Voluntary Disclosures, Corporate Control, and Investment

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Received 22 November 2010; accepted 10 January 2012

ABSTRACT

We examine the valuation and capital allocation roles of voluntary disclosure when managers have private information regarding the firm’s investment opportunities, but an efficient market for corporate control influences their investment decisions. For managers with long-term stakes in the firm, the equilibrium disclosure region is two-tailed: only extreme good news and extreme bad news is disclosed in equilibrium. Moreover, the market’s stock price and investment responses to bad news disclosures are stronger than the responses to good news disclosures, which is consistent with the empirical evidence. We also find that myopic managers are more likely to withold bad news in good economic times when markets can independently assess expected investment returns.

∗University of Houston; †Texas A&M University. Accepted by Abbie Smith. We thank Abbie Smith (Editor) and an anonymous referee for many insightful comments. We also thank Ken Binmore, Ron Dye, Michael Fishman, Frank Gigler, Jonathan Glover, Ilan Guttman, Milton Harris, Thomas Hemmer, Chandra Kanodia, Pierre Liang, Karen Nelson, K. Ramesh, Sri Sridharan, Dan Weiss, workshop participants at Carnegie Mellon University, Hebrew University, IDC, the University of Oklahoma, University of Minnesota, Rice University, and Texas A&M University for helpful comments and discussions. The second author, Nisan Langberg, would like to acknowledge the generous support and assistance of Tel Aviv University and financial support from the Henry Crown Institute of Business Research in Israel during the period in which work on the paper was done and while on leave from the University of Houston.

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1. Introduction

Corporate managers are disciplined and monitored both by shareholders—activist shareholders and/or their elected board of directors—and by the market for corporate control. For example, monitoring top management through the evaluation of its investment and financial policy proposals is a primary function of the board (Van Den Berghe and Levrau [2004], Kumar and Sivaramakrishnan [2008]) and there is a large literature that emphasizes the activist monitoring and value-enhancing role of large shareholders (e.g., Shleifer and Vishny [1986]). Meanwhile, stock prices play a crucial resource allocation role by reflecting managerial inefficiencies, and signaling profitable opportunities for potential acquirers in the market for corporate control (Marris [1964], Manne [1965], Grossman and Hart [1986]). When effective, these control mechanisms discipline managers ex ante to refrain from actions—such as acquisitions, capital expenditures, or labor force adjustments—that are perceived to be value destroying and, when necessary, replace inefficient managements ex post (e.g., Tirole [2006]).

There is thus a natural incentive for managers to engage in strategic information disclosures in order to influence the efficacy of such control mechanisms. Indeed, ample anecdotal evidence suggests that managers often make costly attempts to rationalize their actions and influence the opinions or beliefs of shareholders, analysts, and investors through voluntary disclosures—via the media, letters to shareholders, advertisements, telemarketers’ phone calls, or costly road shows. In particular, managers often justify actions relating to expansions, scale backs, changes in payout policy, and their firms’ competitive strategies (e.g., Soter, Brigham, and Evanson [1996], Bergstein [2002], DeTienne and Hoopes [2004], Lang and Lundholm [2000]).

Such resource allocation implications of voluntary disclosure have received little attention in the literature, which, for the most part, has focused on the valuation implications of disclosure. This literature has established that

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1 More recently, a week prior to Consol Energy Inc.’s announcement of a $1.75 billion public offering on March 22, 2010, management disclosed information in support of its $3.48 billion proposed acquisition: “...As a result of the acquisition, on a pro forma basis, CONSOL Energy will be the largest, and among the fastest growing and lowest cost producers of natural gas in the Appalachian basin. Importantly, the acquisition will give CONSOL Energy a leading position in the strategic Marcellus Shale fairway by tripling its development assets to approximately 750,000 acres with the addition of Dominion’s approximately 500,000 Marcellus Shale acres in Pennsylvania and West Virginia...As we expand our natural gas production, we remain fully committed to utilizing state-of-the-art exploration and production techniques, which enable us to operate efficiently, safely and compatibly with the environment” (Bloomberg report available at http://www.bloomberg.com/apps/news?pid=conewstory&tkr=CXG:US&sid=aEMnbi8JOj2I). Similarly, in its annual financial reports, Yamaha Motors’ CEO explains the adverse business conditions that led to their exit from a number of business segments.

2 See Dye [2001], Verrecchia [2001] for excellent reviews of the voluntary disclosure literature.
managers strategically disclose news that boosts their firms’ stock prices, but withhold news otherwise (Dye [1985], Verrecchia [1983]). To our knowledge, there is little by way of theoretical work on this potential capital allocation role of voluntary disclosures. In this paper, we examine this role by analyzing managers’ disclosure strategies when the manager is governed by an activist shareholder (hereafter, AS).

Our main result is that, not only do managers voluntarily disclose good news to favorably influence the market in equilibrium, but, perhaps surprisingly, they also disclose bad news to achieve investment efficiency. Moreover, the equilibrium response to the disclosure of bad news (in terms of the stock price reaction and change in investment allocation) is stronger than the response to the disclosure of good news (holding fixed the information content); that is, the equilibrium response to the polar disclosure strategy of managers is consistent with an appearance of “overreaction” to bad news.

The literature on corporate investment suggests that managers need not always be driven by maximizing long-term shareholder value at all times in guiding capital allocation by the market (e.g., Stein [1989]). This conflict between shareholder and managerial investment objectives is of course aggravated by managers’ private information on the firms’ economic prospects. Thus, the effectiveness of control mechanisms is constrained by the quality of the information that is strategically transmitted by managers, that is, by the quality of the information that these mechanisms can act upon to ensure that investment levels are efficient (Grossman and Hart [1986], Kumar and Sivaramakrishnan [2008], Kumar and Langberg [2009]). Consequently, investment distortions may not be avoidable unless managers strategically choose to disclose their private information in order to aid capital allocation.

Questions immediately arise as to whether and when managers might engage in such disclosures. Under what circumstances would managers voluntarily disclose their private information if their disclosures (1) affect their firm’s stock price, and (2) affect the allocation of capital to the firm given the disciplining role of activist shareholders? In turn, how would the AS respond to such disclosures in terms of ensuring that capital allocation is efficient given the information disclosed? Can we characterize the market prices and corporate investment levels that emerge in equilibrium?

To address these issues, we consider a model in which a partially informed manager has private information about his/her firm’s prospects.

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3 We discuss the related literature subsequently.

4 Although we do not explicitly model the market for corporate control, our analysis and results also qualitatively apply to a setting in which such a market ensures the ex post efficiency of investment decisions (Kumar and Langberg [2009]).

5 In particular, managers might at times have a preference for larger investment levels (i.e., empire building as in Stulz [1990]), investment levels that resemble those of their industry peers (i.e., herding behavior as in Scharfstein and Stein [1990]), or lower investment levels that preserve the status quo (i.e., enjoying the quiet life as in Bertrand and Mullainathan [2003]).
(e.g., market share, growth in revenues, new contracts) and needs to direct capital allocation in the marketplace according to this information. The manager is controlled by a representative activist shareholder (AS) who is interested in the long-term value of the firm. Neither the AS nor the market knows whether or not the manager is informed and, as is standard in the voluntary disclosure literature, the informed manager can credibly disclose her information if she so chooses (Dye [1985], Verrecchia [1983, 2001]).

In addition to this disclosure, we allow the manager to send a (public) message to the AS—akin to the “cheap talk” message in Crawford and Sobel [1982]—to strategically influence prices and the level of investment chosen by the AS. Such a message serves as another channel of information transmission to the manager to influence the beliefs of the AS, and distinguishes our model from the canonical disclosure literature. More importantly, our model is different from Crawford and Sobel [1982] in that the manager has the option of credibly disclosing her information directly in addition to sending (cheap talk) messages. Clearly, if the manager decides to disclose her private information directly, then her cheap talk message would convey no useful information. However, we show that even when the manager chooses not to disclose her private information directly, the additional message still does not convey any information in equilibrium. Hence, we need only focus on the manager’s disclosure decision from the viewpoint of characterizing the equilibrium information transmission.

In our model, as long as managers care only about investment efficiency (such as when their fortunes are tied to long-term firm value), they will always disclose because this leads to first best investment levels. However, managerial incentives may be driven by considerations other than long-term shareholder value and investment efficiency. We incorporate an element of managerial myopia as a source for such incentives. As in standard voluntary disclosure models, we assume that the manager cares about the short-term price—the price following investment. Moreover, we assume that the manager also cares to some degree about expected long-term value in choosing a disclosure strategy. Such conflicting incentives drive a

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6 See Verrecchia [2001] for a discussion of this assumption. The notion that managers might strategically inflate firm prospects (e.g., to secure favorable financing terms) has been discussed in the literature (e.g., Narayanan [1985], Stein [1989]). Frankel, McNichols, and Wilson [1995] and Lang and Lundholm [2000] provide evidence that managers release good news prior to raising external finance.

7 That is, the equilibrium is not a partition equilibrium in the sense of Crawford and Sobel [1982]. The intuition here is that informed managers choose nondisclosure in equilibrium only to pool with the uninformed manager (Dye [1985]). And, the only way to pool with the uninformed manager is to not just choose nondisclosure but also mimic the uninformed manager’s cheap talk strategy.

8 For example, the manager might be compensated based on short- and long-term performance, or alternatively current shareholders might care about both short- and long-term prices since they might have to sell their shares early for liquidity reasons (see also, Langberg and Sivaramakrishnan [2010] for a discussion of this assumption).
two-tailed disclosure strategy in equilibrium where managers voluntarily disclose good news to favorably influence the market—and investors indeed assign higher valuations and allocate more capital to firms that disclose more favorable news, but they also disclose bad news.

The reason that managers disclose bad news in equilibrium is that, absent its disclosure, there is investment distortion. A manager with some long-term interest in his/her firm cares about this adverse effect of nondisclosure (of bad news) and faces a trade-off in his/her disclosure strategy. On the one hand, voluntary disclosure of bad news prevents substantial deviations between investors’ beliefs and the manager’s private information on firm prospects, thereby improving investment efficiency. On the other hand, by withholding bad news, the manager can pool with nondisclosing firms and avoid adverse short-term price effects. We find that, if the news is sufficiently bad, then long-term investment efficiency gains from disclosure outweigh the short-term adverse price or announcement effects. When news realizations are in the intermediate range, investment distortions are mild enough that they are offset by adverse short-term price reactions to disclosure. Thus, managers possessing “intermediate” news do not disclose in equilibrium, choosing silence when investment distortions are sufficiently small.

Turning to the price response to disclosures, we find that bad news disclosures lead to a precipitous price drop relative to nondisclosure while the market’s response to good news disclosures is smooth. This asymmetry is consistent with empirical evidence on the market’s strong reaction to bad news relative to good news disclosures (e.g., Kothari, Shu, and Wysocki [2009], Skinner [1994]). Intuitively, bad news is voluntarily disclosed only when there is a sufficiently large gap between the manager’s private information and investors’ beliefs that investment distortions from nondisclosures are no longer in the manager’s best interests. Disclosure eliminates this gap, leading to a discrete price drop relative to nondisclosure. On the other hand, good news disclosures are independent of the magnitude of investment efficiency gains and therefore even marginal positive deviations between the manager’s private information and the market beliefs are disclosed—leading to a smooth market upward reaction.

Given that investment efficiency influences the manager’s voluntary disclosure strategy, it is interesting to evaluate this efficiency incentive when

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9 By “investment distortion,” we mean the difference between the first best investment (conditional on the realized firm prospects as observed by the manager) and the level of investment required by investors (based on their Bayesian updated beliefs on firm prospects).

10 An immediate implication of this two-tailed equilibrium result is that the use of equity-based compensation instruments such as restricted equity stock to align managerial incentives with long-term value can induce managers to promote efficient capital allocation by the market when faced with extremely good or bad investment prospects, but not necessarily so when faced with average investment prospects. In the absence of such long-term incentives, the manager’s disclosure strategy is determined purely by short-term price effects, and the disclosure equilibrium will be upper-tailed (Dye [1985]).
the manager's private information has limited effect or when marginal returns on investment vary. To this end, we extend our analysis to examine equilibrium voluntary disclosure strategies when the quality of the firm's investment prospects is public information to some degree—representing public knowledge of the industry or business climate and its impact on investment prospects. We show that congruent managers (i.e., with relatively high long-term stakes in their firms) are less forthcoming with unfavorable information in good times than they are in bad times. Moreover, managers are less likely to disclose unfavorable information when their private information plays a relatively minor role in determining firms' investment quality, as might be the case in high-growth industries and industries with emerging technologies. These implications are potentially empirically testable.

The paper is organized as follows. In section 2, we relate our results briefly to the literature. In section 3, we present the model and in section 4 we derive the basic disclosure equilibrium, characterize the manager's two-tailed disclosure strategy, and derive implications for price response and investment efficiency. In section 5, we extend our analysis to incorporate the value relevance of a public signal. In section 6, we discuss the testable empirical implications of our analysis, and section 7 concludes. All proofs are presented in the appendix.

2. Relation to the Literature

Our study contributes to the literature that delves into the real implications of voluntary disclosures. For example, it has been argued that transparency and voluntary disclosures may lead to information production by market participants (e.g., Fishman and Hagerty [1989], Langberg and Sivaramakrishnan [2008]) and that feedback from financial markets triggered by voluntary disclosures can guide managers' real actions (e.g., Dye and Sridhar [2002], Langberg and Sivaramakrishnan [2010]). In a related vein, Verrecchia [2001] observes that, due to the adverse selection problem in financial markets (e.g., Myers and Majluf [1984], Greenwald, Stiglitz, and Weiss [1984], and Stiglitz and Weiss [1983]), managers might wish to voluntary disclose information to maximize their firms' share price when they intend to issue additional equity for financing operations. Beyer and Guttmann [2010] extend Myers and Majluf [1984] and show that managers can signal the value of their firm's assets in place by reporting biased information when their firm requires external equity financing for an investment.11

We extend this literature by analyzing the role of (credible or "free-of-bias") voluntary disclosures in influencing market opinion when managerial actions are disciplined by the market for corporate control.

11 See also Einhorn and Ziv [2011] for an analysis of managers' disclosure strategy when disclosure is voluntary and not necessarily truthful.
This market discipline introduces an incentive for managers to voluntarily disclose bad news. This incentive is more pronounced when managers are less myopic, when economic conditions are unfavorable, and when their disclosures are more likely to alter the level of investment.

The question as to why managers might release bad news in equilibrium has also received some attention in the literature. In particular, it has been suggested that managers may strategically disclose bad news when bargaining with labor unions (Liberty and Zimmerman [1986]), deterring competition (Donthoh [1989], Darrough and Stoughton [1990]), reducing the exercise price of the options they are given (Aboody and Kasznik [2000]), signaling confidence about future news (Teoh and Hwang [1991]), and triggering feedback from financial analysts (Langberg and Sivaramakrishnan [2010]).

In this paper, managers voluntarily disclose bad news in order to influence investment efficiency. In particular, we present a model in which the presence of an activist shareholder introduces the incentive for managers with long-term interests in their firms to come forward with extreme bad news in order to deploy the appropriate investment strategy.

3. The Model

3.1 PRODUCTION

We consider the investment in a production technology following disclosures made by a manager regarding the prospects of his/her firm’s technology (or investment opportunity) using a two-period model.\(^\text{12}\) The investment \(k\) takes place in the first period and stochastic output is realized in the second period. The investment level is observable by all market participants. Prior to investment, the manager (with some probability \(\lambda\)) privately observes a signal \(x \in X \equiv [0, x_{\text{max}}]\) about the quality of the firm’s investment opportunities with CDF \(F\) and density \(f\) (Dye [1985]). For ease of reference, we also refer to \(x\) as firm quality (i.e., high-quality firms are endowed with more profitable investment opportunities).

Stochastic output is determined by the level of investment \(k\), the cost of investment \(Rk\), where \(R > 1\) is the firm’s gross required rate of return, and the firm’s quality \(x\). For simplicity, we deploy a binary production technology in which the realized gross return on investment is either high (normalized to 1) or low (normalized to 0). In particular, the probability that the return is 1 is given by \(2\pi x \sqrt{k}\) (for some scalar \(\pi > 0\)).\(^\text{13}\) Let, \(y\) denote the

\(^{12}\) The level of investment in our model is disciplined by the market for corporate control as described shortly. For this reason, and as will become clear soon, it is not important for our analysis whether the manager requires funds from an external capital provider or whether the firm has all the resources to finance investments internally.

\(^{13}\) Feasibility of the probability \(2\pi x \sqrt{k}\) in equilibrium requires the assumption \(\pi < \left[ \frac{R}{x_{\text{max}}} \right]\). It will become clear once we derive the levels of investment in equilibrium that this does not qualitatively restrict our analysis.
level of output net of the cost of investment $Rk$, as given by the random variable

$$y = y(x, k) = \begin{cases} 1 - Rk, & \text{w.p. } 2\pi x \sqrt{k} \\ -Rk, & \text{otherwise} \end{cases}$$

and $E(y|x, k) = 2\pi x \sqrt{k} - Rk$. \hfill (1)

3.2 CORPORATE CONTROL AND INVESTMENT

The firm is publicly held and its shares are traded in a frictionless capital (or equity ownership) market. For expositional ease, we assume that all of the firm’s shares are held by a risk-neutral activist shareholder (the AS) who controls and monitors the manager. The payoff to the AS, we denote by $U_{AS}$, is given by the level of output $y$. Namely,

$$U_{AS}(y) = y. \hfill (2)$$

Besides the possibility of credibly revealing $x$ through voluntary disclosure, we build on Crawford and Sobel [1982] and allow communication between the manager and the AS. Although this communication is not relevant following disclosure (since the manager’s type is fully revealed), it might potentially serve as a signal following nondisclosure. Namely, the manager (the sender) can send a (public) message $m \in X$ and the active shareholder (the receiver) decides on the the level of investment $k$. With this setup, we capture the notion that managerial actions such as investment are governed by the market for corporate control (Grossman and Hart [1986], Shleifer and Vishny [1986]).

3.2.1. First-Best Investment. With perfect information about the investment quality $x$, we can define the first-best level of investment $k^*(x)$ that maximizes expected utility of the AS (i.e., expected output) as

$$k^*(x) \equiv \arg \max_k E(y|x, k) \Rightarrow k^*(x) = \left(\frac{\pi x}{R}\right)^2. \hfill (3)$$

The corresponding expected net terminal value of the investment to the AS is

$$E(y|x, k^*(x)) = 2\pi x \sqrt{k^*(x)} - Rk^*(x) = \frac{\pi^2 x^2}{R}. \hfill (4)$$

3.3 PRICES AND MANAGER’S PREFERENCES

Shares of the firm are dynamically traded over time in public security markets. In particular, shares are traded at price $P_1$ at time $t = 1$ after investment takes place (that is, after the activist shareholder sets investment $k$). Short term prices are potentially contingent on the manager’s disclosure of $x$, her (public) message $m$, and the (observed) level of investment $k$ set by AS. A standard assumption in the voluntary disclosure literature is that managers maximize expected firm price following their voluntary disclosure. In these models, there are no future production decisions to
consider, and therefore they do not address the voluntary disclosure incentives when managers can influence the allocation of capital to their firm by voluntarily sharing information with markets.

Our motivation here is to incorporate the resource allocation role of voluntary disclosures in determining the level of corporate investment. For this role to arise in equilibrium, managers must also care about firm value in the long run, after output $y$ is realized. To this end, we consider another round of trade taking place after the terminal payoff is realized, time $t = 2$, at price $P_2$. Following Langberg and Sivaramakrishnan [2010], we assume that managers care about both short-term and long-term prices, with the parameter $\beta \in (0, 1)$ representing the degree to which the manager is concerned with short-term prices (i.e., is myopic). In other words, the objective function that dictates the disclosure strategy choice is

$$U_M(P_1, P_2) = \beta P_1 + (1 - \beta) P_2.$$ (5)

For convenience, we refer to the manager as being (more) “myopic” when $\beta \geq \frac{1}{2}$, and as being (more) “congruent” when $\beta < \frac{1}{2}$.

### 3.4 Timeline

The sequence of events in the two-period model is as follows:

**Period 1**

1) Manager learns $x$ with probability $\lambda$.
2) Manager discloses $x$ or not and sends message $m$ to AS.
3) Investment $k$ takes place as determined by AS.
4) Short-term trade takes place at price $P_1$.

**Period 2**

5) Output $y$ is realized.
6) Long-term trade takes place at price $P_2$.

### 4. Disclosure Equilibrium with Investment

We will examine the Perfect Bayesian equilibrium (PBE) of the game set up by the timeline above. To define a PBE concisely, we establish some notation. Let $\omega \in \Omega = \mathcal{X} \cup \{\emptyset\}$ denote the manager’s information so that $\omega = x \in \mathcal{X}$ when the manager is informed and $\omega = \emptyset$ otherwise. The PBE consists of:

**Manager’s Disclosure and Message Strategy:** The manager’s disclosure and message strategy is $\sigma^w(\omega) = (\delta, m)$, where $\delta \in \{(D, x), ND\}$, with $(D, x)$ denoting a voluntary disclosure of the project quality $x$ by an informed manager, and $ND$ denoting nondisclosure. Note that $\sigma^w(\emptyset) = (ND, \cdot)$ necessarily. For notational ease, we will denote the information available to the AS as $\Phi = (\delta, m)$. In equilibrium, the manager’s strategy is optimal given the market prices and the investment strategy of the AS.
Activist Shareholder’s Investment Strategy: Following the manager’s disclosure and message, the AS chooses investment. In equilibrium, the activist shareholder’s strategy is optimal given information $\Phi = (\delta, m)$ and beliefs regarding the manager’s type. Denote the activist shareholder’s strategy as $\sigma^o(\Phi) = k^o$.

Market Prices: After investment $k$ takes place, the short term price $P_1$ is set. Subsequently, following the realization of output $y$, the market price $P_2$ is set. In equilibrium, $P_2^* = y$, and $P_1^*$ is sequentially rational, that is, it is Bayes-consistent with $(\sigma^{ms}, \sigma^{os})$ given $\Phi = (\delta, m)$ and the beliefs about the manager’s type.

Beliefs: Market participants (including the activist shareholder) form consistent beliefs regarding the manager’s type given $\Phi_1 \in \{(D, x, m) \cup \{ND, x, m\}$ for expositional ease, we will adopt the usual tie-breaking convention that, if the manager is indifferent between disclosure and nondisclosure (i.e., the two choices have the same expected payoffs along the equilibrium path), then the manager chooses to disclose.

4.1 EQUILIBRIUM INVESTMENT AND PRICES

We begin by analyzing the equilibrium market prices at times 1 and 2. Starting from time $t = 2$, after output $y$ is realized, the second period price $P_2^* = y$ simply equals realized output in any equilibrium. The first period price $P_1$ reflects markets’ expectations regarding the second period price (or net output $y$), given the (observed) investment level $k$, and information $\Phi$. That is,

**Lemma 1 (Market Prices).** In any equilibrium $\Sigma^*$, given investment $k$ chosen by the AS, and information $\Phi \in \{(D, x, m) \cup \{ND, x, m\}$, the market prices are:

\[
P_2^* (y) = y \text{ and } P_1^* (\Phi, k) = E(P_2^* | \Phi, k) = E(y | \Phi, k) = 2\pi E(x | \Phi) \sqrt{k} - Rk.
\]

(6)

We turn now to the activist shareholder’s investment strategy given information $\Phi$ and the attendant beliefs about the manager’s type. Because the payoffs of the AS are given by net output $y$ (cf. equation (2)), it is clear that the level of investment $k$ is set to maximize expected net output given information $\Phi$, or investment will be ex post efficient given information $\Phi$.

**Lemma 2 (Ex Post Efficient Investment).** In any voluntary disclosure equilibrium $\Sigma^*$, the AS will set investment to $k^{os} (\Phi)$ where,

\[
k^{os} (\Phi) = \arg\max_k E(y | k, \Phi) = \arg\max_k 2\pi E(x | \Phi) \sqrt{k} - Rk \equiv \left(\frac{\pi E(x | \Phi)}{R}\right)^2
\]

(7)

That is, investment is ex post efficient, $k = k^{os} (\Phi)$, given information $\Phi$. 


One can now be more specific about the level of investment and market prices following disclosure. Namely, given the informed manager’s strategy $\sigma^m(x) = (D, x, m)$, investors observe the manager’s true type. Therefore, we can immediately invoke Lemma 2 to note that equilibrium investment equals first best following disclosure, and this is reflected in equilibrium prices.

**Corollary 1 (Investment and Prices Following Disclosure).** In any voluntary disclosure equilibrium, following disclosure of $x$ investment is first best:

$$k^{os}(\Phi) = k^*(x) = \left(\frac{\pi x}{R}\right)^2,$$

for $\Phi = \langle D, x, m \rangle$, (8)

and prices are given by $P_2^s = y$ and,\(^{14}\)

$$P_1^s(\Phi, k^{os}(\Phi)) = \frac{\pi^2 x^2}{R}, \text{ for } \Phi = \langle D, x, m \rangle.$$ (9)

Next, we analyze the equilibrium level of investment and market prices following nondisclosure. But first we establish that, in equilibrium, the message $m$ following nondisclosure cannot serve as an informative signal regarding whether the manager is informed or her productivity $x$. In other words, in equilibrium the message $m$ does not influence the activist shareholder’s beliefs about the manager’s type, and, therefore, the ex post efficient level of investment following nondisclosure and market prices.

First, note that in equilibrium following nondisclosure the message $m$ cannot perfectly reveal the manager’s type. Indeed, in such a case the level of investment is first best and the manager’s payoff coincides with that obtained from disclosure. We appeal to our tie-breaking convention that the manager would disclose in this case.\(^{15}\) But, it is possible that some managers might wish to partially reveal information through their message $m$ (which, is not possible if they choose to disclose). It is also possible that the uninformed manager may separate herself via her message. We address these possibilities in the next lemma.

**Lemma 3 (Messages Are Uninformative Following Nondisclosure).** In any equilibrium $\Sigma^*_n$, the message $m$ given nondisclosure is uninformative about the manager’s type. That is, for any two messages $m_1$ and $m_2$ ($m_i \in \mathcal{X}, i = 1, 2$), equilibrium beliefs satisfy $E(x|ND, m_1) = E(x|ND, m_2)$.

\(^{14}\) The short-term market price can be defined for any level of investment following disclosure. Namely, $P_1^s(\Phi, k) = 2\pi x\sqrt{k} - Rk$, for $\Phi = \langle D, x, m \rangle$.

\(^{15}\) To see why the manager is indifferent between disclosure and the aforementioned perfectly revealing nondisclosure strategy, consider a manager of type $x'$ who fully reveals his/her type via his/her message $m'$ following nondisclosure, that is, $\sigma^{m*}(x') = (ND, m')$, and $m'$ is perfectly revealing of $x'$. In this case, because the manager’s type is revealed to the AS and investment is always ex post efficient (Lemma 2), it follows that $k^{os}(\Phi) = k^*(x)$, that is, the first best investment. Therefore, the manager’s expected payoffs are identical to what she could have expected to get had she followed the (disclosure) strategy $\sigma^{m*}(x) = (D, x, m)$ for any $m \in \mathcal{X}$.
The intuition in Lemma 3 is as follows. Notice that informed managers choose nondisclosure in equilibrium only to pool with the uninformed manager (Dye [1985]). In our setting, the only way to pool with the uninformed manager is to not just choose nondisclosure but also mimic the uninformed manager’s message strategy. Intuitively, by choosing nondisclosure and a message that differs from that sent by the uninformed manager, the manager of type \( x \) effectively pools with a subset of informed managers and by doing so reveals that she is informed. Equilibrium beliefs, market prices, and investment levels are then set according to the average type in this subset of informed managers. But, according to the standard “unraveling” result (Viscusi [1978], Grossman and Hart [1980], Grossman [1981], Milgrom [1981], Milgrom and Roberts [1986]), informed managers cannot separate themselves from uninformed managers without fully revealing their types. We can now state the following proposition.

**Proposition 1 (Equilibrium Properties).** Without loss of generality, attention can be restricted to equilibria where:

(A) The AS disregards any messages sent by the manager. That is, market prices and the level of investment chosen by the AS are independent of the message \( m \).

(B) The manager of type \( x \) chooses between the two strategies: (i) Disclosure: \( \sigma^m(x) = (D, x) \) and (ii) Nondisclosure: \( \sigma^m(x) = ND \).

Given Proposition 1, we can immediately compute the level of investment chosen by the AS following nondisclosure (given by the ex post efficient investment in Lemma 2) and market prices following nondisclosure.

**Corollary 2 (Investment and Prices Following Nondisclosure).** In any voluntary disclosure equilibrium, investment following nondisclosure is:

\[
k^{**}(\Phi) = \left( \frac{\pi E(x|ND)}{R} \right)^2, \text{ for } \Phi = \langle ND, m \rangle.
\]

and prices are given by \( P_2 = y \) and \( P_1 = \frac{\pi^2 E^2(x|ND)}{R} \), for \( \Phi = \langle ND, m \rangle \).

From this point onward, given the above analysis, we simplify notation by referring to equilibrium investments as \( k(x) \) following disclosure of \( x \), and \( k(ND) \) following nondisclosure instead of \( k^{**}(\Phi) \). Moreover, we refer to prices \( \langle P_1(\Phi, k^{**}(\Phi)) \rangle, P_2 \rangle \) as \( \langle P_1(x), P_2(x) \rangle \) following disclosure of \( x \), and \( \langle P_1(ND), P_2(ND) \rangle \) following nondisclosure (and by disregarding the manager’s message \( m \)).

\[\text{16} \text{ The short-term market price can be defined for any level of investment following nondisclosure. Namely, } P_1(\Phi, k) = 2\pi E(x|ND)\sqrt{k} - Rk, \text{ for } \Phi = \langle ND, m \rangle.\]

\[\text{17} \text{ With this simplified notation, it is worth noting that the disclosure or nondisclosure of } x \text{ affects the Date 2 price only through its effect on the level of investment chosen by the activist shareholder.}\]
4.2 MANAGER’S DISCLOSURE STRATEGY

From the manager’s utility function (see equation (5)), it follows that she will voluntary disclose $x$ in equilibrium if and only if

$$\beta P_1(x) + (1 - \beta) E(P_2(x)|x) > \beta P_1(ND) + (1 - \beta) E(P_2(ND)|x).$$

(12)

Noting that $P_1(x) = E(P_2(x)|x)$, this inequality simplifies to

$$P_1(x) > \beta P_1(ND) + (1 - \beta) E(P_2(ND)|x).$$

(13)

Moreover, disclosure increases long-term value whenever $E(P_2(x)|x) > E(P_2(ND)|x)$, or equivalently whenever $P_1(x) > E(P_2(ND)|x)$. That is, whenever the short term value of the firm following disclosure exceeds the expected long term value of the firm (from the manager’s perspective). The following lemma speaks to this issue.

**LEMMA 4.** Voluntary disclosures increase long-term firm value for any $x$, that is,

$$P_1(x) - E(P_2(ND)|x) = \frac{\pi^2}{R} (x - E(x|ND))^2 > 0 \text{ for } x \neq E(x|ND).$$

(14)

This lemma simply states that the long-term value of the firm once investment is set to its first best level (due to the disclosure of $x$ and Lemma 2) is by definition higher than the long-term value of the firm given a suboptimal level of investment $k(ND)$ associated with its nondisclosure. This long-term benefit can potentially motivate managers having even some long-term stake in their firms to disclose information in equilibrium despite any consequent reduction in the short-term stock price, that is, even if $P_1(x) < P_1(ND)$. We characterize this gain in the following proposition, which follows directly from Lemma 4.

**PROPOSITION 2 (Nonmonotonic Benefit from Disclosure).** The long-term efficiency gain from disclosure is nonmonotonic in the firm’s growth potential $x$. In particular, $P_1(x) - E(P_2(ND)|x)$ is increasing in $|x - E(x|ND)|$.

Intuitively, the investment efficiency gain from voluntary disclosure is attributable to the wedge between the first-best level of investment, as in equation (3), and the level of investment that follows nondisclosure, as in equation (10). Specifically, the more extreme the value of $x$ relative to $E(x|ND)$, the greater is the investment distortion following nondisclosure. Thus, for extreme values of $x$—whether high or low—managers have a greater incentive to disclose because the cost associated with investment distortion would be higher otherwise. This result raises the possibility that a two-tailed disclosure strategy might emerge in equilibrium.

4.3 BENCHMARK CASES

Before we present and characterize the full-fledged voluntary disclosure equilibrium, it is instructive to examine some polar cases of managerial
preferences, namely, the perfectly long-term value maximizing or congruent manager \((\beta = 0)\), and the perfectly myopic manager \((\beta = 1)\).

**Perfectly congruent manager** \((\beta = 0)\). In this case, referring to the disclosure condition (13), the manager’s optimal strategy is to disclose if and only if \(P_1(x) > E(P_2(ND)|x)\). Since this inequality will always hold (Lemma 4), the perfectly congruent manager will always disclose to insure first best resource allocation by the capital provider. That is, there will be a full disclosure equilibrium.

**Perfectly myopic manager** \((\beta = 1)\). Referring to the disclosure condition (13), the manager will disclose if and only if \(P_1(x) > P_1(ND)\). Using the prices in equation (9) and equation (11), this condition can be expressed as

\[
\frac{\pi^2 x^2}{R} > \frac{\pi^2 E^2(x|ND)}{R} \iff x > E(x|ND).
\]

Thus, the perfectly myopic manager will follow an upper-tailed disclosure strategy by disclosing \(x\) when it exceeds a certain threshold \((E(x|ND))\), and not disclose otherwise. The above disclosure cut-off strategy is similar in spirit to that presented in Dye [1985].

### 4.4 Voluntary Disclosure Equilibrium

Consider now the general case in which the manager has both short term and long term incentives, that is, \(\beta \in (0, 1)\). Using the prices in equation (9) and equation (11), appealing to the disclosure condition (13), and noting that \(E(P_2(ND)|x) = \frac{\pi^2 E(x|ND)}{R} [2x - E(x|ND)]\), the manager will disclose \(x\) if and only if

\[
\frac{\pi^2 x^2}{R} > \beta \frac{\pi^2 E^2(x|ND)}{R} + (1 - \beta) \left( \frac{\pi^2 E(x|ND)}{R} [2x - E(x|ND)] \right).
\]

This inequality simplifies to

\[
x^2 > \beta E^2(x|ND) + (1 - \beta) E(x|ND) [2x - E(x|ND)].
\]

or,

\[
\beta \left[ x^2 - E^2(x|ND) \right] + (1 - \beta) [x - E(x|ND)]^2 > 0.
\]

We can now characterize the manager’s disclosure strategy in any equilibrium.

**Lemma 5 (Two-Tailed Disclosure Strategy).** For any short-term nondisclosure price \(P_1(ND) > 0\), and price \(P_1(x) = \frac{\pi^2 x^2}{R}\), a congruent manager (i.e., \(\beta \in (0, \frac{1}{2})\)) will disclose \(x\) whenever (i) \(x > \hat{x}\) or (ii) \(x < \underline{x}\), but a myopic manager (i.e., \(\beta \in (\frac{1}{2}, 1)\)) will disclose \(x\) only when \(x > \hat{x}\), where \(\hat{x} = E(x|ND) = \frac{\sqrt{R P_1(ND)}}{\pi}\), and \(\underline{x} = \max[1 - 2\beta, 0] E(x|ND) = \max[1 - 2\beta, 0] \frac{\sqrt{R P_1(ND)}}{\pi}\).

Lemma 5 allows us to fully characterize all disclosure and nondisclosure regions in terms of the two thresholds \(x\) and \(\hat{x}\). We are now ready to present the disclosure equilibrium and show existence.
CORPORATE CONTROL AND VOLUNTARY DISCLOSURES

PROPOSITION 3 (Voluntary Disclosure of Extreme Values). The manager will voluntarily disclose extreme values of \( x \). Formally, there exists a \( \gamma > 0 \) such that the manager will disclose \( x \in BD \cup GD \) and withhold \( x \in ND \) where

\[
BD = [0, \max \{1 - 2\beta, 0\} \gamma), \text{ bad news disclosure region}
\]

\[
ND = [\max \{1 - 2\beta, 0\} \gamma, \gamma], \text{ nondisclosure region}
\]

\[
GD = (\gamma, \infty), \text{ good news disclosure region}
\]

\[
\gamma = \frac{\lambda \Pr(x \in ND) E(x|x \in ND)}{\lambda \Pr(x \in ND) + (1 - \lambda)} + \frac{(1 - \lambda) E(x)}{\lambda \Pr(x \in ND) + (1 - \lambda)};
\]

short-term market prices are

\[
P_1(x) = \frac{\pi^2 x^2}{R} \text{ for } x \notin ND \text{ and } P_1(ND) = \frac{\pi^2 \gamma^2}{R},
\]

investment levels are

\[
k(x) = \left(\frac{\pi x}{R}\right)^2 \text{ for } x \notin ND \text{ and } k(ND) = \left(\frac{\pi \gamma}{R}\right)^2.
\]

Based on equation (17) and equation (18), we distinguish between bad-news and good-news disclosures. Bad news disclosures occur in the lower tail \( (x \in BD) \) and good news disclosures occur in the upper tail \( (x \in GD) \). Thus, from the market’s perspective, bad news disclosures reflect a “scale down” of investment levels relative to the investment level conditional on nondisclosure, while good news disclosures are associated with higher investment levels.

An immediate implication of this equilibrium is that good-news disclosures lead to a favorable short-term market price reaction.

COROLLARY 3. In equilibrium, the short-term price following disclosure of \( x \in GD \) (i.e., disclosure of good news) is higher relative to the short-term nondisclosure price. Formally, \( P_1(x) > P_1(ND) \) for \( x \in GD \).

This corollary captures the conventional wisdom that managers disclose information that will favorably affect their firm’s stock price. More importantly, in our model, good news disclosures not only increase the short-term stock price but also enhance the efficacy of the allocation of capital by enabling first best investment levels.

Intuitively, this result implies that, when managers with some long-term stake are faced with valuable investment opportunities, they have a natural incentive to disclose that information in order to promote first-best investment levels and benefit from value enhancement. This said, managers also have an incentive to release favorable information, which is consistent with traditional one-tailed disclosure regions derived in the literature (Dye [1985], Verrecchia [1983]).

Interestingly, however, our analysis points to a disclosure behavior that prima facie might appear counterintuitive, as the following corollary demonstrates.
COROLLARY 4. In equilibrium, the short-term price following disclosure of $x \in BD$ (i.e., disclosure of bad news) is lower relative to the short-term nondisclosure price. Formally, $P_1(x) < P_1(ND)$ for $x \in BD$.

A question that naturally arises is why might managers want to disclose bad news, especially given that such disclosures would adversely impact the short-term price. The reason is that the incentive to disclose bad news arises from managers being interested in the long-term value of their firms by enabling first best capital allocation.

Intuitively, favorable value potential realizations for which $P_1(x) > P_1(ND)$ (i.e., $x > E(x|ND)$) are disclosed since they increase the firm’s short-term price and improve investment efficiency. But for unfavorable value potential realizations for which $P_1(x) < P_1(ND)$ (i.e., $x < E(x|ND)$), the manager will only disclose information if she has a sufficient long-term stake in the firm and there are sufficient investment efficiency gains. In other words, a disclosure that reduces the firm’s short-term share price must be justified by a sufficiently high long-term benefit from promoting first best capital allocation. Indeed, the more myopic the manager, the less she weighs this long-term gain from efficient investment. In fact, the bad-news region $BD$ is nonempty only when managers have a sufficient stake in the firm’s long-term performance, that is, are congruent ($\beta \in (0, \frac{1}{2})$).

COROLLARY 5. For a congruent manager (i.e., $\beta < \frac{1}{2}$), the bad-news disclosure set $BD$, the nondisclosure region $ND$, and the good-news disclosure region $GD$ are not empty. For the myopic manager (i.e., $\beta > \frac{1}{2}$), the nondisclosure region $ND$ and the good-news disclosure region $GD$ are not empty, but the bad-news disclosure region $BD$ is empty (i.e., the manager will not disclose bad news).

Figure 1 illustrates this equilibrium.

Notice that, when the manager has purely a long-term interest in the firm, that is, $\beta = 0$ ($1 - 2\beta = 1$), the nondisclosure region collapses and there is disclosure everywhere. This is because disclosure insures investment efficiency and always increases long-term value (Lemma 4).
other extreme, when the manager is relatively more myopic, $\beta \geq \frac{1}{2}$ (or $1 - 2\beta \leq 0$), the bad news disclosure region collapses, and there is only upper-tailed disclosure as in Dye [1985]. In this case, the myopic informed manager uses the camouflage of the uninformed manager for low values of firm quality to benefit from a higher short-term price. In the intermediate region where $0 < \beta < \frac{1}{2}$, the manager discloses at both tails.

Interestingly, not only is the price response to bad-news disclosures negative (relative to the nondisclosure price), but the market’s response to a bad-news disclosure is more abrupt (discontinuous) relative to the positive market response to favorable disclosures.

**Proposition 4 (Price Drop Following Bad News Disclosures).** The market price reaction to disclosure is monotonic in the disclosure content $x$. Moreover, for a congruent manager ($\beta \in (0, \frac{1}{2})$), bad news disclosures are followed by a drop in the short-term market price relative to nondisclosure. In particular, $P_1(ND) - P_1(x) = 4\beta(1 - \beta)$. There is no such discrete jump in the short-term prices following good news disclosures. It follows also that the more myopic the manager (the higher $\beta$), the larger is the price drop upon disclosure of bad news.

Figure 2 illustrates the extreme value disclosure equilibrium for $\beta < \frac{1}{2}$ and illustrates the price drop at the margin where managers prefer to disclose bad news over not disclosing.

This price drop essentially reflects a discrete correction in the investment level relative to the nondisclosure investment level. Indeed, it follows from equation (19) that the investment response to voluntary disclosure in figure 3 mirrors the price response illustrated in figure 2. Referring to figure 3, notice that investments are at their first-best levels in the disclosure regions. In the nondisclosure region, however, the investment level is ex post efficient (given markets’ assessment of the firm’s investment quality), and is fixed at the nondisclosure level over a wide range. Investment levels in the good news disclosure region increase smoothly with news relative to the nondisclosure level beginning at the threshold $\bar{x}$. On the
other hand, there is a sudden and precipitous decline in investment levels associated with unfavorable disclosures relative to the nondisclosure level. This is because bad news disclosures are triggered only when the gap between the manager’s private information and investors’ beliefs is sufficiently large that the consequent investment distortion from nondisclosure imposes a greater cost than the price drop.

**Corollary 6 (Disclosure and Investment).** Investment following disclosure is monotonic in the disclosure content $x$. Moreover, for a congruent manager ($\beta \in (0, \frac{1}{2})$), there is discrete decline in the investment level in response to bad news disclosures relative to the level of investment that follows nondisclosure. In particular, $\frac{k(ND) - k(x)}{k(ND)} = 4\beta(1 - \beta)$. There is no such discrete jump in the investment response to good news disclosures. It follows also that the more myopic the manager (the higher $\beta$), the larger is the investment drop upon disclosure of bad news.

### 4.5 Managerial Myopia, Information Advantage, and Disclosure: An Example

It is clear that the extent of managerial myopia ($\beta$) is an important determinant of disclosure regions that emerge in the equilibrium characterized in Proposition 3 (e.g., see figure 1). Also, as in Dye [1985], the voluntary disclosure region here depends on the manager’s information advantage ($\lambda$). In this section, we present a simple example of the equilibrium to obtain a closed form solution and explore the precise roles of managerial myopia and the manager’s information advantage on the voluntary disclosure equilibrium.

Specifically, we consider the simple case where $x \sim U[0, 1]$. We can then restate the disclosure equilibrium of Proposition 3 as follows:

**Proposition 5.** When $x \sim U[0, 1]$, the disclosure equilibrium is given by (17)-(19) where $\gamma = \frac{-1 + \sqrt{1 + \rho}}{\rho} \in (0, \frac{1}{2})$, for $\rho \equiv \frac{\lambda}{1-\lambda}(1 - \max[1 - 2\beta, 0])^2$. 

![Fig. 3.—Investment distortion.](image)
With this simple stylized characterization of the disclosure equilibrium, we can now examine the impact of the extent of managerial myopia on prices and disclosure regions, and state the following result.

**Corollary 7 (Managerial Myopia).** For the myopic manager, the level of \( \beta \) does not change the disclosure equilibrium (i.e., as long as \( \beta \in (\frac{1}{2}, 1) \)). But, for the congruent manager (\( \beta \in (0, \frac{1}{2}) \)), the less congruent the manager, that is, the higher the \( \beta \), the lower is the nondisclosure short term price, the higher is the likelihood of good news disclosures, and the lower is the likelihood of bad news disclosures.

Intuitively, when the manager is less driven by short-term price performance, there will be more disclosure of unfavorable news. Consequently, investors will be more optimistic following nondisclosure which in turn raises the threshold for good news disclosures leading to less disclosure of good news.

Finally, we can also characterize how disclosure regions change as the probability that the manager is informed, \( \lambda \), increases.

**Corollary 8 (Managerial Information Advantage).** The more informed the manager, that is, the higher \( \lambda \), the lower is the nondisclosure short-term price; the higher is the likelihood of good news disclosures; and, for the congruent manager (\( \beta \in (0, \frac{1}{2}) \)), the lower is the likelihood of bad news disclosures.

Intuitively, the informed manager does not benefit as much by pooling with uninformed managers (by not disclosing) when the likelihood that the manager is informed \( \lambda \) is higher. Upon observing no disclosure, the market places greater weight on the manager being informed and the news being bad than the manager being uninformed. Consequently, the nondisclosure short-term price decreases. Although, this increases investment efficiency on the right tail of the distribution of firm quality \( x \), it pushes down, so to speak, the bad news threshold for the congruent manager (\( \beta < \frac{1}{2} \)). That is, the bad news disclosure region (BD) shrinks as \( \lambda \) increases.

5. **Extension: Voluntary Disclosure with Public Information**

In the previous section, we have considered only the information that the manager has on firm quality. Intuitively, the success of an investment opportunity depends also on the health of the economy, consumers’ propensity to spend, industry trends, and so forth. Suppliers of capital (and managers) would obviously keep abreast of these trends, and would take them into account in their capital allocation decisions. Such public investment-relevant information could affect the disclosure equilibrium of Proposition (3). For instance, since the region of bad news disclosures in equilibrium was motivated by nonmyopic managers’ concerns for investment efficiency, the content of the public information may change equilibrium voluntary disclosure strategies.

Thus, by incorporating a public signal reflecting the impact of the prevailing business or industry climate, we address questions such as: Are firms
more forthcoming with good or bad news in good times versus bad times? How do “myopic” managers behave differently from “congruent” managers when faced with different business conditions? Motivated by these questions, in this section we examine the voluntary disclosure equilibrium when the market and the manager observe a public signal on firm quality prior to the manager’s disclosure of her private information \( x \). We analyze the disclosure equilibrium for various levels of the public signal (e.g., whether the industry is in an expansion or contraction).

Although a complete analysis to address these questions is outside the scope of the paper, our structure lends itself to illustrating the impact of business conditions on the voluntary disclosure strategies that emerge in equilibrium. Modifying our initial structure slightly to accommodate both the public signal and the manager’s private information, we redefine the output \( y \) to depend on firm quality \( p \) and the level of capital invested \( k \) according to

\[
y = y(p, k) = \begin{cases} 1 - Rk, & \text{w.p. } 2\pi p \sqrt{k} \\ -Rk, & \text{otherwise,} \end{cases}
\]

where, firm quality \( p \) depends on a public signal regarding the overall productivity in the industry \( \theta \) and the manager’s private signal \( x \) on firm quality with weights \( 1 - \alpha \) and \( \alpha \), respectively.

\[
p = \alpha x + (1 - \alpha)\theta, \text{ for } \alpha \in (0, 1) \text{ and } \theta \in [0, \theta_{\text{max}}],
\]

where \( \theta_{\text{max}} > 0 \). Our objective is to examine the impact on the voluntary disclosure of varying the content of the public signal, that is, the level of \( \theta \), and the relevance of the public signal, that is, the value of \( \alpha \). Note, for the special case \( \alpha = 1 \), the modified output model in (20) coincides with our earlier output model (1) for which the equilibrium is presented in Proposition 3, and the case where \( \alpha = 0 \) makes the voluntary disclosure issue moot, so we rule this case out.

5.1 PRICES AND INVESTMENT

Notice that there is a one-to-one mapping between \( p \) and \( x \) for any given pair \( (\theta, \alpha) \). That is, by disclosing \( x \) the manager fully reveals \( p \). Thus, equation (21) implies that investment levels follow from equation (7) and prices follow from equation (6), where \( x \) is replaced by the firm quality parameter \( p \).

**Lemma 6.** In any voluntary disclosure equilibrium, when firm quality is given by \( p = \alpha x + (1 - \alpha)\theta \) (as in equation (21)), the long-term price \( P_2 \) is

\[
P_2(\Phi) = y(k(\Phi), p) = \begin{cases} 1 - Rk(\Phi), & \text{w.p. } 2\pi p \sqrt{k(\Phi)} \\ -Rk(\Phi), & \text{otherwise} \end{cases}
\]

where \( \Phi \in \theta \cap \{(D, x) \cup (ND)\} \).
the short-term market prices are
\[ P_1(x) = \frac{\pi^2 p^2}{R}, \quad \text{and} \quad P_1(ND) = \frac{\pi^2 E^2(p|ND)}{R}, \]
and investment levels are
\[ k(x) = \left(\frac{\pi p}{R}\right)^2 \quad \text{and} \quad k(ND) = \left(\frac{\pi E(p|ND)}{R}\right)^2. \]

5.2 DISCLOSURE STRATEGY

Recall from equation (13) that disclosure occurs when
\[ P_1(x) > \beta P_1(ND) + (1 - \beta) E(P_2(ND)|x). \]
Following equation (16), Lemma 6, and equation (21), this can be written as
\[ \beta \left[p^2 - E^2(p|ND)\right] + (1 - \beta) \left[p - E(p|ND)\right]^2 > 0. \]
Therefore, disclosure will occur in equilibrium when:

1) \( p > \bar{p} \) where \( \bar{p} = E(p|ND) \Leftrightarrow \alpha \bar{x} = \alpha E(x|ND). \)
2) \( p < \bar{p} \) where \( \bar{p} = (1 - 2\beta)E(p|ND) \Leftrightarrow \alpha \bar{x} + (1 - \alpha)\theta = (1 - 2\beta) \)
\( \alpha E(x|ND) + (1 - \alpha)\theta \).

Equivalently, in terms of the manager’s private information \( x \) for a given pair \( \theta, \alpha \), disclosure will occur in equilibrium when:

1) \( x > \bar{x} \) where \( \bar{x} = E(x|ND). \)
2) \( x < \bar{x} \) where \( \bar{x} = (1 - 2\beta) E(x|ND) - \frac{2\beta(1 - \alpha)\theta}{\alpha}. \)

5.3 MYOPIC MANAGER AND THE NONRELEVANCE OF PUBLIC INFORMATION

Consider first the myopic manager (\( \beta \in \left(\frac{1}{2}, 1\right) \)). In this case, there is no disclosure of bad news in equilibrium since \( (1 - 2\beta) < 0 \) and therefore \( \bar{x} < 0 \). Thus, the relevant cutoff is given by \( \bar{x} = E(x|ND) \). In this case, because the manager’s private information \( x \) and the public signal \( \theta \) are conditionally independent, neither the value of \( \theta \) nor its relevance \( \alpha \) have an impact on the manager’s disclosure strategy or the voluntary disclosure equilibrium. The following proposition modifies the disclosure equilibrium of Proposition (3) to incorporate the public signal for the myopic manager (proof omitted).

**Proposition 6 (Nonrelevance of Public Information).** When firm quality is given by equation (21), the myopic manager’s (\( \beta \in \left(\frac{1}{2}, 1\right) \)) equilibrium disclosure strategy is not affected by the value \( \theta \) or relevance \( \alpha \) of the public signal. In particular, there exists a \( \gamma > 0 \) such that the manager will disclose \( x \in GD \) and withhold \( x \in \).
\( ND \) where
\[
ND = [0, \gamma]
\]
\[
GD = (\gamma, \infty)
\]
\[
\gamma = \frac{\lambda \Pr(x \in ND) E(x|x \in ND)}{\lambda \Pr(x \in ND) + (1 - \lambda)} + \frac{(1 - \lambda) E(x)}{\lambda \Pr(x \in ND) + (1 - \lambda)},
\]
and short-term market prices are
\[
P_1(x) = \frac{\pi^2 (\alpha x + (1 - \alpha)\theta)^2}{R} \quad \text{for} \ x \notin ND \text{ and } P_1(ND)
\]
\[
= \frac{\pi^2 (\alpha \gamma + (1 - \alpha)\theta)^2}{R}.
\]

5.4 CONGRUENT MANAGER

Consider now the congruent manager \((\beta \in (0, \frac{1}{2}))\). Since \( x = (1 - 2\beta) E(x|x|ND) - \frac{2\beta(1 - \alpha)\theta}{\alpha} \) is potentially positive, the manager might disclose bad news in equilibrium. In particular, the modified disclosure equilibrium with public information is given by the following proposition:

**Proposition 7 (Voluntary Disclosure with Public Information).** *The congruent manager \((\beta \in (0, \frac{1}{2}))\) will voluntarily disclose extreme values of \(x\). Formally, there exists a \( \gamma > 0 \) such that the manager will disclose \( x \in BD \cup GD \) and withhold \( x \in ND \) where*
\[
BD = [0, \delta(\theta, \alpha)] \quad \text{where} \quad \delta(\theta, \alpha) \equiv \max \left(0, (1 - 2\beta)\gamma - \frac{2\beta(1 - \alpha)\theta}{\alpha}\right)
\]
\[
ND = [\delta(\theta, \alpha), \gamma]
\]
\[
GD = (\gamma, \infty)
\]
\[
\gamma = \frac{\lambda \Pr(x \in ND) E(x|x \in ND)}{\lambda \Pr(x \in ND) + (1 - \lambda)} + \frac{(1 - \lambda) E(x)}{\lambda \Pr(x \in ND) + (1 - \lambda)};
\]
short-term market prices are
\[
P_1(x) = \frac{\pi^2 (\alpha x + (1 - \alpha)\theta)^2}{R} \quad \text{for} \ x \notin ND \text{ and } P_1(ND)
\]
\[
= \frac{\pi^2 (\alpha \gamma + (1 - \alpha)\theta)^2}{R}.
\]

The public signal alters the voluntary disclosure equilibrium through two channels when \( \beta < \frac{1}{2} \) (i.e., congruent manager). First, the level of the public signal is important. Empirically, this suggests that voluntary disclosures regarding investment opportunities differ when industry productivity is high, relative to when it is low. Second, the precision of the public signal will affect the investment reaction to disclosure for any given level of the public signal. We further explore these issues next.
5.4.1. Voluntary Disclosure in Good and Bad Times. The equilibrium region of bad news disclosures $BD$, as defined in Proposition 7, suggests that managers will be more likely to withhold bad news in good times, that is, when the industry productivity signal $\theta$ is higher or the relevance of the manager’s private firm quality signal $x$ (captured by $\alpha$) is lower. We confirm these results while revisiting our example with uniformly distributed private information. In particular, we consider the case where $x \sim U[0, 1]$. We can restate the disclosure equilibrium of Proposition 7 as follows:

**Corollary 9.** When $x \sim U[0, 1]$, the disclosure equilibrium for the congruent manager is given by Proposition 7 where $\gamma(\psi)$ is given by

$$
\gamma(\psi) = \frac{-1 - 2 \frac{\lambda}{1 - \lambda} \psi (1 - \beta) + \sqrt{1 + \rho + \frac{4\lambda}{1 - \lambda} \psi ((1 - \beta) + \left(\frac{\lambda}{1 - \lambda}\right) \psi (1 - 2\beta))}}{\rho},
$$

for $\rho \equiv \frac{\lambda}{(1 - \lambda)} (1 - \max (1 - 2\beta, 0))^2$, and $\psi = \frac{2(1 - \alpha)\theta}{\alpha}$ and provided that $\psi < \psi^*$ for some $\psi^* > 0$.

With the voluntary disclosure equilibrium at hand, we can explicitly examine the implications of the level of the public signal on productivity $\theta$ and the value relevance of the manager’s private signal $x$ (as measured by $\alpha$) on the incentives of the manager to come forward with unfavorable news in equilibrium.

**Corollary 10 (Withholding Information in Good Times).** Consider the case where $x \sim U[0, 1]$. It is more likely that the congruent manager withholds bad news the higher the public expectations on productivity, that is, the higher the signal $\theta$. For sufficiently high $\theta$, the congruent manager will no longer disclose bad news in equilibrium. Formally, for any $\beta < \frac{1}{2}$, there exists $\theta'$ such that for all $\theta \leq \theta'$ disclosure of bad news is part of the voluntary disclosure equilibrium, but not for $\theta > \theta'$. Moreover, the equilibrium disclosure thresholds $\langle \overline{x}, \underline{x} \rangle$ are decreasing in $\theta$.

This result is pictorially depicted in figure 4. Intuitively, the investment distortion for a given low $x$ is lower for higher $\theta$ since the probability of project success is also higher. Therefore, the cost of the potential investment distortion is reduced (all else equal). An empirically testable implication here is that managers with sufficient long-term stakes in their firms will not be forthcoming with bad news in good times (good industry/business climate). But they will be more willing to disclose bad news in bad times to eschew investment distortions. In other words, congruent managers are not seemingly as averse to short-term price drops in bad times as they are in good times.

Second, the weight $(1 - \alpha)$ captures the relevance of the public signal. When it is more relevant, that is, $\alpha$ decreases toward zero, then (by definition) the information released by the manager is less value-relevant. The following corollary captures the impact of the public signal’s relevance on the congruent manager’s voluntary disclosure behavior.
COROLLARY 11 (Voluntary Disclosure and the Value Relevance of the Manager’s Information). Consider the case where \( x \sim U[0, 1] \). It is less likely that the congruent manager discloses bad news, the lower the value relevance of the manager’s signal \( x \) (i.e., the lower is \( \alpha \)). Also, for sufficiently low \( \alpha \), the congruent manager will no longer disclose bad news in equilibrium. Formally, for any \( \beta < \frac{1}{2} \), there exists \( \alpha' \) such that for all \( \alpha > \alpha' \) disclosure of bad news is part of the voluntary disclosure equilibrium, but not for \( \alpha < \alpha' \). Moreover, the thresholds \( \langle x, \bar{x} \rangle \) are increasing in \( \alpha \).

This result is pictorially depicted in figure 5. A low disclosure of \( x \) will not move (so to speak) the level of investment or the price as before (when there was no public signal). As the relative relevance of the public signal increases, the congruent manager will tend to withhold bad news to a greater extent since the advantage to the manager of disclosing bad news—in terms of correcting the investment level—is lower as the value relevance of the manager’s signal \( x \) is lower.
6. Empirical Implications

In this section, we discuss some testable empirical implications of our analysis. A central equilibrium result of this paper is that managers with sufficient long-term stakes in their firms (i.e., “congruent” managers) would show greater propensity to disclose bad news relative to myopic managers (see Proposition 3). This implication is empirically testable via an examination of the incentive structures of managers making good versus bad news disclosures about investment prospects. In particular, we posit that managers making bad news disclosures would have a greater proportion of their compensation tied to long-term performance measures and/or restricted stock, other things held fixed.

Our analysis also predicts that even congruent managers, despite their propensity to disclose bad news, would be less forthcoming with bad news in good times (Corollary 10). Thus, when industry and business conditions are bright and shape a firm’s investment prospects to a greater degree, the congruent manager’s own pessimistic investment outlook assumes relatively less importance, thereby dampening the incentive to disclose. Extending the same argument, we posit that bad industry and economic conditions would trigger a wave of bad news disclosures from such managers. In sum, our analysis predicts that the relative frequency of bad disclosures would be anti cyclical. This prediction can be tested by relating the relative frequency of bad versus good disclosures to proxies for business conditions at the industry and macro level.

Next, Corollary 11 establishes that the relative importance of public and private information about investment prospects also influences the propensity of congruent managers to disclose bad news. Going by this result, we would expect that, in industries with emerging technologies and for high growth firms, public signals are less relevant relatively speaking. We therefore predict that congruent managers will be more forthcoming with bad news disclosures in such settings.

From the market’s perspective, an important implication of our analysis is that bad new disclosures about investment prospects will be associated with precipitous and more than proportionate stock price declines—relative to nondisclosure price levels—compared with good news disclosures (Proposition 4 and figure 2). The notion that good news and bad news disclosures are associated with asymmetric price responses has received support in the literature (Kothari, Shu, and Wysocki [2009], Skinner [1994], Soffer, Thiagarajan, and Walther [2000], Anilowski, Feng, and Skinner [2007]).

For example, Skinner [1994] presents evidence that the market looks upon “preemptive” bad news disclosures (made by managers to reduce litigation costs) more negatively than good news disclosures. In more recent work, Kothari, Shu, and Wysocki [2009] argue that managers tend to delay bad news disclosures till the “accumulated bad news is worse than a threshold level,” and therefore bad news disclosures are likely to be greeted
with larger magnitudes of negative stock price reactions than good news disclosures. They provide evidence in support of this argument. We contribute to this literature by offering an equilibrium explanation for why bad news disclosures regarding investment prospects may evoke more precipitous price declines than good news disclosures—an explanation that is unrelated to the relative timing of good news and bad news disclosures, and that is not driven by litigation considerations.

7. Conclusion

The literature on voluntary disclosures has focused primarily on managerial incentives to influence the pricing of their securities—the valuation role of voluntary disclosures. But, managers also have a natural incentive to influence market opinion through strategic voluntary disclosures when they are disciplined by the market for corporate control—the allocation role. This paper contributes to the emerging literature on the real implications of voluntary disclosure by examining the voluntary disclosure incentives of a manager wanting to take advantage of her investment opportunities. We provide an equilibrium analysis of both the valuation and the allocation roles of voluntary disclosures in this specific context.

We show that, in equilibrium, a manager motivated by a combination of both short-term and long-term incentives would choose to disclose not just sufficiently good news but bad news (about the firm’s quality) as well in equilibrium. In our model, disclosure always maximizes long-term value because it triggers first-best investment levels from capital providers (e.g., Grossman and Hart [1986]). Although disclosure of good news clearly has reinforcing short- and long-term price effects, managers face a trade-off when it comes to disclosure of bad news. This is because disclosure of bad news triggers an adverse short-term price effect, but it reduces investment distortions. Consequently, a manager with a sufficiently long-term stake in firm value would prefer investment efficiency over higher short-term prices. We are not aware of any prior work in the literature that has highlighted this novel trade-off between the valuation and the allocation roles of voluntary disclosure and its implications for equilibrium disclosure regions.

Our results offer some interesting empirical implications. We provide an equilibrium rationale for the observed asymmetric price response to good news and bad news disclosures. Our result that bad news disclosures lead to a discrete price drop relative to nondisclosure (there is no corresponding price spike on the good news side) is consistent with empirical evidence on the market’s strong reaction to bad news relative to good news disclosures (e.g., Kothari, Shu, and Wysocki [2009], Skinner [1994]). We also show that, when the market (capital providers) can assess firm quality to some extent based on prevailing business and industry conditions, managers with relatively larger long-term stakes in their firms are more forthcoming with bad news during bad times than during good times. Such managers are also more likely to disclose bad news whenever their private information
is more germane to assessing firm quality than public information (high growth industries, nascent firms, emerging technologies).

APPENDIX

PROOF OF LEMMA 1. The result follows since prices equal expected output in equilibrium.

PROOF OF LEMMA 2. The lemma follows from the observation that the AS wishes to maximize expected output given information F and beliefs about managerial type. Any investment level $k'$ chosen by the AS, such that $k' \neq k(\Phi)$, leads to lower expected net output by definition. Thus, it is optimal for the AS to set investment to its ex post efficient level.

PROOF OF COROLLARY 1. The results follow directly from the fact that disclosures are credible and since investment is ex post efficient in equilibrium (see equation (7)).

PROOF OF LEMMA 3. Consider the strategy $\sigma^m(\varnothing) = (ND, m_1)$ of the uninformed manager in the equilibrium $\Sigma^*$. In this equilibrium, suppose that, for some $x \in \mathcal{X}$, $\sigma^m(x) = (ND, m_2)$, for some $m_1 \neq m_2$ and $E(x|ND, m_1) \neq E(x|ND, m_2)$. This implies that the manager’s message is informative about $x$. Of course complete revelation through message $m_2$ (following nondisclosure) is equivalent in terms of all payoffs and is ruled out by convention. Therefore, the message $m_2$ can only be partially revealing of $x$ in equilibrium.

In any such equilibrium, there exists a set $X_2 \subseteq \mathcal{X}$ such that, for all $x \in X_2$, $\sigma^m(x) = (ND, m_2)$. Let $\{k^o_1, k^o_2\}$ be such that $k^o_i$ is the activist shareholder’s investment response to $\Phi_i \equiv (ND, m_1)$, $i = 1, 2$. It follows from Lemma 2 that

$$k^o_2 = \left(\frac{\pi E(x | x \in X_2)}{R}\right)^2.$$  \hfill (A.1)

Consider $x' \in X_2$ such that $x' > E(x | x \in X_2)$. Because $P^e_2 = y$, the expected period 2 price computed by the manager of type $x'$ is

$$E(P^e_2|x', k^o_2) = 2\pi x'\sqrt{E_2}(1 - Rk^o_2) - [1 - 2\pi x'\sqrt{E_2}]Rk^o_2$$

$$= 2\pi x'\sqrt{E_2} - Rk^o_2.$$  \hfill (A.2)

And because $P^e_1(\Phi_2, k^o_2) = E(P^e_2|\Phi_2, k^o_2)$, the market sets the price $P^e_1$ in the proposed equilibrium as

$$P^e_1(\Phi_2, k^o_2) = 2\pi E(x | x \in X_2)\sqrt{k^o_2} - Rk^o_2$$

And hence, using equation (5), the expected utility of the manager of type $x'$ given public information $\Phi_2$ and investment $k^o_2$ is

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18 We consider mixed strategies by the uninformed manager at the end of the proof.
\[
E \left[ U_M (P_1^* (\Phi_2, k_2^o), P_2^o) \right | x']
= \beta P_1^* + (1 - \beta) E (P_2^o | x', k_2^o)
= 2 \pi \sqrt{k_2^o} \left[ \beta E (x | x \in X_2) + (1 - \beta) x' \right] - R k_2^o
< 2 \pi \sqrt{k_2^o} \left[ \beta x' + (1 - \beta) x' \right] - R k_2^o
= 2 \pi \sqrt{k_2^o} x' - R k_2^o
< 2 \pi \sqrt{k^*(x')} x' - R k^*(x')
= P_1^* (\langle D, x', m \rangle, k^*(x')) \quad \text{[price following disclosure].}
\]

The first inequality follows since \( x' > E (x | x \in X_2) \), and the second inequality follows from the optimality of \( k^*(x) \). Since the utility of the manager following disclosure is \( P_1^* (x') \), the type \( x' \) manager has the incentive to deviate and disclose. Therefore, a partially revealing equilibrium cannot exist. So it must be the case that \( E (x | ND, m) = E (x | ND) \) for any message \( m \).

Finally, we establish that it suffices to consider pure strategies for the uninformed manager. To this end, suppose that the uninformed manager’s strategy is mixed between messages \( m_1 \neq m_2 \) and suppose further by contradiction that \( E (x | ND, m_1) < E (x | ND, m_2) \). This implies that investment levels are \( k_i \) following message \( m_i \) (and nondisclosure) where \( k_i = \left( \frac{\pi E (x | ND, m_i)}{R} \right)^2 \) for \( i = 1, 2 \). If \( E (x | ND, m_2) \leq E (x) \), then the uninformed manager strictly prefers the higher investment level \( k_2 \) over \( k_1 \) and will not play the proposed mixed strategy, that is, we reach a contradiction. Thus, it must be that \( E (x) < E (x | ND, m_2) \). But this implies that there exists \( x' > E (x | ND, m_2) \) that chooses strategy \( ND, m_2 \). But, type \( x' \) can obtain a higher utility by disclosing (higher short-term price and first-best level of investment) and we reach a contradiction.

\[ \square \]

**Proof of Proposition 1.** Follows directly from Lemmas 1–3.

**Proof of Corollary 2.** The prices follow directly from Proposition 1. Namely, observe that

\[
P_2^o (ND) = y (k^*(ND), x) = \begin{cases} 1 - R k^*(ND), & \text{w.p.} \ 2 \pi x \sqrt{k^*(ND)} \ \text{and} \\ -R k^*(ND), & \text{otherwise} \end{cases}, \quad \text{(A.4)}
\]

and,

\[
E (P_2^o (ND) | ND) = E \left[ E (P_2^o (ND) | x, ND) | ND \right].
\]

and

\[
E (P_2^o (ND) | x) = \frac{\pi^2 E (x | ND)}{R} \left[ 2x - E (x | ND) \right].
\]

To see this, note that

\[
k^*(ND) = \left( \frac{\pi E (x | ND)}{R} \right)^2 \quad \text{and}
\]

\[
E (P_2^o (ND) | x) = \begin{cases} [1 - R k^*(ND)] 2 \pi x \sqrt{k^*(ND)} \\ -R k^*(ND) (1 - 2 \pi x \sqrt{k^*(ND)}) \end{cases},
\]
\[ = 2\pi x\sqrt{k^*(ND)} - Rk^*(ND) \]
\[ = 2\pi^2 x E(x|ND) - \frac{\pi^2 E^2(x|ND)}{R} \]
\[ = \frac{\pi^2 E(x|ND)}{R} (2x - E(x|ND)). \]

Therefore,
\[ P_1^*(ND) = E \left[ \frac{\pi^2 E(x|ND)}{R} [2x - E(x|ND)] | ND \right] = \frac{\pi^2 E^2(x|ND)}{R}. \]

PROOF OF LEMMA 4. It follows from Corollary 2 and its proof that
\[ P_1(x) = \frac{\pi^2 x^2}{R}, \text{ and } E(P_2(ND)|x) = \frac{\pi^2 E(x|ND)}{R} [2x - E(x|ND)]. \]

Therefore,
\[ P_1(x) - E(P(ND)|x) = \frac{\pi^2}{R} \left[ x^2 - 2xE(x|ND) + E^2(x|ND) \right] \]
\[ = \frac{\pi^2}{R} \left[ x - E(x|ND) \right]^2 > 0. \]

PROOF OF PROPOSITION 2. Follows directly from Lemma 4.

PROOF OF LEMMA 5. From the condition (16), the manager will disclose if and only if
\[ \beta \left[ x^2 - E^2(x|ND) \right] + (1 - \beta) \left[ x - E(x|ND) \right]^2 > 0. \] (A.5)

Clearly, this condition is satisfied whenever \( x > E(x|ND) = \bar{x} \). Since \( P_1(ND) = \frac{\pi^2 E^2(x|ND)}{R} \), it follows that the manager will disclose whenever
\[ x > \bar{x} = E(x|ND) = \sqrt{\frac{RP_1(ND)}{\pi}}. \]

Next, we can rearrange the expression in equation (A.5) as
\[ \beta \left[ x^2 - E^2(x|ND) \right] + (1 - \beta) \left[ x - E(x|ND) \right]^2 \]
\[ = (x - E(x|ND)) \left[ \beta (x + E(x|ND)) + (1 - \beta) (x - E(x|ND)) \right] \]
\[ = (x - E(x|ND)) \left[ x + 2\beta E(x|ND) - E(x|ND) \right] > 0. \]

Consequently, if \( x < E(x|ND) \), the manager will still disclose if
\[ x - E(x|ND) < -2\beta E(x|ND) \]
\[ \iff x < (1 - 2\beta) E(x|ND) \]
\[ \iff x < (1 - 2\beta) \sqrt{\frac{RP_1(ND)}{\pi}}. \]

Since \( x \geq 0 \), we define \( \bar{x} = \max(1 - 2\beta, 0) \sqrt{\frac{RP_1(ND)}{\pi}}. \)
**Proof of Proposition 3.** For the case $\beta > \frac{1}{2}$, note that $(1 - 2\beta) < 0$, and therefore the lower threshold $x = 0$. Consequently, the manager’s disclosure strategy will necessarily be upper-tailed. The rest of the proof follows directly from Dye [1985], and from the investment response in equation (7).

For the case $\beta < \frac{1}{2}$, we are looking for a solution $\gamma$ that satisfies the condition $\Theta(\gamma^*) = 0$ where $(z \equiv \max [1 - 2\beta, 0])$,

$$\Theta(\gamma) \equiv \gamma - \frac{\lambda \Pr(x \in (yz, \gamma)) E(x \mid x \in (yz, \gamma))}{\lambda \Pr(x \in (yz, \gamma)) + (1-\lambda)} - \frac{(1-\lambda) E(x)}{\lambda \Pr(x \in (yz, \gamma)) + (1-\lambda)}.$$  

This follows since the market will assess the expected firm quality following nondisclosure as

$$E(x \mid ND) = \frac{\lambda \Pr(x \in ND) E(x \mid x \in ND)}{\lambda \Pr(x \in ND) + (1-\lambda)} + \frac{(1-\lambda) E(x)}{\lambda \Pr(x \in ND) + (1-\lambda)}.$$  

For $\gamma = 0$, we have $\Theta(\gamma) = -E(x) < 0$ and for $\gamma = E(x)$ we have

$$\Theta(\gamma) \equiv E(x) - \left[ \frac{\lambda \Pr(x \in (yz, \gamma)) E(x \mid x \in (yz, \gamma))}{\lambda \Pr(x \in (yz, \gamma)) + (1-\lambda)} \Pr(x \in (yz, \gamma)) + (1-\lambda) \right] > E(x) - E(x) = 0.$$  

Thus, by the Intermediate Value Theorem, there exists a solution

$$\Theta(\gamma^*) = 0 \text{ for some } \gamma^* \in (0, E(x)).$$  

**Proof of Corollaries 3–5.** Follows directly from Proposition 3.

**Proof of Proposition 4.** The market price $P_1(x) = \frac{\pi^2 x^2}{R}$ is clearly monotonic in $x$. Moreover, for the congruent manager (i.e., for which $\beta < \frac{1}{2}$), the relative or percentage price drop between the nondisclosure price $P_1(ND) = \frac{\pi^2 \gamma^2}{R}$ and the price following disclosure of bad news $P_1(x) = \frac{\pi^2 (x (1 - 2\beta))^2}{R}$ is given by $4\beta (1 - \beta)$.  

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\[19\] Following Baye’s rule, the market will update the prior $\lambda$ that the manager is informed according to Bayes’s rule:

$$\Pr(\text{Manager} = \text{Informed} \mid ND) = \frac{\lambda \Pr(x \in ND) \Pr(ND)}{\lambda \Pr(x \in ND) + (1-\lambda)}.$$  

(A.6)
**Proof of Corollary 6.** Follows directly from Proposition 4 and Lemma 2.

**Proof of Proposition 5.** The equilibrium of Proposition 3 requires that

\[ BD = [0, \max [1 - 2\beta, 0] \gamma) \]
\[ ND = [\max [1 - 2\beta, 0] \gamma, \gamma] \]
\[ GD = (\gamma, \infty) \]

\[ E(x|ND) = \frac{\lambda \Pr(x \in ND) E(x|x \in ND)}{\lambda \Pr(x \in ND) + (1 - \lambda)} + \frac{(1 - \lambda) E(x)}{\lambda \Pr(x \in ND) + (1 - \lambda)}, \]

where \( \gamma \equiv E(x|ND). \) Letting \( z = \max [1 - 2\beta, 0] \) for convenience, we have

\[ \gamma = \frac{\lambda \gamma (1 - z)}{\lambda \gamma (1 - z) + (1 - \lambda)} + \frac{(1 - \lambda)/2}{\lambda \gamma (1 - z) + (1 - \lambda)}. \]

\[ 2\gamma [\lambda \gamma (1 - z) + (1 - \lambda)] = \lambda \gamma (1 - z) [\gamma (1 + z)] + (1 - \lambda) \]
\[ \iff 2\lambda \gamma^2 (1 - z) + 2\gamma (1 - \lambda) - \lambda \gamma^2 (1 - z) (1 + z) - (1 - \lambda) = 0 \]
\[ \iff \lambda \gamma^2 (1 - z)^2 + 2\gamma (1 - \lambda) - (1 - \lambda) = 0 \]
\[ \iff \frac{\lambda (1 - z)^2}{1 - \lambda} \gamma^2 + 2\gamma - 1 = 0. \]

The proposition follows by letting \( \rho \equiv \frac{\lambda (1 - z)^2}{1 - \lambda} \in (0, \infty). \)  

**Proof of Corollary 7.** Let

\[ \Gamma(\gamma, z) \equiv \frac{\lambda (1 - z)^2}{1 - \lambda} \gamma^2 + 2\gamma - 1. \]

Using the Implicit Function Theorem,

\[ \frac{\partial \gamma}{\partial z} = -\frac{\partial \Gamma / \partial \gamma}{\partial \Gamma / \partial z} = -\frac{2\lambda (1 - z)}{2\lambda (1 - z)^2 \gamma + 1 - \lambda} = \frac{\lambda (1 - z)}{\lambda (1 - z)^2 \gamma + 1 - \lambda} > 0. \]

For \( \beta < \frac{1}{2}, z = (1 - 2\beta) \) increases as \( \beta \) decreases. Thus, the upper threshold or the nondisclosure short-term price, \( \gamma = E(x|ND), \) increases for a more congruent manager (equivalently, the good news disclosure region \( GD \) shrinks). Turning our attention to the lower disclosure threshold \( z\gamma \) (i.e., \( (1 - 2\beta) \gamma) \):

\[ \frac{\partial [z\gamma]}{\partial z} = \gamma + z \frac{\partial \gamma}{\partial z} = \gamma + \frac{\lambda z (1 - z)}{\lambda (1 - z)^2 \gamma + 1 - \lambda} \]
\[ = \gamma \left[ \frac{\lambda (1 - z)^2 \gamma + 1 - \lambda + \lambda z (1 - z)}{\lambda (1 - z)^2 \gamma + 1 - \lambda} \right] > 0. \]

Overall, the nondisclosure interval shifts upward, and the bad news disclosure region expands.
PROOF OF COROLLARY 8. Using the Implicit Function Theorem again,
\[
\frac{\partial \gamma}{\partial \lambda} = -\frac{\partial \Gamma}{\partial \lambda} = \Gamma (\gamma, z) \equiv -\frac{(1 - z)^2}{(1 - \lambda)^2 \gamma^2} < 0.
\]
That is, for any given \(\beta\), the threshold decreases as the probability that the manager is informed increases. Consider the lower threshold for \(\beta < \frac{1}{2}\) (i.e., \(z = 1 - 2\beta > 0\)):
\[
\frac{\partial [z\gamma]}{\partial \lambda} = z \frac{\partial \gamma}{\partial \lambda} < 0.
\]
■

PROOF OF COROLLARY 9. It follows from Proposition 7 and the uniform distribution that
\[
\gamma = \frac{\lambda \left[ 2\beta\gamma + \beta\psi \right] - \lambda \left[ 2\beta\gamma + \beta\psi \right] / 2 + (1 - \lambda) / 2}{\lambda \left[ 2\beta\gamma + \beta\psi \right] + (1 - \lambda)},
\]
where \(\psi \equiv \frac{2(1 - \alpha)\beta}{\alpha}\).

To solve the equilibrium, first assume that, in equilibrium with \(\gamma = \gamma(\psi)\), we have
\[
(1 - 2\beta)\gamma(\psi) > \beta\psi.
\]

We will return and derive the conditions for this assumption to be satisfied in equilibrium. From the above, we have
\[
\gamma = \frac{\lambda \left[ 2\beta\gamma + \beta\psi \right] - \lambda \left[ 2\beta\gamma + \beta\psi \right] / 2 + (1 - \lambda) / 2}{\lambda \left[ 2\beta\gamma + \beta\psi \right] + (1 - \lambda)}.
\]
Thus,
\[
2\gamma \left[ \lambda \left[ 2\beta\gamma + \beta\psi \right] + (1 - \lambda) \right] - \lambda \left[ 2\beta\gamma + \beta\psi \right] \left[ 2\gamma (1 - \beta) - \beta\psi \right] - (1 - \lambda) = 0 \iff
2\gamma \lambda \left[ 2\beta\gamma + \beta\psi \right] + 2\gamma (1 - \lambda) - \lambda \left[ 2\beta\gamma + \beta\psi \right] \left[ 2\gamma (1 - \beta) - \beta\psi \right] - (1 - \lambda) = 0 \iff
4\gamma^2 \lambda \beta + 2\gamma \lambda \beta \psi + 2\gamma (1 - \lambda) - \lambda 2\beta \gamma 2\gamma (1 - \beta) + \lambda \beta \psi (2\gamma (1 - \beta) - 2\beta \gamma) + \lambda \beta^2 \psi^2 - (1 - \lambda) = 0 \iff
4\gamma^2 \lambda \beta + 2\gamma \lambda \beta \psi + 2\gamma (1 - \lambda) - 4\gamma^2 \lambda \beta (1 - \beta) + 2\gamma \lambda \beta \psi (1 - 2\beta) + \lambda \beta^2 \psi^2 - (1 - \lambda) = 0 \iff
\gamma^2 \left( 4\beta^2 \frac{\lambda}{1 - \lambda} \right) + \gamma \left( 2 + 4 \beta \frac{\lambda}{1 - \lambda} \beta (1 - \beta) \right) + \frac{\lambda}{1 - \lambda} \beta^2 \psi^2 - 1 = 0 \iff
\]

The solution to the above quadratic equation is
\[
\gamma(\psi) = \left(-1 + 2\frac{\lambda}{1-\lambda} \beta \psi (1-\beta)\right)
\]
\[
\sum \sqrt{1 + 4\frac{\lambda}{1-\lambda} \beta \psi (1-\beta) + 4 \left(\frac{\lambda}{1-\lambda}\right)^2 \beta^2 \psi^2 (1-2\beta) + 4\beta^2 \frac{\lambda}{1-\lambda}}
\]
\[
\left(4\beta^2 \frac{\lambda}{1-\lambda}\right)
\]
Let \(\rho \equiv \frac{\lambda 4\beta^2}{1-\lambda}\). Thus,
\[
\gamma(\psi) = \left(-1 + 2\frac{\lambda}{1-\lambda} \psi (1-\beta)\right) + \sqrt{1 + \rho + \frac{4\lambda}{1-\lambda} \psi \left((1-\beta) + \left(\frac{\lambda}{1-\lambda}\right) \psi (1-2\beta)\right)}\]
\[
\rho
\]
We need to verify the condition \((1-2\beta)\gamma(\psi) > \beta \psi\). Let,
\[
\Gamma(\gamma, \psi) \equiv \gamma^2 \left(4\beta^2 \frac{\lambda}{1-\lambda}\right) + \gamma \left(2 + 4\frac{\lambda}{1-\lambda} \beta \psi (1-\beta)\right) + \frac{\lambda}{1-\lambda} \beta^2 \psi^2 - 1.
\]
Using the Implicit Function Theorem,
\[
\frac{\partial \gamma}{\partial \psi} = -\frac{\partial \Gamma}{\partial \gamma} \frac{\partial \gamma}{\partial \psi} = -\frac{\gamma^4 \frac{\lambda}{1-\lambda} \beta (1-\beta) + \frac{2\lambda}{1-\lambda} \psi \beta^2}{2\gamma \left(4\beta^2 \frac{\lambda}{1-\lambda}\right) + \left(2 + 4\frac{\lambda}{1-\lambda} \beta \psi (1-\beta)\right)} < 0 \left\{ \text{for case } \beta < \frac{1}{2} \right\}.
\]
This implies that the threshold \(\gamma\) is decreasing in \(\psi\). Moreover, this also implies that the lower threshold \(\max (0, (1-2\beta)\gamma - \beta \psi)\) is also decreasing in \(\psi\). We now verify that the solution \(\gamma(\psi)\) satisfies the assumption that \((1-2\beta)\gamma(\psi) > \beta \psi\). From the monotonicity of \(\gamma(\psi)\) in \(\psi\) documented above, there is a single crossing point \(\psi^*\) such that the assumption is satisfied for all \(\psi < \psi^*\).

**Proof of Corollaries 10 and 11.** We have shown in Corollary 9 that, for \(\psi < \psi^*\), \((1-2\beta)\gamma(\psi) > \beta \psi\) and the disclosure regions are two-tailed. Now, because \(\psi = \frac{2(1-\alpha)\theta}{\lambda}\), an increase in \(\theta\) or a decrease in \(\alpha\) would lead to an increase in \(\psi\). Moreover, because \(\gamma(\psi)\) is a decreasing function, for a given \(\alpha\) level, there exists a lower bound for \(\theta\) above which the condition would be violated, and the voluntary disclosure equilibrium is upper tailed; similarly, for a given \(\theta\), there exists an upper bound for \(\alpha\) below which the
voluntary disclosure equilibrium is again upper tailed. Once $\psi$ exceeds the crossing point $\psi^*$, it follows from above that the equilibrium disclosure threshold no longer depends on $\psi$ and is given by $\gamma = \gamma(\psi^*)$.

Finally, we devote the last part of the proof to verify that earlier comparative statics results have not changed due to the public signal $\theta$ introduced in this section. In particular, note that, by using the Implicit Function Theorem,

$$\frac{\partial \gamma}{\partial \beta} = -\frac{\partial \gamma}{\partial \lambda} \propto -\frac{8\gamma^2 \left( \frac{\beta}{1-\lambda} \right) + \gamma \left( 4 \frac{\lambda}{1-\lambda} \psi - 8 \frac{\lambda}{1-\lambda} \beta \psi \right) + 2 \frac{\lambda}{1-\lambda} \beta \psi^2}{2\gamma \left( 4\beta^2 \frac{\lambda}{1-\lambda} \right) + \left( 2 + 4 \frac{\lambda}{1-\lambda} \beta \psi (1-\beta) \right)} < 0 \left[ \text{for case } \beta < \frac{1}{2} \right].$$

Thus, the threshold $\gamma$ is decreasing in $\beta$. Moreover, this implies, as before, that the lower threshold $\max(0, (1-2\beta)\gamma - \beta \psi)$ is also decreasing in $\beta$.

Similarly,

$$\frac{\partial \gamma}{\partial \lambda} = -\frac{\partial \gamma}{\partial \lambda} \propto -\frac{\gamma^2 (4\beta^2) + \gamma (4\beta \psi (1-\beta)) + \beta^2 \psi^2}{2\gamma \left( 4\beta^2 \frac{\lambda}{1-\lambda} \right) + \left( 2 + 4 \frac{\lambda}{1-\lambda} \beta \psi (1-\beta) \right)} < 0 \left[ \text{for case } \beta < \frac{1}{2} \right].$$

We conclude that the disclosure thresholds both decrease when the manager is more informed. It can be shown here that at the limit when the manager is fully informed, there is full disclosure. \[\square\]

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