# Corporate fraud and investment distortions in efficient capital markets

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*Inefficient investment allocation induced by corporate fraud, where informed insiders strategically manipulate outside investors' beliefs, has been endemic historically and has recently attracted much attention. We reconcile corporate fraud and investment distortions with efficient capital markets, building on shareholder-manager agency conflicts and investment renegotiation in active takeover markets. Because investments that are ex post inefficient are not renegotiation proof, the optimal renegotiation-proof contract induces overstatements by managers, accompanied by overinvestment in low return states and underinvestment in high return states by rational investors. Our framework also helps explain why easy access to external capital appears to facilitate corporate fraud.*

## 1. Introduction

■ Corporate fraud has attracted much attention recently because of prominent cases of corporate malfeasance where insiders were able to attract investment through overly optimistic representations of financial performance and economic prospects (e.g., Worldcom and Enron). An important consequence of this type of fraud is investment inefficiency. Compared to the efficient allocation, there is overinvestment in certain industries or sectors as uninformed investors direct capital flows to not only the firms manipulating their beliefs; however, there is underinvestment in other sectors, as the presence of fraud makes investors generally more cautious.

Manipulation of outside investors by better-informed strategic insiders has a long history, and appears to have existed from the onset of organized trading and investment.[1] A historical review points to the crucial role of asymmetric information and agency conflicts in the manipulation

    [1] Notable early examples include the South Sea Trading Company in the 18th century, U.S. railroad firms in the 1860s, and utilities in midwestern states in the 1920s. See Skeel (2005) for numerous other examples.

of investors' beliefs and the attendant investment distortions. Because of the separation between ownership and control (Berle and Means, 1932), insiders are typically more informed than outsiders about expected returns on investment, especially for new technologies and markets.[2]

However, in spite of considerable anecdotal and more systematic empirical findings emphasizing overinvestment (relative to the first-best) as an attendant cost of corporate fraud,[3] the literature rarely presents frameworks where fraud and overinvestment occur in equilibrium with rational investors. There is in fact a literature arguing that, in efficient capital markets, strategic managerial disclosures aimed at inflating firm prospects should be discounted by rational market participants (e.g., Stein, 1989; Narayanan, 1985). Moreover, models with private information on investment prospects typically predict *under*investment rather than *over*investment, because of the adverse selection problem in equity financing (e.g., Myers and Majluf, 1984; Greenwald, Stiglitz, and Weiss, 1984) or capital rationing by debt markets (e.g., Stiglitz and Weiss, 1983).[4]

Our contribution is to develop a theory of fraud and generalized investment distortions, that is, both under- and overinvestment, with rational expectations and perfect capital markets. Specifically, we construct and characterize an equilibrium with rational investors where under- and overinvestment can both occur with positive probability, that is, low-capital productivity firms receive more than their efficient levels of investment capital, and conversely for high-capital productivity firms.

Our framework is built on two fundamental characteristics of modern corporations operating in well-developed financial markets. First, there are shareholder-management agency conflicts because managers derive private benefits from controlling larger investments and have private information on the investment opportunity set (e.g., Stulz, 1990; Hart, 1995). Second, shareholders cannot credibly precommit to investment that is inefficient *ex post*, because such investment policies are renegotiated in active takeover markets (e.g., Grossman and Hart, 1986).

As is typical in optimal contracting with asymmetric information, the second-best or *ex ante* incentive-efficient contract in our setup requires precommitment to investment policies that are inefficient *ex post* (i.e., conditional on knowing the true state of the world). Relative to the first-best, investment is lowered in the high-productivity state and raised in the low-productivity state. But investment levels that are inefficient *ex post* lead to a transparent undervaluation of the firm and generate a profitable takeover opportunity for any investor who acquires control of the firm and "resets" investment to the *ex post* efficient levels. Indeed, in our setting, any renegotiation-proof investment policy— that is, one that does not generate profitable takeover opportunities—must be *ex post* (or *conditionally*) efficient. We therefore highlight the influence of an active corporate takeover market on the design of incentive contracts between managers and shareholders, and provide an institutionally compelling motivation for the "renegotiation-proofness" problem (see Dewatripont, 1988; Bolton and Dewatripont, 2005).

Our main insight is that the *ex post* investment efficiency constraint substantially affects the information optimally induced from the privately informed agent, and therefore also the equilibrium allocation of capital across productivity states. We find that, for a range of model parameters, the *optimal* renegotiation-proof contract induces misreporting by insiders with a

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[2] For example, Worldcom's claim in 1996—since proved fraudulent—that Internet traffic was doubling every 100 days not only bolstered its own stock price but also apparently induced overinvestment in fiber-optic capacity (see, e.g., Dreazen, 2002; Sidak, 2003). Worldcom's misrepresentations on the rate of growth of Internet traffic had credibility because such information is highly proprietary to carriers: "Worldcom used [the] asymmetry of information to exaggerate the value of its stock by overstating the growth of Internet volumes" (Sidak, 2003).

[3] For instance, during 1996–2000, investment in telecommunication infrastructure to service the nascent internet industry grew from about $62 billion per year to over $135 billion per year (in constant 1996 dollars), and resulted in a glut in fiber-optic cable capacity with utilization rates at only 2.5–3% (Brenner, 2003; page 9). But the literature is now presenting more comprehensive evidence that corporate fraud is associated with overinvestment. For example, Kedia and Philippon (2007) find that firms that commit accounting fraud (i.e., have to restate earnings) also invest more than comparable nonfraudulent firms during the misreporting period.

[4] Other agency models that emphasize the asset substitution problem (Jensen and Meckling, 1976) and the debt overhang problem (Myers, 1977) also lead to underinvestment. See Stein (2003) for a very useful survey.

positive probability. The role for randomized reporting arises in our setting because attempts to induce truth telling through wage contracts (when investment is constrained to be *ex post* efficient) can be costly.

Specifically, truth telling can be induced—even with the constraint of renegotiation-proof investment—by paying the low-productivity agent information-based rents (through high wages) that compensate him for the utility loss from receiving a lower investment level relative to the larger investment allocated to the high-productivity state. But these rents can be high if there is a large difference between the efficient investment level in the high- and low-productivity states. Le Chatelier's principle then suggests that it may be cost effective to induce randomized reporting, because randomization lowers expected wage costs by moving the Bayes-rational (or renegotiation-proof) investment in the direction of the second-best. Indeed, the optimal reporting noise (or probability of misreporting) is determined by trading off the expected cost of investment distortion from the incorrect report with the reduction in the expected wage costs.

In our model, there generally can be randomized reporting by both the low- and the high-productivity manager because the incentive compatibility constraints are binding both upward and downward in the second-best. The reason is that the incentive-efficient contract exploits the trade-offs between wages and utility of control to give zero wages to the high-productivity manager, and make his incentive constraint bind. However, under a range of parameters, it is optimal to induce randomization only by the low-productivity manager. This is because randomization by the high-productivity agent imposes investment distortion costs—because there is underinvestment with positive probability—and *increases* expected wage costs—because the agent receives positive, rather than zero, wages whenever he reports low productivity. Thus, as long as the *a priori* likelihood of the high-productivity state is not too low and the capital productivity in the low state is not too high, it is suboptimal to induce randomization by the high-type agent. But under these conditions, it is still optimal to have the low-type randomize as long as the opportunity cost of investment and the manager's subjective benefits of control are in an intermediate range.

In sum, we establish the existence of, and characterize, a noisy revelation equilibrium where the low-productivity manager inflates his reports with positive probability, and there is overinvestment in the low state but underinvestment in the high state. Comparative statics around this equilibrium reveal that the likelihood of fraud is negatively related to the cost of capital (or external financing), and positively related to the ratio of low- and high-capital productivities. These comparative statics allow us to relate the extent of investment inefficiency to salient model parameters.

The desire to understand the causes and consequences of (corporate) fraud as an equilibrium phenomenon dates at least back to Becker (1968). One strand of the existing literature on fraud emphasizes the role of managerial myopia (e.g., Narayanan, 1985; Stein, 1989; Von Thadden, 1995; Kedia and Philippon, 2005). Such myopia is sometimes motivated through imperfections in long-term incentive contracting. Another strand examines fraud in models with limits on communication (e.g., Dye, 1988; Demski, 1998).[5] And a more recent literature assumes some form of market irrationality in reconciling fraud with market equilibrium (e.g., Jensen, 2005; Bolton, Scheinkman, and Xiong, 2005).[6]

Our analysis differs from the literature, because we emphasize the role of capital markets in the incidence of corporate fraud; in fact, capital markets have played a prominent role in several recent corporate scandals and historically in episodes of fraud. Instead of relying on managerial myopia, limitations on communication and contracting, multitasking, or investor irrationality, we focus on the inability of financial markets to resist value-increasing investments. That well-developed (or relatively frictionless) financial markets will exploit the profit opportunities afforded

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[5] In Dye (1988), there is asymmetric information of two dimensions whereas the informed agent can only provide a one-dimensional report—leading to nontruthful reporting (with some probability). In Demski (1998), earnings management is actually beneficial because that is the only way the manager can communicate the level of future earnings.

[6] See also Guttman, Kadan, and Kandel (2006) for a partially revealing signalling model with costly manipulation of earnings.

by transparent undervaluation of assets is a compelling argument; therefore, our perspective appears to rest on rather unexceptionable foundations.[7]

We organize the remaining article as follows. Section 2 sets out the basic model. Section 3 defines optimal contracting and its equilibrium representation. Section 4 records the first- and second-best benchmark outcomes. Section 5 presents the main results, and Section 6 concludes by discussing some implications of the model. All proofs are placed in the Appendix.

## 2. The model

■ **Technology.** There are three time periods in the model, $t = 0, 1, 2$. The firm has a technology that stochastically converts investment at time $t = 1$, denoted by $k$, to earnings (or output) at time $t = 2$, denoted by $y$. For simplicity, we normalize units so that earnings take only two possible values: a high value, $y = 1$, and a low value, $y = 0$. The probability distribution of earnings is influenced by the firm's capital productivity, $s \in \{s_h, s_\ell\}$, $1 > s_h > s_\ell \geq 0$, and investment ($k$). The probability of high earnings is given by $sf(k)$, where $f = 2\sqrt{k}$ is defined on the feasible investment set $[0, k^{\max}]$, such that $k^{\max} \leq 1/4s_h^2$.[8]

□ **Ownership, control, and managerial preferences.** The firm is controlled by a risk-neutral manager who receives two types of utility from managing the firm. He receives utility from consuming wages, $w$, that are paid at the time of the firm's liquidation. The manager also receives benefits from control that are a mixture of subjective utility and noncontractible pecuniary benefits, and they increase with the size of the capital assets. We represent these benefits by a $b(k) = \psi k$, where $\psi > 0$. Therefore, the manager's utility from possibly uncertain wages and investment level $k$ is $U(w, k) = E(w) + \psi k$. Moreover, the manager has no initial wealth and enjoys limited liability, and therefore wages must be nonnegative. Finally, the manager's reservation utility is normalized to zero.[9]

The firm is publicly held and its shares are traded in a frictionless capital (or equity ownership) market. For simplicity, we assume that all of the firm's shares are held by a risk-neutral active shareholder—hereafter, the original owner. Firm ownership and control, however, may vary over time. In particular, a potential raider may gain control of the firm (we discuss this in detail below). The original owner and the potential raider have a common opportunity cost of investment, namely the gross rate of return $R$.[10]

In the last period, time $t = 2$, output $y$ is realized, wages $w$ are paid to the manager, and the firm is liquidated. The payoff to the owner of the firm at the time of liquidation, given an investment level $k$, output $y$, and wage $w$ is $v(w, k, y) = y - w - Rk$.

□ **Information.** The manager privately observes the productivity of the firm ($s$) before investment takes place, that is, before time $t = 1$. At $t = 0$, the manager and the investors share common prior beliefs about the realizations of $s$; the probability of observing $s_h$ and $s_\ell$ is $\mu$

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[7] More broadly, our article is also related to the agency literature, where constraints on the principal's ability to credibly precommit *ex ante* limit the amount of information he possesses *ex post* (e.g., Crawford and Sobel, 1982; Laffont and Tirole, 1988). Two related applications of this idea are Arya, Glover, and Sunder (1998) and Krasa and Villamil (2000). The focus of these analyses, however, is quite different, because they do not address corporate fraud and its implications for investment.

[8] The feasibility constraint on $k$ will not bind in our setting as long as $s_h^2 < R/2$ and $k^{\max} = \frac{1}{4s_h^2}$, and thus we make this assumption throughout the analysis.

[9] Our results do not rely on the manager having a zero reservation utility. A noisy revelation equilibrium can still be optimal for a strictly positive reservation utility as long as the manager earns rents in equilibrium. Neither does the optimality of a noisy revelation equilibrium rely on the assumption that the manager is risk neutral. Kumar and Langberg (2006) analyze the existence of the noisy revelation equilibrium in the model at hand when the agent is risk averse. The results are quite similar to the ones derived in the current article.

[10] For simplicity, we assume that the firm is unlevered—our results are materially unchanged if we relax this assumption. And although the assumption of a single active shareholder is convenient, our main results are robust to more general ownership patterns.

and $1 - \mu$, respectively. All investors know that the manager will privately observe $s$. Everything else in the model, besides the realization of $s$, is observable and common knowledge.

☐ **Contracting and the market for control.** From an institutional perspective, shareholders delegate the responsibility of wage contracting (with management) to the board of directors. These employment contracts are enforceable in the sense that managers can move the courts to enforce prior wage contracts even though the ownership of the firm changes, as the set of equity holders itself changes.[11] However, owners' ability to credibly *commit* to arbitrary investment in the firm is limited by the possibility of a change in control of the firm *ex post* through a takeover. Because investment at any given point in time is legally the domain of the *current* capital owners, the new owners can choose any desirable (and feasible) investment level.

To see the basic point, imagine a situation where shareholders' planned investment does not maximize efficiency, conditional on knowing the true productivity state. But this (inefficient) investment will lead to a *transparent* undervaluation of the firm's assets, relative to the *ex post* efficient investment level. However, transparent undervaluation is not a viable situation with a frictionless market for control. This is because there is a clear positive net present value (NPV) opportunity for a potential raider to purchase the firm at the undervalued price, reset the investment to the *ex post* efficient level, and sell (or even hold) the firm.

We can incorporate the commitment issues with respect to investment by appealing to the notion of *renegotiation-proof* mechanisms or contracts (see Bolton and Dewatripont, 2005). We will say that the mechanism is *renegotiation proof* if and only if the potential raider cannot benefit from gaining control of the firm and changing investment. Therefore, the investment policy described above is not renegotiation proof because there are incentives for the potential raider to offer an alternative arrangement to increase efficiency—implemented by purchasing the shares of the original owner. We formalize the notion of renegotiation proofness below.

☐ **Incentive mechanisms or contracts.** We will allow the manager to communicate with the owners regarding his private information on the firm's productivity. Owners can therefore design an incentive mechanism (or contract) at the beginning of time $t = 0$ that is contingent on the manager's communication of the productivity and on the publicly observable earnings. This communication occurs later during this time period, following the manager's observation of the actual productivity.

Specifically, the contract is a wage and investment menu. It determines the shareholders' investment policy as a function of the manager's communication; it also determines the manager's wages as a function of his communication and the observed earnings at time $t = 2$. However, the investment may change if the ownership changes (prior to the investment decision). The possibility of investment revision or renegotiation implies that the revelation principle fails to hold in our setting.[12]
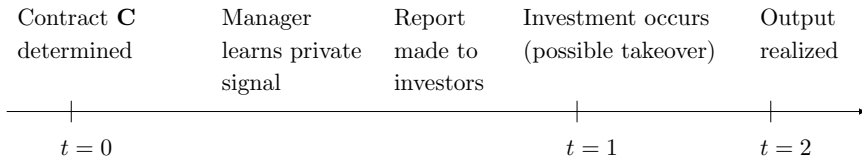
A contract therefore specifies a noisy reporting policy for the manager, contingent on his type. Let $\pi_{jr}$ be the probability that a manager with an actual productivity $j$ reports the productivity $r$, for $j, r \in \{\ell, h\}$. Similarly, the investment policy (or menu) is $k_r$, while the wage policy (or menu) is $\{w_r^+, w_r^0\}_{r=\ell}^h$, where $w_r^+$ ($w_r^0$) denotes the compensation when earnings are positive (zero) and the productivity $r$ was communicated. It is notationally convenient to put $\pi_{jj} \equiv \pi_j$ and

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[11] Indeed, there is much evidence that CEOs are able to successfully enforce their employment contracts, especially the payment of large severance payments, in the event of job termination and the sale of the firm (see, e.g., Murray, 2006; Lublin and Thurm, 2006). The *ex post* inflexibility of these employment contracts seems to surprise even boards who inked the contracts in the first place (Dash, 2006).

[12] This is a well-known result in the literature that studies agency models with adverse selection in the absence of commitment (e.g., Laffont and Tirole, 1987, 1988, 1990; Bester and Strausz, 2007). In general, one may require complex message spaces to characterize the optimal mechanism (e.g., Kumar, 1985; Forges, 1990); however, because we have only *two* possible agent types, we can restrict attention to noisy communications (or randomizations) on the space of types without loss of generality (see Bester and Strausz, 2007).

FIGURE 1

TIMING CONVENTIONS OF THE MODEL

| Contract **C** determined | Manager learns private signal | Report made to investors | Investment occurs (possible takeover) | Output realized |
|---|---|---|---|---|

$t = 0$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $t = 1$ $\qquad\qquad$ $t = 2$

$\pi = (\pi_\ell, \pi_h)$. Similarly, we let $\mathbf{k} \equiv (k_\ell, k_h)$ and $\mathbf{w} = (w_\ell^+, w_\ell^0, w_h^+, w_h^0)$. Thus, a contract is the profile $\mathbf{C} = \{\pi, \mathbf{w}, \mathbf{k}\}$.[13] We denote the set of feasible contracts by $\Omega = [0, 1]^2 \times R_+^4 \times [0, k^{\max}]^2$.

☐ **The takeover process.** At time $t = 1$, after the manager's communication but prior to the investment decision, the potential raider can make a take-it-or-leave-it tender offer to buy the firm from the original owner. If the original investor accepts the offer, then the control of the firm passes to the raider, who is then free to alter any preannounced investment plan, because the raider is the (new) owner of the firm's capital at the time of investment. However, the raider must respect the manager's original wage or employment contract ($\mathbf{w}$), because it is legally enforceable.[14] But if the tender offer fails, then the control of the firm remains with the original owner and the investment plan announced at $t = 0$ is executed.

☐ **Timing conventions.** The timing conventions of the model are described in Figure 1.

## 3. The optimal contract

■ **Admissible contracts and *ex post* investment efficiency.** A contract $\mathbf{C} = \{\pi, \mathbf{w}, \mathbf{k}\}$ is *admissible* if the noisy reporting policy ($\pi$) is incentive compatible for the manager and if the investment menu $\mathbf{k}$ is renegotiation proof. To quantify this notion of admissible contracts, we note first that for any given ($\mathbf{w}, \mathbf{k}$), the manager's payoffs when the true productivity is $j$, but he reports $r$, for $j, r \in \{\ell, h\}$ are

$$U_j(r \mid \mathbf{w}, \mathbf{k}) = 2s_j\sqrt{k_r}\left[w_r^+ - w_r^0\right] + w_r^0 + \psi k_r. \tag{1}$$

Therefore, the manager's expected payoffs when the true productivity is $j$ and he uses the noisy communication policy ($\pi_j, 1 - \pi_j$) are

$$U_j(\pi_j, \mathbf{w}, \mathbf{k}) = \pi_j U_j(j \mid \mathbf{w}, \mathbf{k}) + (1 - \pi_j)U_j(r \mid \mathbf{w}, \mathbf{k}), \quad r, j \in \{\ell, h\}, r \neq j. \tag{2}$$

The manager's reporting policy $\pi \in \mathbf{C}$ is incentive compatible if

$$\pi_j \in \arg\max_{\hat{\pi} \in [0,1]} U_j(\hat{\pi}, \mathbf{w}, \mathbf{k}), j \in \{\ell, h\}.$$

Next, we specify the renegotiation-proofness constraints on $\mathbf{k}$ by excluding opportunities for a profitable takeover *ex post*. Fix some $\mathbf{C}$, and note that the probability of receiving the report $r$

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[13] Notice that the manager's wage contract at time $t = 0$ is not directly contractible on the investment at time $t = 1$. As in the incomplete contracts literature (see Hart and Moore, 1988), we assume that a complete specification of future investment in the corporation is sufficiently complex to make wage contracts contingent on future investment prohibitively costly to enforce. In fact, managerial compensation contracts are not typically contingent on the external investment in the firm (Kole, 1997).

[14] In principle, the raider could also renegotiate the employment contract if it is mutually agreeable. But notice that the possibility of wage contract renegotiation only facilitates takeovers, for any takeover opportunity that is profitable with the constraint of enforcing the prior wage contract of the manager (i.e., $\mathbf{w}$) is at least weakly more profitable with the possibility of a mutually beneficial renegotiation of the wage contract. However, to ease the notational burden and simplify the model specification, we do not consider employment contract renegotiation.

$\in \{\ell, h\}$ (under the given contract) is $q_r(\mathbf{C}) = \mu \pi_{hr} + (1 - \mu)\pi_{\ell r}$. Hence, by Bayes' rule, the conditional expectation of the productivity state $s$, following a report $r$ is

$$E(s \mid \mathbf{C}, r) = \frac{\mu \pi_{hr} s_h + (1 - \mu)\pi_{\ell r} s_\ell}{\mu \pi_{hr} + (1 - \mu)\pi_{\ell r}}. \tag{3}$$

And the market value of the firm following this report is, therefore,

$$V(\mathbf{C}, r) = 2E(s \mid \mathbf{C}, r)\sqrt{k_r}\left[1 - w_r^+ + w_r^0\right] - w_r^0 - Rk_r. \tag{4}$$

Then, there is an opportunity to increase firm value *ex post*—that is, following the report $r \in \{\ell, h\}$—if there exists some investment level $\hat{k}_r$ that improves expected profits relative to the original investment plan ($k_r$) in the contract. That is, if

$$V(\mathbf{C}, r; \hat{k}_r) = 2E(s \mid \mathbf{C}, r)\sqrt{\hat{k}_r}\left[1 - w_r^+ + w_r^0\right] - w_r^0 - R\hat{k}_r > V(\mathbf{C}, r). \tag{5}$$

A renegotiation opportunity exists if, given (5), the potential raider can take over the firm and replace $k_r$ with $\hat{k}_r$. The contract ($\mathbf{C}$) is renegotiation proof if, for each report $r \in \{\ell, h\}$, there exists no $\hat{k}_r$ that is value improving (in the sense of (5)) and implementable through a takeover.

Clearly, given the takeover process, any $\hat{k}_r$ that is value improving is also implementable through a takeover. That is, given any $\hat{k}_r$ that satisfies (5), the potential raider can make a tender offer to purchase the firm's shares at a price that the original owner will accept, revise the investment plan, and make positive expected profits.

The logical culmination of this argument is that the requirement of renegotiation-proof investment will lead to *ex post* or *conditionally efficient*, that is, Bayes-consistent, investment. This is the investment level that maximizes firm value at the information set ($\mathbf{C}$, $r$ ). There is an equivalence between renegotiation-proof investment and *ex post* efficiency through the elimination of profitable takeover opportunities.

*Theorem 1.* If the investment policy $k_r, r \in \{\ell, h\}$, is renegotiation proof (i.e., contractually admissible), then it maximizes the value of the firm conditional on the report $r$, for any given $\mathbf{C}$. That is, $k_r \in \arg\max_{0 \leq k' \leq k^{\max}} V(\mathbf{C}, r; k')$, $r \in \{\ell, h\}$.

Whereas Theorem 1 presents sufficient conditions for *ex post* (or conditional) investment efficiency in the presence of frictionless takeover markets, this result is actually quite robust. For example, it would apply if there was no potential raider and the owner of the firm would simply revise any prior announced investment in light of the new information. With multiple blockholders, the allocation of efficiency gains from revising the investment *ex post* is more complex, but it is difficult to envisage the survival of *ex post* investment inefficiency. In effect, enriching the action space of investors—in this case, allowing them to take over firms to exploit profitable opportunities—results in reducing the *ex ante* incentive efficiency of investment policy in admissible contracts. This outcome is related to Dewatripont and Maskin (1995), who show that it may be optimal to restrict contractual contingencies when renegotiation is possible.

□ **Owner's program.** At time $t = 0$, the initial owner chooses an admissible contract that maximizes the *ex ante* value of the firm. Then, following Theorem 1, the optimal (renegotiation-proof) contract $\hat{\mathbf{C}}^* = (\hat{\pi}^*, \hat{\mathbf{k}}^*, \hat{\mathbf{w}}^*)$ is a solution to the program

$$\max_{\mathbf{C} \in \Omega} \sum_{r \in \{\ell, h\}} [\mu \pi_{hr} + (1 - \mu)\pi_{\ell r}] V(\mathbf{C}, r; k_r) \quad \text{s.t.,} \tag{6}$$

$$V(\mathbf{C}, r; k_r) \geq V(\mathbf{C}, r; \hat{k}_r), \quad \text{for all } \hat{k}_r \in [0, k^{\max}], \text{ and } r \in \{\ell, h\} \tag{7}$$

$$U_j(\pi_j, \mathbf{w}, \mathbf{k}) \geq U_j(\hat{\pi}, \mathbf{w}, \mathbf{k}), \quad \text{for all } \hat{\pi} \in [0, 1], \text{ and } j \in \{\ell, h\} \tag{8}$$

$$k^{\max} \geq k_r \geq 0, w_r^+ \geq 0, \quad w_r^0 \geq 0, \quad r \in \{\ell, h\}. \tag{9}$$

☐ **Equilibrium representation of optimal contracts.** Theorem 1 above implies that in any optimal contract, the investment **k** must be optimal given the posterior (or revised) beliefs of the investors, conditional on the manager's report. That is, **k** must be a Bayes-rational rule. This observation suggests that the optimal contract (cf. equation (6)) may be represented through a slightly modified form of the perfect Bayesian equilibrium (PBE) (see, e.g., Fudenberg and Tirole, 1991)—the modification being required to accommodate the contractually fixed managerial wages.

Specifically, a given wage policy, **w**, defines a noncooperative game with incomplete information with associated strategies $\pi$ (from the manager) and **k** (from the owner). The manager's reporting strategy $\pi$ and the owner's investment response to reports $r \in \{\ell, h\}$, **k**, comprise a PBE if (i) $\pi$ is optimal given (**k**, **w**), (ii) **k** is optimal given **w** and updated beliefs regarding the firm's productivity, denoted by $\beta(s; r)$, and (iii) the beliefs $\beta(s; r)$ are derived using Bayes' rule whenever possible.[15] It is easy to check that any contract $\mathbf{C} = \{\pi, \mathbf{w}, \mathbf{k}\}$ is optimal (in the sense of (6)) iff $(\pi, \mathbf{k})$ is a PBE in the game defined by **w**, where **w** is chosen to maximize firm value, taking as given the menu $(\pi, \mathbf{k})(\mathbf{w})$ that specifies an equilibrium $(\pi, \mathbf{k})$ for each **w**.

## 4. Important benchmark outcomes

■ To facilitate intuition on the basic forces that drive our results, in this section we record two benchmark outcomes. These are: the complete information (or the first-best) outcome; and the information-constrained efficient (or the second-best) outcome, where there is asymmetric information but investors can credibly precommit to any investment policy.

☐ **Complete information.** Suppose there is complete information on the productivity. In this case, the efficient (or the first-best) investment policy is denoted by $k_j^*$, $j = \ell, h$ such that

$$k_j^* = \left(\frac{s_j}{R}\right)^2.$$ (10)

Because $s_h > s_\ell$, it follows that $k_h^* > k_\ell^* > 0$; moreover, expected profits are positive in both productivity states. Consequently, the manager is not given any wage payment here, because he enjoys benefits of $\psi \, k_j^* > 0$ in each state.

*Proposition 1.* With complete information, for each $j \in \{\ell, h\}$, the investment $k_j^*$ is given by (10) and the wage policy sets $w_j^{*+} = w_j^{*0} = 0$.

☐ **Asymmetric information with perfect commitment.** The complete information allocation is transparently not feasible when the manager has private information on capital productivity, because the low-productivity manager can do strictly better by pretending to be the high-productivity manager.

Because owners can credibly precommit to both wage and investment policies (by assumption), the revelation principle applies. Thus, without loss of generality, we can restrict attention to direct mechanisms, where the agent's message space is $\{\ell, h\}$ and truth telling is an optimal strategy. A direct mechanism is specified as the pair (**k**, **w**), and the second-best or optimal mechanism is the solution to the program (P1)

$$\max_{\{\mathbf{w}, \mathbf{k}\} \in R_+^6} \sum_{j=\ell}^{h} \mu_j \left[ 2s_j \sqrt{k_j} \left( 1 - w_j^+ + w_j^0 \right) - w_j^0 - Rk_j \right], \quad \text{s.t.,}$$

$$U_j(j \mid \mathbf{w}, \mathbf{k}) \geq U_j(r \mid \mathbf{w}, \mathbf{k}), \quad r \neq j, \quad r, j \in \{\ell, h\}, \tag{11}$$

$$w_r^+ \geq 0, \quad w_r^0 \geq 0, \quad r \in \{\ell, h\}. \tag{12}$$

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[15] Alternatively, let $(\pi, \mathbf{k})$ be the continuation equilibrium for a given **w**.

Here, (11) are the truth-telling (incentive-compatibility) constraints and (12) are the nonnegativity constraints on managerial wages. We denote the solution to (P1) as $\hat{\delta} \equiv \{\hat{w}_r^+, \hat{w}_r^0, \hat{k}_r\}_{r=\ell}^h$.

Because the marginal productivity of capital becomes unbounded for infinitesimal investment levels, the information-constrained efficient investment levels will be positive in both productivity states. Hence, the manager will obtain positive benefits which cannot be taxed away by the owners due to the nonnegativity restriction on wages. Therefore, the manager's participation constraint will not bind in either productivity state.

Turning to incentive issues, a pooling investment policy where the investment level is the same for each (reported) productivity would trivially be incentive compatible. Consequently, managers will not be paid any incentive wages, that is, $w_j^+ = w_j^0 = 0$, $j \in \{\ell, h\}$. Hence, the optimal pooling level is

$$k^P = \left(\frac{E(s)}{R}\right)^2, \qquad E(s) = \mu s_h + (1 - \mu)s_\ell, \tag{13}$$

However, the pooling investment policy is clearly inefficient. Therefore, and except for a pooling range of parameters that we identify below, it will be optimal to distort wages and investment policy (relative to the first-best) in order to induce information from the managers. Specifically, there will be dispersion in the investment levels across productivity states; that is, $k_h > k_\ell$, but intuition suggests that this dispersion will be lower with asymmetric information relative to the complete information dispersion in order to relax the truth-telling constraint of the low-productivity type; that is, $(k_h - k_\ell) < (k_h^* - k_\ell^*)$.

Of course, a mechanism with $k_h > k_\ell$ is not incentive compatible with zero wages. Some introspection suggests that it would be inefficient to offer a positive wage if the manager reports high productivity. The reason is that such a mechanism will only tighten the truth-telling incentive constraints for the low-productivity manager. Hence, the optimal mechanism will give a positive wage when the manager reports low productivity so as to make him just indifferent between truthfully revealing the true state and pretending to be the high-productivity type. But because the high-productivity manager receives zero wages, he will also be just indifferent between truthful reporting and sending the alternative message.

Therefore, the incentive compatibility constraints are binding for both types in the second-best. The reason is that, in our model, there are two instruments to provide utility to the agent: the monetary wage ($w$) and the benefits of control through investment ($\psi$). The incentive-efficient contract therefore exploits the tradeoffs between wages and the utility of control to give the high-productivity manager zero wages but higher investment allocation. By setting the high-type agent's wages to zero, the optimal mechanism relaxes the low-type agent's truth-telling constraints while also reducing the expected wage costs.[16]

*Theorem 2.* There exists a $\psi^* > 0$ such that under the optimal mechanism $\hat{\delta} \equiv \{\hat{w}_r^+, \hat{w}_r^0, \hat{k}_r\}_{r=\ell}^h$,

$$\hat{w}_h^+ = \hat{w}_h^0 = 0, \text{ and } \hat{w}_\ell^+ = \hat{w}_\ell^0 = \psi(\hat{k}_h - \hat{k}_\ell). \tag{14}$$

Moreover, if $\psi < \psi^*$, then $k_\ell^* < \hat{k}_\ell < \hat{k}_h < k_h^*$, where

$$\hat{k}_h = \left[\frac{s_h}{R + \left(\frac{1-\mu}{\mu}\right)\psi}\right]^2, \hat{k}_\ell = \left[\frac{s_\ell}{R - \psi}\right]^2. \tag{15}$$

Finally, if $\psi \geq \psi^*$, then $\hat{k}_h = \hat{k}_\ell = k^P$.

In this model, $\psi$ is also a measure of the extent of the agency conflict between the shareholders and the manager. Intuition then suggests that it may be too costly to induce revelation if the agency

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[16] This feature distinguishes this model from other mechanism design models with hidden information where only the incentive constraints for one of the agent types are binding (e.g., Laffont and Tirole, 1990).

conflict is "too high," that is, if $\psi$ is sufficiently large. Theorem 2 verifies this argument, indicating that pooling is optimal when $\psi$ exceeds the threshold value $\psi^*$. Otherwise, it is optimal to induce revelation by distorting the investment and wage policy relative to the first-best. Comparing the second-best investment allocation (cf. (15)) with that in the first-best (cf. (10)), we find that investment distortions (in the second-best) are positively related to $\psi$. In particular, investment in the high (low) state of productivity is decreasing (increasing) in $\psi$, for $\psi < \psi^*$.

## 5. Noisy communication equilibria

■ **The role of randomized reporting.** In the optimal renegotiation-proof mechanism (cf. (6)–(9)), the owners have significant influence on the information content or accuracy of the manager's reports through the initial design of the wage policy. A main result of this article is that, for an open set of parameters, there is noisy revelation of the firm's economic state in equilibrium, that is, the optimal renegotiation-proof mechanism induces randomized reporting by the manager.

To motivate the role of randomization by the agent, we note that the set of admissible contracts include contracts that induce a fully revealing separating equilibrium where the manager reveals the true productivity. Due to *ex post* efficiency of investment, any separating equilibrium will imply the first-best investment levels $k_j^*$, $j = \ell, h$. Therefore, a simple example of a renegotiation-proof separating contract is one in which the low-productivity manager receives an output-independent wage $w_\ell$ and the high-productivity manager receives zero wages, such that $w_\ell + \psi k_\ell^* = \psi k_h^*$ (cf. (8)). But whereas this contract attains investment efficiency, it imposes a high-wage cost because the low-productivity manager extracts information-based rents because $w_\ell > 0$. Importantly, the rents are proportional to the difference in the investment allocation across the two states, that is, $w_\ell = \psi(k_h^* - k_\ell^*)$.

On the other extreme, there also exist admissible contracts that induce a pooling equilibrium (with noninformative reports). A simple example for such a contract is one in which the wage and investment policies are independent of the manager's report, such that $\mathbf{w} = \mathbf{0}$ and $k_\ell = k_h = k^P$ (where $k^P$ is quantified in (13)). But, as we noted above, although this pooling contract does not impose a compensation cost, it is costly for the owners in terms of high investment inefficiency, relative to the first-best.

Thus, the tradeoff between efficient allocation of capital and the compensation cost of extracting information determines the optimal information that is induced from the manager.[17] Le Chatelier's principle suggests that it may often be cost effective to influence Bayes-consistent investment policies toward those prescribed by the second-best by inducing randomized reporting from the manager. Such noisy communications will dampen investors' Bayes-rational response to a high-productivity report and amplify it in response to a low-productivity report; the equilibrium investment allocation will, therefore, move toward the second-best allocations (see (15)).

Of course, we need to still determine the optimal *pattern* of randomization across agent types: when is it optimal to have the low- and/or the high-productivity-type agent randomize? We note that in our model, randomization by both agent types (singly or together) may be optimal, because the incentive compatibility constraints bind for both agent types (i.e., upward and downward) in the second-best (see Theorem 2).[18] But for the reasons mentioned in the Introduction, we focus first on showing the existence of, and characterizing, a noisy revelation equilibrium where the low

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[17] See Laffont and Martimort (2002) for further discussion on the respective costs and benefits of separating versus pooling contracts under renegotiation.

[18] In a series of papers, Laffont and Tirole (1987, 1988, 1990) examine a two-period agency problem when hidden action and hidden information are jointly present, under a variety of contracting and commitment assumptions. In their basic model, there are two agent-types: a low-cost agent (the good type) and a high-cost agent (the bad type). Under the first-best, it is the good type that has the incentives to defect. Hence, in the second-best, only the good type's incentive constraints bind. If the principal can only write short-term contracts, then the incentive constraints can be binding for both types. However, if there is long-term contracting with renegotiation, then again only the good type's incentive constraints will bind.

type randomizes (i.e., $0 < \pi_{\ell h} < 1$ and $\pi_{h\ell} = 0$).[19] Subsequently, we provide a range of model parameters for which it is indeed optimal to only have the low-type agent randomize.

☐ **Equilibrium fraud and investment inefficiency.** It is apparent from (1) and (5) that an output-contingent wage policy affects the expected payoffs of both the manager and the shareholders. Thus, output-contingent wages influence not only managerial reports (through (8)) but also the owner's best investment response to these reports (through (7)). To facilitate intuition, we therefore conduct the analysis in two stages. First, we consider wage contracts that are contingent only on the manager's reports, but not on output. We then analyze (in Section 7 below) the more general case of output-contingent contracts.

With output-independent wages, we can write $w_j^+ = w_j^0 \equiv w_j, j \in \{\ell, h\}$. Thus, if $0 < \pi_{\ell h} < 1$, that is, with positive probability the low-productivity manager overreports the firm's productivity (and $\pi_{h\ell} = 0$), then this agent type's incentive constraint (8) is

$$w_h + \psi k_h = w_\ell + \psi k_\ell. \tag{16}$$

Moreover, the renegotiation-proof constraints on investment (7) reduce to

$$k_j = \left( \frac{E(s \mid \mathbf{C}, j)}{R} \right)^2, \; j \in \{\ell, h\}, \tag{17}$$

where $E(s \mid \mathbf{C}, h) = \frac{\mu s_h + (1-\mu)\pi_{\ell h} s_\ell}{\mu + (1-\mu)\pi_{\ell h}}$, and $E(s \mid \mathbf{C}, \ell) = s_\ell$ are given by (3). Therefore, the optimal contract $\hat{\mathbf{C}}^* = (\hat{\pi}^*, \hat{\mathbf{k}}^*, \hat{\mathbf{w}}^*)$, with noisy revelation by the low-type agent and truthful reporting by the high-type agent, is a solution to

$$\max_{\pi_{\ell h}, k_\ell, k_h, w_\ell, w_h} \begin{aligned}[t] &[\mu + (1-\mu)\pi_{\ell h}][E(s \mid \mathbf{C}, h)2\sqrt{k_h} - w_h - Rk_h] \\ &+ (1-\mu)(1-\pi_{\ell h})[E(s \mid \mathbf{C}, \ell)2\sqrt{k_\ell} - w_\ell - Rk_\ell], \end{aligned} \tag{18}$$

$$\text{s.t., (16), (17), and } w_j \geq 0, \; j \in \{\ell, h\}. \tag{19}$$

As in the second-best (cf. Theorem 2), the optimal wage policy here is also to set $\hat{w}_h^* = 0$. This is because the renegotiation-proofness constraints on investment do not affect the optimal policy of trading off lower wages for higher benefits of control (through larger investment) for the high-productivity manager. Thus, constraint (16) reduces to the constraint $w_\ell = \psi(k_h - k_\ell)$.

We now turn to characterization of the noisy revelation equilibrium at hand. Subsequently, we show that such an equilibrium will exist for an open range of model parameters.

*Theorem 3.* The optimal contract $\hat{\mathbf{C}}^* = (\hat{\pi}^*, \hat{\mathbf{k}}^*, \hat{\mathbf{w}}^*)$ with output-independent wages and truth telling by the high-productivity agent (i.e., $\pi_{h\ell} = 0$) is given by

$$\hat{k}_\ell^* = \left( \frac{s_\ell}{R} \right)^2, \; \hat{k}_h^* = \left( \frac{\mu s_h + (1-\mu)\hat{\pi}_{\ell h}^* s_\ell}{R(\mu + (1-\mu)\hat{\pi}_{\ell h}^*)} \right)^2, \; \hat{w}_h^* = 0, \; \hat{w}_\ell^* = \psi(\hat{k}_h^* - \hat{k}_\ell^*). \tag{20}$$

Moreover, in equilibrium, $\hat{\pi}_{\ell h}^*$ is implicitly defined by

$$2E(s \mid \hat{\mathbf{C}}^*, h) - \mu(s_h - s_\ell)\left( \frac{R}{\psi} + 1 \right) = 0, \text{ where } E(s \mid \hat{\mathbf{C}}^*, h) = \frac{\mu s_h + (1-\mu)\hat{\pi}_{\ell h}^* s_\ell}{\mu + (1-\mu)\hat{\pi}_{\ell h}^*}, \tag{21}$$

provided that $0 < \hat{\pi}_{\ell h}^* < 1$.

The optimality condition for $\hat{\pi}_{\ell h}^*$, specified in (21), clarifies that optimal randomization by the low type trades off costs of investment distortion with the wage savings from relaxing the truth-telling constraints of the low-type manager. We note that there are two types of investment distortions (relative to the first-best) induced by the said randomization. First, overstatement of

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FIGURE 2

EQUILIBRIUM REGIONS
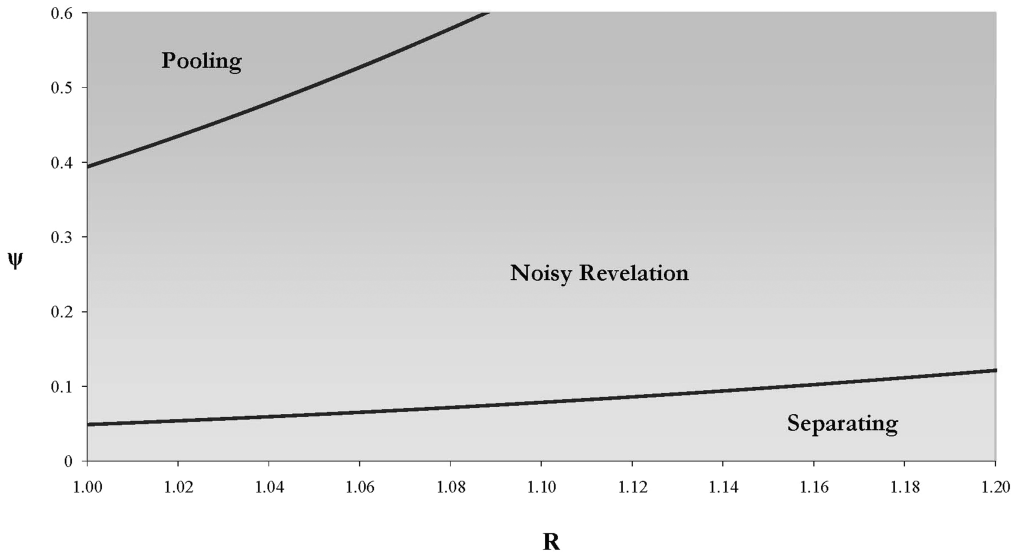


Illustrates the three equilibrium regions. The private benefit of control parameter $\psi$ is the vertical axis while the annual cost of capital $r = R^{1/5}$ is on the horizontal axis (i.e., $r$ represents the annual return on a project spanning over the duration of five years). The parameter values are $s_h = 0.7$, $s_l = 0.0$, and $\mu = 0.1$.

productivity by the low-type manager introduces overinvestment in the low-productivity state. Second, investors rationally respond to randomization by the low-type agent by underinvesting in the high state.

Therefore, the marginal cost of an increase in the probability of over-statement by the low type ($\pi_{\ell h}$) is the cost of overinvestment in the low state, which is positively related to the capital costs $R$. But the incentive gains from randomization are positively related to the manager's subjective benefits of control $\psi$. Hence, the ratio $\frac{R}{\psi}$ can be interpreted as an incentive cost-benefit ratio (related to the investment distortion). The second term in (21) thus reflects the marginal cost of $\pi_{\ell h}$. Meanwhile, expected wage cost is decreasing in the investment gap $k_h - k_\ell$, which is increasing in the posterior expected productivity, given a high report, that is, $E(s \mid \hat{\mathbf{C}}^*, h)$, the quantity in the first term in (21).

We now turn to examine the feasibility of a noisy revelation equilibrium, that is, where $0 < \hat{\pi}_{\ell h}^* < 1$. Intuitively, an interior noisy revelation equilibrium will obtain if the costs of inducing information from the low-productivity manager are neither too low nor too high. For, if these costs are too low, then it would be optimal to induce the correct information, that is, $\pi_{\ell h} = 0$. On the other hand, if these costs are very high, then it may be optimal not to induce any information at all, that is, $\pi_{\ell h} = 1$. It turns out that these conditions, that characterize the information content of the low-type agent's reports in equilibrium, can be succinctly expressed in terms of the incentive cost-benefit ratio $\frac{R}{\psi}$.

*Theorem 4.* There exist positive numbers $1 < \underline{b} < \bar{b}$, such that under the optimal contract $\hat{\mathbf{C}}^*$, the manager reports truthfully (i.e., $\hat{\pi}_{\ell h}^* = 0$) when $\frac{R}{\psi} \geq \bar{b}$; managerial reports are not informative (i.e., $\hat{\pi}_{\ell h}^* = 1$) when $\frac{R}{\psi} \leq \underline{b}$; and, there is noisy revelation (i.e., $0 < \hat{\pi}_{\ell h}^* < 1$) when $\frac{R}{\psi} \in (\underline{b}, \bar{b})$.

Figure 2 illustrates the three equilibrium regions as a function of the private benefit of control and the cost of capital. Furthermore, an immediate implication of Theorem 4 is as follows.

*Corollary 1.* The optimal noisy revelation contract $\hat{\mathbf{C}}^*$, with $0 < \hat{\pi}_{\ell h}^* < 1$, strictly dominates a truth-telling equilibrium and a pooling equilibrium whenever $\frac{R}{\psi} \in (\underline{b}, \bar{b})$.

FIGURE 3

PROBABILITY OF OVER-REPORTING THE LEVEL OF PRODUCTIVITY
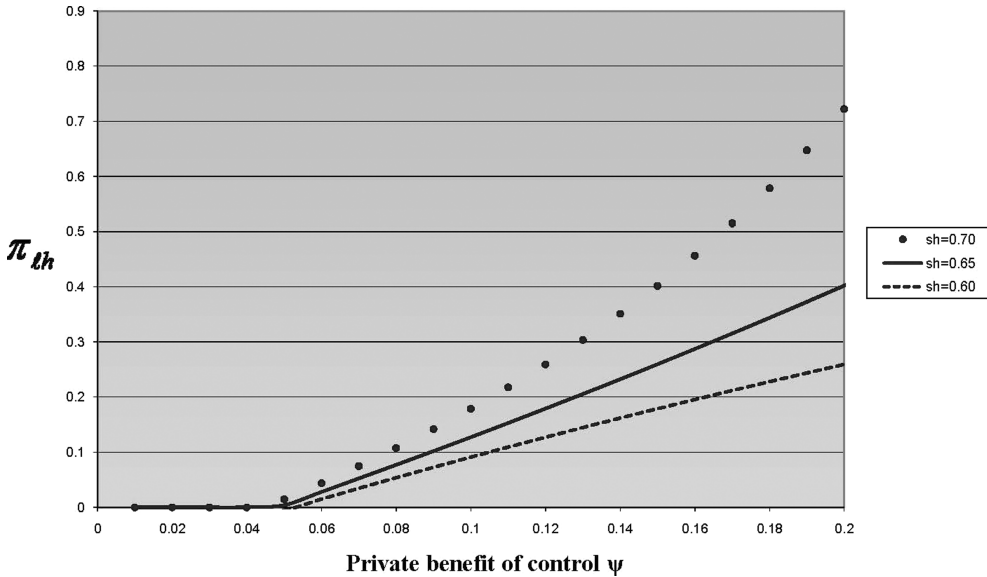


Plots the equilibrium likelihood that the low productivity manager makes a high productivity report, as a function of the private benefit of control parameter $\psi$ (for $R = 1$, and $\mu = 0.1$). Three cases are considered: $\langle s_h, s_l \rangle = \langle 0.7, 0.1 \rangle$, $\langle 0.65, 0.05 \rangle$ and $\langle 0.6, 0 \rangle$. Note that in all three cases $s_h - s_l = 0.60$.

Indeed, from the optimality condition (21), one can calculate analytically the equilibrium $\hat{\pi}_{\ell h}^*$:

$$\hat{\pi}_{\ell h}^* = \left( \frac{\mu}{1-\mu} \right) \frac{2 - \mu \left( 1 - \frac{s_\ell}{s_h} \right) \left( \frac{R}{\psi} + 1 \right)}{\mu \left( 1 - \frac{s_\ell}{s_h} \right) \left( \frac{R}{\psi} + 1 \right) - 2\frac{s_\ell}{s_h}}, \quad \text{for } \frac{R}{\psi} \in (\underline{b}, \bar{b}). \tag{22}$$

From (22), we can deduce the main determinants of $\hat{\pi}_{\ell h}^*$. In general, variations in model parameters that *increase* the manager's propensity to overstate productivity will raise $\hat{\pi}_{\ell h}^*$, because inducing truthfulness becomes more costly. Thus, $\hat{\pi}_{\ell h}^*$ will be increasing in $\psi$, because the manager's incentives for attracting higher investment through misreporting increase with his benefits from control. On the other hand, $\hat{\pi}_{\ell h}^*$ will be negatively related to parameters that increase the cost of investment misallocation due to randomized reporting. Thus, $\hat{\pi}_{\ell h}^*$ will be decreasing in $R$. We can therefore express these comparative statics compactly through variations in the incentive cost-benefit ratio $\frac{R}{\psi}$.

It is also apparent from (22) that $\hat{\pi}_{\ell h}^*$ is increasing in the ratio $\frac{s_\ell}{s_h}$. Effectively, if the lower productivity parameter ($s_\ell$) rises relative to the higher one ($s_h$), then the cost of investment distortion due to randomization by the low-type agent falls, while the incentive benefit increases. But note that an increase (or decrease) in the ratio $\frac{s_\ell}{s_h}$ leaves ambiguous the change in the *difference* of the productivities ($s_h - s_\ell$).

*Proposition 2.* In equilibrium, the probability that the low-type manager overstates productivity, $\hat{\pi}_{\ell h}^*$, is decreasing in the ratio $\frac{R}{\psi}$. Moreover, $\hat{\pi}_{\ell h}^*$ is increasing in the productivity ratio $\frac{s_\ell}{s_h}$; but $\hat{\pi}_{\ell h}^*$ is ambiguously related to ($s_h - s_\ell$).

Figure 3 plots the equilibrium $\hat{\pi}_{\ell h}^*$ against the private benefit parameter $\psi$. Three cases are considered: $s_h = 0.6$, $s_h = 0.65$, and $s_h = 0.7$ (while holding the difference of productivities

fixed, i.e., $(s_h - s_\ell) = 0.6$). The equilibrium likelihood of fraud is increasing in $\psi$ and increasing in the productivity ratio $\frac{s_\ell}{s_h}$, as shown formally in Proposition 2.

In the optimal contract specified in Theorem 3, there is no investment distortion following a low-productivity report (i.e., $\hat{k}_\ell^* = k_\ell^*$), because (with probability 1) only the low-productivity manager sends such a report. But there are investment distortions following a high-productivity report, in both productivity states. To analyze the equilibrium investment distortions, it is useful to compute the level of investment following a high-productivity report by substituting for $\hat{\pi}_{\ell h}^*$ (cf. (22)) in (20) to obtain

$$\hat{k}_h^* = \left( \frac{E(s \mid \hat{\mathbf{C}}^*, h)}{R} \right)^2 \text{ where } E(s \mid \hat{\mathbf{C}}^*, h) = \frac{\mu}{2}(s_h - s_\ell)\left( \frac{R}{\psi} + 1 \right). \tag{23}$$

*Proposition 3.* In equilibrium, investment following a high-productivity report satisfies $\hat{k}_h^* \in (k^P, k_h^*)$. Moreover, $\hat{k}_h^*$ is increasing in $(s_h - s_\ell)$, decreasing in $R$ and $\psi$, and increasing in $\mu$.

The first part of Proposition 3 confirms the intuition that, in the presence of noisy revelation by the low-productivity manager, the optimal investment following a high-productivity report lies between the pooling and the complete information investment levels. Meanwhile, the second part of the proposition indicates that even in the noisy revelation equilibrium with the renegotiation-proofness constraints, the investment response to a high-productivity report maintains certain efficiency features: it is negatively related to the opportunity cost ($R$) and it is positively related to the high-state productivity; moreover, as in the second-best investment policy (cf. (15)), it is negatively related to the manager's benefits of control ($\psi$). However, $\hat{k}_h^*$ also depends negatively on the low-state productivity $s_\ell$, because, for a fixed $s_h$, higher productivity in the low state increases the equilibrium likelihood of overreporting (cf. Proposition 3). By contrast, neither the first-best (cf. Proposition 1) nor the second-best (cf. Theorem 2) investments in the high-productivity state depend on $s_\ell$.

In the noisy revelation equilibrium, investors' posterior expectation for the firm's productivity conditional on a high report are bounded below by the unconditional expected productivity and bounded above by $s_h$, that is, $E(s) < E(s \mid \hat{\mathbf{C}}^*, h) < s_h$. Therefore, with probability $(1 - \mu)\pi_{\ell h}$, there is an overinvestment in the low-productivity state and, with probability $\mu$, there is an underinvestment in the high-productivity state. In relative terms, the extent of overinvestment (underinvestment) following a high-productivity report in the low (high) state is $\frac{\hat{k}_h^*}{k_\ell^*}$ ($\frac{k_h^*}{\hat{k}_h^*}$), namely,

$$\frac{\hat{k}_h^*}{k_\ell^*} = \left( \frac{E(s \mid \hat{\mathbf{C}}^*, h)}{s_\ell} \right)^2 \text{ and } \frac{k_h^*}{\hat{k}_h^*} = \left( \frac{s_h}{E(s \mid \hat{\mathbf{C}}^*, h)} \right)^2. \tag{24}$$

*Proposition 4.* In equilibrium, following a high-productivity report, there is overinvestment (relative to the first-best) if $s = s_\ell$, that is, $\frac{\hat{k}_h^*}{k_\ell^*} > 1$, but there is underinvestment (relative to the first-best), if $s = s_h$, that is, $\frac{k_h^*}{\hat{k}_h^*} > 1$.

In fact, based on (24) and the previous analysis, we can infer the determinants of the two types of investment distortions, in equilibrium. Specifically, from Proposition 2, we know that $\hat{\pi}_{\ell h}^*$ is decreasing in the incentive cost-benefit ratio $\frac{R}{\psi}$, and increasing in the productivity ratio $\frac{s_\ell}{s_h}$. Consequently, investors' posterior expected productivity following a high-productivity report are positively related to $\frac{R}{\psi}$ and negatively related to $\frac{s_\ell}{s_h}$. Thus, we obtain the following implications for relative investment distortions in Proposition 5. Figure 4 plots the investment distortions following a high-productivity report (cf. (24)), as a function of the manager's subjective benefits of control parameter $\psi$.

FIGURE 4

RELATIVE INVESTMENT DISTORTIONS FOLLOWING A HIGH PRODUCTIVITY REPORT



**Private benefit of control ψ**
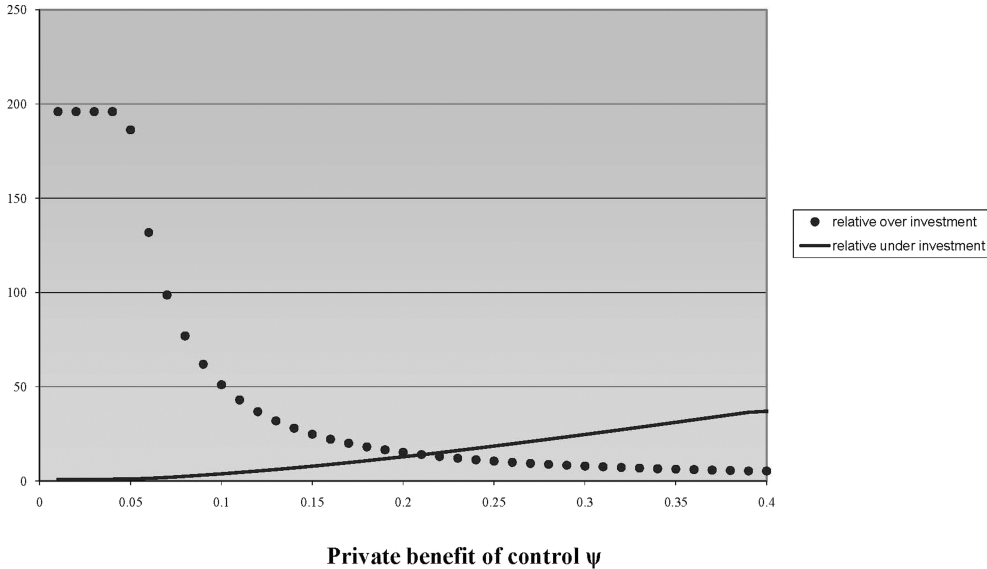
Plots the relative investment distortions following a high productivity report (both under investment and over investment) as a function of the private benefit of control parameter $\psi$ (for $R = 1$, $\langle s_h, s_l \rangle = \langle 0.7, 0.05 \rangle$, and $\mu = 0.1$).

*Proposition 5.* In equilibrium, the relative level of underinvestment (over-investment) $\frac{k_h^*}{k_\ell^*}$ $(\frac{\hat{k}_h^*}{\hat{k}_\ell^*})$ is decreasing (increasing) in $s_h$, increasing (decreasing) in $s_\ell$, and decreasing (increasing) in $\frac{R}{\psi}$.

Finally, the manager's wage compensation following a low-productivity report can be calculated from (20) and (21) as

$$\hat{w}_\ell^* = \psi(\hat{k}_h^* - \hat{k}_\ell^*) = \psi \left[ \left( \frac{E(s \mid \hat{\mathbf{C}}^*, h)}{R} \right)^2 - \left( \frac{s_\ell}{R} \right)^2 \right]. \qquad (25)$$

Thus, consistent with the literature on mechanism design with hidden information, shareholders in our model optimally design the wage policy to purchase more truthfulness from the manager in the low-productivity state in equilibrium (see Eisfeldt and Rampini, 2008; Choe, 1998). Intuitively, a higher wage for a low-productivity report ($w_\ell$) results in more informative managerial disclosures. For example, when the agency conflict is more severe (i.e., there is a higher $\psi$), or when the low-productivity parameter ($s_\ell$) rises relative to the higher one ($s_h$), the likelihood of overreporting by the low-type manager rises, while $w_\ell$ decreases (cf. Proposition 2).

*Proposition 6.* In equilibrium, the wage following a low-productivity report, $\hat{w}_\ell^*$, is decreasing in $\psi$ and $\frac{s_\ell}{s_h}$.

☐ **Output-contingent wage contracts.** In this section, we extend the analysis to allow output-contingent contracts. As we saw earlier, such contracts are not optimal in the benchmark first- or second-best cases (cf. Proposition 1 and Theorem 2). But output-contingent contracts can be potentially optimal, because they can influence investors' investment response to managerial disclosures *ex post*. Specifically, the optimal investment at the information set $(\mathbf{C}, r), r \in \{\ell, h\}$, is

$$k_r = \left( \frac{(1 - (w_r^+ - w_r^0))E(s \mid \mathbf{C}, r)}{R} \right)^2. \qquad (26)$$

Thus, by varying $(w_r^+ - w_r^0)$ *ex ante*, the owner can influence his consistent investment *ex post*. However, using output-contingent wages is also costly, because this wage policy dilutes the owner's claim on realized output in the high-earnings state.

Thus, noisy revelation and the use of output-contingent wages are two costly mechanisms that ameliorate the agency problem that arises due to the renegotiation-proof constraints on investment.[20] In particular, setting $w_h^+ - w_h^0 > 0$ decreases the investment gap $(k_h - k_\ell)$, and relaxes the low-type manager's incentive constraint. However, a positive wage differential $w_\ell^+ - w_\ell^0 > 0$ reduces investment following a low-productivity report, increases the investment gap $(k_h - k_\ell)$, and tightens the low-type manager's incentive constraint. Moreover, setting $w_h^0 > 0$ is not effective in reducing either the investment gap or relaxing the low-type manager's incentive constraints.

Theorem 5 below confirms the intuition that under the optimal contract with output-dependent wages and general randomization by both types, $w_h^0 = 0$ and $w_\ell^+ = w_\ell^0$. Moreover, in the model at hand, with a risk-neutral agent who has a linear benefit of control function, the use of output-contingent wages following a high-productivity report is not optimal, that is, $w_h^+ = w_h^0$. Therefore, in any optimal renegotiation-proof contract (or PBE), the wage policy is not output contingent, that is, $w_j^+ = w_j^0 \ j \in \{\ell, h\}$. Note that the wage contract can certainly be contingent on the reported productivity, in particular Theorem 5 states that $w_\ell^+ = w_\ell^0 \geq w_h^+ = w_h^0 = 0$, in equilibrium.

*Theorem 5.* In any optimal renegotiation-proof contract (or PBE), the equilibrium wages are not output contingent, that is, $w_h^+ = w_h^0 = 0$ and $w_\ell^+ = w_\ell^0$.

We reiterate that Theorem 5 holds while allowing randomized reporting strategies by *both* manager types, that is, while considering strategies $(\pi_{\ell h}, \pi_{h\ell}) \in [0, 1]^2$. To see why the optimal mechanism includes noisy revelation but not a positive wage differential $w_h^+ - w_h^0$, it is useful to consider a relaxed version of the optimal contracting problem with the incentive constraint (27) replacing (8):

$$\left(w_\ell^+ - w_\ell^0\right) s_\ell f(k_\ell) + w_\ell^0 + \psi k_\ell \geq \psi k_h. \tag{27}$$

We commence the proof of Theorem 5 by showing that (8) implies (27), that is, (27) is satisfied whenever (8) is satisfied. Now, considering the relaxed problem, it follows from (26) and (27) that it is suboptimal to set $w_h^0 > 0$, because any such wage policy is strictly dominated by an alternative policy that sets $w_h'^+ = w_h^+ - w_h^0$ and $w_h'^0 = 0$: the alternative policy does not affect the constraint (27), maintains the original investment following a high report (cf. (26)), and lowers expected wages. Similarly, $w_\ell^+ > w_\ell^0$ also cannot be optimal, because an alternative policy that sets $w_\ell'^+ = w_\ell'^0 = w'$, with $w' = (w_\ell^+ - w_\ell^0)s_\ell \ f(k_\ell) + w_\ell^0$, satisfies (27), but improves the *ex post* efficiency of $k_\ell$ without raising the expected wages. It is also the case that (27) binds at the optimum. Next, the objective function—with $w_h^0 = 0, w_\ell^+ = w_\ell^0$, and binding (27)—is such that the first-order condition with respect to $w_h^+$ is proportional to $(1 - w_h^+)$. Therefore, the optimal wage compensation $w_h^+$ is on the boundary, but paying $w_h^+$ at the upper boundary is suboptimal; therefore, $w_h^+ = 0$. We conclude the proof by establishing that the solution to the relaxed problem equals the solution to the original problem, that is, the solution of the relaxed problem satisfies the original incentive constraint (8).

☐ **Generalized randomization.** In Section 5 above, we have examined the optimality of inducing randomized reporting from the low-productivity manager, while assuming that the high-productivity manager reports truthfully. Now, from Theorem 5 it follows that the incentive constraints of both agent types bind at the optimum with *generalized* randomization. But we note that randomization by the high-productivity manager will increase investors' Bayes-rational

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[20] Monotonicity of managerial compensation contracts in performance is an appealing property and is a common simplification in the literature. See, for example, Innes (1990), Nachman and Noe (1994), and DeMarzo and Duffie (1999) for the use of, and potential justification for, such restrictions.

investment response to a low-productivity report, and therefore possibly move equilibrium outcomes toward the second-best. Thus, it may also be optimal (under certain conditions) to induce randomization by the high-type agent by trading off lower expected wages with the cost of investment distortions.

In this section, we therefore examine randomization by both agent types, and present sufficient conditions (on the model parameters) for randomization only by the low-type agent to be optimal. To see the main argument, consider the implications of increasing the probability that the high-productivity manager under reports, that is, $\pi_{h\ell}$. At the margin, this perturbation imposes investment distortion costs that equal $\mu [ r(h, h) - r(h, \ell)]$, where $r(h, h) \equiv s_h f(k_h) - Rk_h$ and $r(h, \ell) \equiv [s_h f(k_\ell) - Rk_\ell - w_\ell ]$ are the expected profits when the high-type manager reports truthfully and falsely, respectively. We also know from Theorem 5 that in any equilibrium, $w_\ell = \psi (k_h - k_\ell)$. But it follows from Bayes' rule that as $\pi_{h\ell}$ rises, investors' posterior expected productivity falls conditional on a high report and rises conditional on a low report; hence, $\frac{\partial k_h}{\partial \pi_{h\ell}} \leq 0$ and $\frac{\partial k_\ell}{\partial \pi_{h\ell}} \geq 0$. Hence, increasing $\pi_{h\ell}$ slightly also yields a marginal benefit of wage reduction that is proportional to $-\frac{\partial w_\ell}{\partial \pi_{h\ell}} (\geq 0)$.

We note that the marginal investment distortion cost, $\mu [ r(h, h) - r(h, \ell)]$, is inversely related to the low-productivity parameter ($s_\ell$) because the optimal $k_\ell$ is increasing in $s_\ell$. That is, the cost of capital misallocation when the true state is $s_h$, but $k_\ell$ is invested, is higher the lower is $s_\ell$. Moreover, the marginal investment cost increases with $\pi_{h\ell}$; and, if $\mu$ is bounded below and $s_\ell$ is not too high, then the objective function is convex in $\pi_{h\ell}$. As a consequence, under the conditions on $\mu$ and $s_\ell$, the optimal $\pi_{h\ell}$ lies on the boundary, that is, $\pi_{h\ell} \in \{0, 1 - \pi_{\ell h}\}$.[21]

However, the boundary solution $\pi_{h\ell} = 1 - \pi_{\ell h}$, as well as the solution $\langle \pi_{\ell h}, \pi_{h\ell} \rangle = \langle 1, 0 \rangle$, implement a pooling equilibrium, that is, reports are not informative and investments are $k_h = k_\ell = k^P$. But, as shown above in Theorem 4, the pooling contract ($\langle \pi_{\ell h}, \pi_{h\ell} \rangle = \langle 1, 0 \rangle$) is *strictly* dominated by the noisy revelation contract whenever $\frac{R}{\psi} \in (\underline{b}, \bar{b})$, or by the truth-telling contract whenever $\frac{R}{\psi} \geq \bar{b}$. Consequently, the upper boundary $\pi_{h\ell} = 1 - \pi_{\ell h}$ is not optimal as long as $\frac{R}{\psi} > \underline{b}$, leaving the solution $\pi_{h\ell} = 0$.

To summarize the immediately preceding arguments, it is optimal to induce truth telling from the high-type agent if the expected production inefficiency cost of under investing in the high-productivity state, that is, investing $k_\ell$ when the true state is $s_h$, is above some threshold. And this expected inefficiency cost (of underinvesting) is inversely related to the low-state productivity ($s_\ell$); that is, the lower is $s_\ell$, the greater is the inefficiency due to underinvestment. But it is positively related to the prior beliefs on the likelihood of the high-productivity state ($\mu$); for example, the expected cost of underinvestment is clearly very low if the prior probability of the high state is itself close to zero. Finally, using previous results, we know that pooling (i.e., $\pi_{\ell h} = 1$) is not optimal whenever the incentive-cost ratio $\frac{R}{\psi}$ is above a specified threshold (see Theorem 4 and Corollary 1). Putting these conditions together, we can specify conditions that ensure that only randomization by the low-type agent is optimal.

*Theorem 6.* There exists $0 < \tilde{s}_\ell < 1$, such that under the optimal contract there is truth telling by the high-productivity manager (i.e., $\pi_{h\ell} = 0$) and randomization by the low-productivity manager (i.e., $0 \leq \pi_{\ell h} < 1$) whenever $\frac{R}{\psi} > \underline{b}$, $0 < s_\ell < \tilde{s}_\ell$, and $\mu > \frac{1}{3}$.

Although Theorem 6 implies that, under the optimal contract, there is noisy revelation ($0 < \pi_{\ell h} < 1$) by the low-productivity agent (as described in Theorem 3) and truth telling by the high-productivity manager if $\frac{R}{\psi} \in (\underline{b}, \bar{b})$, $s_\ell < \tilde{s}_\ell$, and $\mu > \frac{1}{3}$, further exploration of the problem numerically confirms that the conditions on $s_\ell$ and $\mu$ are sufficient but not necessary. In particular, Figure 5 plots firm value as a function of the randomization probabilities $\langle \pi_{\ell h}, \pi_{h\ell} \rangle$, for $\mu = 0.1$,

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[21] As we argue in Theorem 5, a feature of any optimal contract is that $\pi_{h\ell} \leq 1 - \pi_{\ell h}$, as $\pi_{h\ell} = 1 - \pi_{\ell h}$ implements the pooling equilibrium.

FIGURE 5

FIRM VALUE WITH RANDOMIZATION BY BOTH MANAGER TYPES
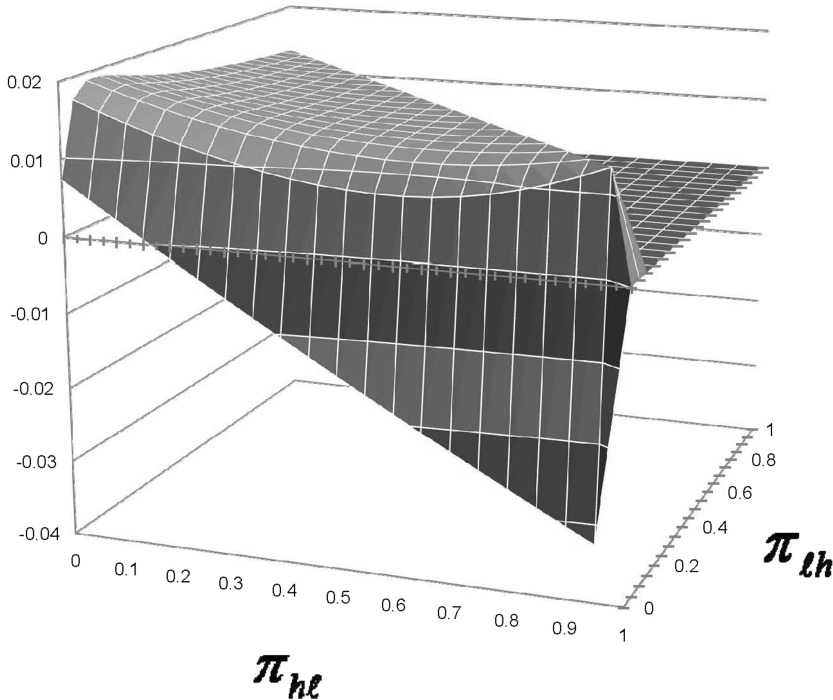


Plots firm value as a function of the randomization probabilities $\langle \pi_{lh}, \pi_{hl} \rangle$, for $\mu = 0.1$, $\psi = 0.1$, $R = 1$, and $\langle s_h, s_l \rangle = \langle 0.7, 0.05 \rangle$. The optimum is located at $\langle \pi_{lh}, \pi_{hl} \rangle = \langle 0.12, 0 \rangle$.

and shows that the optimal general randomization strategy is given by $\langle \pi_{\ell h}, \pi_{h\ell} \rangle = \langle 0.12, 0 \rangle$, consistent with Theorem 3.[22]

## 6. Summary and conclusions

◼　Investment inefficiency in certain firms or sectors, induced by corporate fraud where informed insiders manipulate the beliefs of uninformed investors through exaggerations of economic prospects, has been historically prevalent, and has recently attracted much attention because of some prominent corporate scandals. However, reconciling such investment distortions and corporate fraud with rational capital markets poses obvious challenges. We provide a new theory where corporate fraud (or noisy information revelation) is accompanied by overinvestment in low-return states and under-investment in high-return states. Our framework is based on two important characteristics of modern corporations operating in well-developed financial markets: there are shareholder-management agency conflicts because managers derive private benefits

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[22] Based on the optimal wage policy specified in Theorem 5 and the results of Theorem 6, we can briefly comment on the implications of allowing renegotiation of wage contracts (cf. footnote 12). Suppose that $r = \ell$, in which case the firm is revealed to be a low-productivity firm. Here, $w_\ell$ is renegotiation proof because the owner clearly has no information-based incentives to offer an alternative wage; he also does not have risk-sharing benefits (as in Fudenberg and Tirole, 1990), because the agent is risk neutral. Meanwhile, the agent will not accept any wage lower than $w_\ell$ as long as investment is fixed at $k_\ell^*$ from the *ex post* efficiency constraint. Next, suppose that $r = h$, in which case there is uncertainty *ex post* regarding the firm's productivity. However, the owner can induce further information through renegotiation only by inducing the agent to communicate again (or to choose from a type-based wage menu). Although sequential mechanism design is outside the scope of our model, it is an interesting topic for further research to examine our model in such a setup.

from controlling larger investments and have private information on the investment opportunity set; but shareholders cannot credibly precommit to investment that is inefficient *ex post*, because such investment policies are renegotiated in active takeover markets. For a wide range of model parameters, outside shareholders *optimally* suffer a positive probability of fraud in equilibrium, because the lack of investor commitment increases the incentive compensation costs of inducing truthfulness from informed insiders. Shareholders therefore optimally determine the probability of fraud and the extent of investment inefficiency.

The model predicts that fraud is more likely to occur in firms with easy access to external capital or financing. Historically, there are several examples where firms with easy access to external capital have been subject to corporate fraud and aggressive external financing. For example, the South Sea Company obtained the government's permission to convert a large portion of government debt into the company's shares, and this relatively easy availability of capital appears to have been a major motivation for the company to mislead investors (MacKay, 1980; Faber, 2002). Similarly, in the 1860s, railroad entrepreneurs copied a major contemporaneous financial innovation—namely, the selling of government bonds to the public (rather than only to financial institutions)—to aggressively finance their rapidly growing industry. And, during the 1920s, a period in which it was "easier than ever before to raise huge amounts of capital" (Skeel, 2005), the introduction of closed-end funds played an important role in satisfying the financing needs of corporations (Shleifer, 2000).

Our framework also suggests a link between certain types of technological innovations and the likelihood of corporate fraud. The equilibrium likelihood of fraud is predicted to be higher if there are productivity-enhancing technological innovations that raise the minimum capital productivity (or investment returns) relative to the maximal productivity.[23] That is, the likelihood of fraud is higher if the floor on productivities or investment returns is increased. Innovations that improve infrastructure or operational efficiency arguably bring $s_\ell$ proportionally closer to $s_h$. Therefore, our model provides a perspective on the coincidence of significant increases in fraud with innovations that improved transportation (as in the 1860s) or information processing (as in the 1990s). We reiterate that the relationship between fraud and $(s_h - s_\ell)$, which measures the cross-sectional variations in capital productivities or the uncertainty in investment returns, is theoretically ambiguous. More generally, our analysis provides an interesting agenda for empirical research that systematically examines the relationship between the incidence of fraud at the firm level with the cost of external financing and suitably constructed empirical proxies for the investment opportunity set.

The lack of full commitment to arbitrary investment policies in financial markets is the central friction we have explored in this article. Given the nefarious consequences of this lack of commitment (that we have highlighted above), one would expect that mechanisms that facilitate such commitment should have emerged over time. Indeed, one such example is the corporate charter, which specifies at the time of incorporation the operating guidelines for the corporation. Owners in control of the corporation in the future must follow these guidelines. Corporate charters exhibit substantial cross-sectional differences, especially with respect to the strength of corporate governance (Gompers, Ishii, and Metrick, 2003). For example, whereas some firms provide guidelines that secure the interests of shareholders (*democratic*), other firms provide more freedom to management (*dictatorship*). More relevant to our analysis, corporate charters also differ in the flexibility they grant to management to resist takeovers. *Prima facie*, this type of heterogeneity is puzzling because a common objective of designing corporate charters should be to ensure an efficient operation of the corporation. Our model highlights a somewhat surprising advantage for firms to design charters *ex ante* that impede takeovers and even increase the costs of external financing *ex post*. Impediments to takeovers can weaken the renegotiation-proof constraints on investment, and it may actually be optimal to voluntarily increase the firm's

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[23] We are grateful to a referee for suggesting this point to us.

expected cost of capital under the optimal corporate charter. This topic is clearly an avenue for future research.

## Appendix

■ *Proof of Theorem 1.* Fix any admissible contract $C = \{\pi, \mathbf{k}, \mathbf{w}\}$ and suppose that the report is $r \in \{\ell, h\}$. Then, at the information set $(C, r)$, a takeover strategy by the potential raider is the pair $\Phi = (V', k')$. Here, $V'$ is the tender offer for the firm, and $k'$ is the investment level following a successful tender offer. Next, at the information set $(C, r, V')$, the original owner chooses a strategy, $\xi(V') \in \{0, 1\}$, where "0" denotes the decision to reject the tender offer while "1" denotes the alternative. We recall that if the original owner chooses not to tender his shares, then the investment $k_r \in C$ is executed.

A takeover game $\Gamma(\mathbf{C}, r)$ is a specification of the strategies $(\Phi, \xi)$, where $\xi = \{\xi(V')\}$. We now define a PBE of $\Gamma(\mathbf{C}, r)$. Given the strategies $(\Phi, \xi)$, the expected payoff to the potential raider is

$$\Pi(\Phi, \xi) = \begin{cases} V(\mathbf{C}, r; k') - V' & \text{if } \xi(V') = 1, \\ 0 & \text{else.} \end{cases} \tag{A1}$$

And the expected payoff to the original owner is

$$W(\Phi, \xi) = \begin{cases} V' & \text{if } \xi(V') = 1, \\ V(\mathbf{C}, r) & \text{else.} \end{cases} \tag{A2}$$

Then, the pair $(\Phi, \xi)$ is a PBE of $\Gamma(\mathbf{C}, r)$, if $\Pi(\Phi, \xi) \geq \Pi(\hat{\Phi}, \xi)$, for every $\hat{\Phi} \in R_+ \times [0, k^{\max}]$, and $W(\Phi, \xi) \geq W(\Phi, \hat{\xi})$, for $\hat{\xi} : R_+ \to \{0, 1\}$.

Suppose that for any given $(C, r)$, the contracted $k_r$ is inefficient. Set $k' \in \arg\max_{k' \in [0, k^{\max}]} V(C, r; k')$ and $V' = V(C, r)$. Then, by hypothesis and the definition of $k'$, $V(C, r; k') > V(C, r) = V'$. It is easy to check that the following strategies constitute a PBE: $\Phi = (V(\mathbf{C}, r), k')$, and for the original owner $\xi(V') = 1$ if $V' \geq V(C, r)$ and $\xi(V') = 0$ otherwise. Thus, we have established that as long as the contracted $k_r$ is not *ex post* efficient, there exists a PBE with a successful takeover by the potential raider that "resets" the investment to the *ex post* efficient level following the takeover. *Q.E.D.*

*Proof of Proposition 1.* Because the optimal investments are positive (cf. (10)), $U_j(w = 0, k^*) \geq 0$, $j \in \{\ell, h\}$, because $\psi k_j^* \geq 0$. Hence, the participation constraints of both manager types are satisfied. *Q.E.D.*

*Proof of Theorem 2.* To solve program (P1), we first solve a relaxed program (P1') that ignores the incentive compatibility constraint for the high-type agent (see (11)). We then verify that the solution to (P1') satisfies this constraint. Let $\{\mathbf{w}', \mathbf{k}'\}$ denote the solution to program (P1'), where

$$(\text{P1}') \max_{\{\mathbf{w}, \mathbf{k}\} \in R_+^6} \mu\left[2s_h\sqrt{k_h}\left(1 - w_h^+ + w_h^0\right) - w_h^0 - Rk_h\right]$$

$$+ (1 - \mu)\left[2s_\ell\sqrt{k_\ell}\left(1 - w_\ell^+ + w_\ell^0\right) - w_\ell^0 - Rk_\ell\right], \text{ s.t.,}$$

$$[IC_\ell] \; 2s_\ell\sqrt{k_\ell}\left(w_\ell^+ - w_\ell^0\right) + w_\ell^0 + \psi k_\ell \geq 2s_\ell\sqrt{k_h}\left(w_h^+ - w_h^0\right) + w_h^0 + \psi k_h.$$

The Lagrangian of (P1') is

$$\mathcal{L} = \mu\left[2s_h\sqrt{k_h}\left(1 - w_h^+ + w_h^0\right) - w_h^0 - Rk_h\right] + (1 - \mu)\left[2s_\ell\sqrt{k_\ell}\left(1 - w_\ell^+ + w_\ell^0\right) - w_\ell^0 - Rk_\ell\right]$$

$$+ \lambda\left[2s_\ell\sqrt{k_\ell}\left(w_\ell^+ - w_\ell^0\right) + w_\ell^0 + \psi k_\ell - \left(2s_\ell\sqrt{k_h}\left(w_h^+ - w_h^0\right) + w_h^0 + \psi k_h\right)\right]$$

$$+ \sum_{j=\ell}^{h} \theta_j^+ w_j^+ + \sum_{j=\ell}^{h} \theta_j^0 w_j^0 + \sum_{j=\ell}^{h} \theta_j k_j.$$

For any solution $\{\mathbf{w}', \mathbf{k}'\}$ to (P1'), there exist positive multipliers $(\lambda, \theta_h^+, \theta_\ell^+, \theta_h^0, \theta_\ell^0, \theta_\ell, \theta_h) \geq 0$, such that $\frac{\partial \mathcal{L}}{\partial w_h^+} = \frac{\partial \mathcal{L}}{\partial w_\ell^+} = \frac{\partial \mathcal{L}}{\partial w_h^0} = \frac{\partial \mathcal{L}}{\partial w_\ell^0} = 0$, and

$$\frac{\partial \mathcal{L}}{\partial k_h} = \mu\left[\frac{s_h}{\sqrt{k_h}}\left(1 - w_h^{+'} + w_h^{0'}\right) - R\right] - \lambda\left[\frac{s_h}{\sqrt{k_h}}\left(w_h^{+'} - w_h^{0'}\right) + \psi\right] + \theta_h = 0$$

$$\frac{\partial \mathcal{L}}{\partial k_\ell} = (1 - \mu)\left[\frac{s_\ell}{\sqrt{k_\ell}}\left(1 - w_\ell^{+'} + w_\ell^{0'}\right) - R\right] + \lambda\left[\frac{s_\ell}{\sqrt{k_\ell}}\left(w_\ell^{+'} - w_\ell^{0'}\right) + \psi\right] + \theta_\ell = 0$$

$$0 = \lambda\left[2s_\ell\sqrt{k_\ell}\left(w_\ell^{+'} - w_\ell^{0'}\right) + w_\ell^{0'} + \psi k_\ell' - \left(2s_\ell\sqrt{k_h}\left(w_h^{+'} - w_h^{0'}\right) + w_h^{0'} + \psi k_h'\right)\right]$$

$$0 = \theta_j^+ w_j^{'+} = \theta_j^0 w_j^{'0} = \theta_j k_j' \text{ for } j \in \{\ell, h\}.$$

First note that the solution to program (P1') satisfies $k'_h \geq k'_\ell$. Suppose not, that is, $k'_h < k'_\ell$. Then it must be that $w'^+_\ell = w'^0_\ell = w'^+_h = w'^0_h = 0$, because the incentive compatibility constraint $[IC_\ell]$ is satisfied with slack when

wages are zero, namely, $w' = 0$, and the value of the objective function in program (P1′) is strictly increasing in wages. Moreover, it also implies that $\lambda = \theta_\ell = 0$. Therefore, from the first-order conditions above, $\mu[\frac{s_h}{\sqrt{k'_h}} - R] = -\theta_h$, and $(1-\mu)[\frac{s_\ell}{\sqrt{k'_\ell}} - R] = 0$. It follows that $R = \frac{s_\ell}{\sqrt{k'_\ell}}$ (i.e., $k^*_\ell = k'_\ell$; see Proposition 1), and $\frac{s_h}{\sqrt{k'_h}} \leq R$. But, this implies that $\sqrt{k'_h} \geq \frac{s_h}{R} \geq \frac{s_\ell}{R} = \sqrt{k'_\ell}$. Thus, the assumption that $k'_h < k'_\ell$ contradicts the optimality of $\mathbf{k}'$ and we conclude that $k'_h \geq k'_\ell$.

We claim next that in any solution to (P1′), $w'^+_h = w'^0_h = 0$. Suppose (to the contrary) that max $(w'^+_h, w'^0_h) > 0$. Then, consider an alternative wage contract $\tilde{\mathbf{w}} = (w'^+_\ell, w'^0_\ell, 0, 0)$. The alternative contract $\{\tilde{\mathbf{w}}, \mathbf{k}'\}$ satisfies the constraint $[IC_\ell]$, but strictly improves the objective function. Thus, we reach a contradiction and conclude that $w'^+_h = w'^0_h = 0$. Next, we claim that it is without loss of generality to set $w'^+_\ell = w'^0_\ell$. Suppose not, and consider an alternative wage contract $\check{\mathbf{w}} = (w_\ell, w_\ell, 0, 0)$, where $w_\ell = 2s_\ell\sqrt{k_\ell}(w'^+_\ell - w'^0_\ell) + w'^0_\ell$. The alternative contract $\{\check{\mathbf{w}}, \mathbf{k}'\}$ satisfies the constraint $[IC_\ell]$, and does not affect the value of the objective function (note that $w^+{}'_h = w^0{}'_h = 0$ and $k'_h \geq k'_\ell$ imply that $w_\ell \geq 0$). Therefore, $w'^+_\ell = w'^0_\ell \equiv w'_\ell$. Finally, we claim that the incentive compatibility constraint $[IC_\ell]$, which reduced to $w^0_\ell + \psi k_\ell \geq \psi k_h$, binds under the optimal contract $\{\mathbf{w}', \mathbf{k}'\}$. Suppose (to the contrary) that $w'_\ell + \psi k'_\ell > \psi k'_h$, and consider an alternative wage contract $\hat{\mathbf{w}} = (\hat{w}_\ell, \hat{w}_\ell, 0, 0)$ with lower wage $\hat{w}_\ell = \psi(k'_h - k'_\ell)$. The alternative contract $\{\hat{\mathbf{w}}, \mathbf{k}'\}$ satisfies the constraint $[IC_\ell]$, but achieves a higher value for the objective function. Thus, we reach a contradiction and conclude that $[IC_\ell]$ binds.

Now, the incentive constraint of the high type in the original problem (P1) as given by (11) is

$$[IC_h]\ 2s_h\sqrt{k_h}(w^+_h - w^0_h) + w^0_h + \psi k_h \geq 2s_h\sqrt{k_\ell}(w^+_\ell - w^0_\ell) + w^0_\ell + \psi k_\ell.$$

Note that it follows from the properties of $\{\mathbf{w}', \mathbf{k}'\}$,

$$w'^+_h = w'^0_h = 0, \text{ and } w'^+_\ell = w'^0_\ell \equiv w'_\ell = \psi(k'_h - k'_\ell), \tag{A3}$$

that $[IC_h]$ is satisfied by the solution to the relaxed program (P1′). Thus, $\{\mathbf{w}', \mathbf{k}'\}$ is a solution also to the original problem (P1). Therefore, we conclude that the solution to the original problem (P1), $\{\mathbf{w}', \mathbf{k}'\}$, satisfies (A3) and solves program (A4),

$$\max_{\{\mathbf{k}\}\in R^2_+} \mu\left[2s_h\sqrt{k_h} - Rk_h\right] + (1-\mu)\left[s_\ell f(k_\ell) - Rk_\ell - \psi(k_h - k_\ell)\right],$$
$$\text{s.t. } k_h \geq k_\ell \geq 0. \tag{A4}$$

The Lagrangian of (A4) is

$$\mathcal{L} = \mu\left[2s_h\sqrt{k_h} - Rk_h\right] + (1-\mu)\left[s_\ell f(k_\ell) - Rk_\ell - \psi(k_h - k_\ell)\right] + \lambda[k_h - k_\ell] + \theta k_\ell.$$

For any solution $\mathbf{k}'$ to (A4), there exists positive multipliers $\lambda \geq 0$ and $\theta \geq 0$, such that

$$\frac{\partial\mathcal{L}}{\partial k_h} = \mu\left[\frac{s_h}{\sqrt{k'_h}} - R + \psi\right] - \psi + \lambda = 0$$

$$\frac{\partial\mathcal{L}}{\partial k_\ell} = (1-\mu)\left[\frac{s_\ell}{\sqrt{k'_\ell}} - R + \psi\right] - \lambda + \theta = 0$$

$$\lambda[k'_h - k'_\ell] = \theta k'_\ell = 0.$$

The various cases associated with the set of Kuhn-Tucker conditions are (i) if $\lambda = 0$ (i.e., $k'_h \geq k'_\ell$) and $\theta = 0$ (i.e., $k'_\ell \geq 0$), then $k'_h = [\frac{s_h}{R+(\frac{1-\mu}{\mu})\psi}]^2$, and $k'_\ell = [\frac{s_\ell}{R-\psi}]^2$. But, the constraint $k'_h \geq k'_\ell$ requires that $\psi \leq \frac{\mu R(s_h - s_\ell)}{\mu s_h + s_\ell(1-\mu)}$; (ii) if $\lambda = 0$ (i.e., $k'_h \geq k'_\ell$) and $\theta > 0$ (i.e., $k'_\ell = 0$), then $k'_h = [\frac{s_h}{R+(\frac{1-\mu}{\mu})\psi}]^2$, and $R - \psi - \frac{\theta}{1-\mu} = \frac{s_\ell}{\sqrt{k'_\ell}}$. But, the latter requires that $s_\ell = 0$ and $\theta = (R-\psi)(1-\mu) > 0$ (i.e., $\psi < R$); (iii) if $\lambda > 0$ (i.e., $k'_h = k'_\ell$) and $\theta = 0$ (i.e., $k'_\ell \geq 0$), then $k'_h = [\frac{s_h}{R+(\frac{1-\mu}{\mu})\psi - \frac{\lambda}{\mu}}]^2$, and $k'_\ell = [\frac{s_\ell}{R-\psi+\frac{\lambda}{1-\mu}}]^2$. But, the condition $k'_h = k'_\ell$ implies that $\lambda = (1-\mu)[\psi - \frac{\mu R(s_h-s_\ell)}{\mu s_h + s_\ell(1-\mu)}]$, and the condition $\lambda > 0$ implies that $\psi > \frac{\mu R(s_h - s_\ell)}{\mu s_h + s_\ell(1-\mu)}$. Finally, the solution for $\lambda$ implies that $k'_h = k'_\ell = [\frac{\mu s_h + (1-\mu)s_\ell}{R}]^2$; (iv) if $\lambda > 0$ (i.e., $k'_h = k'_\ell$) and $\theta > 0$ (i.e., $k'_\ell = 0$), then $k'_h = [\frac{s_h}{R+(\frac{1-\mu}{\mu})\psi - \frac{\lambda}{\mu}}]^2 = 0$ and $k'_\ell = [\frac{s_\ell}{R-\psi+\frac{\lambda}{1-\mu} - \frac{\theta}{1-\mu}}]^2 = 0$. But, this requires that $s_h = s_\ell = 0$.

Thus, we conclude that, if $s_h > s_\ell \geq 0$, then $k'_h = [\frac{s_h}{R+(\frac{1-\mu}{\mu})\psi}]^2$, $k'_\ell = [\frac{s_\ell}{R-\psi}]^2$, and $w'_\ell = \psi(k'_h - k'_\ell) > 0$ if $\psi < \frac{\mu R(s_h - s_\ell)}{\mu s_h + s_\ell(1-\mu)}$ ($\equiv \psi^*$), but $k'_h = k'_\ell = [\frac{\mu s_h + (1-\mu)s_\ell}{R}]^2$, and $w'_\ell = 0$ when $\psi \geq \psi^*$. *Q.E.D.*

*Proof of Theorem 3.* For notational ease, put $\pi \equiv \pi_{\ell h}$. Then if $0 < \pi < 1$ and $\pi_{hh} = 1$, the renegotiation-proof constraint on investment implies that $k_\ell = (\frac{s_\ell}{R})^2$ and $k_h = (\frac{\mu s_h + (1-\mu)\pi s_\ell}{R(\mu + (1-\mu)\pi)})^2$. Also, the manager's incentive constraint implies that $w_\ell + \psi k_\ell = w_h + \psi k_h$. As argued in the proof of Theorem 2, one can show that it is optimal to set $w_h = 0$, otherwise a reduction in $w_h$ would increase the objective function without affecting the constraints. Therefore, in equilibrium the incentive compatibility constraint takes the form $w_\ell + \psi k_\ell = \psi k_h$, that is, $\frac{w_\ell}{\psi} = k_h - k_\ell$ (recall that $w^+_\ell = w^0_\ell \equiv w_\ell$).

Rewriting, $k_h = \frac{w_\ell}{\psi} + (\frac{s_\ell}{R})^2$ yields

$$\left(\frac{w_\ell}{\psi} + \left(\frac{s_\ell}{R}\right)^2\right)^{\frac{1}{2}} = \frac{\mu s_h + (1-\mu)\pi s_\ell}{R(\mu + (1-\mu)\pi)}, \text{ and } \pi = \left(\frac{\mu}{1-\mu}\right)\left[\frac{s_h - z}{z - s_\ell}\right], \tag{A5}$$

where $z \equiv R(\frac{w_\ell}{\psi} + (\frac{s_\ell}{R})^2)^{\frac{1}{2}}$, or $w_\ell = \psi((\frac{z}{R})^2 - (\frac{s_\ell}{R})^2)$. We note that $z = E(s \mid r = h)$ and $k_h = (\frac{z}{R})^2$ . Therefore,

$$\begin{aligned}
\frac{\partial \pi}{\partial z} &= -\left(\frac{\mu}{1-\mu}\right)\frac{(s_h - s_\ell)}{(z - s_\ell)^2} \\
\frac{\partial \pi}{\partial w_\ell} &= \left[\frac{\partial \pi}{\partial z}\right]\frac{\partial z}{\partial w_\ell} = -\left(\frac{\mu}{1-\mu}\right)\frac{(s_h - s_\ell)}{(z - s_\ell)^2}\frac{1}{2z}\frac{R^2}{\psi}.
\end{aligned} \tag{A6}$$

Now, the objective function for the optimal contract is

$$OBJ = \mu\left[2s_h\sqrt{k_h} - Rk_h\right] + (1-\mu)\left(\pi\left[2s_\ell\sqrt{k_h} - Rk_h\right] + (1-\pi)\left[2s_\ell\sqrt{k_\ell} - Rk_\ell - w_\ell\right]\right)$$

so that

$$\frac{DOBJ}{dw_\ell} = \frac{\partial OBJ}{\partial k_h}\frac{\partial k_h}{dw_\ell} + \frac{\partial OBJ}{\partial \pi}\frac{\partial \pi}{dw_{ell}} + \frac{\partial OBJ}{\partial w_\ell} = \frac{\partial OBJ}{\partial \pi}\frac{\partial \pi}{\partial w_\ell} + \frac{\partial OBJ}{\partial w_\ell}$$

$$= (1-\mu)(2s_\ell\sqrt{k_h} - Rk_h - 2s_\ell\sqrt{k_\ell} + Rk_\ell + w_\ell)\frac{\partial \pi}{\partial w_\ell} - (1-\mu)(1-\pi).$$

Substitution from (A6) and rearranging terms then gives

$$\begin{aligned}
\frac{dOBJ}{dw_\ell} &\propto (2s_\ell\sqrt{k_h} - Rk_h - 2s_\ell\sqrt{k_\ell} + Rk_\ell + w_\ell)\frac{\partial \pi}{\partial w_\ell} - 1 + \pi \\
&= -\left[-\frac{R}{\psi}(z - s_\ell) + (z + s_\ell)\right]\left(\frac{\mu}{1-\mu}\right)\frac{(s_h - s_\ell)}{(z - s_\ell)}\frac{1}{2z} - 1 + \left(\frac{\mu}{1-\mu}\right)\left[\frac{s_h - z}{z - s_\ell}\right].
\end{aligned}$$

Therefore, the first-order optimality condition for $\pi$ is

$$-\left[-\frac{R}{\psi}(z - s_\ell) + (z + s_\ell)\right]\left(\frac{\mu}{1-\mu}\right)\frac{(s_h - s_\ell)}{(z - s_\ell)}\frac{1}{2z} - 1 + \left(\frac{\mu}{1-\mu}\right)\left[\frac{s_h - z}{z - s_\ell}\right] = 0. \tag{A7}$$

But we can further simplify (A7) as

$$\begin{aligned}
-\psi + \left(\frac{\mu}{2z(1-\mu)}\right)\left[R(s_h - s_\ell) - \psi\frac{(z - s_\ell)\{(z + s_\ell) - (s_h - z)\}}{(z - s_\ell)}\right] &= 0, \text{ or} \\
-2z(1-\mu)\psi + \mu[R(s_h - s_\ell) - \psi\{2z - (s_h - s_\ell)\}] &= 0, \text{ or} \tag{A8} \\
-2z + \mu\left[(s_h - s_\ell)\left(\frac{R}{\psi} + 1\right)\right] &= 0.
\end{aligned}$$

*Q.E.D.*

*Proof of Theorem 4.* We note that solutions to the first-order condition (A8) are feasible only if $z \in (\bar{s}, s_h)$, where, $\bar{s} \equiv \mu s_h + (1-\mu)s_\ell$ . That is,

$$\frac{dOBJ}{dw_\ell} = 0 \iff -2z + \mu\left[(s_h - s_\ell)\left(\frac{R}{\psi} + 1\right)\right] = 0, \text{ for } z \in (\bar{s}, s_h).$$

Thus, the (unique) optimum is interior when

$$z \in (\bar{s}, s_h) \implies \bar{b} \equiv \frac{2s_h}{\mu(s_h - s_\ell)} - 1 > \frac{R}{\psi} > 1 + \frac{2s_\ell}{\mu(s_h - s_\ell)} \equiv \underline{b}. \tag{A9}$$

Moreover, if $\frac{R}{\psi} > \bar{b}$, then $\frac{dOBJ}{dw_\ell} > 0$ for $z \in (\bar{s}, s_h)$, that is, it is optimal to set $z = s_h$ or $\pi = 0$, and if $\frac{R}{\psi} < \underline{b}$, then $\frac{dOBJ}{dw_\ell} < 0$ for $z \in (\bar{s}, s_h)$, that is, it is optimal to set $z = \bar{s}$ or $\pi = 1$. *Q.E.D.*

*Proof of Proposition 2.* The first-order optimality condition for $w_\ell$ is $F \equiv -2z + \mu[(s_h - s_\ell)(\frac{R}{\psi} + 1)] = 0$. We compute

$$\frac{\partial z}{\partial (R/\psi)} = -\frac{\partial F/\partial (R/\psi)}{\partial F/\partial z} = -\frac{\mu(s_h - s_\ell)}{-2} > 0.$$

Using the fact that $\pi = (\frac{\mu}{1-\mu})[\frac{s_h - z}{z - s_\ell}]$, we conclude that $\frac{\partial \pi}{\partial (R/\psi)} < 0$ because $\frac{\partial z}{\partial (R/\psi)} < 0$.

It is useful to rewrite the first-order condition (A8) using $z = \frac{\mu s_h + (1-\mu)\pi s_\ell}{R(\mu + (1-\mu)\pi)}$ as

$$G \equiv -2(\mu + (1-\mu)\pi\lambda) + (\mu + (1-\mu)\pi)\mu\left[(1-\lambda)\left(\frac{R}{\psi} + 1\right)\right] = 0,$$

where $\lambda \equiv \frac{s_\ell}{s_h} \in [0, 1)$. Now,

$$\frac{\partial G}{\partial \lambda} \propto -2(1 - \mu)\pi - (\mu + (1 - \mu)\pi)\mu\left[\left(\frac{R}{\psi} + 1\right)\right] < 0.$$

Moreover,

$$\frac{\partial G}{\partial \pi} \propto -2\lambda + \mu\left[(1 - \lambda)\left(\frac{R}{\psi} + 1\right)\right] > 0 \text{ from (A9).}$$

Hence, $\frac{\partial \pi}{\partial \lambda} > 0$. That $\pi$ is ambiguous in $(s_h - s_\ell)$ follows from the fact that the mapping between $\lambda$ and $(s_h - s_\ell)$ is not one to one. *Q.E.D.*

*Proof of Proposition 3.* This follows immediately from (A9) and the fact that

$$k_h \equiv \left(\frac{\mu\left[(s_h - s_\ell)\left(\frac{R}{\psi} + 1\right)\right]}{2R}\right)^2. \tag{A10}$$

*Q.E.D.*

*Proofs of Propositions 4–6.* Follow directly from equations (21), (24), and (25) in the text. *Q.E.D.*

*Proof of Theorem 5.* In this section, we show that output contingent wages are not optimal. We consider the general case in which $\pi_{\ell h} \equiv \pi \in [0, 1]$ and $\pi_{h\ell} \equiv \rho \in [0, 1]$ at the optimum. Recall that the objective function is given by

$$
\begin{aligned}
OBJ = {} & \mu(1 - \rho)\left[2s_h\sqrt{k_h}\left(1 - w_h^+ + w_h^0\right) - w_h^0 - Rk_h\right] + \mu\rho\left[2s_h\sqrt{k_\ell}\left(1 - w_\ell^+ + w_\ell^0\right) - w_\ell^0 - Rk_h\right] \\
& + (1 - \mu)(1 - \pi)\left[2s_\ell\sqrt{k_\ell}\left(1 - w_\ell^+ + w_\ell^0\right) - w_\ell^0 - Rk_\ell\right] \\
& + (1 - \mu)\pi\left[2s_\ell\sqrt{k_h}\left(1 - w_h^+ + w_h^0\right) - w_h^0 - Rk_h.\right]
\end{aligned}
\tag{A11}
$$

Optimal reporting by the agent (see (8)) implies that

$$\left(w_\ell^+ - w_\ell^0\right)2s_h\sqrt{k_\ell} + w_\ell^0 + \psi k_\ell - \left[\left(w_h^+ - w_h^0\right)2s_h\sqrt{k_h} + w_h^0 + \psi k_h\right] \begin{cases} \le 0, & \text{for } \rho = 0 \\ = 0, & \text{for } \rho \in (0, 1) \\ \ge 0, & \text{for } \rho = 1 \end{cases} \tag{A12}$$

and

$$\left(w_\ell^+ - w_\ell^0\right)2s_\ell\sqrt{k_\ell} + w_\ell^0 + \psi k_\ell - \left[\left(w_h^+ - w_h^0\right)2s_\ell\sqrt{k_h} + w_h^0 + \psi k_h\right] \begin{cases} \ge 0, & \text{for } \pi = 0 \\ = 0, & \text{for } \pi \in (0, 1) \\ \le 0, & \text{for } \pi = 1 \end{cases} \tag{A13}$$

Optimal investments, given $\langle \pi, \rho \rangle$, and wages, (see (7)), are

$$k_\ell = \left(\frac{\left(1 - w_\ell^+ + w_\ell^0\right)y}{R}\right)^2, k_h = \left(\frac{\left(1 - w_h^+ + w_h^0\right)z}{R}\right)^2, \tag{A14}$$

where

$$y = \begin{cases} \dfrac{\mu\rho s_h + (1 - \mu)(1 - \pi)s_\ell}{\mu\rho + (1 - \mu)(1 - \pi)}, & \text{for } \rho \in (0, 1] \\ s_\ell, & \text{for } \rho = 0 \end{cases}, \text{ and } z = \begin{cases} \dfrac{\mu(1 - \rho)s_h + (1 - \mu)\pi s_\ell}{\mu(1 - \rho) + (1 - \mu)\pi}, & \text{for } \pi \in (0, 1] \\ s_h, & \text{for } \pi = 0 \end{cases}. \tag{A15}$$

We restrict attention to monotonic and positive wages,

$$w_r^+ \ge w_r^0 \ge 0 \text{ for } r = \ell, h. \tag{A16}$$

Finally, to insure feasibility and avoid redundancy, we consider randomization strategies,[24]

$$\pi \ge 0, \rho \ge 0, \pi + \rho \le 1. \tag{A17}$$

---

[24] Condition (A17) is equivalent to the requirement that expected productivity is higher following report $r = h$ relative to report $r = \ell$, that is, $E(s \mid r = h) \ge E(s \mid r = \ell)$ (in particular, when $\pi + \rho = 1$, reports are not informative). Because the labeling of a report as $h$ or $\ell$ is arbitrary (for example, both $\pi = \rho = 1$ and $\pi = \rho = 0$ represent a truth-telling equilibrium), it is straightforward to show using a revealed-preference argument that (A17) holds for any optimal contract.

The above constraints are summarized by $F \subset R_+^4 \times [0, 1]^2$ the set of feasible contracts $\langle \mathbf{w}, \pi, \rho \rangle \equiv \langle (w_\ell^+, w_\ell^0, w_h^+, w_h^0), \pi, \rho \rangle$ as follows:

$$F \equiv \{\langle \mathbf{w}, \pi, \rho \rangle : (A12), (A13), (A14), (A16), (A17)\}.$$

Thus, the optimal contract, denoted by $\langle \check{\mathbf{w}}, \check{\pi}, \check{\rho} \rangle$, maximizes $OBJ$ (as in (A11)) over the set $F$, that is,

$$\langle \check{\mathbf{w}}, \check{\pi}, \check{\rho} \rangle \in \arg \max_{\langle \mathbf{w}, \pi, \rho \rangle \in F} OBJ(\mathbf{w}, \pi, \rho). \tag{A18}$$

We must show that $\check{w}_h^+ = \check{w}_h^0 = 0$, and $\check{w}_\ell^+ = \check{w}_\ell^0 = \check{w}$, for some $\check{w} \geq 0$. For notational ease, we will use the generic $f(k)$ (instead of $2\sqrt{k}$) to denote the production function. First note that if $\pi = 1$, then it follows from (A17) that $\rho = 0$, that is, reports are not informative, and that optimal wages are not output contingent. In particular, suppose that $\pi = 1$; then the value of $OBJ$ is (where $k^P$ denotes the optimal investment reopens to uninformative reports, i.e., $k^P \equiv (\frac{\mu s_h + (1-\mu)s_\ell}{R})^2$, as in (13))

$$OBJ = (1 - w_h^+ + w_h^0)(\mu s_h + (1-\mu)s_\ell) f(k_h) - w_h^0 - R k_h$$

$$\leq (\mu s_h + (1-\mu)s_\ell) f(k_h) - R k_h \quad \text{(lower wages)}$$

$$\leq (\mu s_h + (1-\mu)s_\ell) f(k^P) - R k^P \quad \text{(efficient investment when reports are not informative)}$$

$$= OBJ \left( w_\ell^+ = w_\ell^0 = w_h^+ = w_h^0 = 0, \pi = 1, \rho = 0 \right).$$

Moreover, the aforementioned allocation with output-independent wages is feasible, that is,

$$\langle (0, 0, 0, 0), 1, 0 \rangle \in F.$$

We conclude that, if $\pi = 1$ at the optimum, then it is optimal to have output-independent wages. The above analysis also implies that (A13) is binding at an optimum with $\pi = 1$. Thus, we further simplify (A13) as follows:

$$\left(w_\ell^+ - w_\ell^0\right) s_\ell f(k_\ell) + w_\ell^0 + \psi k_\ell - \left[\left(w_h^+ - w_h^0\right) s_\ell f(k_h) + w_h^0 + \psi k_h\right] \begin{cases} \geq 0, & \text{for } \pi = 0 \\ = 0, & \text{for } \pi \in (0, 1]. \end{cases} \tag{A19}$$

This allows us to define $F'$ as the restricted set of feasible contracts we may focus on without loss of generality,

$$F' \equiv \{\langle \mathbf{w}, \pi, \rho \rangle : (A12), (A19), (A14), (A16), (A17)\}.$$

Thus, it suffices to show that there exists wage compensation $\check{w}$, and randomization strategies $\check{\pi}$, and $\check{\rho}$, where $\langle (\check{w}, \check{w}, 0, 0), \check{\pi}, \check{\rho} \rangle \in F'$ and

$$OBJ(\left(w_\ell^+, w_\ell^0, w_h^+, w_h^0\right), \pi, \rho) \leq OBJ(\langle (\check{w}, \check{w}, 0, 0), \check{\pi}, \check{\rho} \rangle), \text{ for any } \langle \mathbf{w}, \pi, \rho \rangle \in F'. \tag{A20}$$

To do so, it is useful to define the set of contracts $\Theta$,

$$\Theta \equiv \{\langle \mathbf{w}, \pi, \rho \rangle : (A21), (A14), (A16), (A17)\}.$$

where

$$\left(w_\ell^+ - w_\ell^0\right) s_\ell f(k_\ell) + w_\ell^0 + \psi k_\ell \geq \psi k_h. \tag{A21}$$

This is useful for two reasons. First, the set $\Theta$ contains all contracts that are feasible (i.e., $F' \subset \Theta$). Second, any contract $\langle \mathbf{w}, \pi, \rho \rangle$ of the form $\langle (w, w, 0, 0), \pi, \rho \rangle$ that (a) belongs to $\Theta$, and (b) satisfies (A21) without slack, is an element in $F'$. Thus, if we show that there exists a contract $\langle \tilde{\mathbf{w}}, \tilde{\pi}, \tilde{\rho} \rangle \equiv \langle (\tilde{w}, \tilde{w}, 0, 0), \tilde{\pi}, \tilde{\rho} \rangle \in \Theta$ that satisfies (A21) without slack, and s.t.,

$$OBJ(\langle \mathbf{w}, \pi, \rho \rangle) \leq OBJ(\langle \tilde{\mathbf{w}}, \tilde{\pi}, \tilde{\rho} \rangle), \text{ for all } \langle \mathbf{w}, \pi, \rho \rangle \in \Theta, \tag{A22}$$

then we have proved (A20) as well, and the result follows.

First, we show that $\tilde{w}_h^0 = 0$. In particular, take any $\langle \mathbf{w}, \pi, \rho \rangle \in \Theta$ such that $w_h^0 > 0$, and consider the alternative contract,

$$\langle \mathbf{w}', \pi, \rho \rangle = \langle \left(w_\ell^+, w_\ell^0, w_h^+ - w_h^0, 0\right), \pi, \rho \rangle, \text{ i.e., } w_h'^+ = w_h^+ - w_h^0, \text{ and } w_h'^0 = 0.$$

Investment levels (as determined by $\langle \mathbf{w}', \pi, \rho \rangle$ and (A14)) are not altered, the constraint (A21) is satisfied under the alternative contract $\langle \mathbf{w}', \pi, \rho \rangle$, and also (A16) and (A17) are satisfied. However, expected wages are lower, and the value of $OBJ$ is increased, under the alternative contract $\langle \mathbf{w}', \pi, \rho \rangle$. Thus, we confirm that $\tilde{w}_h^0 = 0$.

Second, we show that $\tilde{w}_\ell^+ = \tilde{w}_\ell^0$. In particular, take any $\langle \mathbf{w}, \pi, \rho \rangle \in \Theta$, such that $w_h^0 = 0$, but $w_\ell^+ > w_\ell^0$, and consider the alternative contract $\langle \check{\mathbf{w}}, \pi, \rho \rangle = \langle (\check{w}, \check{w}, w_h^+, 0), \pi, \rho \rangle$ (i.e., $\check{w}_\ell^+ = \check{w}_\ell^0 = \check{w}$), where $\check{w} \equiv \left(w_\ell^+ - w_\ell^0\right) s_\ell f(k_\ell) + w_\ell^0$. Investment following a low report increases, in particular, $\check{k}_\ell = (\frac{y}{R})^2 > (\frac{(1-w_\ell^+ + w_\ell^0)y}{R})^2 = k_\ell$ (note that $y$ is the same under both contracts, as we have not changed the randomization strategies $\pi$ and $\rho$). Moreover, investment following a high report is not affected. Thus, (A21) is satisfied under the alternative contract, and $\langle (\check{w}, \check{w}, w_h^+, 0), \pi, \rho \rangle \in \Theta$. Now, the

aforementioned change in investment (i.e., $\check{k}_\ell > k_\ell$) increases investment efficiency, as $\check{k}_\ell = (\frac{v}{R})^2$ is the best response level of investment given reporting strategy $\langle \pi, \rho \rangle$. Namely,

$$\check{k}_\ell \in \arg \max_{k \geq 0} \left( \frac{\mu \rho s_h + (1 - \mu)(1 - \pi) s_\ell}{\mu \rho + (1 - \mu)(1 - \pi)} \right) f(k) - Rk.$$

Moreover, expected wages are smaller under the alternative contract. In particular, the wage following a high report, namely, $w_h^+$, is the same under both contracts, whereas the expected wage payment following a low report is lower under the alternative contract, namely, $E_{\langle \check{\mathbf{w}}, \pi, \rho \rangle}(w \mid r = \ell) \leq E_{\langle \mathbf{w}, \pi, \rho \rangle}(w \mid r = \ell)$, as follows:

$$E_{\langle \check{\mathbf{w}}, \pi, \rho \rangle}(w \mid r = \ell) = [\mu \rho + (1 - \mu)(1 - \pi)] \check{w} \quad \text{(by construction)}$$

$$= [\mu \rho + (1 - \mu)(1 - \pi)] \left( \left( w_\ell^+ - w_\ell^0 \right) s_\ell f(k_\ell) + w_\ell^0 \right)$$

$$\leq \mu \rho \left( \left( w_\ell^+ - w_\ell^0 \right) s_h f(k_\ell) + w_\ell^0 \right) + (1 - \mu)(1 - \pi) \left( \left( w_\ell^+ - w_\ell^0 \right) s_\ell f(k_\ell) + w_\ell^0 \right) \quad (\text{as } s_h > s_\ell)$$

$$= E_{\langle \mathbf{w}, \pi, \rho \rangle}(w \mid r = \ell).$$

Thus, we confirm that $\tilde{w}_\ell^+ = \tilde{w}_\ell^0$.

Third, we show that (A21) binds under $\langle \tilde{\mathbf{w}}, \tilde{\pi}, \tilde{\rho} \rangle$. From the above, we already know that $\tilde{w}_h^0 = 0$, and $\tilde{w}_\ell^+ = \tilde{w}_\ell^0 = \tilde{w}$. Thus, slack in (A21) implies that $\tilde{w} + \psi \tilde{k}_\ell > \psi \tilde{k}_h$ (where investment is determined by $\langle \tilde{\mathbf{w}}, \tilde{\pi}, \tilde{\rho} \rangle$ and (A14)). Then, slack in (A21) must also imply that $\tilde{w} = 0$ (otherwise the alternative contract $\langle (0, 0, \tilde{w}_h^+, 0), \tilde{\pi}, \tilde{\rho} \rangle \in \Theta$ would increase $OBJ$). This, in turn, implies that $\tilde{k}_\ell > \tilde{k}_h$ (under $\langle \tilde{\mathbf{w}}, \tilde{\pi}, \tilde{\rho} \rangle$). But, from $\tilde{k}_\ell > \tilde{k}_h$, $\tilde{\mathbf{w}} = (0, 0, \tilde{w}_h^+, 0)$, and (A14) it follows that $k_\ell = (\frac{v}{R})^2 > k_h = (\frac{(1 - \tilde{w}_h^+)z}{R})^2$. Thus, a reduction in $\tilde{w}_h^+$ to the level $w'^+_h$ that insures $k_\ell = (\frac{v}{R})^2 = k'_h = (\frac{(1 - w'^+_h)z}{R})^2$, that is, the contract $\langle (0, 0, w'^+_h, 0), \tilde{\pi}, \tilde{\rho} \rangle \in \Theta$, would satisfy (A21), reduce expected wages, increase investment efficiency following a high report, and as a result increase the value of $OBJ$. Namely,

$$\mu(1 - \rho) \left[ \left( 1 - w_h^+ \right) s_h f(k_h) - Rk_h \right] + (1 - \mu)\pi \left[ \left( 1 - w_h^+ \right) s_\ell f(k_h) - Rk_h \right] \quad <$$

$$\mu(1 - \rho) \left[ \left( 1 - w'^+_h \right) s_h f(k_h) - Rk_h \right] + (1 - \mu)\pi \left[ \left( 1 - w'^+_h \right) s_\ell f(k_h) - Rk_h \right] \quad < \quad (\text{lower wage})$$

$$\mu(1 - \rho) \left[ \left( 1 - w'^+_h \right) s_h f(k'_h) - Rk'_h \right] + (1 - \mu)\pi \left[ \left( 1 - w'^+_h \right) s_\ell f(k'_h) - Rk'_h \right].$$

The latter inequality follows as $k'_h$ is the best investment response to $w'^+_h$ (and $\tilde{\pi}, \tilde{\rho}$), leading to a contradiction. Therefore, (A21) binds under $\langle \tilde{\mathbf{w}}, \tilde{\pi}, \tilde{\rho} \rangle$.

Importantly, the above results also imply that under contract $\langle \tilde{\mathbf{w}}, \tilde{\pi}, \tilde{\rho} \rangle$,

$$\tilde{w}_h^+ \in \left[ 0, 1 - \frac{y}{z} \right] \subseteq [0, 1] \Leftrightarrow k_\ell = \left( \frac{y}{R} \right)^2 \leq k_h = \left( \frac{(1 - \tilde{w}_h^+)z}{R} \right)^2. \tag{A23}$$

To see this, note that the facts that $\tilde{w}_h^0 = 0$, $\tilde{w}_\ell^+ = \tilde{w}_\ell^0 \geq 0$, and that constraint (A21) binds under $\langle \tilde{\mathbf{w}}, \tilde{\pi}, \tilde{\rho} \rangle$ imply that $\tilde{w}_\ell^0 + \psi k_\ell = \psi k_h$, $k_\ell \leq k_h$, and $\tilde{w}_h^+ \in [0, 1 - \frac{y}{z}]$.

Finally, we show that $\tilde{w}_h^+ = 0$. We proceed by calculating the first-order condition (FOC) of the OBJ with respect to $w_h^+$ while considering $w_h^0 = 0$, $w_\ell^+ = w_\ell^0$, and the binding constraint (A21) (following the above analysis). Namely, for a given $\langle \pi, \rho \rangle$, the value of OBJ as a function of $w_h^+$ is

$$\mu(1 - \rho) \left[ s_h f(k_h) \left( 1 - w_h^+ \right) - Rk_h \right] + \mu \rho \left[ s_h f(k_\ell) - w_\ell^0 - Rk_h \right]$$

$$+ (1 - \mu)(1 - \pi) \left[ s_\ell f(k_\ell) - w_\ell^0 - Rk_\ell \right] + (1 - \mu)\pi \left[ s_\ell f(k_h) \left( 1 - w_h^+ \right) - Rk_h \right],$$

where $k_h$ and $k_\ell$ are given by (A14), and $w_\ell^0 = \psi (k_h - k_\ell)$. It follows from the envelope theorem (and (A14)) that the FOC with respect to $w_h^+$ is given by

$$FOC_{w_h^+} = -(\mu(1 - \rho)s_h + (1 - \mu)\pi s_\ell) f(k_h) - (\mu \rho + (1 - \mu)(1 - \pi)) \frac{\partial w_\ell^0}{\partial w_h^+}$$

$$= -(\mu(1 - \rho)s_h + (1 - \mu)\pi s_\ell) f(k_h) - (\mu \rho + (1 - \mu)(1 - \pi)) \frac{\partial w_\ell^0}{\partial k_h} \frac{\partial k_h}{\partial w_h^+}.$$

Now it follows from $w_\ell^0 = \psi (k_h - k_\ell)$ and (A14) that

$$\frac{\partial w_\ell^0}{\partial k_h} = \psi, \quad k_h = \left( \frac{(1 - w_h^+)z}{R} \right)^2, \quad k_\ell = \left( \frac{y}{R} \right)^2, \quad \text{and} \quad \frac{\partial k_h}{\partial w_h^+} = -2 \left( 1 - w_h^+ \right) \left( \frac{z}{R} \right)^2.$$

Thus (while substituting for $f(k)$)

$$FOC_{w_h^+} = -(\mu(1 - \rho)s_h + (1 - \mu)\pi s_\ell) 2 \left( \frac{(1 - w_h^+)z}{R} \right) + (\mu \rho + (1 - \mu)(1 - \pi)) \psi 2 \left( 1 - w_h^+ \right) \left( \frac{z}{R} \right)$$

$$= 2 \left( 1 - w_h^+ \right) \left( \frac{z}{R} \right) \left[ (\mu \rho + (1 - \mu)(1 - \pi)) \psi \left( \frac{z}{R} \right) - (\mu(1 - \rho)s_h + (1 - \mu)\pi s_\ell) \right].$$

Therefore, the sign of the derivative $FOC_{w_h^+}$ does not depend on $w_h^+$ as long as $w_h^+ \in (0, 1)$. As a result, it must be that $\tilde{w}_h^+ \in \{0, 1 - \frac{y}{z}\}$ (recall from (A23) that we already know that $\tilde{w}_h^+ \in [0, 1 - \frac{y}{z}]$). However, $1 - \tilde{w}_h^+ = \frac{y}{z}$ cannot be optimal. To see this, assume by contradiction that $\tilde{w}_h^0 = 0$, $\tilde{w}_\ell^+ = \tilde{w}_\ell^0$, $\tilde{w}_\ell^0 = \psi(\tilde{k}_h - \tilde{k}_\ell)$ but $1 - \tilde{w}_h^+ = \frac{y}{z}$. This implies that investment does not depend on the report, that is, $\check{k}_h = (\frac{(1-\tilde{w}_h^+)z}{R})^2 = (\frac{y}{R})^2 = \tilde{k}_\ell$, and that the wage compensation following a low-productivity report is zero, namely, $\tilde{w}_\ell^0 = \psi(\check{k}_h - \tilde{k}_\ell) = 0$. But, when investment does not depend on reports, it is efficient to invest the pooling level $k^P$. In particular, such a solution cannot be optimal, as it is dominated by the pooling allocation $w_\ell'^+ = w_\ell'^0 = w_h'^+ = w_h'^0 = 0, \pi' = 1, \rho' = 0$ (i.e., $k_h' = k_\ell' = k^P \equiv \left(\frac{\mu s_h + (1-\mu)s_\ell}{R}\right)^2$). Formally, the value of $OBJ$, with $\tilde{w}_h^0 = 0$, $\tilde{w}_h^+ = 1 - \frac{y}{z}$, $\tilde{w}_\ell^+ = \tilde{w}_\ell^0 = 0$, $\langle \tilde{\pi}, \tilde{\rho}\rangle$, and $k_h = k_\ell = \left(\frac{y}{R}\right)^2$ is given by

$$2\left(\mu(1-\tilde{\rho})s_h + (1-\mu)\tilde{\pi}s_\ell\right)(1-\tilde{w}_h^+)\left(\frac{y}{R}\right) + 2\left(\mu\tilde{\rho}s_h + (1-\mu)(1-\tilde{\pi})s_\ell\right)\left(\frac{y}{R}\right) - R\left(\frac{y}{R}\right)^2 \le$$

$$2\left(\mu(1-\tilde{\rho})s_h + (1-\mu)\tilde{\pi}s_\ell\right)\left(\frac{y}{R}\right) + 2\left(\mu\tilde{\rho}s_h + (1-\mu)(1-\tilde{\pi})s_\ell\right)\left(\frac{y}{R}\right) - R\left(\frac{y}{R}\right)^2 =$$

(lower wages)

$$2\left(\mu s_h + (1-\mu)s_\ell\right)\left(\frac{y}{R}\right) - R\left(\frac{y}{R}\right)^2 \le$$

$$2\left(\mu s_h + (1-\mu)s_\ell\right)\sqrt{k^P} - Rk^P$$

(efficient investment).

Thus, we reach a contradiction and conclude that $\tilde{w}_h^+$ is at the lower boundary $\tilde{w}_h^+ = 0$.

To conclude, so far we have shown that $\langle \tilde{\mathbf{w}}, \tilde{\pi}, \tilde{\rho}\rangle$ has the properties (i) $\tilde{w}_h^+ = \tilde{w}_h^0 = 0$, (ii) $\tilde{w}_\ell^+ = \tilde{w}_\ell^0$, and (iii) the constraint (A21) binds. But, this also implies that $\langle \tilde{\mathbf{w}}, \tilde{\pi}, \tilde{\rho}\rangle \in F'$, and therefore the solution to (A18) is $\langle \check{\mathbf{w}}, \check{\pi}, \check{\rho}\rangle = \langle \tilde{\mathbf{w}}, \tilde{\pi}, \tilde{\rho}\rangle$, which completes the proof. *Q.E.D.*

*Proof of Theorem 6.* From Theorem 5 the optimal contract $\langle \check{\mathbf{w}}, \check{\pi}, \check{\rho}\rangle$ satisfies, $\check{w}_h^+ = \check{w}_h^0 = 0$, $\check{w}_\ell^+ = \check{w}_\ell^0 \equiv \check{w}_\ell \ge 0$, and $\check{w}_\ell^0 = \psi(\check{k}_h - \check{k}_\ell)$. Now, any contract of the form $\langle (w_\ell, w_\ell, 0, 0), \pi, \rho\rangle$ can be represented by the pair of randomization strategies $\langle \pi, \rho\rangle$, where $w_\ell = \psi(k_h - k_\ell)$, and investments are given by (see (7))

$$k_\ell = \left(\frac{y}{R}\right)^2, \quad k_h = \left(\frac{z}{R}\right)^2, \tag{A24}$$

where $y$ and $z$ depend on $\langle \pi, \rho\rangle$ as in (A15). In particular, $z = \frac{\mu(1-\rho)s_h + (1-\mu)\pi s_\ell}{\mu(1-\rho)+(1-\mu)\pi}$, and $y = \frac{\mu\rho s_h + (1-\mu)(1-\pi)s_\ell}{\mu\rho+(1-\mu)(1-\pi)}$. Thus, the optimal contract maximizes the $OBJ$ as in (A25) over the set of randomization strategies $\pi \ge 0, \rho \ge 0, \pi + \rho \le 1$ (see (A17)). For notational ease, we use the generic $f(k)$ to denote the production function.

$$OBJ = \mu(1-\rho)[s_h f(k_h) - Rk_h] + \mu\rho[s_h f(k_\ell) - Rk_\ell - w_\ell]$$
$$+ (1-\mu)\pi[s_\ell f(k_h) - Rk_h] + (1-\mu)(1-\pi)[s_\ell f(k_\ell) - Rk_\ell - w_\ell]. \tag{A25}$$

It follows from the condition $\frac{R}{\psi} > \underline{b}$, and Theorem 3, that the contract $\langle \hat{\pi}^*, 0\rangle$ (as defined in (21)) strictly dominates the contract $\langle 1, 0\rangle$ that implements the pooling equilibrium (i.e., where $k_h = k_\ell = k^P$). Now, returning to the possibility of randomization by both manager types, it is worth noting that the pooling equilibrium allocation is implemented also by the continuum of contracts $\{\langle \pi, 1 - \pi\rangle\}_{\pi \in [0,1]}$. Therefore, when $\frac{R}{\psi} > \underline{b}$, the optimal contract satisfies $\check{\pi} \in [0, 1)$ and $\check{\rho} \in [0, 1 - \check{\pi})$ (see (A17)).

Now, assume by contradiction that at the optimum, $\check{\rho} \in (0, 1 - \check{\pi})$. Then, it must be that the first-order condition with respect to $\rho$ is satisfied, that is, $\frac{dOBJ}{d\rho}|_{\langle \check{\pi}, \check{\rho}\rangle} = 0$, where

$$\frac{dOBJ}{d\rho} = \frac{\partial OBJ}{\partial \rho} + \sum_{j=\{\ell, h\}} \frac{\partial OBJ}{\partial k_j}\frac{\partial k_j}{\partial \rho} + \frac{\partial OBJ}{\partial w_\ell}\frac{\partial w_\ell}{\partial \rho} = \frac{\partial OBJ}{\partial \rho} + \frac{\partial OBJ}{\partial w_\ell}\frac{\partial w_\ell}{\partial \rho}$$

$$= \mu(s_h f(k_\ell) - Rk_\ell - w_\ell - s_h f(k_h) + Rk_h) - (\mu\rho + (1-\mu)(1-\pi))\frac{\partial w_\ell}{\partial \rho}.$$

It must also be the case that $\frac{d^2 OBJ}{d\rho^2}|_{\langle \check{\pi}, \check{\rho}\rangle} \le 0$ for $\langle \check{\pi}, \check{\rho}\rangle$ to be a local maximum. But, we will next provide conditions under which $\frac{d^2 OBJ}{d\rho^2}|_{\langle \pi, \rho\rangle} > 0$ for any pair $\langle \pi, \rho\rangle$ such that $\pi \in [0, 1)$ and $\rho \in [0, 1 - \pi)$, thus ruling out the possibility of an interior solution $\check{\rho} \in (0, 1 - \check{\pi})$. As a first step, we calculate the second derivative,

$$\frac{d^2 OBJ}{d\rho^2} = \frac{\partial}{\partial \rho}\left(\frac{dOBJ}{d\rho}\right) + \frac{\partial}{\partial w_\ell}\left(\frac{dOBJ}{d\rho}\right)\frac{\partial w_\ell}{\partial \rho} + \frac{\partial}{\partial k_\ell}\left(\frac{dOBJ}{d\rho}\right)\frac{\partial k_\ell}{\partial \rho} + \frac{\partial}{\partial k_h}\left(\frac{dOBJ}{d\rho}\right)\frac{\partial k_h}{\partial \rho}$$

$$= -\left[\mu\frac{\partial w_\ell}{\partial \rho} + (\mu\rho + (1-\mu)(1-\pi))\frac{\partial^2 w_\ell}{\partial \rho^2}\right]$$

$$+ \mu\left(-\frac{\partial w_\ell}{\partial \rho} + [s_h f'(k_\ell) - R]\frac{\partial k_\ell}{\partial \rho} - [s_h f'(k_h) - R]\frac{\partial k_h}{\partial \rho}\right).$$

Note that $s_\ell \le y \le z \le s_h$ (as $\pi + \rho \le 1$), and because $k_\ell \le k_h \le k_h^* \equiv (\frac{s_h}{R})^2$, it follows that $[s_h f'(k_\ell) - R] \ge 0$ and $[s_h f'(k_h) - R] \ge 0$ (recall that $f(k) = 2\sqrt{k}$). Moreover, $\frac{\partial k_\ell}{\partial \rho} = \frac{1}{R^2}(\frac{\partial y^2}{\partial \rho})$, $\frac{\partial k_h}{\partial \rho} = \frac{1}{R^2}(\frac{\partial z^2}{\partial \rho})$, and $\frac{\partial w_\ell}{\partial \rho} = \psi(\frac{\partial k_h}{\partial \rho} - \frac{\partial k_\ell}{\partial \rho})$. Next,

we calculate the required derivatives for the case $s_\ell = 0$ and $\mu > \frac{1}{3}$ .

$$\frac{\partial z^2}{\partial \rho} = -2 (\mu s_h)^2 (1-\mu)\pi \frac{(1-\rho)}{(\mu(1-\rho)+(1-\mu)\pi)^3} \leq 0$$

$$\frac{\partial^2 z^2}{\partial \rho^2} = 2 (\mu s_h)^2 (1-\mu)\pi \frac{(1-\mu)\pi - 2\mu(1-\rho)}{(\mu(1-\rho)+(1-\mu)\pi)^4} \leq 0 \text{ (as } \mu > \frac{1}{3} \text{ and } \rho < 1 - \pi \text{),}$$

$$\frac{\partial y^2}{\partial \rho} = 2 (\mu s_h)^2 (1-\mu)(1-\pi) \left( \frac{\rho}{(\mu\rho + (1-\mu)(1-\pi))^3} \right) > 0$$

$$\frac{\partial^2 y^2}{\partial \rho^2} = 2 (\mu s_h)^2 (1-\mu)(1-\pi) \left( \frac{(1-\mu)(1-\pi) - 2\mu\rho}{(\mu\rho + (1-\mu)(1-\pi))^4} \right).$$

Thus, it follows that $\frac{\partial k_\ell}{\partial \rho} > 0$ and $\frac{\partial k_h}{\partial \rho} \leq 0$ and, as a result, a sufficient condition for $\frac{d^2 OBJ}{d\rho^2} > 0$ is

$$\left[ 2\mu \frac{\partial w_\ell}{\partial \rho} + (\mu\rho + (1-\mu)(1-\pi)) \frac{\partial^2 w_\ell}{\partial \rho^2} \right] < 0. \tag{A26}$$

From the above, it follows that $\frac{\partial w_\ell}{\partial \rho} = \frac{\psi}{R^2} (\frac{\partial z^2}{\partial \rho} - \frac{\partial y^2}{\partial \rho}) < 0$, and $\frac{\partial^2 w_\ell}{\partial \rho^2} = \frac{\psi}{R^2} (\frac{\partial^2 z^2}{\partial \rho^2} - \frac{\partial^2 y^2}{\partial \rho^2})$. Note, however, that $\frac{\partial^2 z^2}{\partial \rho^2} \leq 0$, and $\frac{\partial z^2}{\partial \rho} \leq 0$. Therefore, to show (A26), it suffices to show that $[2\mu(-\frac{\psi}{R^2} \frac{\partial y^2}{\partial \rho}) + (\mu\rho + (1-\mu)(1-\pi))(-\frac{\psi}{R^2} \frac{\partial^2 y^2}{\partial \rho^2})] < 0$. In particular,

$$\frac{\psi}{R^2} \left[ 2\mu \left( -\frac{\partial y^2}{\partial \rho} \right) + (\mu\rho + (1-\mu)(1-\pi)) \left( -\frac{\partial^2 y^2}{\partial \rho^2} \right) \right] \propto$$

$$\left[ -2\mu \left( (1-\pi) \left( \frac{\rho}{(\mu\rho + (1-\mu)(1-\pi))^3} \right) \right) - (1-\pi) \left( \frac{(1-\mu)(1-\pi) - 2\mu\rho}{(\mu\rho + (1-\mu)(1-\pi))^3} \right) \right] \propto$$

$$-2\mu\rho - (1-\mu)(1-\pi) + 2\mu\rho =$$

$$-(1-\mu)(1-\pi) < 0.$$

Thus, we have shown that $\frac{d^2 OBJ}{d\rho^2}|_{\langle \pi, \rho \rangle} > 0$ for any pair $\langle \pi, \rho \rangle$ such that $\pi \in [0, 1)$ and $\rho \in [0, 1 - \pi)$, provided that $s_\ell < \tilde{s}_\ell$ and $\mu > \frac{1}{3}$ for some $\tilde{s}_\ell \in (0, s_h)$. This, clearly, rules out the possibility of a solution $\check{\pi} \in [0, 1)$ and $\check{\rho} \in (0, 1 - \check{\pi})$ . And we conclude that $\check{\rho} = 0$, as long as $\frac{R}{\psi} > \underline{b}$, $s_\ell < \tilde{s}_\ell$, and $\mu > \frac{1}{3}$ for some $\tilde{s}_\ell \in (0, s_h)$. Q.E.D.

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