Asset Valuation with known cash flows

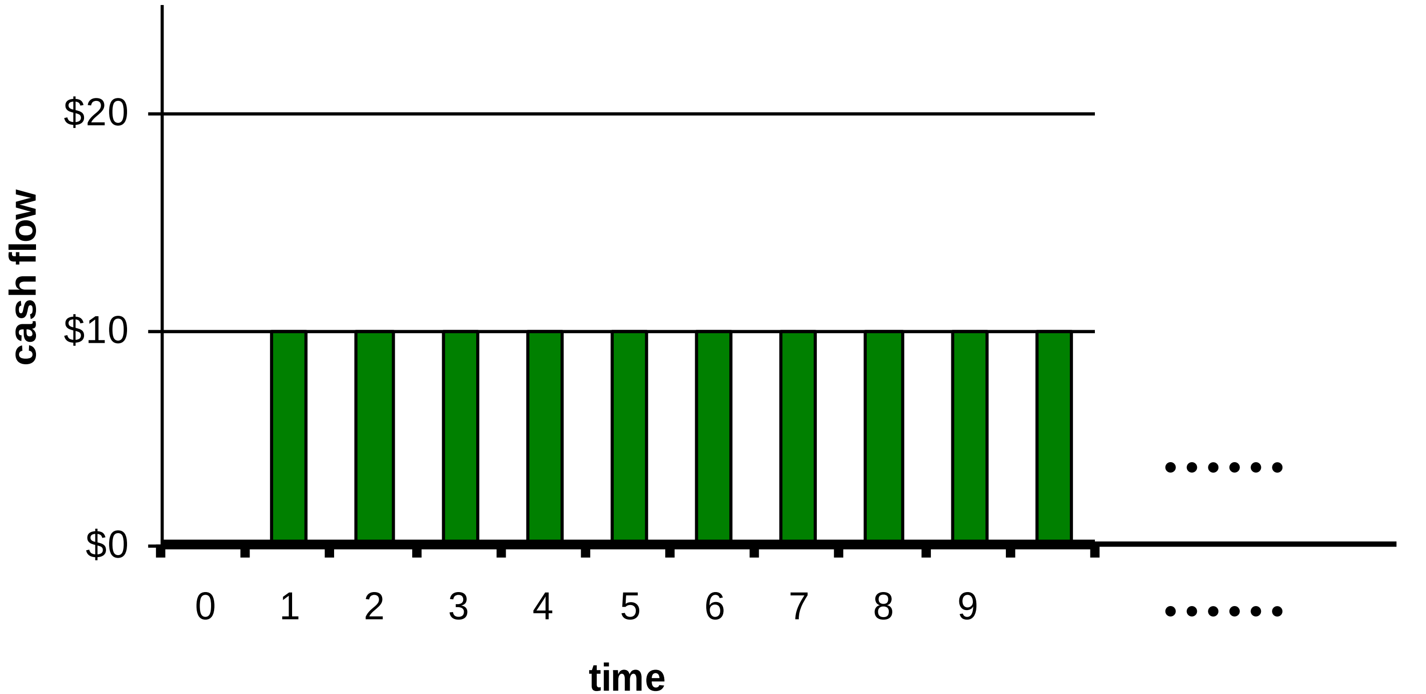
– Annuities and Perpetuities
– care loan, saving for retirement, mortgage

Simple Perpetuity

A perpetuity is a stream of cash flows each of the amount of "CF" dollars, that are received at the end of each period forever

- Note:
  - Cash flows are the **same** over time
  - There is no cash flow today (i.e. you receive the first cash flow one period from now)

Simple Perpetuity



Valuing a perpetuity

The PV of a perpetuity is,

$$PV = \frac{CF}{1+r} + \frac{CF}{(1+r)^2} + \frac{CF}{(1+r)^3} + \ldots$$

$$= \sum_{i=1}^{\infty} \frac{CF}{(1+r)^i} = \frac{CF}{r}$$

Example: You will receive $100 forever beginning the next year. The annual interest rate is 10%. Find PV.

PV = $100/0.1 = **$1,000**

Check:

If we invest $1,000 then we should be able to “replicate” the stream of cash flows generated by the perpetuity. That is by investing $1,000 today we should receive a payment of $100 each year forever.

This is how we can do this:

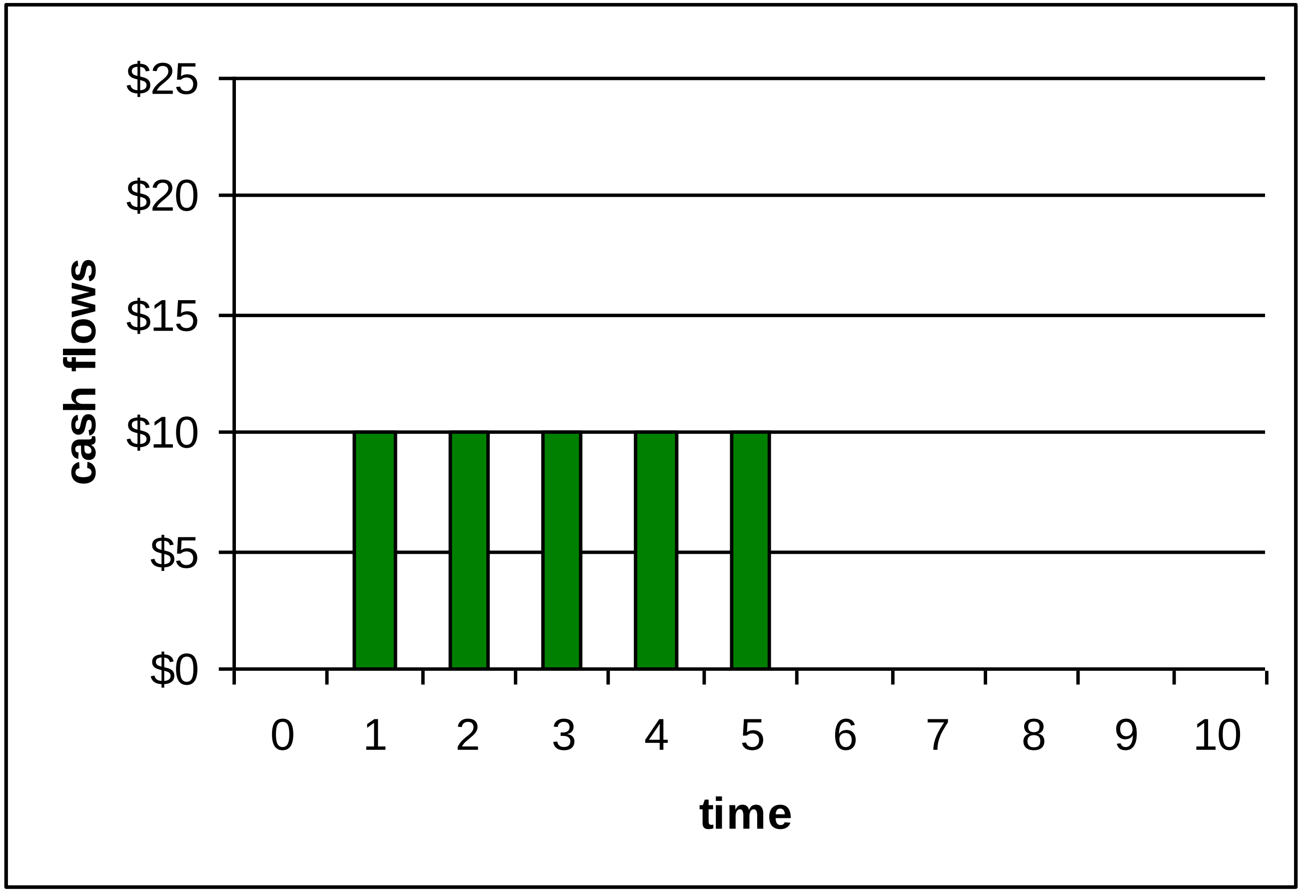
Simple Annuity

An annuity is a stream of cash flows each of the amount of "CF" dollars, that are received at the end of each period for the duration of "n" periods

Note:

- Cash flows are the **same** over time
- There is no cash flow today (i.e. you receive the first cash flow one period from now)

Simple five year Annuity



Simple annuity formula

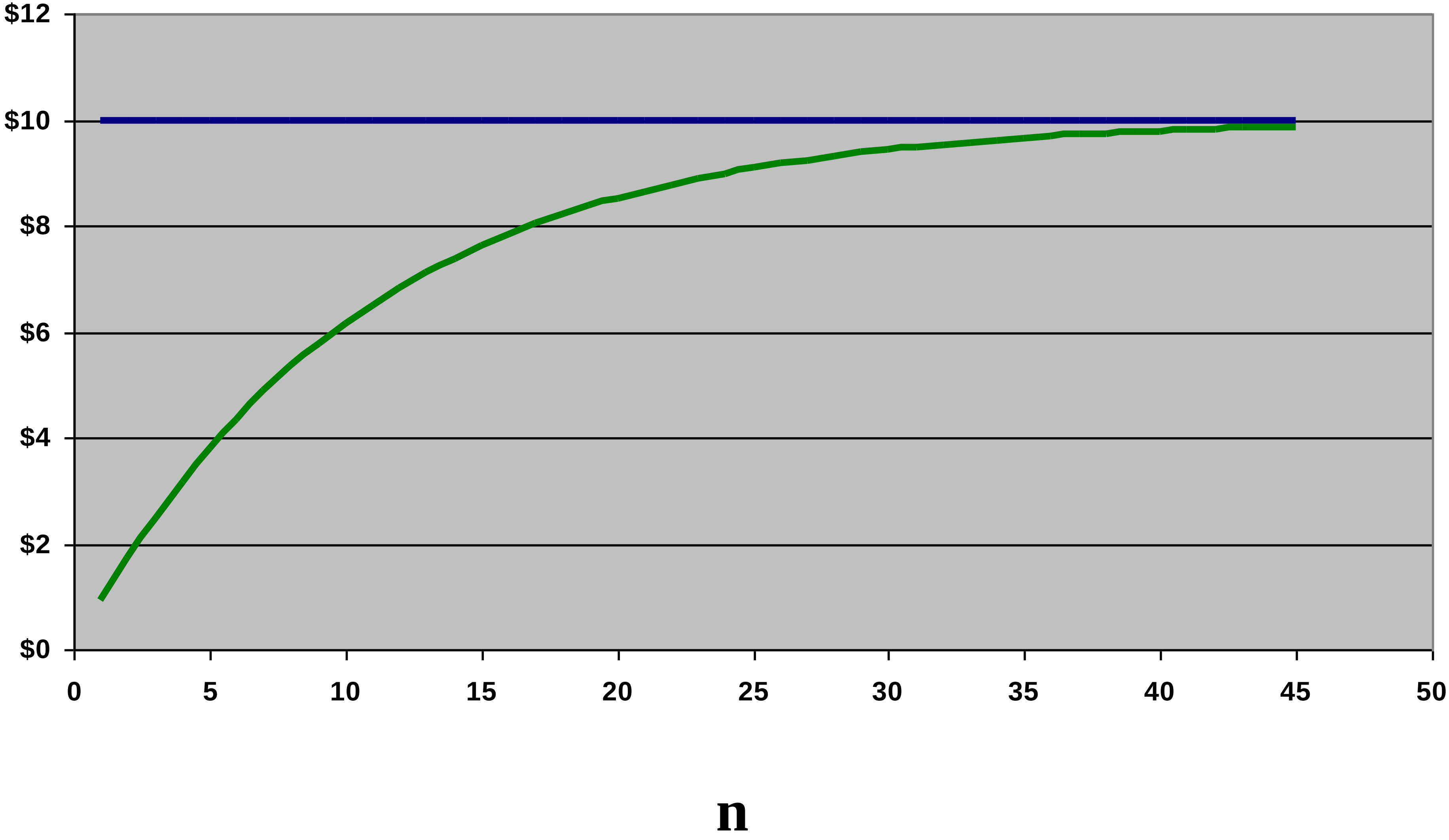
The PV of an annuity for n years is,

$$\mathrm{PV} = \frac{\mathrm{CF}}{1+\mathrm{r}} + \frac{\mathrm{CF}}{(1+\mathrm{r})^2} + \frac{\mathrm{CF}}{(1+\mathrm{r})^3} + \ldots + \frac{\mathrm{CF}}{(1+\mathrm{r})^\mathrm{n}}$$

$$= \sum_{i=1}^{n} \frac{\mathrm{CF}}{(1+\mathrm{r})^\mathrm{i}} = \frac{\mathrm{CF}}{\mathrm{r}}\left(1 - \frac{1}{(1+r)^n}\right)$$

Example: Find the present value of an annuity that pays $500 for the duration of 7 years (beginning at the end of the first year). The annual interest rate is 5%.

"n" year annuity versus perpetuity when r=10%

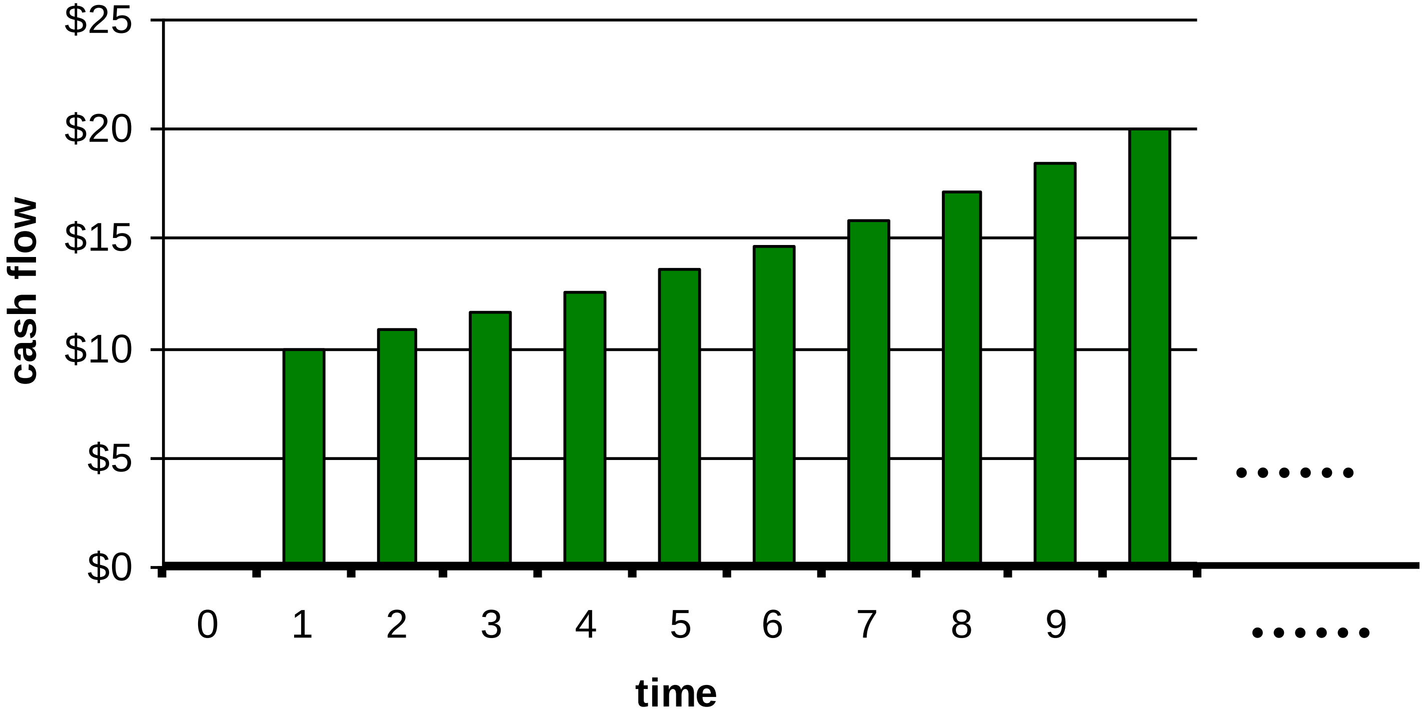


Growing perpetuity

A growing perpetuity is a stream of cash flows that grows over time with growth rate "g" where cash flows are received at the end of each period forever

- Note:
  - Cash flows grow over time with rate "g"
  - There is no cash flow today (i.e. you receive the first cash flow one period from now)

Growing perpetuity with growth rate of 8%



Growing perpetuity formula

- The first cash flow "CF" is received at the end of the first period and is growing at rate "g" afterwards

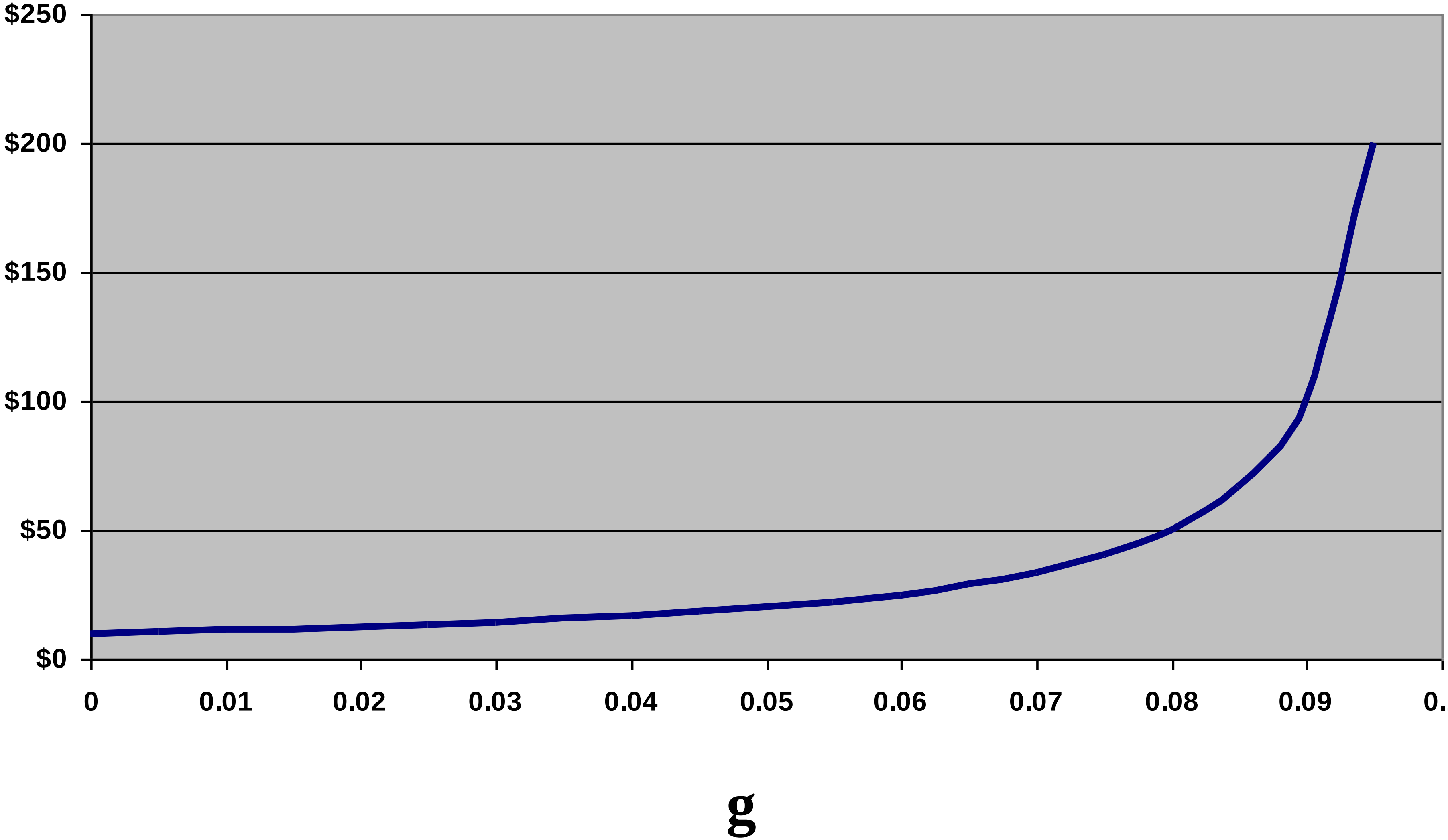
In particular, cash flows look like:

t=0	t=1	t=2	t=3	…..	t=n	…..
	CF	CF(1+g)	$CF(1+g)^2$	…..	$CF(1+g)^{n-1}$	…..

$$PV = \frac{CF}{1+r} + \frac{CF(1+g)}{(1+r)^2} + \frac{CF(1+g)^2}{(1+r)^3} + \ldots$$

$$= \sum_{i=1}^{\infty} \frac{CF(1+g)^{i-1}}{(1+r)^i} = \frac{CF}{r-g}$$

Growing perpetuity with growth rate "g" and interest rate r=10%

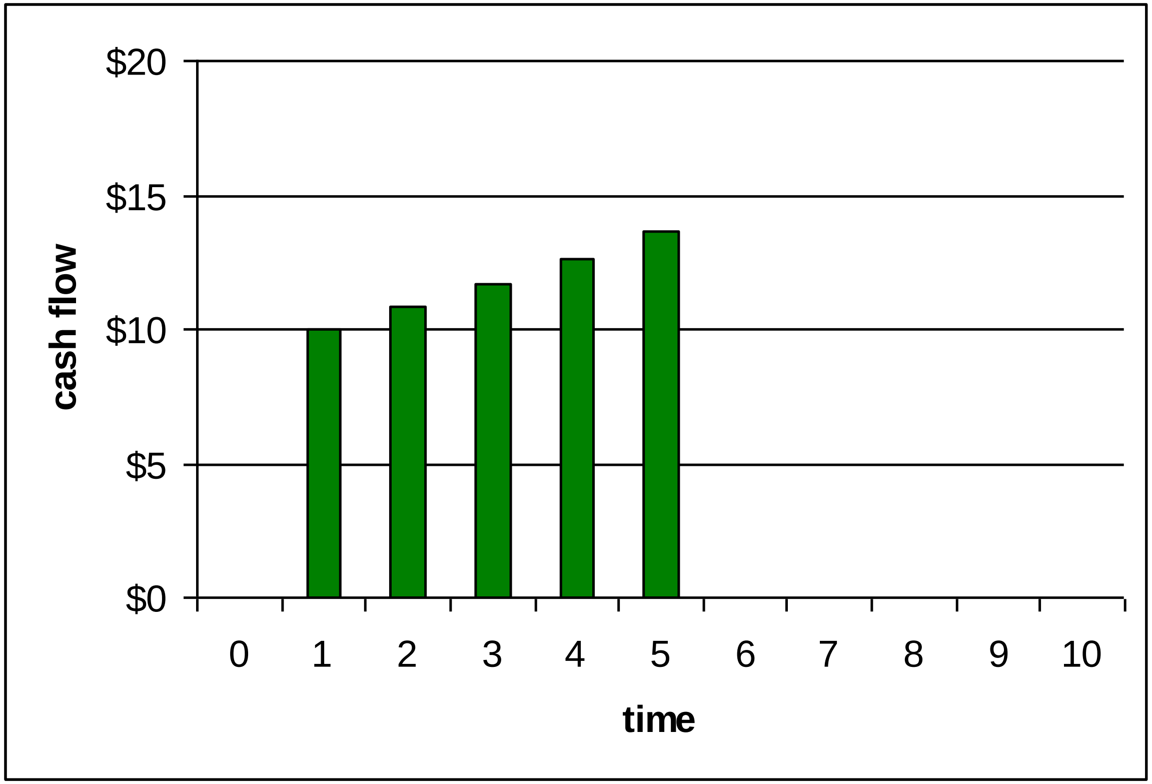


Growing Annuity

A growing annuity is a stream of cash flows that grows over time with growth rate "g" where cash flows are received at the end of each period for the duration of "n" years.

- Note:
  - Cash flows grow over time with rate "g"
  - There is no cash flow today (i.e. you receive the first cash flow one period from now)

Five year growing Annuity with growth rate of 8%



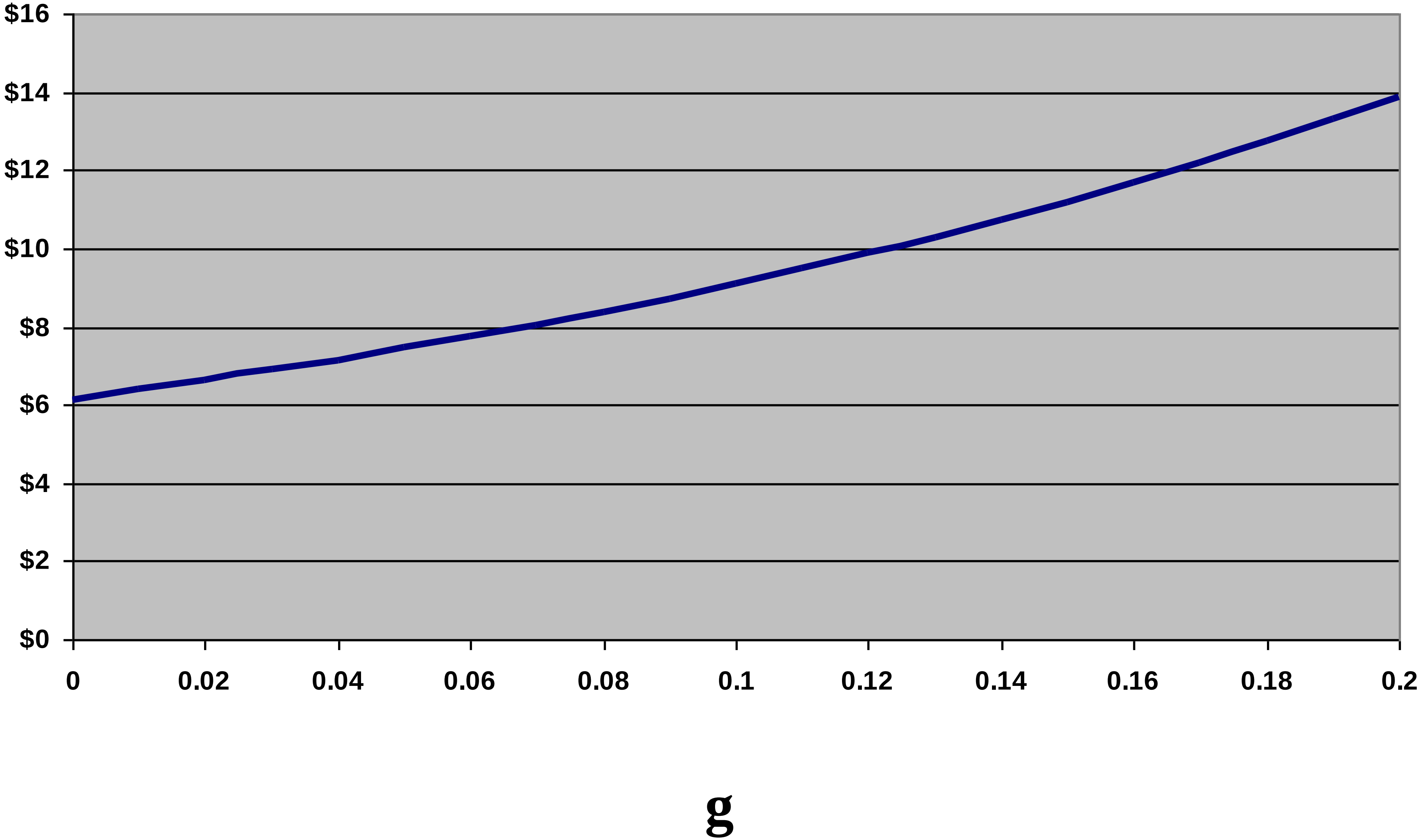
Growing annuity formula

The PV of a growing annuity for n years is,

$$PV = \frac{CF}{1+r} + \frac{CF(1+g)}{(1+r)^2} + \frac{CF(1+g)^2}{(1+r)^3} + \ldots + \frac{CF(1+g)^{n-1}}{(1+r)^n}$$

$$= \sum_{i=1}^{n} \frac{CF(1+g)^{i-1}}{(1+r)^i} = \frac{CF}{r-g}\left(1 - \frac{(1+g)^n}{(1+r)^n}\right)$$

Growing annuity with growth rate "g" and interest rate r=10%



Growing annuity formula for r=g

$$PV = \frac{CF}{1+r} + \frac{CF(1+g)}{(1+r)^2} + \frac{CF(1+g)^2}{(1+r)^3} + \ldots + \frac{CF(1+g)^{n-1}}{(1+r)^n}$$

$$= \frac{CF}{1+r} + \frac{CF}{(1+r)} + \frac{CF}{(1+r)} + \ldots + \frac{CF}{(1+r)}$$

$$= \frac{n \cdot CF}{1+r}$$

- **Example 1**: if you save $1,000 each year for 35 years, how much will you have in your bank account after 35 years if the interest rate is 10%?

- How much would you need to save each year in order to accumulate $300,000 after 35 years?

- What is the present value of your installments if you save $1,000 each year for (a) 35 years and (b) forever?

- What is the present value of your installments if the interest rate changes to 9%?

- What is the future value of your installments if the interest rate changes to 9%?

- **Example 2**: You want to rent an apartment in Houston for one year. The landlord is not willing to reduce the monthly rent of $1,000 but offers the first month for no charge. You can also stay in your old apartment and pay rent of $915 (at the beginning of each month). What should you do? Assume an interest rate of 1% per month.

PV(current rent payments) =

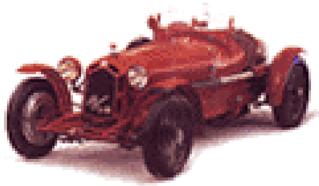
PV(alternative rent payments) =

Would your choice be the same if you got the last month free?

- **Example 3**: You need a parking space for the period of two years. You can either buy a parking space for $10,000 and then sell it in two years for $10,500, or rent a parking space for the period of 2 years. The monthly rent is currently $75 and is expected to rise by 0.5% each month (starting from the next). What should you do? Assume an interest rate of 1% per month.

PV(buy parking space) =

PV(rent parking space) =



- **<u>Example 4</u>**: You have just earned a Federal tax return and are thinking to donate $2,000 to the Museum of Contemporary Art in Houston. In return Museum offers free annual membership ($100 per year paid at the beginning of the year) forever or a growing perpetuity of $70 with growth rate of 3% per year (the first payment of $70 is in one year). What should you do? Assume an interest rate of 7% per year.

PV(free membership offer) =

PV(growing perpetuity) =

- **Example 5**: 30 years ago, André François Raffray agreed to pay the 90 year old Jeanne Calment 2,500 francs ($500) per month (end) until she dies. In return he will receive her apartment when she dies. The apartment is worth $184,000. Suppose the monthly interest rate is 1%. Assuming M. Raffray thought this was a good deal, how long did he think Jeanne Calment would live?

Mr. Raffray will break even if Jeanne Clament lives less than n additional months

This implies that

.

Example 6: An insurance agent offers you the following contract: you pay $5,000 per year (end) for the next 15 years and in return you will receive $7,000 a year (end) for the following 15 years. Suppose interest rates are 9%. Should you buy this contract?

- Example cont'd: suppose that the insurance agent sweetens the deal and says that the payments that you receive will grow at 3% per year. Would you take the contract now?