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USING OPTIMAL POLICY SURFACES TO ANALYZE AGGREGATE INVENTORY TRADEOFFS*

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The marginal cost information needed to implement traditional inventory models is not likely to be available in practice. The most important inventory management issues in practive involve aggregate objectives and constraints while the richest theoretical models deal with single item management. To help resolve these problems, the authors propose that inventory decisions be conceived as policy tradeoffs on a three dimensional response surface showing the optimal relationships among aggregate customer service, workload, and investment. We show that any optimal management decision must result in a point located on the surface. Computational results show that the methodology suggested can make improvements in management policy in four inventories that total more than 78,000 line items.
(INVENTORY/PRODUCTION—PARAMETRIC ANALYSIS; INVENTORY/PRODUCTION—STOCHASTIC MODELS; MILITARY—LOGISTICS)

1. Introduction

In the authors' opinion, a serious gap exists between theory and practice in inventory management. One reason is that the marginal ordering, holding, and shortage costs typically assumed in the theory are difficult, if not impossible, to measure in practice [1], [4], [7], [9], [13], [21], [22], [24], [27], [29]. For example, in Ziegler's survey [29], he concludes that all the suggested approaches to determining ordering costs in the accounting literature result in average rather than marginal costs. The holding cost in practice is mostly composed of the cost of capital, which is a highly subjective measure [1], [14], [16], [21], [29]. The use of shortage costs in inventory models has not been adopted by most practitioners [1], [3], [4], [13], [21], [22], [29] since there is no basis for their measurement in accounting methodology [29].

Another problem in practice is that inventory theory has traditionally emphasized single-item models which provide insufficient insights for the management of multi-item inventories. Most practitioners are primarily concerned instead with aggregate inventory control [1], [2], [18], [21], [27] to meet specific aggregate objectives or constraints for customer service, workload, and investment.

This paper presents an approach to decision making in inventory systems that avoids cost measurement problems and incorporates aggregate objectives and constraints. While traditional theory is based on the objective of cost minimization we propose that inventory decisions be conceived as policy tradeoffs on a three-dimensional response surface, the "optimal policy surface." The axes of the surface are measured in aggregate terms: the percentage of inventory shortages as a measure of customer service; the workload in terms of the number of annual stock replenishment orders; and total investment (the sum of cycle and safety stocks). The surface is optimal in the sense that the number of shortages at any point on the surface is minimal for the corresponding combination of workload and investment values. Aggregate inventory decisions are defined as the selection of some combination of the three variables. We show that decisions resulting in combinations of variables that do not lie on the surface cannot be optimal, regardless of the underlying cost structure of the firm.

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A similar theoretical construct for the case of deterministic demand, the "optimal policy curve," was originally developed by Starr and Miller [24]. We review their ideas in the next section, and then generalize to the stochastic case. Model formulations and solution algorithms are presented, with computational results for four inventories drawn from a large military distribution system. The results show that the concept of the optimal policy surface can be a useful practical tool for inventory decisions.

2. The Optimal Policy Curve

When demand is deterministic, there is an underlying set of optimal relationships in any inventory between aggregate cycle stock investment and workload. This set of relationships may be called an "optimal policy curve." An example is shown in Figure 1, which was derived with Lagrangian multipliers [24], and gives the minimum cycle stock investment for a specified workload or vice versa. Points located below the curve are infeasible combinations of investment and workload, while points above the curve are nonoptimal. For example, a management decision to operate at point A in Figure 1 represents an investment of $350,000 and a workload of 5,000 annual orders. But at point B, workload can be reduced to 3,000 for the same investment. An alternative is to move to point C, where the workload is still 5,000 but investment has been reduced to $225,000.

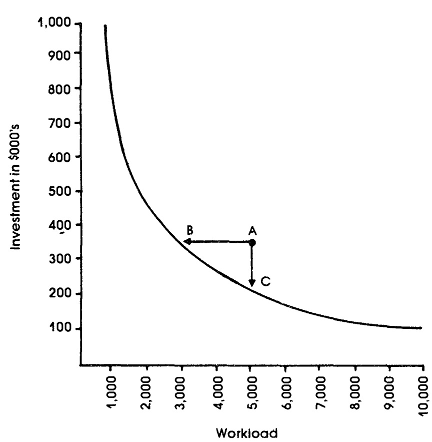

FIGURE 1. The Optimal Policy Curve for Deterministic Demand.

The optimal policy curve is a powerful concept, since it shows exactly how workload and investment may be exchanged for each other. "The executive, with his intimate knowledge of the circumstances of the company, can often quickly converge on the optimal point on the curve for the company without having had to convert his knowledge into the form of carrying and ordering costs—something which can often be done only badly if at all." [24]. There is considerable evidence that the concept of the optimal policy curve has been successful in practice. A simplified computational procedure to derive the optimal policy curve, the "Limit" technique, was developed by Plossl and Wight [21] for the American Production and Inventory Control Society, and the procedure is part of the body of knowledge required to gain certified practitioner status in that organization. Another method of deriving the optimal policy curve was developed by Prichard and Eagle [22]. Other variations of deterministic inventory models which link several items with investment constraints may be found in [11] and [12].

3. The Optimal Policy Surface

With stochastic demand, management decisions are much more complex. Cycle and safety stock investment decisions are interdependent for each line item. Interactions also exist between items, since some aggregate mix of cycle and safety stock investment must be selected and allocated across the items stocked. To treat these complexities, the optimal policy concept must be extended to three dimensions, as illustrated in Figure 2, which was constructed from a sample of 500 line items in a military distribution system. The vertical axis measures customer service in terms of the percentage of annual customer requisitions which are backordered (short). Depending on management objectives, various other measures could be used for the vertical axis, such as the percentage of sales dollars short or the number of shortage occurrences. The investment axis in Figure 2 is stated as the sum of aggregate cycle and safety stocks, while workload is the number of annual stock replenishment orders.

Figure 2 is an optimal policy surface, since it gives the minimal or optimal level of requisitions short for the range of workload and investment shown. For any one of the three variables, the surface also shows the range of optimal combinations of the other two. If management chooses to operate with an aggregate investment of $900,000, the surface shows that requisitions short will vary from 6.13% to 3.42%, depending on the workload decision. If a workload of 3,000 annual orders is selected, requisitions short will vary from 0.75% to 3.71%, depending on the investment decision.

The optimal policy surface provides a sound theoretical basis for aggregate inventory decisions in this sense: any optimal decision must result in a point located on the optimal policy surface. Any point located below the surface is infeasible, and any decision that results in a point located above the surface can be improved by moving back to the surface.

To illustrate, current management policy for the inventory sample in Figure 2 results in the following combination of variables: workload = 3,586, investment = $1,367,000, and requisitions short = 0.89%. For the same workload and investment coordinates, a modest reduction in requisitions short could be made to a level of 0.69%. However, the workload could be cut to less than 2,300 annual orders (a

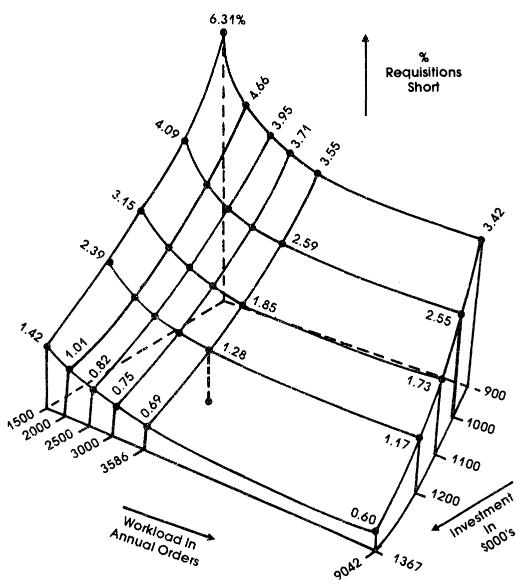

FIGURE 2. The Optimal Policy Surface for Stochastic Demand.

reduction of 35%) and retain current levels of requisitions short and investment. If the investment budget is tight, another choice is to cut investment by about 8.5% to $1,250,000 without changing the other two variables. Other points on the surface would yield simultaneous improvement in all three variables over current policy.

Since customer service objectives depend on a host of complex policy issues in practice [22], the optimal policy surface is useful in quantifying exactly what the firm must pay in terms of workload and investment to meet these objectives. Although cost information is not incorporated in Figure 2, any cost information which the decision-maker is willing to use can be considered after the surface has been constructed. The key point is that the decision-maker does not have to specify marginal cost estimates in order to see the range of tradeoffs in the inventory.

Most of the tradeoffs displayed by the optimal policy surface are straightforward. With a fixed workload, increases in investment simply add safety stock and thereby reduce requisitions short. At a fixed investment level, increases in workload result in an exchange of cycle stock for safety stock, again leading to a reduction in requisitions short. It should be noted that these comments apply to the aggregate behavior only; the effects on individual items can vary considerably.

Most of the axis limits of the surface are also straightforward. At infinite (unconstrained) investment levels, there would be enough safety stock so that the percentage of requisitions short would approach zero for any workload constraint. As investment levels are reduced, safety stock would eventually disappear, so that for a given workload the lowest feasible investment level would be the same as that for the deterministic optimal policy curve. At this limit, requisitions short would, of course, be very large. For a specific investment level there is a similar lower limit on workload, without safety stock, equivalent to that found with the optimal policy curve.

It should be recognized that if budget restrictions are particularly severe, it would be necessary to consider the possibility that the aggregate safety stock level is negative. The model formulated here does not treat this possibility although suitable modifications could accomplish this consideration.

The effects of increases in workload are more complex. The right-hand edge of the surface is the limit of effective constraint on aggregate workload, since a solution with an unconstrained workload will always provide fewer expected requisitions short than would be the case for workload equality constraints larger than the edge. The reason that this limit exists has to do with the two ways in which workload impacts on requisitions short. With a fixed investment constraint, increases in workload are equivalent to increases in the number of exposures to risk of stockout. On the other hand, the increased workload leads to a change in the mix of cycle and safety stock, the reduced need for cycle stock being channeled into increased safety stock.

Thus, as workload increases, the increase in exposure risk tends to increase expected requisitions short while the change in investment mix works in the opposite direction. The net effect is favorable for low to moderate workload levels, but eventually the effect of exposure risk overwhelms the protection afforded by the increased safety stock. Thus we can refer to the right-hand edge of the surface as the edge of optimality since any further increase in workload would only serve to increase requisitions short. It is certainly feasible to choose workload levels beyond the edge, but never optimal.

In the next section, we show how to derive the edge with a Lagrangian model which minimizes the number of shortages subject only to an investment constraint. Since workload is unconstrained, the optimal workloads found by the model serve to define the edge of optimality. Points to the left of the edge can be derived by enriching the same model with a workload constraint. Although details are given only for the requisitions short objective function, extensions to other common objective functions can be made by following the same computational scheme [9].

4. Locating the Edge of Optimality

To locate any single point on the edge of optimality, the objective function is:

$$\text{Min } Z = \sum_i \frac{D_i}{Q_i} \cdot \int_{R_i}^{\infty} \left(\frac{X_i - R_i}{m_i} \right) f(x)\, dx \tag{1}$$

subject to the investment constraint

$$\sum_i \left(\frac{Q_i}{2} + S_i \right) = I \tag{2}$$

where

Z = expected annual number of customer requisitions backordered or short,
D_i = annual sales in dollars for item i,
Q_i = order quantity in dollars for item i,
R_i = reorder point in dollars (sum of safety stock plus leadtime demand stock) for item i,
X_i = leadtime demand in dollars for item i,
m_i = customer requisition size in dollars for item i,
$f(x)$ = probability density function for leadtime demand,
S_i = safety stock in dollars, for item i, and
I = investment constraint in dollars.

The assumptions in this formulation are that the length of the leadtime is constant, and that the customer requisition sizes for each line item are constants and are independent of the level of demand. For this example, we shall also assume that leadtime demand is normally distributed, although the solution procedure applies to other distributions as well.

The next step is to form the Lagrangian function, L:

$$L = \sum_i \frac{D_i}{Q_i} \int_{R_i}^{\infty} \frac{(X_i - R_i)}{m_i} f(x)\, dx + \lambda_I \left[\sum_i \left(\frac{Q_i}{2} + S_i \right) - I \right], \tag{3}$$

where λ_I = the Lagrangian multiplier.

Differentiating with respect to Q_i, S_i, and λ_I, we obtain the first order conditions:

$$\frac{\partial L}{\partial Q_i} = \frac{-D_i}{Q_i^2 m_i} \int_{R_i}^{\infty} (X_i - R_i) f(x)\, dx + \frac{\lambda_I}{2} = 0, \tag{4}$$

$$\frac{\partial L}{\partial S_i} = \frac{-D_i}{Q_i m_i} \int_{R_i}^{\infty} f(x)\, dx + \lambda_I = 0, \quad \text{and} \tag{5}$$

$$\frac{\partial L}{\partial \lambda_I} = \sum_i \left(\frac{Q_i}{2} + S_i \right) - I = 0. \tag{6}$$

Since all model functions are convex, we know that any solution to the first order conditions will be an optimal solution. Unfortunately, there is no direct solution for any of the variables in the problem. The approach followed here is essentially the method of successive approximations which iteratively searches for the simultaneously optimal values of λ_I and the Q_i and S_i for each line item.

Other formulations of stochastic models linking several items with average investment constraints have been proposed by Daeschner [5], Gerson and Brown [10] and Schrady and Choe [23]. To find the optimal solution to these Lagrangian models, the authors proposed either trial and error search [5], [10] or conversion of the problem to a sequence of unconstrained optimization problems using the SUMT technique [23]. Unfortunately, these procedures prove to be tedious and expensive in large applications [9], [23] and become even more difficult when the present model is enriched with a workload constraint, as shown in the next section. Hadley and Whitin [12] have also emphasized the computational difficulties associated with constrained stochastic inventory models.

The method of successive approximations (see [25] for a discussion) can be used to converge rapidly on the optimal value of λ_I which, in turn, can be used to derive the optimal Q_i and S_i values for each line item in the inventory.

Before describing the search algorithm, some simplifying notation is introduced. Let

$$P_i = \int_{R_i}^{\infty} f(x)\,dx, \tag{7}$$

$$E_i = \int_{R_i}^{\infty} (X_i - R_i) f(x)\,dx, \tag{8}$$

$$F_i = D_i / m_i. \tag{9}$$

P_i is the probability of a stockout during one order cycle. E_i is the partial expectation of demand or the expected number of dollars short per order cycle. F_i is the annual frequency of demand for each line item.

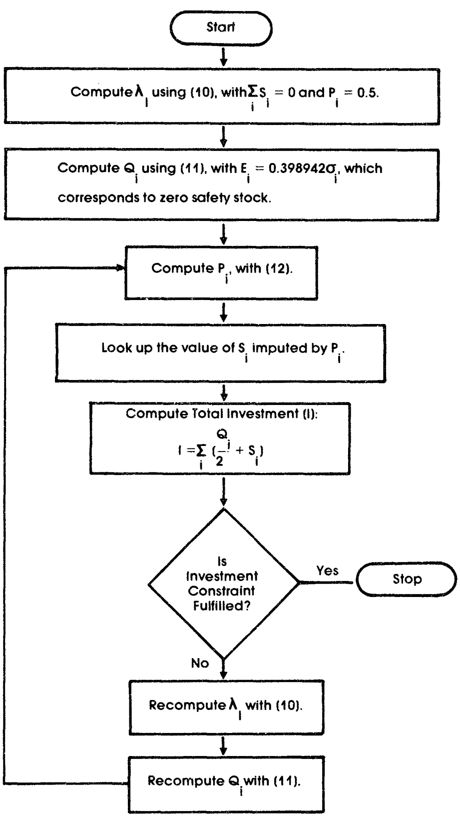

FIGURE 3. The Investment-Constrained Search Algorithm.

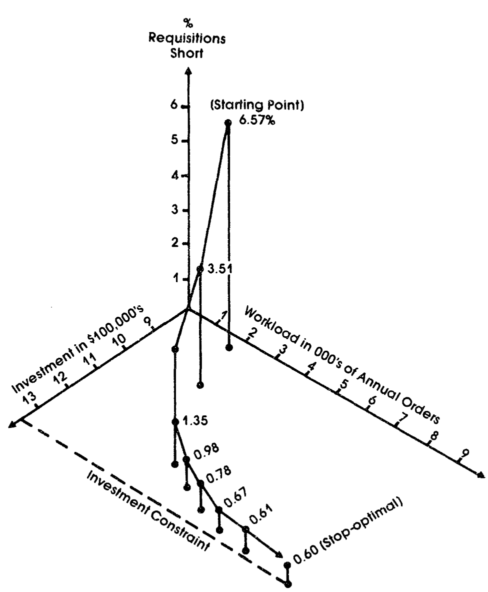

FIGURE 4. Convergence of the Investment-Constrained Search.

Although not derived here, simple algebra provides the equations used in the search:

$$\lambda_I = \sum_i F_i P_i / 2\Big(I - \sum_i S_i\Big), \tag{10}$$

$$Q_i = \sqrt{2F_i E_i/\lambda_I}\,, \tag{11}$$

$$P_i = \lambda_I Q_i / F_i, \tag{12}$$

A summary of the steps in the search algorithm using equations (10)–(12) is shown in Figure 3. We begin with an initial assumption of zero safety stock for each line item, which allows us to use (10) to derive an initial λ_I, which, in turn, determines the initial Q_i's using (11). Equation (12) is next used to calculate appropriate stockout probabilities, P_i, which then determine specific safety stock levels, S_i. This process is repeated until the investment constraint is fulfilled, iteratively updating λ_I, Q_i, P_i, and S_i.

The search algorithm summarized in Figure 3 has been run more than 100 times on data sets ranging in size from 500 to more than 40,000 line items. In every case, the model converged to within 1% of the investment constraint in twelve iterations or less. CPU time in Fortran, Level H, on the IBM 370/155 has been modest, averaging only 0.36 seconds per iteration per 1,000 line items. An example of the way the search algorithm behaves is given in Figure 4. The data used were the same as those used to derive the optimal policy surface in Figure 2. The model assumed an investment constraint of \$1,367,000 and converged to the minimum requisitions short value of 0.60% in 9 iterations. (This point corresponds to the point at the lower right corner of Figure 2.) The curved path followed by the model is representative of all the data sets tested. To complete the edge of optimality, the model was run four more times with the investment levels shown in Figure 2.

5. Locating Interior Points on the Optimal Policy Surface

To locate any interior point on the surface, (to the left of the edge of optimality), a workload constraint is added to (1) and (2):

$$\sum_i \frac{D_i}{Q_i} = W \tag{13}$$

where W = workload constraint in number of annual orders.

The Lagrangian function becomes:

$$L = \sum_i \frac{D_i}{Q_i m_i} \int_{R_i}^{\infty} (X_i - R_i) f(x)\,dx$$
$$+ \lambda_W\Big(\sum_i \frac{D_i}{Q_i} - W\Big) + \lambda_I\Big[\sum_i\Big(\frac{Q_i}{2} + S_i\Big) - I\Big]. \tag{14}$$

Solution of this model leads to identical equations for λ_I and P_i as derived for the simpler model, equations (10) and (12), respectively. To these are added the optimal condition for λ_W,

$$\lambda_W = \frac{1}{W}\left[\frac{\lambda_I \sum_i Q_i}{2} - \sum_i \frac{F_i E_i}{Q_i}\right] \tag{15}$$

and a modified optimal equation for Q_i, which incorporates the effects of both the investment and workload constraints,

$$Q_i = \sqrt{2(F_i E_i + \lambda_W D_i)/\lambda_I} \ . \tag{16}$$

The search strategy employed for this model is similar to the previous case and is outlined in Figure 5. As before, it is assumed initially that no safety stock is maintained for any of the items. This permits, using (10), direct calculation of an initial approximation for λ_I. Note, however, that the equations for λ_W and Q_i are interdependent, preventing their use in the initialization phase. Rearranging equation (12), however, we can derive an equation for Q_i which does not require an estimate of λ_W:

$$Q_i = F_i P_i / \lambda_I = 0.5 F_i / \lambda_I. \tag{17}$$

The Q_i's based on (17) can then be used to provide an initial estimate of λ_W from (15). Thereafter the search progresses by iteratively updating values for Q_i, P_i (and correspondingly S_i), λ_I, and λ_W, using equations (16), (12), (10), and (15), until both the workload and investment constraints are fulfilled.

The model with both constraints has also been run on more than 100 data sets, and has always converged to a point within 1% of both constraints within 30 iterations.

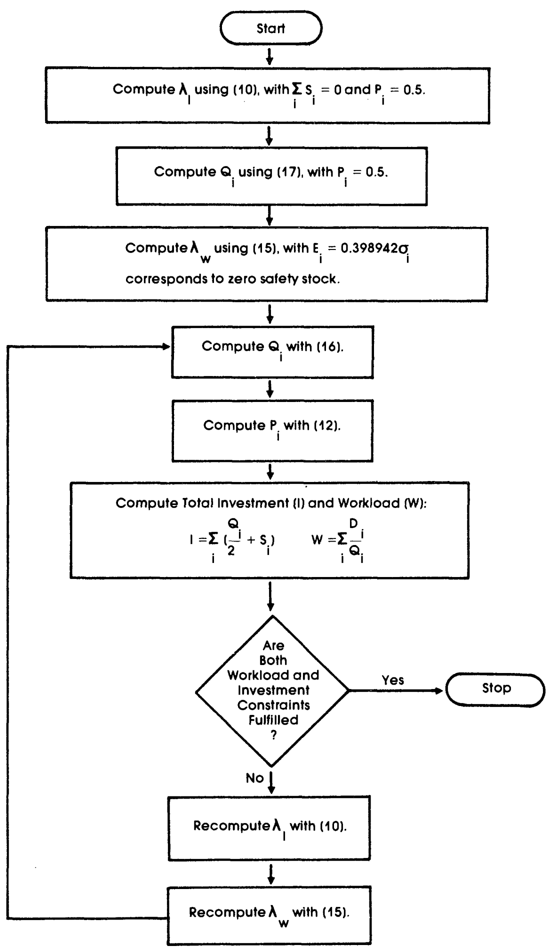

FIGURE 5. The Workload- and Investment-Constrained Search Algorithm.

CPU time has averaged about 0.41 seconds per 1,000 line items. An example of the way the search behaves is given in Figure 6, with a workload constraint of 3,586 orders and an investment constraint of $1,367,000 (refer to those coordinates in Figure 2). After seven iterations, the model reached a requisitions short level of 0.98%. Between that point and the optimum of 0.69%, the model required an additional 23 iterations (which were not plotted individually).

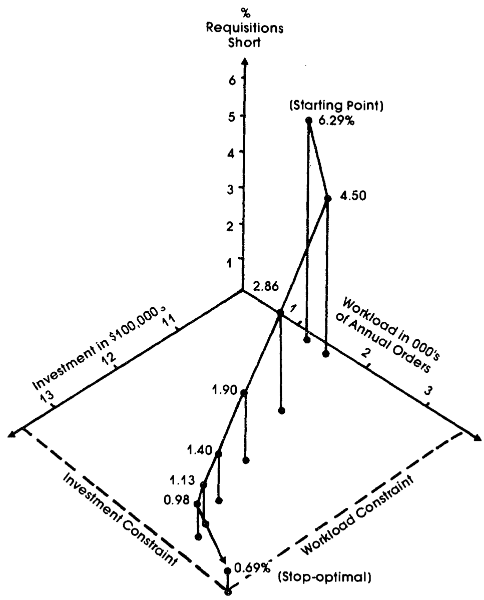

FIGURE 6. Convergence of the Workload- and Investment-Constrained Search.

There are some interesting analogies between the order quantity and safety stock expressions in (16) and (12) and those that would be derived using a well-known classical model with a cost-based objective function. To illustrate, let C_0 be the marginal ordering cost, C_h be the annual inventory carrying cost expressed as a percentage of dollar value, and C_s be the shortage or penalty cost per customer requisition short or backordered. Then the total annual costs for the ith line item are:

$$TC_i = \frac{C_0 D_i}{Q_i} + \frac{C_h Q_i}{2} + C_h S_i + \frac{C_s D_i}{m_i Q_i} \int_{R_i}^{\infty} (X_i - R_i) f(x)\, dx. \tag{18}$$

Solution of this model using classical optimization techniques requires that

$$Q_i = \sqrt{2[F_i E_i + (C_0/C_s)D_i]/(C_h/C_s)} \tag{19}$$

and

$$P_i = (C_h/C_s)Q_i/F_i. \tag{20}$$

A simple comparison of (19) and (20) with (16) and (12) shows that they are equivalent provided that

$$C_0/C_s = \lambda_W, \tag{21}$$

$$C_h/C_s = \lambda_I. \tag{22}$$

Therefore, one way in which the constrained models can be interpreted is that the Lagrangian multipliers act as surrogates for the marginal cost information.

6. An Application of the Models

The models were tested with a sample of 78,180 line items representing the complete inventories at four of thirty stock points in a military distribution system. The line items used in the test represent about 20% of the line items stocked in the system. Order quantities in the system are currently computed with a standard EOQ model. Safety stocks are computed independently of order quantities with a Lagrangian model that minimizes the number of requisitions short for a given safety stock budget. Trial and error procedures are used to: (1) find the single Lagrangian multiplier that allocates safety stock, (2) adjust aggregate workload at each stock point to constraints imposed by personnel budgets, and (3) adjust the sum of cycle and safety stock investment to constraints imposed by budget considerations.

The first step in the test was to find the point on the optimal policy surface (the value of requisitions short) that corresponds to the current workload and investment constraints for each inventory. These results are compared to current policy in Table 1. In every case, current policy could be improved by moving to a position on the optimal policy surface. The results shown in Table 1 are expected values, computed with the assumptions and approximations discussed above in the sections on model development. Since these assumptions and approximations are identical to those used in the current inventory system, the results are strictly comparable. In practice the actual requisitions short achieved using the current policies is normally somewhat higher than that predicted by the model. This difference in predicted and actual performance is due to a number of factors, including the exercise of local control by stock point managers and the existence of line items currently in an out-of-stock status or which have relatively poor current stock positions. Further exploration of the surface showed that workload cuts averaging 25% could be made at each stock point without changing current investment or requisitions short. Some reductions in investment could also be made for the existing workload and requisitions short.

TABLE 1
Comparison of Current Policy to the Optimal Policy Surface

		Expected Value of Requisitions Short		
Stock Point	Line Items Stocked	Current Policy	Optimal Policy	Current Minus Optimal
1	12,262	6.72%	0.78%	5.94%
2	5,039	8.49	2.35	6.14
3	43,882	1.55	0.45	1.10
4	16,997	3.28	0.88	2.40

There appear to be two related reasons for these potential improvements. First, the optimal policy surface is built up from simultaneous solutions for order quantities and safety stocks for each line item. In current policy, these two elements are computed independently of each other. Since standard deviations of leadtime demand in the system are relatively large, we rely on Groff and Muth's conclusion [20] that simultaneous solutions should give better results. Second, the surface gives a better aggregate mix of cycle and safety stock investment than current policy. In current policy, the aggregate mix is roughly 40% cycle stock and 60% safety stock. These percentages are reversed for the corresponding workload and investment coordinates on the optimal policy surface.

7. Conclusions

Given the difficulties in measuring the traditional inventory costs and the strategic advantages of exploring aggregate inventory tradeoffs, we propose that inventory model-builders bypass the use of marginal costs and work directly with those aggregate variables that can be measured—the number of inventory shortages, workload, and investment. Since the response surface that shows the relationships among these variables is optimal, we can state that management decisions should result in points located on the surface. This statement is true, regardless of the particular cost structure of any firm, and provides a sound theoretical basis for decision-making. If objective cost information is available, it can be considered after the surface is constructed. The computational results presented show that the models used to derive the optimal policy surface are efficient, and could make improvements in one inventory system.

References

1. BROWN, R. G., *Decision Rules For Inventory Management*, Holt, New York, 1967.
2. ———, *Materials Management Systems—A Modular Library*, Wiley, New York, 1977.
3. BUCHAN, J. AND KOENIGSBERG, E., *Scientific Inventory Management*, Prentice-Hall, Englewood Cliffs, N.J., 1963.
4. CHURCHMAN, C. W., *Prediction and Optimal Decision—Philosophical Issues of A Science of Values*, Prentice-Hall, Englewood Cliffs, N.J., 1961.
5. DAESCHNER, W. E., *Models For Multi-Item Inventory Systems Under Constraints*, Unpublished Ph.D. Dissertation, Naval Postgraduate School, 1975.
6. EVERETT, H., "Generalized Lagrange Multiplier Method for Solving Problems of Optimum Allocation of Resources," *Operations Res*., Vol. 1, No. 3 (May–June, 1963).
7. EAGLE, R. H., *Optimal Inventory Decisions Under Constraints*, Unpublished Ph.D. Dissertation, University of North Carolina, 1968.
8. FETTER, R. B. AND DALLECK, W. C., *Decision Models For Inventory Management*, Irwin, Homewood, Ill., 1961.
9. GARDNER, E. S., *Aggregate Inventory Models: Theory and Applications*, Unpublished Ph.D. Dissertation, University of North Carolina, 1978.
10. GERSON, G. AND BROWN, R. G., "Decision Rules for Equal Shortage Policies," *Naval Res. Logist. Quart.*, Vol. 17, No. 3 (September, 1970).
11. GROFF, G. K. AND MUTH, J. F., *Operations Management: Analysis For Decisions*, Irwin, Homewood, Ill., 1972.
12. HADLEY, G. AND Whitin, T. M., *Analysis of Inventory Systems*, Prentice-Hall, Englewood Cliffs, N.J., 1963.
13. HANSMANN, F., *Operations Research In Production and Inventory Control*, Wiley, New York, 1962.
14. JOHNSON, R. W., *Financial Management*, Allyn and Bacon, Boston, Mass., 1971.
15. JONES, C. H., "The Manager's Use of Simulation Models," *1970 APICS Conference Proceedings*, American Production and Inventory Control Society, Washington, D.C.
16. LAMBERT, D. M., *"The Development of An Inventory Costing Methodology,"* National Council of Physical Distribution Management, Chicago, Ill., 1975.
17. MAGEE, J. F., AND BOODMAN, D. M., *Production Planning and Inventory Control*, 2nd ed., McGraw-Hill, New York, 1967.
18. MATHER AND PLOSSL, INC., *News Note* #19, Atlanta, Georgia (May, 1978).
19. MAYER, R. R., "The Interrelationship Between Lot Sizes and Safety Stock in Inventory Control," *Indust. Engr.* Vol. 16, No. 4 (July–August, 1965).
20. PARKER, L. L., "Economical Reorder Quantities and Reorder Points With Uncertain Demand," *Naval Res. Logist. Quart.*, Vol. 11, No. 4 (December 1964).
21. PLOSSL, G. W. AND WIGHT, O. W., *Production and Inventory Control*, Prentice-Hall, Englewood Cliffs, N.J., 1967.
22. PRICHARD, J. W. AND EAGLE, R. H., *Modern Inventory Management*, Wiley, New York, 1965.
23. SCHRADY, D. A. AND CHOE, V. E., "Models for Multi-Item Continuous Review Inventory Policies Subject to Constraints," *Naval Res. Logist. Quart.* Vol. 8, No. 4 (December, 1971).

24. STARR, M. K. AND MILLER, D. W., *Inventory Control: Theory and Practice*, Prentice-Hall, Englewood Cliffs, N.J., 1962.
25. WAGNER, H. M., *Principles of Operations Research*, Prentice-Hall, Englewood Cliffs, N.J., 1969.
26. WHITIN, T. M., *The Theory of Inventory Management*, Princeton Univ. Press, Princeton, N.J., 1953.
27. WIGHT, O. W., *Production and Inventory Management In The Computer Age*, Cahners, Boston, Mass., 1974.
28. WILDE, D. AND BEIGHTLER, C., *Foundations of Optimization*, Prentice-Hall, Englewood Cliffs, N.J., 1967.
29. ZIEGLER, R. E., *Criteria for Measurement of the Cost Parameters of an Economic Order Quantity Inventory Model*, Unpublished Ph.D. Dissertation, University of North Carolina, 1973.