Optimal Incentive Contracts and Information Cascades

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We examine information aggregation regarding industry capital productivity from privately informed managers in a dynamic model with optimal incentive contracts. Information cascades always occur if managers enjoy limited liability: when beliefs regarding productivity become endogenously extreme (optimistic or pessimistic), learning stops. There is no learning if initial beliefs are extreme, or if agency conflicts are severe. In contrast to the literature, cascades occur even when signals have unbounded precision or when there are rich action spaces. Relaxing limited liability constraints is not sufficient to avoid cascades; we provide sufficient conditions for efficient information aggregation through incentive contracts. (*JEL* G32, D23)

Investment uncertainty in firms typically has both project-specific and common (or aggregate) components. Consider, for example, investment in new business opportunities that are created by technological innovations, changes in underlying buyer preferences, or the development of new markets. In each of these cases, undoubtedly of substantial empirical significance, there will be initial uncertainty regarding the long-run economic viability of the technological or market innovation that is common to all firms in the industry. Indeed, a long line of literature documents the investment distortions (at the level of the industry) generated by this type of uncertainty because of extreme—optimistic or pessimistic—initial

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beliefs. Whereas optimistic beliefs often lead to overinvestment and overcapacity in the incipient stages (Kindleberger 1978; MacKay 1980; Garber 2001; Sidak 2003), pessimistic beliefs can lead to underinvestment (Juma 2014).[1]

There is typically asymmetric information regarding this aggregate (or industry-wide) uncertainty. Some agents, such as the managers of the first movers in the new opportunities, have private signals that are correlated with the true industry productivity.[2] Because the observed decisions of these agents, such as their investment and financing choices, generally will be correlated with their private signals, there is the possibility of efficient aggregation of dispersed and private information based on observational learning (see, e.g., Smith and Sørensen 2001).

However, the recent literature on observational learning highlights the possibility of information cascades, wherein agents completely ignore their useful private information and herd on wrong decisions (Scharfstein and Stein 1990; Bannerjee 1992; Bikhchandani, Hirshleifer, and Welch [BHW] 1992). But the herding literature generally does not consider dynamic information aggregation when informed agents can be offered optimal incentive contracts for inducing information on the,ir private signals. As pointed out by Ottaviani and Sorensen (2000), information cascades occur in the statistical herding models of Bannerjee (1992) and BHW (1992), where agents take optimal actions based on private signals of bounded precision and decisions of other agents, and in the reputational herding model considered by Scharfstein and Stein (1990), where allocations are efficient ex post.

On the other hand, in the context of firms, it is realistic to expect that uninformed investors will interface with privately informed managers through incentive contracts, as is considered in the large agency literature.[3] In particular, there is an intuition that information cascades may be avoided through the design of appropriate contracts by uninformed outsiders who can precommit to investment responses that give insiders

[1] Historical examples of over-investment due to optimistic priors include the South Sea Trading Company in the 18th century; the development of the railroad industries in Britain and the U.S. in the 1830s and the 1860s, respectively; the growth of power Utilities in the U.S. in the 1920s; and, the internet and telecommunication industries world-wide in the 1990s. Meanwhile, Juma (2014) argues that *pessimistic* beliefs among development experts (in the 1950s) regarding the innovation and entrepreneurial capacity of newly independent developing economies led to underinvestment in new technologies in these societies, which had long term negative effects on their economic growth.

[2] Recent examples of such asymmetric information include the private information of firms such as WorldCom regarding the growth of internet traffic in the 1990s (Sidak 2003) and that of mortgage lenders and investment banks in the securitization of mortgages during 2002-2006 (Coffee 2010).

[3] The theory of incentives and mechanism design is a mature field and, as such, there are a number of excellent surveys available. A non-exhaustive list includes Hart and Holmstrom (1987), Mookherjee (2006), Myerson (2008), and Martimort (2006).

incentives for information transmission.[4] But there are also well known institutional constraints on the design of optimal incentive contracts, such as limited liability or nonnegativity constraints on managerial compensation, which may restrict the ability of incentive contracts to induce private information. However, the literature has not examined whether optimal incentive contracts, designed under plausible institutional constraints, can achieve dynamic information aggregation and avoid information cascades. In particular, the literature on the effects of asymmetric information on investment policies of firms typically considers situations in which there is private information on project-specific, and not on industry (or aggregate), productivity.[5]

In this paper, we examine the dynamic aggregation of private information on industry capital productivity when firms enter over time and equity owners of each firm design compensation- and investment-based incentive contracts to induce information from their managers who are privy to signals correlated with the industry productivity. Consistent with our focus on the firm, we impose limited liability (or nonnegativity) constraints on managerial compensation. We characterize the interim efficient investment and managerial compensation contracts (Holmstrom and Myerson 1983) in this dynamic game when owners design contracts based on the observed history of previous contracting outcomes. Strikingly, we find that learning on the unknown industry productivity stops endogenously along the equilibrium path for all initial conditions; hence, with positive probability there is herding on the wrong investment level (compared with the complete information outcome) along the equilibrium path.

Intuitively, as learning proceeds and beliefs become more precise, the net benefit of inducing additional information through costly incentive-provision falls. To fix ideas, suppose that the true industry productivity can be high or low. If, at some stage, the markets' posterior beliefs regarding the unknown productivity are very optimistic or pessimistic—that is, the probability of high productivity is close to one or zero—then the expected learning from an additional signal is small. However, incentive provision through investment and compensation distortions is costly; hence, pooling contracts become endogenously optimal over time. We reiterate that this result occurs even though we impose no restrictions on the principal's ability to credibly precommit to investment and only

[4] The literature presents suffcient conditions to rule out information cascadaes — such as, unboundedly informative signals (Smith and Sorensen 2001) or rich action spaces (Lee 1993). Scharfstein and Stein (1990) and Kumar and Langbeg (2013) consider contract design with observational learning but do not allow precommitment with respect to the investment response of the uninformed outside investors.

[5] Papers in this tradition include Stiglitz and Weiss (1983), Myers and Majluf (1984), Greenwald, Stiglitz, and Weiss (1984), DeMeza and Webb (1987), Stein (1989), Harris and Raviv (1996), Martin (2009), and Kumar and Langberg (2009).

impose the economically reasonable constraint that there is no unlimited liability on managers.

In sum, the herding outcomes presented in the literature (e.g., Scharfstein and Stein 1990; Bannerjee 1992; BHW 1992) appear to be robust to incentive contracting with precommitment in an economically reasonable model of incentive contracting in firms. An important aspect of our analysis is that initial beliefs and past industry productivity-related disclosures not only determine the financial markets' expectations but also affect the information content of future disclosures through their influence on the design of optimal incentive contracts. In particular, if the initial beliefs regarding industry productivity are extreme—optimistic or pessimistic—then no learning occurs through contracting and there is a greater likelihood of herding on the wrong investment level. Namely, when the true productivity is low (high), there will be over- (under-) investment at the aggregate or industry-level because the initial beliefs will not be corrected based on managerial signals, even with an infinite sequence of incentive contracts. In a similar vein, because the signals are stochastically related to the true productivity state, managers may receive good (bad) signals even if the true productivity is low (high). A sequence of good (bad) signals can therefore endogenously induce market optimism (pessimism) along the equilibrium path that does not get corrected, even if negative (positive) signals eventually appear, because contracts endogenously enter the "pooling region" with extreme beliefs. These possibilities appear empirically consistent with the historically observed confluence of extreme beliefs and aggregate industry investment distortions that we have already noted. However, in our framework, the herding outcomes arise along perfect Bayesian equilibrium paths with incentive-efficient contracting, and not because of "irrational exuberance" (Greenspan 1996), "overreactions to innovations" (Shiller 2005), or misplaced pessimism (Juma 2014).

When the initial beliefs and the agency conflicts are moderate, there is learning along the equilibrium path even though learning eventually stops as beliefs endogenously enter the pooling region. Our model allows us to relate the "learning region" of beliefs to salient model parameters. We find that the learning region is positively related to the cost of capital. Thus, expected learning is lower and the aggregate investment distortions are likely to be higher when the cost of capital is low because of stock market booms or greater influx of funds in the capital market, consistent with what we actually tend to observe (Kindleberger 1978; Garber 2001). Meanwhile, the learning region is positively related to the productivity gap between the high and low states, because the gains from inducing information increase as the expected costs of investment distortions rise. Finally, the learning region is also positively related to the precision of managers' signals with respect to the unknown productivity.

As noted already, in herding models without contracting there exist sufficient conditions to ensure efficient information aggregation through observational learning: information cascades can be avoided if signals are of unbounded precision (Smith and Sørensen 2001) or the action space is continuous (Lee 1993).[6] Thus, there is an intuition that incentive contracting will avoid information cascades under weaker sufficiency conditions. Somewhat surprisingly, we find that the crucial determinant of dynamic information aggregation is not the information content of the signals per se but whether it is efficient to induce signals from informed agents. In particular, if the agency conflict between the informed and uninformed parties is sufficiently severe, or if the initial beliefs are extreme, then there may be no information transmission, even when signals have unbounded precision or are continuously distributed.

The herding outcomes with optimal incentive contracting raise a general issue: which conditions on agents' preferences and the information structure are sufficient to ensure that incentive contracting will result in complete learning, that is, will avoid information cascades? There may be a conjecture that relaxing limited liability constraints on the optimal contract design will eliminate herding. But this is not true, because additional restrictions on preferences and information structure need to be imposed. The sufficient conditions for asymptotically complete learning are related (but are not equivalent) to conditions used in the optimal contracting literatures for adverse selection and moral hazard, even though there are no unobservable actions in our model. To guarantee complete learning asymptotically, one requires not only the Spence-Riley single-crossing property (SCP) but also the monotone hazard rate property of the posterior beliefs with the monotone and concave likelihood ratio properties of signals (conditional on the state). The former condition on posterior beliefs is more restrictive than the monotone hazard rate property of the prior distribution of types (Myerson 1981; Maskin and Riley 1984).[7] Meanwhile, the monotone and concave likelihood ratio properties of noisy signals are used by, for example, Jewitt (1988).[8] Essentially, these conditions are sufficient to ensure that the history-dependent interim-efficient decision rules are completely separating with respect to the informed agents' private signals.

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[6] Private beliefs are bounded if the likelihood ratio implied by individual signals is finite and strictly bounded away from zero. That is, with bounded beliefs the information content of signals (at the margin) is bounded above. In contrast, with unbounded private beliefs agents can receive signals with unbounded information content.

[7] The monotone hazard rate property of the prior distribution of types plays an important role in guaranteeing the monotonicity of the optimal decision rule in models with adverse selection.

[8] In Jewitt (1988) these conditions are used in validating the first order approach for the derivation of optimal contracts in moral hazard settings.

To our knowledge, this is the first analysis of the interim (or incentive) efficiency of information cascades. Our study bridges the gap between the herding and incentive contracting literatures. Our analysis helps explain why information cascades can occur with optimal contracting in agency models that are widely considered in the finance and economics literatures and clarifies the conditions needed in these models to ensure complete learning through incentive contracts. In particular, we find that incentive contracts are likely to avoid information cascades and herding on wrong decisions when agents' preferences satisfy properties that are also conducive to the existence of signaling equilibria: investment decisions in new industries or markets undergoing structural changes will be efficient in the long run precisely under conditions in which there may exist signaling equilibria through dividends (Bhattacharya 1979; John and Williams 1985), insiders' equity (Leland and Pyle 1977), or higher debt (Ross 1977; Heinkel 1982). Our results also provide a novel perspective on learning unknown industry productivity, an issue that has been of long-standing interest in the literature (Zeira 1987, 1994; Rob 1991). Finally, although there is a literature on the implications of limited liability for optimal contracting (e.g., Demougin and Garvie 1991), our analysis is among the first to relate limited liability to long-term information aggregation and herding.

1. Learning Unknown Industry Productivity

1.1 Environment

Suppose that a new technology is introduced and deployed by sequence of firms that enter the industry, but its productivity—that is, its expected rate of return on investment—is unknown. Firms enter the industry sequentially and make investments. Each firm is controlled by a manager who receives a private noisy signal on the industry productivity. If these signals were aggregated over time, then by the law of large numbers the true industry productivity would become known asymptotically. However, managers may not truthfully reveal these signals to outside investors (the equity owners) because of an intrinsic conflict of interest: managers obtain private benefits from investment that are (axiomatically) not valued by the investors. However, the equity owners of each firm can write an incentive contract with their manager to induce information. But whereas the owners can precommit to their investment response to their manager's signals, they realistically cannot precommit to investment in firms that may enter subsequently.[9] There is perfect recall (Kuhn 1953) in that all players know the prior contracting outcomes or decisions of firms

[9] Such long term commitment may be available to a social planner in a productive economy, but is not credible when that economy is organized through ownership of individual firms through equity markets.

that have previously entered. Thus, there is observational learning (Bannerjee 1992; BHW 1992) in contract design because owners can optimally design their incentive contracts, which then determine the investment and managerial compensation decision of their firm, based on their observations of prior investment and compensation decisions in the industry. The central question we examine is: whether the sequence of optimally designed contracts will lead to asymptotically complete learning on the industry productivity.

We model the situation just described in the following fashion. There are a countably infinite number of stages—each stage representing entry by a firm—indexed by $n \in \mathcal{N}$. To each stage is associated a manager-owner pair (x_n, y_n), who together decide on an investment level a_n chosen from a prespecified feasible set $\mathcal{A} \subseteq \mathcal{R}_+$. Let x_n manage or control a production technology owned by y_n; this technology stochastically relates capital investment a_n to output $z_n = [\theta(\nu_h - \nu_\ell) + \nu_\ell]f(a_n)$, where f is twice continuously differentiable, strictly increasing and strictly concave, whereas $\theta \in \{0, 1\}$ is the unknown productivity parameter, and $\nu_h > \nu_\ell \geq 0$.

For convenience, we label the sequence of managers $\{x_n\}_{n=1}^{\infty}$ and firms $\{y_n\}_{n=1}^{\infty}$ as type-X and type-Y agents, respectively (we will also refer to the type-X agent as the *agent* and to the type- Y agent as the *principal*). Each individual's payoff depends on its type (X or Y), the investment a_n, an underlying state $\theta \in \Theta = \{0, 1\}$, and the monetary transfers between them $w_n \in \mathcal{W} \subseteq \mathcal{R}$. In the situation at hand, w_n is naturally interpreted as wages or compensation paid by the principal to the agent. We assume that the payoffs for type-X and type-Y agents are given, respectively, by[10]

$$U^X(a_n, w_n, \theta) = u^X(a_n, \theta) + w_n = w_n + \varphi a_n, \tag{1}$$

$$U^Y(a_n, w_n, \theta) = u^Y(a_n, \theta) - w_n = [\theta(\nu_h - \nu_\ell) + \nu_\ell]f(a_n) - Ra_n - w_n. \tag{2}$$

Here, the parameter $0 < \varphi < 1$ models the private benefits of control or utility from "empire building" (Stulz 1990; Hart 1995) and $R > 1$ is the cost of capital. The preference specification of the agent in (1) is standard in this literature and is adapted from Harris and Raviv (1996). Note that from the perspective of the principal the complete-information optimal level of investment $a^{fb}(\theta)$ maximizes expected output net of financing costs, that is, satisfies $[\theta(\nu_h - \nu_\ell) + \nu_\ell]f'(a^{fb}(\theta)) = R$.

The true state θ is unknown, and all agents and principals have the common prior belief $\mu_0 = \Pr(\theta = 1)$. A central question is whether and what the agents learn about the true state over time. At each stage n,

[10] We allow for the possibility that the agents cannot agree or decide on an action, in which case they each receive zero utility.

private signals $s_n \in \mathcal{S}$ are received by the type-X agent or manager. That is, there is asymmetric information between the informed agent and uninformed principal. The signals $\{s_n\}_{n=1}^{\infty}$ are generated independently conditional on θ according to the nondegenerate probability measures G_θ, $\theta \in \Theta$, on the common support $\mathcal{S}$, and such that G_0, G_1 are nonidentical ($G_i(s) = \Pr(s_n = s \mid \theta = i)$, for $i = 1, 2$). These assumptions imply that signals are at least somewhat informative but are not perfectly revealing of θ. We will denote the probability mass functions with $g_0(s)$, $g_1(s)$.[11]

We now describe the contracting environment, define the equilibrium, and relate the information revelation along the equilibrium path to asymptotic learning.

1.2 Contracting

We focus on bilaterally interim efficient decision rules at each stage $n \in \mathcal{N}$.[12] The information set at n is $\phi_n = (s_n, h_n) \in \Phi_n$, where h_n is the observed profile of decisions $d_i = (a_i, w_i)$ for $i = 1, \ldots, n-1$ (or $h_n = (d_1, \ldots, d_{n-1}) = ((a_1, w_1), \ldots, (a_{n-1}, w_{n-1})))$. A decision rule is the mapping $\delta_n : \Phi_n \to \Delta[\mathcal{A} \times \mathcal{W}]$ (the space of probability measures on $\mathcal{A} \times \mathcal{W}$). Now let $\mu_n = \Pr(\theta = 1 \mid h_n)$. Then, by Bayes' rule, the private beliefs of type-X agent x_n, given the signal s_n, are

$$p_n(s_n; \mu_n) \equiv \Pr(\theta = 1 \mid s_n, h_n) = \frac{g_1(s_n)\mu_n}{g_1(s_n)\mu_n + g_0(s_n)(1 - \mu_n)}. \tag{3}$$

The expected utility of x_n when $s_n = s$, but when state s' is reported and the decision rule $\delta_n(s_n = s', h_n)$ is used, is

$$\begin{aligned} &V_n^X(s, s'; h_n, \delta_n) \\ &\quad \equiv \int_{\mathcal{A} \times \mathcal{W}} \big[p_n(s; \mu_n) U^X(a, w, 1) + (1 - p_n(s; \mu_n)) U^X(a, w, 0)\big] dQ(\delta_n(s', h_n)). \end{aligned} \tag{4}$$

Here, the conditional distribution of allocations $d_n = (a_n, w_n)$ following report s' as governed by $\delta_n(\phi_n)$ is denoted by $Q(\delta_n(\phi_n))$ (that is, the projection of $\delta_n(\phi_n)$ on $\Delta[\mathcal{A} \times \mathcal{W}_n]$). A decision rule δ_n is (Bayesian) incentive compatible if for every $s, s' \in \mathcal{S}$

$$V_n^X(s, s; h_n, \delta_n) \geq V_n^X(s, s'; h_n, \delta_n), \; s \neq s'. \tag{IC_n}$$

[11] We conduct the analysis by considering a binary distribution of signals but subsequently in the appendix we extend the analysis to show that our results also hold for the case of continuous signals. When $\mathcal{S}$ is continuous, the notation used above stands for the probability density functions.

[12] The concept of interim efficiency is introduced and discussed generally in Holmstrom and Myerson (1983).

(For notational ease, let $V_n^X(s; h_n, \delta_n) \equiv V_n^X(s, s; h_n, \delta_n)$). In addition, δ_n is acceptable (or individually rational) for x_n if

$$V_n^X(s_n; h_n, \delta_n) \geq 0, \forall s \in \mathcal{S}. \qquad (IR_n)$$

δ_n is admissible if it is incentive compatible and acceptable.

Note, however, that admissibility of contracts imposes no restrictions on the monetary transfers among the two parties. In particular, it is possible theoretically that an admissible contract involves negative transfer to the agent. To see this, note that the individual rationality constraint for the agent (cf. (IR_n)) will be satisfied as long as $w_n + \varphi a_n \geq 0$; in particular, if $a_n > 0$, then $-\varphi a_n \leq w_n < 0$ will satisfy this constraint. This possibility is clearly unappealing in many institutional settings, especially in the context of the firm in which managerial or worker compensation is generally nonnegative. To accommodate these institutional constraints, we will impose an additional limited liability constraint, which requires that the monetary transfers to the agent be nonnegative (Demougin and Garvie, 1991). Specifically, δ_n is feasible if it admissible and, in addition, $\delta_n : \Phi_n \to \Delta[\mathcal{A} \times \mathcal{W}_+]$, where $\mathcal{W}_+$ is the set of nonnegative monetary transfers.

Next, to define the notion of interim efficient contracts concisely, we first compute the expected utility of y_n conditional on h_n and signal s_n (reported truthfully) as

$$V_n^Y(s; h_n, \delta_n) = \int_{\mathcal{A}\times\mathcal{W}} \left[p_n(s; \mu_n)U^Y(a, w, 1) + (1 - p_n(s; \mu_n))U^Y(a, w, 0)\right]dQ(\delta_n(s, h_n)). \tag{5}$$

Meanwhile, the expected utility of y_n conditional on h_n, (that is, before signal s_n is reported), namely, $V_n^Y(h_n, \delta_n)$, is computed as follows. Let $\gamma_n(s; h_n)$ denote the probability density function of $s \in \mathcal{S}$ conditional on h_n; that is,

$$\gamma_n(s; h_n) \equiv \gamma_n(s) = g_1(s)\mu_n + g_0(s)(1 - \mu_n). \tag{6}$$

Then the expected utility of y_n under the decision rule δ_n is

$$V_n^Y(h_n, \delta_n) = \int_{\mathcal{S}} V_n^Y(s; h_n, \delta_n)\gamma_n(s)ds. \tag{7}$$

δ_n^* is interim efficient if it is feasible and maximizes $V_n^Y(h_n, \delta_n)$.[13]

[13] In general, interim efficient rules maximize a welfare function that is a weighted average of the expected utilities of the two agents (x_n, y_n), where the weights may depend on ϕ_n (Holmstrom and Myerson 1983). Our formulation conforms to the standard principal-agent contracting framework.

By the revelation principle any feasible decision rule can be implemented through a direct mechanism or contract with the message space $\mathcal{M}_n = \mathcal{S}$ (with truth-telling on signals) for x_n and the (possibly randomized) message-contingent decision rule $\delta_n(m_n, h_n) \in \Delta[\mathcal{A} \times \mathcal{W}_+]$, $m_n \in \mathcal{M}_n$ (see, e.g., Jackson 2003). With a straightforward adaptation of notation, the incentive compatibility conditions (4)-(IC_n) can be expressed as $V_n^X(s; h_n, \delta_n) \geq V_n^X(s, s'; h_n, \delta_n)$, $s \neq s' \in \mathcal{S}$; that is, δ_n is incentive compatible if it induces truth-telling with $m_n = s_n$. Similarly, acceptability (for x_n) requires $V_n^X(s; h_n, \delta_n) \geq 0$ for each $s \in \mathcal{S}$. The interim efficient contract $\mathbf{C}_n^*(\phi_n) = \delta_n^*(\phi_n)$ is then a feasible contract that maximizes $V_n^Y(h_n, \delta_n)$. Hence, $Q(\delta_n^*(\phi_n))$ is the distribution of decisions in the efficient contract, and we will denote its support by $\Delta_n^*(\phi_n)$.

1.3 Contracts and learning

By construction, the various generations of players (x_n, y_n) observe the prior profile of contract outcomes or decisions, that is, $\{(a_t, w_t)\}_{t \leq n-1}$. This is consistent with the literature on information cascades because, from the viewpoint of observational learning, it is the information content of prior decisions (rather than any inter-agent messages) that is relevant (Bannerjee 1992; BHW 1992). But the information gleaned from the optimal decisions can range from complete revelation, where the optimal decision rule δ_n^* is invertible in the signal s_n, to no revelation, when the decision rule is identical for all signal types.

A sequence of bilateral contracts $\{\delta_n\}_{n=1}^{\infty}$ generates a stochastic process of investment and compensation decisions $\{(a_n, w_n)\}_{n=1}^{\infty}$, where the randomization occurs through the realization of signals $\{s_n\}_{n=1}^{\infty}$ (and the possibly randomized decision rule). A perfect Bayesian equilibrium (or just "equilibrium") in the game described already is then specified by a sequence of optimal bilateral contracts $\delta^* = \{\delta_n^*\}_{n=1}^{\infty}$ and the sequence of communication strategies (for x_n) $\mathbf{m}^* = \{m_n^*\}_{n=1}^{\infty}$ where $m_n^* = s_n$, $n \in \mathcal{N}$. It is straightforward to show that a $\sigma_n^* = (\delta_n^*, m_n^*)$ is an equilibrium because, by construction $\delta_n^*(m_n, h_n)$ and $m_n^* = s_n$ are sequentially rational strategies.[14]

An equilibrium is then the profile $\sigma^* = \{\sigma_n^*\}_{n=1}^{\infty}$. (We label the continuation equilibrium at the root h_n by $\sigma^* \mid_{h_n}$.) We let $\mathcal{P}_{\sigma^*}$ denote the probability measure generated by σ^*, whereas the Bayes-consistent beliefs $\Pr(\theta = 1 \mid h_n)$ recursively generated by this measure are denoted by $\mu_n^{\sigma^*}$. Then, learning is asymptotically complete along σ^* if

$$\lim_{n \to \infty} \Pr(\mu_n^{\sigma^*} = \theta \mid \theta) = 1. \tag{8}$$

[14] To complete the specification of the PBE, we assume that upon observing any market outcome $c_n \notin \mathbf{C}_{n*}(\phi_n)$, for any n, $\mu_{n+1} = \mu_n$.

That is, with probability one the posterior measure is degenerate and the expectations of θ are consistent (with the true $\theta \in \{0, 1\}$). And learning is asymptotically incomplete if $\lim_{n\to\infty} \Pr(\mu_n^{\sigma^*} = \theta \mid \theta) < 1$.

It is apparent that the learning over time on θ will depend on the amount of information revealed by the contracting outcomes along the equilibrium path. The following categorization of the informational content of contracting outcomes, which follows the norms in the literature, is useful. The optimal contract is separating, denoted by $\mathbf{C}_n^{S*}(\phi_n)$, if the contracting outcome $d_n = (a_n, w_n)$ is completely revealing of the true signal realization, that is,

$$\Pr(s_n = s' \mid d_n \in \Delta_n^*(m_n = s', h_n), h_n) = 1, \ \forall s' \in \mathcal{S}. \tag{9}$$

However, in a pooling contract (denoted by $\mathbf{C}_n^{P*}(\phi_n)$), the contracting outcomes have no information content regarding the informed player's signals; that is, for all $s, s' \in \mathcal{S}$,

$$\Pr(s_n = s \mid d_n \in \Delta_n^*(m_n = s', h_n), h_n) = \gamma_n(s; h_n). \tag{10}$$

In sum, in our model, the observed history of prior contracting outcomes will have information content with respect to the true productivity parameter θ only if at least some of the previous contracts were separating or bunching contracts. On the other hand, the posterior and prior beliefs are unchanged if the contracts are pooling. That is, if at any information set h_n, the optimal contract is pooling, then $\mu_{n+1}^{\sigma^*} = \mu_n^{\sigma^*}$, and hence every $\mathbf{C}_n^*(\cdot, h_{n+i})$, $i = 1, 2..$, along $\sigma^* \mid_{h_n}$ also will be pooling.

Proposition 1. Along any σ^*, for any h_n, $n \in \mathcal{N}$, if $\mathbf{C}_n^*(\cdot, h_n) = \mathbf{C}_n^{P*}(\cdot, h_n)$, then $\mu_{n+i}^{\sigma^*} = \mu_n^{\sigma^*}$, $i \in \mathcal{N}$, with probability one.

Thus, pooling optimal contracts will rule out complete learning, that is, allow herding. Conversely, if with strictly positive probability the optimal contracts reveal some new information, which will certainly be the case if they are separating, then learning will be complete.[15]

Proposition 2. Along any σ^*, for any h_n, $n \in \mathcal{N}$, if $\mathbf{C}_n^*(\cdot, h_n) = \mathbf{C}_n^{P*}(\cdot, h_n)$ and $0 < \mu_n^{\sigma^*} < 1$, then learning is asymptotically incomplete along $\sigma^* \mid_{h_n}$. However, if $\mathbf{C}_n^*(\cdot, h_n) = \mathbf{C}_n^{S*}(\cdot, h_n)$ for every h_n, $n \in \mathcal{N}$, then learning is asymptotically complete along σ^*.

[15] In general actions can generate noisy information. Namely, a contract is bunching, if d_n is not invertible (or not completely revealing) but is informative of the true signal because the uninformed player's posterior beliefs are different from its prior beliefs. We note that the implications of a sequence of bunching contracts for complete learning requires careful analysis. If, for example, bunching involves a partitioning of the type space where the partition size is uniformly bounded, then bunching will lead to complete learning. However, there can be more complex trajectories, where the pooling interval expands to cover the type space asymptotically, where complete learning may not result. For tractability, we will therefore focus on complete pooling and separating contracts. This will not materially limit our analysis because the optimal contracts are either pooling or separating.

Of course, there is a large literature that examines sufficient conditions for the optimal contract to be strictly monotone in hidden types, that is, be separating, or involve bunching in types (see, e.g., Fudenberg and Tirole 1991). It is worthwhile to reiterate the important differences between our set up, which builds on the canonical herding model, and the standard nonlinear pricing problem with hidden information. First, the private information, that is, the signal s_n, itself does not affect the payoffs of either the informed or the uninformed agent; rather, signals influence expected payoffs only through their effect on the Bayes estimate (or posteriors) on the unknown state parameter θ. Second, there is a distinction between the posterior beliefs on θ, which are endogenous because they depend on the outcome of the contracting game, and the likelihood that the (type-X) agent will receive a given signal, which is fixed exogenously by the signal structure $\{G_1(s), G_0(s)\}$. Finally, the standard (or textbook) mechanism design formulation with hidden information does not impose limited liability constraints in the form of monetary transfers between the informed and uninformed parties. As we noted already, although these constraints may not be relevant or be usually nonbinding in some settings (see Section 2.2), they are natural in the context of the firm. As we will see, whether or not the limited liability constraints are binding plays an important role in the possibility of herding with optimal contracting.

We now return to the agency model of the firm specified in Section 1.1 and analyze the long-run learning outcomes.

1.4 Evolution of beliefs

To facilitate intuition, we consider the case of a binary signal space $\mathcal{S} = \{H, L\}$. This case allows us to exposit our main points. (However, in the Appendix we extend the analysis to allow for continuous signals.) It is also convenient to use the following symmetric signal distribution: prior to the investment, x_n receives a private signal $s_n \in \mathcal{S} = \{H, L\}$ with $\alpha \equiv g_1(H) = g_0(L) > \frac{1}{2}$. Hence, conditional on $\mu_n = \Pr(\theta = 1 \mid h_n)$, γ_n^H is given by

$$\gamma_n^H \equiv \Pr(s_n = H \mid h_n) = \alpha\mu_n + (1-\alpha)(1-\mu_n). \tag{11}$$

And the posterior-type distribution $\{p_n^H, p_n^L\}$ conditional on s_n is given by

$$p_n^H = \Pr(\theta = 1 \mid s_n = H, \mu_n) = \frac{\Pr(\theta = 1, s_n = H \mid \mu_n)}{\Pr(\theta = 1, s_n = H \mid \mu_n) + \Pr(\theta = 0, s_n = H \mid \mu_n)}$$
$$= \frac{\mu_n\alpha}{\mu_n\alpha + (1-\mu_n)(1-\alpha)}. \tag{12}$$

$$p_n^L = \Pr(\theta = 1 \mid s_n = L, \mu_n) = \frac{\mu_n(1-\alpha)}{\mu_n(1-\alpha) + (1-\mu_n)\alpha}. \tag{13}$$

Furthermore, the posterior expected productivity conditional on the signal $s_n = s$ is

$$\psi_n(s) \equiv E(\theta \mid s_n = s, \mu_n)(\nu_h - \nu_\ell) + \nu_\ell = (\nu_h - \nu_\ell)p_n^s + \nu_\ell, s \in \{H, L\}. \quad (14)$$

Along the equilibrium path the posterior beliefs μ_n on the unknown productivity state θ are informationally sufficient to represent history h_n. Hence, for notational convenience, we will subsequently denote the optimal contracts by $\mathbf{C}_n^*(\mu_n)$ (where we suppress the dependence of the decision on the agent's message).

1.5 Asymptotic learning

The contract between x_n and y_n specifies a menu of investment levels and wage payments $\langle a_n^j, w_n^j \rangle_{j=L}^H \in \mathcal{R}_+^2$. Here, in the optimal separating contract only the incentive constraint for the low-type agent is binding. Hence,

$$\mathbf{C}_n^{S*}(\mu_n) \in \arg \max_{\langle a_n^j, w_n^j \rangle_{j=L}^H \in \mathcal{A} \times \mathcal{W}_n} \Big\{ \gamma_n^H [\psi_n(H) f(a_n^H) - R a_n^H - w_n^H] + (1 - \gamma_n^H)\big[\psi_n(L) f(a_n^L) - R a_n^L - w_n^L\big] \Big\}, \quad (15)$$

subject to

$$w_n^L - w_n^H = \varphi\big(a_n^H - a_n^L\big), \quad (16)$$

$$w_n^L + \varphi a_n^L \geq 0, \quad (17)$$

$$w_n^L \geq 0, w_n^H \geq 0. \quad (18)$$

Here, (16) is the binding incentive (IC) constraint for the low-type agent; (17) is the acceptability or individual rationality (IR) constraint for the low-type agent; and (18) are limited liability (LL) constraints. Note that if the IR constraint is satisfied for the low-type agent, then it follows that (16) is also satisfied for the high-type agent.

Importantly, because of the LL constraints, the IR constraint need not be binding for the low-type agent, unlike the standard optimal contract for incomplete information, where the IR constraint is binding for a boundary type (see, e.g., Fundenberg and Tirole 1991). In particular, suppose that there were no LL constraints here. Then if the IR constraint were nonbinding for the low-type agent, the principal could lower w_n^L and w_n^H by the same amount, which would leave the IC constraint (16) unaffected, and strictly improve on the candidate contract. However, this argument is no longer valid with LL constraints because lowering wages may violate the LL constraints. Indeed, if the IR constraint (17) were binding, then $w_n^j = -\varphi a_n^j, j = H, L$ (using (16)) so that the LL constraints would be only satisfied when there is no investment in the firm for any communication from the manager, which is generally inefficient.

The possibility of binding LL constraints and nonbinding IR constraints substantially affects information transmission in the optimal contract; in particular, pooling becomes optimal for an open set of parameters. We analyze the optimal contract by first characterizing a candidate separating contract and then examining its optimality. It follows from (15)-(18) that the optimal separating contract $\mathbf{C}_n^{S*}(\mu_n)$ is given by

$$\begin{aligned} f'(a_n^H) &= \frac{1}{\psi_n(s)(H)}\left(\frac{\left(1-\gamma_n^H\right)\varphi}{\gamma_n^H}+R\right), \\ f'(a_n^L) &= \frac{R-\varphi}{\psi_n(s)(L)}, w_n^L = \varphi\left(a_n^H - a_n^L\right),\ w_n^H{=}0. \end{aligned} \tag{19}$$

Intuitively, the optimal separating contract relaxes the low-type agent's incentive constraints by setting the high-type agent's wages to be zero—the minimum feasible under the LL constraints. Note that the IR constraint for the high-type agent is still nonbinding because it receives positive utility φa_n^H. Meanwhile, the low-type agent's wages are determined by the IC constraint (16); substituting these wages in the objective function (15), then yields the optimal investment levels specified in (19).

Now, the candidate separating contract in (19) is feasible iff $a_n^H > a_n^L$, that is,

$$\frac{1}{\psi_n(s)(H)}\left(\frac{\left(1-\gamma_n^H\right)\varphi}{\gamma_n^H}+R\right) < \frac{R-\varphi}{\psi_n(s)(L)}. \tag{20}$$

With some manipulation (see the Appendix), (20) can be restated as (where $\rho \equiv \frac{\nu_\ell}{\nu_h - \nu_\ell} > 0$)

$$Z(\mu_n) \equiv \frac{\mu_n + \rho}{\left[\frac{(1-\alpha)\mu_n}{(1-\alpha)\mu_n + \alpha(1-\mu_n)}\right] + \rho} > \frac{R}{R-\varphi}. \tag{21}$$

Otherwise, the optimal pooling contract $\mathbf{C}_n^{P*}(\mu_n)$ is given by zero wages and action a_n^{pool} such that

$$f'(a_n^H) = f'(a_n^L) = f'(a_n^{pool}) \equiv \frac{R}{[E(\theta \mid \mu_n)(\nu_h - \nu_\ell) + \nu_\ell]}, w_n^L = w_n^H = 0. \tag{22}$$

Therefore, pooling is optimal when beliefs μ_n are sufficiently high or sufficiently low or when the private benefit of control parameter φ (cost of capital R) is sufficiently high (low). Let $\overline{\varphi} > 0$ be the critical level of private benefit of control such that pooling is optimal for all μ when $\varphi > \overline{\varphi}$. And for levels of private benefit of control below the critical level, let $\mu^- < \mu^+$ be the solutions to the equation $Z(\mu_n) = \frac{R}{R-\varphi}$.[16]

Proposition 3. For any $0 < \mu_n < 1$ and for any $\phi_n, n \in \mathcal{N}$:

1. If $\varphi < \overline{\varphi}$ and $\mu_n \in (\mu^-, \mu^+)$, then the equilibrium is a separating equilibrium, that is, $\mathbf{C}_n^*(\mu_n) = \mathbf{C}_n^{S*}(\mu_n)$ as given by (19).
2. If (i) $\varphi < \overline{\varphi}$ and $\mu_n \notin (\mu^-, \mu^+)$ or if (ii) $\varphi > \overline{\varphi}$, then the equilibrium is a pooling equilibrium, that is, $\mathbf{C}_n^*(\mu_n) = \mathbf{C}_n^{P*}(\mu_n)$ as given by (22).

It is instructive to consider the implications of Proposition 3 for the endogenous or path-dependent information content of contracting outcomes and consequently for the possibility of information cascades or herding. If the agency conflict between the owners and the managers, represented in our model by the managers' private benefit of control parameter φ, is high (that is, $\varphi > \overline{\varphi}$), then no information is gleaned from managers in the optimal contracts and herding occurs irrespective of the initial beliefs on the industry productivity (μ_0). However, when the agency conflicts are not very high ($\varphi < \overline{\varphi}$), then the information generated by the optimal contracts depends on the initial beliefs. If the initial beliefs are either optimistic ($\mu_0 > \mu^+$) or pessimistic ($\mu_0 < \mu^-$), then again there is no information revelation through optimal contracts because pooling is optimal.[17] Hence, information cascades arise axiomatically. Finally, if the agency conflicts are not too high and the initial beliefs are in the intermediate range ($\mu^- < \mu_0 < \mu^+$), then optimal contracting induces information from managers until the endogenously generated posterior beliefs become extreme; for example, when μ_n approaches the optimistic belief boundary μ^+. At this point, pooling becomes optimal and further learning stops, resulting in incomplete asymptotic learning or cascades.

Although it follows from Proposition 3 that the optimal contract can be either pooling or separating for any given stage n—depending on the severity of the agency conflict and the endogenous current level of expected productivity—over time we expect learning to stop as beliefs enter the pooling regions. But because Bayesian posterior beliefs are martingales it follows from the martingale convergence theorem (see, e.g., Billingsley 1979) that beliefs must eventually enter these regions.

[16] The critical level of private benefit of control is defined by $\overline{\varphi} = \left(\frac{Z^{\max}-1}{Z^{\max}}\right)R$, where $Z^{\max} = \max(\mu \in (0,1) : Z(\mu))$. Pooling occurs at the extreme levels of μ_n because $Z(\mu_n)$ approaches the value 1 as μ_n becomes extreme. Formally, $\lim_{\mu\to 0} Z(\mu) = \lim_{\mu\to 1} Z(\mu) = 1 < \frac{R}{R-\varphi}$.

[17] Note, in the special case $\rho = 0$ (i.e., when $\nu_\ell = 0$) we obtain $\mu^- = 0$ and pooling is optimal for sufficiently high priors on θ, but there is separation for low priors on θ (provided that $\varphi < \overline{\varphi}$).

Hence, learning can not be asymptotically complete in this model with probability 1 and cascades must eventually arise.[18]

Theorem 1. With probability one, there exists a finite stage n^* such that the contract is pooling (that is, $\mathbf{C}_n^*(\mu_n) = \mathbf{C}_n^{P*}(\mu_n)$) for all $n \geq n^*$. That is, learning is asymptotically incomplete along the σ^* specified in Proposition 3.

In sum, contracts become endogenously pooling once learning proceeds and beliefs become more optimistic ($\mu_n > \mu^+$) or pessimistic ($\mu_n < \mu^-$). The intuition here is that as beliefs become more extreme in the sense of approaching the boundary points (0 or 1), the net benefit of inducing additional information through costly incentive-provision falls. To fix ideas, suppose that μ_n is close to one, so the financial markets are almost sure that the true productivity state is good. Because this optimism about the true state is the *prior* belief guiding the design of the optimal contract, the expected learning (or change in beliefs) from inducing information is small, and hence pooling is optimal. We note that the limited liability constraints are important in this argument because they constrain the design of the incentive contract; in particular, because of the nonnegativity constraints on wages, the burden of truth-telling incentives falls on investment distortions (relative to the complete information efficient investment levels).

It is useful to examine the determinants of the separating region of posterior beliefs (μ^-, μ^+). In general, while learning is asymptotically incomplete with probability one in this model, the extent of information induced by optimal incentive contracting depends on the size of the separating region.

Corollary 1. If $\varphi < \overline{\varphi}$, then the boundary μ^+ (μ^-) is increasing (decreasing) in R, α and v_h, and decreasing (increasing) in φ and v_ℓ. Moreover, the critical value $\overline{\varphi}$ is increasing in R, α and v_h, and decreasing in v_ℓ.

The boundary points μ^- and μ^+ are depicted in the three graphs in Figure 1 as a function of the private benefit of control parameter φ. Consistent with the intuition above, as the private benefit of control increases, the upper bound μ^+ decreases, the lower bound μ^- increases, and the optimal contract becomes a pooling contract if the agency conflict between the owners and the manager is sufficiently high. Intuitively, the separating contract becomes more costly to implement as the agency

[18] The optimality of pooling extends to cases that include more general contracts. In particular, if payoffs could be contingent on the firm's output or even future firms' outputs, then pooling will still be an optimal equilibrium outcome. Intuitively, since wages following a high productivity report are already set to zero under the optimal contract and since there is limited liability pooling remains a robust equilibrium outcome.

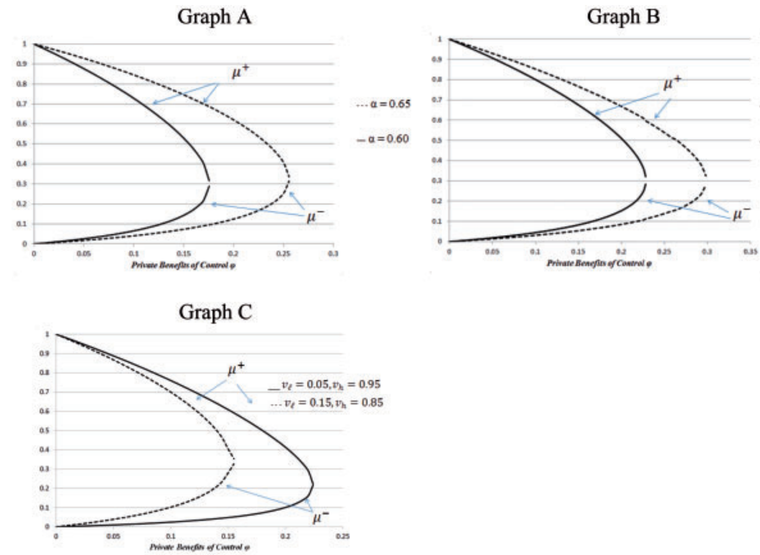

Figure 1
Optimal contract – boundaries on beliefs.
In all three graphs critical values of $\mu^-$and μ^+ are depicted as a function of the private benefit of control parameter φ. In Graph A, two levels of precision of signals ($\alpha = 0.6,\, \textit{and}\; 0.65$) are depicted, where the remaining parameters are ($v_\ell = 0.1,\, v_h = 0.8,\, R = 1$). In Graph B, two levels of the cost of capital ($R = 1.3,\, \textit{and}\; 1.7$) are depicted, where the remaining parameters are ($v_\ell = 0.1,\, v_h = 0.8,\, \alpha = 0.6$). In Graph C, different productivity gaps are depicted, where the remaining parameters are ($\alpha = 0.6,\, R = 1$).

conflict rises, because for any given $\left(a_n^H - a_n^L\right)$, the low-type manager has to be provided higher wages for truthful communication, which is apparent from (19). Thus, one would expect pooling to be optimal for sufficiently high φ, which is verified by Proposition 3.

In Figure 1, we graphically present the boundaries μ^- and μ^+ as functions of the cost of capital (R), the the precision of the signals α, and the productivity levels $\langle v_\ell, v_h \rangle$. Part A of the figure shows that, keeping fixed the level of agency conflict (φ), the separating region (μ^-, μ^+) expands as α rises, that is, the signals s_n become more precise with respect to the unknown productivity state θ. This is consistent with intuition because more precise signals, ceteris paribus, increase the benefit from the separating equilibrium. Meanwhile, part B of the figure shows that the separating region is positively related to the cost of capital R. As the cost of capital rises, the owners' investment response to high signals is muted (because capital has become more costly by assumption); hence, the cost of inducing truthful revelation from the low-type managers falls and the optimal separating region expands. Conversely, the pooling region expands and the expected information generation through optimal contracts falls when the cost of capital is low; that is, there are stock market booms and greater influx of funds in capital markets, other things held

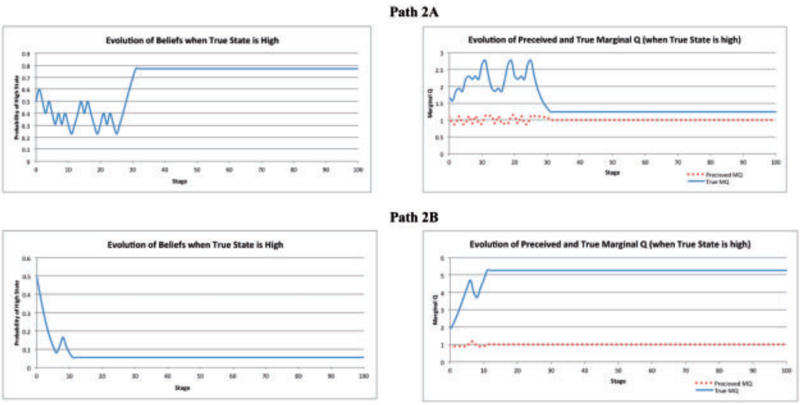

Figure 2
Learning unknown industry productivity and marginal Q (when true productivity is high).
Two random paths are simulated for the high productivity state, that is, $\theta = 1$. For each path, the left-hand-side graph depicts the evolution of beliefs, and the right-hand-side graph depicts the evolution of the perceived marginal Q by investors and the true marginal Q. The parameters are ($\alpha = 0.6$, R $= 1$, $\varphi = 0.1$, $v_\ell = 0.1$, $v_h = 0.7$) , where $\mu^- = 0.077$, and $\mu^+ = 0.719$.

fixed. Finally, part C of the figure shows that the separating region expands when the productivity gap expands (that is, following an increase in v_h or/and an increase in v_ℓ). Intuitively, this follows because a separating contract is more beneficial when the investment distortion from pooling is higher.

Figure 2 presents two simulated equilibrium paths for beliefs μ_n when the true state is high ($\theta = 1$), and Figure 3 does so when the true state is low ($\theta = 0$). Both figures depict the path of beliefs, the marginal returns on investment or path of marginal-Q's as perceived by the market, and the path of true marginal-Q, while taking into account the true state of productivity. In particular,

$$\text{Perceived marginal-}Q \text{ in stage } n \text{ is } \psi_n(s) f'(a_n^s) \text{ for } s \in \{L, H\},$$

while to calculate the true marginal-Q one replaces $\psi_n(s)$ with the true productivity $[\theta(\nu_h - \nu_\ell) + \nu_\ell]$. As one can see from the figures, the beliefs on productivity are distorted and the perceived marginal-Q's deviate from the marginal cost of capital R (which is normalized to one in the figures) along the equilibrium path and in the limit. Figure 2 shows that learning is asymptotically incomplete because posterior beliefs hit the bounds μ^+ (in Figure 2A) and μ^- (Figure 2B). However, in Figure 2A, beliefs follow trajectories toward the true state, and the limiting expected value of θ is relatively close to the true state. However, in Figure 2B, the trajectories of beliefs are in the "wrong" direction and converge to values that are substantially different from the true state. Because the underlying parameters are the same, Figures 2A and 2B graphically demonstrate the

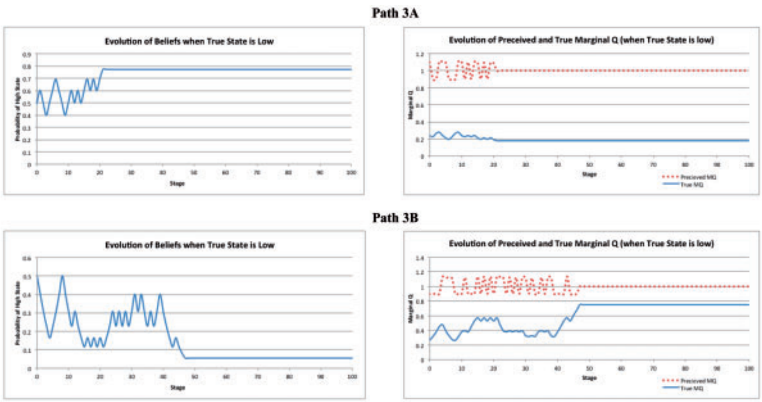

Figure 3
Learning unknown industry productivity and marginal Q (when true productivity is low).
Two random paths are simulated for the high productivity state, that is, $\theta = 0$. For each path, the left-hand-side graph depicts the evolution of beliefs, and the right-hand-side graph depicts the evolution of the perceived marginal Q by investors and the true marginal Q. The parameters are ($\alpha = 0.6$, $R = 1$, $\varphi = 0.1$, $v_\ell = 0.1$, $v_h = 0.7$) , where $\mu^- = 0.077$, and $\mu^+ = 0.719$.

history- or path-dependence of the asymptotic learning outcomes. The two paths in Figure 3 present situations similar to Figure 2, except when the true productivity state is low.

1.6 The role of limited liability

The optimal contract in Proposition 3 trades off the benefits from efficient investment with the cost of rent extraction by the informed agent: pooling is optimal when the cost of eliciting information exceeds the benefit from efficient investment. Thus, the optimality of the pooling contract is directly related to the limited liability of the agent. In particular, suppose that the agent could make transfers to the principal, that is, $w_n^j \in \mathbf{R}$ for $j = L, H$. Individual rationality requires that such transfers satisfy $w_n^j + \varphi a_n^j \geq 0$ for $j = L, H$. The optimal contract without the limited liability constraint is a separating contract for all beliefs μ_n and is given by[19]

$$f'(a_n^H) = \frac{R-\varphi}{\psi_n(H)},\ f'(a_n^L) = \frac{R-\varphi}{\psi_n(L)},\ w_n^L = -\varphi a_n^L,\ w_n^H = -\varphi a_n^H. \tag{23}$$

Under the optimal contract in (23) the agent "pays" for her private benefits from control and earns zero rents. Although the limited liability restriction on wages is realistic, it suggests that in economic settings that allow for substantial transfers from the agent to the principal complete

[19] As noted above, in the absence of the LL constraints, the IC and IR constraints are binding for the low-type manager in the optimal contract. Thus, it follows from (16) and (17) above that $w_n^L = -\varphi a_n^L$ and $w_n^H = -\varphi a_n^H$. Substituting these conditions in the principal's objective function (15) above then yields (23).

learning is more likely. A natural, and more general, question that arises here is which conditions on preferences and information structure would be sufficient for incentive contracts to efficiently aggregate information over time? Addressing this issue will not only help in interpreting the pooling results with respect to capital investment seen above but also help in relating our analysis to the contracting and signaling literatures.

Based on (23), when monetary transfers between the two parties are unrestricted, complete learning will occur along the equilibrium path in the model at hand (see Proposition 2). However, as we show in the subsequent section, this need not be the case in general, when considering a more general adverse selection model. That is, relaxing the limited liability constraint by itself is not generally a sufficient condition to guarantee complete learning asymptotically; additional conditions are needed. For example, we can quickly see from a trivial modification of one of our assumptions for the model at hand that signals must be informative for separation to be optimal.[20] More generally, without limited liability one must impose restrictions on preferences and the distribution of signals to guarantee complete learning. In the next section, we present such sufficient conditions that, along with the relaxation of the limited liability constraints, eliminate information cascades in the presence of optimal incentive contracting.

2. Conditions for Complete Learning

We consider the following general version of the agency model specified in Section 1. There are a countably infinite number of stages—each stage representing entry by a firm—indexed by $n \in \mathcal{N}$. To each stage is associated a pair (x_n, y_n) of players, who together decide on an action a_n chosen from a pre-specified feasible set $\mathcal{A} \subseteq \mathcal{R}_+$. The players' welfare depends on the action, monetary transfers between them w_n, and an unknown parameter $\theta \in \{0, 1\}$. Consistent with the mechanism design and nonlinear pricing literatures, we will focus attention on quasilinear preferences (cf. (1) and (2)):

$$U^X(a_n, w_n, \theta) = u^X(a_n, \theta) + w_n,\ U^Y(a_n, w_n, \theta) = u^Y(a_n, \theta) - w_n. \qquad (25)$$

[20] Suppose that there is no information content in the signal regarding the unknown productivity θ, specifically $\alpha = \frac{1}{2}$. It follows then (see (12)-(13)) that $p_n^H = p_n^L = \mu_n$, that is, receiving a high (low) signal does not change beliefs on whether $\theta = 1$ or 0. Consequently, the posterior expectation of productivity is independent of the signal, since (cf. (14)):

$$\psi_n(s_n = H) = \psi_n(s_n = L) = \mu_n v_h + (1 - \mu_n) v_\ell \qquad (24)$$

However, it follows from (23)-(24) that a separating contract is not sustainable since in any such candidate contract the optimal action is identical across the signals, i.e., $f'(a_n^H) = f'(a_n^L) = f'(a_n^{pool})$.

Here, for each θ, $u^j(a_n, \cdot), j = X, Y$, are concave and twice continuously differentiable functions on the interior of $\mathcal{A}$. We will adopt the convention that $u^i(a, 1) \geq u^i(a, 0)$, for every $a \in \mathcal{A}$, and where the monotonicity is strict for at least one of the agents. This is essentially a labeling convention because it interprets the high state to be "good." Next, at each stage n, private signals $s_n \in \mathcal{S}$ are received by the type-X player; the relation of these signals to θ is exactly as specified in Section 1.1. Furthermore, the contracting environment is the same as specified in Section 1.2.

Our objective is to identify conditions that are sufficient for the optimal contract to be separating when there are no limited liability constraints, that is, when only the incentive and acceptability (or individual rationality) constraints apply. Thus, these conditions will ensure asymptotically complete learning and rule out information cascades (in the absence of limited liability constraints). In the terminology of Section 1.2, δ_n^* is the optimal (or interim efficient) contract if it is admissible—that is, satisfies the incentive compatibility and acceptability conditions—and maximizes $V_n^Y(h_n, \delta_n)$ (cf. (7)).

2.1 Conditions on preferences and signal structure

We will utilize the following conditions to ensure the optimality of separating contracts.

The preferences $u^i(a, \theta), i = X, Y$ satisfy the weak (strict) single-crossing property (SCP) if $u_1^X(a, 1) \geq (>)\, u_1^X(a, 0)$ for every $a \in \mathcal{A}$.

The SCP is the well-known Spence-Mirrleees condition (with binary types) and plays a basic role in theory of sorting and models of incomplete information (Maskin and Riley 1984; Milgrom and Shannon 1994). Next, fix any information set h_n, $n \in \mathcal{N}$, such that the posterior beliefs are μ_n. (Because the posterior beliefs μ_n are informationally sufficient for the history data h_n, we suppress h for notational ease.) Recall that, the cumulative distribution of signals at stage n is $\Gamma_n(s_n = s; \mu_n) = G_1(s)\mu_n + G_0(s)(1 - \mu_n)$ with the associated hazard rate

$$\frac{1 - \Gamma_n(s; \mu_n)}{\gamma_n(s; \mu_n)} = \frac{1 - (G_1(s)\mu_n + G_0(s)(1 - \mu_n))}{g_1(s)\mu_n + g_0(s)(1 - \mu_n)}. \tag{26}$$

The signal structure $\{G_1(s), G_0(s)\}$ satisfies the posterior monotone hazard rate property (PMHRP) if, for any given $\mu_n \in [0, 1]$, $\frac{\partial\left(\frac{1-\Gamma_n(s;\mu_n)}{\gamma_n(s;\mu_n)}\right)}{\partial s} \leq 0$, for each $s \in \mathcal{S}$.

In the literature, the (closely related) monotone hazard property (MHRP) on the distribution of types is widely used (Myerson 1981). However, the PMHRP is different than requiring the MHRP on $G_\theta(s)$

for each θ.[21] Finally, we also require the monotonicity and concavity restrictions on the likelihood ratio of signals (conditional on the unknown parameter θ).

The signal structure $\{G_1(s), G_0(s)\}$ satisfies the

1. monotone likelihood ratio property (MLRP) if $\frac{g_1(s)}{g_0(s)}$ is strictly increasing in s,[22] and
2. concave likelihood ratio property (CLRP) if $\frac{g_1(s)}{g_0(s)}$ is concave in s.

The MLRP is used in the literature on hidden actions or moral hazard (along with other conditions) to justify the first-order approach to the analysis of the principal-agent problem (Rogerson 1985). Jewitt (1988) uses both the MLRP and the CLRP in developing another set of conditions to justify the first-order approach. This connection with moral hazard models is noteworthy because, although there are no hidden actions in our setup, agents do receive noisy signals on unobserved parameters (that is, θ). However, in moral hazard models the noisy signal (output) is publicly observable and endogenously generated by the agent's unobservable action; here, the noisy signal (s) is private information and generated exogenously by the unobservable state.

We now examine the application of these conditions for two polar assumptions on the signal space $\mathcal{S}$: when signals are binary, and when continuous; that is, $\mathcal{S}$ is an interval on the real line.

Section 2.1.1. Binary signals. Suppose first that $\mathcal{S} = \{H, L\}$. For notational ease, let for $j \in \{H, L\}$, $\gamma_n^j = \gamma_n(s_n = j; \mu_n)$ (cf. Equation (6)), $p_n^j = p_n(s_n = j; \mu_n)$ (cf. Equation (3)), and $\delta_n(m_n = j, h_n) = (a_n^j, w_n^j)$. Then the optimal contract solves the optimization problem

$$\mathbf{C}_n^*(\mu_n) \in \arg \max_{\langle a_n^j, w_n^j \rangle_{j=L}^H \in \mathcal{A} \times \mathcal{W}_n} \sum_{j=L}^{H} \gamma_n^j \left[p_n^j U^Y(a_n^j, w_n^j, 1) + (1 - p_n^j) U^Y(a_n^j, w_n^j, 0) \right], \tag{27}$$

[21] The following is an example where the PMHRP is more restrictive than the MHRP for $G_1(s)$ and $G_0(s)$. Let, for $\lambda > 2$ and $s \geq 0$, $g_1(s) = (\lambda - 1)\exp(-(\lambda - 1)s)$, $g_0(s) = \lambda \exp(-\lambda s)$, and $g_1(s) = g_0(s) = 0$ when $s < 0$. In this case, the hazard rates are constant, i.e., $\frac{1-G_1(s)}{g_1(s)} = (\lambda - 1)^{-1}$ and $\frac{1-G_o(s)}{g_0(s)} = \lambda$; thus, the monotone hazard rate property is trivially satisfied for $G_1(s)$ and $G_0(s)$. However, the ratio

$$\frac{1 - \Gamma_n(s; \mu_n)}{\gamma_n(s; \mu_n)} = \frac{\gamma_n(s; \mu_n)}{\lambda \gamma_n(s; \mu_n) - \exp(-\lambda s)\mu_n}$$

is not a constant. Nevertheless, straightforward computations show that the PMHRP is satisfied strictly in this case.

[22] Note that when the signal space is binary, i.e., $s_n \in \{H, L\}$, then the MLRP applies if $g_1(s = H) > g_0(s = H)$.

subject to

$$p_n^j U^X(a_n^j, w_n^j, 1) + (1 - p_n^j) U^X(a_n^j, w_n^j, 0) \geq \\ p_n^j U^X(a_n^k, w_n^k, 1) + (1 - p_n^j) U^X(a_n^k, w_n^k, 0),\ j, k \in \{H, L\}, \tag{28}$$

$$p_n^j U^X(a_n^j, w_n^j, 1) + (1 - p_n^j) U^X(a_n^j, w_n^j, 0) \geq 0, j \in \{H, L\}. \tag{29}$$

In the standard adverse selection model with finite types, the SCP ensures that the incentive compatibility constraints are binding only in one direction and the individual rationality constraint is also only binding for the boundary type. However, because of the noisy signal structure in our setting, one also has to require the MLRP for these properties of the optimal contract to apply. But, note that the conditions CLRP, and PMHRP are trivially satisfied in the binary case, whereas the MLRP is satisfied if signals are strictly informative, that is, $g_1(H) > g_0(L)$ and $g_0(L) > g_0(H)$.

We now characterize the sufficient conditions for the optimal contract to be separating, that is, $a_n^{H*} \neq a_n^{L*}$ *for all* possible histories of observed contracting outcomes h_n and, hence, for all posterior beliefs $\mu_n \in [0, 1]$. In the standard fashion, we do so first by verifying that the incentive (IC) and individual rationality (IR) constraints bind only for one type of agent-X.

Proposition 4. If the SCP holds for X at least weakly and the MLRP applies, then in any optimal separating contract $\mathbf{C}_n^{S*}(\mu_n)$ the incentive and individual rationality constraints (28) and (29) are binding for the low type agent, that is, $s_n = L$.

Using Proposition 4, we can substitute the binding IR and IC constraints for the low type in the objective function (27) to eliminate w_n^L and w_n^H. It is then straightforward to show that in any optimal contract $a_n^{H*} > a_n^{L*}$ if the SCP holds for both agents, with the condition being strict for at least one of the players, and the MLRP applies as well.

Theorem 2. Suppose that the SCP holds for both players, with the condition applying strictly for at least one player,and the MLRP applies. If there are no limited liability constraints, then in any σ^*, $\mathbf{C}_n^*(\mu_n) = \mathbf{C}_n^{S*}(\mu_n)$ for every μ_n. Hence, learning is asymptotically complete along σ^*.

We note that in the model considered in Section 1, the preferences of both players satisfy concavity and twice differentiability in a (see (1), (2)). However, the preferences of the managers do not depend on the unknown state θ, and hence the SCP applies only weakly, but the SCP applies strictly for the owners. Moreover, the MLRP is satisfied whenever $\alpha > \frac{1}{2}$. As pointed out in Section 1.6, complete learning occurs if the managers can be forced to "pay" the utility value of their private benefits of control (while still satisfying the individual rationality constraints).[23] Thus, the sufficient conditions of Theorem 2 apply in the version of the agency model considered in Section 1.6.

Section 2.1.2. Continuous signals. Suppose now that $\mathcal{S} = [s^L, s^H] \subseteq \mathcal{R}$.[24] Thus, at each $n \in \mathcal{N}$ the decision rule is $\delta_n(s) = (a_n(s), w_n(s))$, and given any μ_n, the expected utility of x_n when it receives the signal s and reports $s' \in \mathcal{S}$ is:

$$V_n^X(s, s'; \mu_n, \delta_n) \equiv p_n(s; \mu_n)u^X(a_n(s'), 1) + (1 - p_n(s; \mu_n))u^X(a_n(s'), 0) + w(s'). \tag{30}$$

Under our assumptions on u^i, the optimal decision rule will be piecewise continuously differentiable (e.g., Hadley and Kemp 1971). Maximizing (30) over s' at a point of differentiability yields the necessary and sufficient conditions for local incentive compatibility

$$\frac{\partial V_n^X(s, s; \mu_n, \delta_n)}{\partial s'} = 0, \frac{\partial^2 V_n^X(s, s; \mu_n, \delta_n)}{(\partial s')^2} \leq 0. \tag{31}$$

We show explicitly in the Appendix that if the SCP applies for player X, then a decision rule $\delta_n(s)$ is incentive compatible if and only if $a_n(\cdot)$ is a monotone-increasing function. The MLRP is also useful in ensuring monotonicity of the optimal action in our model. But while the role of the MLRP in the moral hazard literature is to ensure that the optimal sharing rule for risk-averse agents is nonincreasing in output, here its role is to ensure that the private posterior beliefs of the informed agent for the high state (that is, $\theta = 1$) are nondecreasing in the signal (s), which in turn yields a convenient monotonicity (in s) of the informed agents' indirect expected utility. However, to ensure that the optimal action rule $a_n^*(s)$ is monotone strictly increasing we require also that the SCP apply for both players and at least strictly for one player; in addition, we also need the CLRP and the PMHRP. We note that in justifying the first-order approach to the optimal contract in the moral hazard problem, Jewitt (1988) uses the MLRP and CLRP to ensure that the inverse of the agent's marginal utility with the optimal sharing rule is nondecreasing and concave. Here, the MLRP, CLRP, PMHRP, and SCP of both players are jointly used to ensure the desired comparative static property of the optimal decision rule.[25]

[23] The implication of SCP applying only weakly for the informed managers in the agency model considered above is that the IC constraint is binding for both types (see (16)) and if the IR constraint is binding for the low type, then it is also binding for the high type (see (23)).

[24] For expositional convenience, we first restrict attention to non-randomized contracts; we then show that under the identified sufficient conditions for non-pooling decision rules, this restriction is not binding.

[25] We reiterate that the CLRP and PMHRP are trivially satisfied in the case of a binary signal space, so that Theorem 3 below indeed specializes to Theorem 2 for the binary case.

Theorem 3. Suppose that $\mathcal{S} = [s^L, s^H] \subseteq \mathcal{R}$, the SCP holds for both players, with the condition applying strictly for at least one player, the MLRP, CLRP, and the PMHRP hold. If there are no limited liability constraints, then in any σ^*, $\mathbf{C}_n^*(\mu_n) = \mathbf{C}_n^{S*}(\mu_n)$ for every μ_n. Hence, learning is asymptotically complete along σ^*.

2.2 Example: Complete learning of product value

In this section we demonstrate that the sufficient conditions for complete learning previously identified are satisfied for the well-known model of price discrimination (Mussa and Rosen 1978; Kumar 2002, 2006). We nest this model into our framework to consider unknown product value and show that the optimal contract implies complete learning. Suppose that an innovation generates a new product whose true value to consumers is unknown and is represented by an unknown value parameter $\theta \in \{0, 1\}$. There are infinite stages in which each stage buyers and sellers (x_n, y_n) transact in the product market. In each stage the seller is a monopolist who provides a menu of high and low price-quality pairs $\langle a_n^j, w_n^j \rangle_{j=L}^H$, where $(a_n^H, a_n^L) \in R_+^2 = \mathcal{A}$, and $(w_n^H, w_n^L) \in R^2 = \mathcal{W}_n$. The unit cost of production is $\beta(a_n)^2/2$. The buyer can choose one of the price-quality pairs or reject the menu. Thus, at each stage the buyer purchases only one unit of the product. In the buyer agrees to purchase, then the payoffs to the players are

$$U^X(a_n, w_n, \theta) = \theta a_n - w_n, \tag{32}$$

$$U^Y(a_n, w_n, \theta) = w_n - \frac{\beta(a_n)^2}{2}. \tag{33}$$

Notice that the buyers' utility from the good depends on two parameters: the quality of the product, which is specific to the stage or generation n, and the underlying value parameter θ, which is invariant across the various offers of product qualities. As before, we assume that in every generation n, x_n receives a private signal $s_n \in \{H, L\}$ regarding θ with the probability $\alpha \equiv g_1^H = g_0^L > \frac{1}{2}$. The optimal contracting problem is therefore

$$\mathbf{C}_n^*(\mu_n) \in \arg \max_{\langle a_n^j, w_n^j \rangle_{j=L}^H \in \mathcal{A} \times \mathcal{W}_n} \sum_{j=L}^{H} \gamma_n^j [w_n^j - \frac{\beta(a_n^j)^2}{2}], \tag{34}$$

subject to the buyer's IC and IR constraints:

$$p_n^j a_n^j - w_n^j \geq p_n^j a_n^k - w_n^k, \; j, k \in \{H, L\}, \tag{35}$$

$$p_n^j a_n^j - w_n^j \geq 0, \; j \in \{H, L\}. \tag{36}$$

It is well known that inducing separation thus requires distorting the price-quality offer for the low-type consumer relative to the complete information optimal offer; in the extreme, it may be optimal to "price out" such consumer types by setting $\langle a_n^{L*}, w_n^{L*}\rangle = \langle 0, 0\rangle$ (Mussa and Rosen 1978). The following Proposition characterizes the optimal contract $\mathbf{C}_n^*(\mu_n)$.

Proposition 5. For any $0 < \mu < 1$ and for any $\phi_n, n \in \mathcal{N}$ (where $\mu(\alpha) \equiv \max(0, \frac{\alpha^2+\alpha-1}{2\alpha-1})$)[26]

1. if $\mu_n \in (\mu(\alpha), 1)$, then $\mathbf{C}_n^*(\mu_n)$ is separating with

$$\left\langle a_n^H = \frac{p_n^H}{\beta}, w_n^H = p_n^H a_n^H - a_n^L\left(p_n^H - p_n^L\right)\right\rangle, \tag{37}$$

$$\left\langle a_n^L = \frac{p_n^L}{\beta} - \frac{\gamma_n^H\left(p_n^H - p_n^L\right)}{\beta\left(1-\gamma_n^H\right)}, w_n^L = p_n^L a_n^L\right\rangle, \text{ and} \tag{38}$$

2. if $\mu_n \in (0, \mu(\alpha))$, then $\mathbf{C}_n^*(\mu_n)$ is separating with the pricing out of the low-type consumer:

$$\left\langle a_n^H = \frac{p_n^H}{\beta}, w_n^H = p_n^H a_n^H\right\rangle, \left\langle a_n^L = 0, w_n^L = 0\right\rangle. \tag{39}$$

Proposition 5 indicates that separation is optimal for all posterior beliefs on the underlying state.

Corollary 2. Learning is asymptotically complete along the σ^* specified in Proposition 5.

Figure 4 presents two random paths simulated when the true productivity state is low, that is, $\theta = 0$. For each path, the left-hand-side graph depicts the evolution of beliefs, and the right-hand-side graph depicts the menu of product qualities (a_n^L, a_n^H) offered by the optimal contract. One can see that although the quality levels offered converge to each other over time as uncertainty on the true product value is removed, pooling is never attained. Consequently, learning is asymptotically complete and the efficient product quality is offered in the limit.

From (32), we see that the preferences for both the buyers and the sellers satisfy concavity and twice differentiability in a. Moreover, $u_1^X(a_n, \theta) = \theta$, so that $u_1^X(a_n, 1) > u_1^X(a_n, 0)$, and hence the SCP applies strictly for the buyers, whereas the SCP applies weakly for the sellers. And the MLRP holds because $\alpha > \frac{1}{2}$. Hence, consistent with Proposition 4,

[26] We remark that $\mu(\alpha) = 0$ if $\alpha \in (0.5, \frac{\sqrt{5}-1}{2})$ and $\mu(\alpha) \in (0, 1)$ if $\alpha \in (\frac{\sqrt{5}-1}{2}, 1)$.

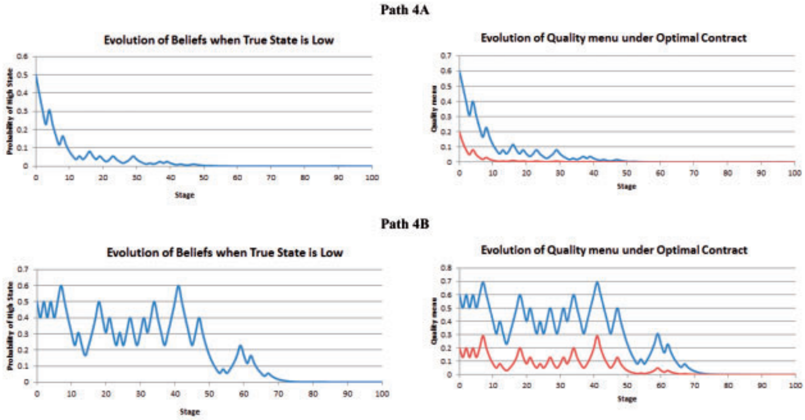

Figure 4
Learning unknown product innovation value (when true value is low).
Two random paths are simulated for the low productivity state, that is, $\theta = 0$. For each path, the left-hand-side graph depicts the evolution of beliefs, and the right-hand-side graph depicts the menu of product qualities offered by the optimal contract (a_n^L, a_n^H) . The parameters are($\alpha = 0.6$, $\beta = 1$).

the IR constraint is binding for buyers with the low signal, so that $w_n^L = p_n^L a_n^L$. As in the literature on price discrimination (see, e.g., Varian 1989), there are no meaningful constraints on the monetary transfers, namely, product prices, other than those given by the IR constraints. In sum, this example also satisfies the sufficient conditions for complete learning in Theorem 2.

2.3 Robustness of incentive efficient information cascades

In this section we examine the role of unbounded beliefs in inducing informational cascades with efficient incentive contracts. It has been pointed out in the herding literature that there can be asymptotically complete learning when beliefs are unbounded, even if there is herding when beliefs are bounded (Smith and Sorensen 2001; Acemoglu et al. 2010).[27] Intuitively, a sufficiently precise signal at some point along the equilibrium path will lead to an action that is sensitive to agent's private information, thereby disturbing the informational cascade. But, in our

[27] When the support of the private beliefs is $[\underline{p}, \overline{p}] \subseteq [0, 1]$, we will say that private beliefs are bounded if $\underline{p} > 0$ and $\overline{p} < 1$; but they are unbounded if $[\underline{p}, \overline{p}] = [0, 1]$. In particular, we will say that private beliefs are bounded and unbounded if, respectively,

$$\begin{aligned} \inf_{s \in \mathcal{S}} \frac{g_1(s)}{g_0(s)} &= \xi^- \ \& \ \sup_{s \in \mathcal{S}} \frac{g_1(s)}{g_0(s)} = \xi^+, \\ \inf_{s \in \mathcal{S}} \frac{g_1(s)}{g_0(s)} &= 0 \ \& \ \sup_{s \in \mathcal{S}} \frac{g_1(s)}{g_0(s)} = \infty. \end{aligned} \tag{40}$$

setting the sensitivity of actions to private information depends on the optimal stage contract. Thus, the question of interest is whether the optimality of a pooling contract with bounded beliefs will unravel if there are unbounded beliefs?

Notice that the sufficient conditions for asymptotically complete learning apply independent of the range of the posterior beliefs; that is, they hold for both bounded and unbounded beliefs (cf. (40)). Namely, in the case that signals are binary it is critical that signals are strictly informative, but not whether or not beliefs are bounded. Basically, the issue is not whether the type-X agent has arbitrarily high or low private (posterior) beliefs on θ; rather, the question is whether this agent has the incentives to reveal such information to the type-Y agent?

According to the equilibrium detailed in Proposition 3 there is incomplete learning of industry productivity even when the precision of signals become perfectly precise in the limit. In particular, the precision of the signal is infinite when $\frac{g_1^H}{g_0^H} = \frac{\alpha}{1-\alpha}$ becomes unbounded, that is, $\alpha \uparrow 1$. However, we have (for $\rho > 0$)

$$\lim_{\alpha\uparrow 1}\left(\lim_{\mu\to 0} Z(\mu)\right) = \lim_{\alpha\uparrow 1}\left(\lim_{\mu\to 1} Z(\mu)\right) = 1. \tag{41}$$

That is, pooling in the extreme region of posterior beliefs is robust to unbounded private beliefs when a high or low signal becomes almost perfectly revealing of the true state θ. Moreover, even at the point $\alpha = 1$ (that is, perfectly informative signals) incomplete learning is possible, namely, the optimal contract becomes one of pooling once expectations on the state of productivity are sufficiently low.[28] This illustrates the situation in which unbounded private beliefs do not lead to asymptotically complete learning because of the conflict of interest between the two parties. On the other hand, bounded beliefs do not imply incomplete learning in the limit as evident by the example for complete learning considered in Section 2.2. Thus, although more informative signals might increase the likelihood of an optimal separating stage contract (cf. Corollary 1) it does not imply complete learning, and low signal precision also does not imply incomplete learning.

3. Summary and Conclusions

We analyze the conditions for efficient dynamic aggregation of information when there is observational learning with endogenous design of incentive contracts, a problem that has received limited attention in the literature. Although the aggregation of dispersed information has been a central concern in the design of economic and political institutions, the

[28] Namely, when $\mu_n < \left(\frac{\varphi}{R-\varphi}\right)\rho$.

efficiency of information aggregation largely has been examined in an individualistic setting in which individual agents choose their decisions in some optimal fashion based on their private information. This is the setting, for example, in the Arrow-Debreu framework and in the recent literature on observational learning, which directs attention to the possibility that agents completely ignore their useful private information and herd on wrong decisions if they make decisions sequentially after observing decisions by other agents. However, decisions by informed agents often are not taken in isolation but involve interactions with other uninformed agents; these interactions are governed by contractual relationships that provide incentives for information dissemination. Moreover, there is a dynamic informational externality because the information dissemination at any stage influences the incentive contract design in the future and thereby affects the dynamic aggregation of dispersed information.

Although, intuitively, one expects that the ability to design incentives for truthful communication of private information should make (asymptotically) complete learning more likely, somewhat surprisingly we find that in plausible economic environments conflicts of interest among group members can rule out any information aggregation along the equilibrium path and there may be information cascades even with unbounded beliefs and rich action spaces. Alternatively, information can be asymptotically complete even with bounded (private) beliefs. The crucial determinant of information aggregation when decisions are made sequentially by collections of agents through incentive contracting is not the information content of the signals per se but whether it is incentive efficient to induce signals from informed agents along the equilibrium path.

We derive sufficient conditions on preferences and signal structures that ensure complete learning in the long run. These conditions are related (but are not equivalent) to conditions used in the optimal contracting literatures for adverse selection and moral hazard, even though there are no unobservable actions in our model. In particular, to guarantee complete learning asymptotically, one requires not only the Spence-Riley single crossing property but also the monotone hazard rate property of the posterior beliefs with the monotone and concave likelihood ratio properties of signals (conditional on the state). We illustrate the application of these conditions to a model of learning unknown product quality through intertemporal price-quality discrimination.

Our study highlights the bidirectional interaction between institutional or contract design and observational learning. That is, institutional design is influenced by observations of previous decisions or outcomes from similar existing institutions but the efficiency of information aggregation—or the asymptotic learning outcomes—depend on efficiency of the institutions to address the conflicts of interests amongst agents. In particular, we show that information aggregation and efficient resource allocation can be challenged when institutions do not incorporate the dynamic informational externality that arises from their influence on future information transmission and learning. Analyzing the design of institutions that function optimally (in the sense of incentive efficiency), while also incorporating this externality is an important topic for future research.

Appendix A

Proof of Proposition 1. By definition each σ^* specifies for each $n \in \mathcal{N}$ the decision rule $\delta_n^*(m_n, h_n)$ and the corresponding support $\Delta_n^*(s, h_n)$ of the conditional distribution $Q(\delta_n^*(m_n, h_n))$ of decision d_n. Now, for any decision at stage $n \in \mathcal{N}$, $d_n \equiv (a_n, w_n) \in \mathcal{A} \times \mathcal{W}_n$, define $\Omega_n^*(d_n; h_n) = \{s \in \mathcal{S} \mid d_n \in \Delta_n^*(s, h_n)\}$. Then by Bayes' rule,

$$\mu_{n+1} = \Pr(\theta = 1 \mid d_n, h_n) = \frac{\mu_n \left(\int\limits_{\Omega_n^*(d_n;h_n)} g_1(s)ds \right)}{\mu_n \left(\int\limits_{\Omega_n^*(d_n;h_n)} g_1(s)ds \right) + (1-\mu_n) \left(\int\limits_{\Omega_n^*(d_n;h_n)} g_0(s)ds \right)}. \tag{A.1}$$

But if $\mathbf{C}_n^*(h_n) = \mathbf{C}_n^{P*}(h_n)$, then by definition $\Omega_n^*(d_n; h_n) = \mathcal{S}$. Hence, from (A.1), it follows that

$$\mu_{n+1} = \Pr(\theta = 1 \mid d_n, h_n) = \frac{\mu_n \left(\int\limits_{\mathcal{S}} g_1(s)ds \right)}{\mu_n \left(\int\limits_{\mathcal{S}} g_1(s)ds \right) + (1-\mu_n) \left(\int\limits_{\mathcal{S}} g_0(s)ds \right)} \tag{A.2}$$

$$= \frac{\mu_n}{\mu_n + (1-\mu_n)} = \mu_n.$$

Because this applies for any deterministic $\delta_n^*(m_n, h_n)$, it follows that the statement is true for any randomized decision rule as well. ∎

Proof of Proposition 2. Note that if $\mathbf{C}_n^* = \mathbf{C}_n^{S*}(h_n)$, then by definition, $\Omega_n^*(d_n; h_n) \subset \mathcal{S}$ for every $d_n \in \mathcal{A} \times \mathcal{W}_n$. And because the probability measures $G_1(s)$ and $G_0(s)$ are nonidentical (conditional on θ) by assumption, it follows from (A.1) and (A.2) that for any h_n, $n \in \mathcal{N}$,

$$\Pr(\mu_{n+1}^{\sigma^*} = \mu_n^{\sigma^*}) = \begin{cases} 1 & \text{if } \mathbf{C}_n^*(h_n) = \mathbf{C}_n^{P*}(h_n) \\ 0 & \text{if } \mathbf{C}_n^*(h_n) = \mathbf{C}_n^{S*}(h_n). \end{cases} \tag{A.3}$$

Thus, if h_n, $n \in \mathcal{N}$, is such that $\mu_n^{\sigma^*} < 1$, for $\mathbf{C}_n^*(h_n) = \mathbf{C}_n^{P*}(h_n)$, then $\mu_{n+i}^{\sigma^*} = \mu_n^{\sigma^*}$, $i = 1, 2, \ldots$, with probability 1. Hence, the first part of the Proposition then follows from the definition of asymptotic complete learning (cf. (8)), because

$$\lim_{n\to\infty} \Pr(\mu_n^{\sigma^*} = \theta \mid \theta) < 1. \tag{A.4}$$

Next, suppose that $\mathbf{C}_n^*(h_n) = \mathbf{C}_n^{S*}(h_n)$ for each $n \in \mathcal{N}$. Then conditional on θ, the sequence of posterior beliefs $\{\mu_n^{\sigma^*}\}$ is independent under the probability measures $G_1(s)$ and $G_0(s)$. Furthermore, by the definition of conditional expectations, $\{\mu_n^{\sigma^*}\}$ are Martingales. It follows from the Martingale convergence theorem that $\{\mu_n^{\sigma^*}\}$ must converge to some integrable μ^{σ^*}, and because $G_1(s)$ and $G_0(s)$ are mutually singular (see, e.g., Billingsley 1979, 409–410), it must be the case that $\mu^{\sigma^*} = 1$ if $\theta = 1$ (or $\mu^{\sigma^*} = 0$ if $\theta = 0$). ■

Proof of Proposition 3. The objective function is

$$\mathbf{OBJ} = \gamma_n^H\big(\psi_n(H)f(a_n^H) - Ra_n^H\big) + \big(1 - \gamma_n^H\big)\big(\psi_n(L)f(a_n^L) - Ra_n^L - \varphi\big(a_n^H - a_n^L\big)\big).$$

The first-order conditions are

$$\frac{\partial \mathbf{OBJ}}{\partial a_n^H} = \gamma_n^H\big(\psi_n(H)f'(a_n^H) - R\big) - \big(1 - \gamma_n^H\big)\varphi = 0$$
$$\frac{\partial \mathbf{OBJ}}{\partial a_n^L} = \psi_n(L)f'(a_n^L) - R + \varphi = 0.$$

Separating is feasible for intermediate μ_n. To see this, note that the interior levels of investment in the low and high states (as provided by the first-order conditions above) coincide when

$$\frac{R - \varphi}{\psi_n(L)} = \frac{1}{\psi_n(H)}\left[R + \left(\frac{1 - \gamma_n^H}{\gamma_n^H}\right)\varphi\right] = \frac{R}{\mu_n(\nu_h - \nu_\ell) + \nu_\ell}.$$

Thus, separating is feasible when

$$Z(\mu_n) \equiv \frac{\mu_n + \rho}{\left[\frac{(1-\alpha)\mu_n}{(1-\alpha)\mu_n + \alpha(1-\mu_n)}\right] + \rho} > \frac{R}{R - \varphi}. \tag{A.5}$$

The conclusions of the proposition then follow from the foregoing, where $\overline{\varphi} = \left(\frac{Z^{\max} - 1}{Z^{\max}}\right)R$ and $Z^{\max} = \max(\mu \in (0, 1) : Z(\mu))$. Pooling occurs at the extreme levels of μ_n because $Z(\mu_n)$ approaches the value one as μ_n becomes extreme. Formally,

$$\lim_{\mu \to 0} Z(\mu) = \lim_{\mu \to 1} Z(\mu) = 1 < \frac{R}{R - \varphi}. \tag{A.6}$$

■

Proof of Theorem 1. Suppose first that $\mu_0 \notin (\mu^-, \mu^+)$. Then, from Proposition 3, $n^* = 1$. Meanwhile, suppose that $\mu_0 \in (\mu^-, \mu^+)$ and the true productivity state is $\theta = 0$. If there were separating contracts for all n (that is, n^* is undefined), then it follows from Proposition 2 that $\lim_{n\to\infty} \mu_n^{\sigma^*} = 0$. Then define $n^* = \inf_n\{n \mid \mu_n^{\sigma^*} < \mu^-\}$, which is well defined because (for a specified set of model parameters) μ^- is a given positive real number. But by Proposition 3, $\mu_{n+i}^{\sigma^*} = \mu_n^{\sigma^*}$, $i = 1, 2, ..$, for all $n \geq n^*$, which contradicts the hypothesis that $\lim_{n\to\infty} \mu_n^{\sigma^*} = 0$. Similarly, if $\mu_0 \in (\mu^-, \mu^+)$ and the true productivity state is $\theta = 1$, then define $n^* = \inf_n\{n \mid \mu_n^{\sigma^*} > \mu^+\}$. ■

Proof of Corollary 1. This follows from Proposition 3 and the implicit function theorem. ■

Proof of Proposition 4. We first show that if the incentive compatible constraint for the low type is binding, then the corresponding incentive constraint for the high type is satisfied. Now define for $j, k \in \{H, L\}$

$$\Delta_n^{j,k} \equiv p_n^j u^X(a_n^j, 1) + (1 - p_n^j)u^X(a_n^j, 0) + w_n^j$$
$$- \big[p_n^j u^X(a_n^k, 1) + (1 - p_n^j)u^X(a_n^k, 0) + w_n^k\big].$$

Note that if the IC constraint is binding for the low type then $\Delta_n^{L,H} = 0$. Hence, $\Delta_n^{H,L} \geq 0$ if $\Delta_n^{H,L} + \Delta_n^{L,H} \geq 0$. However,

$$\Delta_n^{H,L} + \Delta_n^{L,H} = (p_n^H - p_n^L)\big[(u^X(a_n^H, 1) - u^X(a_n^H, 0)) - \big(u^X(a_n^L, 1) - u^X(a_n^L, 0)\big)\big]. \tag{A.7}$$

But $(p_n^H - p_n^L) > 0$ from the MLRP and the expression inside the brackets in (A.7) can be rewritten as

$$(u^X(a_n^H, 1) - u^X(a_n^L, 1)) - (u^X(a_n^H, 0) - u^X(a_n^L, 0)) \geq 0$$

using the SCP. Thus, if the IC constraint binds for the low type, it is satisfied for the high type in the presence of SCP and MLRP, and therefore we can reduce the problem and disregard the IC constraint for the high type, while solving for the optimal contract. That is, the candidate contract is optimal in the reduced problem (that is, without the IC constraint for the high type) as well.

Now, in the reduced problem, suppose that the IR constraint is nonbinding for the low type, that is,

$$p_n^L u^X(a_n^L, 1) + (1 - p_n^L) u^X(a_n^L, 0) + w_n^L > 0.$$

Note that this implies also that the IR constraint for type-H is nonbinding because $\Delta_n^{H,L} \geq 0$. Thus, the principal can reduce w_n^j, $j = H, L$ by the same small amount $\epsilon > 0$. Consequently, in the reduced problem, this leaves the IR constraints and the IC constraint of the low type unaffected, which contradicts the assumption that the candidate contract is optimal. ■

Proof of Theorem 2. Using Proposition 4, we find

$$-w_n^L = p_n^L u^X(a_n^L, 1) + (1 - p_n^L) u^X(a_n^L, 0)$$
$$-w_n^H = p_n^L u^X(a_n^H, 1) + (1 - p_n^L) u^X(a_n^H, 0).$$

Substituting these in the objective function can eliminate the transfers $w_n^j, j = H, L$, and we write the optimal contracting problem as

$$\max_{\langle a_n^j \rangle_{j=L}^H \in \mathcal{A}} \{\gamma_n^H \big[p_n^H u^Y(a_n^H, 1) + (1 - p_n^H) u^Y(a_n^H, 0) + p_n^L u^X(a_n^H, 1) + (1 - p_n^L) u^X(a_n^H, 0)\big]$$
$$+ (1 - \gamma_n^H)[\sum_{i=X}^{Y} p_n^L u^i(a_n^L, 1) + (1 - p_n^L) u^i(a_n^L, 0)]\}. \tag{A.8}$$

Under the assumed conditions on $u^i(a_n^j, \cdot), j = H, L$, the objective function (A.8) is concave, and the first-order conditions for interior optima are

$$\sum_{i=X}^{Y} \big[p_n^H u_1^i(a_n^H, 1) + (1 - p_n^H) u_1^i(a_n^H, 0)\big] = 0, \tag{A.9}$$

$$\sum_{i=X}^{Y} \big[p_n^L u_1^i(a_n^L, 1) + (1 - p_n^L) u_1^i(a_n^L, 0)\big] = 0. \tag{A.10}$$

Suppose that $a_n^{H*} = a_n^{L*} = a$. Then subtracting (A.10) (evaluated at $a_n^L = a$) from (A.9) (also evaluated at $a_n^H = a$), we obtain

$$\frac{\partial OBJ}{\partial a_n^H} \mid_{a_n^H = a} = (p_n^H - p_n^L)[\sum_{i=X}^{Y} (u_1^i(a, 1) - u_1^i(a, 0))] > 0 \tag{A.11}$$

using the SCP (that holds for both players, but applies strictly for at least one player) and the MLRP. Thus, $a_n^{H*} > a_n^{L*}$ in the optimal contract. ■

Proof of Theorem 3. As usual (Mirrlees [1971] and onward), we use the indirect utility function (with truth-telling) to eliminate transfers in the optimal decision rule. Put

$$J_n^X(s; \mu_n, \delta_n) \equiv \max_{s' \in \mathcal{S}} V_n^X(s, s'; \mu_n, \delta_n) = V_n^X(s, s; \mu_n, \delta_n). \tag{A.12}$$

If the SCP applies, then a decision rule $\delta_n(s)$ is incentive compatible if and only if $a_n(\cdot)$ is a monotone increasing function, that is, $\dot{a}_n(s) \geq 0$ (a.e.). Using (A.12) and applying the envelope theorem yields

$$\frac{dJ_n^X(s;\mu_n,\delta_n)}{ds} \equiv \dot{J}_n^X(a(s),s;\mu_n) = \dot{p}_n(s;\mu_n)[u^X(a_n(s),1) - u^X(a_n(s),0)]. \tag{A.13}$$

But the MLRP implies that $\dot{p}_n(s;\mu_n,\delta_n) > 0$. This, along with the fact that u^X is increasing in θ, implies that $\dot{J}_n^X(a(s),s;\mu_n) > 0$. It follows from (A.13) that the participation constraint for the type-X agent binds only for $s_n = s^L$, and from the fundamental theorem of calculus,

$$J_n^X(s;\mu_n,\delta_n) = \int_{s^L}^{s} \dot{J}_n^X(a(r),r;\mu_n)dr. \tag{A.14}$$

Meanwhile, the expected utility of y_n with the decision rule $\delta_n(s)$ is

$$V_n^Y(s;\mu_n,\delta_n) = \left[p_n(s;\mu_n)U^Y(a_n(s),w_n(s),1) + (1-p_n(s;\mu_n))U^Y(a_n(s),w_n(s),0)\right]. \tag{A.15}$$

To derive the sufficient conditions for $a_n(\cdot)$ to be monotone, we examine the relaxed problem:

$$\max_{\delta_n(\cdot)\in\mathcal{A}\times\mathcal{W}} \int_{\mathcal{S}} V_n^Y(s;\mu_n,\delta_n)\gamma_n(s;\mu_n)ds, \text{ s.t. (A.14)}. \tag{A.16}$$

Now put

$$K_n^i(a_n(s),s;\mu_n) \equiv p_n(s;\mu_n)u^i(a_n(s),1) + (1-p_n(s;\mu_n))u^i(a_n(s),0),\ i = X, Y. \tag{A.17}$$

This formulation allows us to eliminate transfers (that is, $w_n(s)$), and we write (from (A.12), (A.15), and (A.17))

$$V_n^Y(s;\mu_n,\delta_n) = K_n^Y(a_n(s),s;\mu_n) + K_n^X(a_n(s),s;\mu_n) - J_n^X(s;\mu_n,\delta_n). \tag{A.18}$$

Next, by substituting the incentive compatibility constraint (A.14) into (A.18), we can rewrite the relaxed problem as

$$\begin{aligned}
&\max_{a_n(\cdot)\in\mathcal{A}} \int_{\mathcal{S}} \left[K_n^Y(a_n(s),s;\mu_n) + K_n^X(a_n(s),s;\mu_n) - \int_{s^L}^{s} \dot{J}_n^X(a(r),r;\mu_n)dr\right]\gamma_n(s;\mu_n)ds \\
&= \max_{a_n(\cdot)\in\mathcal{A}} \int_{\mathcal{S}} \left[\begin{array}{l} K_n^Y(a_n(s),s;\mu_n) + K_n^X(a_n(s),s;\mu_n) - \\ \dfrac{1-\Gamma_n(s;\mu_n)}{\gamma_n(s;\mu_n)} \dot{J}_n^X(a_n(s),s;\mu_n) \end{array}\right]\gamma_n(s;\mu_n)ds
\end{aligned} \tag{A19}$$

(applying integration by parts). The optimality condition for the optimal action is therefore

$$\sum_{i=X}^{Y} \frac{\partial K_n^i(a_n(s),s;\mu_n)}{\partial a_n} = \frac{1-\Gamma_n(s;\mu_n)}{\gamma_n(s;\mu_n)} \frac{\partial \dot{J}_n^X(a_n(s),s;\mu_n)}{\partial a_n}, \tag{A.20}$$

where, from (A.17),

$$\sum_{i=X}^{Y} \frac{\partial K_n^i(a_n(s),s;\mu_n)}{\partial a_n} = \sum_{i=X}^{Y} \left[p_n(s;\mu_n)u_1^i(a_n(s),1) + (1-p_n(s;\mu_n))u_1^i(a_n(s),0)\right].$$

A straightforward application of the implicit function theorem on (A.20) yields

$$\begin{aligned}
\dot{a}_n^*(s) \propto \sum_{i=X}^{Y} \frac{\partial^2 K_n^i(a_n(s),s;\mu_n)}{\partial a_n \partial s} - \frac{\partial \dfrac{1-\Gamma_n(s;\mu_n)}{\gamma_n(s;\mu_n)}}{\partial s} \frac{\partial \dot{J}_n^X(a_n(s),s;\mu_n)}{\partial a_n} \\
- \frac{1-\Gamma_n(s;\mu_n)}{\gamma_n(s;\mu_n)} \frac{\partial^2 \dot{J}_n^X(a_n(s),s;\mu_n)}{\partial a_n \partial s}.
\end{aligned} \tag{A.21}$$

But if the MLRP applies and the SCP applies for both players (and strictly for at least one player), then

$$\sum_{i=X}^{Y} \frac{\partial^2 K_n^i(a_n(s), s; \mu_n)}{\partial a_n \partial s} = \sum_{i=X}^{Y} \dot{p}_n(s; \mu_n)[u_1^i(a_n(s), 1) - u_1^i(a_n(s), 0)] > 0. \tag{A.22}$$

Next, if the MLRP and SCP apply for $i = X$, then

$$\frac{\partial \dot{J}_n^X(a_n(s), s; \mu_n)}{\partial a_n} = \dot{p}_n(s; \mu_n)[u_1^X(a_n(s), 1) - u_1^X(a_n(s), 0)] \geq 0. \tag{A.23}$$

Hence, in addition to MLRP and the SCP condition, if the PMHRP also applies, then

$$\frac{\partial \frac{1-\Gamma_n(s;\mu_n)}{\gamma_n(s;\mu_n)}}{\partial s} \frac{\partial \dot{J}_n^X(a_n(s), s; \mu_n)}{\partial a_n} \leq 0.$$

Finally, if the CLRP holds, then $\ddot{p}_n(s; \mu_n) \leq 0$, and hence,

$$\frac{\partial^2 \dot{J}_n^X(a_n(s), s; \mu_n)}{\partial a_n \partial s} = \ddot{p}_n(s; \mu_n)[u^X(a_n(s), 1) - u^X(a_n(s), 0)] \leq 0. \tag{A.24}$$

It then follows from (A.21)-(A.24) that $\dot{a}_n^*(s) > 0$ for each s in the interior of $\mathcal{S}$. Hence, $\delta_n^*(s, h_n)$ is separating. ■

Proof of Proposition 5. Because the individual rationality constraint is binding for the low type consumer, we have, $p_n^L a_n^L = w_n^L$; and becase the incentive compatibility constraints are only binding downwards, $a_n^H(p_n^H - p_n^L) + w_n^L = w_n^H$. Substituting these conditions in the objective function and ignoring, for the moment, the nonnegativity restrictions on a_n, the optimal contract maximizes:

$$\mathbf{OBJ} = \gamma_n^H\left(p_n^H a_n^H - p_n^H a_n^L + p_n^L a_n^L - \frac{\beta (a_n^H)^2}{2}\right) + \left(1 - \gamma_n^H\right)\left(p_n^L a_n^L - \frac{\beta (a_n^L)^2}{2}\right). \tag{A.25}$$

The first-order conditions are

$$\begin{aligned} \frac{\partial \mathbf{OBJ}}{\partial a_n^H} &= \gamma_n^H\left(p_n^H - \beta a_n^H\right) = 0 \\ \frac{\partial \mathbf{OBJ}}{\partial a_n^L} &= -\gamma_n^H\left(p_n^H - p_n^L\right) + \left(1 - \gamma_n^H\right)\left(p_n^L - \beta a_n^L\right) = 0. \end{aligned} \tag{A.26}$$

Solving these and acknowledging the nonnegativity constraints on a_n, we obtain the following characterization of the optimal contract:

$$\begin{aligned} a_n^H &= \frac{p_n^H}{\beta} \\ a_n^L &= \begin{cases} \dfrac{p_n^L}{\beta} - \dfrac{\gamma_n^H\left(p_n^H - p_n^L\right)}{\beta\left(1 - \gamma_n^H\right)} & \text{if } p_n^L > \gamma_n^H p_n^H \\ 0 & \text{if } p_n^L < \gamma_n^H p_n^H \end{cases} \\ p_n^L a_n^L - w_n^L &= 0 \\ p_n^H a_n^H - w_n^H &= p_n^H a_n^L - w_n^L. \end{aligned} \tag{A27}$$

Then substituting (12) and (13) in (A.27) and simplifying yields the conclusions of the proposition. ■

Appendix B: Learning Unkown Investment Productivity with Continuous Signals

Here, we consider the agency moel of Section 1.1 when the distribution of signals $G_1(s)$ and $G_0(s)$ is continuous with the support $\mathcal{S} = [s^L, s^H]$. Here, the private beliefs that $\theta = 1$ of x_n given μ_n and the signal s_n are given by (3) and $\gamma_n(s_n = s; h_n) = g_1(s)\mu_n + g_0(s)(1 - \mu_n)$. The bilateral contract between x_n and y_n specifies a menu of investment levels and wage payments $\langle a_n(\cdot), w_n(\cdot)\rangle : \mathcal{S} \to \mathcal{R}_+^2$. In the optimal separating contract the upward incentive constraint is binding. Hence, the optimal contract solves

$$\mathbf{C}_n^*(\mu_n) \in \arg \max_{a_n(s), w_n(s)} \int \gamma_n(s; h_n)\big[\psi_n(s)f(a_n(s)) - Ra_n(s) - w_n(s)\big]ds,$$

subject to

$$w_n(s) + \varphi a_n(s) = w_n(s') + \varphi a_n(s') \text{ for all } s, s' \in \mathcal{S}. \tag{B.1}$$

(where $\psi_n(s)$ is defined in (14)). Because wages in (B.1) are determined by highest action a_n, one can show that under the optimal contract there exists a maximum level of investment $\overline{a}_n$ such that:

$$\langle a_n(s), w_n(s)\rangle = \begin{cases} \langle \overline{a}_n, 0\rangle \text{ for } s \in [\overline{s}(\overline{a}_n), s^H] \\ \langle a_n^*(s), \varphi\big(\overline{a}_n - a_n^*(s)\big)\rangle \text{ for } s \in [s^L, \overline{s}(\overline{a}_n)], \end{cases} \tag{B.2}$$

where $a_n^*(s)$ is given by

$$f'(a_n^*(s)) = \frac{R - \varphi}{\psi_n(s)}, \; s \in [s^L, \overline{s}(\overline{a}_n)] \tag{B.3}$$

and $\overline{s}(\overline{a}_n)$ satisfies

$$\psi_n(\overline{s}(\overline{a}_n)) = \frac{R - \varphi}{f'(\overline{a}_n).} \tag{B.4}$$

One can now analyze the optimal $\overline{a}_n^*$, where $\overline{s}(\overline{a}_n)$ and $a_n^*(s)$ are given by (B.3) and (B.4). Pooling occurs when the solution is $\overline{a}_n = a_n^P$ defined by

$$f'(a_n^P)[E(\theta \mid \mu_n)(\nu_h - \nu_\ell) + \nu_\ell] - R = 0. \tag{B.5}$$

We conclude

Proposition 6. There exist $0 \leq \breve{\mu}^- < \breve{\mu}^+ \leq 1$ such that for any $\phi_n, n \in \mathcal{N}$:

1. if $\varphi \leq \check{\varphi}$ and $\mu_n \in \left(\breve{\mu}^-, \breve{\mu}^+\right)$, then $C_n^*(\mu_n) = C_n^{S*}(\mu_n)$ and is characterized by (B.2)-(B.4) for $\overline{a}_n^* > a_n^P$, and
2. if (1) $\varphi \leq \check{\varphi}$ and $\mu_n \notin \left(\breve{\mu}^-, \breve{\mu}^+\right)$ or (2) $\varphi > \check{\varphi}$, then $C_n^*(\mu_n) = C_n^{P*}(\mu_n)$ and is characterized by (B.5), where $w_n(s) = 0$ for $s \in S$.

Proof of Proposition 6. The optimization problem is

$$\mathbf{OBJ} = \max_{a_n(s), w_n(s)} \int \gamma_n(s; h_n)\big[\psi_n(s)f(a_n(s)) - Ra_n(s) - w_n(s)\big]ds,$$

subject to

$$w_n(s) + \varphi a_n(s) = w_n(s') + \varphi a_n(s') \text{ for all } s, s' \in [s^L, s^H]. \tag{B.6}$$

Because wages in (B.6) are determined by highest action a_n, let $\overline{a}_n \equiv \max_{s \in S}(a_n(s))$, and let the set of signals that correspond to the highest action be given by $\overline{S} \equiv \{s \in S : a_n(s) = \overline{a}_n\}$. Then it is optimal to set $w_n(s) = \varphi(\overline{a}_n - a_n(s))$ for all $s \in \mathcal{S}$ (that is, $w_n(s) = 0$ for all $s \in \overline{S}$).

Thus, we can write **OBJ** as

$$\mathbf{OBJ} = \max_{a_n(s), w_n(s)} \int_{\mathcal{S}-\overline{\mathcal{S}}} \gamma_n(s; h_n)[\psi_n(s)f(a_n(s)) - Ra_n(s) - \varphi(\overline{a}_n - a_n(s))]ds$$
$$+ \int_{\overline{\mathcal{S}}} \gamma_n(s; h_n)\big[\psi_n(s)f(\overline{a}_n) - R\overline{a}_n]ds.$$

Moreover, for any given $\overline{a}_n$, optimality requires that

$$f'(a_n^*(s)) = \frac{R-\varphi}{\psi_n(s)} \text{ for } s \in \mathcal{S}-\overline{\mathcal{S}}.$$

Because $\psi_n(s)$ is monotone in the signal s, it is optimal to set $\overline{\mathcal{S}} = (\overline{s}(\overline{a}_n), s_h)$, where $\overline{s}(\overline{a}_n)$ satisfies

$$\psi_n(\overline{s}(\overline{a}_n)) = \frac{R-\varphi}{f'(\overline{a}_n).}$$

Next, the level of $\overline{a}_n$ is given by the solution to the problem (where $\overline{s}(\overline{a}_n)$ and $a_n^*(s)$ are given by above)

$$\overline{a}_n \in \operatorname*{argmax}_a \int_{s^L}^{\overline{s}(\overline{a}_n)} \gamma_n(s; h_n)[\psi(s)f(a_n^*(s)) - Ra_n^*(s) - \varphi\big(\overline{a}_n - a_n^*(s))\big]ds$$
$$+ \int_{\overline{s}(\overline{a}_n)}^{s^H} \gamma_n(s; h_n)[\psi(s)f(\overline{a}_n) - R\overline{a}_n]ds.$$

The first-order conditions for optimality are

$$0 = -\varphi\int_{s^L}^{\overline{s}(\overline{a}_n)} \gamma_n(s; h_n)ds + \int_{\overline{s}(\overline{a}_n)}^{s^H} \gamma_n(s; h_n)[\psi(s)f'(\overline{a}_n) - R]ds.$$

The second-order conditions for optimality are

$$\begin{aligned} 0 &= -\varphi\overline{s}'(\overline{a}_n)\gamma_n(\overline{s}(\overline{a}_n); h_n) + \int_{\overline{s}(\overline{a}_n)}^{s^H} \gamma_n(s; h_n)[\psi(s)f''(\overline{a}_n)]ds \\ &\quad - \overline{s}'(\overline{a}_n)\gamma_n(\overline{s}(\overline{a}_n); h_n)[\psi(\overline{s}(\overline{a}_n))f'(\overline{a}_n) - R] \\ &= \int_{\overline{s}(\overline{a}_n)}^{s^H} \gamma_n(s; h_n)[\psi(s)f''(\overline{a}_n)]ds < 0. \end{aligned}$$

The first-order condition simplifies to

$$0 = -\varphi\frac{\Pr(s_n < \overline{s}(\overline{a}_n) \mid h_n)}{\Pr(s_n > \overline{s}(\overline{a}_n) \mid h_n)} + f'(\overline{a}_n)E(s_n \mid h_n, s_n > \overline{s}(\overline{a}_n)) - R.$$

By definition $f'(\overline{a}_n) = \frac{R-\varphi}{\psi_n(\overline{s}(\overline{a}_n))}$, and we can write the first-order condition for an interior solution as

$$R + \varphi\frac{\Pr(s_n < \overline{s}(\overline{a}_n) \mid h_n)}{\Pr(s_n > \overline{s}(\overline{a}_n) \mid h_n)} = [R-\varphi]\frac{E(s_n \mid h_n, s_n > \overline{s}(\overline{a}_n))}{\psi_n(\overline{s}(\overline{a}_n))}.$$

Pooling occurs at the corner solution $\overline{a}_n = a_n^{pool}$ defined by

$$[E(\theta \mid \mu_n)(\nu_h - \nu_\ell) + \nu_\ell]f'(a_n^{pool}) - R = 0$$

and is optimal when

$$\frac{R}{R-\varphi} > \frac{[E(\theta \mid \mu_n)(\nu_h - \nu_\ell) + \nu_\ell]f'(a_n^{pool})}{\psi_n(s^L)}$$

or

$$\frac{R}{R-\varphi} > \frac{(\nu_h - \nu_\ell)\mu_n + \nu_\ell}{(\nu_h - \nu_\ell)p_n(s^L;\mu_n) + \nu_\ell}$$
$$= \frac{[(\nu_h - \nu_\ell)\mu_n + \nu_\ell]\left[\left(g_1(s^L) - g_0(s^L)\right)\mu_n + g_0(s^L)\right]}{(\nu_h g_1(s^L) - \nu_\ell g_0(s^L))\mu_n + \nu_\ell g_0(s^L)}$$
$$\equiv \check{Z}(\mu_n).$$

Thus, $\lim_{\mu_n \to 0^+} \check{Z}(\mu_n) = \lim_{\mu_n \to 1^-} \check{Z}(\mu_n) = 1 < \frac{R}{R-\varphi}$, and pooling is optimal at extreme levels of expected productivity or when the agency conflict is sufficienlty severe, $\varphi > \check{\varphi}$. Let, $\check{\varphi} \equiv \frac{R(\check{Z}_{\max}-1)}{\check{Z}_{\max}}$ and $\check{Z}_{\max} \equiv \max\left(\mu \in (0,1) : \check{Z}(\mu)\right)$. ■

Hence, it follows from Proposition 6 that also with continuous signals learning cannot be asymptotically complete in this model because information aggregation ceases once posterior beliefs enter the pooling region.

Corollary 3. Learning is asymptotically incomplete along the σ^* specified in Proposition 6.

Proof of Corollary 3. Follows directly from the above analysis. ■

References

Acemoglu, D., M. Dahleh, I. Lobel, and A. Ozdaglar. 2010. Bayesian learning in social networks. *Review of Economic Studies* 77:1–34.

Bannerjee, V. 1992. A simple model of herd behavior. *Quarterly Journal of Economics* 107:797–817.

Bhattacharya, S. 1979. Imperfect information, dividend policy and ‘the bird in the hand’ fallacy. *Bell Journal of Economics* 10:259–70.

Bikhchandani, S., D. Hirshleifer, and I. Welch. 1992. A theory of fads, fashion, custom, and cultural change as informational cascades. *Journal of Political Economy* 100:992–1026.

Billingsley, P. 1979. *Probability and measure*. New York: Wiley.

Coffee, J. 2010. Ratings reform: The good, the bad, and the ugly. Working Paper, Columbia University.

DeMeza, D., and D. Webb. 1987. Too much investment: A problem of asymmetric information. *Quarterly Journal of Economics* 102:281–92.

Demougin, D., and D. Garvie. 1991. Contractual design with correlated information under limited liability. *RAND Journal of Economics* 22:477–89.

Fudenberg, D., and J. Tirole. 1991. *Game theory*. Cambridge: MIT Press.

Garber, P. 2001. *Famous first bubbles: The fundamentals of early manias*. Cambridge: MIT Press.

Greenspan, A. 1996. The challenge of central banking in a democratic society. *Francis Boyer Lecture at the American Enterprise Institute*. Washington, D.C.

Greenwald, B., J. Stiglitz, and A. Weiss. 1984. Informational imperfections in the capital market and macroeconomic fluctuations. *American Economic Review* 74:194–99.

Hadley, G., and M. Kemp. 1971. *Variational methods in economics*. Amsterdam: North-Holland.

Harris, M., and A. Raviv. 1996. The capital budgeting process: Incentives and information. *Journal of Finance* 51:1139–74.

Hart, O. 1995. *Firm, contracts and financial structure*. London: Oxford University Press.

Hart, O., and B. Holmstrom. 1987. The theory of contracts. In T. Bewley (Ed.), *Advances in Economic Theory*. Cambridge: Cambridge University Press.

Heinkel, R. 1982. A theory of capital structure relevance under imperfect information. *Journal of Finance* 37:1141–50.

Holmstrom, B., and R. Myerson. 1983. Efficient and durable decisions with incomplete information. *Econometrica* 51:1799–899.

Jackson, M. 2003. Mechanism theory. In U. Derigs (Ed.), *Optimization and operations research*. UK: EOLSS Publishers.

Jewitt, I. 1988. Justifying the first-order approach to principal-agent problems. *Econometrica* 56:1177–90.

John, K., and J. Williams. 1985. Dividends, dilution, and taxes: A signalling equilibrium. *Journal of Finance* 40:1053–70.

Juma, C. 2014. Complexity, innovation, and development: Schumpeter revisited. *Policy and Complex Systems* 1:4–21.

Kindleberger, C. 1978. *Manias, panics, and crashes*. New York: Wiley.

Kuhn, H. W. 1953. Extensive games and the problem of information. In H. W. Kuhn, and W. W. Tucker (Eds.), *Contributions to the theory of games*, vol. 2. Princeton: Princeton University Press.

Kumar, P. 2002. Price and quality discrimination in durable goods monopoly with resale trading. *International Journal of Industrial Organization* 30:1313–39.

———. 2006. Intertemporal price-quality discrimination and the Coase conjecture. *Journal of Mathematical Economics* 42:896–940.

Kumar, P., and N. Langberg. 2009. Corporate fraud and overinvestment in efficient capital markets. *RAND Journal of Economics* 40:144–72.

———. 2013. Information manipulation and rational investment booms and busts. *Journal of Monetary Economics* 60:408– 25.

Lee, I. 1993. On the convergence of informational cascades. *Journal of Economic Theory* 61:395–411.

Leland, H., and D. Pyle. 1977. Informational asymmetries, financial structure, and financial intermediation. *Journal of Finance* 32:371–87.

MacKay, C. 1980. *Extraordinary popular delusions and the madness of crowds*. New York: Three Rivers Press.

Martimort, D. 2006. Contract theory. In L. Blume and S. Durlauf (Eds.), *The new Palgrave*. UK: Macmillan.

Martin, A. 2009. A model of collateral, investment, and adverse selection. *Journal of Economic Theory* 144:1572–88.

Maskin, E., and J. Riley. 1984. Monopoly with incomplete information. *RAND Journal of Economics* 15: 171–96.

Milgrom, P., and C. Shannon. 1994. Monotone comparative statics. *Econometrica* 62:157–80.

Mirrlees, J. 1971. An exploration in the theory of optimal income taxation. *Review of Economic Studies* 38:175–203.

Mookherjee, D. 2006. Decentralization, hierarchies and incentives: A mechanism design perspective. *Journal of Economic Literature* 44:367–90.

Mussa, M., and S. Rosen. 1978. Monopoly and product quality. *Journal of Economic Theory* 18:301–17.

Myers, S., and N. Majluf. 1984. Corporate financing and investment decisions when firms have information that investors do not have. *Journal of Financial Economics* 13:187–222.

Myerson, R. 1981. Optimal auction design. *Mathematics of Operations Research* 6:58–63.

———. 2008. Perspectives on mechanism design in economic theory. *American Economic Review* 98: 586–603.

Ottaviani, M., and P. Sorensen. 2000. Herd behavior and investment: Comment. *American Economic Review* 90:695–704.

Rob, R. 1991. Learning and capacity expansion under demand uncertainty. *Review of Economic Studies* 58:655–75.

Rogerson, W. 1985. The first-order approach to principal-agent problems. *Econometrica* 53:1357–67.

Ross, S. 1977. The determination of financial structure: The incentive–signalling approach. *Bell Journal of Economics* 8:23–40.

Scharfstein, D., and J. Stein. 1990. Herd behavior and investment. *American Economic Review* 80:465–79.

Shiller, R. 2005. *Irrational exuberance*. Princeton: Princeton University Press.

Sidak, J. 2003. The Failure of good intentions: The Worldcom fraud and the collapse of American telecommunications after regulation. *Yale Journal on Regulation* 20:207–68.

Smith, L., and P. Sørensen. 2001. Pathological outcomes of observational learning. *Econometrica* 68: 371–98.

Stein, J. 1989. Efficient capital markets, inefficient firms: A model of myopic corporate behavior. *Quarterly Journal of Economics* 104:655–69.

Stiglitz, J., and A. Weiss. 1983. Incentive effects of terminations: Applications to the credit and labor markets. *American Economic Review* 73:912–27.

Stulz, R. 1990. Managerial discretion and optimal financing policies. *Journal of Financial Economics* 26: 3–27.

Varian, H. 1989. Price discrimination. In R. Schmalensee and R. Willig (Eds.), *Handbook of industrial organization*, vol. 1. Amsterdam: North-Holland.

Zeira, J. 1987. Investment as a process of search. *Journal of Political Economy* 95:204–10.

———. 1994. Informational cycles. *Review of Economic Studies* 61:31–44.