

Chapter 4 Exponential Smoothing

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Simple exponential smoothing is a short-range forecasting tool that assumes a reasonably stable mean in the data with no trend (consistent growth or decline). If the data are nonseasonal, use the SIMPLE model. If seasonal, you have two choices: SIMPLEADDSEAS for additive seasonality and SIMPLEMULSEAS for multiplicative seasonality. The seasonal models perform seasonal adjustment, forecast the adjusted data with simple smoothing, then reverse the seasonal adjustment in the forecasts.

To forecast a trend in nonseasonal data, try the TREND model, which allows you to compare several different types of trend before committing to a forecast. If the data contain both a trend and seasonality, again there are two choices, depending on the type of seasonality: TRENDADDSEAS and TRENDMULSEAS.

4.1 Simple exponential smoothing, nonseasonal data (SIMPLE)

Exponential smoothing models are ubiquitous in business forecasting, especially in budgeting and inventory control applications. Such models are relatively simple, easy to understand, and easy to implement. Smoothing models also compare quite favorably in accuracy to complex forecasting models. One of the surprising things scientists have learned about forecasting in recent years is that complex models are not necessarily more accurate than simple models.

The simplest form of exponential smoothing is called, appropriately enough, simple smoothing. Simple smoothing is used for short-range forecasting, usually just one month into the future. The model assumes that the data fluctuate around a reasonably stable mean (no trend or consistent pattern of growth).

Figure 4-1 illustrates an application of simple exponential smoothing at the International Airport in Victoria, Texas. The airport has been open for a year and the data are the monthly numbers of passengers embarked. There is not yet enough data to estimate a seasonal pattern, so we will use SIMPLE, the nonseasonal version of simple exponential smoothing.

To get the forecasting process started, SIMPLE automatically sets the first forecast (E26) equal to the average of the number of warm-up data specified in cell D9. The number of warm-up data is 6, so the first forecast of 30.0 is the average of the data for months 1-6. If you don't like the first forecast, replace the formula in E26 with a value. Thereafter the forecasts are updated as follows: In column F, each forecast error is equal to actual data minus the forecast for that period. In column E, each forecast is equal to the previous forecast plus a fraction of the previous error. This fraction is found in cell D8 and is called the smoothing weight. The model works much like an automatic pilot, a cruise control on an automobile, or a thermostat. If a given forecast is too low, the forecast error is positive, and the next forecast is increased by a fraction of the error. If a given forecast is too high, the forecast error is negative, and the next forecast is reduced by a fraction of the error. If we get lucky and a forecast is perfect, the error is zero and there is no change in the next forecast.

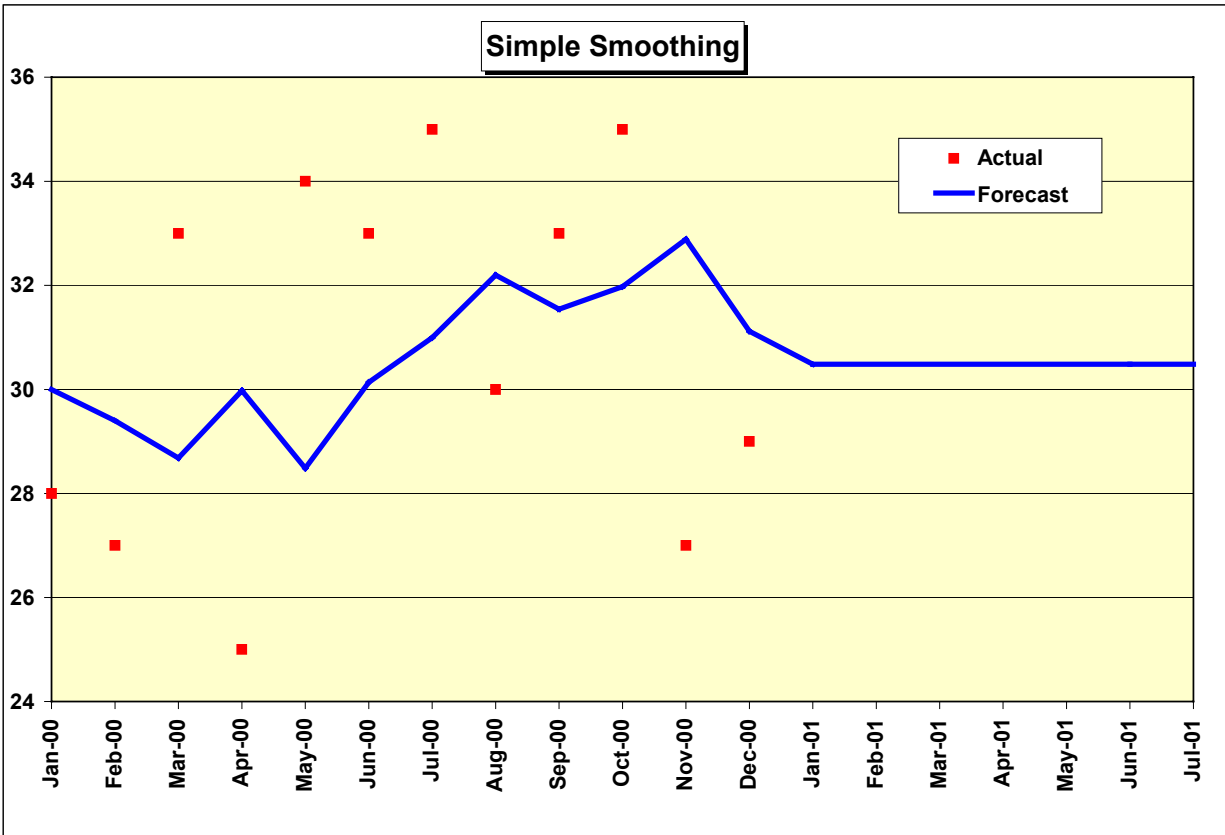
A total of 12 data observations are entered in Figure 4-1. The model automatically makes forecasts through the last period specified in cell D10. For months 13-24, the forecasts are constant as shown in Figure 2-2. Remember that the model assumes no trend, so the only option is to project the last forecast for every period in the future.

The model computes two mean forecast error measures. The MSE is the mean-squared-error and the MAD is the mean of the absolute errors (called the mean absolute deviation). Both are commonly used in practice. The MSE gives more weight to large errors, while the MAD is easier to interpret.

Figure 4-1

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	SIMPLE.XLS					INTERMEDIATE CALCULATIONS:					DATA TABLE:		
2	Simple exponential smoothing.					RMSE (Square root of MSE)			3.91		Select K6..L16.		
3						3 X RMSE			11.74		Select Data Table.		
4											Enter column input cell = D8.		
5						Warm-up SSE			91.81		Weight	MSE	
6						Warm-up MSE			15.30			11.87	
7	INPUT:					Forecasting SSE			163.05		0.10	11.41	
8	Smoothing weight			0.30		Fcst SSE - Warm-up SSE			71.24		0.20	11.58	
9	Nbr. Of warm-up data			6		Nbr. of forecast periods			6		0.30	11.87	
10	Last period to forecast			24		Forecasting MSE			11.87		0.40	12.24	
11											0.50	12.74	
12	OUTPUT:					Warm-up Sum abs. err.			22.07		0.60	13.41	
13	Number of data			12		Warm-up MAD			3.68		0.70	14.29	
14	Nbr. of outliers			0		Forecasting Sum abs. err.			40.76		0.80	15.39	
15	Warm-up MSE			15.30		Fcst Sum abs. - Warm-up Sum			18.69		0.90	16.73	
16	Forecasting MSE			11.87		Nbr. of forecast periods			6		1.00	18.33	
17	Warm-up MAD			3.68		Forecasting MAD			3.11		Note: MSE values are based		
18	Forecasting MAD			3.11		Last forecast at period			24		on forecasting periods.		
19													
20													
21													Sum
22												Avg.	absolute
23											Period	data	errors
24	Month	Month	Period	Actual					Outlier	Sum			
25	& year	nbr.	nbr.	data	Fcst	Error			Indicator	nbr out			
26	Jan-00	1	1	28	30.00	-2.00			0	0	1	28.00	2
27	Feb-00	2	2	27	29.40	-2.40			0	0	2	27.50	4.4
28	Mar-00	3	3	33	28.68	4.32			0	0	3	29.33	8.72
29	Apr-00	4	4	25	29.98	-4.98			0	0	4	28.25	13.696
30	May-00	5	5	34	28.48	5.52			0	0	5	29.40	19.2128
31	Jun-00	6	6	33	30.14	2.86			0	0	6	30.00	22.07456
32	Jul-00	7	7	35	31.00	4.00			0	0	7	30.71	26.07779
33	Aug-00	8	8	30	32.20	-2.20			0	0	8	30.63	28.27553
34	Sep-00	9	9	33	31.54	1.46			0	0	9	30.89	29.73711
35	Oct-00	10	10	35	31.98	3.02			0	0	10	31.30	32.76022
36	Nov-00	11	11	27	32.88	-5.88			0	0	11	30.91	38.64405
37	Dec-00	12	12	29	31.12	-2.12			0	0	12	30.75	40.76272
38	Jan-01	13	#N/A		30.48	#N/A			0	0	13	#N/A	0
39	Feb-01	14	#N/A		30.48	#N/A			0	0	14	#N/A	0
40	Mar-01	15	#N/A		30.48	#N/A			0	0	15	#N/A	0

Figure 4-2



Both MSE and MAD are computed for two samples of the data. The first sample (periods 1-6) is called the warm-up sample. This sample is used to "fit" the forecasting model, that is to get the model started by computing the first forecast and running for a while to get "warmed up." The second part of the data (periods 7-12) is used to test the model and is called the forecasting sample. Accuracy in the warm-up sample is really irrelevant. Accuracy in the forecasting sample is more important because the pattern of the data often changes over time. The forecasting sample is used to evaluate how well the model tracks such changes. There are no statistical rules on where to divide the data into warm-up and forecasting samples. There may not be enough data to have two samples. A good rule of thumb is to put at least six nonseasonal data points or two complete seasons of seasonal data in the warm-up. If there is less data than this, there is no need to bother with two samples. In a long time series, it is common in practice to simply divide the data in half. If you don't want to bother with a warm-up sample, set the number of warm-up data equal to the total number of data. The forecasting MSE and MAD will then be set to zero.

How do you choose the weight in cell D8? A range of trial values must be tested. The best-fitting weight is the one that gives the best MSE or MAD in the warm-up sample. There are two factors that interact to determine the best-fitting weight. One is the amount of noise or randomness in the series. The greater the noise, the smaller the weight must be to avoid overreaction to purely random fluctuations in the time series. The second factor is the stability of the mean. If the mean is relatively constant, the weight must be small. If the mean is changing, the weight must be large to keep up with the changes. Weights can be selected from the range 0 - 1 although we recommend a minimum weight of 0.3 in practice. Smaller values result in a very sluggish response to changes in the mean of the time series. In practice, smoothing weights in the range of 0.30 to 0.50 are commonly used. Weights in this range offer a reasonable compromise between sensitivity to noise and responsiveness.

An Excel data table is available in columns K and L to assist in selecting smoothing weights. Column K displays smoothing weights from 0.10 to 1.00 in increments of 0.10 while column L displays the corresponding MSE for the forecasting sample. Follow the instructions at the top of the data table to update MSE values. The weights in column K can be changed. You can also edit the formula in L6 to compute MAD rather than MSE results. The formula in Cell L6 is "`=I10`". If you want MAD results, change this to "`=I17`".

Two other graphs in the SIMPLE workbook assist in evaluation of the forecast model. The error graph (Figure 4-3) compares individual forecast errors to control limits. These limits are established at plus and minus three standard deviations from zero. The standard deviation is an estimate based on the square root of the MSE, called the RMSE for root-mean-squared-error. The probability is less than 1% that individual errors will exceed the control limits if the mean of the data is unchanged. The "outlier" count in cell D14 of Figure 4-1 is the number of errors that went outside control limits. Finally, the MSE graph (Figure 4-4) is a bar chart of MSE values for alternative smoothing weights. Since the data are not changing, the best weight is relatively small.

Figure 4-3

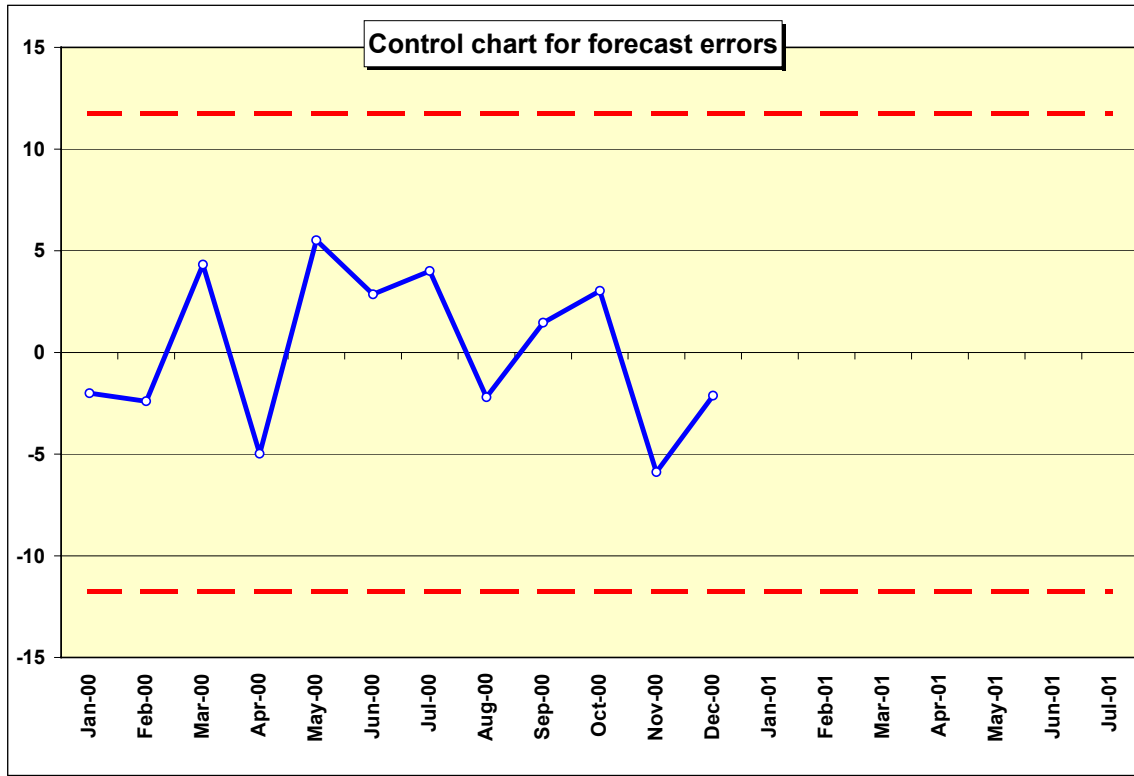
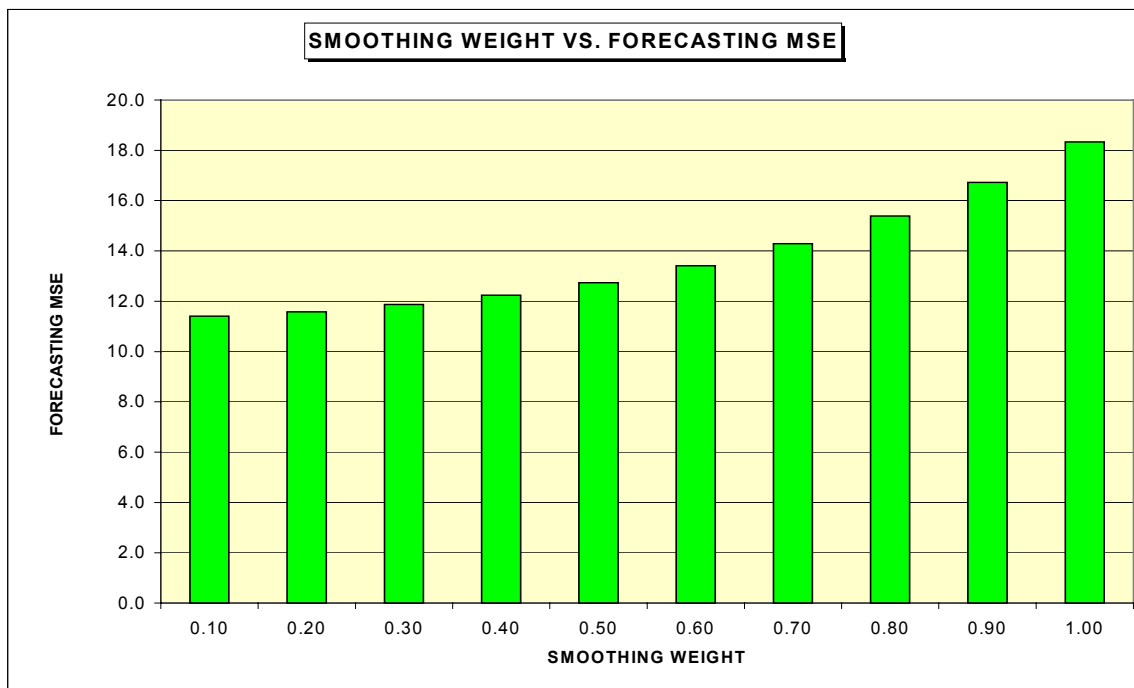


Figure 4-4



The forecasting model in SIMPLE is based on two equations that are updated at the end of each time period:

$$\begin{aligned} \text{Forecast error} &= \text{actual data} - \text{current forecast} \\ \text{Next forecast} &= \text{current forecast} + (\text{weight} \times \text{error}) \end{aligned}$$

A little algebra shows that this model is equivalent to another model found in many textbooks and in practice:

$$\text{Next forecast} = (\text{weight} \times \text{actual data}) + [(1 - \text{weight}) \times \text{current forecast}]$$

The mathematical formulation of the simple smoothing model is:

$$e_t = X_t - \hat{X}_{t-1}(1) \quad (4-1)$$

$$S_t = S_{t-1} + h_1 e_t \quad (4-2)$$

$$\hat{X}_t(m) = S_t \quad (4-3)$$

We compute the forecast error e_t , defined as actual data X_t minus the forecast. The letter t refers to the time period. The current period, for example the month, is t , while last period is designated $t-1$.

$\hat{X}_{t-1}(1)$ is the forecast made at the end of $t-1$ for 1 step ahead, that is the forecast made last month for this month. S_t is the new level or the weighted average of the time series at the end of t . The new level is equal to the level at the end of $t-1$ plus a fraction of the error. The fraction h_1 is the smoothing weight or parameter. Finally, in equation (4-3), the forecast at the end of t for m (any number) of steps ahead is equal to the smoothed level at the end of t .

Many companies forecast with moving averages. It may be surprising that simple exponential smoothing is equivalent to a weighted moving average in which the weights decline exponentially with time. Here are the effective weights assigned by the model to past data, with the smoothing weight denoted by h_1 :

Most recent data observation	h_1
Data in previous period	$h_1(1 - h_1)$
Data two periods ago	$h_1(1 - h_1)^2$
Data three periods ago	$h_1(1 - h_1)^3$

In the Victoria data, the weight assigned to December, 2000, is 0.30, the weight for November, 2000, is $0.30(1-0.30) = 0.21$, and so on. Every data observation gets some weight although the weights decline rapidly with time.

4.2 Simple exponential smoothing with seasonality (SIMPLEADDSEAS and SIMPEMULSEAS)

If the data are seasonal, with no trend, use SIMPLEADDSEAS for additive seasonality or SIMPEMULSEAS for multiplicative seasonality. In many cases, the type of seasonality is not obvious, even after testing, so it may be wise to experiment with both models before committing to a forecast.

SIMPLEADDSEAS is a combination of the ADDITMON and SIMPLE models. Data entry is done in the first tab, named Additive (identical to ADDITMON and not shown here). Seasonally-adjusted data from the first tab is automatically used in the second tab, named Simple, shown in Figure 4-5. The data are highly seasonal sales of chlorine tablets at a supply store for home swimming pools. Column D repeats seasonally-adjusted data from the Additive tab, while column E repeats the seasonal indices. In column F, simple smoothing is applied to the seasonally-adjusted data. Errors in column G are equal to seasonally-adjusted data minus seasonally-adjusted forecasts. In column H, seasonally-adjusted forecasts are converted to final forecasts by adding back the appropriate seasonal index—that is, seasonal adjustment in the forecasts is reversed.

Two forecast graphs are included in the SIMPLEADDSEAS workbook (Figures 4-6 and 4-7). The first graph shows seasonally-adjusted data and seasonally-adjusted forecasts, while the second shows original data and final forecasts. The graphs demonstrate that seasonal adjustment made an enormous reduction in variance in the chlorine-tablet data. The workbook also includes an error graph (not shown here) like the SIMPLE model. Which errors are plotted, seasonally-adjusted or errors based on the final forecasts? The answer is both. Seasonally-adjusted errors and errors based on the final forecasts are identical.

SIMPEMULSEAS is a combination of the MULTIMON and SIMPLE models and is not shown here. All calculations and worksheet operations are similar to SIMPLEADDSEAS except that seasonality is multiplicative.

Figure 4-5

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	SIMPLEADDSEAS					INTERMEDIATE CALCULATIONS:						DATA TABLE:	
2	Simple smoothing with additive seasonality					RMSE (Square root of MSE)					10.87	Select L6..M16.	
3	Note: data entry is done in the additive tab.					3 X RMSE					32.61	Select Data Table.	
4												Enter column input cell	
5						Warm-up SSE					1417.94	Weight	MSE
6						Warm-up MSE					118.16		55.93
7	INPUT:					Forecasting SSE					2760.19	0.10	58.89
8	Smoothing weight			0.30		Fcst SSE - Warm-up SSE					1342.25	0.20	55.82
9	Nbr. Of warm-up data			12		Nbr. of forecast periods					24	0.30	55.93
10	Last period to forecast			48		Forecasting MSE					55.93	0.40	57.83
11												0.50	60.87
12	OUTPUT:					Warm-up Sum abs. err.					113.10	0.60	64.75
13	Number of data			36		Warm-up MAD					9.42	0.70	69.36
14	Nbr. of outliers			0		Forecasting Sum abs. err.					264.54	0.80	74.71
15	Warm-up MSE			118.16		Fcst Sum abs. - Warm-up Sum abs.					151.45	0.90	80.89
16	Forecasting MSE			55.93		Nbr. of forecast periods					24	1.00	88.11
17	Warm-up MAD			9.42		Forecasting MAD					6.31	Note: MSE values are on forecasting periods.	
18	Forecasting MAD			6.31		Last forecast at period					48		
19													
20													
21													
22													
23												Avg.	data
24	Month	Month	Period	S. Adjuste	Seas	S. Adjusted	Final	Original	Outlier	Sum			
25	& year	nbr.	nbr.	Data	Index	Fcst	Error	Fcst	Data	Indicator	nbr out		
26	Jan-99	1	1	87.11	-55.11	79.75	7.36	24.64	32.00	0	0	1	87.11
27	Feb-99	2	2	79.48	-55.48	81.96	-2.48	26.48	24.00	0	0	2	83.29
28	Mar-99	3	3	65.65	-20.65	81.21	-15.57	60.57	45.00	0	0	3	77.41
29	Apr-99	4	4	63.94	-3.94	76.54	-12.61	72.61	60.00	0	0	4	74.04
30	May-99	5	5	90.48	-36.48	72.76	17.72	36.28	54.00	0	0	5	77.33
31	Jun-99	6	6	71.23	78.77	78.08	-6.85	156.85	150.00	0	0	6	76.31
32	Jul-99	7	7	93.47	80.53	76.02	17.44	156.56	174.00	0	0	7	78.76
33	Aug-99	8	8	91.02	112.98	81.26	9.77	194.23	204.00	0	0	8	80.30
34	Sep-99	9	9	72.61	-17.61	84.19	-11.58	66.58	55.00	0	0	9	79.44
35	Oct-99	10	10	76.06	-9.06	80.71	-4.65	71.65	67.00	0	0	10	79.10
36	Nov-99	11	11	80.11	-22.11	79.32	0.79	57.21	58.00	0	0	11	79.19
37	Dec-99	12	12	85.86	-51.86	79.55	6.30	27.70	34.00	0	0	12	79.75
38	Jan-00	13	13	84.11	-55.11	81.44	2.66	26.34	29.00	0	0	13	80.09
39	Feb-00	14	14	87.48	-55.48	82.24	5.24	26.76	32.00	0	0	14	80.61
40	Mar-00	15	15	79.65	-20.65	83.81	-4.17	63.17	59.00	0	0	15	80.55

Figure 4-6

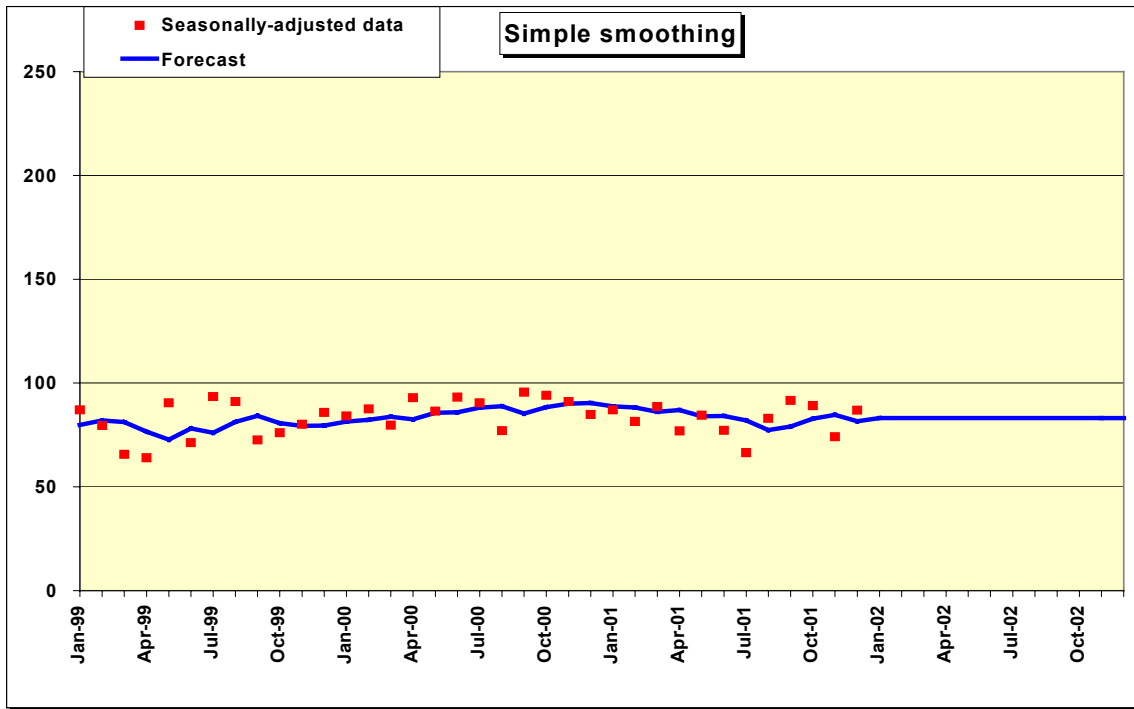
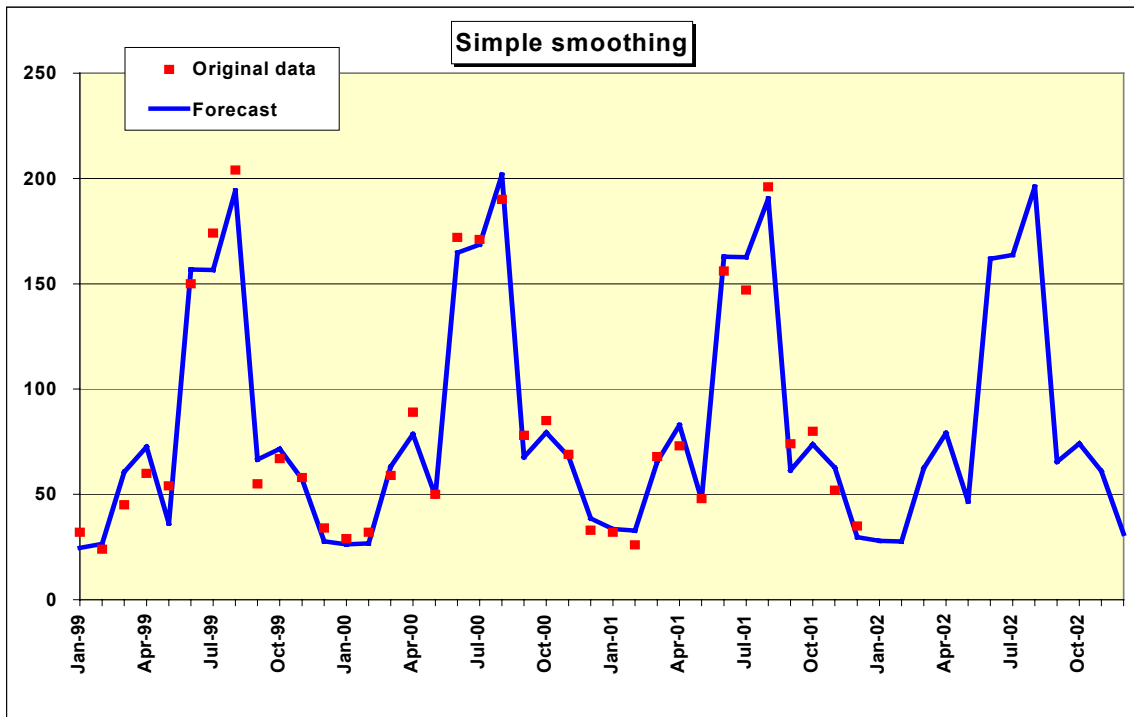


Figure 4-7

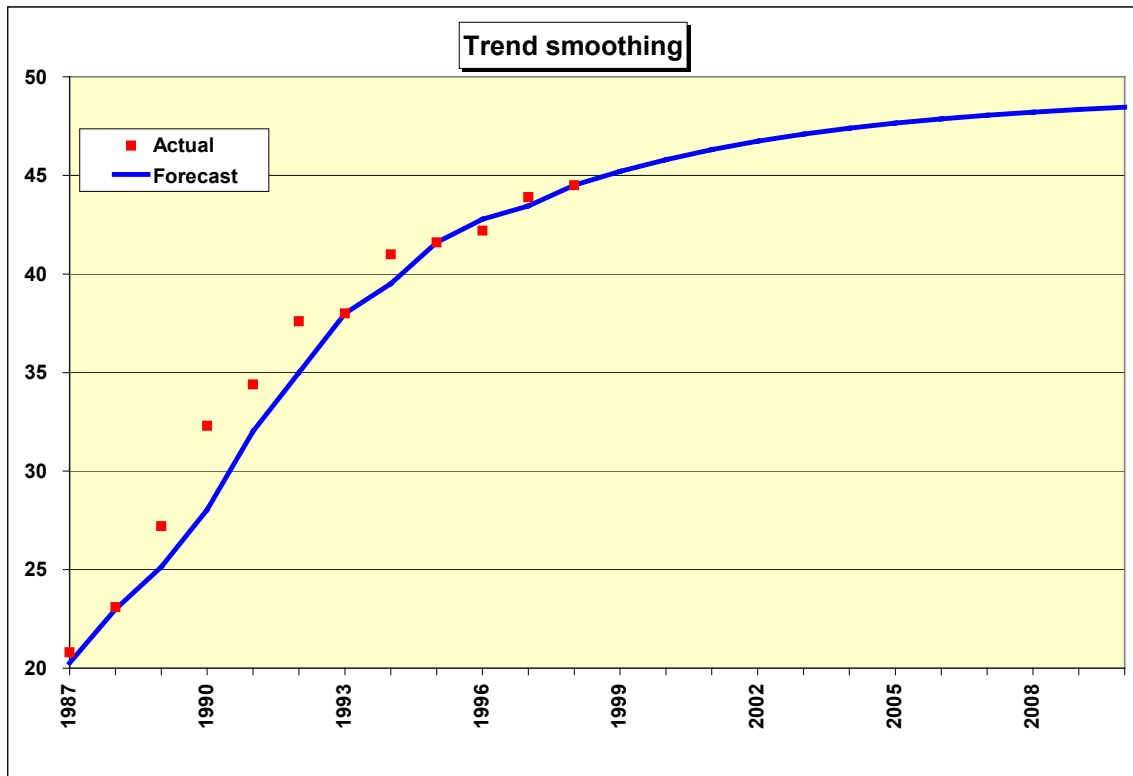


4.3 Trend smoothing, nonseasonal data (TREND)

Exponential smoothing with a trend works much like simple smoothing except that two components must be updated each period: level and trend. The level is a smoothed estimate of the value of the data at the end of each period. The trend is a smoothed estimate of average growth at the end of each period.

To explain this type of forecasting, let's review an application at Alief Precision Arms, a company that manufactures high-quality replicas of the Colt Single-Action Army revolver and other revolvers from the nineteenth century. Alief was founded in 1987 and, as shown in Figure 4-8, experienced rapid growth through about 1994. Since 1994, growth has slowed and this is reflected in the forecasts.

Figure 4-8



TREND in Figure 4-9 computes several different types of trend forecasts. This worksheet can produce a linear or straight-line trend, a damped trend in which the amount of growth declines each period in the future (as in Figure 4-8), or an exponential trend in which the amount of growth increases each period in the future.

To get started, initial values for level and trend are computed in cells G22 and H22. The model sets the initial trend equal to the average of the first four differences among the data. These differences are (23.1 - 20.8), (27.2 - 23.1), (32.3 - 27.2), and (34.4 - 32.3). The average difference or initial trend is 3.4. This value is our estimate of the average growth per period at the beginning of the data. The initial level is the first data observation minus the initial trend or $20.8 - 3.4 = 17.4$.

The forecasting system works as follows:

$$\begin{aligned} \text{Forecast error} &= \text{Actual data} - \text{current forecast} \\ \text{Current level} &= \text{Current forecast} + (\text{level weight} \times \text{error}) \\ \text{Current trend} &= (\text{Trend modifier} \times \text{previous trend}) + (\text{trend weight} \times \text{error}) \\ \text{Next forecast} &= \text{Current level} + (\text{trend modifier} \times \text{current trend}) \end{aligned}$$

Level and trend are independent components of the forecasting model and require separate smoothing weights. Experience shows that the level weight is usually much larger than the trend weight. Typical level weights range anywhere from 0.10 to 0.90, while trend weights are usually small, in the range of 0.05 to 0.20. The trend modifier is usually in the range 0.70 to 1.00. If the trend modifier is less than 1.00, the effect is to reduce the amount of growth extrapolated into the future. If the modifier equals 1.00, we have a linear trend with a constant amount of growth each period in the future. If the modifier exceeds 1.00, growth accelerates, a dangerous assumption more than a few time periods into the future.

Let's work through the computations at the end of 1987. The forecast error in 1987 is data minus forecast or $20.80 - 20.26 = 0.54$. The current level is the forecast for 1987 plus the level weight times the error, or $20.26 + 0.46 \times 0.54 = 20.51$. The current trend is the trend modifier times the previous trend plus the trend weight times the error, or $0.84 \times 3.40 + 0.10 \times 0.54 = 2.92$. The forecast for 1988 is the current level plus the trend modifier times the current trend or $20.51 + 0.84 \times 2.92 = 22.96$.

Now look at the forecasts for more than one period ahead. Let n be the number of periods ahead. To forecast more than one period into the future, the formula is:

$$\text{Forecast for } n > 1 = (\text{previous forecast}) + [(\text{trend modifier})^n] \times (\text{final computed trend estimate})$$

Let's forecast the years 2000 - 2003, or 2 - 4 years into the future. The previous forecast needed to get started is the 1999 forecast of 45.20. The final computed trend estimate was 0.83 at the end of 1998 (it is a coincidence that the final trend value and trend modifier are so close). The forecasts are:

Figure 4-9

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	TREND									INTERMEDIATE CALCULATIONS:			
2	Exponential smoothing with damped trend									RMSE (Square root of MSE)		2.45	
3										3 X RMSE		7.34	
4										Avg. of first 4 differences		3.4	
5										Warm-up SSE		35.92	
6										Warm-up MSE		5.986613	
7										Forecasting SSE		38.64	
8										Fcst SSE - Warm-up SSE		2.72	
9										Nbr. of forecast periods		6	
10	INPUT:				OUTPUT:				Forecasting MSE		0.45		
11	Level weight			0.46		Number of data		12		Warm-up Sum abs. err.		12.11	
12	Trend weight			0.10		Number of outliers		0		Warm-up MAD		2.01853	
13	Trend modifier			0.84		Warm-up MSE		5.99		Forecasting Sum abs. err.		14.63	
14	Number of warm-up data			6		Forecasting MSE		0.45		Fcst Sum abs. - Warm-up Sum a		2.52	
15	Last period to forecast			24		Warm-up MAD		2.02		Nbr. of forecast periods		6	
16										Forecasting MAD		0.42	
17										Forecasting MAD		0.42	
18										Last forecast at period		24	
19													
20	Month	Month	Period								Outlier	Sum	Period
21	& year	nbr.	nbr.	Data	Fcst	Error	Level	Trend			Indicator	nbr out	nbr.
22							17.40	3.40					
23	1987	1	1	20.8	20.26	0.54	20.51	2.92			0	0	1
24	1988	2	2	23.1	22.96	0.14	23.03	2.47			0	0	2
25	1989	3	3	27.2	25.10	2.10	26.07	2.29			0	0	3
26	1990	4	4	32.3	28.00	4.30	29.99	2.37			0	0	4
27	1991	5	5	34.4	31.98	2.42	33.10	2.24			0	0	5
28	1992	6	6	37.6	34.98	2.62	36.19	2.15			0	0	6
29	1993	7	7	38.0	38.00	0.00	38.00	1.81			0	0	7
30	1994	8	8	41.0	39.52	1.48	40.21	1.67			0	0	8
31	1995	9	9	41.6	41.61	-0.01	41.61	1.41			0	0	9
32	1996	10	10	42.2	42.79	-0.59	42.52	1.12			0	0	10
33	1997	11	11	43.9	43.47	0.43	43.67	0.99			0	0	11
34	1998	12	12	44.5	44.50	0.00	44.50	0.83			0	0	12
35	1999	13	#N/A		45.20	#N/A	44.50	0.83			#N/A	#N/A	13
36	2000	14	#N/A		45.79	#N/A	44.50	0.83			#N/A	#N/A	14
37	2001	15	#N/A		46.29	#N/A	44.50	0.83			#N/A	#N/A	15
38	2002	16	#N/A		46.71	#N/A	44.50	0.83			#N/A	#N/A	16
39	2003	17	#N/A		47.06	#N/A	44.50	0.83			#N/A	#N/A	17
40	2004	18	#N/A		47.36	#N/A	44.50	0.83			#N/A	#N/A	18

$$\begin{aligned} \text{Forecast for 2 years ahead (2000)} &= 45.20 + .84^2 \times 0.83 = 45.79 \\ \text{Forecast for 3 years ahead (2001)} &= 45.79 + .84^3 \times 0.83 = 46.29 \\ \text{Forecast for 4 years ahead (2002)} &= 46.29 + .84^4 \times 0.83 = 46.71 \\ \text{Forecast for 5 years ahead (2003)} &= 46.71 + .84^5 \times 0.83 = 47.06 \end{aligned}$$

The trend modifier is a fractional number. Raising a fractional number to a power produces smaller numbers as we move farther into the future. The result is called a damped trend because the amount of trend added to each new forecast declines. This makes sense because growth cannot continue indefinitely. The damped trend was selected by Alief management because it reflects slowing growth, probably the best that can be expected given political and economic conditions in the firearms market at the end of 1998.

By changing the trend modifier, you can produce different kinds of trend. A modifier equal to 1.0 yields a linear trend, where the amount of growth in the forecasts is constant beyond the end of the data. In the example, change the trend modifier to 1.0 to produce a graph showing growth that runs well above the last few data observations. Optimists can also use a modifier greater than 1.0 to produce an exponential trend in which the amount of growth gets larger each time period.

TREND requires that you choose the best combination of three parameters: level weight, trend weight, and trend modifier. There are numerous ways to do this in Excel. The quickest is to use the Excel Solver. Suppose that we want to choose parameters that minimize the forecasting MAD in the Alief data. Under the Tools menu, select Solver. In the menu box, set the target cell to \$H\$15 (the forecasting MAD). In the "Equal To" row, select Min. In the entry area for "By Changing Cells", enter \$D\$11..\$D\$13 (the parameters). Finally, click on Solve. The result is as shown in Figure 4-6. A word of caution is in order here. Sometimes the Solver returns extreme parameters, such as a level or trend weight of zero. We recommend that limits be imposed on the parameters: 0.10-0.90 for the level weight, 0.05-0.20 for the trend weight, and 0.70-1.00 for the trend modifier.

Another approach is to set the trend modifier equal to 1.0. and use the data table starting at the top of column Q in TREND to find the best combination of level and trend parameters, the combination that minimizes the MSE or MAD. Search over the range 0.10-0.90 for the level weight in increments of 0.10. Search over the range 0.05-0.20 for the trend weight in increments of 0.05. Then fix the level and trend parameters and try alternative values of the trend modifier in the range 0.70 to 1.00 in increments of 0.05. Once you find the best trend modifier, run the data table again, then do another search for the trend modifier. Keep going until the forecasting MSE or MAD stabilizes. Great precision is not necessary. TREND is a robust model, relatively insensitive to smoothing parameters provided that they are approximately correct.

The mathematical formulation of the model in TREND, originally developed by Gardner and McKenzie,¹ is:

$$\mathbf{e}_t = \mathbf{X}_t - \hat{\mathbf{X}}_{t-1}(\mathbf{1}) \quad (4-4)$$

$$\mathbf{S}_t = \hat{\mathbf{X}}_{t-1}(\mathbf{1}) + \mathbf{h}_1 \mathbf{e}_t \quad (4-5)$$

$$\mathbf{T}_t = \phi \mathbf{T}_{t-1} + \mathbf{h}_2 \mathbf{e}_t \quad (4-6)$$

$$\hat{\mathbf{X}}_t(\mathbf{m}) = \mathbf{S}_t + \sum_{i=1}^m \phi^i \mathbf{T}_t \quad (4-7)$$

Compared to the simple model, there is one additional smoothing parameter, \mathbf{h}_2 , for the trend, denoted \mathbf{T}_t . The trend modification parameter is ϕ . In the trend equation (4-6), notice that the old trend is multiplied by ϕ , while in the forecast equation (4-7), the new trend is multiplied by ϕ^i .

4.4 Trend smoothing with seasonality (TRENDADDSEAS and TRENDMULSEAS)

If the data are seasonal and trending, use TRENDADDSEAS for additive seasonality or TRENDMULSEAS for multiplicative seasonality. TRENDADDSEAS is a combination of the ADDITMON and TREND models. Data entry is done in the first tab, named Additive (identical to ADDITMON and not shown here). Seasonally-adjusted data from the first tab is automatically used in the second tab, named Trend, shown in Figure 4-10. The data are the Houston-London yields discussed in Chapter 2. Column D repeats seasonally-adjusted data from the Additive tab, while column E repeats the seasonal indices. In columns F-I, trend smoothing is applied to the seasonally-adjusted data. Errors in column G are equal to seasonally-adjusted data minus seasonally-adjusted forecasts. In column J, seasonally-adjusted forecasts are converted to final forecasts by adding back the appropriate seasonal index—that is, seasonal adjustment in the forecasts is reversed.

Two forecast graphs are included in the TRENDADDSEAS workbook (Figures 4-11 and 4-12). The first graph shows seasonally-adjusted data and seasonally-adjusted forecasts, while the second shows original data and final forecasts.

TRENDMULSEAS is a combination of the MULTIMON and TREND models and is not shown here. All calculations and worksheet operations are similar to TRENDADDSEAS except that seasonality is multiplicative.

¹Everette S. Gardner, Jr. and E. McKenzie, "Forecasting Trends in Time Series," *Management Science*, Vol. 31, No. 10 (October, 1985), pp. 1237-1246.

Figure 4-10

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	TRENDADDSEAS									INTERMEDIATE CALCULATIONS:			
2	Trend smoothing with damped trend and additive seasonality.									RMSE (Square root of MSE)		0.00235997	
3	Note: data entry is done in the additive tab.									3 X RMSE		0.00707992	
4										Avg. of first 4 differences		-0.00029177	
5										Warm-up SSE		0.00016708	
6										Warm-up MSE		0.00000557	
7										Forecasting SSE		0.00034162	
8										Fcst SSE - Warm-up SSE		0.00017454	
9										Nbr. of forecast periods		30	
10	INPUT:					OUTPUT:				Forecasting MSE			
11	Level weight				0.30	Number of data		60		Warm-up Sum abs. err.		0.05553602	
12	Trend weight				0.05	Number of outliers		0		Warm-up MAD		0.00185120	
13	Trend modifier				0.90	Warm-up MSE		0.00000557		Forecasting Sum abs. err.		0.11603014	
14	Number of warm-up data				30	Forecasting MSE		0.00000582		Fcst Sum abs. - Warm-up Sum at		0.06049411	
15	Last period to forecast				72	Warm-up MAD		0.00185120		Nbr. of forecast periods		30	
16						Forecasting MAD		0.00201647		Forecasting MAD		0.00201647	
17										Last forecast at period		72	
18													
19													
20	Month	Month	Period	S. Adjuste	Seas	S. Adjusted				Final	Original	Outlier	Sum
21	& year	nbr.	nbr.	Data	Index	Fcst	Error	Level	Trend	Fcst	Data	Indicator	nbr out
22								0.05365	-0.00029				
23	Jan-97	1	1	0.0534	-0.00357	0.05339	-0.00003	0.05338	-0.00026	0.04982	0.04979	0	0
24	Feb-97	2	2	0.0540	-0.00845	0.05314	0.00084	0.05339	-0.00020	0.04468	0.04552	0	0
25	Mar-97	3	3	0.0539	-0.00829	0.05321	0.00071	0.05343	-0.00014	0.04492	0.04563	0	0
26	Apr-97	4	4	0.0523	-0.00484	0.05330	-0.00097	0.05301	-0.00018	0.04845	0.04748	0	0
27	May-97	5	5	0.0522	-0.00171	0.05285	-0.00066	0.05265	-0.00019	0.05114	0.05048	0	0
28	Jun-97	6	6	0.0545	0.00010	0.05248	0.00202	0.05309	-0.00007	0.05258	0.05460	0	0
29	Jul-97	7	7	0.0549	0.01273	0.05302	0.00190	0.05359	0.00003	0.06575	0.06765	0	0
30	Aug-97	8	8	0.0542	0.01370	0.05362	0.00059	0.05380	0.00006	0.06732	0.06791	0	0
31	Sep-97	9	9	0.0514	0.00436	0.05385	-0.00245	0.05312	-0.00007	0.05821	0.05576	0	0
32	Oct-97	10	10	0.0522	0.00102	0.05305	-0.00082	0.05280	-0.00010	0.05407	0.05325	0	0
33	Nov-97	11	11	0.0503	-0.00334	0.05271	-0.00237	0.05200	-0.00021	0.04937	0.04700	0	0
34	Dec-97	12	12	0.0508	-0.00170	0.05181	-0.00103	0.05150	-0.00024	0.05011	0.04908	0	0
35	Jan-98	13	13	0.0508	-0.00357	0.05128	-0.00046	0.05114	-0.00024	0.04771	0.04725	0	0
36	Feb-98	14	14	0.0484	-0.00845	0.05092	-0.00253	0.05017	-0.00034	0.04247	0.03994	0	0
37	Mar-98	15	15	0.0487	-0.00829	0.04986	-0.00119	0.04950	-0.00037	0.04157	0.04038	0	0
38	Apr-98	16	16	0.0518	-0.00484	0.04917	0.00263	0.04996	-0.00020	0.04432	0.04695	0	0
39	May-98	17	17	0.0479	-0.00171	0.04977	-0.00183	0.04923	-0.00027	0.04807	0.04624	0	0
40	Jun-98	18	18	0.0485	0.00010	0.04898	-0.00052	0.04883	-0.00027	0.04908	0.04856	0	0

Figure 4-11

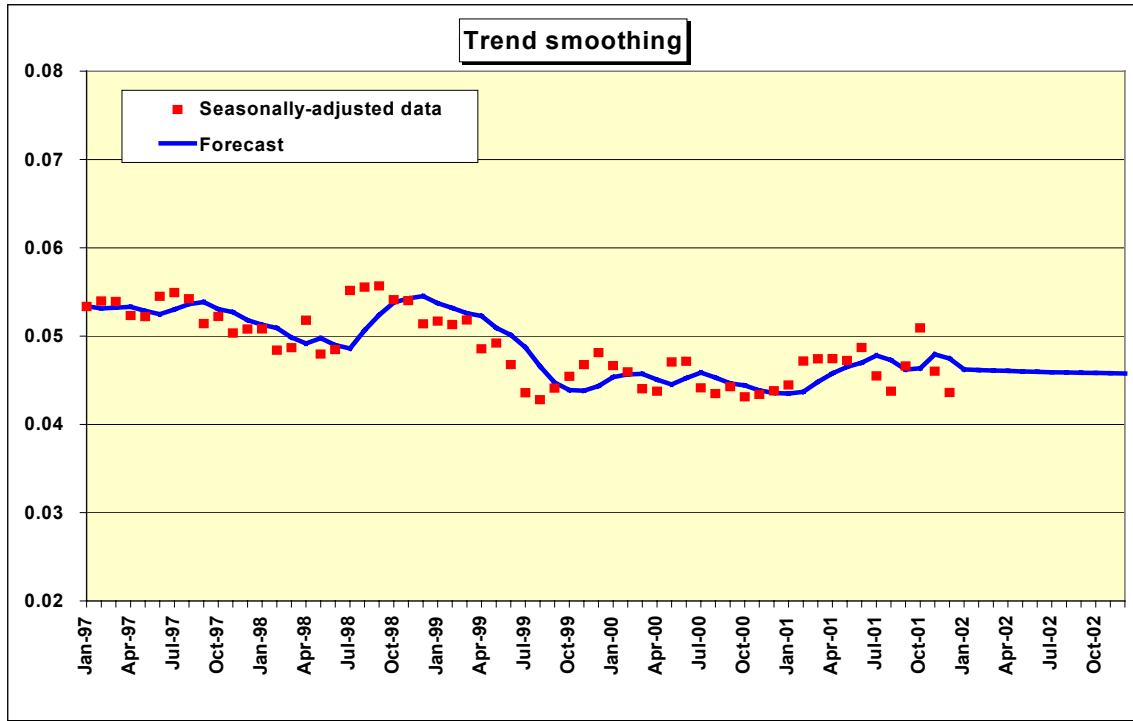


Figure 4-12

