Bridge, Focus, Attack, or Stimulate: Retail Category Management Strategies with a Store Brand

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Abstract. We investigate a monopolist retailer’s category management strategy where the main strategic decisions are how to horizontally position a store brand relative to the incumbent national brands and how to price the store and national brands for retail category profit maximization. We analyze a market composed of two consumer segments with differing tastes and heterogeneity with respect to willingness to pay and a product category consisting of two competing national brands and one store brand. We find that contrary to the existing literature, it is not always optimal for a retailer to position its store brand against the leading national brand; instead there are many situations where it is best to position the store brand close to the weaker national brand or to position it in the “middle” so it appeals to both national brands’ target segments. In the process we identify four distinct category management strategies that a retailer can use with a store brand. In three of these the optimal store brand price is the brand’s monopoly price, while in the remaining one strategy the price is lower. We also suggest an easy to implement means for a retailer to determine which strategy is best to use, depending on the particular competitive environment present before the introduction of the store brand and the relative quality of the store brand. We find that the store brand entry is most beneficial to the retailer when the national brands are moderately differentiated. Finally we show that introducing a store brand not only allows the retailer to garner a higher share of the channel profits through higher retail margins, but also often provides the retailer the benefit of increases in national brand unit sales as well as incremental sales from the store brand.

Key words. store brands, retailing, category management, positioning, channel strategy, game theory, strategic pricing

JEL Classification: M310

1. Introduction

Store brands now account for one of every five items sold every day in U.S. supermarkets, drug chains and mass merchandisers. They represent more than $50 billion of current business at retail, are achieving new levels of growth every year, (Private Label Manufacturers Association, www.plma.com) and have been expanding into non-grocery categories, such as personal computers (Wall Street Journal, May 3, 2002). Yet, optimal pricing and positioning of store brands still pose a challenging problem as witnessed by the following two quotes: “A retailer’s pricing strategy should be based upon the role of private label. A
number of people now realize they don’t have the answer to that question.” (Bill Bishop, President of Willard Bishop Consulting, Progressive Grocer, 2000) “A lot of things still get put together in the old way. For example, ‘Tide is doing well, so we ought to emulate Tide. We will be 10% to 15% below Tide and make a better profit.’ That is an answer, but in a world where retail is rapidly changing it may not be the optimal answer.” (Ron Lunde, a consultant and former retailer, Progressive Grocer, 2000).

Given the growing importance of store brands and how to manage them, it is not surprising that marketing scholars have paid more attention to this issue in the recent years. 1 For example Scott Morton and Zettelmeyer (2004, SZ hereafter) and Sayman et al. (2002, SHR hereafter) analyze the issue of store brand positioning 2 and suggest that, in general, a store brand should be positioned as close to the leading national brand as possible, a la “Emulate Tide.” SHR further show that store brand introduction under this positioning strategy has an asymmetric impact on the incumbent national brands, causing a greater decrease in wholesale price and a greater increase in retail margin for the leading national brand than for the secondary national brand.

Interestingly, the few published empirical studies on store brand positioning do not suggest the “Emulate Tide” strategy is the universally used strategy. For instance, SHR analyzed 75 product categories in two grocery chains and found this strategy was followed in less than 1/3 of the categories. Similarly, SZ surveyed two stores and found only 15–20% of the store brands matched a major national brand in size, shape, color, lettering and art although 63–65% of the store brands were placed next to a major national brand on the store shelves. Pauwels and Srinivasan’s (2004) investigation of four product categories in Dominick’s Finer Foods suggests that store brands typically compete more closely with second-tier national brands than against premium national brands, contrary to the results of the two theoretical studies. These findings, as well as SHR’s analysis of secondary data, all suggest that retailers might be pursuing a variety of different positioning strategies for store brands.

Our own observation of three additional product categories in the Dominick’s database further supports the possible presence of multiple store brand strategies across product categories. For instance, the changes in manufacturer revenue in Table 1 show that the major “victim” of the store brand entry in one instance is the premium national brand manufacturer (Welch’s cranberry juice), and in other instances is the non-premium national brand manufacturer (Kellogg’s raisin bran cereal and Geisha canned tuna). The table also shows store brand introduction can affect national brands in a variety of ways—increasing the wholesale and retail prices with little impact on the quantity sold (Total Raisin Bran cereal), decreasing the wholesale and retail prices as well as the quantity (Chicken of the Sea tuna), decreasing the wholesale and retail prices while increasing the quantity (Bumble Bee tuna), or causing little changes in the wholesale and retail prices while significantly decreasing the quantity (Welch’s cranberry juice).

1 We follow the lead of most of the literature and use the term “store brand” to refer to all merchandise sold under a retail store’s private label. That label can be the store’s own name or a brand name created exclusively by the retailer for that store.

2 As in these studies, we define positioning strictly as horizontal positioning. As for vertical positioning of store brands, Raju et al. (1995) as well as our analysis show a retailer is always better off with higher levels of store brand quality as long as the incremental cost is not too high.
### Table 1. The impact of store brand entry on incumbent national brands

<table>
<thead>
<tr>
<th>Product category</th>
<th>Brand</th>
<th>Change in wholesale price (%)</th>
<th>Change in retail price (%)</th>
<th>Change in quantity sold (%)</th>
<th>Change in manufacture revenue (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canned Tuna (Solid White)</td>
<td>Chicken of the Sea</td>
<td>−11.4</td>
<td>−10.43</td>
<td>−11.53</td>
<td>−21.67</td>
</tr>
<tr>
<td></td>
<td>Geisha</td>
<td>−6.91</td>
<td>−7.28</td>
<td>−27.33</td>
<td>−32.35</td>
</tr>
<tr>
<td></td>
<td>3 Diamond</td>
<td>−2.04</td>
<td>−12.01</td>
<td>+6.73</td>
<td>+4.55</td>
</tr>
<tr>
<td></td>
<td>StarKist∗∗</td>
<td>−10.27</td>
<td>−9.3</td>
<td>+15.32</td>
<td>+3.47</td>
</tr>
<tr>
<td></td>
<td>Bumble Bee</td>
<td>−14.21</td>
<td>−10.37</td>
<td>+42.19</td>
<td>+21.98</td>
</tr>
<tr>
<td>Raisin Bran Cereal</td>
<td>Total∗∗</td>
<td>+2.62</td>
<td>+2.69</td>
<td>−0.75</td>
<td>+1.85</td>
</tr>
<tr>
<td></td>
<td>Kellogg</td>
<td>−5.2</td>
<td>−3.22</td>
<td>−4.78</td>
<td>−9.73</td>
</tr>
<tr>
<td></td>
<td>Post</td>
<td>−3.52</td>
<td>−2.99</td>
<td>+17.77</td>
<td>+13.62</td>
</tr>
<tr>
<td>Frozen Cranberry Juice</td>
<td>Welch’s∗∗</td>
<td>−0.7</td>
<td>−1.34</td>
<td>−19.99</td>
<td>−20.56</td>
</tr>
<tr>
<td></td>
<td>Tropicana</td>
<td>−17.01</td>
<td>−17.96</td>
<td>+23.03</td>
<td>+2.1</td>
</tr>
</tbody>
</table>

*The numbers in this table represent % changes from the average for the period until 24 weeks prior to the time of the store brand introduction to the average for the period since 24 weeks after the store brand introduction. For raisin bran cereal, however, the “before store brand introduction” period was defined as the period until 12 weeks prior to the time of store brand introduction, due to limited number of observations.

∗∗Premium national brand (characterized by the highest retail price following Pauwels and Srinivasan’s 2004 definition).

SHR attribute the discrepancies between their theoretical analysis conclusion and empirical evidence to factors such as a high cost involved in imitating the leading national brands, the presence of a price-sensitive segment, and the lack of sufficient convexity of the function that links horizontal differentiation into cross price sensitivity of demand. While accepting these as plausible explanations, our study demonstrates that there exist more fundamental strategic forces in retail category management that lead to multiple store brand strategies. We do this by analyzing a parsimonious two-manufacturer, one-retailer game theoretic model to explore the following research questions:

1. Excluding the situations noted by SHR which preclude a retailer from positioning the store brand against the leading national brand, is it otherwise always optimal to follow SHR’s and SZ’s suggestion to position the store brand as close as possible to the leading national brand?
2. If positioning against the leading national brand is not always optimal, what are the other alternative strategies?
3. Under what conditions should a retailer use each of the alternative strategies?

Our model builds on the relative strengths of SZ’s and SHR’s analytic models. In order to ensure transparent connections between the underlying market characteristics and the demand structure, we derive demand functions from an explicit buyer behavior model that is identical to SZ’s. Then, we follow the lead of SHR and relax three of SZ’s restrictions on strategic alternatives by allowing the retailer to (a) position the store brand anywhere between the two national brands (instead of selecting one of two pre-specified positions), (b) have stronger bargaining power against a store brand manufacturer (instead of assuming
the same bargaining power as against national brand manufacturers), and (c) carry two
(instead of one) national brands in addition to a store brand. In this way, our model does
not impose any new assumptions that have not been used in the existing studies, yet does
enable us to expand their findings and provide richer insights into the retailer’s category
management strategies with a store brand.

Our analysis produces three key results. First, in contrast to the extant analytic literature
that suggests a retailer position the store brand as close as possible to the leading national
brand, we show that the retailer often earns higher category profits by positioning the
store brand either close to the weaker national brand or roughly halfway between the two
national brands. Interestingly, the existence of diverse optimal store brand positions does
not require asymmetric sizes between consumer segments nor a store brand manufacturer’s
inability to fully imitate a national brand. This leads to our second major finding, i.e.,
these variations in optimal store brand position reflect four distinct types of retail category
management strategies with a store brand. We label these strategies “Bridge”, “Focus”,
“Attack”, and “Stimulate” and describe each with specific positioning and pricing actions.
Third, we identify the conditions under which each of the four strategies is optimal for a
retailer. These conditions are described in terms of measurable real world factors, thereby
enabling academics to empirically test our conclusions and practitioners to directly apply
our suggestions to their specific circumstances.

We note that these new insights are due in large part to two factors. First our derived
demand function has multiple regions with different slopes, a characteristic empirically
observed in brand competition within categories. Faced by such a demand structure, an
important strategic issue for all the channel members is to ensure their strategic actions
let them “play in” the demand region most favorable to themselves. In this sense, one can
consider a store brand as a strategic tool that a retailer can use to reshape the competitive en-
vironment within the category for its own advantage. Second our approach is less restrictive
than previous work and thus we are able to identify new strategic alternatives.

Consistent with previous analytical studies we find a major benefit of introducing a store
brand comes from the retailer increasing its share of the channel profits associated with the
national brand sales. However, a retailer can also benefit from increases in national brand
unit sales caused by a lower retail price and profits generated from the store brand sales. The
relative magnitudes of these different benefits systematically depend upon the underlying
market conditions. Therefore, it is imperative that a retailer understands the linkages between
the underlying market conditions and the optimality of each of the category management
strategies and their impact on the market outcomes.

2. Model

In this section, we develop a model of buyer behavior and market structure and use this model
to derive demand functions. This demand derivation approach is similar to SZ’s and allows
us to ensure that the demand models before and after the store brand introduction represent
the same underlying market environment. In Section 3, we analyze this model by applying
a set of rules of the game that are comparable to SHR’s to obtain equilibrium solutions.

3 By allowing the retailer to add a third brand, we implicitly assume no binding constraint on shelf space.
2.1. Consumer utility model

Our model of consumer behavior is based on Desai’s (2001) utility function for horizontally and vertically differentiated products. Specifically:

\[ U_{ij} = \beta_i v_j - k_i t_{ij} - p_j \]  

Equation (1) indicates consumers are heterogeneous in three ways—willingness to pay for quality (\( \beta_i \)), cost of mismatch (\( k_i \)) and degree of product mismatch (\( t_{ij} \)). In addition, “to model the possibility that higher valuation consumers also have stronger taste preferences,” Desai assumes if \( \beta_H > \beta_L \) then \( k_H \geq k_L \). Thus, \( \beta_i \) and \( k_i \) are either positively correlated (\( k_H > k_L \)) or independent (\( k_H = k_L \)). As we discuss below, these two situations lead to two alternative utility models for our study.

Assume first that \( \beta_i \) and \( k_i \) are perfectly positively correlated. Next, define a rescaled mismatch variable, \( m_{ij} = t_{ij}/b \) where \( b \) is the constant that perfectly maps \( k_i \) into \( \beta_i \), i.e. \( \beta_i = bk_i \). Substituting this into equation (1), the utility function simplifies to:

\[ U_{ij} = \beta_i (v_j - m_{ij}) - p_j \]  

Alternatively, let \( \beta_i \) and \( k_i \) be independent (\( k_H = k_L = k \)). Then rescale \( t_{ij} \) into \( m_{ij} = k t_{ij} \) so that \( t_{ij} = m_{ij}/k \). Substituting this into equation (1), the utility function simplifies to:

\[ U_{ij} = \beta_i v_j - m_{ij} - p_j \]  

without loss of generality.  

Equations (2) and (3) represent two alternative models of consumer behavior. In words, equation (2) implies that a consumer who highly values quality will also highly disvalue a horizontal mismatch (i.e., highly value a horizontal fit) between her ideal point and the horizontal brand position. This is identical to the consumer utility model used by SZ. In contrast, equation (3) states that regardless of how much (little) a person values quality, this person always disvalues horizontal mismatch (i.e., values horizontal fit) to the same degree. For parsimony, we provide a full discussion of our analyses only for the first model in this paper. However, we later discuss the results obtained from the latter model in order

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We thank one of the anonymous reviewers for suggesting this utility formulation.
to examine the robustness of our main conclusions to alternative assumptions of consumer behavior.

2.2. **Underlying market structure**

We assume a market composed of two consumer segments, each with its own ideal point. Within each segment consumers are heterogeneous with respect to willingness to pay, $\beta$, which is uniformly distributed between 0 and 1 with a density of 1. We assume each segment is targeted by one national brand and the characteristics of the brand fit its target segment’s tastes perfectly. More technically, national brand $j$ (hereafter referred to as NB$_j$) is positioned at segment $j$’s ideal point ($j = 1, 2$). Let parameter $d$ ($0 \leq d \leq 1$) denote the “perceived distance” between the positions of NB$_1$ and NB$_2$ in the product space. A higher $d$ simultaneously implies greater horizontal differentiation between the NB’s in the minds of the consumers and a greater mismatch between NB$_1$ and segment 3-$j$’s tastes. If a store brand (SB) also exists in the category, its position relative to the NB’s is captured by $x$, $0 \leq x \leq 1$. $xd$ and $(1-x)d$ capture the perceived distances from the SB’s position to NB$_1$ and NB$_2$, respectively. $x < .5$ implies the SB is positioned closer to NB$_1$ than to NB$_2$.

5 We do not assume a third, unique target segment for SB. This rules out the (realistic but less interesting) possibility that the store brand position is determined by the unique consumer needs not met by the NB’s.

6 We later relax this assumption and discuss the impact on our results.

The above formulation of market structure relaxes SZ’s assumption of equal quality of the two NB’s and thus reflects the reality that SB’s are sometimes perceived to have higher quality than some NB’s (Chintagunta, 2002; Harrison, 2000; USDA, 2000). It also allows for three brands to be sold at one time and allows the SB to be positioned anywhere on the line between the two NB’s. On the other hand, our assumption of an equal mass of consumers in each segment is a simplification of SZ’s model of two segments with unequal masses.

For a given set of horizontal product positions ($d$ and $x$) and quality levels ($1$, $\alpha$ and $\alpha_S$), one can determine each consumer’s utility by using equation (2). This leads to the following utility functions for NB$_1$, NB$_2$ and SB for consumer $i$ in segment 1:

\begin{align*}
U_{i1} &= \beta_i - p_1 \quad (4) \\
U_{i2} &= \beta_i (\alpha - d) - p_2 \quad (5) \\
U_{iS} &= \beta_i (\alpha_S - xd) - p_S, \quad (6)
\end{align*}

where the second subscript refers to the brand, and $p_1$, $p_2$ and $p_S$ are the retail prices of NB$_1$, NB$_2$ and the SB, respectively. Likewise, the utility functions for consumer $i$ in segment 2...
are as follows:  

\begin{align*}
U_{i1} &= \beta_i (1 - d) - p_1 \\
U_{i2} &= \beta_i (\alpha) - p_2 \\
U_{iS} &= \beta_i (\alpha_S - d(1 - x)) - p_S,
\end{align*}

Note that when \( \beta_i \) is close to 0, consumer \( i \)'s net utility depends almost entirely upon prices and little on the products' quality levels and positions, indicating high price sensitivity and little brand loyalty. As is standard, we assume the utility for the outside option is zero for all customers.

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7 Note that consumer \( i \) in segment 1 is not the same consumer \( i \) found in segment 2.
8 Note that, in our second buyer behavior model (equation (3)), every consumer values a good fit equally regardless of their willingness to pay for quality. Thus, even low willingness to pay consumers exhibit “loyalty” and thus are less price sensitive under this second buyer behavior model.
2.3. Derivation of demand

We make the typical assumption that each consumer purchases either one unit of the brand that yields the highest positive net utility or nothing if none of the brands yields positive net utility. Then, the market demand for each brand can be derived from equations (4) to (9) in a straightforward four-step process. As a first step, we determine the rank order of the three brands in terms of gross utility (i.e., the non-price component of utility) for each segment. For each segment, let A denote the brand with the highest gross utility, B the next highest, C the lowest, and D the no purchase option. Note that the identity of A, B and C is a function of the category characteristics, i.e., $\alpha$, $\alpha_S$, $d$ and $x$.

Second, for each segment we determine the six values of $\beta_i$ that represent the marginal consumers who are indifferent between a pair of alternatives (including the no purchase option), using equations (4)–(9). For instance, the marginal consumer in segment 1 who is indifferent between purchasing NB1 and SB can be identified by equating equations (4) and (6) and solving for $\beta_{1NS} = (p_1 - p_S)/(1 - \alpha_S + xd)$, where the first subscript denotes the segment and the next two the identity of the two brands. Similarly, the marginal consumer in segment 2, who is indifferent between NB2 and no purchase, is identified by equating equation 8 to zero. In this way, we determine the six critical $\beta$’s for segment $h = 1, 2$, labeled as $\beta_{hAB}^*, \beta_{hAC}^*, \beta_{hAD}^*, \ldots, \beta_{hCD}^*$. Note that these six values are functions of prices as well as $\alpha$, $\alpha_S$, $d$ and $x$.

Third, we determine the within-segment demand for each brand by partitioning the unit line representing each segment as shown in Figure 1(a). To partition the market segment into discrete groups of consumers making different brand choices, we use the following Lemma (proof in Appendix 1, which is available at http://www.fuqua.duke.edu/faculty/alpha/staelin.htm):

**Lemma 1.** For a given set of values for $\alpha$, $\alpha_S$, $x$ and $d$, there exists a rank order of the competing brands in terms of their gross utility in each segment. If for a given set of prices a consumer with a certain level of willingness to pay purchases a particular brand, another consumer with lower willingness to pay will never buy a higher ordered brand at that set of prices.

In the context of our illustrative example, Lemma 1 implies that if a consumer with $\beta_H$ purchases brand B (second highest gross utility) instead of brand A (highest gross utility) because of B’s relatively low price, another consumer with $\beta_L (< \beta_H)$ will not purchase A, either.

Applying this principle to all pairs of choice alternatives and the fact that $\beta_i$ is distributed uniformly between zero and one, the demand for each brand from segment $h$ is derived as follows:

\[
q_{hA} = 1 - \min[1, \max(\beta_{hAB}^*, \beta_{hAC}^*, \beta_{hAD}^*, 0)] \tag{10}
\]

\[
q_{hB} = 1 - q_{hA} - \min[1 - q_{hA}, \max(\beta_{hBC}^*, \beta_{hBD}^*, 0)] \tag{11}
\]

\[
q_{hC} = 1 - q_{hA} - q_{hB} - \min[1 - q_{hA} - q_{hB}, \max(\beta_{hCD}^*, 0)]. \tag{12}
\]

9 In analyzing our second model of buyer behavior, the ranking is done with respect to just quality. Thus for this model of buyer behavior NB1 is ranked first in both segments as long as $\alpha$ and $\alpha_S$ are less than 1.
Since the $\beta^*$'s are simple linear functions of prices, equations 10–12 represent linear demand functions. However, note that as prices change, the rank order of $\beta_{hAB}^*$, ..., and $\beta_{hCD}^*$ might also change, leading to multiple linear regions in the above demand functions, i.e., a kinked demand system. Specifically there are seven possible ways of partitioning a segment as shown in Table 2. (The table also displays the three possible ways of partitioning the segment if only two competing brands, A and B exist.) The fourth and final step of demand derivation is to simply sum up the brand demands in the two segments. This creates a maximum of $7 \times 7 = 49$ ($3 \times 3 = 9$) possible regions for the three (two) brand situation.

### 2.4. Demand characteristics

Figure 2(a) displays an example of NB1’s demand function derived through this four step process for $\alpha = 1, \alpha_S = .8, d = .25, x = .5, p_2 = .5, \text{ and } p_S = .35$. Note that the demand function is continuous and downward sloping in own price but also kinked with five regions of linear demand with different slopes. These varying slopes reflect the effect of $p_1$ on the patterns of brand competition in each segment. In region 1, $p_1$ is sufficiently lower than $p_2$ and $p_S$ so that NB1 is the only brand purchased by consumers in both segments. In region 2, $p_1$ is still low enough to attract price sensitive (low $\beta$) consumers of segment 2 to buy NB1, but segment 2 consumers with higher $\beta$’s are now willing to pay the higher price ($p_2 = .5$) to purchase NB2 since it fits their tastes better. In region 3, $p_1$ is too high for NB1 to attract any buyer from segment 2 but still low enough to keep the segment 1 buyers from switching to the SB. In region 4, the high $p_1$ causes some price sensitive consumers in segment 1 to switch from NB1 to the SB. Finally, in region 5, $p_1$ is so high that NB1 loses all its segment 1 customers to the SB.

This type of kinked demand has been used in other studies analyzing spatial models (e.g., Salop, 1979; Vandenbosch and Weinberg, 1995; Chiang et al., 2003). Empirical studies also show the presence of kinked demand in an oligopoly (Bhaskar et al., 1991; Awh and Primeaux 1992; Dickson and Urbany, 1994). More importantly, a study by ACNielsen...
Figure 2. Demand characteristics.

(Shr) finds kinked demand a key characteristic for a national brand and a store brand competing in the same product category. Specifically, as Figure 2(b) shows, there exists a price range over which price changes cause active switching from one brand to the other. Outside this range, demand is much less sensitive to price changes. Our model captures this demand characteristic very well. In contrast SHR’s demand function (which is linear over
the entire price range) is not able to reflect this varying price sensitivity. Our formulation also implies that the competitive environment depends in part on the specific price levels since the slopes differ over the range of prices. It is this changing competitive environment that allows us to provide new insights into inter-brand strategic interactions within a category and broaden our understanding of optimal category management with a store brand.

2.5. Rules of the game

To date two different approaches have been used to model the game between channel members while studying the introduction of a store brand. SZ use a bargaining model first proposed by Shaffer and Zettelmeyer (2002) that implicitly assumes the channel members are able to perfectly coordinate the channel (i.e., charge the channel profit maximizing retail prices) both before and after the introduction of the SB. SHR assume no channel coordination and independent profit maximization by each channel member. Given our interest in category management (which includes not only the positioning decision, but also the pricing strategy) and our belief that most channels are not fully coordinated, we use SHR’s approach.

We treat the horizontal and vertical positions of the NB’s ($d$ and $\alpha$) and the quality level of the SB ($\alpha_S$) as exogenous factors characterizing the market environment. Then, within this environment, we apply the following sequence of moves:

1. The retailer, if it offers a SB, selects the SB’s position, $x$, relative to the NB’s.
2. The two NB manufacturers set their respective wholesale prices, $w_1$ and $w_2$, to maximize their own respective profits taking into consideration the retailer’s reaction to the wholesale price changes.
3. The retailer sets retail prices for NB1, NB2 and the SB conditional on wholesale prices, $w_1$ and $w_2$, to maximize the retailer category profits.

As in previous studies (Raju et al., 1995; Vandenbosch and Weinberg, 1995; SHR, 2002; SZ, 2004), we assume the marginal cost of production is zero regardless of the quality level for all three brands. While this assumption may be unrealistic, the first three studies mentioned above show their main results remain qualitatively unchanged when the production cost is assumed to be an increasing function of quality. In addition, since the SB is still a commodity until it has the store label, SB manufacturers have little market power (Connor and Peterson, 1992; Mills, 1995; Ailawadi and Harlam, 2004). Consequently, we assume the SB is not subject to double marginalization and, thus, is obtained by the retailer at the manufacturer’s cost ($c = 0$) (Raju et al., 1995; SHR, 2002). This makes the SB almost always cheaper for the retailer to acquire than a NB although the SB could be of higher quality than a NB, which is consistent with previous empirical observations (USDA, 2000; Chintagunta, 2002).
These assumptions lead to the following objective functions for the manufacturers and the retailer:

\[ \Pi_{M1} = w_1 q_1, \quad (13) \]
\[ \Pi_{M2} = w_2 q_2, \quad \text{and} \quad (14) \]
\[ \Pi_R = (p_1 - w_1)q_1 + (p_2 - w_2)q_2 + pSqS, \quad (15) \]

where \( p_S \) and \( q_S \) are held to zero before SB introduction.\(^{10}\)

3. Model analysis

The kinked demand functions derived from our model are not continuously differentiable over the entire range of prices. Consequently, using the standard mathematical method (e.g., McGuire and Staelin, 1983) for obtaining the equilibrium conditions requires solving the problem for each region of the kinked demand curve and then comparing all local optimums and corner solutions to find the global optimum. This approach can be very complicated even with a simpler demand structure as shown by Chiang et al. (2003). Given this complexity we start our model analysis with two polar cases for \( d \) (i.e., \( d = 0 \) and \( d \) large enough to insure the two NB’s don’t compete), which can be easily solved using the standard mathematical approach. Later, we generalize our analysis to the entire range of \( d \) by taking a more efficient approach of quadratic programming and numerical optimization.

3.1. When \( d = 0 \)

\( d = 0 \) represents an environment where there are no taste differences among consumers and no horizontal differentiation between the NB’s. Since the SB position is assumed to be between the two NB’s, it, too, is positioned at the ideal point of the one consumer segment which is twice the density of the single segment in our general case. Since there is no horizontal differentiation between the available brands, consumers only consider quality and price when deciding which brand to buy. Thus, the retailer only needs to decide what to charge for each brand. This simplification allows us to analyze the model mathematically (Details are shown in Technical Appendix 1, which, along with Technical Appendix 2 and 3, is available at http://www.fuqua.duke.edu/faculty/alpha/staelin.htm). The closed form solutions are presented in Table 3.\(^{11}\)

We note three main results from Table 3. First, before the introduction of the SB, the premium national brand (NB\(_1\)) has a higher wholesale price, retail price, and quantity sold than the weaker national brand (NB\(_2\)), allowing manufacturer 1 to earn more profits than manufacturer 2. This indicates the weaker national brand with no horizontal differentiation must offer a significantly lower price in order to motivate some low willingness to pay consumers to buy the brand. As \( \alpha \) approaches 1, the wholesale prices and, thus, profits for both manufacturers go to zero as expected for perfectly substitutable products. In this

\(^{10}\) This profit function does not consider any fixed costs associated with introducing a store brand.
\(^{11}\) Note that since \( d = 0 \), our two models of buyer behavior are identical. Thus, the Table 3 results hold for both models of buyer behavior.
Table 3. Equilibrium results for vertically differentiated product category (d = 0).

<table>
<thead>
<tr>
<th></th>
<th>Before SB</th>
<th>After SB (1 &gt; α &gt; αS)</th>
<th>After SB (1 &gt; αS ≥ α)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_1 )</td>
<td>( \frac{2(1-\alpha)}{4-\alpha} )</td>
<td>( \frac{2(1-\alpha)(1-\alphaS)}{4-\alpha-3\alphaS} )</td>
<td>( 1-\alpha )</td>
</tr>
<tr>
<td>( w_2 )</td>
<td>( \frac{\alpha(1-\alpha)}{4-\alpha} )</td>
<td>( \frac{\alpha(1-\alpha)(1-\alphaS)}{4-\alpha-3\alphaS} )</td>
<td>( \frac{\alpha}{2} )</td>
</tr>
<tr>
<td>( p_1 )</td>
<td>( \frac{4\alpha-\alphaS}{4-\alpha} )</td>
<td>( \frac{6-\alphaS-3\alphaS}{4-\alpha-3\alphaS} )</td>
<td>( 3-\alpha )</td>
</tr>
<tr>
<td>( p_2 )</td>
<td>( \frac{\alpha S-2\alphaS}{4-\alphaS} )</td>
<td>( \frac{5\alpha S-3\alphaS}{4(4-\alpha-3\alphaS)} )</td>
<td>( \frac{\alpha S}{3} )</td>
</tr>
<tr>
<td>( q_1 )</td>
<td>( \frac{2}{4-\alpha} )</td>
<td>( \frac{2(1-\alphaS)}{4-\alphaS} )</td>
<td>( \frac{1}{3} )</td>
</tr>
<tr>
<td>( q_2 )</td>
<td>( \frac{1}{4-\alpha} )</td>
<td>( \frac{1-\alphaS}{4-\alphaS} )</td>
<td>0</td>
</tr>
<tr>
<td>( q_3 )</td>
<td>( \frac{1}{4-\alphaS} )</td>
<td>( \frac{1-\alphaS}{4-\alphaS} )</td>
<td>( \frac{1}{3} )</td>
</tr>
<tr>
<td>( \Pi_{M1} )</td>
<td>( \frac{4(1-\alpha)(1-\alphaS)}{4-\alphaS} )</td>
<td>( \frac{4(1-\alpha)(1-\alphaS)}{4-\alphaS} )</td>
<td>( 1-\alphaS )</td>
</tr>
<tr>
<td>( \Pi_{M2} )</td>
<td>( \frac{\alpha(1-\alpha)}{4-\alphaS} )</td>
<td>( \frac{\alpha(1-\alpha)(1-\alphaS)}{4(4-\alpha-3\alphaS)} )</td>
<td>0</td>
</tr>
<tr>
<td>( \Pi_R )</td>
<td>( \frac{4\alpha S-3\alphaS}{4-\alphaS} )</td>
<td>( \frac{4+5\alpha S-2\alphaS-2\alphaS^2-8\alphaS+3\alphaS^2+11\alphaS^2}{2(4-\alpha-3\alphaS)} )</td>
<td>( 1+3\alphaS )</td>
</tr>
</tbody>
</table>

Demand structure before SB

\[ q_1 = 2\left(1 - \frac{p_1 - p_2}{1-\alpha}\right), \quad q_2 = 2\left(\frac{p_1 - p_2}{1-\alpha} - \frac{p_2}{\alpha}\right) \]

Demand structure after SB (1 > α > αS)

\[ q_1 = 2\left(1 - \frac{p_1 - p_2}{1-\alpha}\right), \quad q_2 = 2\left(\frac{p_1 - p_2}{1-\alpha} - \frac{p_2 - p_1}{\alpha - \alphaS}\right), \quad q_3 = 2\left(\frac{p_2 - p_S}{\alpha - \alphaS} - \frac{p_2}{\alphaS}\right) \]

Demand structure after SB (1 > αS ≥ α)

\[ q_1 = 2\left(1 - \frac{p_1 - p_2}{1-\alphaS}\right), \quad q_2 = 0, \quad q_3 = 2\left(\frac{p_1 - p_2}{1-\alphaS} - \frac{p_2}{\alphaS}\right). \]

situation, the retailer extracts all the profits in the channel. This shows how the competitive environment between the two NB’s affects the retailer’s ability to increase its share of channel profits.

Second, comparing the “Before SB” column and the next column, we note that when the SB has the lowest level of quality (1 > α > αS), the SB entry leads to a larger percent decrease in the wholesale price for NB2 than for NB1 while the quantities for NB’s decrease proportionally. Consequently, the profit for manufacturer 2 is more severely affected by the SB entry. As expected, the negative impact of the SB entry on manufacturer profits increases as αS becomes larger. In particular, as αS approaches α, w2 and \( \Pi_{M2} \) are driven toward zero. Interestingly we note that in these situations the retailer always prices the SB at its monopoly price, \( \alphaS/2 \), which is a function of its own quality level, but is not influenced by the quality levels of the NB’s.

Third, as seen in the last column of Table 3, when the quality level of the SB exceeds that of NB2 (1 > αS ≥ α), the retailer completely displaces the lower quality NB2, again pricing the SB at its monopoly price. As before, the SB entry results in decreases in \( w_1, p_1 \) and \( q_1 \), making manufacturer 1 worse off. Nevertheless, it is clear that the main “victim” of the SB entry is still NB2 as is the case when α > αS. This result is in agreement with Pauwels and Srinivasan’s (2004) empirical finding that a SB introduction affects a second
tier NB much more than a premium NB. This suggests that perhaps the product categories analyzed by Pauwels and Srinivasan consist mainly of vertically differentiated NB’s with little horizontal differentiation.

The above results reflect how the SB interacts with the two NB’s when \( d = 0 \). Note that when \( 1 > \alpha > \alpha_S \), the SB only directly affects the demand for NB2 (i.e., the demand function for NB1 does not contain \( p_S \)). However, NB1 is still impacted by the SB entry since the SB’s “attack” on NB2 causes the NB2 manufacturer to react (by lowering \( w_2 \)) and this reaction directly affects NB1. In this way, the SB is able to “stimulate” the NB1 manufacturer to react. As a result, wholesale prices decrease and retail margins increase for both NB’s, enhancing the retailer’s category profits. We label this the “Stimulate” strategy since the retailer uses the SB to stimulate the price competition between the two NBs.

In contrast, when \( 1 > \alpha_s \geq \alpha \), the SB enters the category as the medium quality brand, competing directly against both NB’s. In this case, the retailer directly attacks both NB’s with the SB. We label this the “Attack Both” strategy. Within this parameter range, the retailer finds it best to displace NB2, since the SB is of higher quality and thus can garner a higher retail price. Consequently, the retailer only carries two brands.

### 3.2. When \( d \) is large.

We next consider the situation where the two NB’s are highly differentiated horizontally. With a sufficiently large \( d \), we can safely assume that before the introduction of the SB, NBj is the most preferred brand (before price) in segment \( j \) and neither its manufacturer nor the retailer finds it profitable to try to sell it to segment \( 3-j \). This allows each NB to operate at the bilateral monopoly solution in its target segment resulting in the prices and profits as shown in the column labeled “Before SB” in Table 4.

A sufficiently large value of \( d \) also simplifies the after-SB situation since Lemma 1 implies only four of the 49 possible demand situations can occur in this case. They are:

1. NB1 and SB sold in segment 1; NB2 sold in segment 2
2. NB1 sold in segment 1; NB2 and SB sold in segment 2
3. NB1 and SB sold in segment 1; NB2 and SB sold in segment 2
4. NB1 sold in segment 1; SB sold in segment 2 (possible only if \( 1 > \alpha_s \geq \alpha \))

Given this smaller number of demand regions, it is feasible to solve the game mathematically for each region and check for boundary conditions. (Details are available in Technical Appendix 2.) The closed form solutions are provided in Table 4. These solutions are functions of the SB position, \( x \), and, thus, provide initial answers to our research questions regarding whether or not the retailer should always position the SB as close as possible to NB1 and if not, where the retailer should position the SB and why.

From Table 4(a) we see that there exist five distinct possibilities for the equilibrium after the SB introduction when \( d \) is large. These five possible sets of equilibrium solutions

---

12 We again note that when \( d \) is large enough to preclude inter-segment competition, our two models of buyer behavior are identical. Thus Table 4 results apply to both cases.
Table 4. Equilibrium results for high level of horizontal differentiation (large $d$).

### a. When $\alpha > \alpha_S$

<table>
<thead>
<tr>
<th>Before</th>
<th>FO1</th>
<th>BG1</th>
<th>BGb</th>
<th>After SB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1-\alpha_g + \alpha d}{2}$</td>
<td>$\frac{1-\alpha_g + \alpha d}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$w_2$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{\alpha d}{2}$</td>
<td>$\frac{\alpha d}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$p_1$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1-\alpha_g + \alpha d}{4}$</td>
<td>$\frac{1-\alpha_g + \alpha d}{4}$</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>$p_2$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{\alpha d}{4}$</td>
<td>$\frac{\alpha d}{4}$</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>$p_S$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{\alpha d}{4}$</td>
<td>$\frac{\alpha d}{4}$</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>$q_1$</td>
<td>$1 - p_1$</td>
<td>$1 - \frac{p_1 - p_3}{\alpha_g - (1 - x)d}$</td>
<td>$1 - \frac{p_1 - p_3}{\alpha_g - (1 - x)d}$</td>
<td>$1 - p_1$</td>
</tr>
<tr>
<td>$q_2$</td>
<td>$1 - \frac{p_1 - p_3}{\alpha_g - (1 - x)d}$</td>
<td>$1 - \frac{p_1 - p_3}{\alpha_g - (1 - x)d}$</td>
<td>$1 - \frac{p_1 - p_3}{\alpha_g - (1 - x)d}$</td>
<td>$1 - p_1$</td>
</tr>
<tr>
<td>$q_S$</td>
<td>$\frac{p_1 - p_3}{\alpha_g - (1 - x)d}$</td>
<td>$\frac{p_1 - p_3}{\alpha_g - (1 - x)d}$</td>
<td>$\frac{p_1 - p_3}{\alpha_g - (1 - x)d}$</td>
<td>$1 - \frac{p_1 - p_3}{\alpha_g - (1 - x)d}$</td>
</tr>
</tbody>
</table>

### b. When $\alpha_S > \alpha$

<table>
<thead>
<tr>
<th>Before</th>
<th>FO1</th>
<th>BG1</th>
<th>BGb</th>
<th>After SB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1-\alpha_g + \alpha d}{2}$</td>
<td>$\frac{1-\alpha_g + \alpha d}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$w_2$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{\alpha d}{2}$</td>
<td>$\frac{\alpha d}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$p_1$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1-\alpha_g + \alpha d}{4}$</td>
<td>$\frac{1-\alpha_g + \alpha d}{4}$</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>$p_2$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{\alpha d}{4}$</td>
<td>$\frac{\alpha d}{4}$</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>$p_S$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{\alpha d}{4}$</td>
<td>$\frac{\alpha d}{4}$</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>$q_1$</td>
<td>$1 - p_1$</td>
<td>$\frac{1}{\alpha_g - (1 - x)d}$</td>
<td>$\frac{1}{\alpha_g - (1 - x)d}$</td>
<td>$1 - p_1$</td>
</tr>
<tr>
<td>$q_2$</td>
<td>$\frac{1}{\alpha_g - (1 - x)d}$</td>
<td>$\frac{1}{\alpha_g - (1 - x)d}$</td>
<td>$\frac{1}{\alpha_g - (1 - x)d}$</td>
<td>$1 - p_1$</td>
</tr>
<tr>
<td>$q_S$</td>
<td>$\frac{p_1 - p_3}{\alpha_g - (1 - x)d}$</td>
<td>$\frac{p_1 - p_3}{\alpha_g - (1 - x)d}$</td>
<td>$\frac{p_1 - p_3}{\alpha_g - (1 - x)d}$</td>
<td>$1 - \frac{p_1 - p_3}{\alpha_g - (1 - x)d}$</td>
</tr>
</tbody>
</table>

$\bar{w}_1 = \begin{cases} \frac{\alpha_g - (1 - x)d}{\alpha_g - (1 - x)d + \alpha d - \alpha_g + \alpha d} & \text{if } \alpha_S < \alpha < \alpha_g \\ \frac{\alpha_g - (1 - x)d}{\alpha_g - (1 - x)d + \alpha d - \alpha_g + \alpha d} & \text{if } \alpha_g < \alpha \end{cases}$

BGj: “Bridge” by attacking NB$_j$ and threatening NB$_{3-j}$; BG: “Bridge” by attacking both NB’s; FOj: “Focus” by attacking NB$_j$ and leaving NB$_{3-j}$ unthreatened.
represent five conceptually different category management strategies available to the retailer when introducing a SB to a category characterized by highly differentiated NB’s. We illustrate these five different strategies in Figure 3, which plots the retailer’s equilibrium profits as a function of the SB’s position.

The first example shown in Figure 3(a) is for the parameter setting $\alpha = .95$, $\alpha_S = .85$, and $d = .65$. Starting from $x = 0$, the first region ($0 \leq x \leq .2$) maps to the equilibrium solution in the “FO1” column of Table 4(a). In this region, the SB is positioned close to NB1 and far from NB2. Consequently the SB only affects the demand for NB1 and not NB2. (Table 4(a) shows $w_2$, $p_2$ and $q_2$ remain at the bilateral monopoly solution as found for the “Before SB” condition). We label this strategy of positioning the SB to attack one NB while leaving the other NB unaffected “Focus”. Since the SB attacks NB1 and does not impact NB2, we call the strategy “Focus 1” (FO1). The downward slope of the retail profit function in the region of $0 \leq x \leq .2$ indicates that the retailer using the FO1 strategy finds it optimal to position the SB as close as possible to NB1 (i.e., $x = 0$) and charge the monopoly price for the SB.

In the second region of Figure 3(a) ($0.2 < x \leq .3$), the SB still only attacks NB1. However, since the SB now is positioned closer to NB2, the SB, by lowering its price below the monopoly price, is able to pose a competitive threat to NB2. Perceiving this competitive threat, the NB2 manufacturer lowers $w_2$ below the bilateral monopoly wholesale price of $\alpha/2$ in order to ensure that none of its customers in segment 2 buy the SB. Consequently, although NB2 remains the only brand selling to segment 2, the retailer successfully introduces competition to NB2 and benefits from the increased retail margin and sales volume due to the lower wholesale and retail prices of NB2.13 We label this strategy of positioning the SB between the two NB’s to lower the previously charged monopoly prices of both NB’s as “Bridge” (as in bridge the gap between the two highly differentiated NB’s). The specific strategy used in this region is “Bridge-Attack 1 and Threaten 2” (BG1), which has NB1 as the target of the SB attack while the SB threatens NB2. The closed form solution for this strategy is presented in the (BG1) column of Table 4.

The third region of Figure 3(a) ($0.3 < x < .7$) shows another variant of the Bridge strategy labeled “Bridge-Attack Both” (BGb). In this situation, the retailer positions the SB roughly half way between the two NB’s, again setting the SB price below the monopoly price so that the SB attracts consumers from both segments. The retailer’s strategy in the fourth region ($0.7 \leq x \leq .8$) is “Bridge-Attack 2 and Threaten 1” (BG2) which is conceptually analogous to BG1. However, the identities of the NB being attacked and the NB being threatened are reversed. The fifth region of Figure 3(a) ($0.8 < x \leq 1$) represent the “Focus 2” (FO2) strategy, which is the exact mirror image of the FO1 strategy in the first region. The positive slope of the retail category profit function in this region indicates that the retailer should position the SB as close to NB2 as possible (i.e., $x = 1$), when using the FO2 strategy.

Faced by such a profit function as Figure 3(a), the retailer must then choose among these five strategies and within the chosen strategy, the optimal positioning. For the particular parameter setting assumed in Figure 3(a), the retailer’s category profits are maximized when

13 Chiang, Chhajed, and Hess (2003) capture a similar effect by showing that a manufacturer can profitably introduce a direct channel as a threat to an indirect channel without taking away any sales form the latter.
In the situation shown in Figure 3(a), the SB has a reasonably high quality SB \(\alpha_s = .85\), making the Bridge strategy more profitable than Focus. However, if \(\alpha_s\) becomes smaller and/or \(d\) becomes larger, the competitive pressure on the NB's from the SB positioned in the middle will get weaker, making the Bridge strategy less profitable or possibly even infeasible. Figure 3(b) shows such an example. Here \(\alpha\) and \(d\) remain the same as in Figure 3(a), but...
but the SB quality parameter, $\alpha_S$, is reduced to .5. Now the only way the retailer can bridge the gap is to lower the SB price to such a degree that this strategy becomes less profitable than focusing on one of the NB’s and charging the SB’s monopoly price while leaving the other NB unaffected. Thus the optimal SB position is $x = 0$ or 1.

The optimality of the Focus strategy shown in Figure 3(b) is somewhat consistent with the conclusions of SHR and SZ, who suggest that the retailer should generally position the SB as close to the leading NB as possible. Note, however, that our closed form solutions in Table 4(a) indicate FO1 and FO2 lead to the exactly same retail category profits, making the retailer indifferent between targeting the stronger or weaker national brand. This difference is partially due to how the three different research teams model the asymmetry between the NB’s. In our model, the asymmetry comes from a quality difference between the two NB’s while holding the size of the two segments the same. In contrast, SZ assume identical product quality but more consumers in segment 1, while SHR’s demand specification implicitly assumes both types of asymmetries.

To see the implications of these assumptions, look at Figure 3(c), which is based upon the same parameter values as in Figure 3(b) but with double the mass of consumers in segment 2. This represents a situation where NB1 is the leading NB in terms of quality but targets a niche market. Consequently NB2 is the leading brand in terms of market share before the SB entry. In such a situation, Figure 3(c) clearly shows that the retailer is better off by selecting the FO2 strategy and positioning the SB at $x = 1$, contrary to SHR’s conclusion. On the other hand, if we assume a larger mass of consumers in segment 1 relative to segment 2, the optimal strategy will be FO1 with $x = 0$ as found by SHR and SZ. This implies when considering where to position the SB, the retailer must first understand how the “leading NB” is defined.

Note that the examples shown in Figure 3 are based on the assumption that the two NB’s have higher quality than the SB ($\alpha > \alpha_S$). Table 4(b) shows that if the SB’s quality is higher than the weaker NB ($\alpha_S > \alpha$), the retailer can completely displace NB2 with the SB by positioning the SB close to NB2 (i.e., $x$ is close to 1), resulting in $q_2 = 0$. In this way, we find situations similar to SZ where only two brands are sold even after the SB introduction, although we do not require shelf space scarcity to obtain this result. More importantly, when $\alpha_S > \alpha$, FO2 is more profitable than FO1 and BG2 can be more profitable than BG1. Consequently, if $\alpha_S > \alpha$ and $d$ is too large for the retailer to use the BG strategy profitably, the optimal strategy is FO2 with $x = 1$.

3.3. General case

3.3.1. Analysis method. Our analyses of the two polar cases for $d$ provide initial answers to the three research questions we listed in the beginning of the paper. First, we have shown that it is not always optimal for the retailer to position the SB as close to the leading NB as possible. Second, we find there are three generic SB positioning strategies (i.e., close to the leading NB, close to the weaker NB, or in the middle) and two generic SB pricing strategies (same as or below the monopoly price) that, in combination, yield a number of distinct category management strategies. Third, the choice of the best category strategy depends on the degree of horizontal differentiation of the two NB’s as well as the relative quality levels of the three competing brands.
We next generalize these findings by analyzing situations between the two polar cases. Due to the complexity associated with our kinked demand structure, we take a numerical approach for obtaining equilibrium solutions for the general case. Specifically, we first specify proper boundary conditions for each demand region and then use a quadratic programming algorithm to solve for the profit maximizing set of quantities (and thus prices) for the region conditional upon given levels of \( w_1, w_2, x, d, \alpha \) and \( \alpha_S \). We then select the solution from the region that yields the global maximum category profits. This efficiently yields the retailer’s optimal category pricing response to a particular vector of wholesale prices and brand positions. Once the optimal retailer pricing response is identified, we are able to obtain the manufacturer level equilibrium solutions as well as the optimal store brand positions using a numerical optimization procedure described in Technical Appendix 3.

Note that the equilibrium solution obtained from this numerical procedure is equivalent to an analytically derived solution although it only pertains to the specific set of parameter values used. Consequently, in order to produce sufficiently generalizable results, we perform our analysis following an “experimental design” over a wide range of feasible values for the three key model parameters, \( d, \alpha \) and \( \alpha_S \). A preliminary analysis revealed that the closed form solutions found in Table 4 hold for \( d \geq .65 \). Therefore, we vary \( d \) between 0 and .65, letting it take four levels, .1, .175, .25 and .5. For the other two parameters, we found it sufficient to vary \( \alpha \) across the four values .7, .8, .9, and 1 and \( \alpha_S \) across the six values, .5, .6, .7, .8, .9 and 1. This allows us to span the relevant “Before SB” situations by analyzing 16 (4 levels of \( d \) times 4 levels of \( \alpha \)) different parameter settings and the relevant “After SB” situations by analyzing, 96 (4 \( \times \) 4 \( \times \) 6) different cases.

### 3.3.2. Results

Table 5 shows the conditions under which the retailer finds it optimal to utilize each of the four strategies over this wider range of settings analyzed numerically as well as the polar extremes analyzed mathematically. The table shows that the numerically obtained solutions blend in nicely with the mathematically obtained solutions. For instance, the mapping of optimal strategies for \( d = .5 \) is similar to that for \( d = .65 \), the only difference being that the retailer can use the Bridge strategy with a lower quality SB since the “gap” is smaller. In the same way, the mapping of optimal strategies for \( d = .1 \) is very similar to that for \( d = 0 \). Thus, the implications of the mathematical analysis for \( d = 0 \) generalize to low values of \( d \). For the remaining two levels of \( d \), the mapping reflects a gradual transition between more extreme values of \( d \).

Interestingly, when we use our second model of buyer behavior (i.e., where consumers’ willingness to pay for quality is independent of their willingness to pay for product match), we find the same general pattern of strategies. The only difference is the correspondence

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14 The retailer’s profit can be expressed as a quadratic function of prices (or quantities).

15 We solved the manufacturer level pricing game in two ways, once with the NB1 manufacturer being the Stackelberg leader to ensure the existence of a unique equilibrium and once by assuming a Bertrand Nash game. Except for the fact that in a few instances we did not find a unique equilibrium in the Bertrand game, the two game rules produced very similar results. Thus we only report the Stackelberg game results.

16 Since our second model of buyer behavior is less competitive we found that values of \( d > .40 \) yielded monopoly solutions. Consequently we let \( d \) take on the values .40, .175, .1 and .05 when analyzing this model.
### Table 5. Effects of \(d\), \(\alpha\) and \(\alpha_S\) on optimal category management strategy.

<table>
<thead>
<tr>
<th>(d)</th>
<th>(\alpha)</th>
<th>(\alpha_S = .5)</th>
<th>(\alpha_S = .6)</th>
<th>(\alpha_S = .7)</th>
<th>(\alpha_S = .8)</th>
<th>(\alpha_S = .9)</th>
<th>(\alpha_S = 1)</th>
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<td>ST2</td>
<td>ST2</td>
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<td>Ab*</td>
<td>.90</td>
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<tr>
<td></td>
<td>.8</td>
<td>ST2</td>
<td>ST2</td>
<td>Ab*</td>
<td>Ab*</td>
<td>Ab*</td>
<td>ST2</td>
<td>.93</td>
</tr>
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<td>ST2</td>
<td>ST2</td>
<td>ST2</td>
<td>Ab*</td>
<td>.97</td>
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<td>Ab*</td>
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<td>Ab*</td>
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The case of \(d = 0\) and \(\alpha = 1\) is not included since, in this perfect competition situation, there is no need for a SB.

*NB2 is completely displaced.

**Based on closed form solutions.

BGj: “Bridge” by attacking NBj and threatening NB3-j; BGb: “Bridge” by attacking both NB’s; FO: “Focus” by attacking one NB and leaving the other NB unthreatened; STj: “Stimulate” NBj to attack NB3-j; Aj: “Attack” NBj while threatening NB3-j; Ab: “Attack” both NB’s.

between the specific parameter values and the optimality of a given strategy. This difference occurs because the same parameter values imply different buyer behavior and thus different demand characteristics which in turn lead to different levels of competition between the two NB’s. Since the optimality of a given strategy depends on this level of competition, we next introduce a measure of competitive intensity which not only can be used to relate our findings from the two different buyer behavior models, but also provide testable hypotheses on when each of the identified strategies is the best.

### 3.4. Impact of competitive intensity on optimal SB strategies

Conceptually, competitive intensity prior to the introduction of the SB depends on the degrees of vertical and horizontal differentiation between the two NB’s, the consumers’ taste
distribution, and the consumers’ choice rules. One measure that captures all these factors, is
easy to obtain empirically and analytically, and has theoretical underpinnings is the percent
of current buyers who find both NB’s to be viable options (i.e., who receive positive net
utilities from both NB’s) at the equilibrium retail prices. Such a measure is conceptually
similar to the proportion of consumers whose consideration sets contain multiple brands.
When this measure equals zero, each brand enjoys a “monopoly” position since there are
no viable alternatives for the current customers other than the one they purchased. Likewise
when this measure equals 100%, all current buyers are potential switchers, finding both
brands acceptable.

This measure of competitive intensity is given in the last column of Table 5 for various
values of \( d \) and \( \alpha \) based on equation (2) buyer behavior. As shown in the table, this measure
varies with both the horizontal and vertical differentiation between the NB’s. This same
general pattern was found for the alternative buyer behavior model (equation (3)), although
the mapping from competitive intensity to specific values of \( d \) and \( \alpha \) differed in a systematic
fashion.

More importantly, as seen in Table 6, we found a very strong correspondence for both
sets of buyer behavior results between the competitive intensity measure and the optimal
category management strategy. Thus, under both buyer behavior models the retailer uses
Focus or Bridge when our competitive intensity measure is zero. For moderate levels of
competitive intensity (i.e., for values greater than zero but less than .6), the results from both

<table>
<thead>
<tr>
<th>Table 6. Category conditions and retailer actions associated with each category management strategy.</th>
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<tbody>
<tr>
<td>Strategy</td>
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</table>
| Focus FO          | Competitive intensity: Low \(\alpha_{S}\)
                   | Product quality: 0 or 1 near middle
                   |                               | Always 1 (.73 on average)       |
| Bridge BG         | Competitive intensity: Low \(\alpha_{S}\)
                   | Product quality: 0 or 1 near middle
                   |                               | Always 1 (.73 on average)       |
| Attack A1/A2**    | Competitive intensity: Moderate \(\alpha_{S}\)
                   | Product quality: low \(\alpha_{S}\)
                   |                               | .97 on average (.88~1.05)       |
|                  | Competitive intensity: Moderate \(\alpha_{S}\)
                   | Product quality: low \(\alpha_{S}\)
                   |                               | .97 on average (.88~1.05)       |
|                  | Competitive intensity: High \(\alpha_{S}\)
                   | Product quality: high \(\alpha_{S}\)
                   |                               | .94 on average (.89~1.05)       |
| Attack A2         | Competitive intensity: Moderate \(\alpha_{S}\)
                   | Product quality: low \(\alpha_{S}\)
                   |                               | .97 on average (.88~1.05)       |
|                  | Competitive intensity: Moderate \(\alpha_{S}\)
                   | Product quality: low \(\alpha_{S}\)
                   |                               | .97 on average (.88~1.05)       |
|                  | Competitive intensity: High \(\alpha_{S}\)
                   | Product quality: high \(\alpha_{S}\)
                   |                               | .94 on average (.89~1.05)       |
| Attack Ab         | Competitive intensity: Moderate \(\alpha_{S}\)
                   | Product quality: low \(\alpha_{S}\)
                   |                               | .97 on average (.88~1.05)       |
|                  | Competitive intensity: Moderate \(\alpha_{S}\)
                   | Product quality: low \(\alpha_{S}\)
                   |                               | .97 on average (.88~1.05)       |
|                  | Competitive intensity: High \(\alpha_{S}\)
                   | Product quality: high \(\alpha_{S}\)
                   |                               | .94 on average (.89~1.05)       |
| Attack ST1        | Competitive intensity: Moderate \(\alpha_{S}\)
                   | Product quality: low \(\alpha_{S}\)
                   |                               | .97 on average (.88~1.05)       |
|                  | Competitive intensity: Moderate \(\alpha_{S}\)
                   | Product quality: low \(\alpha_{S}\)
                   |                               | .97 on average (.88~1.05)       |
|                  | Competitive intensity: High \(\alpha_{S}\)
                   | Product quality: high \(\alpha_{S}\)
                   |                               | .94 on average (.89~1.05)       |
| Attack ST2        | Competitive intensity: Moderate \(\alpha_{S}\)
                   | Product quality: low \(\alpha_{S}\)
                   |                               | .97 on average (.88~1.05)       |
|                  | Competitive intensity: Moderate \(\alpha_{S}\)
                   | Product quality: low \(\alpha_{S}\)
                   |                               | .97 on average (.88~1.05)       |
|                  | Competitive intensity: High \(\alpha_{S}\)
                   | Product quality: high \(\alpha_{S}\)
                   |                               | .94 on average (.89~1.05)       |

\* Ratio of the equilibrium SB price to what the retailer would charge if the SB is in monopoly \((p_{S}^{MON}/p_{MON}^{*})\).
\** When A1 is the optimal strategy, A2 is also optimal.
\*** Ab or ST2 is the optimal strategy when there exists little or no horizontal differentiation between
the NBs. Consequently, the retailer’s choice of \(x\) has little or no impact on both SB position and retail
category profits.
buyer behavior models indicate that the retailer finds it best to attack the weaker NB and threaten the stronger NB (A2) unless both the weaker NB and the SB are very low quality (ST1) or there is no quality difference in the two NB’s (A1 or A2). When the competitive intensity is high prior to the SB introduction (i.e., greater than .6), both models have the retailer using the Stimulate strategy against the weaker NB (ST2) if the quality level of the SB is the lowest of the three brands, and the Attack Both (Ab) strategy if the SB is the middle quality brand.

Table 6 also summarizes the actual positioning and pricing actions associated with each strategy. As in the polar cases, we find many instances where the optimal SB position is closer to the weaker NB. In addition, as seen before, the SB is almost always priced close to its monopoly price except when the retailer is using the Bridge strategy. Here we note that the SB is priced low enough to entice customers from both market segments to consider the SB. The degree to which the price must be lowered depends on the degree of differentiation between the two NB’s and the quality level of the SB.

### 3.5. Impact of store brand introduction

We further analyzed our results to better understand the sources of the retailer’s benefit of introducing a SB by decomposing the retailer’s profit enhancement in the 138 different cells found in Table 5 in two different ways. The results are summarized in Table 7(a).

The first three columns of Table 7(a) show the relative contribution of each of three sources to the retailer’s profit improvement due to the introduction of a SB. The relative contribution of the first source, improved retail margins from the NB’s, is remarkably stable.
across different levels of competitive intensity. In contrast, the relative contributions of the other two sources, changes in the NB quantity and the increases due to the SB sales, are highly sensitive to the level of competitive intensity. When this measure is greater than .6, the SB’s entry cannibalizes sales from the two NB’s, resulting in a net decrease in retail profits from these products. However, this loss is more than compensated for by the profits from the SB sales. In contrast, when the NB’s are at least moderately differentiated (competitive intensity is less than .6), the SB’s entry leads to an increased NB quantity, allowing the retailer to benefit positively from all three sources. In any case, for our setting the largest contribution, on average, comes from the SB sales, which is in contrast to SHR and SZ. Their models suggest the major benefit comes from a reduction of the NB manufacturers’ margins.

Another way of understanding the impact of the SB on retailer category profits is to decompose this impact into changes in total category profits and changes in the retailer’s share of channel profits. As seen in the last two columns of Table 7(a), we find the majority of the retail profit improvement comes from the increased share of the total channel profits for the retailer. This is due to the fact that the SB’s entry not only increases the retailer’s margins on the NB’s but also allows the retailer to keep 100% of channel profit associated with selling SB. In addition, we find the SB entry normally enhances channel coordination thereby increasing the total channel profits especially when the competitive intensity is moderate to low. (In a few cases when the competitive intensity is very high, a small decrease in total channel profits is observed after SB’s entry.\footnote{In contrast, Narasimhan and Wilcox (1998) find the SB introduction always decreases total channel profits. Their result is due to their assumption of fixed total category demand, which rules out the presence of the double marginalization problem.}) This indicates that the SB’s entry generally improves channel coordination but this effect is small or even negative when there already exists intense competition between the NB’s.

Table 7(b) provides more details on how these increases in profits occur. We note that in almost every case NB\textsubscript{2} is affected more than NB\textsubscript{1}, both in terms of wholesale price and retail price. This is counter to the findings of SHR but consistent with the empirical finding of Pauwels and Srinivasan (2004). The only exceptions are found when the retailer uses FO\textsubscript{1} or ST\textsubscript{1}. In the latter case the retailer not only positions its SB near NB\textsubscript{1}, but also lowers the retail price of NB\textsubscript{1} more than for NB\textsubscript{2} in order to have NB\textsubscript{1} attack NB\textsubscript{2}’s target market. In short, the diverse set of retail category management strategies provides intuitive explanations for why the empirically observed impact of a SB entry on existing NB’s varies across categories. Finally we see that the SB entry is most beneficial to the retailer when the NB’s are moderately differentiated since such a condition provides the opportunity for the SB to significantly increase competitive pressure on both NB’s.

4. Discussion

Our main message is that it is not always optimal for a retailer to position its SB against the leading NB; instead it is often optimal to position the SB close to the weaker NB or to introduce a SB that appeals to both market segments. In deciding where to position the
SB and how to price each of the brands, the retailer needs to consider how to “play in” the most favorable demand region thereby reshaping the pattern of intra-category brand competition. We identify four distinct types of category management strategies. We also demonstrate that the optimal strategy depends upon the degree of competitive intensity between the two NB’s prior to the introduction of the SB and the relative quality levels of the three brands. Moreover, these relationships are robust to two different assumptions of buyer behavior. Since both our measure of competitive intensity and product quality are easy to obtain empirically, we believe our results as summarized in Table 6 can be readily tested empirically and implemented in practice.

Consistent with previous analytical studies, we find a major benefit of introducing a store brand is increased retail margins on the NB’s. However, we show the retailer can also benefit from increases in NB unit sales caused by a lower retail price and profits generated from the SB sales. In fact we find this latter source of profits to be the largest of the three sources in many situations. We also find that the retailer normally finds it best to price its SB near its monopoly price. The only exception to this is when the NB’s are highly differentiated and the retailer finds it best to use the Bridge strategy in order to impact both NB’s. In addition we find a SB is most beneficial when there was only moderate competitive intensity between the two NBs prior to the introduction of the SB.

One might ask why our results are different from those of SZ’s and SHR’s despite our model’s similarities to theirs. The answer to this question rests in the few new features incorporated in our model. Although SZ’s market model is similar to ours, they only allow the retailer to carry two brands in the category and to position the store brand at one of the two segment ideal points (i.e., \( x = 0 \) or \( x = 1 \)). In addition, they assume retail prices are set with complete channel coordination and the resulting channel profits are split between the retailer and the manufacturers via a bargaining game. In contrast, we follow SHR’s lead and assume double marginalization within the channel.

The difference between our results and SHR’s is a result of the two studies using different demand models, since all other aspects of the analysis remain almost identical. These two demand models differ in two important ways. First, we derive a demand structure from an explicit market model. This yields a kinked demand function. In contrast SHR assume a demand model that is continuously linear over the total range of prices. As a result they were not able to uncover the four category management strategies reported in this paper. Second, SHR’s demand specification restricts their cross price effects to be no higher than \( 1/3 \) of the own price effects. In contrast we allow the cross price effects to vary from zero to one in relation to the own price effects. Consequently their analyzed markets do not capture the competitive intensity found in situations with highly substitutable brands, thereby limiting the generalizability of their results. In contrast, our results span the total competitive environment.

Interestingly, when we assume complete channel coordination, we also find the optimal strategy is always to sell just two brands and that, if a NB is displaced, it is always the weaker NB. However, the optimal SB position, if it is introduced, is \( x = 1 \), not \( x = 0 \) as concluded by SZ.

By assuming a larger size for segment 1 and limiting the competitive intensity to low levels in our model, our analysis replicated SHR’s result of optimal SB position at \( x = 0 \).
With this said, any generalization of our findings also must be done after considering the
simplifying assumptions in our model. For one, our model consists of two discrete segments
of consumers in terms of tastes. This is an abstraction of a more realistic situation where
consumer tastes follow a continuous bimodal distribution. As long as the two modes are
sufficiently pronounced (i.e., well-defined clusters of consumer tastes exists), we believe
our findings should hold. However, it is not clear how well our results will apply to the
case of a more uniform distribution of tastes. Analyzing such a case will require not only
changing the consumer taste distribution assumption but also determining the equilibrium
number and positions of national brands before the SB entry.

We also acknowledge that our model results do not capture the size asymmetry between
the two consumer segments, which is considered in SZ’s and SHR’s models. When we
reanalyzed our model by allowing for this target market size asymmetry (while holding
$\alpha = 1$) we found a few differences in the optimal product positioning ($x^*$) but little difference
in our basic findings on optimal category management strategy. Thus, we do not believe
this set of assumptions is driving our main results.

Our model is also silent on specifically how a retailer goes about positioning the SB in
the real world. Clearly the retailer can use such tools as local advertising, shelf location,
and package design to ‘locate’ the store brand in the minds of the consumer. In addition,
the retailer might be able to specify certain product characteristics the SB manufacturer
must deliver. For example Food Lion, a major regional grocery chain, carries two major
national brands of yogurt, Dannon and Yoplait, as well as its own SB. Dannon has fruit
at the bottom and comes in 8 oz. containers. Yoplait blends the fruit with the yogurt and
comes in a distinctive 6 oz. container. The SB comes in a container very similar to Dannon’s
8 oz. container, but the yogurt is blended with the fruit. Casual empiricism would classify
this SB position as being in the middle of two differentiated NB’s. One might also argue
that a retailer cannot always position the SB at any desired position due to the retailer’s
need for establishing a consistent image for all of its SB’s. However, horizontal features are
usually category-specific as seen in the yogurt example while a storewide image is often
vertically defined (quality level). Therefore, the category specific SB positioning strategy
and the storewide SB positioning strategy do not have to be linked.

Finally, our analysis is from a monopolist retailer’s point of view, ignoring the possible
role of SB’s as a competitive strategic tool against other retailers. Considering competing
retailers, each with the ability to introduce a store brand, raises a number of interesting
questions regarding a retailer’s store brand strategy. If one retailer introduces a store brand,
what would be the best reaction by the competing retailer? What would be the resulting
equilibrium? Could it be that the rapid proliferation of store brands is partially due to retail
competition following the pattern of the “prisoner’s dilemma”? These questions, as well
as the empirical testing of the implications of our findings using our proposed measure of
competitive intensity, represent interesting future research directions.

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References

Appendix 1
Proof of Lemma 1

Define $\gamma_j = v_j - m_j$, where $v_j$ and $m_j$ measure the quality and the degree of mismatch for brand $j$ perceived by the consumers of a certain segment. Suppose brand $a$ is evaluated higher than brand $b$ by the buyers of this segment before prices are considered. Thus:

$$\gamma_a > \gamma_b.$$  \hspace{1cm} (A1)

Consider two buyers, $h$ and $i$, belonging to this segment. Let buyer $h$ have higher willingness to pay than buyer $i$, i.e.,:

$$\beta_h > \beta_i.$$  \hspace{1cm} (A2)

If buyer $h$ purchases brand $b$, it must be because $U_{hh} = \beta_h \gamma_b - p_b > U_{ha} = \beta_h \gamma_a - p_a$. Thus,

$$\beta_h < \frac{p_a - p_b}{\gamma_a - \gamma_b}. \hspace{1cm} (A3)$$

From Equations A2 and A3, we get

$$\beta_i < \frac{p_a - p_b}{\gamma_a - \gamma_b}. \hspace{1cm} (A4)$$

From (A4), one can see that

$$p_a - p_b > \beta_i (\gamma_a - \gamma_b),$$

which can be rearranged as

$$\beta_i \gamma_b - p_b > \beta_i \gamma_a - p_a.$$  \hspace{1cm} (A5)

Thus,

$$U_{ih} = \beta_i \gamma_b - p_b > U_{ia} = \beta_i \gamma_a - p_a.$$  \hspace{1cm} (A6)

Therefore, buyer $i$, who has lower willingness to pay than buyer $h$, will not buy brand $a$, the higher ordered brand of the two.
Technical Appendix 1
Derivation of Closed Form Solutions for $d = 0$

Before the SB entry, the demand functions for the two NBs’ are:

\[ q_1 = 2(1 - \frac{p_1 - p_2}{1 - \alpha}) \quad \text{and} \quad q_2 = 2\left(\frac{p_1 - p_2}{1 - \alpha} - \frac{P_2}{\alpha}\right). \quad (1-1) \]

By substituting these demand functions into the retailer’s profit function,

\[ \Pi_R = (p_1 - w_1)q_1 + (p_2 - w_2)q_2, \]

and solving the resultant first order conditions, we obtain the following reaction functions for the retailer:

\[ p_1^* = \frac{1 + w_1}{2} \quad (1-3) \]
\[ p_2^* = \frac{\alpha + w_2}{2} \quad (1-4) \]

By substituting these reactions functions into the manufacturers’ profit functions,

\[ \Pi_{M1} = w_1 q_1 \quad \text{and} \quad \Pi_{M2} = w_2 q_2, \]

and solving the resultant first order conditions, we obtain the equilibrium solutions shown in Table 3.

Next, we consider the case after the SB entry, when $1 > \alpha > \alpha_S$. With three brands available for the retailer to include in its assortment, there exist 7 possible demand structures, as indicated in Table 2. However, at equilibrium, all three brands receive positive quantities because it can be shown that 1) the retailer profits are higher with SB than without SB in the assortment and that 2) a manufacturer has an obvious incentive to set its wholesale price low enough to avoid selling zero quantity. Therefore, the relevant demand structure is as follows:

\[ q_1 = 2(1 - \frac{p_1 - p_2}{1 - \alpha}) \quad (1-5) \]
\[ q_2 = 2\left(\frac{p_1 - p_2}{1 - \alpha} - \frac{p_1 - p_2 - p_S}{\alpha - \alpha_S}\right) \quad (1-6) \]
\[ q_S = 2\left(\frac{p_2 - p_S}{\alpha - \alpha_S} - \frac{p_S}{\alpha_S}\right) \quad (1-7) \]

which leads to the equilibrium solutions presented in Table 3 via the standard mathematical process outlined above.

For the case of $1 > \alpha_S \geq \alpha$, once again it is true that 1) the retailer profits are higher with SB than without SB in the assortment and that 2) a manufacturer has an obvious incentive to set its wholesale price low enough to avoid selling zero quantity. This leads to the following demand structure:
\[ q_1 = 2(1 - \frac{p_1 - p_S}{1 - \alpha_S}) \quad (1-8) \]
\[ q_2 = 2\frac{p_S - p_2 - p_2}{\alpha - \alpha} \quad (1-9) \]
\[ q_S = 2\frac{p_1 - p_S - p_s - p_2}{1 - \alpha_S - \alpha} \quad (1-10) \]

From this demand structure, we derive (via the same mathematical approach used above) the following equilibrium solution:

\[ w_1 = \frac{1 - \alpha_s}{2}, \quad w_2 = 0, \quad p_1 = \frac{3 - \alpha_s}{4}, \quad p_2 = \frac{\alpha}{2}, \quad p_s = \frac{\alpha_s}{2}, \quad q_1 = q_s = \frac{1}{4}, \quad \text{and} \quad q_2 = 0. \]

This indicates that, even though manufacturer 2 tries its best \( w_2 = 0 \) to sell a positive quantity, the retailer finds it optimal to sell only NB1 and SB and, thus, eliminates NB2 from its assortment. Given this, it is not surprising that the exactly same equilibrium solution can be derived from the demand structure composed of NB1 and SB only as follows:

\[ q_1 = 2(1 - \frac{p_1 - p_S}{1 - \alpha_S}) \text{ and } q_s = 2\frac{p_1 - p_S - p_s - p_2}{1 - \alpha_S - \alpha}. \]
Technical Appendix 2
Derivation of Closed Form Solutions for a Sufficiently Large $d$

In this technical appendix, we use the following assumptions:

1. $\alpha, \alpha_s, d \leq 1$
2. $d$ is sufficiently large to ensure that NB1 is the most preferred in segment 1 and NB2 is the most preferred in segment 2 (i.e., $1 - d < \alpha$ and $\alpha_s < \alpha$).
3. $d$ is sufficiently large to ensure that each manufacturer focuses on its own target segment (ruling out NB1 selling in seg. 2 and vice versa). Therefore, before the SB entry, the equilibrium is localized monopoly.
4. $\alpha_s > d/2$ so that SB can sell to both segments, if the retailer chooses to do so.

The equilibrium condition before the SB entry is two localized bilateral monopolies. The solutions are easily obtained from $q_1 = 1 - p_1$ for NB1 and $q_2 = 1 - \frac{p_2}{\alpha}$ for NB2.

When SB is introduced, if R chooses to sell SB only to segment 1, the demand structure is:

\begin{align*}
q_1 &= 1 - \frac{p_1 - p_s}{1 - \alpha_s + xd}, \\
q_2 &= 1 - \frac{p_2}{\alpha}, \\
q_s &= \frac{p_1 - p_s}{1 - \alpha_s + xd} - \frac{p_s}{\alpha_s - xd}
\end{align*}

Then,

\[\Pi_R = q_1(p_1 - w_1) + q_2(p_2 - w_2) + q_s p_s.\]

Solving the FOC’s yields the following reactions functions:

\begin{align*}
p_1^* &= \frac{1 + w_1}{2}, \\
p_2^* &= \frac{\alpha + w_2}{2}, \\
p_s^* &= \frac{\alpha_s - xd}{2}
\end{align*}

Let’s call this (the set of Eq. 2-5, 2-6 and 2-7) “A1” reaction.

If R chooses to sell SB only to segment 2, the demand structure is:

\[q_1 = 1 - p_1,\]
\[
q_z = 1 - \frac{p_z - p_s}{\alpha - \alpha_s + (1-x)d}, \quad (2-9)
\]
\[
q_s = \frac{p_z - p_s}{\alpha - \alpha_s + (1-x)d} - \frac{p_s}{\alpha_s - (1-x)d} \quad (2-10)
\]

Solving the FOC's under this demand structure yields the following reactions functions:

\[
p_1^* = \frac{1 + w_1}{2} \quad \text{(same as 2-5)}
\]
\[
p_2^* = \frac{\alpha + w_2}{2} \quad \text{(same as 2-6)}
\]
\[
p_s^* = \frac{\alpha_s - (1-x)d}{2} \quad (2-11)
\]

Let's call this (the set of Eq. 2-5, 2-6 and 2-11) “A2” reaction.

If R chooses to sell SB to both segments, the demand structure is:

\[
q_1 = 1 - \frac{p_1 - p_s}{1 - \alpha_s + xd}, \quad (2-12)
\]
\[
q_2 = 1 - \frac{p_2 - p_s}{\alpha - \alpha_s + (1-x)d}, \quad (2-13)
\]
\[
q_s = \frac{p_1 - p_s}{1 - \alpha_s + xd} - \frac{p_s}{\alpha_s - xd} + \frac{p_2 - p_s}{\alpha - \alpha_s + (1-x)d} - \frac{p_s}{\alpha_s - (1-x)d} \quad (2-14)
\]

Solving the FOC's under this demand structure, we get:

\[
p_1^{**} = \frac{2\alpha_s - \alpha_s d - d + 2\alpha_s xd + xd^2 - 2x^2d^2}{2(2\alpha_s - d)} + \frac{w_1}{2} \quad (2-15)
\]
\[
p_2^{**} = \frac{2\alpha_s - \alpha_s d - d + 2\alpha_s xd + xd^2 - 2x^2d^2}{2(2\alpha_s - d)} + \frac{w_2}{2} \quad (2-16)
\]
\[
p_s^{**} = \frac{\alpha_s(\alpha_s - d) + xd^2(1-x)}{2\alpha_s - d} \quad (2-17)
\]

Let’s call this “AB” reaction.

Now, R’s choice among A1, A2, and AB depends upon 1) whether a particular reaction is feasible (i.e., the resulting demand structure is as intended by R) and 2) whether a particular reaction is more profitable than the other two.

In order for A1 to be feasible, the following conditions must hold:

\[
\frac{p_1^* - p_s^*}{1 - \alpha_s + xd} > \frac{p_s^*}{\alpha_s - xd} \quad (2-18)
\]

(i.e., marginal consumer between NB1 and SB has higher \(\beta\) than marg. consumer between SB and no purchase. This ensures SB sells to segment 1.)
\[
\frac{p_2^* - p_S^*}{\alpha - \alpha_s + (1-x)d} < \frac{p_S^*}{\alpha_s - (1-x)d}
\]  \tag{2-19}

(i.e., marginal consumer between NB2 and SB has lower \(\beta\) than marg. consumer between SB and no purchase. This ensures SB doesn’t sell to segment 2)

Plugging (2-5) and (2-7) into (2-18), one can easily show that (2-18) is true for any positive \(w_1\).

Plugging (2-6) and (2-7) into (2-19) yields:
\[
w_2 < \frac{\alpha d(1-2x)}{\alpha_s - (1-x)d} = A
\]  \tag{2-20}

Note that the above condition can exist only if \(x<1/2\) (since \(w_2>0\) and \(0<x<1\)).

In order for \(A2\) to be feasible, the following conditions must hold:
\[
\frac{p_2^{**} - p_S^{**}}{\alpha - \alpha_s + (1-x)d} > \frac{p_S^{**}}{\alpha_s - (1-x)d}
\]  \tag{2-21}

(i.e., marginal consumer between NB2 and SB has higher \(\beta\) than marg. consumer between SB and no purchase. This ensures SB sells to segment 2.)

Plugging (2-6) and (2-11) into (2-21), one can easily show that (2-21) is true for any positive \(w_2\).

Plugging (2-5) and (2-11) into (2-22) yields:
\[
w_1 < \frac{(2x-1)d}{\alpha_s - xd} = B
\]  \tag{2-23}

Note that the above condition can exist only if \(x>1/2\) (since \(w_1>0\) and \(0<x<1\)).

Eq. 2-20 and 2-23 jointly imply that when SB is positioned at \(x = 1/2\) (\& \(d\) is constrained to be smaller than \(\alpha_s\)), NB manufacturers can stop SB attacking both only by setting \(w = 0\). Thus, AB is the only relevant reaction function at \(x=1/2\).

In general, for AB to be feasible, the following conditions must hold:
\[
\frac{p_1^{***} - p_S^{***}}{1-\alpha_s + XD} < \frac{p_S^{***}}{\alpha_s - XD}
\]  \tag{2-24}

(i.e., marginal consumer between NB1 and SB has lower \(\beta\) than marg. consumer between SB and no purchase. This ensures SB doesn’t sell to segment 1)

Plugging (2-6) and (2-11) into (2-24), one can easily show that (2-24) is true for any positive \(w_2\).

Plugging (2-5) and (2-11) into (2-25) yields:
\[
w_1 < \frac{(2x-1)d}{\alpha_s - xd} = B
\]  \tag{2-23}

Note that the above condition can exist only if \(x>1/2\) (since \(w_1>0\) and \(0<x<1\)).

Eq. 2-20 and 2-23 jointly imply that when SB is positioned at \(x = 1/2\) (\& \(d\) is constrained to be smaller than \(\alpha_s\)), NB manufacturers can stop SB attacking both only by setting \(w = 0\). Thus, AB is the only relevant reaction function at \(x=1/2\).

In general, for AB to be feasible, the following conditions must hold:
\[
\frac{p_2^{***} - p_S^{***}}{\alpha - \alpha_s + (1-x)d} > \frac{p_S^{***}}{\alpha_s - (1-x)d}
\]  \tag{2-25}
(i.e., marginal consumer between NB2 and SB has higher $\beta$ than marg. consumer between SB and no purchase. This ensures SB sells to segment 2.)

By plugging (2-15), (2-16) and (2-17) into (2-24) and (2-25), we obtain:

$$w_1 > \frac{d(2x-1)(1-\alpha_s+xd)}{2\alpha_s-d} = C$$

$$w_2 > \frac{d(1-2x)(\alpha-\alpha_s+(1-x)d)}{2\alpha_s-d} = D$$

(2-26)

(2-27)

Note that these two conditions are guaranteed to hold for $x = \frac{1}{2}$. They also imply that $R$ should be able to employ AB as long as $x$ is reasonably in the middle.

Comparing eq. 2-20 and 2-27 reveals $A > D$. Therefore, if $x < \frac{1}{2}$, $R$’s reaction is:

1. $A1$ if $w_2 < D$
2. $A1$ or AB depending on profits if $D \leq w_2 \leq A$, and
3. AB if $w_2 > A$.

Further investigating condition 2 above, we compare

$$\Pi_s = q_1(p_1-w_1) + q_2(p_2-w_2) + q_s p_s$$

between A1 reaction and AB reaction. This reveals:

$R$ prefers $A1$ if $w_2 < \overline{w}_2$ and AB if $w_2 > \overline{w}_2$ where

$$\overline{w}_2 = \frac{(1-2x)d\sqrt{\alpha(\alpha_s-(1-x)d)(2\alpha_s-d)(\alpha-\alpha_s+(1-x)d)}}{(\alpha_s-(1-x)d)(2\alpha_s-d)}$$

(2-28)

It can be shown that $D < \overline{w}_2 < A$.

Therefore, when $x < \frac{1}{2}$, $R$’s best reaction is

$$A1 \text{ for } w_2 \leq \overline{w}_2 \text{ and AB for } w_2 > \overline{w}_2.$$  

Knowing the retailer’s reaction function, we now solve the Bertrand Nash game between the two M’s.

For M1, if $w_2 \leq \overline{w}_2$, M1 knows the retailer’s reaction is A1. Thus, its profit function is

$$\Pi_{M1} = w_1(1-\frac{p_1^* - p_s^*}{1-\alpha_s + xd}).$$

From the resulting FOC, we derive, $w_1^* = \frac{1-\alpha_s + xd}{2}$.

Similarly, by solving the FOC from

$$\Pi_{M2} = w_2(1-\frac{p_2^*}{\alpha}),$$

we get, $w_2^* = \frac{\alpha}{2}$.
Thus, if \( w_2^* = \frac{\alpha}{2} \leq w_2 \),
\[
w_1^* = \frac{1 - \alpha_s + xd}{2},
\]
\[
w_2^* = \frac{\alpha}{2},
\]
\[
p_1 = \frac{1 + w_1^*}{2} = \frac{3 - \alpha_s + xd}{4},
p_2 = \frac{\alpha + w_2^*}{2} = \frac{3\alpha}{4}
\] and \( p_s = \frac{\alpha_s - xd}{2} \)
is the Nash equilibrium. This is the “**FOCUS 1**” solution. Note that the resulting quantities are \( q_1 = q_2 = q_s = .25 \). Also note that R’s profit under Focus 1 is maximized at \( x = 0 \) since retail margin on NB1 and SB are decreasing in \( x \). Thus, if R is to employ Focus 1, it should position SB at \( x = 0 \).

If \( w_2 > \bar{w}_2 \), M1 knows the retailer’s reaction is AB. Thus, its profit function is
\[
\Pi_{M1} = w_1(1 - \frac{p_1^{***} - p_s^{***}}{1 - \alpha_s + xd}),
\]
the FOC of which leads to \( w_1^{***} = \frac{1 - \alpha_s + xd}{2} \).

For M2, \( \Pi_{M2} = w_2(1 - \frac{p_2^{***} - p_s^{***}}{\alpha - \alpha_s + (1-x)d}) \), the FOC of which yields
\[
w_2^{***} = \frac{\alpha - \alpha_s + (1-x)d}{2}.
\]

Thus,
\[
p_1^{***} = \frac{6\alpha_s - 2\alpha_s^2 - 3d - \alpha_s d + 6\alpha_s xd + xd^2 - 4x^2 d^2}{4(2\alpha_s - d)},
\]
\[
p_2^{***} = \frac{6\alpha\alpha_s - 2\alpha_s^2 + 5\alpha_s d - 3\alpha d - 3d^2 - 6\alpha_s xd + 7xd^2 - 4x^2 d^2}{4(2\alpha_s - d)},
\] and
\[
p_s^{***} = \frac{\alpha_s(\alpha_s - d) + xd(1-x)}{2\alpha_s - d}.
\]

This is the “**ATTACK BOTH**” solution, which is the Nash equilibrium if \( w_2^{***} > \bar{w}_2 \).

Note that the resulting quantities are \( q_1 = q_2 = .25 \) and \( q_s = .5 \) (Total quantity of 1, as often seen in our numerical analysis).

Note that the conditions for FOCUS 1 (\( w_2^* \leq \bar{w}_2 \)) and for ATTACK BOTH (\( w_2^{***} \geq \bar{w}_2 \)) are not collectively exhaustive, since there exists a range of \( x \) that leads to
\[
w_2^{***} = \frac{\alpha - \alpha_s + (1-x)d}{2} < w_2^* = \frac{\alpha}{2}.
\]
Consequently, there exist the following three possibilities:

1) \( w_2^*, w_2^{***} \leq \bar{w}_2 \): FOCUS 1 is the solution.

2) \( \bar{w}_2 \leq w_2^*, w_2^{***} \): ATTACK BOTH is the solution.

3) \( w_2^{***} \leq \bar{w}_2 \leq w_2^*: \) The interior solutions are not feasible.
Under condition 3), in the region where \( w_2 \leq \bar{w}_2 \) (SB is not selling to seg. 2), M2 will keep raising \( w_2 \) toward \( w_2^* \). In the region where \( w_2 \geq \bar{w}_1 \) (SB sells to both segments), M2 will keep lowering \( w_2 \) toward \( w_2^{**} \). The end result is that \( w_2 = \bar{w}_2 \), where \( w_2 \) is set just low enough to keep SB from attacking segment 2.

Thus, this is the “ATTACK 1 and THREATEN 2” solution.

For \( x > 1/2 \), comparing eq. 2-23 and 2-26 reveals \( B > C \). Therefore, R’s reaction is:

1. A2 if \( w_1 < C \)
2. A2 or AB depending on profits if \( C \leq w_1 \leq B \), and
3. AB if \( w_1 > B \).

Further investigating condition 2 above, we compare

\[
\Pi_R = q_1(p_1 - w_1) + q_2(p_2 - w_2) + q_3p_3
\]

between A2 reaction and AB reaction. This reveals:

R prefers A2 if \( w_1 < \bar{w}_1 \) and AB if \( w_1 > \bar{w}_1 \) where

\[
\bar{w}_1 = \frac{(2x-1)d\sqrt{(\alpha_s - xd)(2\alpha_s - d)(1-\alpha_s + xd)}}{(\alpha_s - xd)(2\alpha_s - d)} \tag{2-29}
\]

It can be shown that \( C < \bar{w}_1 < B \).

Therefore, when \( x > 1/2 \), R’s best reaction is

\[
\text{A2 for } w_1 \leq \bar{w}_1 \text{ and AB for } w_1 > \bar{w}_1.
\]

Knowing the retailer’s reaction function, we now solve the Bertrand Nash game between the two M’s.

For M2, if \( w_1 \leq \bar{w}_1 \), M2 knows the retailer’s reaction is A2. Thus, its profit function is

\[
\Pi_{M2} = w_2(1 - \frac{p_2^{**} - p_s^{**}}{\alpha - \alpha_s + (1-x)d}).
\]

From the resulting FOC, we derive,

\[
w_2^{**} = \frac{\alpha - \alpha_s + (1-x)d}{2}.
\]

Similarly, by solving the FOC from \( \Pi_{M1} = w_1(1 - p_1^{**}) \), we get, \( w_1^{**} = \frac{1}{2} \).

Thus, if \( w_1^{**} = \frac{1}{2} \leq \bar{w}_1 \),

\[
w_1^{**} = \frac{1}{2}, \quad w_2^{**} = \frac{\alpha - \alpha_s + (1-x)d}{2},
\]
is the Nash equilibrium. This is the “FOCUS 2” solution. Note that the resulting quantities are \( q_1 = q_2 = q_S = .25 \). Also note that R’s profit under Focus 2 is maximized at \( x = 1 \) since retail margin on NB2 and SB are increasing in \( x \). Thus, if R is to employ Focus 2, it should position SB at \( x = 1 \).

The “ATTACK BOTH” solution, derived earlier, holds for \( x > 1/2 \) if \( w_1^{***} > \bar{w}_1 \).

\[
p_1^{***} = \frac{3}{4}, \quad p_2^{***} = \frac{3\alpha - \alpha_S + (1-x)d}{4} \quad \text{and} \quad p_S^{***} = \frac{\alpha_S - (1-x)d}{2}
\]

is the Nash equilibrium. This is the “FOCUS 2” solution. Note that the resulting quantities are \( q_1 = q_2 = q_S = .25 \). Also note that R’s profit under Focus 2 is maximized at \( x = 1 \) since retail margin on NB2 and SB are increasing in \( x \). Thus, if R is to employ Focus 2, it should position SB at \( x = 1 \).

The “ATTACK BOTH” solution, derived earlier, holds for \( x > 1/2 \) if \( w_1^{***} > \bar{w}_1 \).

\[
w_1^{***} = 1 - \frac{\alpha_S + xd}{2}, \quad w_2^{***} = \frac{\alpha - \alpha_S + (1-x)d}{2},
\]

\[
p_1^{***} = \frac{6\alpha_S - 2\alpha_S^2 - 3d - \alpha_S d + 6\alpha_S xd + xd^2 - 4x^2 d^2}{4(2\alpha_S - d)},
\]

\[
p_2^{***} = \frac{6\alpha_S - 2\alpha_S^2 + 5\alpha_S d - 3ad - 3d^2 - 6\alpha_S xd + 7xd^2 - 4x^2 d^2}{4(2\alpha_S - d)}, \quad \text{and}
\]

\[
p_S^{***} = \frac{\alpha_s(\alpha_S - d) + xd(1-x)}{2\alpha_S - d},
\]

with \( q_1 = q_2 = .25 \) and \( q_S = .5 \).

Since it is possible that \( w_1^{***} = \frac{1 - \alpha_S + xd}{2} < w_1^{**} = \frac{1}{2} \), there exist the following three possibilities:

1) \( w_1^{**}, w_1^{***} \leq \bar{w}_1 \): FOCUS 2 is the solution.
2) \( w_1 \leq w_1^{**}, w_1^{***} \): ATTACK BOTH is the solution.
3) \( w_1^{***} \leq \bar{w}_1 \leq w_1^{**} \): The interior solutions are not feasible.

Under condition 3), in the region where \( w_1 \leq \bar{w}_1 \) (SB is not selling to seg. 1), M1 will keep raising \( w_1 \) toward \( w_1^{**} \). In the region where \( w_1 \geq \bar{w}_1 \) (SB sells to both segments), M1 will keep lowering \( w_1 \) toward \( w_1^{***} \). The end result is that \( w_1 = \bar{w}_1 \), where \( w_1 \) is set just low enough to keep SB from attacking segment 1.

Thus, this is the “ATTACK 2 and THREATEN 1” solution.

The solutions for the case of \( 1 > \alpha_S \geq \alpha \) can be obtained through a similar process.
Numerical Algorithm for Equilibrium Analysis

The algorithm consists of four major routines solving, respectively, (1) the retailer’s pricing problem, (2) NB₂ manufacturer’s pricing problem, (3) NB₁ manufacturer’s pricing problem, and (4) the retailer’s store brand positioning problem – all for a given set of environmental parameters, \( \alpha \), \( \alpha_s \) and \( d \).

Routine I -- optimize the retailer’s pricing decision

This routine produces numerical answers to the following question: for any given pair of wholesale prices, how should the retailer set prices for NB₁, NB₂ and SB? The most straightforward way of solving this problem is to maximize retail profit over retail prices, using Equation 10-12 as the demand functions. However, since Equation 10-12 are not “well behaved”, no standard optimization procedure can guarantee global optimal. To get around this problem, we reframe the problem using reverse demand functions as a mathematically equivalent decision of how many units of each brand to sell to each segment, i.e., making six quantity decisions \([Q_{11}, Q_{21}, Q_{31}, Q_{12}, Q_{22}, Q_{32}]\). There exist 49 scenarios regarding which of the six quantities are non-negative (as implied by Table 2), each associated with a set of constraints on retail prices following Equation 10-12. For each of the 49 scenarios, the retailer’s optimization problem is characterized by linear constraints and a quadratic objective function as follows:

\[
\text{Maximize} \left[ f'(i) \cdot q - q' \cdot H(i) \cdot q \right],
\]

subject to: \( A_1(i) \cdot q \leq b_1(i) \), \( A_2(i) \cdot q = b_2(i) \), and \( q \geq 0 \)

where \( q' = [Q_{11}, Q_{21}, Q_{31}, Q_{12}, Q_{22}, Q_{32}] \). \( H(i), f(i), A_j(i) \) and \( b_j(i) \) are unique to scenario \( i \). The objective function can be rearranged as \( (f'(i) - q' \cdot H(i)) \cdot q \) in which the term \( (f'(i) - q' \cdot H(i)) \) represents the retail margin vector. The three constraints are the segment size constraint, the equal price constraint between the two segments, and the non-negative quantity constraint, respectively.
For illustration, take a scenario where the brand before price valuation ranking is NB1 > NB2 > SB in both segments, and all three brands have positive sales in both segments. The corresponding $H(i), f(i), A_1(i), A_2(i), b_1(i)$ and $b_2(i)$ are as follows:

$$H = \begin{bmatrix}
\gamma_{11} & \gamma_{21} & \gamma_{31} & 0 & 0 & 0 \\
\gamma_{21} & \gamma_{21} & \gamma_{31} & 0 & 0 & 0 \\
\gamma_{31} & \gamma_{31} & \gamma_{31} & 0 & 0 & 0 \\
0 & 0 & 0 & \gamma_{12} & \gamma_{22} & \gamma_{32} \\
0 & 0 & 0 & \gamma_{22} & \gamma_{22} & \gamma_{32} \\
0 & 0 & 0 & \gamma_{32} & \gamma_{32} & \gamma_{32}
\end{bmatrix},$$

$$f^* = [\gamma_{11} - w_1, \gamma_{21} - w_2, \gamma_{31} - w_3, \gamma_{12} - w_1, \gamma_{22} - w_2, \gamma_{32} - w_3],$$

$$A_1 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1
\end{bmatrix},$$

$$A_2 = \begin{bmatrix}
\gamma_{11} & \gamma_{21} & \gamma_{31} & -\gamma_{12} & -\gamma_{22} & -\gamma_{32} \\
\gamma_{21} & \gamma_{21} & \gamma_{31} & -\gamma_{22} & -\gamma_{22} & -\gamma_{32} \\
\gamma_{31} & \gamma_{31} & \gamma_{31} & -\gamma_{32} & -\gamma_{32} & -\gamma_{32}
\end{bmatrix},$$

$$b_1 = \begin{bmatrix} 1 \\
1
\end{bmatrix}, \text{ and } b_2 = \begin{bmatrix}
\gamma_{11} - \gamma_{12} \\
\gamma_{21} - \gamma_{22} \\
\gamma_{31} - \gamma_{32}
\end{bmatrix},$$

where $\gamma_{ij} = v_i - m_{ij}$ (i.e., quality minus mismatch) for brand $i$ in segment $j$. We solve this optimization problem for each scenario using the standard quadratic programming routine with guaranteed global optimality of the solution. (We use the QUADPROG routine from MATLAB 6.0, and codes are available on request.) Finally, we compare the optimization outputs from all the 49 scenarios, and pick the one that leads to the highest retail profit as the solution to the retailer’s pricing problem.

**Routine II -- optimize NB2 manufacturer’s pricing decision**

For any given wholesale price of NB1, the manufacturer of NB2 searches over its strategy space and picks the wholesale price that leads to the highest profit level, taking into account that retail prices will be determined through Routine I, for any given pair of wholesale prices. We mimic this search behavior by conducting a
numerical grid search. In order to guarantee that this routine produces accurate results, the search is conducted over the entire feasible range of $w_2$, from 0 (marginal cost) to 1 (the highest reservation price for any consumer), in steps of 1/2000, resulting in a near-continuous search over the range.

**Routine III -- optimize NB$_1$ manufacturer’s pricing decision**
Routine III is similar to II. We conduct an exhaustive search over a bounded and discrete strategy space for the manufacturer of NB$_1$, taking into account that $w_2$ is determined through Routine II, in which Routine I is called to determine the retail prices for each pair of wholesale prices.

**Routine IV – optimize store brand positioning**
We search over the entire feasible range of SB positioning ($0 \leq x \leq 1$) taking 21 steps with an increment of 0.05. Our investigation of the retailer profit function with respect to $x$ confirms that this discrete strategy space is fine enough to ensure practically the same results as its continuous counterpart would produce.

Routine IV is the outmost loop in the sense that, when evaluating the profitability of each feasible store brand position, Routine IV calls Routine III as a subroutine to determine the NB$_1$ manufacturer’s response, $w_1$. In a similar way, during each run of Routine III, Routine II is called to determine the NB$_2$ manufacturer’s response to $w_1$ and each run of Routine II calls Routine I as a subroutine to determine optimal retail prices for each pair of wholesale prices.