Optimal Tactics for Close-Support Operations—III. Degraded Intelligence and Communications

J. Hess

Department of Economics
University of Southern California
Los Angeles, California

Harriet Kagiwada

Radar Systems Group,
Hughes Aircraft Company,
Canoga Park, California

R. Kalaba

Departments of Economics and Biomedical Engineering
University of Southern California
Los Angeles, California

K. Spingarn

Space and Communications Group
Hughes Aircraft Company
Los Angeles, California

and

C. Tsokos

Department of Mathematics
University of South Florida
Tampa, Florida

ABSTRACT

A new generation of C^3 (command, control, and communication) models for military cybernetics is developed. Recursive equations for the solution of the C^3 problem are derived for an amphibious campaign with linear time-varying dynamics. Air and ground commanders are assumed to have no intelligence and no communications. Numerical results are given for the optimal decision rules.

0096-3003/80/030217 + 12$01.75

INTRODUCTION

Consider a C^3 (command, control, and communication) problem in which there are two subordinate commanders, both striving to coordinate their decisions to attain the tactical objective set down by superordinate headquarters. In an amphibious campaign, the blue naval force lands ground troops and provides close support. The objective is to move inland a certain distance in a specified time. It is desired to attain the objective at minimum expected cost. The primary output of mathematical models of C^3 is optimal decision rules for force commitments to be employed by subordinate headquarters.

Some general concepts of C^3 are discussed in Refs. [1], [2], and [3]. Recursive equations for the solution of the C^3 problem are derived for linear dynamics with quadratic costs in Refs. [4] and [5]. In the latter references, the blue naval air and ground commanders are assumed to have perfect intelligence with degraded communication between them. In this paper, it is assumed that the blue commanders have no intelligence and no communication. The blue air commander has no intelligence concerning the red air strength commitment for the coming day and has no communication with the blue ground commander. Similarly, the blue ground commander has no intelligence concerning the red ground strength commitment for the coming day and has no communication with the blue air commander.

The general equations for optimal tactics with no intelligence and no communications are derived for a perturbation model followed by the recursive equation solution. The equations may be considered to provide either the optimal decisions for the total campaign with main objective s_0, or the optimal decision increments for a perturbed objective s_0. Numerical results are given for campaigns with time-invariant and time-varying dynamics.

C^3 MODEL

The C^3 model to be derived is a perturbation model with linear time-varying dynamics and quadratic costs. It is assumed that the scenario for the main forces has already been planned. Thus only perturbations about the planned scenario are considered. For example, assume that the main objective is to move inland a distance of 200 miles in 21 days. The perturbed objective might be to move inland an additional 21 miles. The red air and ground strengths, p and q, are the perturbation strength increments (or decrements) about the main strengths in the 21-day campaign.

Let N be the duration of the campaign, and let the distance to the perturbed objective be s_0. Consider K days remaining with the front line at the perturbed position s. The new perturbed position of the front line is

$$S = S(s, p, q, \alpha, \beta), \tag{1}$$

where

S = new perturbed position increment of the front line about the planned position with $K-1$ days remaining,

s = current perturbed position increment of the front line about the planned position with K days remaining,

p, q = red air and ground perturbed strength increments (or decrements) about the main strengths, respectively, with K days remaining,

α, β = blue naval air and ground perturbed strength increments (or decrements) about the main strengths, respectively, with K days remaining.

The daily cost increment (or decrement) is given by

$$C = C(s, p, q, \alpha, \beta, K). \tag{2}$$

An additional cost is assessed if the front line at the end of the campaign is at some perturbed position increment s other than s_0. This terminal cost is

$$\phi = \phi(s). \tag{3}$$

The red air and ground commanders make the decision to employ strength increments p and q respectively each day. The decision making of the enemy is simplified by assuming that p and q are random variables with joint probability density function

$$w = w(p, q). \tag{4}$$

Furthermore, assuming that p and q are independent random variables, the joint probability function can be expressed as the product

$$w(p, q) = P(p)Q(q). \tag{5}$$

The minimum expected cost is defined by

$h_K(s)$ = the expected cost increment (or decrement) of a campaign beginning with the front line at s, of duration K, and employing an optimal sequence of decisions,

$$K=0,1,2,\ldots,N, \qquad \text{all } s \tag{6}$$

Using Bellman's principle of optimality [6], the functions $h_{K+1}(s)$ and $h_K(s)$ are related by the recurrence equation

$$h_{K+1}(s)=\min_{\alpha,\beta}\int\int\{C(s,p,q,\alpha,\beta,K)+h_K[S(s,p,q,\alpha,\beta)]\}P(p)Q(q)\,dp\,dq,$$

$$K=0,1,2,\ldots,N-1. \tag{7}$$

All integrals on p and q are from $-\infty$ to ∞. When no time remains,

$$h_0(s)=\phi(s). \tag{8}$$

The conditions for obtaining the minimum are

$$0=\frac{\partial}{\partial\alpha}\int\int B_{K+1}(s,p,q,\alpha,\beta)P(p)Q(q)\,dp\,dq, \tag{9}$$

$$0=\frac{\partial}{\partial\beta}\int\int B_{K+1}(s,p,q,\alpha,\beta)P(p)Q(q)\,dp\,dq, \tag{10}$$

where

$$B_{K+1}(s,p,q,\alpha,\beta)=C(s,p,q,\alpha,\beta,K)+h_K[S(s,p,q,\alpha,\beta)]. \tag{11}$$

Equations (9) and (10) can then be written

$$0=\int\int\{C_\alpha(s,p,q,\alpha,\beta,K)+h_K'[S(s,p,q,\alpha,\beta)]S_\alpha(s,p,q,\alpha,\beta)\}P(p)Q(q)\,dp\,dq, \tag{12}$$

$$0=\int\int\{C_\beta(s,p,q,\alpha,\beta,K)+h_K'[S(s,p,q,\alpha,\beta)]S_\beta(s,p,q,\alpha,\beta)\}P(p)Q(q)\,dp\,dq, \tag{13}$$

$$K=0,1,2,\ldots,N-1,$$

where $\alpha=\alpha(K+1,s)$ and $\beta=\beta(K+1,s)$.

Equations (12) and (13) are the dynamic headquarters-by-headquarters optimality conditions. They state that at every decision-making opportunity, each headquarters is to make the decision which reduces the marginal expected cost of the remainder of the process to zero.

For linear time-varying dynamics with quadratic costs, the new perturbed position of the front line is assumed to be a function of the old perturbed position plus a linear combination of the strengths

$$S = s + C_1\alpha + C_2\beta - C_3 p - C_4 q, \tag{14}$$

where

$$C_1 = C_1(K), \quad C_2 = C_2(K), \quad C_3 = C_3(K), \quad C_4 = C_4(K).$$

The perturbed daily cost is assumed to be proportional to the losses, which in turn are proportional to the strengths utilized. The ground losses are reduced by the air strength for close-support missions. Thus the daily cost is assumed to be

$$C = C_5\alpha + (C_6 - C_7\alpha)\beta + \tfrac{1}{2}C_8\alpha^2 + \tfrac{1}{2}C_9\beta^2, \tag{15}$$

where

$$C_5 = C_5(K), \quad C_6 = C_6(K), \quad C_7 = C_7(K), \quad C_8 = C_8(K), \quad C_9 = C_9(K).$$

The terms in α^2 and β^2 in the above cost expression serve to limit the force commitments made each day. To assure convexity we assume that $C_8 > 0$ and $C_7^2 < C_8 C_9$.

The terminal cost is assumed to be

$$\phi(s) = \lambda(s - s_0)^2. \tag{16}$$

From general control-theoretical considerations, the minimum expected cost has the form

$$h_K(s) = \gamma_K + \delta_K s + \epsilon_K s^2, \tag{17}$$

where the coefficients γ_K, δ_K, and ϵ_K are computed for K stages remaining. Differentiating Eqs. (14), (15), and (17) and substituting into Eqs. (12) and

(13), the following equations are obtained:

$$0=\int\int\{(C_5-C_7\beta)+C_8\alpha+[\delta_K+2\epsilon_K(s+C_1\alpha+C_2\beta-C_3p-C_4q)]C_1\}$$
$$\times P(p)Q(q)\,dp\,dq, \tag{18}$$

$$0=\int\int\{(C_6-C_7\alpha)+C_9\beta+[\delta_K+2\epsilon_K(s+C_1\alpha+C_2\beta-C_3p-C_4q)]C_2\}$$
$$\times P(p)Q(q)\,dp\,dq, \tag{19}$$

which are a set of linear algebraic equations for α and β. Clearly α and β are linear in s.

RECURSIVE EQUATIONS

The recursive equations for the solution of Eqs. (6) and (7) are derived as follows. Making use of the equations

$$\int P(p)\,dp=1, \qquad \int Q(q)\,dq=1, \tag{20}$$

$$\int\int pP(p)Q(q)\,dp\,dq=\bar{p}, \tag{21}$$

$$\int\int qP(p)Q(q)\,dp\,dq=\bar{q}, \tag{22}$$

Eqs. (18) and (19) can be integrated and rearranged to obtain

$$-[C_5+\delta_KC_1+2\epsilon_KC_1s-2\epsilon_KC_1C_3\bar{p}-2\epsilon_KC_1C_4\bar{q}]$$
$$=(2\epsilon_KC_1^2+C_8)\alpha+(2\epsilon_KC_1C_2-C_7)\beta, \tag{23}$$

$$-[C_6+\delta_KC_2+2\epsilon_KC_2s-2\epsilon_KC_2C_3\bar{p}-2\epsilon_KC_2C_4\bar{q}]$$
$$=(2\epsilon_KC_1C_2-C_7)\alpha+(2\epsilon_KC_2^2+C_9)\beta. \tag{24}$$

Solving the simultaneous equations (23) and (24) for α and β, it can be shown that

$$\alpha(K+1,s)=u_K+v_K s, \tag{25}$$

$$\beta(K+1,s)=x_K+y_K s, \tag{26}$$

where

$$u_K=(-B_5B_1+B_2B_3)/D, \tag{27}$$

$$v_K=-2\epsilon_K(C_2C_7+C_1C_9)/D, \tag{28}$$

$$x_K=(-B_4B_3+B_2B_1)/D, \tag{29}$$

$$y_K=-2\epsilon_K(C_1C_7+C_2C_8)/D, \tag{30}$$

$$B_1=C_5+\delta_K C_1-2\epsilon_K C_1C_3\bar{p}-2\epsilon_K C_1C_4\bar{q}, \tag{31}$$

$$B_2=2\epsilon_K C_1C_2-C_7, \tag{32}$$

$$B_3=C_6+\delta_K C_2-2\epsilon_K C_2C_3\bar{p}-2\epsilon_K C_2C_4\bar{q}, \tag{33}$$

$$B_4=2\epsilon_K C_1^2+C_8, \tag{34}$$

$$B_5=2\epsilon_K C_2^2+C_9, \tag{35}$$

$$D=B_4B_5-B_2^2. \tag{36}$$

Substituting the optimal α and β into Eq. (14), the new position of the front line becomes

$$\begin{aligned} S&=s+C_1(u_K+v_K s)+C_2(x_K+y_K s)-C_3p-C_4q\\ &=C_1u_K+C_2x_K+(1+C_1v_K+C_2y_K)s-C_3p-C_4q. \end{aligned} \tag{37}$$

The minimum expected cost, $h_{K+1}(s)$, is obtained by substituting Eqs.

(14), (15), and (17) into Eq. (7):

$$\begin{aligned} h_{K+1}(s) = \int\int \Big\{ & C_5(u_K+v_K s)+\left[C_6-C_7(u_K+v_K s)\right](x_K+y_K s) \\ & +\tfrac{1}{2}C_8(u_K+v_K s)^2+\tfrac{1}{2}C_9(x_K+y_K s)^2+\gamma_K \\ & +\delta_K\left[s+C_1(u_K+v_K s)+C_2(x_K+y_K s) \right. \\ & \left. -C_3 p-C_4 q\right] \\ & +\epsilon_K\left[s+C_1(u_K+v_K s)+C_2(x_K+y_K s)-C_3 p-C_4 q\right]^2\Big\} \\ & \times P(p)Q(q)\,dp\,dq. \end{aligned} \tag{38}$$

To simplify the notation, express Eq. (38) in the form

$$\begin{aligned} h_{K+1}(s) &= \gamma_{K+1}+\delta_{K+1}s+\epsilon_{K+1}s^2 \\ &= \int\int \Big\{ C_5 u_K + C_5 v_K s \\ &\quad + C_6 x_K - C_7 u_K x_K + \left[C_6\, y_K - C_7(u_K y_K + v_K x_K)\right]s - C_7 v_K y_K s^2 \\ &\quad + \tfrac{1}{2}\left(C_8 u_K^2 + C_9 x_K^2\right) + (u_K v_K C_8 + x_K y_K C_9)s + \tfrac{1}{2}\left(C_8 v_K^2 + C_9\, y_K^2\right)s^2 \\ &\quad + \gamma_K + \delta_K(A_1 + A_2 s + A_3 p + A_4 q) \\ &\quad + \epsilon_K\left[(A_1 + A_3 p + A_4 q)^2 + 2A_2(A_1 + A_3 p + A_4 q)s + A_2^2 s^2\right]\Big\} \\ &\quad \times P(p)Q(q)\,dp\,dq. \end{aligned} \tag{39}$$

Integrating and setting the coefficients of the constant terms, s, and s^2 equal, it can be shown that

$$\begin{aligned} \gamma_{K+1} = {} & C_5 u_K + C_6 x_K - C_7 u_K x_K + \tfrac{1}{2}\left(C_8 u_K^2 + C_9 x_K^2\right) + \gamma_K + \delta_K\left(A_1 + A_3 \bar{p} + A_4 \bar{q}\right) \\ & + \epsilon_K\left[A_1^2 + 2\left(A_1 A_3 \bar{p} + A_1 A_4 \bar{q} + A_3 A_4 \overline{pq}\right) + A_3^2\, \overline{p^2} + A_4^2\, \overline{q^2}\right], \end{aligned} \tag{40}$$

$$\begin{aligned} \delta_{K+1} = {} & C_5 v_K + C_6\, y_K - C_7(u_K y_K + v_K x_K) + u_K v_K C_8 + x_K y_K C_9 + \delta_K A_2 \\ & + 2\epsilon_K A_2\left(A_1 + A_3 \bar{p} + A_4 \bar{q}\right), \end{aligned} \tag{41}$$

$$\epsilon_{K+1} = -C_7 v_K y_K + \tfrac{1}{2}\left(C_8 v_K^2 + C_9\, y_K^2\right) + \epsilon_K A_2^2, \tag{42}$$

where

$$A_1 = C_1 u_K + C_2 x_K, \tag{43}$$

$$A_2 = 1 + C_1 v_K + C_2\, y_K, \tag{44}$$

$$A_3 = -C_3, \tag{45}$$

$$A_4 = -C_4. \tag{46}$$

The second moments of p and q are

$$\overline{p^2} = \bar{p}^2 + \sigma_p^2, \qquad \overline{q^2} = \bar{q}^2 + \sigma_q^2, \tag{47}$$

where $\bar{p}$ and $\bar{q}$ are the average values and σ_p and σ_q are the standard deviations of p and q.

The recursive relations are given by Eqs. (40), (41), and (42) supplemented by Eqs. (27) to (30) and (31) to (36). These equations may be considered to provide either:

(1) the optimal daily decisions, α and β, for the total campaign with the main objective s_0, or

(2) the optimal daily decision increments, α and β, for the perturbed objective s_0.

NUMERICAL RESULTS

Numerical results were obtained using the recursive equations derived in the previous paragraphs. At each stage the coefficients in the equations for the optimal blue air strength, α, and the optimal blue ground strength, β, are computed. The new position of the front line is computed in Eq. (37) using the optimal decisions.

Consider a campaign with the following duration, N, and additional distance to be covered, s_0:

$$N = 21 \text{ days}, \qquad s_0 = 21 \text{ miles}.$$

The average red strengths and standard deviations are assumed to be

$$\bar{p} = \bar{q} = 1, \qquad \sigma_p = \sigma_q = 0.5.$$

Assume the coefficients in Eqs. (14), (15), and (16) are constants with the exception of C_5:

$$C_1=0.1, \qquad C_4=1, \qquad C_8=0.02,$$
$$C_2=1, \qquad C_6=0.1, \qquad C_9=0.02,$$
$$C_3=0.1, \qquad C_7=0.01, \qquad \lambda=1.$$

The coefficient C_5 in the cost equation is proportional to the red anti-air strength and may vary during the course of the campaign as the red anti-air fortifications are destroyed. Then for various choices of $C_5(K)$ the optimal air and ground decisions, α and β for $K=21$, 11, and 1 stages to go are as shown in Table 1.

In Case I, the coefficient C_5 is constant. Considering the campaign as a whole with 21 days and 21 miles to go, and starting with the front line at $s=0$, if the red air and ground strengths are constant and equal to their average values of one, i.e., $p=\bar{p}=1$ and $q=\bar{q}=1$, then the optimal blue air and ground strengths are also constant and given by

$$\alpha=1.359 \quad \text{and} \quad \beta=1.961.$$

The daily cost given by Eq. (15) is

$$C=C_5\alpha+(C_6-C_7\alpha)\beta+\tfrac{1}{2}C_8\alpha^2+\tfrac{1}{2}C_9\beta^2$$
$$=0.00679+0.169+0.018+0.038$$
$$=0.233.$$

TABLE 1

Case I, C_5 constant	Case II, Time-varying C_5
$C_5=0.005$	$C_5(K)=0.005+0.0005K$
$K=21$:	$K=21$:
$\alpha=1.359-2.573\times10^{-2}s$	$\alpha=0.7776-2.573\times10^{-2}s$
$\beta=1.961-4.503\times10^{-2}s$	$\beta=1.891-4.503\times10^{-2}s$
$K=11$:	$K=11$:
$\alpha=1.848-4.911\times10^{-2}s$	$\alpha=1.547-4.911\times10^{-2}s$
$\beta=2.818-8.594\times10^{-2}s$	$\beta=2.748-8.594\times10^{-2}s$
$K=1$:	$K=1$:
$\alpha=12.07-0.5369s$	$\alpha=12.04-0.5369s$
$\beta=20.70-0.9396s$	$\beta=20.70-0.9396s$

The front line will move forward at the increment of approximately one mile per day.

In Case II, $C_5(K)$ decreases as the campaign progresses. If the red air and ground strengths are equal to their average values, then $\alpha=0.7776$ and $\beta=1.819$ at the beginning of the campaign, and they gradually increase to $\alpha=1.444$ and $\beta=2.153$ at the end of the campaign. The front line moves forward at the increment of 0.8 miles per day at the beginning of the campaign, which gradually increases to 1.2 miles per day at the end of the campaign. The front line moves forward at a lower rate at the beginning of the campaign where the coefficients of the daily cost are higher, and moves forward at a higher rate near the end of the campaign where the coefficients are lower.

It should be noted that in the examples given, the optimal decisions are the perturbed strength increments about the main strengths, and the positions are the perturbed position increments of the front line about the planned position. Furthermore, α, β, and the minimum expected cost increments are all positive.

The optimal decisions correspond closely to those obtained for perfect intelligence with no communications in Ref. [5]. In the latter case the optimal decisions have the form

$$\alpha(p)=u'_K+v'_Ks+w'_Kp,$$

$$\beta(q)=x'_K+y'_Ks+z'_Kq.$$

For no intelligence, as discussed in this paper, $w'_K=z'_K=0$, and the coefficients of s in Eqs. (25) and (26) are numerically the same as in the above equations, i.e., $v_K=v'_K$ and $y_K=y'_K$. If the red air and ground strengths are equal to their average values, then the optimal decisions for perfect intelligence with no communications are identical to those with no intelligence and no communications, assuming the coefficients C_i given above are the same. The optimal decisions for no intelligence and no communications do not depend directly on p and q. The minimum expected cost for the latter case is higher, however, and increases much more rapidly as the standard deviations, σ_p and σ_q, are increased. The effect of increasing the standard deviations is to increase γ_{K+1} in Eq. (40).

In this paper the air and ground commanders were assumed to have no intelligence and no communications. Future papers will consider perfect intelligence and perfect communications, and detailed comparisons will be made so that the effects of intelligence and communications on tactics and costs can be assessed.

REFERENCES

1 H. Kagiwada and R. Kalaba, C^3 Modeling, USNPS Workshop, 1978.
2 V. V. Druzhinin and D. S. Kontorov, *Concept, Algorithm, Decision*, translated and published by USAF, 1972.
3 R. Thrall, C. Tsokos, and J. Turner, *Decision Information for Tactical Command and Control*, R. M. Thrall and Associates, Houston, 1977.
4 J. Hess, H. Kagiwada, R. Kalaba, K. Spingarn, and C. Tsokos, Optimal tactics for close support operations, I. Degraded communications, *J. Optimization Theory Appl.*, to appear.
5 J. Hess, H. Kagiwada, R. Kalaba, K. Spingarn, and C. Tsokos, Optimal tactics for close-support operations—II. Degraded communications with linear time-varying dynamics, *Appl. Math. and Comp.*, 5:199–212 (1979).
6 R. Bellman and R. Kalaba, *Dynamic Programming and Modern Control Theory*, Academic, New York, 1966.