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Team Decision Theory and Integral Equations

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Abstract. The coordination of decisions under uncertainty in a team leads to optimality conditions that are integral equations. A specific example of a two-division firm is developed to illustrate these conditions. Numerical imbedding techniques are used to solve the firm's decision problem. Extensions toward more general techniques and applications are indicated.

Key Words. Team decision theory, command and control, organization theory, integral equations, numerical methods.

1. Introduction

Organizations face decision problems that are more complex than the problems of a single agent. As Radner (Ref. 1) points out, there may be differences among members of the organization with respect to possibilities of action, information, and preferences. In addition, there may be uncertainties about the actions of other members, making coordination difficult. The theory of team decisions ignores the possibility of internal differences in preferences in order to concentrate on the study of how communication helps coordinate the decisions of individual members.

In this paper, we look at the specific problem of computing optimal decision rules for a team with a given communication or information system. The optimal decision rules must satisfy a system of integral equations which can be quite complicated in general (nonlinear, infinite limits of integration, with multiple variables of integration). Previous works in team theory have circumvented these difficulties by selecting problems with known solutions.

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Algorithms for the solution of the general problem have not yet been developed, but we have taken a first step in that direction by applying a parametric imbedding technique to a specific numerical problem which does not have a closed-form solution.

Section 2 summarizes the decision problem of a multimember team. Section 3 looks at an economic application of the theory with simplifying assumptions that allow a closed-form solution. In Section 4, this example is modified to present the more typical computationally complex team problem. Section 5 presents numerical results for this team decision problem and conclusions and discussions are found in Section 6.

2. Team Decision Theory

A team is an organization whose members share a single, well-defined objective function. Such a harmonious group has but one problem: how are individual activities coordinated in an optimal fashion? Team decision theory explores such problems when the organization is uncertain about its environment and when information about environment differs among team members. The decision problem reduces to the selection of rules of action that coordinate the interdependent activities of the teammates to maximize the expected payoff of the team.

Organizations are seldom harmonious and a game-theoretic model may seem more appropriate than a team-theoretic model. Since team theory is an element of a more general normative approach to the problem of *organization*, Marschak and Radner (Ref. 2) emphasized harmony in order to study the use of communication in the task of coordinating actions.[5] Team decision theory is an extension of Bayesian statistical decision theory to a multimember organization. The basic difference between these two decision theories is that the information provided each team member may be different. In statistical decision theory, the action may consist of several components, but each component decision is based on the same information.

The team consists of n decision makers or teammates, indexed by $i=1,2,\ldots,n$. The basic elements of the team decision problems are as follows:

$\theta\in\Theta\subseteq R^l$, the unknown state of nature;
$A=(a_1,\ldots,a_n)\in\mathscr{A}\subseteq R^n$, the actions of the teammates;
$U(A,\theta)$, the team's utility function;
$Y=(y_1,y_2,\ldots,y_n)\in\mathscr{Y}\subseteq R^n$, the information of the teammates[6];
$f(\theta)$, the team's prior probability density of θ;
$f(Y|\theta)$, *the team's conditional prior probability density of* Y, given θ;
$\alpha(Y)=(\alpha_1(y_1),\ldots,\alpha_n(y_n))\in\Delta$, the team's decision function.[7]

The crucial points to notice are these: (a) there is only one utility function, agreed upon by all members; (b) the utility function is not necessarily separable; that is, in general, $U_{a_ia_j}\neq 0$; (c) there is only one pair of probability densities, $f(\theta)$ and $f(Y|\theta)$, agreed upon by all members; (d) the information of the ith teammate y_i is different from the jth teammate's information y_j; and (e) since the ith teammate knows only y_i, his action depends only on y_i; i.e.,

$$a_i=\alpha_i(y_i).$$

Each teammate wants to select decision rules that are coordinated to maximize the team's expected utility

$$W[\alpha]=\int_\Theta\int_y U(\alpha(Y),\theta)f(Y|\theta)f(\theta)\,dY\,d\theta. \qquad (1)$$

How can we characterize the optimal decision rules for the teammates? Let $\alpha^*(Y)$ denote the optimal decision rule: that is,

$$W[\alpha^*]\geq W[\alpha]$$

for all decision rules $\alpha\in\Delta$. Any arbitrary decision rule can be written as $\alpha_i^*(y_i)+\delta_i\gamma_i(y_i)$, where δ_i is an arbitrary scalar and $\gamma_i(y_i)$ is a function of the ith teammate's information. Thus, any team decision rule can be expressed as

$$\alpha(Y)=\alpha^*(Y)+D\gamma(Y),$$

[5] The normative theory might be divided into three stages: (i) an organization must create a group objective function by constitutionally *aggregating* individual objectives (see Arrow, Ref. 3; DeGroot, Ref. 4, and Dalkey, Ref. 5); (ii) individuals must be motivated to act with the group objective in mind (see Groves, Ref. 6); and (iii) optimal strategies must be specified for individuals (this is the basic problem investigated in team decision theory).

[6] The information that the ith teammate uses may come from two sources, a personal observation of the environment or a message from another teammate that summarizes his knowledge about the environment. Hence, it may seem more natural to make each component y_i a vector itself; but this will significantly complicate the results that follow (see Section 6 for further discussion). One might imagine that the vector of information has been reduced to a single *statistic*.

[7] The function space Δ is presumed to be some complete normed linear vector space. The only important distinction that we want to make is that the ith component function $\alpha_i()$ depends only on y_i.

where D is a diagonal matrix with elements $\delta_1, \delta_2, \ldots, \delta_n$ along the diagonal and

$$\gamma(Y) = (\gamma_1(y_1), \gamma_2(y_2), \ldots, \gamma_n(y_n)) \in \Delta.$$

The optimality of $\alpha^*(Y)$ can be expressed by the marginal conditions

$$\{\partial W[\alpha^* + D\gamma]/\partial\delta_i\}_{\delta_1 = \cdots = \delta_n = 0} = 0, \qquad i = 1, 2, \ldots, n, \tag{2}$$

for all $\gamma \in \Delta$. Radner (Ref. 7) has shown that these conditions (2) can be expressed as follows.

Theorem 2.1. *Person-by-Person Optimality.* If $\alpha^*(Y)$ is the optimal team decision rule, then it must satisfy the following equations:

$$0 = \int_{\mathcal{Y}_1} \cdots (i) \cdots \int_{\mathcal{Y}_n} \int_{\Theta} U_{a_i}(\alpha^*(Y), \theta) f(Y(i), \theta \mid y_i)\, d\theta\, dY(i) \tag{3}$$

for all $y_i \in \mathcal{Y}_i$. Here

$$f(Y(i), \theta | y_i) = f(Y|\theta) f(\theta) / f_i(y_i)$$

is the posterior probability of θ and

$$Y(i) \equiv (y_1, \ldots, y_{i-1}, y_{i+1}, \ldots, y_n),$$

given y_i.[8] The equations (3) can be written succinctly as

$$E\{U_{a_i}(\alpha^*(Y), \theta) | y_i\} = 0 \qquad \text{for all } y_i \in \mathcal{Y}_i. \tag{4}$$

The optimality conditions are called *person-by-person*, because they have the interpretation that the ith teammate, assuming that his colleagues are using their best decision rules, picks a decision rule such that his posterior expected marginal utility equals zero no matter what information he might receive.

The optimality conditions for the single-agent, statistical decision problem are similar to (3), but not nearly so complicated. In statistical decision theory, the single decision maker has the privilege of waiting until the information is obtained, modifying his probability judgements using Bayes' rule, and then selecting a single action to maximize posterior expected utility. The ith teammate cannot delay computation of his entire decision function, because other teammates must know his decision rule in order to predict his actions and thus coordinate their decisions. The person-by-person optimality conditions must be solved simultaneously before information is gathered.

[8] Radner (Ref. 7) has also shown that differentiability and concavity of $U(A, \theta)$ in A for every θ makes (3) sufficient as well as necessary. We implicitly assume throughout that U is concave and differentiable.

The person-by-person optimality conditions (3) are, in the most general case, a nonlinear interdependent system of integral equations. In some important special cases, the solution is relatively easy. Most analytic contributions to team theory have concentrated on these specific solutions. However, for slightly different problems, the computation (on the back of an envelope or empirically) of $\alpha^*(Y)$ is nontrivial. The application of team theory to more realistic problems requires the development of algorithms for the solution of person-by-person optimality conditions. The next section introduces an example that has a closed-form solution, and the following sections look at a more complicated case.

3. Multidivisional Firm: Quadratic–Gaussian Example

Suppose that a firm consists of two autonomous divisions that produce different commodities in the amounts a_1 and a_2, respectively. The commodities are sold in a competitive market at prices P_1 and P_2. Because of random variations in supply and demand, the prices are not known precisely until the instant the commodities are sold. Each division separately gathers information about the market it sells in and uses this information to help select its quantity of output. Let y_i be the *price forecast*, which the ith division uses in its decision making.[9] Suppose that the divisions believe that prices are distributed jointly Gaussian-normal with zero means[10] and a covariance matrix

$$\begin{bmatrix} 1 & r \\ r & 1 \end{bmatrix}.$$

Assume that the price forecast y_i is distributed Gaussian-normal with mean of P_1 and variance equal to 1.

The firm's total revenue is

$$P_1 a_1 + P_2 a_2.$$

Suppose that the total cost to the firm of producing quantities a_1 and a_2 is

$$c(a_1, a_2) = \tfrac{1}{2} c_{11} a_1^2 + c_{12} a_1 a_2 + \tfrac{1}{2} c_{22} a_2^2.$$

[9] There is no communication between divisions in these examples.

[10] The fact that expected prices are zero is unimportant and is made to simplify calculations.

Notice that, since c_{12} is nonzero, there is an interdependence between the production in two divisions.[11] The firm's objective function is the expected profit

$$E\{P_1\alpha_1(y_1)+P_2\alpha_2(y_2)-\tfrac{1}{2}c_{11}\alpha_1(y_1)^2-c_{12}\alpha_1(y_1)\alpha_2(y_2)-\tfrac{1}{2}c_{22}\alpha_2(y_2)^2\}.$$

The person-by-person optimality conditions which the output decision rules $\alpha_1^*(y_1)$ and $\alpha_2^*(y_2)$ must satisfy are:

$$0=\int_{-\infty}^{\infty} P_1 f(P_1|y_1)\,dP_1-c_{11}\alpha_1(y_1)-c_{12}\int_{-\infty}^{\infty}\alpha_2(y_2)f(y_2|y_1)\,dy_2, \quad \text{for } -\infty<y_1<\infty; \tag{5}$$

$$0=\int_{-\infty}^{\infty} P_2 f(P_2, y_2)\,dP_2-c_{22}\alpha_2(y_2)-c_{12}\int_{-\infty}^{\infty}\alpha_1(y_1)f(y_1|y_2)\,dy_1, \quad \text{for } -\beta<y_2<\infty. \tag{6}$$

The posterior probability densities can be calculated easily, with the result that

$$f(P_1|y_1)\sim N(\tfrac{1}{2}y_1;1), \qquad f(P_2|y_2)\sim N(\tfrac{1}{2}y_2;1),$$
$$f(y_2|y_1)\sim N(r/2y_1;\sqrt{[1-(r/2)^2]}), \qquad f(y_1|y_2)\sim N(r/2y_2;\sqrt{[1-(r/2)^2]}).$$

Equations (5) and (6) constitute a system of linear Fredholm integral equations of the second type. While their solution may not be self-evident, because the regression of P_i on y_i is linear, as is the regression of y_j on y_i, one can easily verify that the optimal output decision rules will be linear functions of the price forecasts[12]:

$$\alpha_1^*(y_1)=\beta_1 y_1, \qquad \alpha_2^*(y_2)=\beta_2 y_2.$$

The optimality conditions (5) and (6) imply that the slope coefficients β_1 and β_2 must satisfy the linear equations

$$0=\tfrac{1}{2}-c_{11}\beta_1-(r/2)c_{12}\beta_2, \tag{7}$$
$$0=\tfrac{1}{2}-(r/2)\beta_1-c_{22}\beta_2. \tag{8}$$

[11] The coefficients are assumed to satisfy the following restrictions:

$$c_{11}>0, \qquad c_{22}>0, \qquad c_{11}c_{22}>c_{12}^2,$$

so that costs are convex in output.

[12] See Radner (Ref. 7).

The optimal team output rules relating forecasts of market prices to quantities produced are

$$\alpha_1^*(y_1)=(2c_{22}-rc_{12})y_1/(4c_{11}c_{22}-r^2c_{12}^2), \tag{9}$$
$$\alpha_2^*(y_2)=(2c_{11}-rc_{12})y_2/(4c_{11}c_{22}-r^2c_{12}^2). \tag{10}$$

Much of the work on team decision theory has used the quadratic–Gaussian assumptions of this example, primarily because the optimal decision rules are known to be linear in the information variables. The quadratic utility function might be justified as a second-order approximation of a more general utility function, but it has some theoretical shortcomings (see Arrow, Ref. 3). The use of Gaussian-normal probability densities also has defects. Prices should always be nonnegative, yet with Gaussian densities there is always a finite probability that the price is negative.

4. Multidivisional Firm: Quadratic–Uniform Example

The discussion of the previous paragraph suggests that a probability density for prices to be picked that is not Gaussian-normal. In this section, we will develop an example where prices and price forecasts are (jointly) uniformly distributed over a compact set. We will continue to assume that the cost function is quadratic in order to prevent the appearance of nonlinear integral equations.

Assume that the firm believes that the relative prices of its two commodities are fixed, but is uncertain about the price level. That is, the price vector that will occur is $(\bar{P}_1\theta, \bar{P}_2\theta)$, where θ is a random variable and $\bar{P}_1$ and $\bar{P}_2$ are fixed numbers. The expected profit of the firm is

$$E\{\theta(\bar{P}_1\alpha_1(y_1)+\bar{P}_2\alpha_2(y_2))-\tfrac{1}{2}c_{11}\alpha_1(y_1)^2-c_{12}\alpha_1(y_1)\alpha_2(y_2)-\tfrac{1}{2}c_{22}\alpha_2(y_2)^2\}.$$

Suppose that the price level θ and the price forecasts of the individual divisions y_1 and y_2 are uniformly distributed with the joint probability density

$$f(\theta, y_1, y_2)=\begin{cases}3, & 0\le y_1\le\theta\le 1,\ 0\le y_2\le\theta\le 1,\\ 0, & \text{elsewhere}.\end{cases} \tag{11}$$

One can calculate the needed posterior probability densities without much difficulty. These are given here in closed form:

$$f(\theta|y_i)=\begin{cases}2\theta/(1-y_i^2), & y_i\le\theta\le 1,\\ 0, & \text{elsewhere};\end{cases} \tag{12}$$

$$f(y_j\,|\,y_i)=\begin{cases}2(1-\max[y_1,y_2])/(1-y_i^2), & 0\le y_j\le 1,\\ 0, & \text{elsewhere.}\end{cases}\tag{13}$$

Using these posterior probability densities, the optimal decision rules $\alpha_1^*(y_1)$ and $\alpha_2^*(y_2)$ must satisfy the following system of linear Fredholm equations:

$$\begin{aligned}\alpha_1^*(y_1)&=\frac{\bar P_1}{c_{11}}\int_{y_1}^1\theta\frac{2\theta}{1-y_1^2}\,d\theta-\frac{c_{12}}{c_{11}}\int_0^1\alpha_2^*(y_2)\frac{2(1-\max[y_1,y_2])}{1-y_1^2}\,dy_2\\ &=\frac{2}{3}\frac{\bar P_1}{c_{11}}\frac{1-y_1^3}{1-y_1^2}-\frac{c_{12}}{c_{11}}\int_0^1\alpha_2^*(y_2)\frac{2(1-\max[y_1,y_2])}{1-y_1^2}\,dy_2,\end{aligned}\tag{14}$$

$$\begin{aligned}\alpha_2^*(y_2)&=\frac{\bar P_2}{c_{22}}\int_{y_2}^1\alpha\frac{2\theta}{1-y_2^2}\,d\theta-\frac{c_{12}}{c_{22}}\int_0^1\alpha_1^*(y_1)\frac{2(1-\max[y_1,y_2])}{1-y_2^2}\,dy_1\\ &=\frac{2}{3}\frac{\bar P_2}{c_{22}}\frac{1-y_2^3}{1-y_2^2}-\frac{c_{12}}{c_{22}}\int_0^1\alpha_2^*(y_1)\frac{2(1-\max[y_1,y_2])}{1-y_2^2}\,dy_1.\end{aligned}\tag{15}$$

5. Numerical Solution

The integral equations (14) and (15) of the quadratic–uniform team example are of the following form:

$$u(t)=g_1(t)+\int_0^1 K_1(y,t)v(y)\,dy,\qquad 0\le t\le 1,$$

$$v(t)=g_2(t)+\int_0^1 K_2(y,t)u(y)\,dy,\qquad 0\le t\le 1.$$

The forcing functions are identical, except for multiplication by a scalar

$$g_1(t)=kg_2(t).$$

Similarly, the kernels differ only by multiplication by a constant,[13]

$$K_1(y,t)=hK_2(y,t).$$

The kernels can be written in the following form:

$$K_i(y,t)=\begin{cases}\beta_i(t), & 0\le y\le t,\\ \gamma_i(t)\delta_i(y), & t\le y\le 1.\end{cases}$$

[13] Here, $k=(\bar P_1/c_{11})(c_{22}/\bar P_2)$ and $h=c_{22}/c_{11}$.

That is, the kernels are semidegenerate. In the following, we want to apply an imbedding technique for semidegenerate kernels to compute the solution of (14) and (15).

At this point in time, our numerical algorithm can handle only a single integral equation. As a result, the numerical problem was simplified by assuming that the multiplicative constants k and h were equal to one, or

$$c_{11}=c_{22}\qquad\text{and } \bar P_1=\bar P_2.$$

In this symmetric case, both integral equations are identical, so it must be true that

$$u(t)=v(t)\qquad\text{for } 0\le t\le 1.$$

Hence the problem reduces to the solution of the following single integral equation:

$$u(t)=g_1(t)+\int_0^1 K_1(y,t)u(y)\,dy.\tag{16}$$

By selecting

$$c_{11}=c_{12}=1\qquad\text{and } \bar P_1=1,$$

we can make this, more specifically,

$$u(y_1)=\frac{2}{3}\frac{1-y_1^3}{1-y_1^2}-\int_0^1\frac{2(1-\max[y_1,y_2])}{1-y_1^2}u(y_2)\,dy_2.\tag{17}$$

The numerical algorithm is based on the solution of a class of problems indexed by an upper limit of integration x:

$$u(t,x)=g(t)+\int_0^x K(t,y)u(y,x)\,dy,\qquad 0\le t\le x,\tag{18}$$

where the kernel K and the inhomogeneous term g are given, and the function u is to be determined for $0\le t\le x$. The upper limit has been written as x, for we intend to study the solution u primarily as a function of x for a fixed value of t, $x\ge t$. This also accounts for the fact that u is written as $u(t,x)$, rather than $u(t)$, in Eq. (18).

We assume that the kernel has the semidegenerate form

$$K(t,y)=\begin{cases}\sum_{i=1}^M\alpha_i(t)\beta_i(y), & 0\le y\le t,\\ \sum_{i=1}^N\gamma_i(t)\delta_i(y), & t\le y\le x.\end{cases}\tag{19}$$

In the event that the kernel K is not given in the form displayed in Eq. (19), it may be possible to approximate it by an appropriate series, e.g., a sum of powers, or Legendre polynomials, or a trigonometric series.

Our aim is to transform the given integral equation into an initial-value problem, i.e., a system of ordinary differential equations with known initial conditions. Modern digital, analog, and hybrid computers can solve several thousand such simultaneous differential equations with speed and accuracy.

A derivation of an initial value for determining the function $u(t, x)$ is given by Kagiwada and Kalaba (Ref. 9). Let us now summarize the derived Cauchy system. The functions $\{e_m\}$ and $\{r_{mn}\}$ are determined by the differential equations

$$e'_m(x) = \left[g(x) + \sum_{i=1}^{M} \alpha_i(x)e_i(x)\right] \cdot \left[\beta_m(x) + \sum_{j=1}^{N} r_{mj}(x)\delta_j(x)\right], \tag{20}$$

$$r'_{mn}(x) = \left[\gamma_n(x) + \sum_{i=1}^{M} \alpha_i(x)r_{in}(x)\right] \cdot \left[\beta_m(x) + \sum_{j=1}^{N} r_{mj}(x)\delta_j(x)\right], \tag{21}$$

where

$$m = 1, 2, \ldots, M, \qquad n = 1, 2, \ldots, N, \qquad x \geq 0,$$

together with the initial conditions at $x = 0$ given by

$$e_m(0) = 0, \qquad m = 1, 2, \ldots, M, \tag{22}$$

$$r_{mn}(0) = 0, \qquad m = 1, 2, \ldots, M, \qquad n = 1, 2, \ldots, N. \tag{23}$$

These equations are integrated from $x = 0$ to $x = t$. At $x = t$, the differential equations for u and J_n are adjoined; these are Eqs. (24) and (25) below. If the original integral equation had an upper limit of integration T, then the solution $u(t, T)$ is computed by integrating Eqs. (20), (21), (24), and (25) from $x = t$ to $x = T$ for each $0 \leq t \leq T$. Specifically,

$$u_x(t, x) = \Phi(t, x)u(x, x), \tag{24}$$

$$J'_n(t, x) = \Phi(t, x)J_n(x, x), \tag{25}$$

with

$$n = 1, 2, \ldots, N, \qquad 0 \leq t \leq x.$$

The function Φ is expressed in terms of $J_1, J_2, \ldots, J_n$ as follows:

$$\Phi(t, x) = \sum_{j=1}^{N} J_j(t, x)\delta_j(x), \tag{26}$$

where

$$J'_n = \partial J_n/\partial x,$$

and $u(x, x)$ and $J_n(x, x)$ are expressed in terms of e and r as follows:

$$u(x, x) = g(x) + \sum_{i=1}^{M} \alpha_i(x)e_i(x), \tag{27}$$

$$J_n(x, x) = \gamma_n(y) + \sum_{i=1}^{M} \alpha_i(x)r_{in}(x). \tag{28}$$

The initial conditions on the function u and J_n are given in Eqs. (27) and (28) for $x = t$; i.e.,

$$[u(t, x)]_{x=t} = u(t, t) = g(t) + \sum_{i=1}^{M} \alpha_i(t)e_i(t) \tag{29}$$

$$[J_n(t, x)]_{x=1} = J_n(t, t) = \delta_n(t) + \sum_{i=1}^{M} \alpha_i(t)r_{in}(t), \tag{30}$$

with

$$n = 1, 2, \ldots, N, \qquad t > 0.$$

A FORTRAN program for such a numerical solution is given in Kagiwada and Kalaba (Ref. 9). The numerical results of this program for the integral equation (17) are given in Table 1. As a check on the results, it was shown that the solution points satisfy the trapezoidal approximation of the integral equation (17) with an accuracy of up to the fourth significant figure.

The numerical solution indicates that the optimal team decision rule for output is a monotonically increasing function of the price forecast. That is,

Table 1. Numerical solution for optimal decision function (integration stepsize = 0.01).

y_i	$\alpha_i^*(y_i)$
0.0	0.3026
0.1	0.3081
0.2	0.3224
0.3	0.3434
0.4	0.3695
0.5	0.3996
0.6	0.4328
0.7	0.4685
0.8	0.5062
0.9	0.5455
1.0	0.5863

when a division feels that the price level is going to be low, then it should produce a relatively small quantity of its good; when the price level is forecast to be high, the division will produce large amounts of its good. Also, the numerical solution has a distinctly convex shape, so that output is more sensitive to price forecasts when y_i is large than when it is small. When price forecasts are large, a bigger gamble can be taken.

6. Conclusions

The development of techniques for specifying optimal decisions when there is uncertainty and information about the environment has had a major impact in many areas, from the management of corporations to the actions of military command. The application of this statistical decision theory was greatly facilitated by the development of conjugate probability functions, nonlinear optimization procedures, etc. Team decision theory promises to open new frontiers by investigating the multiperson decision problems involving communication and coordination. Team theory has existed as an analytic tool since the work of Marschak in the early 1950's (Refs. 10–11). Yet, the number of applications of team theory is small (see Beckmann, Ref. 12; McGuire, Ref. 13; and Kriebel, Ref. 14). This is undoubtedly due to the difficulties of formulating and solving the basic team decision problems. Numerical techniques have not been applied to the team problem; hence, most studies have been restricted to the examples which have well-known closed-form solutions.

In this paper, we have attempted to show how one step can be taken in the direction of generality; the optimality conditions of team decision theory were shown to be amenable to solution by techniques of parametric imbedding. Many more such steps will have to be added before numerical solutions of more difficult team problems can be found routinely. We only need to point out that the solution procedure of Section 5 was dependent upon the quadratic assumption on utility functions, the compactness of the interval of integration, the symmetry assumptions that reduced a system of integral equations to a single integral equation, and the assumption of a scalar information variable.

One justification of numerical studies is that numerical solutions may lead to insights which can be translated into heuristic rules of thumb. We agree that the numerical approach to team decision theory will reinforce analytic conclusions with respect to optimal organizing and may even suggest regularities that should be investigated with analytic techniques. But more subtly, the authors feel that the numerical techniques may provide new

analytic approaches to the study of team theory. In particular, the parametric imbedding solution technique may help us analyze the theoretical relationships between adjustment of parameters and changes in decision rules.

Finally, *procedures* for selecting several decision rules to optimize a single objective function are the topics of investigation in the theory of decentralized optimization. The theory of decentralized optimization pioneered by Hurwicz, Arrow, Malinvaud, Dantzig, and Wolfe should provide further tools for investigating the solution of team problems.[14] The authors have begun some preliminary studies along this line, and hope to tie them into the numerical algorithm discussed in this paper.

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[14] See Arrow and Hess (Ref. 15) or Heal (Ref. 16).

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Confidence Structures in Decision Making[1]

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Abstract. Decision making is defined in terms of four elements: the set of decisions, the set of outcomes for each decision, a set-valued criterion function, and the decision maker's value judgment for each outcome. Various confidence structures are defined, which give the decision maker's confidence of a given decision leading to a particular outcome. The relation of certain confidence structures to Bayesian decision making and to membership functions in fuzzy set theory is established. A number of schemes are discussed for arriving at *best* decisions, and some new types of domination structures are introduced.

Key Words. Confidence structures, domination structures, chance constraint formulation, multicriteria decision making, hierarchy of decision processes.

1. Introduction

We consider the process of decision making to be composed of four elements:

(i) the set of all feasible alternatives (decisions) X with elements denoted by x;

(ii) the set of all possible outcomes $Y(x) \subset R^m$ for each feasible alternative $x \in X$;

(iii) the criterion function $f(\cdot): x \mapsto Y(x)$, a set-valued function that measures the *value* of a decision;

(iv) the decision maker's *value judgment* or *preference* for each outcome.

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