Dimensional Analysis of Airline Quality

Everette S. Gardner Jr.

Decision and Information Sciences Department, C. T. Bauer College of Business, University of Houston,
4800 Calhoun, Houston, Texas 77204-6021, egardner@uh.edu

Dimensional analysis is widely used in physics and engineering to contribute to modeling systems in which variables are measured in incompatible units. I used dimensional analysis to rank the airlines in overall quality based on US Department of Transportation data: on-time arrivals, denied boardings, mishandled baggage, and customer complaints. The results conflict with the well-known airline quality rating (AQR), published annually since 1991 by Bowen and Headley. Weighted averages of quality data in the AQR are dimensionally incorrect and produce rankings that are virtually independent of on-time arrivals. For example, the 2001 AQR ranks Alaska Airlines first in overall quality, despite the worst on-time performance in the industry. Dimensional analysis places Alaska Airlines near the bottom of the industry, seventh in overall quality.

Key words: industries: transportation, shipping; decision analysis: applications.
History: This paper was refereed.

In physics and engineering, dimensional analysis is a standard method for reducing such physical properties as energy and acceleration to their fundamental dimensions of length, mass, and time. In general, dimensional analysis facilitates mathematical modeling, usually by reducing the number of variables, and avoids the nuisance of incompatible units. For example, using dimensional analysis, one expresses acceleration as length or distance per unit of time squared. Whether one measures distance in English or metric units does not matter.

In operations research, Epstein (1957) appears to have been the first to recognize the value of dimensional analysis. He presented a general procedure, based on Bridgman's (1922) tutorial for physicists, for computing dimensionless indices to help people to choose between competing engineering designs. Naddor (1966) demonstrated that dimensional analysis can simplify solutions and help people to interpret the behavior of queuing, inventory lot-sizing, and linear programming models. Since Naddor's work appeared, only a few applications of dimensional analysis in operations research have been published. These include inventory modeling (Sivazlian 1971, Ehrhardt 1979, Silver 1983, Ehrhardt and Mosier 1984, Vignaux and Jain 1988), warehouse layout (Mahoney and Ventura 1985), logistics networks (Daganzo 1987), facility location (Starr 1996), supplier performance evaluation (Willis et al. 1993, Li et al. 1997), and selection of industrial robots (Braglia and Gabbrielli 2000).

As Naddor (1966) observed, it is unfortunate that more operations researchers are not aware of dimensional analysis. Huntley (1951) lists a number of ways

that physicists can use dimensional analysis as an analytical tool:

(1) In aiding the memory to reconstitute forgotten formulae and equations;

(2) In checking algebraic errors, which are revealed by the lack of dimensional homogeneity of equations;

(3) In providing a conversion factor for changing one system of units to another;

(4) In interpreting the behavior of scale models and capitalizing on the information gained from them;

(5) In selecting experiments that can yield significant information and avoiding redundant experiments; and

(6) In obtaining partial solutions of problems too complex for ordinary mathematical analysis.

I repeat Huntley's list because operations researchers can use it as well.

I demonstrate several of Huntley's points in using dimensional analysis to evaluate airline quality. The results differ substantially from the airline quality rating (AQR), published annually since 1991 by Bowen and Headley (2003). The AQR ranks US domestic airlines in overall quality using weighted averages of various data published by the US Department of Transportation (DOT). Since its inception, the AQR annual report has received a great deal of attention in the news media. Bowen and Headley (2003) list television programs, newspapers, and magazines that have featured the AQR.

The AQR weighted averages are dimensionally incorrect and seriously misleading. I will illustrate the problem using Naddor's (1966) notation: let $[A]$ represent the dimension of some quantity A. If $A + B = C$,

then $[A] = [B] = [C]$. This rule is referred to as dimensional homogeneity, that is, "it makes no sense to add apples and oranges." The problem with the AQR is that it is based on adding apples and oranges. The overall quality rating is the sum of a percentage of on-time arrivals, a rate of mishandled bags per 1,000 passengers, a rate of denied boardings per 10,000, and a rate of customer complaints per 100,000.

Perhaps the most implausible AQR was that for 2001; Bowen and Headley ranked Alaska Airlines first in overall quality despite an on-time arrival rate of 69 percent, the worst in the industry. I developed a dimensionless value function for ranking airlines. Using this value function, I placed Alaska Airlines near the bottom of the industry for 2001, seventh in overall quality.

Department of Transportation Airline Quality Data

From 1991 through 1998, Bowen and Headley (1991, 1992, 1993, 1994, 1995, 1996, 1997, 1998) based the AQR on a weighted average of 19 quality factors collected from the National Transportation Safety Board, the Federal Aviation Administration, and other sources within the DOT. Many of these factors were not clearly related to quality from the viewpoint of the customer, for example, corporate bond ratings, average seat-mile cost, and number of aircraft in the carriers' fleets. Beginning with the 1999 report, Bowen and Headley (1999, 2000, 2001, 2002, 2003) simplified the weighted average to include only four quality factors: on-time arrivals, involuntary denied boardings, mishandled baggage, and customer complaints (Table 1). I reexamined the data for 1998 through 2002 (corresponding to the 1999 through 2003 AQR reports). I did not reexamine earlier data because I did not believe that models based on so many dubious quality measures would produce meaningful results.

I took all the data for 1998 through 2002 from the DOT's Air Travel Consumer Reports (US Department of Transportation 2003) for the US carriers that have at least one percent of total domestic scheduled-service passenger revenues. The on-time arrival percentage covers nonstop flights between points within the United States. Flights are considered to be on time if they arrive at the gate within 15 minutes of the scheduled times shown in the carriers' computerized reservations systems. Canceled and diverted flights are counted as late.

On-time arrival performance has always been controversial because the numbers can be biased by air-traffic-control policies and by the geographic regions in which an airline concentrates its flights. The numbers are also difficult to interpret because little public information is available on the specific causes of delays and cancellations. In hopes of providing more useful reports, the DOT (US Department of Transportation 2002) required carriers to collect the causes of airline cancellations and delays starting in June 2003. The new rules create four broad categories for reporting the causes of cancellations: (1) circumstances within the control of the carrier, (2) extreme weather, (3) security, and (4) a catch-all "National Aviation System" category, including nonextreme weather, airport operations, heavy traffic volume, and air-traffic control. Carriers report delays in the same way except that they use an additional category for data on delays caused by late incoming aircraft from a previous flight. The DOT imposed the new rules over the opposition of the industry trade group, the Air Transport Association, which believes that causal data are proprietary and should be released to the public only in aggregate form, with no identification of individual carriers.

The second quality factor, involuntary denied boardings, is the number of passengers per 10,000 who hold confirmed reservations and are denied boarding on (bumped from) flights because they are oversold. The DOT does not count canceled, delayed, or diverted flights as denied boardings. It includes both domestic and international flight segments that originate in the United States. In involuntary denied boardings, the DOT counts many that are actually voluntary because it includes passengers who volunteer to take other flights in return for compensation. No public data exists on the number of passengers who receive compensation, although the practice is common in the industry. Another problem in denied-boardings data is that the DOT counts passengers who purchased discounted tickets that specifically rule out compensation for denied boardings.

Mishandled baggage is the number of enplaned passengers per 1,000 who report loss, damage, delay, or pilferage. The figures include all such reports, regardless of whether the airline ultimately found the bags in good condition or compensated the passengers. The DOT does not consider the validity of mishandled baggage reports in reporting the figures.

Finally, customer complaints are the number of passengers per 100,000 who file complaints with the DOT. About 24 percent of the complaints in 2002 were concerned with cancellations, delays, or other deviations from flight schedules. Another 19 percent were about customer service in general with no further explanation. About 16 percent dealt with oversales, reservations, ticketing, and boarding, and 14 percent dealt with baggage handling. The remaining complaints concerned a host of miscellaneous problems. The DOT excludes safety issues from customer complaints because they refer them to the Federal Aviation Administration.

	Airline	Overall rank		DOT statistics			
		AQR	Dim. anal.	On-time arrival %	Denied boardings	Mishandled baggage	Customer complaints
1998	Alaska	8	8	0.72	1.30	7.27	0.54
	America West	6	9	0.69	1.14	3.88	2.11
	American	3	4	0.80	0.46	4.40	1.14
	Continental	2	1	0.77	0.14	4.06	1.02
	Delta	4	5	0.80	1.31	4.27	0.79
	Northwest	9	6	0.71	0.30	6.63	2.21
	Southwest	5	3	0.81	1.73	4.53	0.25
	TWA	7	10	0.78	2.61	5.39	1.29
	United	10	7	0.74	0.57	7.79	1.28
	US Airways	1	2	0.79	0.22	4.09	0.84
1999	Alaska	5	6	0.71	0.91	5.75	1.64
	America West	8	10	0.70	1.39	4.52	3.73
	American	7	4	0.74	0.43	5.21	3.50
	Continental	2	3	0.77	0.34	4.42	2.62
	Delta	3	8	0.78	1.53	4.39	1.82
	Northwest	4	1	0.80	0.18	4.81	2.93
	Southwest	1	2	0.80	1.38	4.22	0.40
	TWA	9	7	0.81	0.73	5.38	3.45
	United	10	9	0.74	0.90	7.01	2.66
	US Airways	6	5	0.71	0.52	5.08	3.15
2000	Alaska	2	6	0.68	1.41	3.48	2.04
	America West	10	10	0.66	1.12	6.62	7.51
	American	6	4	0.73	0.42	5.50	3.54
	Continental	7	7	0.78	1.80	5.35	2.84
	Delta	1	1	0.75	0.33	4.49	2.01
	Northwest	5	3	0.77	0.57	5.24	2.61
	Southwest	3	2	0.75	1.89	5.00	0.47
	TWA	8	8	0.77	2.54	6.06	3.47
	United	9	9	0.61	1.43	6.57	5.30
	US Airways	4	5	0.72	0.65	4.76	2.59
2001	Alaska	1	7	0.69	1.36	3.00	1.27
	America West	7	5	0.75	0.38	4.22	3.72
	American	6	4	0.76	0.36	4.60	2.51
	American Eagle	10	6	0.71	0.43	7.36	1.70
	Continental	8	9	0.81	1.51	4.29	2.23
	Delta	5	8	0.78	0.77	4.11	2.16
	Northwest	3	3	0.80	0.45	4.19	1.97
	Southwest	4	2	0.82	1.50	4.77	0.38
	TWA	11	11	0.81	1.83	6.35	2.54
	United	9	10	0.74	0.92	5.07	3.24
	US Airways	2	1	0.78	0.34	3.86	1.87
2002	Alaska	2	6	0.78	1.17	2.63	0.91
	America West	4	1	0.83	0.20	3.55	1.63
	American	6	5	0.84	0.31	4.27	1.29
	American Eagle	10	3	0.79	0.19	9.81	0.60
	Continental	5	7	0.84	0.87	3.14	1.14
	Delta	7	10	0.80	1.11	3.57	1.37
	Northwest	9	8	0.81	0.60	4.52	1.45
	Southwest	3	4	0.83	1.09	3.52	0.33
	United	8	9	0.84	0.69	3.76	1.71
	US Airways	1	2	0.83	0.35	2.95	1.13
	AQR Weight			8.63	−8.03	−7.92	−7.17

Table 1: In the airline quality ratings (AQR), Bowen and Headley rank airlines using weighted averages of four DOT quality factors. Airlines are ranked if they have at least one percent of total domestic scheduled-service passenger revenues. Bowen and Headley ranked 10 airlines each year except in 2001, when they ranked 11 airlines. The reason is that American Eagle reached the one percent revenue goal for the first time in 2001. TWA merged with American in 2001, leaving 10 airlines in 2002. The AQR weighted averages are dimensionally incorrect because the data are measured on four different scales. I based alternative rankings on dimensional analysis, a method of combining multiple criteria into a single performance measure. Dimensional analysis rankings do not depend on scale and they are often very different from the AQR. For example, Bowen and Headley ranked Alaska Airlines first in overall quality in 2001, despite the worst on-time performance in the industry; dimensional analysis ranked Alaska Airlines seventh.

In most industries, customer complaints are the most important measure of quality. Evans and Lindsay (2002) discuss the importance of complaint data and how to use it in quality management. In my opinion, customer complaint data are the least important of the DOT quality measures for the airline industry. The proportion of customers who complain is so small and oddly distributed over time that it is difficult to accept complaints as representative of the population of airline passengers. In 2002, more than 510 million enplanements occurred (for the 10 airlines listed in Table 1), but only 6,229 customers complained to the DOT. Thus one in 82,000 customers complained, compared to an average rate of one in 25 customers in many other industries (Evans and Lindsay 2002). It seems likely that most customers complain directly to the airlines rather than to the DOT, but there is no public information on direct complaints. The tiny sample of those who did complain to the DOT includes much double-counting. Most baggage complaints relate to problems already counted in the rate of mishandled baggage, while most complaints about oversales are duplicated in the rate for involuntary denied boardings.

The terrorist attacks certainly distorted DOT data from September through December 2001, although adjustments in DOT reports are inconsistent. For example, in the on-time arrival report for the month of September, the DOT excluded the 11th through the 30th but it used the entire month in the final percentage for the year. For mishandled bags, the DOT excluded September 11th through the 30th for both the month and the year. For denied boardings and customer complaints, the DOT included all of September for both the month and the year. The DOT receives most customer complaints via mail, and the mail was disrupted in the Washington, DC area because of the anthrax attacks during the last third of 2001. The number of complaints for this period was substantially lower than that for the same period in 2000, and it is not clear whether some airlines were affected disproportionately. Nevertheless, Bowen and Headley used the DOT data, and I did the same for the sake of consistency.

AQR Methodology

The AQR (Bowen and Headley 2003) is a weighted average of DOT quality data, with the weights derived from a survey of 65 airline-industry experts regarding their opinion "as to what consumers would rate as important (on a scale of 1 to 10) in judging airline quality." Bowen and Headley do not specify when they performed this survey but the weights have not been changed since the original AQR report in 1991 (Bowen and Headley 1991).

Bowen and Headley give the weights signs that reflect the "direction of impact that the criteria should have on the consumer's rating of airline quality." They give the on-time arrival weight (8.63) a positive sign and all other weights negative signs. All the weights are similar in magnitude. They weight involuntary denied boardings at −8.03, mishandled bags at –7.92, and customer complaints at −7.17. In the AQR reports, they do not disclose the identity of the experts, how they selected them, if the experts knew how their opinions would be used, or why they surveyed experts rather than passengers themselves to determine weights. Another reason to question the original survey is that the absolute weights are so similar. Resetting all the weights to unity (retaining the signs) makes no significant difference in AQR rankings from 1998 through 2002.

The annual AQR quality rankings for 1998 through 2002 are based on a weighted average of a percentage (on-time arrivals), a rate per 1,000 passengers (denied boardings), a rate per 10,000 passengers (mishandled baggage), and a rate per 100,000 passengers (customer complaints) (Table 1). This jumble of dimensions often produces illogical rankings.

For example, the 2001 AQR rankings were extremely controversial in the airline industry. The only quality factor in which Alaska Airlines led the industry was in the rate of mishandled bags. Alaska was last in on-time arrivals yet ranked first in AQR overall quality. How did this ranking come about? On-time arrivals in the AQR weighted average are fractional numbers less than one while the other numbers are much larger. Because the weights for the four factors are almost equal, on-time arrivals are virtually irrelevant in average scores. Sensitivity analysis shows that Alaska Airlines' 2001 on-time arrivals could have fallen from 69 to 45 percent and it still would have ranked first in the AQR. Even if all airlines except Alaska were never late, that is, if they achieved 100 percent on-time arrivals for all flights in 2001 and Alaska achieved only 69 percent on-time arrivals, it still would have ranked first.

Similar results occur in the AQR rankings in other years. For example, in 2002, US Airways ranked first in the AQR. This carrier's on-time arrivals could have fallen from 83.4 to 50 percent, and it still would have ranked first. If all other carriers had achieved 100 percent on-time arrivals for the entire year, US Airways still would have ranked first with a rate of 83.4 percent.

Dimensional Analysis

The analytical problem in ranking the airlines using DOT quality data seems complex at first glance although the solution is simple. We must identify a

		2002 DOT statistics				Dimensional analysis		
	Rate	On-time arrivals	Denied boardings	Mishandled baggage	Customer complaints	Value function	Weighted avg. of logs	AQR average
America West	Original	0.829	0.200	3.550	1.630	0.716	−0.145	−1.138
Southwest		0.826	1.090	3.520	0.330	0.652	−0.186	−1.029
Ratio						**1.098**	0.781	1.106
Difference						0.064	**0.041**	−0.109
America West	Per 100	82.900	0.020	0.036	0.002	71,632,247	7.855	20.711
Southwest		82.600	0.109	0.035	0.000	65,237,445	7.814	20.614
Ratio						**1.098**	1.005	1.005
Difference						6,394,802	**0.041**	0.097
Weight		1	−1	−1	−1			

Table 2: By changing the size of the units of measurement in ranking a pair of airlines, one changes their ranking. In the first comparison, between America West and Southwest, we use the original DOT statistics and equal absolute weights. The AQR average (last column) gives Southwest the larger score. In the second comparison, all statistics are rescaled to rates per hundred and the AQR average reverses the rankings. In dimensional analysis, rankings are consistent. Scores in the value-function column are the products of quality measurements, with each raised to a power equal to its weight. Another dimensionally correct way to rank the airlines is to use the weighted average of logged quality measures.

special kind of function, a value function that combines measurements of multiple quality factors into a single index of overall performance. To prevent dimensional problems, we must find a value function that satisfies the condition that the ratio of the numbers measuring any two examples of the same quality factor shall not depend on the size of the units in which the measurement was made. In the terminology of dimensional analysis, this condition is called the "absolute significance of relative magnitude" (Bridgman 1922). For example, the statement that one airline has twice as many mishandled bags as another has absolute significance, independent of the units in which mishandled bags are stated. The condition of absolute significance of relative magnitude is essential to all scientific systems of measurement but is really nothing more than common sense. Surely any sound value function cannot depend on units of measure.

The condition places definite restrictions on the form that the value function may take. In a comparison of any two airlines, we can prove (appendix) that the required value function is a geometric weighted average defined as the product of quality measurements, with each measurement raised to an exponent equal to its weight. We can use only ratio-scale values for both exponents and measurements. Exponents are positive for desirable quality factors and negative for undesirable factors.

To construct a dimensionally correct quality ranking of the airlines, all we must do is sort their value functions. A little algebra shows that we obtain an equivalent ranking when we score the airlines using a weighted average of the logarithms of the quality measures. Other researchers have developed more complex but equivalent value-ranking procedures based on dimensional analysis (Willis et al. 1993, Li et al. 1997).

To understand the implications of dimensional analysis, it is helpful to study the effects of rescaling the airline quality measurements. It may seem that one way to obtain sensible rankings using the AQR weighted averages is to rescale all quality measurements to the same rate of occurrence, say per hundred. In the original AQR rankings, on-time arrivals were irrelevant. With rates per hundred, we have just the opposite result: airline rankings depend solely on on-time arrivals and everything else is irrelevant. The reason is that the rate of on-time arrivals per hundred is enormous compared to the other rates (Table 2).

We face precisely the same problem in the AQR when we state all quality measurements in any other common rate. What if we reduce the differences in magnitude by changing on-time arrivals to late arrivals, again with all measurements at the same rate? This idea is of no help because the rate of late arrivals becomes the only relevant measurement. We could present other examples, but clearly we could develop many alternative AQR rankings by rescaling selected individual measurements to overwhelm the others.

These scaling problems do not exist in the value function derived from dimensional analysis. When we use the original DOT data with equal absolute weights, the value function gives overall quality scores of 0.716 for America West and 0.652 for Southwest (Table 2). The scores have no particular numeric interpretation except to say that America West provides better quality than Southwest (under the given assumptions and weights). When we rescale the data to rates per hundred, the overall scores

grow dramatically but the ratio of the scores does not change. For any pair of airlines, the ratio of overall quality scores cannot change no matter how we scale the quality measurements, a result guaranteed by the condition of absolute significance of relative magnitude placed on the value function. It follows that overall quality rankings of any number of airlines cannot depend on the size of units of measure.

The value function can produce rather unwieldy numbers (Table 2). It may be more convenient to base rankings on the weighted averages of logarithms of quality measurements. The same conclusions hold except that in log-scale the difference between scores, rather than the ratio of scores, is unaffected by the size of units of measure.

I show the airline quality rankings from 1998 through 2002 in Table 1. Anyone who wants to reproduce the calculations should be aware that Bowen and Headley compute the AQR weighted averages monthly. At year end, for unstated reasons they compute an unweighted average of monthly averages to obtain their final rankings. For all data in Table 1, there is no significant difference in final AQR rankings

regardless of whether we use an average of monthly averages or an average of final data for the year. The same comment is true for dimensional analysis.

Sensitivity Analysis

I examined the sensitivity of 2002 airline quality rankings to weights in the value function (Table 3). Using the original AQR weights and dimensional analysis, I ranked the top three airlines as America West, US Airways, and American Eagle. Next, I tested 24 additional combinations of weights as follows: 1 and 2 for on-time arrivals, −1 and −2 for denied boardings and mishandled baggage, and 0, −1, and −2 for customer complaints. By assigning zero weight to customer complaints, I excluded this factor from the value function for the reasons I explained above.

Sensitivity analysis reveals that a group of five airlines (America West, US Airways, American, American Eagle, and Southwest) dominated the industry in overall quality in 2002. The top three airlines always came from this group, and no reasonable combination of weights would have admitted any other airline to the top three. America West's

Dimensional analysis weights				Top three airlines		
On-time arrival %	Denied boardings	Mishandled baggage	Customer complaints	1	2	3
8.63	−8.03	−7.92	−7.17	America West	US Airways	American Eagle
1	−1	−1	0	America West	US Airways	American
1	−1	−1	−1	America West	US Airways	American Eagle
2	−1	−1	0	America West	US Airways	American
2	−1	−1	−1	US Airways	America West	American Eagle
1	−2	−1	0	America West	US Airways	American Eagle
1	−2	−1	−1	American Eagle	America West	US Airways
2	−2	−1	0	America West	US Airways	American Eagle
2	−2	−1	−1	America West	American Eagle	US Airways
1	−1	−2	0	America West	US Airways	American
1	−1	−2	−1	US Airways	America West	Southwest
2	−1	−2	0	America West	US Airways	American
2	−1	−2	−1	US Airways	America West	Southwest
1	−2	−1	−2	American Eagle	America West	Southwest
2	−2	−1	−2	American Eagle	America West	US Airways
1	−2	−2	−2	American Eagle	America West	US Airways
2	−2	−2	−2	America West	US Airways	American Eagle
1	−1	−1	−2	Southwest	American Eagle	US Airways
2	−1	−1	−2	Southwest	American Eagle	US Airways
1	−2	−2	0	America West	US Airways	American
1	−2	−2	−1	America West	US Airways	American Eagle
2	−2	−2	0	America West	US Airways	American
2	−2	−2	−1	America West	US Airways	American
1	−1	−2	−2	Southwest	US Airways	America West
2	−1	−2	−2	Southwest	US Airways	America West

Table 3: This table shows the sensitivity of airline quality rankings based on dimensional analysis. If we restrict weights to 1 and 2 for on-time arrivals, −1 and −2 for denied boardings and mishandled baggage, and 0, −1, and −2 for customer complaints, we obtain 24 combinations. America West's performance was less sensitive to weights than its competitors' performance, and America West ranked among the top three airlines in 22 cases. Customer complaint data are questionable. When customer complaints are excluded by using a weight of zero, America West and US Airways always ranked first and second.

performance was the most consistent or least sensitive to weights. America West ranked among the top three airlines in 22 cases, among the top two in 20 cases, and first in 13 cases. My most important conclusion is that omission of customer complaints made the rankings highly insensitive to weights on the remaining quality factors. Without customer complaints, America West and US Airways always ranked first and second, respectively, with American in third place in six of eight cases.

Other Quality Rating Systems

One of the referees for this paper asked this question: Are the dimensional mistakes in the AQR unique or a general phenomenon? This is a difficult question to answer. I examined a nonrandom sample of 100 Web sites containing quality ratings. Most ratings came from simple consumer-opinion surveys, but numerous multiple-criteria ratings included no details on how the ratings were computed.

Rather than follow up on all of these, I decided to focus on the two best-known quality critics, Consumers Union (publisher of *Consumer Reports*) and J. D. Power and Associates. Even though Consumers Union is a not-for-profit organization, the person I contacted refused to answer questions about how any of its quality ratings are computed, stating that they wished to avoid arguments. In an attempt to get answers, I became a member of Consumers Union, which did nothing to improve the quality of its responses. In contrast, J. D. Power was extremely helpful and provided detailed explanations of the methodology for several of its famous automotive quality studies. The studies were designed to avoid dimensional problems and I could see no reason to take issue with the results.

Conclusions

Dimensional analysis is always based on one elementary principle, that of dimensional homogeneity. Quantities may be added or subtracted only when they have the same dimensions. The AQR has violated this principle since 1991, producing indefensible airline rankings.

DOT airline quality data are limited in scope and open to criticism for a variety of reasons. However, the DOT publishes the only industrywide data on airline quality, and I expect that researchers will continue to rank the airlines using that data. My point is that dimensional analysis provides the correct value function for quality rankings using criteria with different dimensions and varying relative importance. Anyone who disagrees with the rankings is free to make his or her own decisions about relevant data and weights to use in the value function.

Dimensional analysis is simple, robust, and flexible both in defining performance measures and in determining the numerical standards for evaluation. Furthermore, dimensional analysis can deal with both tangible and intangible decision criteria so long as the numbers are ratio-scaled. In the context of physics, Langhaar (1951) went so far as to argue that dimensional analysis can produce at least a partial solution to nearly any problem. This may seem improbable in operations research, but certainly we have many opportunities for dimensional reasoning.

Appendix

My purpose in this appendix is to identify a quality value function that does not depend on units of measure. To simplify the analysis, I consider only systems with four quality metrics, but the results are easily generalized.

Let α_1, β_1, γ_1, δ_1 represent a set of quality metrics for an airline. Let α_2, β_2, γ_2, δ_2 represent the same metrics for an alternative airline. The relative value of the first airline's quality is described by the unknown function $f(\alpha_1, \beta_1, \gamma_1, \delta_1)$. Relative value for the second airline is described by $f(\alpha_2, \beta_2, \gamma_2, \delta_2)$. The choice of the value function f is made on the basis of the following proposed axiom: The ratio $f(\alpha_1, \beta_1, \gamma_1, \delta_1)/f(\alpha_2, \beta_2, \gamma_2, \delta_2)$ shall not depend on the units in which any metric is stated.

To determine the form of the value function that satisfies this axiom, we follow Bridgman (1922). The basic approach is to analyze the effects of changing the size of the units of measure. First, make the units in which α is measured $1/w$th as large. Then, the number measuring α will be w times as large or $w\alpha$. In the same way, make the number measuring β $1/x$th as large, so the measuring number becomes $x\beta$. This process produces the following relation:

$$\frac{f(\alpha_1, \beta_1, \gamma_1, \delta_1)}{f(\alpha_2, \beta_2, \gamma_2, \delta_2)} = \frac{f(w\alpha_1, x\beta_1, y\gamma_1, z\delta_1)}{f(w\alpha_2, x\beta_2, y\gamma_2, z\delta_2)}. \quad (1)$$

It is important to note that our axiom requires this relation to hold for all values of $\alpha_1, \beta_1, \gamma_1, \delta_1$ for all $\alpha_2, \beta_2, \gamma_2, \delta_2$ and for all w, x, y, z.

To solve for the unknown function f, rewrite as follows:

$$f(w\alpha_1, x\beta_1, y\gamma_1, z\delta_1) = f(w\alpha_2, x\beta_2, y\gamma_2, z\delta_2)$$
$$\times \frac{f(\alpha_1, \beta_1, \gamma_1, \delta_1)}{f(\alpha_2, \beta_2, \gamma_2, \delta_2)}. \quad (2)$$

Next, differentiate partially with respect to w. Let f_1 represent the partial derivative of the function with respect to the first argument. This yields the following:

$$\alpha_1 f_1(w\alpha_1, x\beta_1, y\gamma_1, z\delta_1) = \alpha_2 f_1(w\alpha_2, x\beta_2, y\gamma_2, z\delta_2)$$
$$\times \frac{f(\alpha_1, \beta_1, \gamma_1, \delta_1)}{f(\alpha_2, \beta_2, \gamma_2, \delta_2)}. \quad (3)$$

Now put w, x, y, z all equal to 1. This produces

$$\alpha_1 \frac{f_1(\alpha_1, \beta_1, \gamma_1, \delta_1)}{f(\alpha_1, \beta_1, \gamma_1, \delta_1)} = \alpha_2 \frac{f_1(\alpha_1, \beta_1, \gamma_1, \delta_1)}{f(\alpha_2, \beta_2, \gamma_2, \delta_2)}. \tag{4}$$

Equation (4) is to hold for all values of α_1, β_1, γ_1, δ_1 and α_2, β_2, γ_2, δ_2. Hence, keeping α_2, β_2, γ_2, δ_2 constant and allowing α_1, β_1, γ_1, δ_1 to vary, we have

$$\frac{\alpha}{f}\frac{\partial f}{\partial \alpha} = \text{Const} \tag{5}$$

or

$$\frac{1}{f}\frac{\partial f}{\partial \alpha} = \frac{\text{Const}}{\alpha}, \tag{6}$$

which integrates to $f = C_1 \alpha^{\text{Const}}$. The factor C_1 is a function of the other parameters β, γ, δ.

Next, repeat the process above, differentiating partially with respect to x, y, z in turn, and integrating. The final result reveals the required value function:

$$f = C\alpha^a \beta^b \gamma^c \delta^d. \tag{7}$$

In (7), the exponents a, b, c, and d are weights. The coefficient C is almost always chosen to be unity.

Acknowledgments

References

Bowen, Brent, Dean Headley. 1991. The Airline Quality Rating 1991. Retrieved May 1, 2003 from http://www.unomaha.edu/~unoai/aqr/.

Bowen, Brent, Dean Headley. 1992. The Airline Quality Rating 1992. Retrieved May 1, 2003 from http://www.unomaha.edu/~unoai/aqr/.

Bowen, Brent, Dean Headley. 1993. The Airline Quality Rating 1993. Retrieved May 1, 2003 from http://www.unomaha.edu/~unoai/aqr/.

Bowen, Brent, Dean Headley. 1994. The Airline Quality Rating 1994. Retrieved May 1, 2003 from http://www.unomaha.edu/~unoai/aqr/.

Bowen, Brent, Dean Headley. 1995. The Airline Quality Rating 1995. Retrieved May 1, 2003 from http://www.unomaha.edu/~unoai/aqr/.

Bowen, Brent, Dean Headley. 1996. The Airline Quality Rating 1996. Retrieved May 1, 2003 from http://www.unomaha.edu/~unoai/aqr/.

Bowen, Brent, Dean Headley. 1997. The Airline Quality Rating 1997. Retrieved May 1, 2003 from http://www.unomaha.edu/~unoai/aqr/.

Bowen, Brent, Dean Headley. 1998. The Airline Quality Rating 1998. Retrieved May 1, 2003 from http://www.unomaha.edu/~unoai/aqr/.

Bowen, Brent, Dean Headley. 1999. The Airline Quality Rating 1999. Retrieved May 1, 2003 from http://www.unomaha.edu/~unoai/aqr/.

Bowen, Brent, Dean Headley. 2000. The Airline Quality Rating 2000. Retrieved May 1, 2003 from http://www.unomaha.edu/~unoai/aqr/.

Bowen, Brent, Dean Headley. 2001. The Airline Quality Rating 2001. Retrieved May 1, 2003 from http://www.unomaha.edu/~unoai/aqr/.

Bowen, Brent, Dean Headley. 2002. The Airline Quality Rating 2002. Retrieved May 1, 2003 from http://www.unomaha.edu/~unoai/aqr/.

Bowen, Brent, Dean Headley. 2003. The Airline Quality Rating 2003. Retrieved May 1, 2003 from http://www.unomaha.edu/~unoai/aqr/.

Braglia, Marcello, Roberto Gabbrielli. 2000. Dimensional analysis for investment selection in industrial robots. *Internat. J. Production Res.* **38**(18) 4843–4848.

Bridgman, P. W. 1922. *Dimensional Analysis.* Yale University Press, New Haven, CT.

Daganzo, Carlos F. 1987. The break-bulk role of terminals in many-to-many logistics networks. *Oper. Res.* **35**(4) 543–555.

Ehrhardt, Richard. 1979. The power approximation for computing (s, S) inventory policies. *Management Sci.* **25**(8) 777–786.

Ehrhardt, Richard, C. Mosier. 1984. A revision of the power approximation for computing (s, S) inventory policies. *Management Sci.* **30**(5) 618–622.

Epstein, L. Ivan. 1957. A proposed measure for determining the value of a system. *Oper. Res.* **5**(2) 297–299.

Evans, James R., William L. Lindsay. 2002. *The Management and Control of Quality.* Southwestern, Cincinnati, OH.

Huntley, H. E. 1951. *Dimensional Analysis.* Rinehart and Company, New York.

Langhaar, Henry L. 1951. *Dimensional Analysis and Theory of Models.* John Wiley and Sons, New York.

Li, C. C., Y. P. Fun, J. S. Hung. 1997. A new measure for supplier performance evaluation. *IIE Trans.* **29**(2) 753–758.

Mahoney, J. F., J. A. Ventura. 1985. Dimensional analysis in an industrial engineering setting. *IIE Trans.* **17**(2) 198–203.

Naddor, Eliezer. 1966. Dimensions in operations research. *Oper. Res.* **14**(1) 508–514.

Silver, Edward A. 1983. An example of indifference curves in inventory control. *IIE Trans.* **15**(1) 79–80.

Sivazlian, B. D. 1971. Dimensional and computational analysis in stationary (s, S) inventory problems with gamma-distributed demand. *Management Sci.* **17**(6) 307–311.

Starr, Martin K. 1996. *Operations Management: A Systems Approach.* Boyd and Fraser Publishing Company, Danvers, MA.

US Department of Transportation. 2002. Reporting the causes of airline delays and cancellations. Docket no. OST 2000-8164, November 25, **67**(227) 70535–70545.

US Department of Transportation. 2003. Air travel consumer reports, 1998–2002. Retrieved May 1, 2003 from www.airconsumer.ost.dot.gov/reports/atcr02.htm.

Vignaux, G. A., Sudha Jain. 1988. An approximate inventory model based on dimensional analysis. *Asia-Pacific J. Oper. Res.* **5**(2) 117–123.

Willis, T. Hillman, C. Richard Huston, Frank Pohlkamp. 1993. Evaluation measures of just-in-time supplier performance. *Production and Inventory Management J.* **32**(3) 1–6.