Chapter 7
ARBITRAGE in FX MARKETS

Triangular & Covered (IRP)Arbitrage

Arbitrage in FX Markets

Arbitrage Definition

It is an activity that takes advantages of *pricing mistakes* in financial assets in one or more markets. It involves *no risk* and *no capital of your own*.

Elements:

- Pricing mistake
- No Capital
- No Risk

• There are 3 types of arbitrage

(1) Local (sets uniform rates across banks)

(2) Triangular (sets cross rates)

(3) Covered (sets forward rates)

Elements of Arbitrage:

- Pricing mistake
- No Capital
- No Risk

Remark: The definition presents the ideal view of (riskless) arbitrage. "Arbitrage," in the real world, involves some risk. We will call this arbitrage *pseudo arbitrage.*

• Big academic literature on the limitations of arbitrage in real world settings ("*limits of arbitrage*").

Local Arbitrage (One good, one market)

Example: Suppose two banks have the following bid-ask FX quotes:

	Bank A		Bank B	
USD/GBP	1.50	**1.51**	**1.53**	1.55

Sketch of Local Arbitrage strategy:

(1) Borrow USD 1.51

(2) Buy GBP 1 from **Bank A** at **USD 1.51**

(3) Sell GBP 1 to **Bank B** at **USD 1.53**

(4) Return USD 1.51 & make a **USD .02 profit** (1.31% per USD borrowed)

Note I: All steps should be done simultaneously. Otherwise, there is risk!

Note II: **Bank A** and **Bank B** will notice a book imbalance

Bank A: All activity at $\mathbf{S_{A,ask}}$ (buy GBP orders at 1.51)

Bank B: All activity at $\mathbf{S_{B,bid}}$ (sell GBP orders at 1.53).

⇒ Both banks will adjust the quotes. Say,

Bank A adjusts $S_{A,ask}$ = 1.54 USD/GBP ($S_{A,ask}$ ↑). ¶

Triangular Arbitrage (Two related goods, one market)
Triangular arbitrage is a process where two related goods set a third price.

• In FX Markets, triangular arbitrage sets FX *cross rates*.

• Cross rates do not involve the USD. Most currencies are quoted against the USD. Thus, cross-rates are calculated from USD quotations.

Example: A JPY/GBP quote is derived from

$S_{\text{JPY}/USD,t}$ (say, 100 JPY/USD)

$S_{USD/\text{GBP},t}$ (say, 1.60 USD/GBP)

• Cross-rates are calculated in a way that avoids triangular arbitrage. For example, using above quotes:

$$S_{\text{JPY/GBP},t} = S_{\text{JPY}/USD,t} * S_{USD/\text{GBP},t} \text{ (= 160 JPY/GBP)}$$

Example: Suppose Bank One gives the following quotes:

$S_{\text{JPY}/USD,t}$ = 100 JPY/USD

$S_{USD/\text{GBP},t}$ = 1.60 USD/GBP

$S_{\text{JPY/GBP},t}$ = 140 JPY/GBP

Taking the first two quotes ⇒ Implied (no-arbitrage) JPY/GBP:

$$S^{I}_{\text{JPY/GBP},t} = S_{\text{JPY}/USD,t} * S_{USD/\text{GBP},t} = \textbf{160 JPY/GBP} > S_{\text{JPY/GBP},t}$$

At S_t = 140 JPY/GBP, Bank One ***undervalues*** the GBP against the JPY (with respect to the first two quotes). ⟸ ***Pricing mistake!***

Sketch of Triangular Arbitrage (Key: Buy undervalued GBP with the overvalued JPY):

(1) Borrow JPY 140

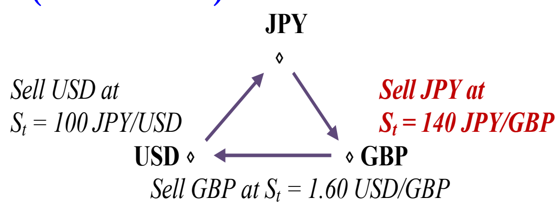
(2) Sell the JPY/Buy GBP at $S_{JPY/GBP,t}$ = 140 JPY/GBP. Get GBP 1.

(3) Sell GBP/Buy USD (at $S_{USD/\text{GBP},t}$ = 1.60 USD/GBP). Get USD 1.60.

(4) Sell USD/Buy JPY (at $S_{\text{JPY}/USD,t}$= 100 JPY/USD). Get JPY 160

⇒ Profit: Π = JPY 20 (= 20/140 =**14.29%** per JPY borrowed).

Example (continuation):



<u>Note</u>: Bank One will notice a book imbalance: All the activity involves **selling JPY for GBP**, **selling GPB for USD**, & **selling USD for JPY**.

Bank One will adjust quotes:

$S_{JPY/GBP,t}$ ↑ (say, $S_{JPY/GBP,t}$ = 145 JPY/GBP).
$S_{USD/GBP,t}$ ↓ (say, $S_{USD/GBP,t}$ = 1.56 USD/GBP) ⎫ ⇒ $S^{I}_{JPY/GBP,t}$ ↓
$S_{JPY/USD,t}$ ↓ (say, $S_{JPY/USD,t}$ = 93 JPY/USD). ⎭

There is convergence between $S^{I}_{JPY/GBP,t}$ & $S_{JPY/GBP,t}$:

$$S^{I}_{JPY/GBP,t}\downarrow (= S_{JPY/USD,t}\downarrow * S_{USD/GBP,t}\downarrow) \Leftrightarrow S_{JPY/GBP,t}\uparrow$$

<u>Remark</u>: Again, all steps should be done at the same time.

Example (continuation):

<u>Note</u>: It does not matter which currency you borrow in step (1). Recall the pricing mistake: Bank One *undervalues* the GBP against the JPY (with respect to the first two quotes):

$$S^{I}_{JPY/GBP,t} = \textbf{160 JPY/GBP} > S_{JPY/GBP,t} = 140 \text{ JPY/GBP}$$

Sketch of Triangular Arbitrage (<u>Key</u>: Buy undervalued GBP with the overvalued JPY). Simultaneously we do the following steps:

(1) Borrow USD 1
(2) Sell the USD/Buy JPY at $S_{JPY/USD,t}$ = 100 JPY/USD. Get JPY 100.
(3) Sell JPY/Buy GBP (at $S_{JPY/GBP,t}$ = 140 JPY/GBP).
    ⇒ Get GBP 0.7143
(4) Sell GPB/Buy USD (at $S_{USD/GBP,t}$ = 1.60 USD/GBP).
    ⇒ Get USD 1.1429

Profit: Π = USD 0.1429 (**14.29%** per USD borrowed).

Covered Interest Arbitrage (4 instruments: 2 goods per market & 2 markets)

From Bloomberg:
- Brazilian bonds yield **10%**
- Japanese bonds yield **1%**

Q: Why wouldn't capital flow to Brazil from Japan?
A: FX risk.

⇒ The only way to avoid FX risk is to be *covered* with a forward FX contract.

Intuition: Today, at $t = 0$, we have the following data:
i_{JPY} = **1%** for 1 year (T = 1 year)
i_{BRL} = **10%** for 1 year (T = 1 year)
$\boldsymbol{S_t}$ = **.025 BRL/JPY**

Carry Trade: A speculative strategy to take "advantage" of interest rate differentials.

Today (time $t = 0$), we do the following:
(1) Borrow JPY 1,000 at **1%** for 1 year. (At T = 1 year, repay JPY 1,010.)
(2) Convert to BRL at **.025 BRL/JPY**. Get BRL 25.
(3) Deposit BRL 25 at **10%** for 1 year. (At T = 1 year, receive BRL 27.50.)

At time T = 1 year, we do the final step:
(4) Exchange BRL 27.50 for JPY at $\boldsymbol{S_{T=1-yr}}$

⇒ Π = BRL 27.50 / $\boldsymbol{S_{T=1-yr}}$ – JPY 1010

Problem carry trade: It is risky ⇒ today ($t = 0$), $\boldsymbol{S_{T=1-yr}}$ is unknown

Profits are a function of an unknown price at time $t = 0$:

Π = BRL 27.50 / $\boldsymbol{S_{T=1-yr}}$ – JPY 1010

• Scenarios for $\boldsymbol{S_{T=1-yr}}$:

- $\boldsymbol{S_{T=1-yr}}$ = **0.02 BRL/JPY**. Then,

Π = BRL 27.50 / (**0.02 BRL/JPY**) – JPY 1010 = **JPY 365**

- $\boldsymbol{S_{T=1-yr}}$ = **0.03 BRL/JPY**. Then,

Π = BRL 27.50 / (**0.03 BRL/JPY**) – JPY 1010 = **JPY -93.33**

Note: The break-even $\boldsymbol{S_{T=1-yr}}$ is

$\boldsymbol{S^{BE}_{T=1-yr}}$ = BRL 27.50/JPY 1010 = 0.027227723 BRL/JPY.

That is, if $\boldsymbol{S_{T=1-yr}}$ < 0.027227723 BRL/JPY carry trade is profitable.

• We can cover ourselves and eliminate all uncertainty (& risk) with a forward contract.

• Carry trade with cover.

Suppose at $t = 0$, a bank offers $\mathbf{F_{t,1\text{-}year}}$ = **.026 BRL/JPY**.

Then, at time T = 1 year, we do the final step:

(4') Exchange BRL 27.50 for JPY at **.026 BRL/JPY**.

⇒ We get **JPY 1057.6923** (= BRL 27.50/**.026 BRL/JPY**).

⇒ Π = **JPY 1057.6923** – JPY 1010 = JPY 47.8

or 4.78% per JPY borrowed.

Now, instead of borrowing **JPY 1,000**, we will try to borrow **JPY 10 billion** (and make a **JPY 478M profit**) or more.

Obviously, no bank will offer a **.026 BRL/JPY** forward contract!

⇒ Banks will offer $F_{t,T=1-yr}$ contracts that produce $\Pi \leq 0$.

Interest Rate Parity Theorem

Q: How do banks price FX forward contracts?

A: In such a way that arbitrageurs cannot take advantage of their quotes.

To price a forward contract, banks consider covered arbitrage strategies.

Notation:

i_d = domestic nominal T days interest rate (annualized).

i_f = foreign nominal T days interest rate (annualized).

S_t = time t spot rate (direct quote, for example USD/GBP).

$F_{t,T}$ = forward rate for delivery at date T, at time t.

Note: In developed markets (like the US), *all* interest rates are quoted on annualized basis.

Now, consider the following (*covered*) strategy:

1. At $t = 0$, borrow from a foreign bank FC 1 for T days.
   ⇒ At time T, We pay the foreign bank FC: $(1 + i_f * T/360)$.

2. At $t = 0$, exchange FC 1 = DC S_t.

3. At $t = 0$, deposit DC S_t in a domestic bank for T days.
   ⇒ At time T, receive DC: $S_t * (1 + i_d * T/360)$.

4. At $t = 0$, buy a T-day forward contract to exchange DC for FC at a $F_{t,T}$.
   ⇒ At time T, exchange (in DC) $S_t\,(1 + i_d * T/360)$ for FC, using $F_{t,T}$.
   ⇒ We get FC: $S_t * (1 + i_d * T/360)/F_{t,T}$.

This strategy will **not be profitable** if, at time T, what we receive in FC is less or equal to what we have to pay in FC. That is, arbitrage will force:

$$S_t * (1 + i_d * \frac{T}{360})/F_{t,T} = (1 + i_f * \frac{T}{360}).$$

Solving for $F_{t,T}$, we get: $$F_{t,T} = S_t * \frac{\left(1 + i_d * \frac{T}{360}\right)}{\left(1 + i_f * \frac{T}{360}\right)}$$

$$F_{t,T} = S_t * \frac{\left(1 + i_d * \frac{T}{360}\right)}{\left(1 + i_f * \frac{T}{360}\right)}$$

This equation represents the *Interest Rate Parity Theorem* (***IRPT*** or just ***IRP***).

It is common to use the following linear IRPT approximation:

$$F_{t,T} \approx S_t * [1 + (i_d - i_f) * \frac{T}{360}].$$

This linear approximation is very accurate for small differences in $(i_d - i_f)$.

Example: Using IRPT.
S_t = 106 JPY/USD.
$i_{d=JPY}$ = .034.
$i_{f=USD}$ = .050.
T = 1 year
⇒ $F^{IRP}_{t,1-yr}$ = **106 JPY/USD** * (1+.034)/(1+.050) = 104.384 JPY/USD.
Using the linear approximation:
$F^{IRP}_{t,1-yr}$ ≈ **106 JPY/USD** * (1 – .016) = **104.304 JPY/USD**.

Example 1: Violation of IRPT at work.
S_t = 106 JPY/USD.
$i_{d=JPY}$ = .034.
$i_{f=USD}$ = .050.
$F^{IRP}_{t,1-yr}$ = **106 JPY/USD** * (1 – .016) = **104.304 JPY/USD**.

Suppose Bank A offers: $F^{A}_{t,1-yr}$ = **100 JPY/USD**.

$F^{A}_{t,1-yr}$ = **100 JPY/USD** < $F^{IRP}_{t,1-yr}$ (***pricing mistake!***)
⇒ Bank A *undervalues* the forward USD against the JPY.

We take advantage of Bank A's mistake: Buy USD/Sell JPY forward.

Sketch of a covered arbitrage strategy:
(1) Borrow USD 1 from a U.S. bank for one year at 5%.
(2) Exchange the USD for JPY at S_t = 106 JPY/USD.
(3) Deposit the JPY in a Japanese bank at 3.4%.
(4) **Cover.** Buy USD forward/Sell JPY forward at $F^{A}_{t,1-yr}$ = **100 JPY/USD**

Example 1 (continuation):

t =today | T = 1 year

Borrow 1 USD | **5%** | USD 1.05

Deposit JPY 106 | **3.4%** | JPY 109.6

Cash flows at time T = 1 year,

(i) We get: **JPY 106** * (1 + .034)/(**100 JPY/USD**) = **USD 1.096**

(ii) We pay: USD 1 * (1 + .05) = USD 1.05

Π = **USD 1.096** – USD 1.05 = **USD .046**

That is, after one year, the U.S. investor realizes a risk-free profit of **USD .046** per USD borrowed (4.6% per unit borrowed).

Note: Arbitrage will force Bank A's quote to quickly converge to

$F^{IRP}_{t,1-yr}$ = **104.304 JPY/USD**. ¶

Example 2: Violation of IRPT 2.

Now, suppose Bank X offers: $F^{X}_{t,1-yr}$ = **110 JPY/USD**.

$F^{X}_{t,1-yr}$ = 110 JPY/USD > $F^{IRP}_{t,1-yr}$ (***a pricing mistake!)***

⇒ The forward USD is *overvalued* against the JPY.

We take advantage of Bank X's overvaluation: Sell USD forward at $F^{X}_{t,1-yr}$.

Sketch of a covered arbitrage strategy:

(1) Borrow JPY 1 for one year at 3.4%.

(2) Exchange the JPY for USD at S_t = 106 JPY/USD (get USD 1/106)

(3) Deposit the USD at 5% for one year.

(4) **Cover.** Sell USD/Buy JPY forward at $F^{X}_{t,1-yr}$= **110 JPY/USD**.

Cash flows at T = 1 year:

(i) We get: USD 1/106 * (1 + .05) * (**110 JPY/USD**) = **JPY 1.0896**

(ii) We pay: JPY 1 * (1 + .034) = JPY 1.034

Π = **JPY 1.0896** – JPY 1.034 = **JPY .0556** (or 5.56% per JPY borrowed)

The Forward Premium and the IRPT

Reconsider the linearized IRPT. That is,

$$F_{t,T} \approx S_t * [1 + (i_d - i_f) * \frac{T}{360}].$$

A little algebra gives us:

$$\frac{F_{t,T} - S_t}{S_t} * \frac{360}{T} \approx (i_d - i_f)$$

Let T = 360. Then,

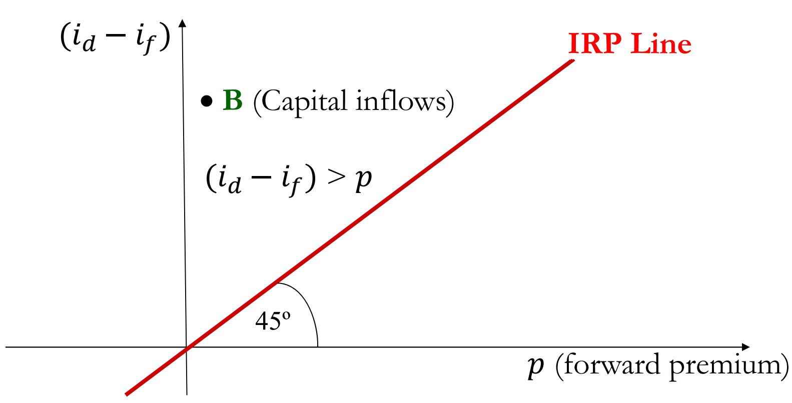
$$p \approx i_d - i_f$$

Note: p is the annualized % gain/loss of buying FC spot and selling it forward. Then,

- $p + i_f$: Annualized return from converting DC to FC & investing in FC (covered) for T days.
- i_d: Opportunity cost of borrowing DC (to buy FC at S_t).

Equilibrium: p exactly compensates $(i_d - i_f)$ → No arbitrage
→ No capital flows.

Equilibrium: $p \approx (i_d - i_f)$ ⇒ IRP Line

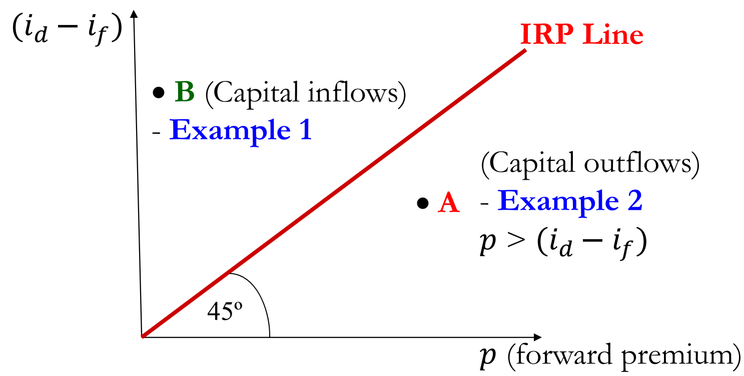


Example: Go back to **Example 1** (point **B**)

p = [$(F_{t,T} - S_t)/S_t$] * 360/T = [(100 − 106)/106] * 360/360 = - 0.0566

p = - 0.0566 < $(i_{d=JPY} - i_{f=USD})$ = - 0.016 ⇒ Arbitrage (pricing mistake!)
⇒ Capital flows to domestic country

Equilibrium: $p \approx (i_d - i_f)$ $\Rightarrow$ IRP Line



Consider point **A** (like in **Example 2**):

p = [($F_{t,T} - S_t$)/S_t] * 360/T = [(**110** − **106**)/**106**] * 360/360 = 0.0377

p = 0.0377 > $(i_{d=JPY} - i_{f=USD})$ = - 0.016 (or $p + i_f > i_d$),

$\Rightarrow$ Borrow at i_d & invest at i_f: Capital fly to the foreign country!

Intuition: What an investor pays to finance the foreign investment, i_d, is more than compensated by the high forward premium, p, plus i_f.

IRPT: Assumptions

Behind steps (1) to (4), we have implicitly assumed:

(1) *Funding is available*. Step (1) can be executed.

(2) *Free capital mobility*. Step (2) and, later, Step (4) can be implemented.

(3) *No default/country risk*. Steps (3) & (4) are safe.

(4) *No significant frictions*. Typical examples: transaction costs & taxes. Small transactions costs are OK, as long as they do not impede arbitrage.

We are also implicitly assuming that the forward contract for the desired maturity T is available. This may not be true.

In general, the forward market is liquid for short maturities (up to 1 year).

For many currencies, say from emerging market, the forward market may be liquid for much shorter maturities (up to 30 days).

IRP Application: Synthetic Forward Rates
A trader is not able to find a specific forward currency contract.
This trader can replicate $F_{t,T}$ using a spot currency contract combined with borrowing and lending.
This replication is done using the IRP equation.

Example: Replicating a USD/GBP 10-year forward contract.
$i_{\text{USD},10-\text{yr}}$ = 6%
$i_{\text{GBP},10-\text{yr}}$ = 8%
S_t **= 1.60 USD/GBP**
T = 10 years.

Ignoring transactions costs, she creates a 10-year (*implicit quote*) forward quote:
1) Borrow USD 1 at 6% for 10 years
2) Convert to GBP at 1.60 USD/GBP
3) Invest in GBP at 8% for 10 years

Transactions to create a 10-year (implicit) forward quote:
1) Borrow USD 1 at 6% for 10 years.
2) Convert to GBP at **1.60 USD/GBP** (GBP 0.625)
3) Invest in GBP at 8% for 10 years.

Cash flows in 10 years:
(1) Trader will receive **GBP 1.34933** (= GBP 0.625 * 1.08^{10})
(2) Trader will have to repay **USD 1.79085** (= 1.06^{10})

⇒ Implicit Exchange in 10 years: **GBP 1.34933** for **USD 1.79085**

We have created an implicit forward quote:
USD 1.79085/ **GBP 1.34933** = **1.3272 USD/GBP**. ¶

Or

$$F_{t,10-year}^{implicit} = S_t * [(1 + i_{d,10-yr})/(1 + i_{f,10-yr})]^{10}$$
$$= 1.60 \text{ USD/GBP} * [1.06/1.08]^{10} = 1.3272 \text{ USD/GBP}. ¶$$

Synthetic forward contracts are very useful for exotic currencies.

IRPT: Evidence

Starting from Frenkel and Levich (1975), there is a lot of evidence that supports IRPT.

Taylor (1989): Strong support for IRP using **10' intervals**.

Akram, Rice and Sarno (2008, 2009): Short-lived (from **30" up to 4'**) departures from IRP, with a profit range of 0.0002-0.0006 per unit.

Overall, we see a fairly efficient market, with data close to the IRPT line.

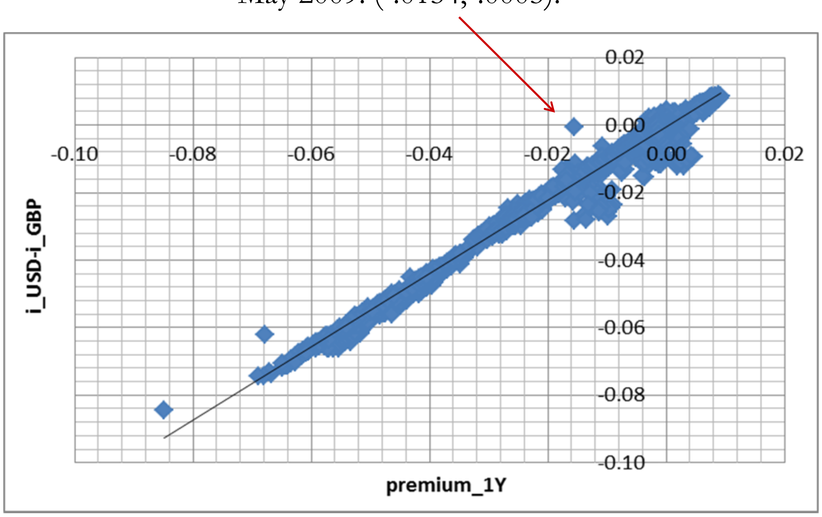
But, there are moments with significant deviations from the IRPT line. These situations reflect violations of IRPT's assumptions.

For example, during the 2008-2009 financial crisis. Probable cause: Funding constraints –Step (1) in trouble!

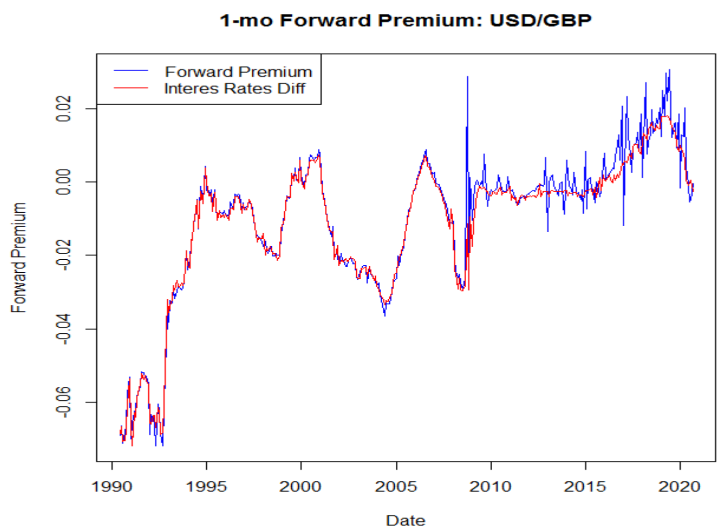
Evidence that **IRP since financial crisis is not holding** as expected.

IRPT: Evidence

May 2009: (-.0154,-.0005).



IRPT: Evidence



Almost perfect fit until 2008. Since 2008-2009, IRPT the fit is not that good. One explanation, the interest rates used are no longer "risk-free."