Quoting interest rates

- Compounded annual percentage rate (APR)
- Effective annual yield (EAY)

Mortgages

- Payments/Principal and interest
- Refinancing

Quoting interest rates

"the CD offers a 6% A.P.R. compounded quarterly"

- Okay…so what is the annual interest rate?
  - A.P.R. – annual percentage rate
  - quarterly interest rate is 6% / 4 = 1.5%
  - annual interest rate (or effective annual yield – EAY) is $(1.015)^4 = 6.14\%$

Quoting interest rates

- Let r = A.P.R. and k = *# of compounding intervals*.
- EAY is given by, $1+EAY = (1+r/k)^k$
  - *Always make sure that you get EAY close to r*
- Example: A.P.R. = 6%

k	1+EAY
1	1.06
2	$(1+0.06/2)^2 = 1.0609$
4	$(1+0.06/4)^4 = 1.0614$
12	$(1+0.06/12)^{12} = 1.0617$
365	$(1+0.06/365)^{365} =$ 1.06183
8,760	$(1+0.06/8760)^{8760} = 1.061836$

Continuous compounding

- What if interest is compounded very frequently or continuously?

$$1+\text{EAY}=\lim_{k\to\infty}\left(1+\frac{r}{k}\right)^{k}=e^{r}\quad(e\approx 2.7)$$

Example: Assume 6% A.P.R. compounded continuously, what is the EAY?

$$1+\text{EAY}=e^{0.06}=1.061837$$

Car loan (example): You want to buy a $20,000 car but you have only enough for the down payment of $4,000. You can borrow from a bank at 9% APR compounded monthly or from "Houston Dealers" who offers financing at 6% APR for a 30 month loan but then you must forego a $1,000 discount. What should you do?

- Bank: pay a high interest rate on a smaller loan
- Dealer: pay a lower interest rate on a larger loan

monthly payment **bank**:

monthly payment **dealer**:

- Given your credit history, the bank changes your rate to 12%. What should you do?

monthly payment **bank**:

Example (continued): Suppose now that you had $10,000 to pay on the car (that is $6,000 above the required down payment). You can save your money in the bank and earn 7% APR. Should you save your extra money or use it to pay for the car (consider the 12% APR case for this part)?

We should consider two alternatives,

Calculating Mortgage Payments

Example: You have managed to save $50,000 and are buying a house for $250,000. You are offered a 9% APR (compounded monthly) 30 year mortgage. What is your monthly payment?

monthly payment:

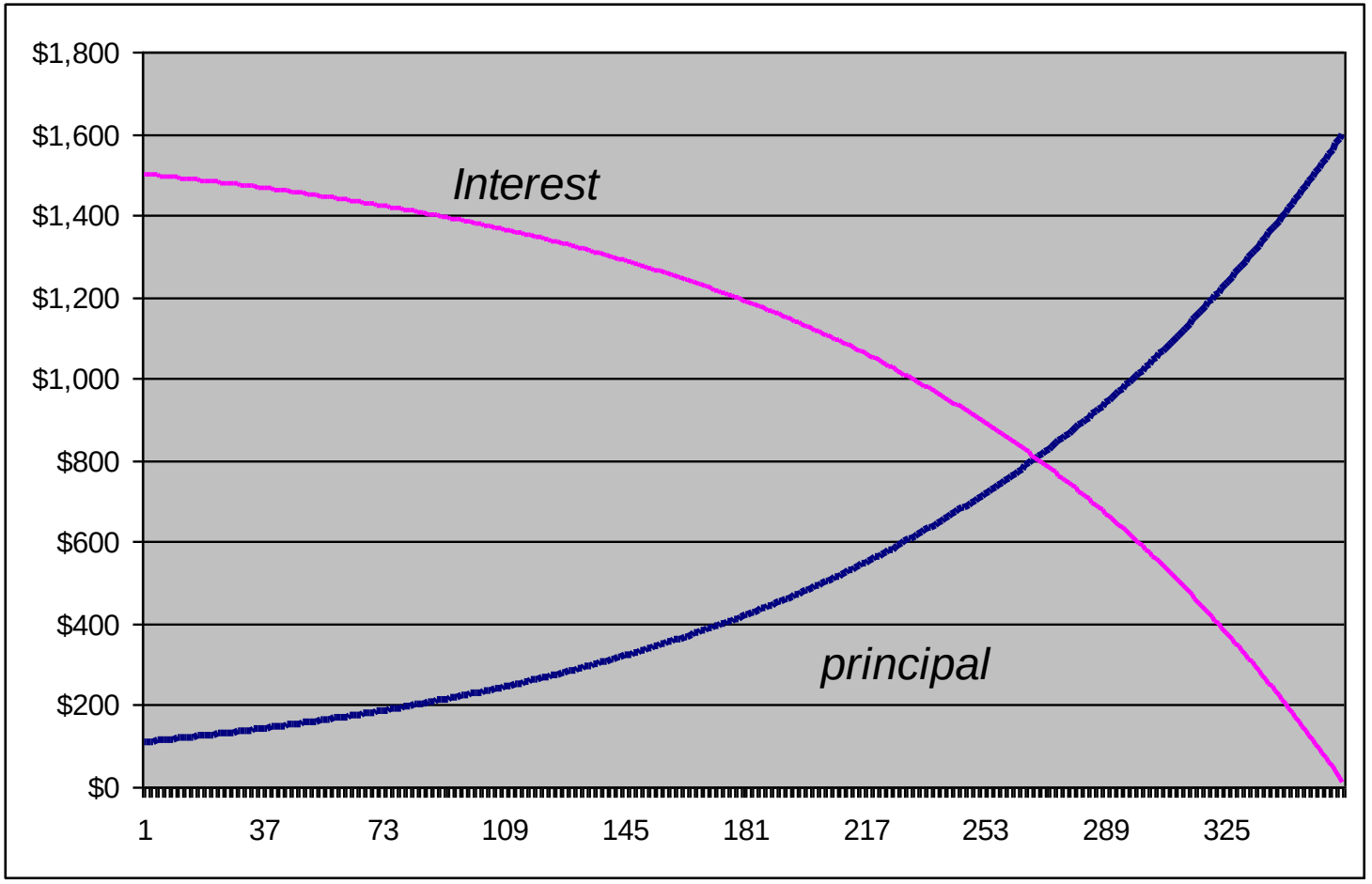
- How long would it take you to pay back the mortgage if you are willing to cut back on your monthly expenses and increase the monthly payment by $200?

Calculating “interest” and “principal”.

Example (continued)

time	Loan outstanding (beginning month)	Payment (end month)	Interest (end month)	Principal (end month)
1	200,000			
2				
3				
…	…	…	…	…
360				

Composition of mortgage payments



Loan Outstanding

Example: Consider the previous 9% APR (compounded monthly) 30 year mortgage for $200,000. What is the remaining principal (loan outstanding) on your mortgage after 10 and 20 years?

Outstanding loan:

- Suppose that after ten years the interest rate on mortgages drops to 8% APR. (1) What is the loan outstanding at that time and (2) what is the PV of the remaining mortgage payments after 10 years.

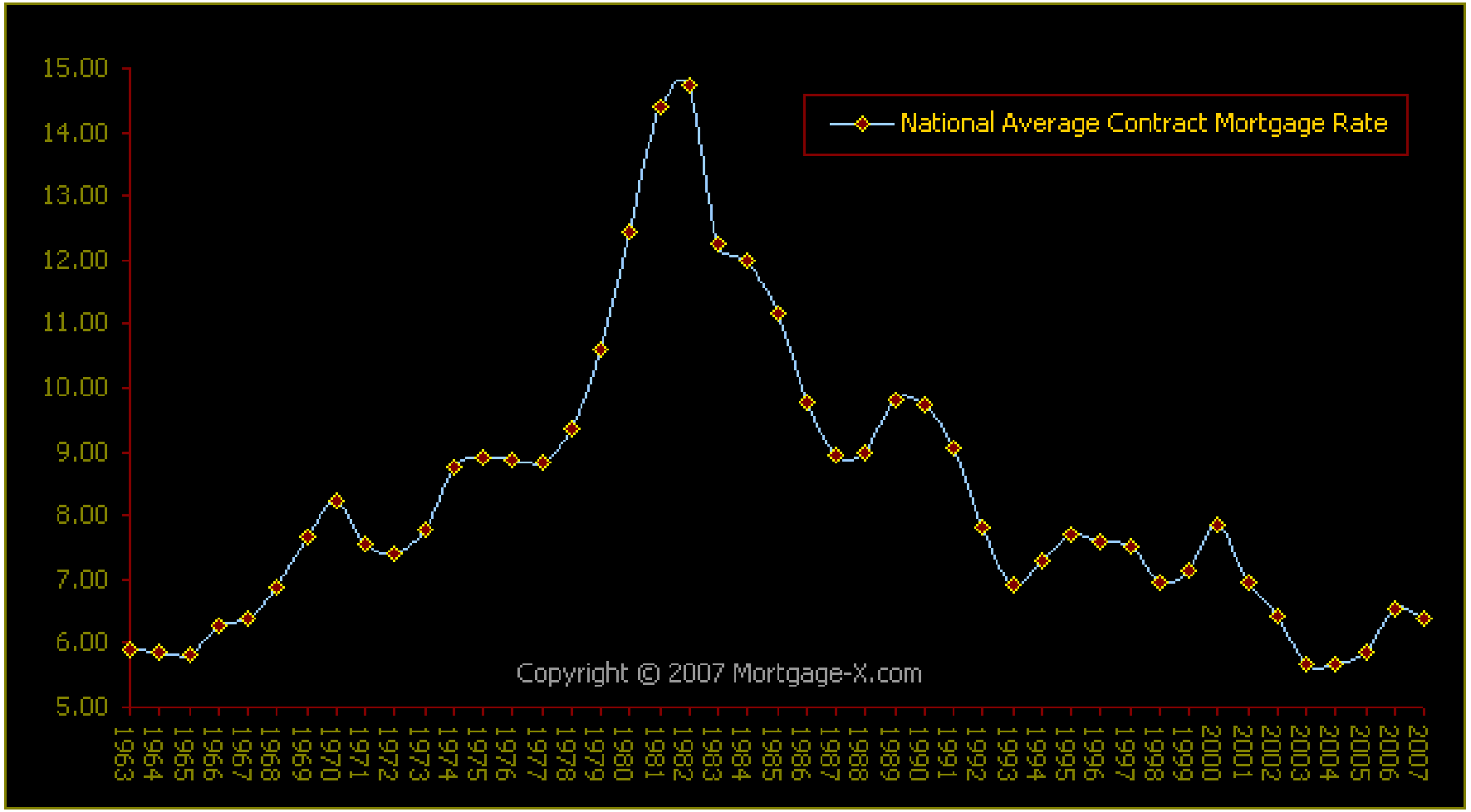
Refinancing

- How does it work?
  - You are allowed to close the mortgage by paying the **outstanding principal** at any point in time (this is called "*the option to refinance*"). But, you must pay refinancing fees when singing the new mortgage.
  - The **outstanding principal** is determined by the mortgage contract and is usually specified to be the present value of remaining payments under the mortgage interest rate (this is not the market interest rate at the time of refinancing).

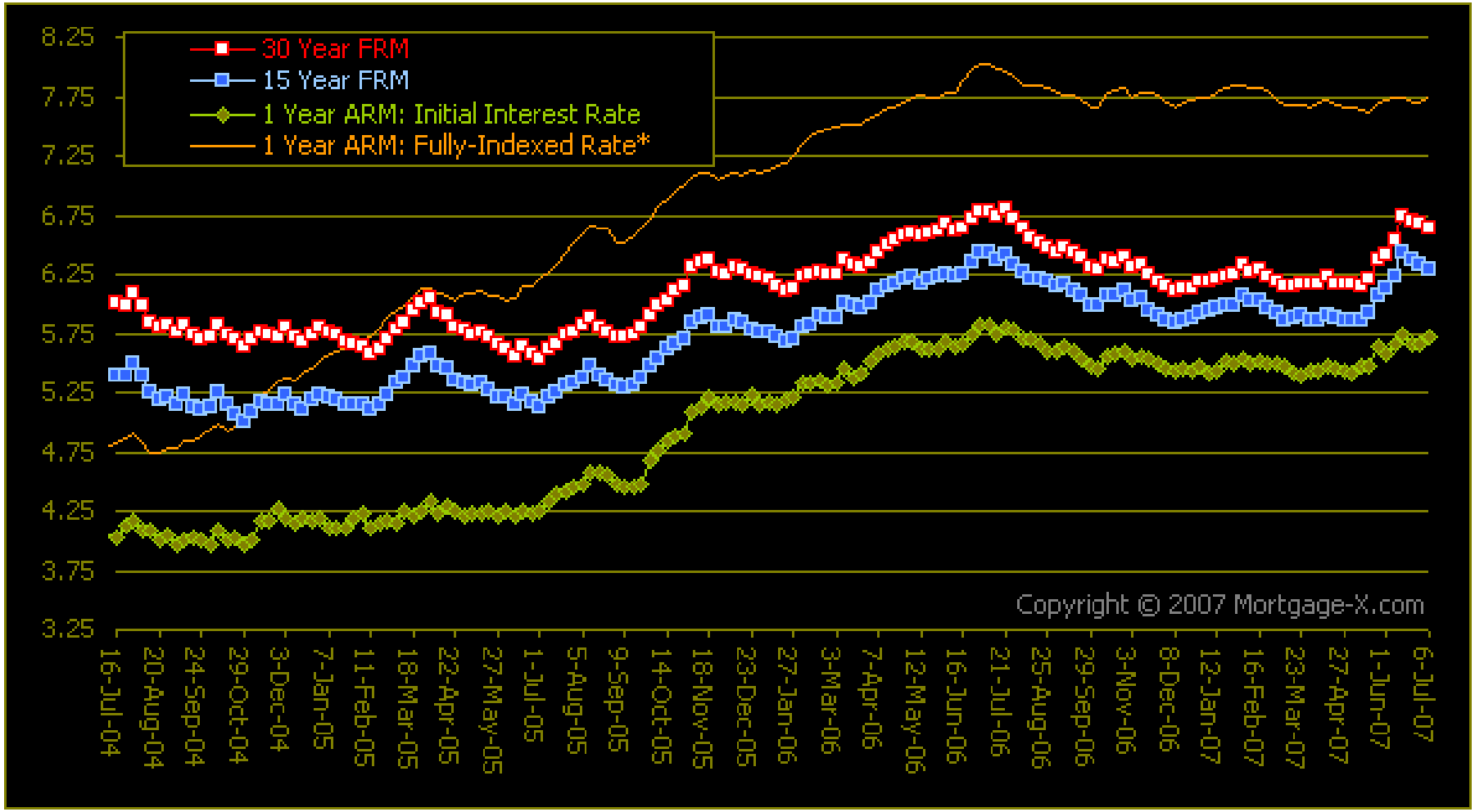
Why should I ever refinance?

When should I refinance?

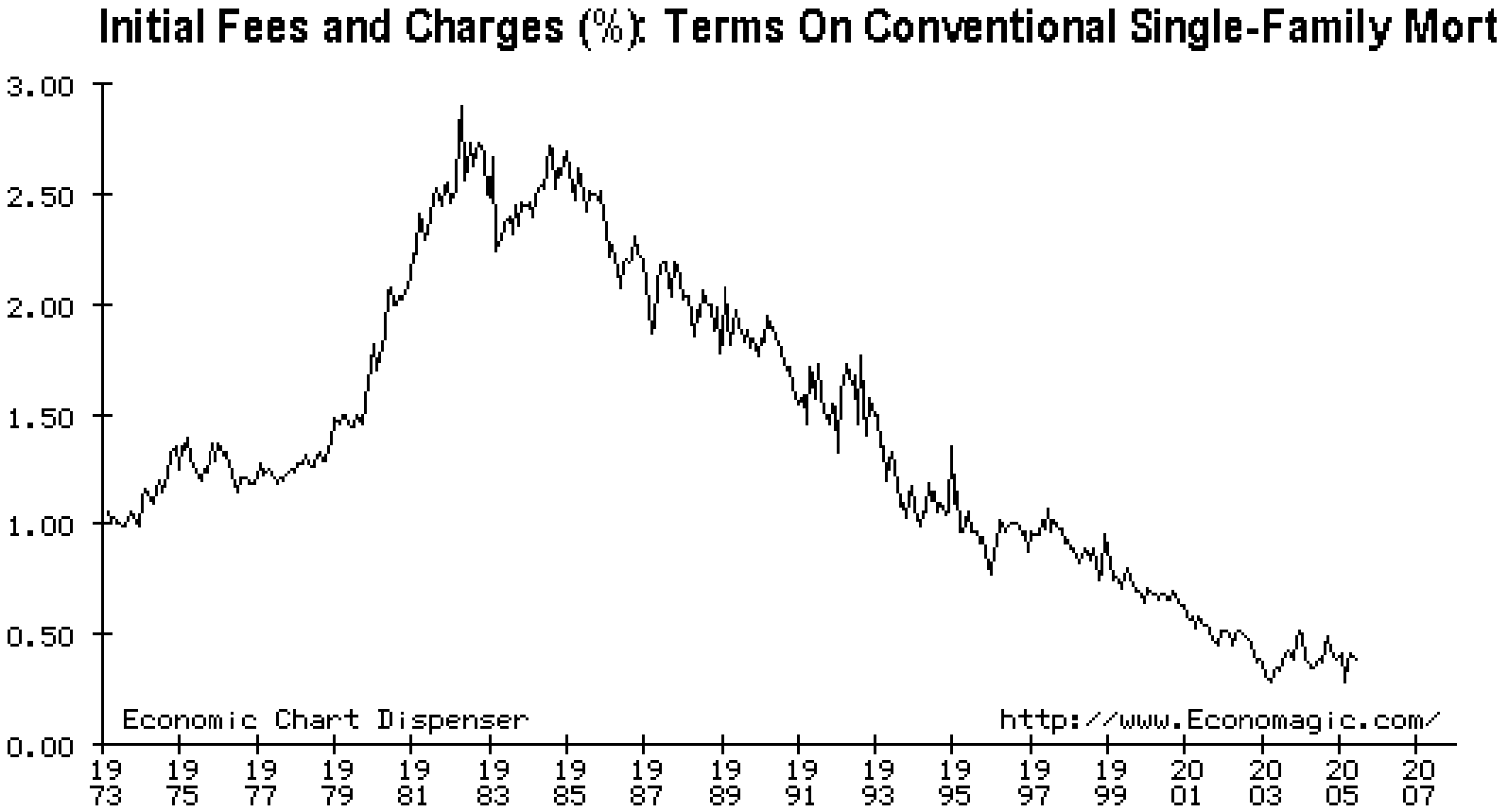
Mortgage rates in the U.S. 1963-2007



FRM & ARM in the U.S. 2004-2007



How costly is it to refinance?



Mortgage example (continued): After 5 years of payments, mortgage interest rates dropped from 9% to 8.75% for a 30 year term. The refinancing fee is $2,000. Should you refinance [assume that you will stay in the house for the next 30 years and that interest rates will not change for the next 30 years]?

In order to refinance we must

When should we refinance?

- loan outstanding after 5 years (300 remaining payments)

- PV of payments of old mortgage,

- By refinancing you gain

Mortgage example (continued): Assume that you are planning to move and sell your house in 5 years. Should you still refinance?

Why does this matter?

- loan outstanding in 5 years from now (year 10, 240 remaining payments)

- PV of payments of old mortgage,

- By refinancing you gain