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Comments on a patented bootstrapping method for forecasting intermittent demand

Willemain, Smart, and Schwarz (2004) developed a patented forecasting method for intermittent demand data. Their method is not model-based, but instead is a heuristic that combines a Markov process, bootstrapping, and jittering to simulate an entire distribution for lead time demand rather than a single forecast. The authors claimed significant improvements in forecast accuracy over simple exponential smoothing (Brown, 1959) and the method in Croston (1972). Unfortunately, Willemain et al. (2004) did not use the correct lead time demand distribution for either simple exponential smoothing or the Croston method, nor did they consider the published modifications to the Croston method.

For simple exponential smoothing, the authors assumed independent and identically distributed demands and estimated the standard deviation using $s\sqrt{L}$, where s is the one-step-ahead standard deviation and L is the number of time periods in the lead time. This expression was originally suggested by Brown (1959), but is seriously misleading. For simple exponential smoothing, the correct multiplier for the standard deviation s was derived using a multiple source of error (MSOE) state space model by Johnston and Harrison (1986) and a single source of error (SSOE) state space model by Snyder, Koehler, and Ord (1999):

$$f(\alpha, L) = \sqrt{(L + \alpha(L-1)L(1 + \alpha(2L-1)/6))}\ , \tag{1}$$

where α is the smoothing parameter. The effect of the correct multiplier is significant for any value of α at any lead time greater than one period. For example, with $\alpha = 0.2$ at a lead time of six periods, the correct standard deviation is 50% larger than $s\sqrt{L}$, and with $\alpha = 0.4$ it is over twice as large. The use of Eq. (1) to compute the correct multiplier for s would certainly change the performance of simple exponential smoothing in the Willemain et al. (2004) study.

Another critical assumption made by Willemain et al. (2004) is that lead time demand for simple exponential smoothing has a normal distribution. This assumption is not necessary, and alternatives should have been considered. One can easily use an SSOE model to bootstrap a lead time demand distribution. A parametric bootstrap procedure was proposed by Snyder et al. (2002), and it is straightforward to make this procedure an empirical bootstrap as was done for L-period-ahead forecasts in Hyndman, Koehler, Snyder, and Grose (2002).

For the Croston method, Willemain et al. (2004) used the same assumptions as for simple exponential smoothing. These assumptions have no basis in Croston (1972), who did not attempt to find the lead time demand for a fixed L periods into the future. Croston's replenishment level is simply designed to cover the next nonzero demand. It is the nonzero demands that are assumed to be independent and identically distributed. But even this assumption is not valid because the Croston method uses simple exponential smoothing to revise estimates for these demands. Once again, the standard deviation of lead time demand will not be $s\sqrt{L}$.

Several modifications to the Croston method should have been considered by Willemain et al. (2004). Alternative variance expressions are found in Johnston and Boylan (1996) and Sani and Kingsman (1997). In Syntetos and Boylan (2001), Croston's

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estimate of mean demand per unit time is shown to be biased and corrected equations are given. In Snyder (2002), an underlying model for intermittent demand was identified and the Croston method was modified to match the model. Snyder (2002) also gave a procedure for simulating intermittent lead time demand. Yet another modified version of Croston's method is found in Eaves and Kingsman (2004), which was published about the same time as the work of Willemain et al. (2004). The results of Eaves and Kingsman (2004) should be considered in any future research because their empirical results are convincing. The method espoused by Eaves and Kingsman (2004) was also used in Syntetos and Boylan (2005) and produced encouraging results.

A likely response to these comments is that Willemain et al. (2004) were simply making comparisons to methods and procedures commonly used in practice. That may be the case, but a paper in a scientific journal must consider prior research, and must test likely alternatives to any new method. We hope that a good comparison among forecasting methods for intermittent demand will be conducted.

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