ARBITRAGE in FX Markets

Triangular & Covered (IRP)Arbitrage

Arbitrage in FX Markets

Arbitrage Definition

It is an activity that takes advantages of pricing mistakes in financial instruments in one or more markets. It involves *no risk* and *no capital of your own*.

- There are 3 kinds of arbitrage

  (1) Local (sets uniform rates across banks)

  (2) Triangular (sets cross rates)

  (3) Covered (sets forward rates)

Note: The definition presents the ideal view of (riskless) arbitrage. "Arbitrage," in the real world, involves some risk. We'll call this arbitrage *pseudo arbitrage*.

Local Arbitrage (One good, one market)

Example: Suppose two banks have the following bid-ask FX quotes:

	Bank A		Bank B	
USD/GBP	1.50	1.51	1.53	1.55

Sketch of Local Arbitrage strategy:
(1) Borrow USD 1.51
(2) Buy a GBP from Bank A (at ask price $S^A_{t,ask}$ = USD 1.51)
(3) Sell GBP to Bank B (at bid price $S^B_{t,bid}$ = USD 1.53)
(4) Return USD 1.51 and make a USD .02 profit (1.31%)

Note I: All steps should be done simultaneously. Otherwise, there is risk! (Prices might change).

Note II: Bank A and Bank B will notice a book imbalance. Bank A will see all activity at $S^A_{t,ask}$ (buy GBP orders) and Bank B will see all the activity at $S^B_{t,bid}$ (sell GBP orders). They will adjust the quotes. Say,

Bank A increases $S^A_{t,ask}$ and Bank B decreases $S^B_{t,bid}$ ($<S^A_{t,ask}$). ¶

Triangular Arbitrage (Two related goods, one market)

Triangular arbitrage is a process where two related goods set a third price.

• In the FX Markets, triangular arbitrage sets FX cross rates.

• Cross rates are exchange rates that do not involve the USD. Most currencies are quoted against the USD. Thus, cross-rates are calculated from USD quotations.

• The cross-rates are calculated in such a way that arbitrageurs cannot take advantage of the quoted prices. Otherwise, triangular arbitrage strategies would be possible.

Example: Suppose Bank One gives the following quotes:

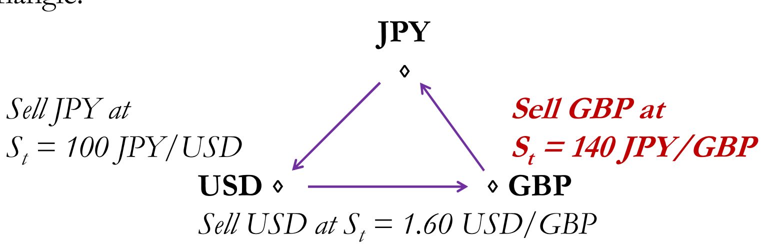
S_t = 100 JPY/USD
S_t = 1.60 USD/GBP
S_t = 140 JPY/GBP

Take the first two quotes. Then, the no-arbitrage JPY/GBP quote should be S^{NA}_t = 160 JPY/GBP

At S_t = 140 JPY/GBP, Bank One undervalues the GBP against the JPY (with respect to the first two quotes). <= This is the pricing mistake!

Sketch of Triangular Arbitrage (Key: Buy undervalued GBP with the overvalued JPY):
(1) Borrow USD 1
(2) **Sell USD for JPY 100 (at S_t = 100 JPY/USD)**. Get JPY 100.
(3) Sell JPY for GBP (at S_t = 140 JPY/GBP). Get GBP 0.7143
(4) Sell GPB for USD (at S_t = 1.60 USD/GBP). Get USD 1.1429
=> Profit: USD 0.1429 (14.29% per USD borrowed).

Example (continuation):
• The triangle:



<u>Note</u>: Bank One will notice a book imbalance: all the activity involves selling USD for JPY, selling JPY for GBP, etc. Bank One will adjust the quotes. For example, Bank One sets: S_t = 160 JPY/GBP. ¶

• Again, all the steps in the triangular arbitrage strategy should be done at the same time. (No risk!)
• It does not matter which currency you borrow (USD, GBP, JPY) in step (1). As long as the strategy involves the step **Sell JPY/Buy GBP** (following the arrows in the triangle above!), you should get the same profit as a %.

Covered Interest Arbitrage (4 instruments: 2 goods per market and 2 markets)

Open the third section of the WSJ: Brazilian bonds yield 10% and Japanese bonds 1%.

Q: Why wouldn't capital flow to Brazil from Japan?
A: FX risk: Once JPY are exchanged for BRL (Brazilian reals), there is no guarantee that the BRL will not depreciate against the JPY.

$\Rightarrow$ The only way to avoid this FX risk is to be covered with a forward FX contract.

Intuition: Let's suppose today, at t=0, we have the following data:
i_{JPY} = 1% for 1 year (T=1 year)
i_{BRL} = 10% for 1 year (T=1 year)
S_t = .025 BRL/JPY

Strategy to take "advantage" of the interest rate differential: *Carry Trade*
Today, at time t=0, we do the following:

(1) Borrow JPY 1000 at 1% for 1 year.
(At T=1 year, we will need to repay JPY 1010.)
(2) Convert to BRL at .025 BRL/JPY. Get BRL 25.
(3) Deposit BRL 25 at 10% for 1 year.
(At T=1 year, we will receive BRL 27.50.)

At time T=1 year, we do the final step:

(4) Exchange BRL 27.50 for JPY at $S_{T=1\text{-year}}$
$\Rightarrow$ Profit = BRL 27.50 * $S_{T=1\text{-year}}$ – JPY 1010

Problem with Carry Trades: Risk $\Rightarrow$ Today, we do not know $S_{T=1\text{-year}}$

Suppose at t=0, Bank Z offers $F^{Z}_{t,1\text{-year}}$ = .026 BRL/JPY.

Then, at time T=1 year, we do the final step:

(4) Exchange BRL 27.50 for JPY at 026 BRL/JPY.

Cash flows at time T=1 year:

(i) We get JPY 1057.6923

(ii) We pay JPY 1010

$\Rightarrow$ Profit = JPY 1057.6923 – JPY 1010 = JPY 47.8

Now, instead of borrowing JPY 1000, we will try to borrow JPY 10 billion (and make a JPY 480M profit) or more.

Obviously, no bank will offer a .026 JPY/BRL forward contract!

$\Rightarrow$ Banks will offer $F_{t,1\text{-year}}$ contracts that produce non-positive profits for arbitrageurs.

Interest Rate Parity Theorem

Q: How do banks price FX forward contracts?

A: In such a way that arbitrageurs cannot take advantage of their quotes.

To price a forward contract, banks consider covered arbitrage strategies.

Notation:

i_d = domestic nominal T days interest rate (annualized).

i_f = foreign nominal T days interest rate (annualized).

S_t = time t spot rate (direct quote, for example USD/GBP).

$F_{t,T}$ = forward rate for delivery at date T, at time t.

Note: In developed markets (like the US), *all* interest rates are quoted on annualized basis.

Now, consider the following (*covered*) strategy:

1. At t=0, borrow from a foreign bank 1 unit of a FC for T days.
   ⇒ At time T, We pay the foreign bank $(1+i_f \text{ x } T/360)$ units of the FC.

2. At t=0, exchange FC 1 = DC S_t.

3. Deposit DC S_t in a domestic bank for T days.
   ⇒ At time T, we receive DC $S_t(1+i_d \text{x} T/360)$.

4. At t=0, buy a T-day forward contract to exchange DC for FC at a $F_{t,T}$.
   ⇒ At time T, we exchange the DC $S_t(1+i_d \text{x} T/360)$ for FC, using $F_{t,T}$.
   ⇒ We get $S_t(1+i_d \text{ x } T/360)/F_{t,T}$ units of foreign currency.

This strategy will not be profitable if, at time T, what we receive in FC is less or equal to what we have to pay in FC. That is, arbitrage will force: :

$$S_t\,(1 + i_d \text{ x } T/360)/F_{t,T} = (1 + i_f \text{ x } T/360).$$

Solving for $F_{t,T}$, we get: $F_{t,T} = \dfrac{S_t\,(1 + i_d \text{ x } T/360)}{(1 + i_f \text{ x } T/360)}$

$$F_{t,T} = S_t \frac{(1+i_d \text{ x } T/360)}{(1+i_f \text{ x } T/360)}$$

This equation represents the *Interest Rate Parity Theorem* (*IRPT* or just *IRP*).

It is common to use the following linear IRPT approximation:

$$F_{t,T} \approx S_t\,[1 + (i_d - i_f) \text{ x } T/360].$$

This linear approximation is quite accurate for small differences in $i_d - i_f$.

Notes:

◊ Steps **(1)** and **(4)** simultaneously done produce a FX swap transaction! In this case, we buy the FC forward at $F_{t,T}$ and go sell the FC at S_t. We can think of $(F_{t,T} - S_t)$ as a profit from the FX swap.

◊ We get the same IRPT equation if we start the covered strategy by **(1)** borrowing DC at i_d; **(2)** exchanging DC for FC at S_t; **(3)** depositing the FC at i_f; and **(4)** selling the FC forward at $F_{t,T}$.

Example: Using IRPT.
S_t = 106 JPY/USD.
$i_{d=JPY}$ = .034.
$i_{f=USD}$ = .050.
T = 1 year

$\Rightarrow F_{t,1\text{-year}}$ = 106 JPY/USD x (1+.034)/(1+.050) = 104.384 JPY/USD.

Using the linear approximation:

$F_{t,1\text{-year}} \approx$ 106 JPY/USD x (1 – .016) = 104.304 JPY/USD.

Note: If Bank A sets $F^A_{t,1\text{-year}}$ = 104.38 JPY/USD arbitrageurs cannot profit from Bank A's quotes.¶

Arbitrageurs can profit from any violation of IRPT. Bank A can make two pricing mistakes:

$F^A_{t,1\text{-year}} < F_{t,1\text{-year-IRP}}$ – i.e., the forward FC is undervalued.
$F^A_{t,1\text{-year}} > F_{t,1\text{-year-IRP}}$ – i.e., the forward FC is overvalued.

Example 1: Violation of IRPT at work (forward FC undervalued).
S_t = 106 JPY/USD.
$i_{d=JPY}$ = .034.
$i_{f=USD}$ = .050.
$F_{t,\text{one-year-IRP}}$ = 106 JPY/USD x (1 - .016) = 104.304 JPY/USD.

Suppose Bank A offers: $F^A_{t,1\text{-year}}$ = 100 JPY/USD.

$F^A_{t,1\text{-year}}$ = 100 JPY/USD < $F_{t,1\text{-year-IRP}}$ (a pricing mistake!)

$\Rightarrow$ The forward USD is undervalued against the JPY.

Let's take advantage of Bank A's mistake: Buy USD forward.

Sketch of a covered arbitrage strategy:
(1) Borrow USD 1 from a U.S. bank for one year at 5%.
(2) Exchange the USD for JPY at S_t = 106 JPY/USD
(3) Deposit the JPY in a Japanese bank at 3.4%.
(4) Cover. Buy USD forward (Sell forward JPY) at $F^A_{t,1\text{-yr}}$=100 JPY/USD

Example 1 (continuation):

t=today		T = 1 year
Borrow 1 USD	**5%**	USD 1.05
Deposit JPY 106	**3.4%**	JPY 109.6

Cash flows at time T=1 year,
(i) We get: JPY 106 x (1+.034)/(100 JPY/USD) = USD 1.096
(ii) We pay: USD 1 x (1+.05) = USD 1.05
Profit = Π = USD 1.096 – USD 1.05 = USD .046

That is, after one year, the U.S. investor realizes a risk-free profit of USD. 046 per USD borrowed (4.6% per unit borrowed).

Note: Arbitrage will force Bank A's quote to quickly converge to $F_{t,1\text{-yr-IRP}}$ = 104.3 JPY/USD. ¶

Example 2: Violation of IRPT 2 (forward FC overvalued).
Now, suppose Bank X offers: $F^X_{t,1\text{-year}}$ = 110 JPY/USD.

$F^X_{t,1\text{-year}}$ = 110 JPY/USD > $F_{t,1\text{-year-IRP}}$ (a pricing mistake!)
⇒ The forward USD is overvalued against the JPY.

Let's take advantage of Bank X's overvaluation: Sell USD forward.

Sketch of a covered arbitrage strategy:
(1) Borrow JPY 1 from for one year at 3.4%.
(2) Exchange the JPY for USD at S_t = 106 JPY/USD
(3) Deposit the USD at 5% for one year.
(4) Cover. Sell USD forward (Buy forward JPY) at $F^X_{t,1\text{-yr}}$=110 JPY/USD.

Cash flows at T=1 year:
(i) We get: USD 1/106 x (1+.05) x (110 JPY/USD) = JPY 1.0896
(ii) We pay: JPY 1 x (1+.034) = JPY 1.034
Π = JPY1.0896 – JPY 1.034 = JPY .0556 (or 5.56% per JPY borrowed)

The Forward Premium and the IRPT

Reconstder the linearized IRPT. That is,

$$F_{t,T} \approx S_t\,[1 + (i_d - i_f) \times T/360].$$

A little algebra gives us:

$$(F_{t,T} - S_t)/S_t \times 360/T \approx (i_d - i_f)$$

Let T=360. Then,

$$p \approx i_d - i_f.$$

<u>Note</u>: p measures the annualized % gain/loss of buying FC spot and selling it forward. The opportunity cost of doing this is given by i_d-i_f.

Equilibrium: p exactly compensates (i_d - i_f) → No arbitrage
→ No capital flows.

Example: Go back to **Example 1**

$p = [(F_{t,T} - S_t)/S_t] \times 360/T = [(100 - 106)/106] \times 360/360 = -0.0566$

$p = -0.0566 < (i_d - i_f) = -0.016$ ⇒ Arbitrage (pricing mistake!)
⇒ Capital inflows to Japan

IRPT: Assumptions

Behind steps (1) to (4), we have implicitly assumed:

(1) Funding is available. Step (1) can be executed.

(2) Free capital mobility. Step (2) and later (4) can be implemented.

(3) No default/country risk. Step (3) and (4) are safe.

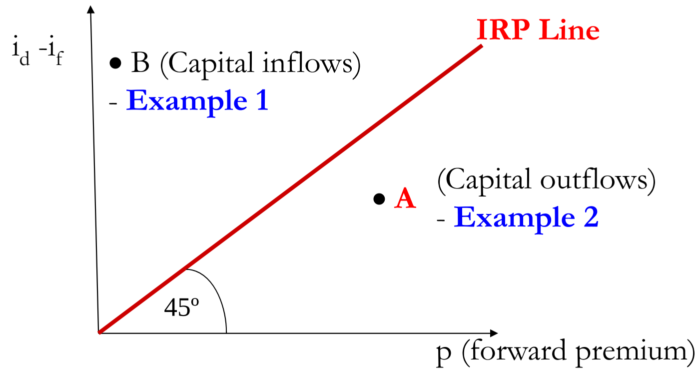
(4) Absence of significant frictions. Typical examples: transaction costs & taxes. Small transactions costs are OK, as long as they do not impede arbitrage.

We are also implicitly assuming that the forward contract for the desired maturity T is available. This may not be true.

In general, the forward market is liquid for short maturities (up to 1 year).

For many currencies, say from emerging market, the forward market may be liquid for much shorter maturities (up to 30 days).

Under the linear approximation, we have the IRP Line



Consider point **A** (like in **Example 2**): $p > i_d - i_f$ (or $p + i_f > i_d$),

$\Rightarrow$ Borrow at i_d & invest at i_f: Capital fly to the foreign country!

Intuition: What an investor pays to finance the foreign investment, i_d, is more than compensated by the high forward premium, p, plus i_f .

IRPT with Bid-Ask Spreads

Exchange rates and interest rates are quoted with bid-ask spreads.

Consider a trader in the interbank market:

She will have to buy or borrow at the other party's ask price.

She will sell or lend at the bid price.

There are two roads to take for arbitrageurs:

(1) borrow domestic currency

(2) borrow foreign currency.

• Bid's Bound: Borrow Domestic Currency

(1) A trader borrows DC 1 at time t=0, and repays $1+i_{ask,d}$ at time=T.

(2) Using the borrowed DC 1, she can buy spot FC at $(1/S_{ask,t})$.

(3) She deposits the FC at the foreign interest rate, $i_{bid,f}$.

(4) She sells the FC forward for T days at $F_{bid,t,T}$

This strategy would yield, in terms of DC:

$$(1/S_{ask,t})\ (1+i_{bid,f})\ F_{bid,t,T}.$$

In equilibrium, this strategy should yield no profit. That is,

$$(1/S_{ask,t})\ (1+i_{bid,f})\ F_{bid,t,T} \leq (1+i_{ask,d}).$$

Solving for $F_{bid,t,T}$,

$$F_{bid,t,T} \leq S_{ask,t}\ [(1+i_{ask,d})/(1+i_{bid,f})] = U_{bid}.$$

• Ask's Bound: Borrow Foreign Currency

(1) The trader borrows FC 1 at time t=0, and repay $1+i_{ask,f}$.

(2) Using the borrowed FC 1, she can buy spot DC at $S_{ask,t}$.

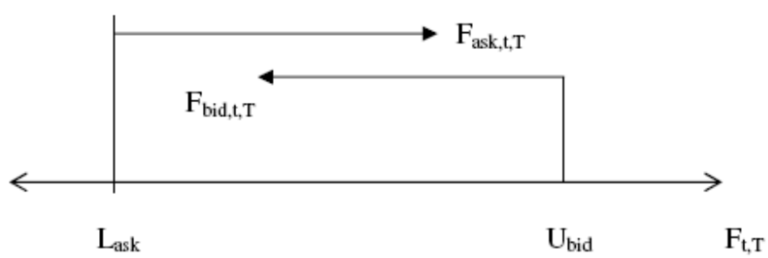
(3) She deposits the DC at the domestic interest rate, $i_{bid,d}$.

(4) She buys the FC forward for T days at $F_{ask,t,T}$

Following a similar procedure as the one detailed above, we get:

$$F_{ask,t,T} \geq S_{bid,t}\ [(1+i_{bid,d})/(1+i_{ask,f})] = L_{ask}.$$

Graph 7.2: Trading bounds for the Forward bid and the Forward ask.



Example: IRPT bounds at work.

Data: S_t = 1.6540-1.6620 USD/GBP

i_{USD} = 7¼-½,

i_{GBP} = 8 1/8–3/8,

$F_{t,one\text{-}year}$= 1.6400-1.6450 USD/GBP.

Check if there is an arbitrage opportunity (we need to check the bid's bound and ask's bound).

i) Bid's bound covered arbitrage strategy:

1) Borrow USD 1 at 7.50% for 1 year

$\Rightarrow$ Repay USD 1.07500 in 1 year.

2) Convert to GBP & get GBP 1/1.6620 = GBP 0.6017

3) Deposit GBP 0.6017 at 8.125%

4) Sell GBP forward at 1.64 USD/GBP

$\Rightarrow$ we get (1/1.6620) x (1 + .08125)x1.64 = USD 1.06694

$\Rightarrow$ No arbitrage: For each USD borrowed, we lose USD .00806.

Example (continuation):

ii) Ask's bound covered arbitrage strategy:

1) Borrow GBP 1 at 8.375% for 1 year => we will repay GBP 1.08375.

2) Convert to USD & get USD 1.6540

3) Deposit USD 1.6540 at 7.250%

4) Buy GBP forward at 1.645 USD/GBP

$\Rightarrow$ we get 1.6540x(1 + .07250)x(1/1.6450) = GBP 1.07837

$\Rightarrow$ No arbitrage: For each GBP borrowed, we lose GBP 0.0054.

Note: The bid-ask forward quote is consistent with no arbitrage. That is, the forward quote is within the IRPT bounds. Check:

$U_{bid} = S_{ask,t}[(1+i_{ask,d})/(1+i_{bid,f})]$ = 1.6620x[1.0750/1.08125]
= 1.6524 USD/GBP $\geq F_{bid,t,T}$ = 1.6400 USD/GBP.

$L_{ask} = S_{bid,t}[(1+i_{bid,d})/(1+i_{ask,f})]$ = 1.6540x[1.0725/1.08375]
= 1.6368 USD/GBP $\leq F_{ask,t,T}$ = 1.6450 USD/GBP. ¶

Synthetic Forward Rates

A trader is not able to find a specific forward currency contract.
This trader might be able to replicate the forward contract using a spot currency contract combined with borrowing and lending.
This replication is done using the IRP equation.

Example: Replicating a USD/GBP 10-year forward contract.

$i_{USD,10\text{-}yr} = 6\%$
$i_{GBP,10\text{-}yr} = 8\%$
$S_t = 1.60$ USD/GBP
T = 10 years.

Ignoring transactions costs, she creates a 10-year (implicit quote) forward quote:

1) Borrow USD 1 at 6% for 10 years
2) Convert to GBP at 1.60 USD/GBP
3) Invest in GBP at 8% for 10 years

Transactions to create a 10-year (implicit) forward quote:

1) Borrow USD 1 at 6%
2) Convert to GBP at 1.60 USD/GBP (GBP 0.625)
3) Invest in GBP at 8%

Cash flows in 10 years:
(1) Trader will receive GBP 1.34933 ($=1.08^{10}/1.60$)
(2) Trader will have to repay USD 1.79085 ($= 1.06^{10}$)

We have created an implicit forward quote:
USD 1.79085/ GBP 1.34933 = 1.3272 USD/GBP. ¶
Or

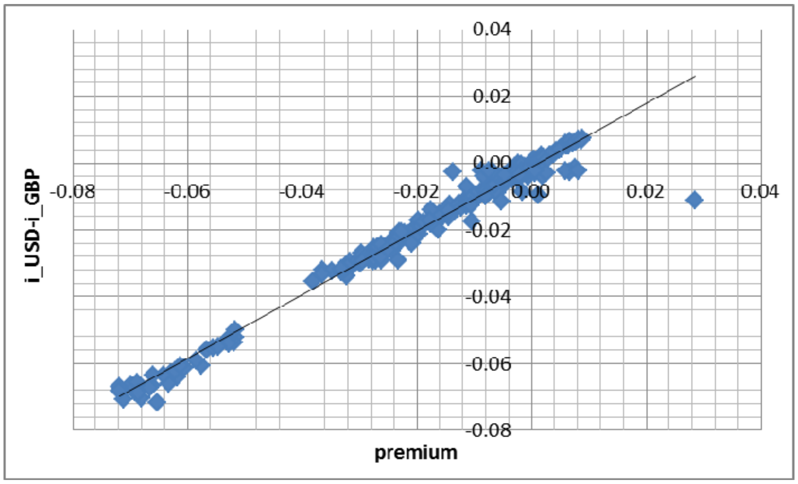
$$F_{t,10\text{-}year} = S_t\,[(1+i_{d,10\text{-}year})/(1+i_{f,10\text{-}year})]^{10}$$
$$= 1.60 \text{ USD/GBP } [1.06/1.08]^{10} = 1.3272 \text{ USD/GBP. ¶}$$

Synthetic forward contracts are very useful for exotic currencies.

IRPT: Evidence

Starting from Frenkel and Levich (1975), there is a lot of evidence that supports IRPT.

Graph 7.2: IRPT Line – USD/GBP (monthly, 1990-2015)



IRPT: Evidence

Taylor (1989): Strong support for IRPT using 10' intervals.

Akram, Rice and Sarno (2008, 2009): Using tick-by-tick data, show that there are short-lived (from 30 seconds up to 4 minutes) departures from IRP, with a potential profit range of 0.0002-0.0006 per unit.

Overall, the short-lived nature and small profit range point out to a fairly efficient market, with the data close to the IRPT line.

But, there are situations where we see significant deviations from the IRPT line. These situations reflect violations of IRPT's assumptions

For example, during the 2007-2008 financial crisis there were violations of IRPT. Probable cause: funding constraints –Step (1) in trouble!