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Problem Set #3
Mortgages, Term Structure of Interest Rates, Gordon Growth Model

Start by solving the following practice questions in chapter 23 (page 644): 4,5,6 a,b,c [read the definition of spot rate before answering the questions- we used the notation y_i in class instead of r_i], and questions in chapter 4 (page 78): 5,6,7,8

1. **[Mortgage]** Mr. Bond is taking a 15 year mortgage loan of \$100,000 with 6% APR (compounded monthly) to purchase a house.
 - a) Calculate the monthly mortgage payment.
 - b) Calculate the outstanding loan after 10 years of payments (that is just after the 120th payment).
 - c) Break the monthly payment into interest and principle for the 121st month.
 - d) After 10 years you discover that interest rates have dropped. In particular, a 5 year mortgage loan charges 5.4% APR (compounded monthly). According to the mortgage contract, you can close the mortgage (at any time) by paying the outstanding loan. Should you refinance in this case if the refinancing fees are \$500?
 - e) Suppose that you go ahead and refinance and that the loan you take on the new mortgage is just enough to cover the outstanding loan (you have the money to currently pay for the refinancing fee). Calculate the payment under the new mortgage. How much money are you gaining (or losing) each month by refinancing? What is the present value of your gains (or losses)?

2. **[Yield Curve]** Consider a two year coupon bond with coupon rate 6% paid annually and face value of \$10,000. What is the price of this coupon bond if the YTM on a one year STRIP bond is $y_1 = 4\%$ and the YTM on a two year STRIP bond is $y_2 = 5\%$? What would the price be if the YTM's of both the one year and the two year STRIP bonds were $y_1 = y_2 = 6\%$ instead?
3. **[Yield Curve]** You are given the following information about STRIP prices (pure discount bonds that pay \$100 at maturity): $B_1 = \$95$, $B_2 = \$90$, $B_3 = \$85$, where B_j is the price of a j year STRIP.
- a) Given these market prices, calculate the price of a three year annuity that pays \$1,000 at the end of each year?
- b) Calculate the three-year yield curve (i.e. y_1, y_2, y_3) implied by the above prices.
- c) Suppose that instead of B_1 you are given the price $P(1) = \$105$ of a one year coupon bond with coupon rate of 2% paid annually and face value \$110 (while prices B_2 and B_3 remain the same). Find the yield curve.
- d) Suppose now that instead of B_2 you are given the price $P(2) = \$550$ of a two year coupon bond with coupon rate of 2% paid annually and face value \$600 (while prices B_1 and B_3 remain the same). Find the yield curve.
4. **[Forward Rates]** As the investment manager for Lemon County, you are given the following information about the yields of U.S. Treasury STRIPS (which are zero-coupon bonds that pay \$100 at maturity): $y_1 = 3\%$, $y_2 = 4\%$, $y_{29} = 6.1\%$, and $y_{30} = 6.0\%$, where y_j is the yield-to-maturity on a j -year STRIP. Assume that all cash flows are riskless and we can borrow and lend at the stated rates. You buy 50,000 30-year strips and partially finance these by issuing (that is selling) 5,000 2-year strips.
- a) How much money do you need today for this transaction?

- b) How much money do you need in two years from now to complete this transaction?
- c) What are the forward rates of interest $f_{1,2}$ and $f_{29,30}$?

5. **[Forward Rates]** We see the following yield curve for discount, or zero-coupon, bonds:

| Maturity | Yield to Maturity |
|----------|-------------------|
| 1 year | 6% |
| 2 years | 7% |
| 3 years | 8% |

- a) Find the three year yield curve, y_1, y_2, y_3 .
- b) If the *fair* price for a *four-year* annuity paying \$100 per year is \$334.57, what should be y_4 ?
- c) Suppose that you wish to lock in right now (time zero) a loan rate for borrowing \$20 million in year 2 for one year. What discount bond holdings would construct this loan *now*? That is, you will be buying and selling different bonds. How many bonds of what maturity will you buy? How many bonds of what maturity will you sell?
- d) What is the forward rate of interest between year 2 and year 3?

6. [**Gordon Growth Model**] You have been hired to value Lagoon Inc., a company specialized in lakefills. The stock is currently trading at \$120 per share. The firm is 100% equity financed and the total number of shares outstanding is 2 million. You expect the firm to generate \$24 million in earnings next year. In the foreseeable future you assume that Lagoon Inc. continues to payout 75% of its earnings as dividends. You decide to use the Gordon growth model and make all the assumptions necessary for that model to hold. The appropriate opportunity cost of capital for Lagoon Inc. is 8%.

- a) Calculate next year's earnings per share and dividend per share.
- b) What growth rate of Lagoon Inc.'s earnings justifies the current price of its equity?
- c) From the growth rate you have calculated earlier, find the return on book equity (ROE).
- d) What is the current book value of equity of Lagoon Inc.?
- e) How will the value of Lagoon Inc.'s stock change if the management reduces the payout ratio to 50%? Why does the price per share increase/decrease (explain)?