A Top-Down Approach to Modeling US Navy Inventories
Author(s): Everette S. Gardner, Jr.
Reviewed work(s):
Source: *Interfaces*, Vol. 17, No. 4 (Jul. - Aug., 1987), pp. 1-7
Published by: INFORMS
Stable URL: http://www.jstor.org/stable/25060979
Accessed: 14/11/2011 15:07

A Top-Down Approach to Modeling US Navy Inventories

EVERETTE S. GARDNER, JR.

College of Business Administration
University of Houston
Houston, Texas 77004

For a military distribution system with a fixed investment budget, I developed trade-off curves between two aggregate variables: reordering work load and customer service. The curves showed that reallocating investment funds from safety stocks to cycle stocks would cut reordering work load by 20 percent, with no impact on customer service. Implementation of this idea yielded annual cost savings of $2 million.

In an earlier article in this journal [Gardner 1980], I argued that most of the theory available for inventory decisions is difficult, if not impossible, to implement. In theory, marginal costs are readily available for ordering, holding, and shortages of stock. In practice, marginal costs are elusive. Another problem is that theory usually deals with the behavior of individual items while managers are concerned with total inventory performance.

One way to resolve these problems is to take a top-down or aggregate approach to inventory modeling. Examples of this approach are found in Gardner and Dannenbring [1979], Peterson and Silver [1979], and Brown [1982]. The aim is to provide managers with trade-off curves or response surfaces showing the aggregate relationships among several variables: (1) customer service, defined as the number of inventory shortages per unit time, (2) the lump-sum investment in inventories on the balance sheet, and (3) reordering or stock replenishment work load. The relationships among these variables can be analyzed without any knowledge of marginal costs.

The top-down approach to inventory

0091-2102/87/1704/0001$01.25
This paper was refereed.

INVENTORY/PRODUCTION — PARAMETRIC ANALYSIS
INVENTORY/PRODUCTION — STOCHASTIC MODELS
MILITARY — LOGISTICS

modeling often reveals surprising trade-off options. I directed an analysis of inventories stocked at US Naval Supply Centers; the resulting trade-off curves showed that reordering work load could be cut by more than 20 percent with no perceptible impact on customer service. The basic idea is straightforward: reallocate funds from safety stocks to cycle stocks, while keeping total investment constant. This recommendation was implemented by the Navy with annual cost savings of $2 million.

The Navy Inventory System

The Navy operates eight supply centers in the United States. Each stocks two types of inventories, wholesale and retail. Wholesale inventories are primarily repair parts unique to Navy equipments and are managed under "push" decision rules. These rules do not allow the supply centers to make reorder decisions. Instead, managers at central inventory control sites determine system-wide requirements, procure material from industry, and push allocations to the supply centers.

Most of the theory available for inventory decisions is difficult to implement.

Retail material has applications common to all military services and is procured from industry by agencies outside the Navy, such as the General Services Administration and the Defense Logistics Agency. In contrast to the wholesale push system, retail inventories are managed under "pull" concepts. Each supply center makes its own reorder decisions based on local demand. These decisions are subject to budget constraints and policy guidelines established by the Naval Supply Systems Command. The Navy Fleet Material Support Office, where my research was carried out, also provides computer software, operations research support, and financial management assistance to the supply centers.

My analysis concentrated on the retail segment of supply center inventories. On average, each center stocks about 80,000 retail items worth about $25 million. As discussed in the appendix, retail order quantities are computed with a version of the EOQ model. Safety stocks are computed to minimize the number of requisitions short. The costs of ordering, holding, and shortages of stock are used as decision variables. These costs are varied until order quantities and safety stocks are obtained that meet budget constraints and customer service goals.

For many years each center has operated with a budget constraint requiring average inventory investment to be approximately 2.5 months of stock. This is an aggregate constraint computed by summing the monthly value of annual demand across the inventory and multiplying the sum by 2.5. The 2.5-month constraint is inflexible because it is established by law. However, flexibility exists to change the way the constraint is allocated to safety stocks and cycle stocks. Prior to this research, 1.5 months were allocated to safety stocks and 1.0 to cycle stocks (cycle stocks are estimated to be one-half order quantities). Again, these are aggregate constraints. For individual items, allocations to safety and cycle stocks vary considerably.

The customer service goal for this system requires that each supply center fill at least 85 percent of customer requisitions immediately from stock. A requisition is defined as a demand for a single inventory item and may be for any number of units of that item. The goal is based on Department of Defense policy. Numerous studies have shown the goal is reasonable, given budget constraints and the availability of back-up stocks at the General Services Administration, the Defense Logistics Agency, and other sources. The goal may seem low, but it is based on a stringent measure of customer service. The aim is to satisfy each requisition completely as soon as it is received, regardless of the number of units demanded. Partial shipments are not considered in measuring attainment of the goal.

Trade-Off Analysis

Budget constraints and the customer service goals narrow the trade-off options available in retail inventories. From the management scientist's point of view, the trade-off problem can be stated as follows: at each supply center, find the exchange curve between customer service and reordering work load at a fixed investment of 2.5 months of stock.

The Navy's retail inventory model is not suited to this kind of analysis. The only way to find the exchange curves with this model is to conduct tedious trial-and-error experiments with ordering, holding, and shortage costs. Another problem is that the model is imbedded in a number of large multipurpose programs that are difficult to use for trade-off analysis. These programs would also be difficult to rewrite to accommodate any new model proposal.

Fortunately, I found that the existing model has an alternative formulation, much easier to use for trade-off analysis. The alternative is a Lagrangian model (see the appendix for details). The objective function is to minimize the aggregate number of customer requisitions short. There are two aggregate constraints, reordering work load and total investment. The Lagrangian multipliers corresponding to these constraints are actually imputed marginal cost estimates. If these costs are used in existing models,

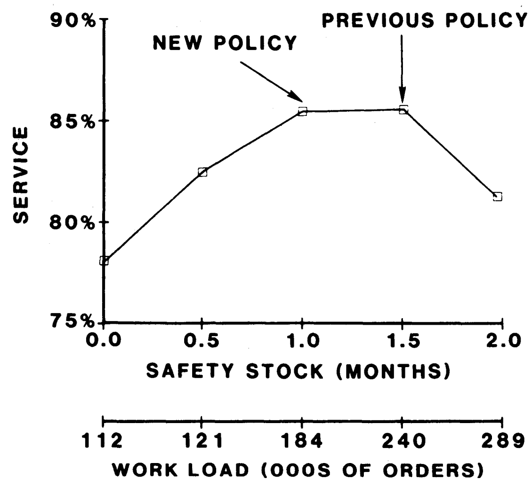

Figure 1: Trade-offs between work load and service with a fixed-inventory investment. The curve shows trade-offs at the Naval supply center in San Diego, with an aggregate investment constraint of 2.5 months of stock. Under previous policy, management allocated 1.5 months to safety stock, with the remaining 1.0 months in cycle stock. This policy yielded a work load of 240,000 reorders per year. A better policy is to put only 1.0 months in safety stock. Cycle stock can be increased to 1.5 months, which reduces work load to 184,000 reorders. The curve is almost flat between these two points, so there is little change in customer service.

the service, work load, and investment results are the same as the Lagrangian model.

The Lagrangian model was programmed as a research tool for studying inventory trade-offs. By solving the Lagrangian model for a range of work load constraints at a fixed investment constraint of 2.5 months, exchange curves were developed.

An example of an exchange curve for the supply center in San Diego, California, is shown in Figure 1. The curve has customer service, the percentage of requisitions filled immediately from stock, on the vertical axis. On the horizontal axis, the scale shows safety-stock investment in months. Remember that total investment is fixed at 2.5 months. Thus cycle stock at each point on the horizontal axis is 2.5 minus safety stock.

At the origin on the horizontal scale, there is no safety stock. The entire investment of 2.5 months is allocated to cycle stock, resulting in a service percentage of 78 percent. It may be difficult to understand why a service percentage this large can be obtained with no safety stock. The reason is that the number of shortages per unit time is the product of two factors, the number of reorder cycles and the expected shortage quantity per cycle. When the entire investment is allocated to cycle stock, the number of reorder cycles is minimized.

Moving to the right on the horizontal scale, safety stock increases while cycle stock decreases in order to maintain the fixed total investment of 2.5 months. At the extreme right, safety stock is 2.0 months while cycle stock is only 0.5 months. As safety stock increases, there is an improvement in customer service for a time. Eventually service reaches a plateau, followed by a decline, because the number of reorder cycles becomes overwhelming.

The work load or number of reorders corresponding to each investment allocation strategy is listed below the horizontal axis. The previous allocation was 1.5 months safety stock and 1.0 months cycle stock, yielding a work load of 240,000 reorders each year. Work load can be cut to 184,000 reorders by changing the allocation to 1.0 months safety stock and 1.5 months cycle stock. The exchange curve is almost flat between these points so the work load cut can be made with negligible impact on customer service.

Reordering work load could be cut by more than 20 percent with no perceptible impact on customer service.

In this example, a 50 percent increase in cycle stocks yields only a 23 percent decrease in reordering work load. There are two reasons for this difference in percentages. First, the relationship between order quantities and work load is nonlinear and subject to diminishing marginal returns. Second, these are aggregate figures. The effects for individual items vary considerably.

The San Diego trade-off curve is not unique. At each supply center, a similar curve was developed, showing that the investment allocation should be changed. There is no way to tell from these curves what the "best" allocation should be. However, we can identify the minimum safety stock necessary to meet the 85 percent customer service goal. The minimum safety stock varied somewhat at each center, ranging from 0.8 to 1.1 months of stock. I decided to recommend that safety stocks be set at one month at each center to simplify budget administration.

The trade-off curves are based on steady-state projections of service, work load, and investment. Before implementing the change in safety stocks, a simulation of the inventory system was conducted to be certain that the model assumptions were reasonable. For a discussion of assumptions in this type of inventory modeling, see Hadley and Whitin [1983], Peterson and Silver [1979], or Gardner [1983, 1984]. Data for the simulation were generated from empirical distributions of demand history collected over a period of several years at the supply centers. The simulation included a wide range of real-world complications, such as delays in processing reorders, variable lead times caused by transportation problems and shortages of stock at the inventory sources supplying the Navy, and beginning on-hand stock balances that differed from steady-state assumptions. The simulation was validated by comparing simulated service, work load, and investment to known historical values. The correspondence was excellent. Next, the change in investment allocation was simulated, and this confirmed the steady-state results from the Lagrangian model. No negative impact from the new investment allocation could be found.

Each center stocks about 80,000 retail items worth about $25 million.

Implementation

Implementation of this research was a simple matter. At each supply center, the Lagrangian multipliers were used to determine new cost variables yielding one month of safety stock in the existing inventory model. Prior to this research, the total number of reorders placed by the centers was about 840,000 per year. I predicted this figure would be reduced to about 670,000 orders. Navy accountants estimated a savings in manpower costs of about $2 million per year.

There is no question that these savings were actually achieved. The Naval Supply Systems Command made sure of the savings by cutting the operating expense budgets at each supply center. At the time of this writing, one year after implementation, the inventory system has behaved as planned. Work load is down by more than 20 percent, while customer service and inventory investment have remained constant.

Conclusion

This research should have potential applications in other physical distribution systems. The inventory model outlined in the appendix is relatively simple and can be reformulated to handle other objective functions. The information required to run the model is minimal: annual demand estimates, unit prices, and means and variances of lead time demands.

Acknowledgment

This article is based on an official US

Navy study directed by the author while serving as Commander, Supply Corps, US Navy and Director of Operations Analysis at the Navy Fleet Material Support Office, Mechanicsburg, Pennsylvania.

References

Brown, R. G. 1982, *Advanced Service Parts Inventory Control*, second edition, Materials Management Systems, Incorporated, Norwich, Vermont.

Gardner, E. S. 1980, "Inventory theory and the gods of Olympus," *Interfaces*, Vol. 10, No. 4 (August), pp. 42–45.

Gardner, E. S. 1983, "Approximate decision rules for continuous review inventory systems," *Naval Research Logistics Quarterly*, Vol. 20. No. 4 (March), pp. 59–68.

Gardner, E. S. 1984, "A comparison of investment allocation strategies for distribution inventories," *Decision Sciences*, Vol. 15, No. 1 (Winter), pp. 22–30.

Gardner, E. S. and Dannenbring, D. G. 1979, "Using optimal policy surfaces to analyze aggregate inventory trade-offs," *Management Science*, Vol. 25, No. 8 (August), pp. 709–720.

Hadley, G. and Whitin, T. M. 1983, *Analysis of Inventory Systems*, Prentice-Hall, Englewood Cliffs, New Jersey.

Peterson, R. and Silver, E. A. 1979, *Decision Systems for Inventory Management and Production Planning*, John Wiley and Sons, New York.

APPENDIX — Model Formulations

The inventory model used by the Navy is based on the following total cost expression for each line item stocked:

$$TC_i = \frac{C_o D_i}{Q_i} + \frac{C_h Q_i}{2} + C_h S_i + \frac{C_s D_i}{m_i Q_i} \int_{R_i}^{\infty} (x_i - R_i)\, f(x)\, d(x). \tag{1}$$

The variables are

C_o = ordering cost,
C_h = annual inventory carrying cost as a percentage of dollar value,
C_s = shortage or penalty cost per customer requisition backordered,
D_i = annual demand in dollars,
Q_i = order quantity in dollars,
R_i = reorder point in dollars,
S_i = safety stock in dollars,
x_i = lead time demand in dollars,
m_i = customer requisition size in dollars, assumed constant, and
$f(x)$ = probability density function for lead time demand, assumed normal, although the solution algorithm can be adapted to other distributions.

D_i / Q_i is the number of orders per year, so the first term in (1) is the total ordering cost. The second and third terms are total holding costs for cycle and safety stocks, respectively. In the final term, the integral is the partial expectation of demand or the expected number of dollars short during one order cycle. Thus the fraction multiplied by the integral yields total shortage costs per year.

Before presenting the solution, the following simplifying notation is introduced:

$$P_i = \int_{R_i}^{\infty} f(x)\, d(x), \tag{2}$$

$$E_i = \int_{R_i}^{\infty} (x_i - R_i)\, f(x)\, d(x), \tag{3}$$

$$F_i = D_i / m_i. \tag{4}$$

P_i is the probability of a shortage occurrence during one order cycle, E_i is the partial expectation discussed above, and F_i is the annual frequency of demand.

Solution of (1) using classical optimization techniques requires that the following expressions hold for each inventory item:

$$Q_i = [2[F_i E_i + (C_o/C_s)\, D_i]/(C_h/C_s)]^{1/2}, \tag{5}$$

$$P_i = (C_h/C_s)\, Q_i/F_i. \tag{6}$$

Another way to model this inventory system is to formulate a Lagrangian optimization. Rather than optimize costs for a single item, we optimize customer service for all items under a set of aggregate constraints. The objective is to minimize the number of requisitions short.

$$\text{Minimize } Z = \sum_i (F_i E_i)/Q_i \qquad (7)$$

subject to the investment constraint

$$\sum_i (Q_i/2 + S_i) = I, \qquad (8)$$

and the work load constraint

$$\sum_i (D_i/Q_i) = W. \qquad (9)$$

In (8), the investment constraint I is the sum of aggregate cycle and safety stocks. In (9), the work load constraint W is the aggregate number of orders per year.

After forming the Lagrangian function, differentiating, and simplifying the first-order conditions, the following solution is obtained. L_I and L_W are the Lagrangian multipliers associated with the investment and work load constraints, respectively:

$$Q_i = \left[2(F_i E_i + L_W D_i)/L_I\right]^{1/2}, \qquad (10)$$

$$P_i = L_I Q_i/F_i, \qquad (11)$$

$$L_I = \sum_i F_i P_i/(2[I - \sum_i S_i]), \qquad (12)$$

$$L_W = (1/W) \left[[(L_I \sum_i Q_i)/2] - [\sum_i (F_i E_i)/Q_i]\right]. \qquad (13)$$

The cost-based expressions for Q_i and P_i in (5) and (6) are equivalent to the Lagrangian expressions in (10) and (11), provided that $C_h/C_s = L_I$ and $C_o/C_s = L_W$. Thus the Lagrangian multipliers can be used to determine cost variables for the existing inventory model. The values of the Lagrangian multipliers are found using the method of successive approximations. See Gardner [1983] or Gardner and Dannenbring [1979] for details.

A letter from R. M. Moore, Captain, SC, USN, Commanding Officer, Department of the Navy, Navy Fleet Material Support Office, 5450 Carlisle Pike, P. O. Box 2010, Mechanicsburg, Pennsylvania 17055-0787, states, "This letter is to confirm the accuracy of the article, 'A Top-Down Approach to Modeling US Navy Inventories,' by Everette S. Gardner, Jr. This article is approved for publication."