage).



Hedging with FX Options

• Hedging with Options is Simple

Situation 1: Underlying position: long in foreign currency.
Hedging position: long in foreign currency ***puts***.

Situation 2: Underlying position: short in foreign currency.
Hedging position: long in foreign currency ***calls***.

OP = underlying position (UP) + hedging position (HP-options)
Value of OP = Value of UP + Value of HP + Transactions Costs (TC)

Profit from OP = ΔUP + Δ HP-options + TC

• Advantage of options over futures:
⇒ Options simply expire if S_t moves in a beneficial way.

• Price of the asymmetric advantage of options: The TC (insurance cost).

• We will present a simple example, where the size of the hedging position is equal to the hedging options (A Naive or Basic Approach)

Example: A U.S. investor is long GBP 1 million.
She hedges using Dec put options with **X = USD 1.60** (ATM).

Underlying position: V_0 = GBP 1,000,000.
$S_{t=0}$ = 1.60 USD/GBP.
Size of the PHLX contract: GBP 10,000.
X = USD 1.60
$P_{t=0}$ = premium of Dec put = USD .05.

TC = Cost of Dec puts = 1,000,000 x USD .05 = **USD 50,000**.
Number of contracts = GBP 1,000,000/ GBP 10,000 = 100 contracts.

On December S_t = 1.50 USD/GBP ⇒ option is exercised (put is ITM)

ΔUP = V_0 x (S_t – S_0) = GBP 1M (1.50 – **1.60**) USD/GBP = - USD 0.1M.
ΔHP = V_0 x (**X** – S_t) = GBP 1M x (**1.60** – 1.50) USD/GBP = USD 0.1M.
ΔOP = -USD 100,000 + USD 100,000 – **USD 50,000** = -USD 50,000. ¶

Example:
If at T, S_T = 1.80 USD/GBP => option is not exercised (put is OTM).

ΔUP = GBP 1M x (1.80 – **1.60**) USD/GBP = USD 0.2M
ΔHP = 0 (No exercise)
ΔOP = USD 200,000 - **USD 50,000** = USD 150,000. ¶

The price of this asymmetry is the premium: USD 50,000 (a sunk cost!).

FX Options: Hedging Strategies

• Hedging strategies with options can be more sophisticated:

⇒ Investors can play with several exercise prices with options only.

Example: Hedgers can use:

- Out-of-the-money (least expensive)
- At-the-money (expensive)
- In-the-money options (most expensive)

• Same *trade-off* of car insurance:

- Low premium (high deductible)/low floor or high cap: *Cheap*
- High premium (low deductible)/high floor or low cap: *Expensive*

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Example: It is February 2, 2011.
UP = Long bond position EUR 1,000,000.
HP = EUR Mar put options: **X = 134** and **X = 136**.
S_t = 1.3554 USD/EUR.

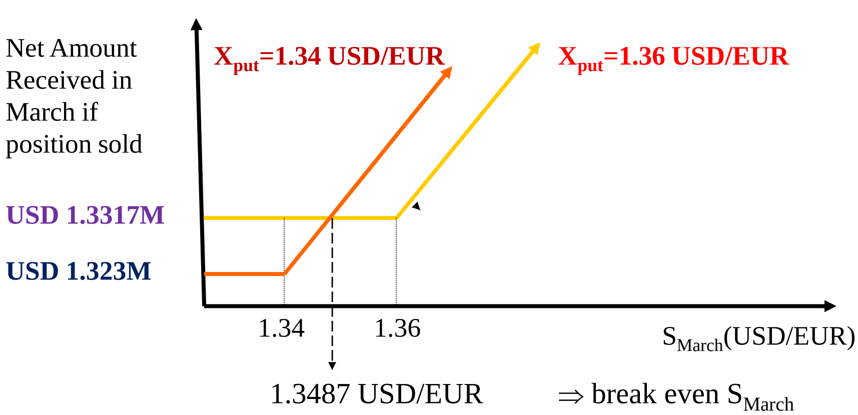
(A) Out-of-the-money Mar 134 put.
Total cost = USD .0170 x 1,000,000 = USD 17,000
Floor = **1.34 USD/EUR** x EUR 1,000,000 = USD 1,340,000.
Net Floor = USD 1.34M – USD .017M = **USD 1.323M**

(B) In-the-money Mar 136 put.
Total cost = USD .0283 x 1,000,000 = USD 28,300
Floor = **1.36 USD/EUR** x EUR 1,000,000 = USD 1,360,000
Net Floor = USD 1.36M – USD .0283M = USD 1.3317M

- As usual with options, under both instruments there is some uncertainty about the final cash flows. ¶

- Both FX options limit FX risk:
  - **X_{put}=1.34 USD/EUR** ⇒ floor: **USD 1.323M** (cost: USD .017 M)
  - **X_{put}=1.36 USD/EUR** ⇒ floor: **USD 1.3317M** (cost: USD .0283M)

Typical trade-off: A higher minimum (floor) amount for the UP (USD 1,060,000) is achieved by paying a higher premium (USD 28,300).



Exotic Options

Exotic options: options with two or more option features.

Example: a compound option (an option on an option).

Two popular exotic options: knock-outs and knock-ins.

• Barrier Options: Knock-outs/ Knock-ins

Barrier options: the payoff depends on whether S_t reaches a certain level during a certain period of time.

Knock-out: A standard option with an "insurance rider" in the form of a second, out-of-the-money strike price.

This "out-strike" is a stop-loss order: if the out-of-the-money X is crossed by S_t, the option contract ceases to exist.

Knock-ins: the option contract does not exist unless and until S_t crosses the out-of-the-money "in-strike" price.

Example: Knock-out FX options

Consider the following European option:

1.65 USD/GBP March GBP call knock-out **1.75** USD/GBP.

S_t = 1.60 USD/GBP.

If in March S_t= 1.70 USD/GBP, the option is exercised

⇒ writer profits: USD (1.65-1.70) + premium per GBP sold.

If in March $S_t \geq$ **1.75** USD/GBP, the option is cancelled

⇒ writer profits are the premium. ¶

Q: Why would anybody buy one of these exotic options?

A: They are cheaper.

Example (continuation): Knock-out put FX options.

UP = Long bond position EUR 1,000,000.

HP = Mar put options: X_{put}=1.34 USD/EUR with X_{KO} = 1.30 USD/EUR

$S_{t=March}$ (in USD/EUR)	Value long position
$S_{t=March} \geq 1.34$	EUR 1M x S_t .
$1.34 \geq S_{t=March} \geq 1.30$	EUR 1M x 1.34 USD/EUR.
$S_{t=March} < X_{KO} = 1.30$	EUR 1M x S_t .