Forward interest rates

- Forward contracts
- From the yield curve to forward interest rates and back using the no-arbitrage principle

Designing future financial transactions

Notation from last time:

B_j - *price of a STRIP that matures at time j*

y_j - ytm *of a STRIP that matures at time j*

Example: Your client will need to borrow $20M in exactly 3 years and would like to repay this in exactly 4 years (i.e. one year later), without being exposed to interest rate risk.

Can you recommend a trading strategy that will help your client?

What are *forward* contracts

- A forward contract is an **obligation** to borrow (or invest) a prespecified amount at a prespecified interest rate (**forward rate**) for a prespecified time. For example, to borrow $1M in two years from now at a forward rate of 5%, for one year.
- The forward rate is set such that the forward contract costs **nothing** today.
- Forward rates are often quoted by banks.

The **Eurodollar** futures contract, developed and introduced by CME® in 1981, represents an interest rate on a three-month deposit of $1 million. The Eurodollar futures contract is now the most actively traded futures contract in the world. Open interest in the contract recently surpassed four million.

Notation we will use to denote forward rates

$\mathbf{f_{i,i+1}}$ *or sometimes just* $\mathbf{f_{i+1}}$

– today's forward rate of interest between time i and time i+1.

$f_{3,4}$ or f_4 stand for the interest rate currently charged for a one year loan between time 3 and time 4.

If $f_{3,4}$ =4% then this means that a loan of $100 received at time 3 requires a repayment of $104 one year later (at time 4).

Where do forward rates come from?

Lets consider our clients' request…can we calculate the forward rate implied by the market prices below?

j	1	2	3	4
B_j	$95.24	$89.00	$82.78	$76.29

To answer this question we create a synthetic forward and apply no-arbitrage pricing. Trade the above securities to create:

- inflow of $20M at time t=3
- outflow of $X at time t=4
- no other payments

Creating the synthetic forward contract

Creating a positive payment of $20M at time t=3

Financing the payment of $16.556M at time t=0

The implied forward interest rate is:

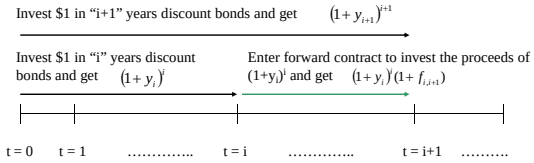
Summary of client's transactions and cash flows

year	0	1	2	3	4
Buying 3 year bonds					
Selling 4 year bonds					
Total					

Formula for calculating forward rates

The forward rate between time $t=i$ and $t=i+1$, satisfies

$$(1+y_i)^i(1+f_{i,i+1}) = (1+y_{i+1})^{i+1} \text{ and } f_{i,i+1} = \frac{B_i}{B_{i+1}} - 1$$



Back to our example…

Using the discount bond prices below (and ytm's), calculate the forward rate of interest for borrowing from time 3 to time 4

j	1	2	3	4
B_j	95.24	89.00	82.78	76.29
y_j	0.05	0.06	0.065	0.07

using prices:

using ytm:

Arbitrage pricing & Forward Rates

Example: Suppose that you can buy or sell bonds at the bond prices from the previous example. In bank "Golden Opportunities Forever©" they quoted a forward rate of $f_{3,4}$ = 7.5% instead of 8.5%. Can you earn arbitrage profits?

Arbitrage trading Strategy

Borrow forward $1M at the bank rate 7.5% from time 3 to time 4.

Sell 3 year bonds in the amount of

Buy 4 year bonds in the amount of

Your arbitrage profit is

Summary of transactions and cash flows

	t = 0	t = 3	t = 4
Borrow $1M forward			
Sell 3 year bonds today			
Buy 4 year bonds today			
Total			