Unspanned Macroeconomic Risks in Oil Futures

Job Market Paper

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Abstract

This paper constructs a macro-finance model for commodity futures. I document a feedback relationship between crude oil prices and real economic activity. The channel from real activity to oil prices is unspanned – meaning not identified in current futures prices – consistent with oil futures as a hedge asset against supply shocks. Unspanned macroeconomic risks have first order effects on risk premiums and the value of real options. The model also yields a precise estimate of the cost of carry that is strongly related to physical inventories.

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1 Introduction

Commodity futures are claims on direct inputs into production and consumption, and are among the most active markets in the world.[1] Understanding the interaction of the real economy with futures prices can shed light on equilibrium production and consumption, forecasting, hedging, risk premiums, and the values of real options. There is a gap in the current literature between time series analyses of commodity prices and the economy, which are silent on risk premiums and the term structure of futures prices, and pricing models that contain no macroeconomic data.

This paper fills the gap by developing a macro-finance model for futures – to my knowledge, the first – that incorporates both pricing and macroeconomic factors. The approach is tractable and nests previous models as special cases. It can be applied to any commodity and any set of economic data. I estimate models with state variables that span oil futures prices, economic activity and physical oil inventories. I concentrate on oil because it is the single most important commodity to the U.S. and world economy as reflected in its trading volume, media coverage and academic and industry attention.

I find evidence of both macro-to-futures and futures-to-macro links. There is a negative feedback relationship between oil prices and U.S. economic activity and a strong relationship between the slope of the futures curve and physical oil inventories. The estimates suggest that previous models miss a large component of risk premiums and the values of real options. The estimates also imply that the cost of carry – the quantity that ties futures prices to physical inventories – is pinned down precisely by the data.

A key empirical question that this paper raises is which macroeconomic risks are spanned

[1] In October 2014 the average trading volume across the two benchmark crude oil futures, WTI and Brent, was $120 billion per day compared to $129 billion per day for all NYSE and NASDAQ stocks.

by commodity futures and which are material but unspanned. An unspanned macro risk is a state variable that is relevant to expected returns and/or price forecasts, but is not identified in contemporaneous futures prices. Vector autoregressions (VARs) that include a single commodity price cannot address this question. Previous pricing models for commodity derivatives assume that all relevant risks in the economy are spanned, which imposes strong restrictions on the joint behavior of macro information, futures prices, and futures returns. In particular, perfect spanning implies that conditional on current futures prices no other information can forecast oil prices or returns *or* the macroeconomic variables. I find that this restriction is strongly rejected in the data.

A high spot price of oil forecasts lower real economic activity, consistent with the findings of Hamilton (1983), Bernanke et al. (1997) and Kilian (2009) that oil shocks forecast recessions. The effect is conditional on the market's forecast of how long the shock will last: persistent shocks to oil prices lower real activity persistently, while transient shocks lower real activity transiently.

Conversely, a high level of real activity forecasts higher oil prices. Although shocks to real activity dissipate in less than a year the market forecasts that their effect on oil prices persists for decades, perhaps because oil is a nonrenewable resource. Moreover, economic activity forecasts oil prices over and above the information in oil futures – equivalently, the spot risk premium in oil futures has an unspanned procyclical component. This finding aligns with those of Ludvigson and Ng (2009), Duffee (2011) and Joslin, Priebsch and Singleton (2014) of unspanned countercyclical risk premia in bonds, but in the opposite direction. I argue that the results are consistent with oil futures as a hedge asset against supply shocks. Allowing the spot risk premium to vary with unspanned real activity raises its monthly volatility almost tenfold, which suggests that spanned-risk pricing models may miss much of the variation in commodity risk premiums.

By definition unspanned macro risks cannot affect the prices of financial options or other derivatives, but they may affect the valuation and exercise of real options. In a calibrated example I find that adding unspanned macro risk increases the value of a hypothetical oil well over the corresponding spanned-risk model by 35% to 400% depending on the well's current cost of extraction. There are two channels by which unspanned macro risks raise real option values: their dynamics with futures prices and their risk premiums. In the example the dynamics effect dominates, while the effect of unspanned risk premiums on real option values is much smaller.

The model estimates further imply that the cost of carry is pinned down precisely by the data. The cost of carry is the marginal cost of physical storage for one period, equal to storage costs plus the risk free rate minus the convenience yield. In empirical studies, the basis – the percent spread between the first- and second-nearest maturity futures prices – is often used as a proxy for the cost of carry. I find that the cost of carry differs significantly from the basis. Its monthly standard deviation is 40% lower and it mean reverts more than twice as slowly. The cost of carry is more strongly associated with inventories than the basis is, and in a horse race of forecasting changes in inventories it drives out the basis entirely. Moreover, the cost of carry estimated from North Sea oil futures drives out the U.S. basis as a predictor of U.S. inventories. These results suggest that the additional variation in the basis over and above the cost of carry is not linked to physical storage.

There are two strands of prior literature in commodity futures that this paper builds on. In the first, commodity futures prices are modeled as affine functions of latent state variables. Classic examples are Gibson and Schwartz (1990), Schwartz (1997), and Casassus and Collin-Dufresne (2005). More recent examples include Casassus, Liu and Tang (2013) and Hamilton and Wu (2014). None of the previous studies of this type incorporate explicit macroeconomic data. Second, these models implicitly assume that all relevant information in the economy is spanned by futures prices and no other information can contribute incremental forecasting power. I find that real economic activity has material effects on risk premiums and forecasts of oil prices that are unspanned in the futures curve.

The second strand uses VARs to explore the time series relations of oil prices with the real economy; examples include Hamilton (1983); Hamilton (2003); Kilian (2009); Alquist and Kilian (2010); Kilian and Vega (2011). These studies generally include a single state variable based on the spot price of oil. A limitation of this approach is that it does not incorporate the full set of futures prices of different maturities. The model in this paper imposes the additional assumption that risk premiums are "essentially affine" in the state variables which lets us bring the full futures curve to bear on returns, price forecasts, and the spanning of macroeconomic risks.

2 Data and Forecasting Regressions

In this section I describe the data, which consist of crude oil futures prices and macroeconomic data, and investigate to what extent the macroeconomic data are spanned by futures prices. In addition to being an empirical question in its own right, the distinction between spanned and unspanned risks drives the modelling strategy. I conclude that, first, two linear factors suffice to summarize the oil futures curve, and second, the macro factors, in particular economic activity, contain relevant information that is unspanned by oil futures.

2.1 Futures Price Data

I use closing prices for West Texas Intermediate (WTI) oil futures with maturities of one to twelve months, on the last business day of each month from January 1986 to July 2013. Prices of futures with maturities longer than twelve months are often missing in the early years; the results do not change if I include longer maturities.

The futures price data is denoted

$$f_t^j = \log(F_t^j),\ j = 1...J,\ t = 1...T$$

$$f_t = \left[\begin{array}{cccc} f_t^1 & f_t^2 & ... & f_t^J \end{array}\right]'$$

where F_t^j is the closing price at end of month t of the future that expires in month $t+j$, $t = 1$ corresponds to 1/1986, $T = 331$ corresponds to 7/2013, and $J = 12$.

2.2 Macro Factors

I use the Chicago Fed National Activity Index, hereafter labelled GRO, as the first macro factor. The index is released toward the end of each month and is a weighted combination of 74 U.S. economic indicators, similar in spirit to the real economic activity indexes of Stock and Watson (1999) and Ludvigson and Ng (2009). The index is intended as a forward-looking indicator of U.S. economic activity and is used in macro-finance models of bond yields (cf. Joslin, Priebsch and Singleton (2014)). From January 2001 onward I use the real-time values of the index, although the results using the revised values are very similar. I also use changes in the Conference Board's Leading Economic Index (LEI) as an alternative proxy and obtain similar results.

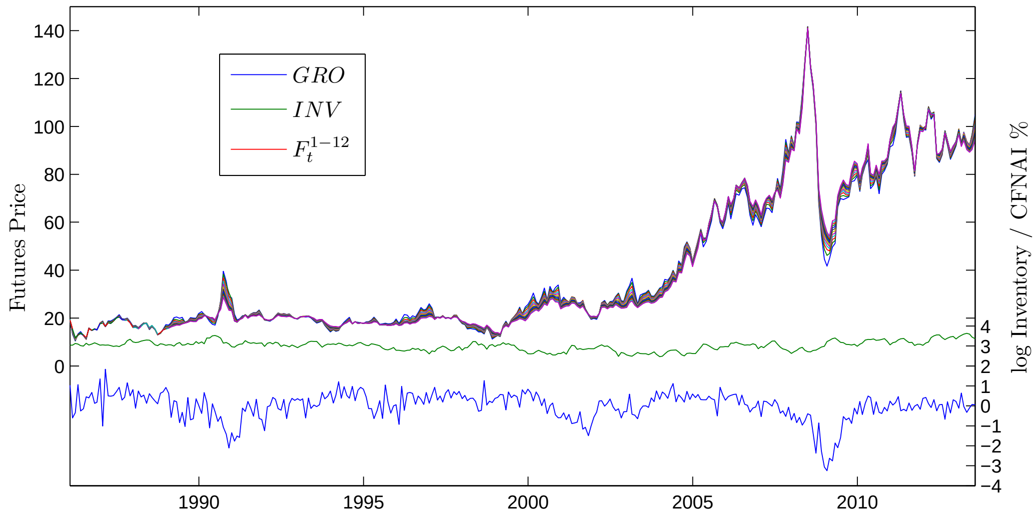

Figure 1: The figure plots the time series of futures prices for Nymex crude oil F_t^{1-12}, the Chicago Fed National Activity Index GRO, and the log of the EIA's monthly U.S. oil inventory INV.

The second macro factor is the inventory, or quantity in readily available storage. I use the log of the Energy Information Administration (EIA)'s "Total Stocks of Commercial Crude Oil excluding the Strategic Petroleum Reserve" as a measure of the readily available U.S. inventory of crude oil, hereafter labelled INV. The EIA's storage report is released weekly, and I use the most recent data as of the last business day of each month. The macro factors are thus $M_t = [GRO_t, INV_t]'$. Figure 1 plots the time series of the twelve constant-maturity log futures prices, overlaid with the time series of GRO and INV.

2.3 Evidence for Unspanned Macro Risks

Previous commodity pricing models assume complete spanning of the state variables and are estimated using financial data only. As Duffee (2011) and Joslin, Le and Singleton

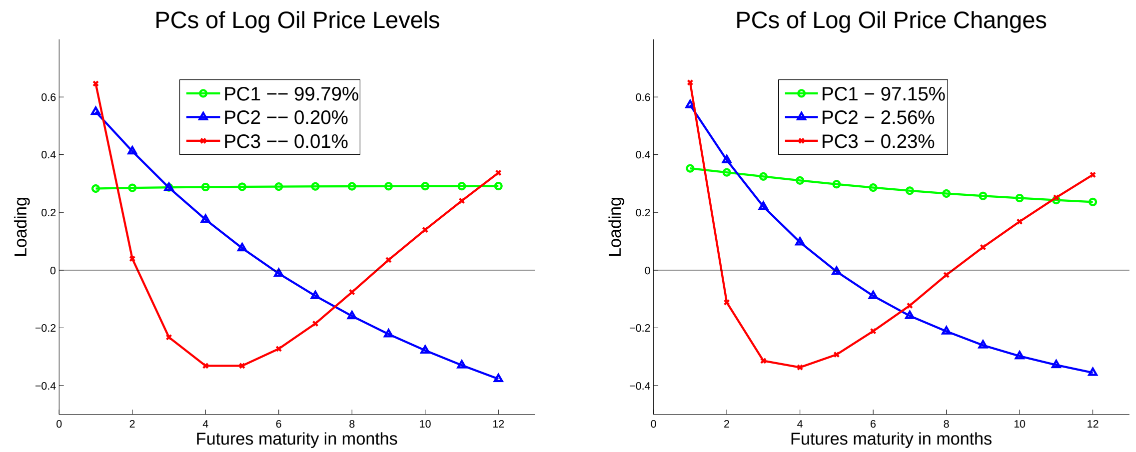

Figure 2: Loadings of the first three principal components of the levels (panel A) and changes (panel B) of log oil futures prices, monthly from 1/1986 - 7/2013. The legend displays the fraction of total variance explained by each of the principal components.

(2013) observe in the context of bond yields, this assumption has strong implications for the joint behavior of futures prices and the economy. First, it implies that the state vector can be rotated so that the state variables equal the prices of arbitrary linearly independent portfolios of futures contracts. Second, it implies that those portfolios capture log futures prices up to idiosyncratic pricing errors. Third, it implies that all relevant information is fully summarized by those portfolios' prices and no other information can contribute incremental forecasting power.

I first document that the first two principal components, level and slope, account for well over 99% of the variation in both levels and changes of log futures prices. There are more than two sources of aggregate uncertainty in the world, so the natural hypothesis is that some relevant economic state variables are unspanned by oil futures.

A) Oil futures prices display a low dimensional factor structure

Figure 2 plots the loadings of the first three principal components (PCs) of log oil futures prices. The figure also displays the fraction of the variance that is accounted for by the PCs. The first two PCs – level and slope – account for 99.9% of the variation in log price levels and 99.7% of variation in log price changes.

Second, I find that the macro factors M_t, in particular the economic activity index GRO, are not well summarized by futures prices.

B) M_t is mostly unspanned by oil futures

I project M_t on the time series of the first five principal components of log oil futures prices, and label the residual UM_t:

$$M_t = \alpha + \gamma_{1-5} PC_t^{1-5} + UM_t$$

The R^2 of the projection is 14.5% for GRO and 30.0% for INV. If I instead project M_t on the log prices of all 12 futures contracts, the R^2 is 18.9% for GRO and 30.9% for INV.

These results are consistent with the hypothesis that M_t is unspanned by oil futures. However, M_t might be measured with error or some component of M_t might be irrelevant to the oil market. In this case the test of whether M_t is unspanned is not the projection R^2, but whether the residuals UM_t are economically meaningful over and above the information in futures prices. Third, I find that UM_t contributes incremental forecasting power for oil prices and returns.

C) M_t forecasts prices and returns over and above information in the futures curve

Table 1 Panel A shows the results of forecasting log returns to oil futures contracts using the first five principal components PC^{1-5} and then adding the residual UM_t. The results suggest that the economic growth time series contributes additional forecasting power over and above information in the futures curve. For log returns to the one month and twelve month futures contracts over a one month holding period, the coefficient on $UGRO_t$ is positive and significant at the 5% level. The adjusted R^2s increase from 0.7% and 0.3% to 3.0% and 2.7% respectively. The coefficients of returns on $UGRO$ increase over holding periods of three and six months, though they are not significant statistically, and the adjusted R^2 increases in every case.

Panel B shows the results of forecasting changes in the level and slope factors of the oil futures curve using PC^{1-5} and then adding the macro factors. The results again suggest that UM_t has incremental forecasting power. Higher levels of growth forecast a higher level to the futures curve and the coefficient increases with the forecast horizon. The incremental increase in the adjusted R^2 for the change in the level factor is significant over one, six and twelve month horizons. Similar results obtain for both returns and PC changes if I use all 12 log futures prices in place of the principal components. The results are not driven by the big swings in 2008-2009: Appendix F shows the results are similar when I estimate on a subsample that ends in 2007.

Previous studies of oil prices and the macroeconomy generally use VARs. The most closely related to my results is Kilian and Vega (2011), who generally find no consistent predictive power of macroeconomic news for the spot price of crude oil at a monthly horizon and conclude that relevant information is more or less immediately reflected in the spot price.

Table 1: Panel A shows the results of forecasting returns to oil futures. Panel B shows the results of forecasting changes in the level and slope, $PC1$ and $PC2$ respectively, of the oil futures curve. The data are monthly from from 1/1986 to 7/2013. The forecasting variables are the first five principal components of log futures prices, PC^{1-5}, and the residuals of the Chicago Fed National Activity Index and U.S. oil inventory projected on PC^{1-5}. The standard errors are Hansen-Hodrick. $^{*}: p < 0.10$, $^{**}: p < 0.05$, $^{***}: p < 0.01$.

Panel A: Forecasting Returns to Futures

$$r_{t \to t+horizon} = \alpha + \beta_{1-5} PC_t^{1-5} + \beta_{UGRO,UINV} UM_t + \epsilon_t$$

Horizon	1 month		6 months		12 months	
Futures maturity	1 m	12 m	1 m	12 m	1 m	12 m
β_{UGRO}	0.025**	0.017**	0.032	0.023	0.042	0.033
	(0.010)	(0.007)	(0.039)	(0.030)	(0.055)	(0.043)
β_{UINV}	0.028	0.003	-0.170	-0.328	-0.112	-0.542
	(0.087)	(0.059)	(0.510)	(0.385)	(0.860)	(0.664)
Adjusted $R^2(PC^{1-5})$	0.7%	0.3%	6.9%	0.7%	13.0%	1.3%
Adj. $R^2(PC^{1-5} + UM_t)$	3.0%	2.7%	7.1%	2.0%	13.2%	3.2%
F-ratio	9.6***	4.9***	1.4	3.0**	2.2*	4.0**

Panel B: Forecasting Changes in Level and Slope

$$\Delta PC^{(n)}_{t \to t+horizon} = \alpha + \beta_{1-5} PC_t^{1-5} + \beta_{UGRO,UINV} UM_t + \epsilon_t$$

Horizon	1 month		6 months		12 months	
Principal Component	Level	Slope	Level	Slope	Level	Slope
β_{UGRO}	0.068**	-0.008	0.091	0.001	0.071	-0.002
	(0.028)	(0.005)	(0.144)	(0.016)	(0.107)	(0.009)
β_{UINV}	0.040	-0.032	-1.615	-0.124	-1.047	-0.073
	(0.236)	(0.047)	(2.267)	(0.191)	(1.386)	(0.162)
Adjusted $R^2(PC^{1-5})$	-0.6%	8.0%	4.0%	50.0%	2.2%	34.3%
Adj. $R^2(PC^{1-5} + UM_t)$	1.7%	8.5%	5.2%	50.0%	3.0%	34.0%
F-ratio	9.5***	1.9	6.1**	0.9	4.6**	0.4

Kilian and Vega examine a variety of macroeconomic data that are mostly backward looking. They do not examine the Chicago Fed National Activity Index, and for the most closely related forward-looking time series that they examine – the Conference Board's Leading Economic Index – they find that it does in fact forecast oil prices at a monthly horizon ($p < 0.01$) and I replicate their finding in untabulated results.

Another way in which our approaches differ is that Kilian and Vega (2011) and others regress monthly changes in the spot price on their macro factors. This approach lumps together the spanned and unspanned effects of macroeconomic news. Specifically, GRO is positively correlated with the futures price f_t^1 and negatively correlated with the basis $f_t^2 - f_t^1$. Thus a high realization of GRO is accompanied by a more downward-sloping curve, and the latter naturally forecasts a lower realization of $\Delta s = s_{t+1} - s_t$. If we instead regress the log return $rf = f_{t+1}^1 - f_t^2$ on $UGRO$ as in Table 1 Panel A then we isolate the incremental effect of the growth index on the forecast.

3 Model

Motivated by the empirical findings in Section 2, in this section I develop a macro-finance model for commodity futures that admits unspanned macroeconomic risks. Macro-finance models that investigate the interaction of bond markets with the real economy are an active area of research starting with Ang and Piazzesi (2003). Let X_t denote a vector of N state variables that summarize the economy. X_t includes macroeconomic risk factors such as expected economic growth, and factors specific to the commodity such as hedging pressure, inventories, and expectations of supply and demand. The stochastic discount factor is given by

$$e^{\Lambda_t' \epsilon_{t+1}} = e^{(\Lambda_0 + \Lambda_1 X_t)' \epsilon_{t+1}} \tag{1}$$

The state vector follows a Gaussian VAR,

$$X_{t+1} = K_{0X}^{\mathbb{P}} + K_{1X}^{\mathbb{P}} X_t + \Sigma_X \epsilon_{t+1}^{\mathbb{P}} \tag{2}$$

where $\epsilon_{t+1}^{\mathbb{P}} \sim N(0, 1_N)$.

Previous models such as Gibson and Schwartz (1990); Schwartz (1997); Casassus and Collin-Dufresne (2005) assume that X_t is spanned i.e. fully reflected in contemporaneous futures prices. As is well known for bond yields (Duffie and Kan (1996)), Appendix B shows that the spanning assumption implies that X_t can be replaced by an arbitrary set of linear combinations of log futures prices:

$$\mathcal{P}_t^N = W f_t$$

where W is any full rank real valued $N \times J$ matrix. This assumption has the following implications for the interaction of the futures market and the macroeconomy:

1. Futures prices are described up to idiosyncratic errors by the N factors $\mathcal{P}_t^N$.

2. The projection of X_t on $\mathcal{P}_t^N$ has R^2 of one.

3. Conditional on $\mathcal{P}_t^N$, no other information forecasts X_t or futures prices or returns.

I instead assume that a subspace of X_t is spanned, while its complement is unspanned but observed by the econometrician. Suppose that contemporaneous futures prices are determined by a set of linear combinations $L_t = V f_t$ where V is a real valued $N_L \times J$ matrix and

$N_L < N$. That is, the spot price and its evolution under the risk neutral measure are:

$$s_t = \delta_0 + \delta_1' L_t \tag{3}$$

$$L_{t+1} = K_{0L}^{\mathbb{Q}} + K_{1L}^{\mathbb{Q}} L_t + \Sigma_L \epsilon_{t+1}^{\mathbb{Q}} \tag{4}$$

where $\epsilon_{t+1}^{\mathbb{Q}} \sim N(0, 1_{N_L})$ and $\Sigma_L = V\Sigma_X$. The spanned components L_t may be observed or latent.

Finally, I assume that the unspanned components $L_t^{\perp}$ of X_t are observed by the econometrician. There are $N_M = N - N_L$ of these factors. Label them $UM_t = L_t^{\perp}$ – the unspanned components of observed macroeconomic information – and rewrite

$$\begin{bmatrix} L_t \\ UM_t \end{bmatrix} = K_0^{\mathbb{P}} + K_1^{\mathbb{P}} \begin{bmatrix} L_t \\ UM_t \end{bmatrix} + \Sigma \epsilon_{t+1}^{\mathbb{P}}$$

By construction, the factors UM_t are not spanned by contemporaneous futures prices: this specification is in the class of macro-finance models explored by Diebold, Rudebusch and Aruoba (2006); Duffee (2011); Joslin, Priebsch and Singleton (2014) for bonds. By the same rationale as before, we can replace L_t with N_L linear combinations of log prices,

$$\mathcal{P}_t^L = W_L f_t$$

where W_L is any full rank $N_L \times J$ real valued matrix. In contrast to the spanned-risk formulation, this model has the implications that:

1. Futures prices are described up to idiosyncratic errors by $N_L < N$ factors.

2. The projection of X_t on $\mathcal{P}_t^L$ has R^2 less than one.

3. Conditional on $\mathcal{P}_t^L$, other information may forecast X_t or futures prices or returns.

Motivated by the variance decompositions in the previous section, I assume the number of spanned state variables $N_L = 2$. Appendix B describes the parametrization and estimation of the model. After estimating, I rotate and translate so that the state variables are the model implied spot price and cost of carry $(s_t,\, c_t)$ and the macroeconomic series $M_t = SM_t + UM_t$.[2]

The model can then be described in just two equations:

1) the law of motion for the state variables:

$$\begin{bmatrix} s_{t+1} \\ c_{t+1} \\ M_{t+1} \end{bmatrix} = \begin{bmatrix} K_{0sc}^{\mathbb{P}} \\ K_{0M}^{\mathbb{P}} \end{bmatrix} + \begin{bmatrix} K_{sc,sc}^{\mathbb{P}} & K_{sc,M}^{\mathbb{P}} \\ K_{M,sc}^{\mathbb{P}} & K_{MM}^{\mathbb{P}} \end{bmatrix} \begin{bmatrix} s_t \\ c_t \\ M_t \end{bmatrix} + \Sigma \epsilon_{t+1}^{\mathbb{P}} \tag{5}$$

2) the dynamics of $(s_t,\, c_t)$ under the risk neutral measure:

$$\begin{bmatrix} s_{t+1} \\ c_{t+1} \end{bmatrix} = K_0^{\mathbb{Q}} + K_1^{\mathbb{Q}} \begin{bmatrix} s_t \\ c_t \end{bmatrix} + \Sigma_{sc} \epsilon_{t+1}^{\mathbb{Q}} \tag{6}$$

where

- s_t is the spot price and c_t is the one-period cost of carry
- M_t are the macro state variables
- $K_{0sc}^{\mathbb{P}}$, $K_{0M}^{\mathbb{P}}$ are 2×1 and $N_M \times 1$ real valued matrices
- $K_{sc,sc}^{\mathbb{P}}$, $K_{sc,M}^{\mathbb{P}}$, $K_{M,sc}^{\mathbb{P}}$, $K_{MM}^{\mathbb{P}}$ are 2×2 , $2 \times N_M$, $N_M \times 2$ and $N_M \times N_M$ real valued matrices

[2] Appendix A.2 presents the definitions of s_t and c_t.

- $K_0^{\mathbb{Q}}$, $K_1^{\mathbb{Q}}$ are 2×1 and 2×2 real valued matrices

- Σ is $N_M + 2 \times N_M + 2$, lower triangular, and Σ_{sc} is the upper left 2×2 submatrix of Σ.

The model is a canonical form, that is, any affine model with two spanned state variables and $N_M \geq 0$ macroeconomic variables can be written in the form above. Extending the model to more than two spanned state variables is straightforward.

The assumption of constant volatility in the model is a strong one, because the volatility of commodity futures markets does vary over time. To the extent that such variation is spanned by futures contracts its effects on forecasts or risk premiums will be reflected in the reduced-form factors. That is, spanned time-varying volatility is compatible with the model because the model is agnostic with respect to the spanned state variables.[3]

4 Model Estimates

This section presents the estimates of the macro-finance model with two spanned ("pricing") factors and two macroeconomic factors: the monthly Chicago Fed National Activity Index (GRO) and log U.S. oil inventories (INV).

Figure 3 Panel A plots the spanned and unspanned components of GRO as well as the log spot price. We see that essentially all of the monthly and yearly variation in GRO appears in the unspanned component. Figure 3 Panel B plots the spanned and unspanned components of log oil inventories INV. Compared to GRO, much more of the monthly and yearly variation in INV is spanned by futures prices. The spanned component of inventory loads exclusively, and strongly, on the cost of carry.

[3]The model of Trolle and Schwartz (2009) features unspanned stochastic volatility (USV), where time varying volatility in the spot price is not reflected in current futures prices. USV can be incorporated in my model at the cost of losing some tractability. I leave this extension for future work.

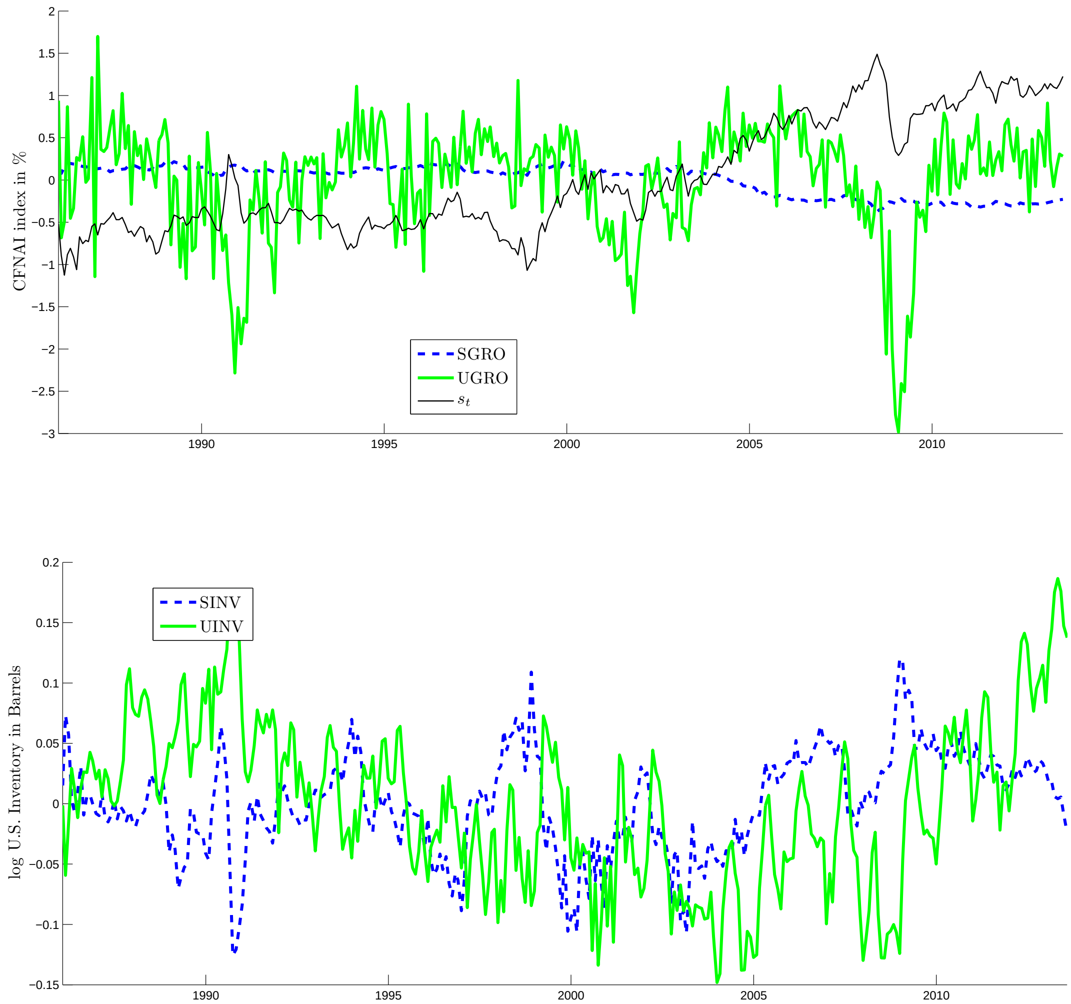

Figure 3: Panel A plots the components of the monthly Chicago Fed National Activity Index that are spanned ($SGRO$) and unspanned ($UGRO$) by oil futures prices, as well as the log spot price of oil s_t. Panel B plots the components of monthly log U.S. oil inventories that are spanned ($SINV$) and unspanned ($UINV$) by oil futures prices.

Table 2: Maximum likelihood (ML) estimate of the macro-finance model for Nymex crude oil futures. s, c are the spot price and annualized cost of carry respectively. GRO is the monthly Chicago Fed National Activity Index. INV is the log of the private U.S. crude oil inventory as reported by the EIA. The coefficients are over a monthly horizon, and the state variables are de-meaned. ML standard errors are in parentheses. Coefficients in bold are significant at the 5% level.

Historical ($\mathbb{P}$) Measure

	$K_0^{\mathbb{P}}$	$K_1^{\mathbb{P}}$: s_t	c_t	GRO_t	INV_t
s_{t+1}	0.008	**0.994**	0.061	**0.025**	0.038
	(0.006)	(0.009)	(0.033)	(0.008)	(0.082)
c_{t+1}	-0.007	0.016	**0.874**	-0.015	-0.045
	(0.005)	(0.008)	(0.031)	(0.008)	(0.078)
GRO_{t+1}	0.001	**-0.115**	0.051	**0.618**	-0.194
	(0.030)	(0.046)	(0.171)	(0.043)	(0.433)
INV_{t+1}	0.002	0.000	**0.032**	-0.002	**0.888**
	(0.002)	(0.002)	(0.009)	(0.002)	(0.023)

Risk Neutral ($\mathbb{Q}$) Measure

	$K_0^{\mathbb{Q}}$	$K_1^{\mathbb{Q}}$: s_t	c_t
s_{t+1}	-0.004	**1.000**	**0.083**
	(0.008)	(0.004)	(0.011)
c_{t+1}	0.001	-0.004	**0.892**
	(0.012)	(0.010)	(0.029)

Shock Volatilities
[off-diagonal = % correlations]

	s	c	GRO	INV
s	0.103			
c	-81%	0.057		
GRO	5%	-2%	0.530	
INV	-22%	27%	4%	0.028

Table 2 presents the maximum likelihood estimate of the model using the monthly data from January 1986 to July 2013.

4.1 Model Fit

The risk neutral parameters $K_0^{\mathbb{Q}}$, $K_1^{\mathbb{Q}}$ are estimated relatively precisely for two reasons. First, they use the entire futures curve data; on each date, the set of futures prices represents a "snapshot" of risk-neutral expectations. Second, a linear factor model holds closely in the futures curve: our two affine factors fit the futures curve closely, so the pricing errors relative to the fitted model are small and idiosyncratic. By contrast, the historical measure parameters $K_0^{\mathbb{P}}$, $K_1^{\mathbb{P}}$, Σ are estimated less precisely because their estimation uses only the time series of the state variables. Each month represents a single observation, and the R^2 of the best-fit forecast is low. Risk prices $\Lambda = K^{\mathbb{P}} - K^{\mathbb{Q}}$ inherit the lower precision of $K_0^{\mathbb{P}}$, $K_1^{\mathbb{P}}$.

The values of both s_t and c_t are precisely estimated. Figure 4 plots the fitted values of s_t and c_t with 95% confidence intervals, which are too small to see. A flip side of this observation is that s_t and c_t as affine state variables do a good job of summarizing oil futures prices[4]. The principal components that we used in Section 2 do the best possible job by construction, and it is not guaranteed that s_t and c_t with their AR(1) structure will perform as well. I find that s_t, c_t explain 99.97% of variation in futures prices and the residuals (pricing errors) explain 0.03% of the total variation.

These observations hold for the canonical model with no auxiliary restrictions. But because they are a product of the strong factor structure in the data, they also hold for other

[4] This observation does not contradict the conclusions of Schwartz (1997) and Casassus and Collin-Dufresne (2005) that a three-factor model is necessary to summarize commodity futures prices. The three-factor models in those papers have two latent factors – spot price and convenience yield – and a spanned interest rate that is estimated separately. Interest rates are very slow moving compared to futures prices, so they contribute almost no extra explanatory power.

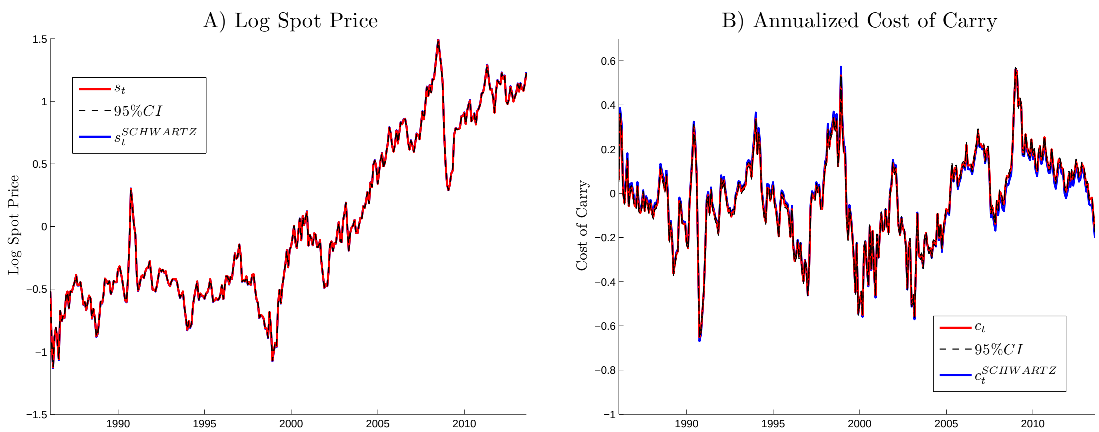

Figure 4: The figure plots the fitted values of the log spot price s_t and the annualized cost of carry c_t in Nymex oil futures from the canonical model, with 95% (±two standard errors) confidence intervals. The figure also plots the fitted values of s_t and c_t from the Schwartz (1997) two factor model for comparison.

models. Figure 4 also plots s_t and c_t implied by the two factor model of Schwartz (1997), which corresponds to our model with no macro factors and five auxiliary restrictions:

- The rate of mean reversion of c_t under $\mathbb{Q}$ does not depend on s_t i.e. the lower left entry of $K_1^{\mathbb{Q}}$ is zero

- Risk premiums are non time varying i.e. $K_1^{\mathbb{P}} = K_1^{\mathbb{Q}}$.

These restrictions change the estimated historical measure dynamics materially and the risk neutral dynamics slightly, but leave the fitted values of s_t and c_t virtually unchanged.

4.2 Historical Dynamics

The models of Schwartz (1997) and Schwartz and Smith (2000) impose that s_t is unit-root. Without that restriction and using data from 1990 to 2003, Casassus and Collin-Dufresne (2005) estimate that s_t has a long run mean to which it reverts with a halflife of around two years, so the expected spot price of oil in ten years is essentially constant. Our unrestricted estimate which adds ten years of subsequent data is more consistent with Schwartz (1997). The AR(1) coefficient for s_t of 0.994, which is close to the largest eigenvalue of $K_1^{\mathbb{P}}$, is very close to unity.[5]

The cost of carry reverts to a slightly negative mean with a half-life of five months. Shocks to the spot price and the cost of carry are strongly negatively correlated ($\rho = -81\%$), so a higher spot price is accompanied by a more downward sloping curve, but spot price shocks are essentially permanent while the cost of carry shock decays within a few years. As a result, about half of a typical move in the oil spot price disappears after two to three years, while the other half is expected to persist effectively forever.

4.2.1 Oil Futures and Economic Activity

Shocks to economic activity are almost uncorrelated with shocks to the spot price and the cost of carry. Looking down the third column of the transition matrix, a one percent shock to economic activity predicts a 2.5% higher spot oil price the next month but only a 1.5% lower cost of carry. Thus, the effects of economic activity on oil prices are forecast by the market to be persistent – higher activity raises both the short run and the expected long run price of oil.

Looking across the third row of the transition matrix, a higher spot price of oil predicts lower economic activity. A higher cost of carry – higher expected prices in future – forecasts slightly higher economic activity, but the effect is not significant, and c_t also forecasts a higher spot price. The impulse response functions in section 4.2.3 make clear that the net

[5] The estimates all use nominal futures prices. Inflation was relatively constant over the period from 1986 to 2013, relative to the movement in oil prices, so it does not drive the high AR(1) coefficient of s_t. Using futures prices deflated by the CPI or PPI does not materially change any of the results.

effect of c_t on GRO is negative. As a result, a shock to the spot price that the market expects to persist has a more negative effect on growth than a shock that is expected to be transitory. These results are not driven by the big swings in 2008-2009: Appendix F shows the results are similar when I estimate on a subsample that ends in 2007.

Taken together, there is a negative feedback relationship between the spot price of oil and growth. A positive growth shock forecasts persistent higher oil prices, while a positive oil price shock forecasts slower growth, and the effect is stronger for oil price shocks that the market expects to persist.

4.2.2 Oil Futures and Inventories

Shocks to log inventories are negatively correlated with the spot price and positively correlated with the cost of carry. Both of these observations are consistent with the Theory of Storage – higher inventories signal that the market is moving up the supply-of-storage curve. The correlation between shocks to inventory and the cost of carry (27%) is relatively modest; in the frictionless storage model of Working (1949) and others, INV_t and c_t are collinear. Looking across the bottom row of the transition matrix, a higher cost of carry strongly predicts higher inventories the next month. This relationship further suggests adjustment costs in physical storage: the futures curve adjusts to relevant information first and inventories respond with a lag.

Looking down the last column of the transition matrix, unspanned crude oil inventory does not forecast any of the other variables. In particular, periods of higher inventory do not have much effect on the forecast of either the spot price or the cost of carry. This finding is consistent with the fundamental drivers of oil inventory such as precautionary storage and expected physical supply and demand being fully spanned by oil futures prices.

4.2.3 Impulse Response Functions

Figure 5 plots the impulse response functions (IRFs) to shocks to oil prices and economic activity. The ordering of the variables for the impulse response functions is (GRO, s_t, c_t, INV). GRO is first because innovations in the unspanned component, which dominates the variation in GRO, can be thought of as exogenous to contemporaneous oil prices and inventories. We analyze s_t and c_t simultaneously so their relative ordering is not important. Finally, it is intuitive and also supported by the estimates and regressions that the oil futures curve adjusts to new information faster than physical inventory does.

Panel A plots the response to a unit shock to the log spot price, which is correlated with a negative shock to the cost of carry and a more downward-sloping curve. A unit shock to s_t means a doubling of the spot price of oil. About half of the increase decays within two years, while the other half is effectively permanent, and forecasts an economic activity index that is 0.2% lower effectively forever. This effect is material: the index averaged -1.66% in 2009 during the depths of the financial crisis, while it averaged 0.02% in 2006. The higher spot price and lower cost of carry also produce a fall in inventories.

Panel B plots the response to a joint shock to s_t and c_t such that the spot price is expected to fully revert to the pre-shock baseline. The response of economic activity is transient as well, and in fact GRO recovers to the baseline faster than s_t does. Comparing to Panel A, which only differs in the size of the shock to c_t, makes clear that the net effect of c_t on expected growth is negative. Note that the fact that the forecast of the long-run spot price is unchanged in Panel B does not mean that long maturity futures prices will be unchanged – the two are equivalent only in the case that oil risk premiums are non time varying. Thus, a VAR that includes a long-maturity futures price or spread will not in general recover the correct dynamics of the state variables.

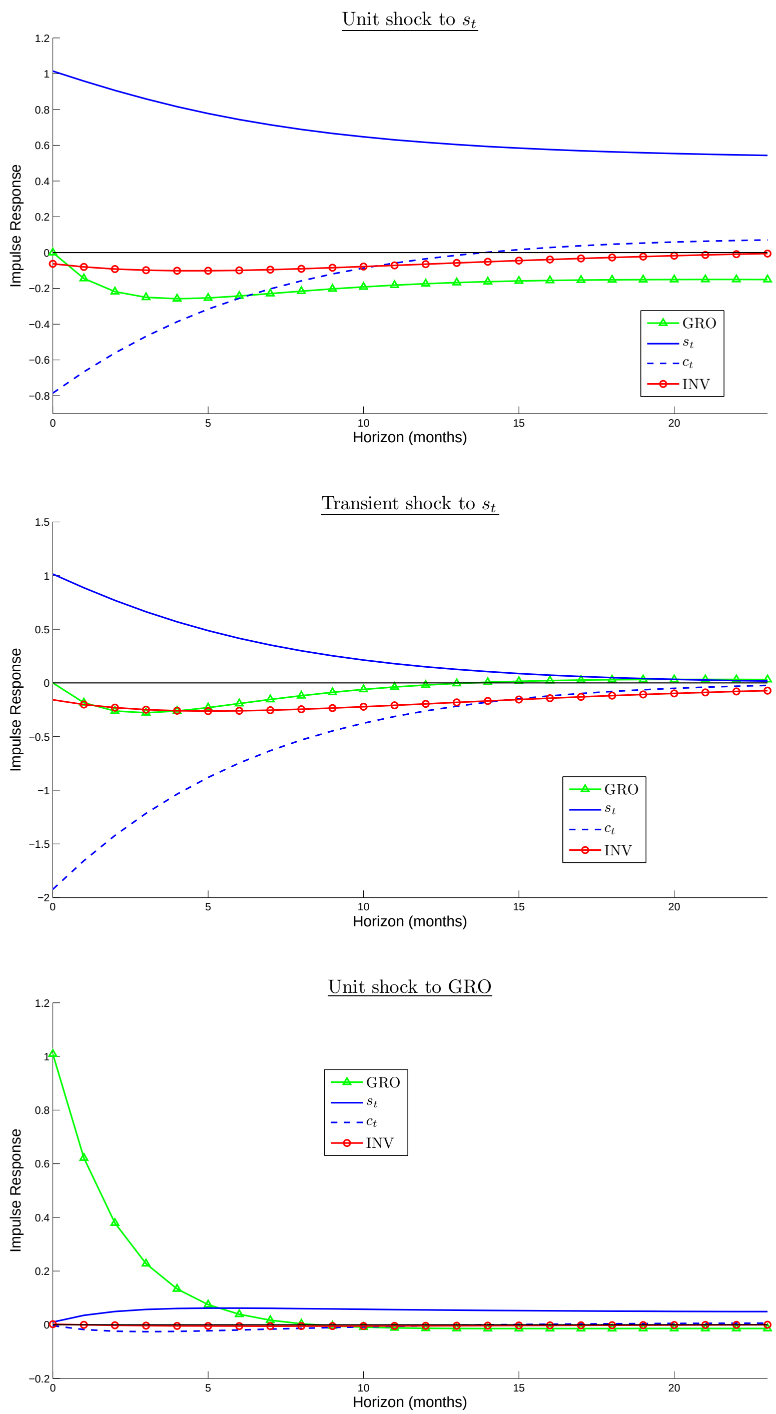

Figure 5: Panel A shows the impulse response functions (IRFs) of the four state variables to a unit shock to the log spot price of oil s_t. Panel B shows the IRFs for a transient shock for which the spot price of oil fully reverts to the baseline. Panel C shows the IRFs for a unit shock to economic growth, GRO. The order of the variables is (GRO, s_t, c_t, INV).

Panel C plots the response to a shock to economic activity. The index mean reverts rapidly and the shock decays back to the baseline within a year. However, a transient shock to *GRO* produces a near permanently higher spot price of oil – perhaps because oil is a nonrenewable resource. The magnitude of the effect is large: a one-period shock to economic activity of one percent produces a spot price of oil that is 5.1% higher than the baseline, ten years later.

4.3 Risk Neutral Dynamics

By definition, c_t is the annualized forecast of the change in the spot price under the $\mathbb{Q}$-measure. Thus, the top row of $K_1^{\mathbb{Q}}$ by definition equals $[1, \frac{1}{12}]$. Any mean reversion of the spot price under the risk neutral measure is thus rotated into the bottom-left entry of $K_1^{\mathbb{Q}}$. In our estimate the entry is small (-0.004) and not statistically significant, so the risk neutral forecast of c_t is almost uncorrelated with the level of s_t. This implies that the shape of the futures curve does not vary with the price level, which corroborates our observation in Section 2 that the first two principal components explain the vast majority of the variation in log futures prices. The cost of carry mean-reverts under the risk neutral measure at a similar speed to that under the physical measure.

4.4 Risk Premiums

Several recent studies (for example Erb and Harvey (2006); Etula (Forthcoming); Gorton, Hayashi and Rouwenhorst (2013); Yang (2013)) investigate the returns to commodity futures by sorting cross sections of individual near-maturity contracts on different commodities. Singleton (2013) and Hamilton and Wu (Forthcoming) run return forecasting regressions for individual futures maturities. The model in this paper contributes to this literature as well,

Table 3: Maximum Likelihood (ML) estimates of risk premiums in the macro-finance model for U.S. crude oil futures. s, c are the spot price and annualized cost of carry respectively. GRO and INV are the Chicago Fed National Activity Index and log U.S. crude oil inventory respectively. The coefficients are standardized to reflect a one standard deviation change in each variable over a monthly horizon, and the state variables are de-meaned. ML standard errors are in parentheses. Coefficients in bold are significant at the 5% level.

$$\begin{bmatrix} \Lambda^s \\ \Lambda^c \end{bmatrix}_t = \Lambda_0 + \Lambda_1 \begin{bmatrix} s_t & c_t & M_t \end{bmatrix}'$$

	Λ_0	Λ_1			
		s	c	GRO	INV
Λ^s	0.012	-0.001	-0.001	**0.013**	0.001
	(0.014)	(0.002)	(0.003)	(0.004)	(0.002)
Λ^c	-0.010	0.003	-0.001	-0.008	-0.001
	(0.017)	(0.002)	(0.004)	(0.004)	(0.002)

as it offers a simple way to utilize the full futures curve data for each commodity.

Szymanowska et al. (2013) decompose futures returns into two components. The spot premium corresponds to going long near-maturity futures, while the term premium corresponds to going long near-maturity futures and short distant-maturity futures (a term spread position). Appendix A describes the correspondence of their decomposition to the risk premiums in my model. Their spot premium equals the risk premium attached to the spot price plus a small convexity term, while their term premium equals the risk premium attached to the cost of carry minus the conditional expected cost of carry.

Table 3 displays the estimates of the parameters governing risk premiums. The unconditional spot risk premium for oil is positive, while the unconditional cost of carry premium is negative. Only one entry in the time-varying loadings of risk premiums Λ_1 is statistically significant: higher economic activity is associated with a higher spot risk premium in oil. Similar results obtain using the monthly change in the Conference Board's Leading Eco-

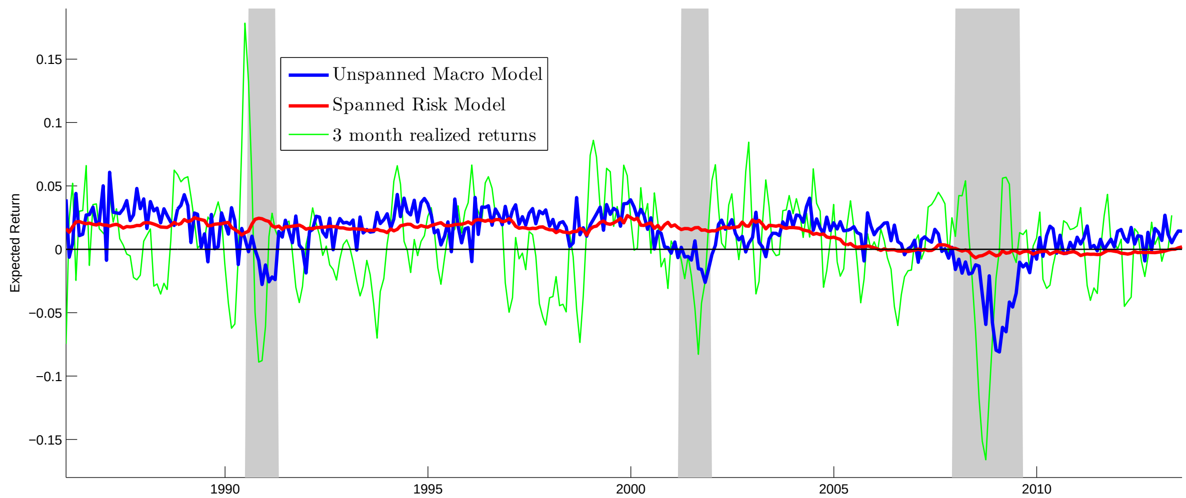

Figure 6: The figure compares the model implied spot premium for the macro-finance model versus the two-factor model which enforces spanning. Also displayed is the average return to oil futures over the subsequent three month horizon. NBER recessions are shaded in grey.

nomic Index as an alternate proxy for growth, including longer maturity futures prices, or estimating on a subsample that stops in 2007 and omits the wide swings of 2008-9.

The effect of economic activity on the spot risk premium in oil futures is material. Figure 6 plots the implied spot premiums for the macro-finance model and the two-factor nested model that enforces spanning[6], as well as the average realized returns for oil futures in the sample over the following three months. The model predictions differ most noticeably during 1990-1991, 2001-2002 and 2008-2009: slumps in real activity forecast lower oil prices. The risk premium for exposure to the spot price of oil is procyclical. The unspanned procyclical component dominates the variation in the model-implied spot risk premium; the standard deviation of changes in Λ_t^s in the unspanned macro model is 1.5% per month compared to 0.16% per month in the spanned-risk model, an increase in volatility of almost ten times.

[6]That is, the unrestricted two factor model with $UM_t = \emptyset$.

This forecast is attached to the unspanned component of real activity because it is not reflected in the futures curve at the time. Per the estimates in Table 2, a fall in *GRO* is weakly correlated with a rise, not a fall, in the cost of carry. In other words, in economic downturns the oil futures curve "fails" to forecast the subsequent fall in the spot price. This observation aligns with the findings of unspanned countercyclical risk premia in bonds by Ludvigson and Ng (2009), Duffee (2011) and Joslin, Priebsch and Singleton (2014), but in the opposite direction.

A potential explanation is as follows. Conditional on news about economic growth, higher oil prices reflect negative supply shocks (Kilian (2009)) and forecast lower growth (Hamilton (1983); Hamilton (2003)). In an ICAPM where the state variables are total wealth and the quantity of oil in the world, we have

$$\Lambda_t^s = \beta_t^{mkt} \lambda_t^{mkt} - \lambda_t^{oil}$$

where λ_t^{mkt}, λ_t^{oil} are the risk premiums of shocks to the market portfolio and to oil futures prices as the replicating portfolio for the quantity of oil in the world, respectively. The negative sign in front of λ_t^{oil} reflects that going long oil futures is a hedge against supply shocks. The magnitude of λ_t^{oil} is plausibly countercyclical: in economic slumps when uncertainty is greater or their consumption-investment tradeoff is steeper, investors have more demand for the oil hedge and the risk premium is more negative. As a result Λ_t^s is procyclical, but since oil futures have a positive beta with the market portfolio the unconditional spot risk premium may be positive or negative.

In addition, both the regressions and model estimates indicate that innovations to growth are unspanned by contemporaneous futures prices. The existence of state variables that are meaningful for bond risk premiums yet unspanned by bond prices is an active question in the term structure literature (Duffee (2011); Joslin, Priebsch and Singleton (2014)). In our setting, it corresponds to the state variable having offsetting effects on oil risk premiums and the oil price forecast. This is again consistent with oil futures as a hedge against oil supply shocks. A negative growth shock raises risk premiums including λ_t^{oil}, which raises futures prices ceteris paribus. At the same time, it forecasts reduced demand and a lower spot price of oil, which acts to lower futures prices. If these two effects are of comparable magnitude and expected duration then innovations in growth could be unspanned or – more likely – the net effect on the futures curve could be small enough that we do not detect it.

4.4.1 Positive vs Negative Growth Regimes

The effects of economic activity on the spot risk premium in oil appear to be concentrated in economic downturns. To investigate this possibility I split GRO into two components: GRO^+ equals GRO in months when its value is positive and zero otherwise, while GRO^- equals GRO in months when its value is negative, and zero otherwise. This split lets the coefficients of risk premiums GRO differ when the world is in a positive-growth regime versus a negative-growth regime.

Table (4) presents the estimated risk prices when GRO is split in this way. The size of the coefficients on GRO^+ and GRO^- are not directly comparable to the previous table because all of the coefficients are standardized to reflect a one standard deviation change, and GRO^+ and GRO^- naturally have lower standard deviations than GRO. The message of the table is that the response of the spot risk premium to growth shocks is symmetric on the upside and the downside: the coefficient on GRO^- is 0.008 per month compared to 0.007 per month for GRO^+. Contrary to our impression from Figure 6, the effect of growth

Table 4: Estimates of risk premiums in the macro-finance model in which s, c are the spot price and annualized cost of carry in oil futures and GRO^{+} and GRO^{-} are the monthly Chicago Fed National Activity Index in months when the index is positive and negative respectively. The coefficients are standardized to reflect a one standard deviation change in each variable over a monthly horizon, and the state variables are de-meaned. ML standard errors are in parentheses. Coefficients in bold are significant at the 5% level.

$$\begin{bmatrix} \Lambda_s \\ \Lambda_c \end{bmatrix}_t = \Lambda_0 + \Lambda_1 \begin{bmatrix} s_t & c_t & M_t \end{bmatrix}'$$

	Λ_0	Λ_1			
		s	c	GRO^{+}	GRO^{-}
Λ_s	0.013	-0.001	-0.001	0.007	**0.008**
	(0.017)	(0.002)	(0.002)	(0.006)	(0.004)
Λ_c	-0.011	0.003	-0.002	-0.003	-0.005
	(0.020)	(0.002)	(0.004)	(0.006)	(0.004)

on the oil price forecast is relatively symmetric in good times versus bad.

5 The Cost of Carry

The model estimates say that s_t and c_t are precisely pinned down by the data. Empirical studies often proxy for these quantities with futures prices and spreads. Fama and French (1987); Gorton, Hayashi and Rouwenhorst (2013); Singleton (2013) and others use some version of the log spread or "basis", $f_t^2 - f_t^1$, as a proxy for the cost of carry.[7]

By definition, c_t equals the difference between the model implied values of f_t^1 and s_t. The model fits an AR(1) process to (s_t, c_t) using the full futures curve at each date. The basis does not model the behavior or number of state variables but instead assumes that the errors on f_t^1 and f_t^2 are zero and that $f_t^2 - f_t^1$ is closely correlated with c_t.

[7] Fama and French (1987) and others subtract a short term Treasury bill rate from the spread. The short rate is slow moving relative to the spread, and defining the basis as $\left(f_t^2 - f_t^1\right) - r_t^f$ gives the same results.

The normalization and estimation in this paper, based on the recent advances of Joslin, Singleton and Zhu (2011) in term structure modelling, are exceedingly stable and tractable; estimates converge in a few seconds. Thus it is practical to use the model estimated c_t instead of the basis. There are several reasons why fitting the model could yield more accurate estimates of the cost of carry. First, producers and consumers plan and hedge their activities more than two months in advance, in which case longer dated futures prices will contain relevant information. Second, market microstructure issues like congestion at delivery points or financial order flows could add noise to individual prices. Third, because the cost of carry is mean reverting, when the futures curve slopes in either direction $f_t^2 - f_t^1$ is biased toward the mean as a proxy for $f_t^1 - s_t$.

Figure 7 Panel A plots the model implied log spot price s_t against the one month futures log price f_t^1 in the sample. The two are almost collinear with a correlation in levels (monthly changes) of 0.999 (0.995). Thus, the one month futures prices is a close proxy for the spot price although they occasionally differ by as much as 5%.

Figure 7 Panel B plots the model implied cost of carry c_t against the annualized basis. The two series have similar unconditional averages but their correlation in levels (monthly changes) is 0.79 (0.60) and their values differ significantly throughout the sample. In particular, c_t is much more slow moving. Monthly innovations in c_t have a standard deviation of 10.0% compared to 16.9% for the basis. The AR(1) coefficient of c_{t+1} on c_t is 0.88 (half-life of 5.5 months) compared to 0.74 (half-life of 2.3 months) for the basis, and the differences in variances and AR(1) coefficients are significant at the 1% and 5% level respectively. Thus, c_t implies that the net convenience yield for oil varies less and returns to its mean more slowly than the basis implies.

Whether c_t or the basis is a better measure of the cost of carry is answered by linking them

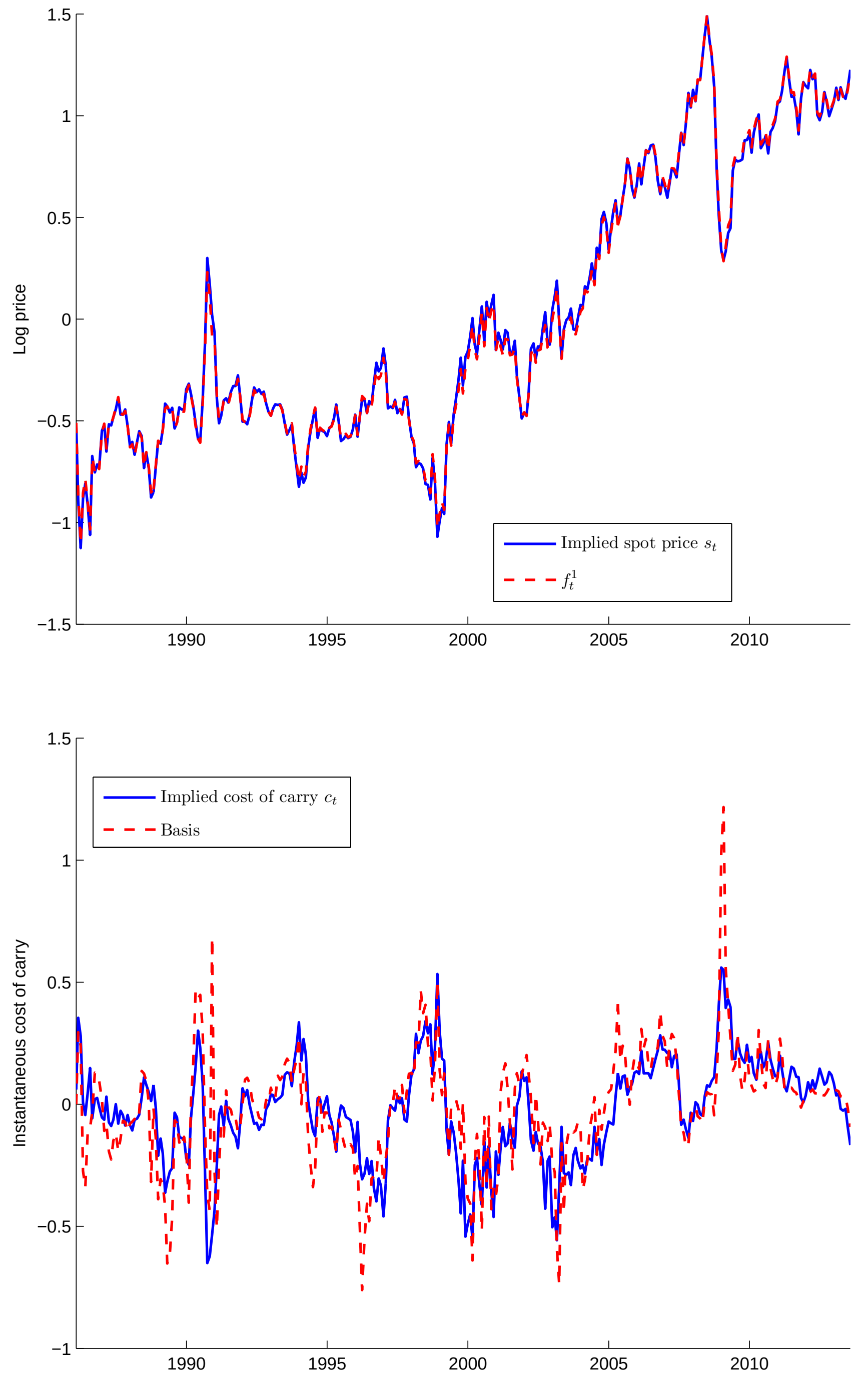

Figure 7: Panel A plots the model implied log spot price of oil s_t and the one-month log futures price f_t^1. Panel B plots the model implied cost of carry c_t and the annualized basis $12 \times (f_t^2 - f_t^1)$.

Table 5: Comparison between the model implied cost of carry c_t and the basis $12 \times (f_t^2 - f_t^1)$ as predictors of log U.S. inventories of crude oil INV. The standard errors are Newey-West with six lags. $^*: p < 0.10$, $^{**}: p < 0.05$, $^{***}: p < 0.01$.

Panel A: Cost of Carry from WTI Futures, 1/1986-7/2013

	ΔINV_{t+1}		
c_t	0.031***		0.038***
	(0.008)		(0.010)
$basis_t$		0.016*	-0.009
		(0.009)	(0.012)
INV_t	-0.112^{***}	-0.088^{***}	-0.112^{***}
	(0.022)	(0.023)	(0.022)
$adj.\,R^2$	6.4%	3.9%	6.2%
T	330	330	330

Panel B: Cost of Carry from Brent Futures, 1/1990-7/2013

	ΔINV_{t+1}		
c_t^{BRENT}	0.039***	-0.027	0.035**
	(0.011)	(0.037)	(0.015)
c_t		0.062**	
		(0.030)	
$basis_t$			0.004
			(0.013)
INV_t	-0.105^{***}	-0.127^{***}	-0.107^{***}
	(0.026)	(0.023)	(0.028)
$adj.\,R^2$	6.3%	7.4%	6.0%
T	281	281	281

with the inventory data. In models of storage without adjustment costs, the cost of carry and the quantity stored are perfectly correlated. c_t is modestly more correlated with contemporaneous inventories INV_t than the basis is: $corr(c_t, INV_t)$ is 0.52 while $corr(basis_t, INV_t)$ is 0.42, and the difference in correlations is significant at the 10% level. Table 5 Panel A presents a horse race regressing the change in inventories on c_t, the basis, and the current inventory level. We see that c_t is a stronger predictor of future inventories, and in the joint regression c_t drives out the basis entirely. Thus, c_t is more tightly linked to both present and future storage decisions than the basis is. Similar results obtain if I winsorize the basis at the 1% or 5% level in both tails, indicating that the stronger performance of c_t is not driven by a few extreme observations in the basis.

To further investigate the validity of the model implied cost of carry, I estimate the model using the prices of Brent crude oil futures from one to twelve months maturity, from January 1990 to July 2013. The WTI contract delivers oil in Cushing, Oklahoma, while the Brent contract delivers oil on shipboard in the North Sea approximately 4,500 miles away. The two markets are naturally linked, but can diverge materially. The correlation of the WTI basis with the Brent basis is 77.8%, while the correlation of c_t with c_t^{BRENT} is 95.2%. Thus, there is considerable market-specific variation in the basis, while the slopes of the futures curves are more closely linked. The question is whether the market-specific variation in the basis is economically meaningful in relation to storage. Table (5) Panel B, Column 1 shows that c_t^{BRENT} is a strong predictor of future U.S. inventories. Column 2 shows that c_t^{BRENT} is driven out entirely by c_t computed from U.S. oil futures, so some variation between the markets is relevant to storage. But Column 3 shows that c_t^{BRENT} completely drives out the U.S. futures basis as a predictor of U.S. inventories. These results suggest, first, that the model's estimation of c_t picks up as much or more of the market-specific variation in the cost of carry than the basis does, and second, that the additional market-specific variation in the basis is not related to storage.

6 Real Options

Firms' capacity to adjust investment or production ex post – their real options – make up a substantial part of firm value, and evaluating and managing these adjustments is a primary role of firm management (Pindyck (1988); Berk, Green and Naik (2004)). Previous studies (Brennan and Schwartz (1985); Schwartz (1997); Casassus and Collin-Dufresne (2005)) have explored what commodity derivatives can tell us about the valuation of real options. These previous studies make the strong assumption that the economy is spanned by commodity futures. One exception is Trolle and Schwartz (2009), in which a latent stochastic volatility factor is unspanned by futures but identified in options prices.

By contrast, if a factor M_t is unspanned in the sense of this paper then it cannot be identified from commodity futures or options data. Unspanned factors of this type are still relevant to real options, however, when the option payoff depends on M_t. For example, an oil well is often presented as the right to pump oil out of the ground at a fixed cost per barrel, analogous to a purely financial option. But for a real oil well, the costs of extraction are uncertain. Moel and Tufano (2002) find that for gold mines, changes in extraction costs over time are a significant predictor of mine openings and closings after controlling for the spot price, convenience yield and price volatility of gold.

More generally, commodity prices are only one element of a firm's decision process. For example, in an airline's decision to purchase more fuel efficient planes, the cost savings will vary with oil prices while revenues will vary with aggregate economic activity. Pindyck

(1993) makes this argument and points out that the risk premiums of all risk factors will also affect real option valuation[8].

To illustrate the effects of unspanned macroeconomic risks on real options valuation, I model an oil well as a ten year strip of European options on an oil field that produces 1000 barrels of oil per month when open. The oil is extracted at lifting cost l_t and sold at the spot price s_t each month that it is open. Thus, it is open whenever $s_t > l_t$. In the model that is used to generate the data, the log lifting cost l_t has both spanned and unspanned components plus idiosyncratic noise. The dynamics of the state variables (s, c, GRO) are a simplified version of the estimates presented earlier. Appendix E describes the setting in detail. I assume that the unspanned macro variable GRO carries a non time varying risk premium λ.

Figure 8 plots the value of wells with different current lifting costs L_0 using different models. The lower two lines represent option values for spanned-risk models in which all relevant risks are assumed to be spanned by oil futures. This means l_t must be a linear combination of s_t and c_t plus an error term (Joslin, Le and Singleton (2013)). Whether the error term is modelled as an i.i.d. or AR(1) process is essentially irrelevant to option value.

The upper two lines represent option values for unspanned-risk models. We see that the spanned-risk models miss a large component of option value due to the contribution of unspanned macro risk. To emphasize, the monthly volatility of shocks to l_t in the spanned-risk models is the same as it is in the unspanned-risk models. The difference is that l_t's dependence on GRO adds persistent time variation in lifting costs that can interact with the spot price and cost of carry (recall the impulse response functions presented in Section

[8] *"... this effect [of uncertainty on option value and exercise] is magnified when fluctuations in construction costs are correlated with the economy, or, in the context of the Capital Asset Pricing Model, when the 'beta' of cost is high... [A] higher beta raises the discount rate applied to expected future costs, which raises the value of the investment opportunity as well as the benefit from waiting rather than investing now."*

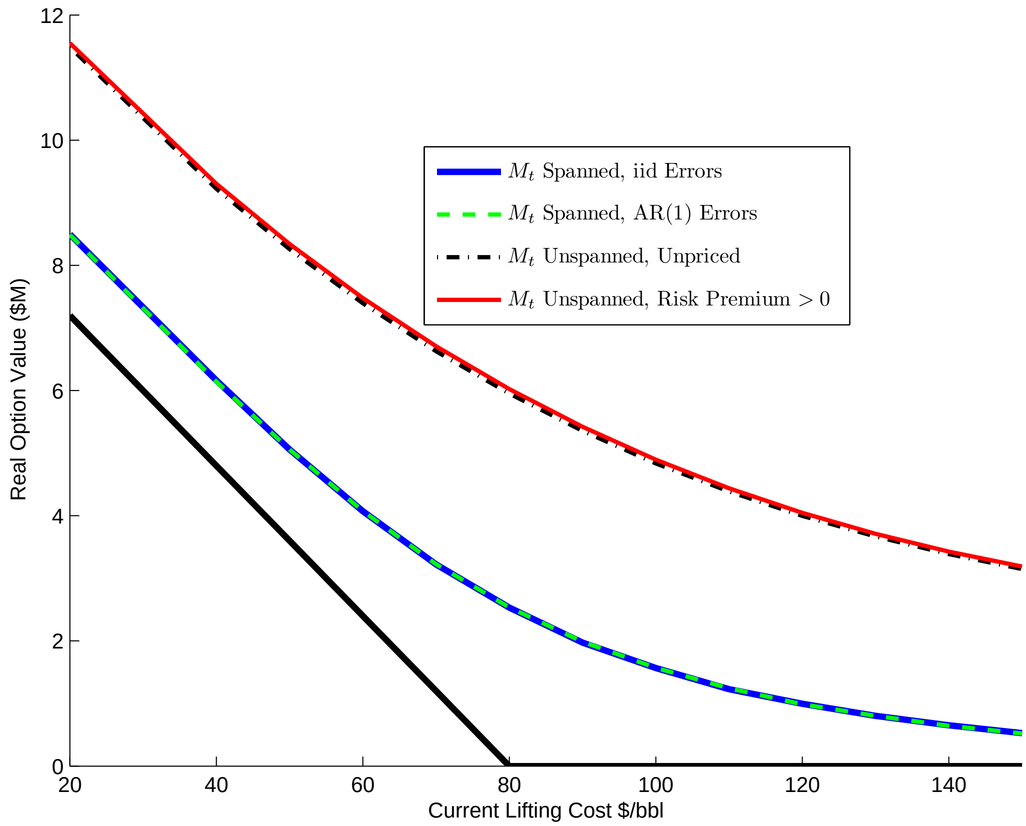

Figure 8: Examples of real options valuation with unspanned risks. An oil well is modelled as a strip of European options that are exercised when the stochastic log extraction cost l_t is less than the log spot price s_t. l_t covaries with s_t and the unspanned macro risk GRO_t. The current spot price of oil is \$80, and the x-axis indexes the current lifting cost L_0 of different oil wells.

4.2.3). This addition has a large effect on option valuation: Adding the unpriced ($\lambda = 0$) unspanned macro risk raises the real option value by 35% for an 'in the money' well with current lifting cost = \$20 per barrel and 405% for an 'out of the money' well with current lifting cost of \$150 per barrel.

The risk premium (Pindyck) effect is that the option value is higher when GRO, and hence L_t, carries a positive risk premium ($\lambda > 0$). This effect on valuation is present but minor in the example, increasing the well's value by only 0.99% for the 'in the money' well with $L_0 = 20$ and by 1.27% for the 'out of the money' well with $L_0 = 150$.

7 Conclusion

This paper develops an affine macro-finance model for futures that admits unspanned macroeconomic variables. The model includes many existing commodity futures models as special cases, and represents a middle ground between studies that use vector autoregressions (VARs) on the one hand and affine latent-factor models on the other. The model can be applied to any commodity and any set of macro factors. I apply it to crude oil futures prices and investigate their interaction with economic growth and oil inventories. I find novel evidence that higher real activity forecasts higher oil prices and that this forecast is unspanned in contemporaneous futures prices. At the same time, higher oil prices forecast lower real activity, especially when the price increase is forecast by the market to be persistent. Thus, there is a negative feedback relationship between oil prices and the economy. The implied spot risk premium in the model estimate differs, particularly in recessions, from the spot risk premium in a model that omits growth as an unspanned risk factor. These results highlight the importance of using information beyond that contained in the futures curve when studying futures returns and price forecasts.

The model also has applications to real options valuation. By assumption, the unspanned macro factors do not affect the prices of commodity futures or any other financial derivatives. However, when the payoff of a real option such as an oil well depends on macroeconomic factors beyond the oil price, then those factors can have a large effect on option value and exercise. In a calibrated example I show that both the dynamics and the risk premiums of unspanned macro risks have large effects on real options valuation.

The model estimates imply that the spot price and cost of carry in the oil market are precisely pinned down by futures prices. The model cost of carry differs significantly from the basis, which is commonly used as a proxy for the cost of carry. In particular, the model cost of carry is 40% less volatile month-to-month, and reverts to its mean more than twice as slowly as the basis does. The model cost of carry is more strongly related to both current and future oil inventories than the basis, and the cost of carry based on North Sea futures is more strongly related to U.S. inventories than the U.S. basis. Thus, the model estimates imply that the convenience yield is much less volatile than the basis is and that we obtain a better measure by fitting a pricing model to the full futures curve than we do from a single calendar spread.

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A Model Specification and Risk Premiums

Consider a Gaussian model where the log spot price s_t of a commodity is a function of N_L spanned state variables L_t, which may be latent or observed, and N_M unspanned state variables M_t that are observed:

$$\begin{aligned}
\begin{bmatrix} L_{t+1} \\ M_{t+1} \end{bmatrix} &= K^{\mathbb{P}}_{0X} + K^{\mathbb{P}}_{1X} X_t + \Sigma_X \epsilon^{\mathbb{P}}_{t+1} \\
L_{t+1} &= K^{\mathbb{Q}}_{0L} + K^{\mathbb{Q}}_{1L} L_t + \Sigma_L \epsilon^{\mathbb{Q}}_{t+1} \\
s_t &= \delta_0 + \delta_1' L_t
\end{aligned} \tag{7}$$

where

- $\mathbb{P}$ denotes dynamics under the historical or data generating measure
- $\mathbb{Q}$ denotes dynamics under the risk neutral measure
- $\epsilon^{\mathbb{Q}}_{L,t+1} \sim N(0,\, I_{N_L})$, $\epsilon^{\mathbb{P}}_{t+1} \sim N(0,\, I_N)$
- Σ_L is the top left $N_L \times N_L$ block of Σ_X; Σ_L, Σ_X are lower triangular

The model is written in discrete time but all results follow in continuous time as well. (7) is equivalent to specifying the equation for s_t and the $\mathbb{P}$-dynamics plus a lognormal affine discount factor with 'essentially affine' prices of risk as in Duffee (2002). For $N_M = 0$ the model includes existing models such as Gibson and Schwartz (1990); Schwartz (1997); Schwartz and Smith (2000) as special cases (see Appendix D). Standard recursions show that (7) implies affine log prices for futures,

$$f_t = A + BX_t \tag{8}$$

$$f_t = \left[\begin{array}{cccc} f_t^1 & f_t^2 & \ldots & f_t^J \end{array} \right]'$$

where f_t^j is the price of a j period future and J is the number of futures with different maturities.

Estimating the model as written presents difficulties; with two latent factors and two macro factors there are 40 free parameters. Different sets of parameter values are observationally equivalent due to rotational indeterminacy of the latent factors. Discussing models of the form (7) for bond yields, Hamilton and Wu (2012) refer to *"tremendous numerical challenges in estimating the necessary parameters from the data due to highly nonlinear and badly behaved likelihood surfaces."* In general, affine models for futures identify the model by specifying dynamics that are less general than (7) and risk prices that are zero or non time varying.

Joslin, Priebsch and Singleton (2014) note that if N_L linear combinations of bond yields are measured without error, then any model of yields of the form (7) implies a model with observable factors in place of the latent factors. They construct a minimal parametrization where no sets of parameters are redundant - models in the "JPS form" are unique. Thus the likelihood surface is well behaved and contains a single global maximum. Their results hold to a very close approximation if the linear combinations of yields are observed with relatively small and idiosyncratic errors.

Section B demonstrates the same result for futures pricing: if N_L linear combinations of log futures prices are measured without error,

$$\mathcal{P}_t = W f_t \tag{9}$$

for any full rank $N_L \times J$ matrix W, then any model of the form (7) is observationally equivalent to a unique model of the form

$$\begin{bmatrix} \Delta\mathcal{P}_{t+1} \\ \Delta UM_{t+1} \end{bmatrix} = \begin{aligned} \Delta Z_{t+1} &= K_0^{\mathbb{P}} + K_1^{\mathbb{P}} Z_t + \Sigma_Z \epsilon_{t+1}^{\mathbb{P}} \\ \Delta\mathcal{P}_{t+1} &= K_0^{\mathbb{Q}} + K_1^{\mathbb{Q}} \mathcal{P}_t + \Sigma_{\mathcal{P}} \epsilon_{t+1}^{\mathbb{Q}} \\ s_t &= \rho_0 + \rho_1 \mathcal{P}_t \end{aligned} \tag{10}$$

parametrized by $\theta = (\lambda^{\mathbb{Q}},\ p_\infty,\ \Sigma_Z,\ K_0^{\mathbb{P}},\ K_1^{\mathbb{P}})$, where

- λ^Q are the N_L ordered eigenvalues of $K_1^{\mathbb{Q}}$
- p_∞ is a scalar intercept
- Σ_Z is the lower triangular Cholesky decomposition of the covariance matrix of innovations in the state variables
- $\Sigma_{\mathcal{P}}\Sigma'_{\mathcal{P}} = [\Sigma_Z\Sigma'_Z]_{N_L}$, the top left $N_L \times N_L$ block of $\Sigma_Z\Sigma'_Z$

A.1 $\mathcal{P}_t$ Measured Without Error

In this paper I assume that while each of the log futures maturities is observed with iid measurement error, the pricing factors $\mathcal{P}_t^1$ and $\mathcal{P}_t^2$ are measured without error.

$$f_t^j = A_j + B_j\mathcal{P}_t + \nu_t^j,\ \nu_t^j \sim N(0, \zeta_j^2)$$

The use of the first two PCs of log price levels is not important: in unreported results I find that all estimates and results are effectively identical using other alternatives such as the first two PCs of log price changes or of returns, or a priori weights such as

$$W = \begin{bmatrix} 1 & ... & 1 \\ 0 & ... & 12 \end{bmatrix}$$

The identifying assumption that N_L linear combinations of yields are measured without error is common in the bond yields literature beginning with Chen and Scott (1993). Given the model parameters, values of the latent factors at each date are then extracted by inverting the relation (8). The same assumption is used to identify previous latent factor models for commodity futures (see Gibson and Schwartz (1990); Casassus and Collin-Dufresne (2005); Hamilton and Wu (2014)). In unreported results I find that all estimates and results are effectively identical if the pricing factors are estimated via the Kalman filter.

A.2 Rotating to s_t and c_t

Once the model is estimated in the JPS form, I rotate $(\mathcal{P}_t^1, \mathcal{P}_t^2)$ to be the model implied log spot price and instantaneous cost of carry, $(s_t,\, c_t)$. For s_t this is immediate:

$$s_t = \rho_0 + \rho_1 \mathcal{P}_t$$

For c_t the definition is as follows. Any agent with access to a storage technology can buy the spot commodity, sell a one month future, store for one month and make delivery. Add up all the costs and benefits of doing so (including interest, costs of storage, and convenience yield) and express them as quantity c_t where the total cost in dollar terms $= S_t(e^{c_t} - 1)$.

Then in the absence of arbitrage it must be the case that

$$F_t^1 = S_t e^{c_t}$$

$$f_t^1 = s_t + c_t = E^{\mathbb{Q}}[s_{t+1}] + \frac{1}{2}\sigma_s^2$$

$$c_t = E^{\mathbb{Q}}[\Delta s_{t+1}] + \frac{1}{2}\sigma_s^2$$

$$= \rho_1[K_0^{\mathbb{Q}} + K_1^{\mathbb{Q}}\mathcal{P}_t] + \frac{1}{2}\sigma_s^2$$

A.3 Risk premiums and s_t, c_t:

Szymanowska et al. (2013) define the per-period log basis $y_t^n \equiv f_t^n - s_t$, and define two risk premiums based on different futures trading strategies; the spot premium $\pi_{s,t}$ and the term premium $\pi_{y,t}^n$.

The spot premium is defined as

$$\pi_{s,t} \equiv E_t[s_{t+1} - s_t] - y_t^1$$

$$= E_t[s_{t+1}] - f_t^1 = E_t^{\mathbb{P}}[s_{t+1}] - E_t^{\mathbb{Q}}[s_{t+1}] - \frac{1}{2}\sigma_s^2$$

$$\Rightarrow \pi_{s,t} = \Lambda_{s_t} - \frac{1}{2}\sigma_s^2$$

The term premium is defined as

$$\pi_{y,t}^n \equiv y_t^1 + (n-1)E_t\left[y_{t+1}^{n-1}\right] - ny_t^n$$

$$= f_t^1 + (n-1)E_t\left[f_{t+1}^{n-1}\right] - nf_t^n$$

The one month term premium is always zero, because storing for one month is riskless.

$$\pi_{y,t}^{(1)} = f_t^1 + 0 - f_t^1 = 0$$

$$\pi_{y,t}^{(2)} = c_t + E_t^{\mathbb{P}}[c_{t+1}] - 2E_t^{\mathbb{Q}}[s_{t+2} - s_{t+1} + s_{t+1} - s_t]$$

$$= E_t^{\mathbb{P}}[c_{t+1}] - E_t^{\mathbb{Q}}[c_{t+1}] - E_t^{\mathbb{Q}}[s_{t+2} - s_{t+1}] - c_t$$

$$\Rightarrow \pi_{y,t}^{(2)} = \Lambda_{c_t} - \left(c_t + E_t^{\mathbb{Q}}[c_{t+1}]\right)$$

Thus the spot premium and term premium of Szymanowska et al. (2013) each have a natural expression in our affine framework. The spot premium is exactly the risk premium attached to shocks to the log spot price s_t plus a small constant. The term premium is the risk premium attached to shocks to the cost of carry minus the (risk-neutral) total expected cost of carry.

B JPS Parametrization

I assume that N_L linear combinations of log futures prices are measured without error,

$$\mathcal{P}_t^L = W f_t$$

for any full-rank real valued $N_L \times J$ matrix W, and show that any model of the form

$$\begin{aligned} \begin{bmatrix} \Delta L_{t+1} \\ \Delta M_{t+1} \end{bmatrix} = \Delta X_{t+1} &= K_{0X}^{\mathbb{P}} + K_{1X}^{\mathbb{P}} X_t + \Sigma_X \epsilon_{t+1}^{\mathbb{P}} \\ \Delta L_{t+1} &= K_{0L}^{\mathbb{Q}} + K_{1L}^{\mathbb{Q}} X_t + \Sigma_L \epsilon_{L,t+1}^{\mathbb{Q}} \\ s_t &= \delta_0 + \delta_1' X_t \end{aligned} \tag{11}$$

is observationally equivalent to a unique model of the form

$$\begin{aligned} \begin{bmatrix} \Delta \mathcal{P}_{t+1}^L \\ \Delta M_{t+1} \end{bmatrix} = \Delta Z_{t+1} &= K_0^{\mathbb{P}} + K_1^{\mathbb{P}} Z_t + \Sigma_Z \epsilon_{Z,t+1}^{\mathbb{P}} \\ \Delta \mathcal{P}_{t+1}^L &= K_0^{\mathbb{Q}} + K_1^{\mathbb{Q}} Z_t + \Sigma_{\mathcal{P}} \epsilon_{t+1}^{\mathbb{Q}} \\ s_t &= \rho_0 + \rho_1' Z_t \end{aligned} \tag{12}$$

which is parametrized by $\theta = (\lambda^{\mathbb{Q}},\ p_\infty,\ \Sigma_Z,\ K_0^{\mathbb{P}},\ K_1^{\mathbb{P}})$.

The proof that follows is essentially the same as that of Joslin, Priebsch and Singleton (2014). Joslin, Singleton and Zhu (2011) demonstrates the result for all cases including zero, repeated and complex eigenvalues.

Assume the model (11) under consideration is nonredundant, that is, there is no observationally equivalent model with fewer than N state variables. If there is such a model, switch to it and proceed.

B.1 Observational Equivalence

Given any model of the form (11), the $J \times 1$ vector of log futures prices f_t is affine in L_t,

$$f_t = A_L + B_L L_t$$

Hence the set of N_L linear combinations of futures prices, $\mathcal{P}_t^L$, is as well:

$$\mathcal{P}_t^L = W_L f_t = W_L A_L + W_L B_L L_t$$

Assume that the N_L ordered elements of $\lambda^{\mathbb{Q}}$, the eigenvalues of $K_{1L}^{\mathbb{Q}}$, are real, distinct and nonzero. There exists a matrix C such that $K_{1L}^{\mathbb{Q}} = C\text{diag}(\lambda^{\mathbb{Q}})C^{-1}$. Define $D = Cdiag(\delta_1)C^{-1}$, $D^{-1} = Cdiag(\delta_1)^{-1}C^{-1}$ and

$$Y_t = D[L_t + \left(K_{1L}^{\mathbb{Q}}\right)^{-1} K_{0L}^{\mathbb{Q}}]$$

$$\Rightarrow \; L_t = D^{-1}Y_t - \left(K_{1L}^{\mathbb{Q}}\right)^{-1} K_{0L}^{\mathbb{Q}}$$

Then

$$\Delta Y_{t+1} = D\Delta L_{t+1}$$

$$= D[K_{0L}^{\mathbb{Q}} + K_{1L}^{\mathbb{Q}}(D^{-1}Y_t - \left(K_{1L}^{\mathbb{Q}}\right)^{-1} K_{0L}^{\mathbb{Q}}) + \Sigma_L \epsilon_{L,t+1}^{\mathbb{Q}}]$$

$$= \text{diag}(\lambda^{\mathbb{Q}})Y_t + D\Sigma_L \epsilon_{L,t+1}^{\mathbb{Q}}$$

and

$$\begin{bmatrix} \Delta Y_{t+1} \\ \Delta M_{t+1} \end{bmatrix} = \begin{bmatrix} D & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} [K_{0X}^{\mathbb{P}} + K_{1X}^{\mathbb{P}} (\begin{bmatrix} D^{-1} & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \begin{bmatrix} Y_t \\ M_t \end{bmatrix} - \begin{bmatrix} \left(K_{1L}^{\mathbb{Q}}\right)^{-1} K_{0L}^{\mathbb{Q}} \\ 0 \end{bmatrix}) + \Sigma_X \epsilon_{t+1}^{\mathbb{P}}]$$

$$= K_{0Y}^{\mathbb{P}} + K_{1Y}^{\mathbb{P}} \begin{bmatrix} Y_t \\ M_t \end{bmatrix} + \begin{bmatrix} D & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \Sigma_X \epsilon_{t+1}^{\mathbb{P}}$$

and

$$p_t = \delta_0 + \delta_1' L_t = \delta_0 + \delta_1' D^{-1} Y_t - \delta_1' \left(K_{1L}^{\mathbb{Q}}\right)^{-1} K_{0L}^{\mathbb{Q}} = p_\infty + \iota \cdot Y_t$$

where ι is a row of N_L ones.

$$f_t = A_Y + B_Y Y_t$$

$$\mathcal{P}_t^L = W f_t = W A_Y + W B_Y Y_t$$

The model is nonredundant $\Rightarrow$ WB_Y is invertible:

$$Y_t = (WB_Y)^{-1} \mathcal{P}_t^L - (WB_Y)^{-1} W A_Y$$

$$\cdot \mathcal{P}_{t+1}^L = W B_Y \Delta Y_{t+1} = W B_Y \mathrm{diag}(\lambda^{\mathbb{Q}}) [(WB_Y)^{-1} \mathcal{P}_t^L - (WB_Y)^{-1} W A_Y] + W B_Y D \Sigma_L \epsilon_{L,t+1}^{\mathbb{Q}}$$

$$= K_0^{\mathbb{Q}} + K_1^{\mathbb{Q}} \mathcal{P}_t^L + \Sigma_{\mathcal{P}} \epsilon_{t+1}^{\mathbb{Q}}$$

Further,

$$\Delta Z_{t+1} = \begin{bmatrix} \cdot\mathcal{P}^L_{t+1} \\ \Delta M_{t+1} \end{bmatrix} = \begin{bmatrix} WB_Y & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \begin{bmatrix} \Delta Y_{t+1} \\ \Delta M_{t+1} \end{bmatrix}$$

$$= \begin{bmatrix} WB_Y & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \left(K^{\mathbb{P}}_{0Y} + K^{\mathbb{P}}_{1Y} \begin{bmatrix} Y_t \\ M_t \end{bmatrix} + \begin{bmatrix} D & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \Sigma_X \epsilon^{\mathbb{P}}_{t+1} \right)$$

$$= K^{\mathbb{P}}_0 + K^{\mathbb{P}}_1 Z_t + \Sigma_Z \epsilon^{\mathbb{P}}_{t+1}$$

$$p_t = p_\infty + \iota \cdot Y_t = p_\infty + \iota \cdot (WB_Y)^{-1}\mathcal{P}^L_t - \iota \cdot (WB_Y)^{-1} W A_Y = \rho_0 + \rho_1' \mathcal{P}^L_t$$

QED. Collecting the formulas: given any model of the form (7), there is an observationally equivalent model of the form (10), parametrized by $\theta = (\lambda^{\mathbb{Q}},\ p_\infty,\ \Sigma_Z,\ K^{\mathbb{P}}_0,\ K^{\mathbb{P}}_1)$, where

- $D = C diag(\delta_1)^{-1} C^{-1}$
- $\Sigma_Z = \begin{bmatrix} WB_Y D & 0 \\ 0 & I_{\mathcal{M}} \end{bmatrix} \Sigma_X,\ \Sigma_{\mathcal{P}} = [\Sigma_Z]_{\mathcal{L}\mathcal{L}}$
- $B_Y = \begin{bmatrix} \iota'[I_{\mathcal{L}+\mathcal{M}} + \text{diag}(\lambda^{\mathbb{Q}})] \\ \vdots \\ \iota'[I_{\mathcal{L}+\mathcal{M}} + \text{diag}(\lambda^{\mathbb{Q}})]^J \end{bmatrix}$
- $A_Y = \begin{bmatrix} p_\infty + \frac{1}{2}\iota'\Sigma_{\mathcal{P}}\Sigma_{\mathcal{P}}'\iota \\ \vdots \\ A_{Y,J-1} + \frac{1}{2} B_{Y,J-1}\Sigma_{\mathcal{P}}\Sigma_{\mathcal{P}}' B_{Y,J-1}' \end{bmatrix}$
- $K^{\mathbb{Q}}_1 = WB_Y \text{diag}(\lambda^{\mathbb{Q}})(WB_Y)^{-1},\ K^{\mathbb{Q}}_0 = -K^{\mathbb{Q}}_1 W A_Y$
- $\rho_0 = p_\infty - \iota \cdot (WB_Y)^{-1} W A_Y,\ \rho_1' = \iota \cdot (WB_Y)^{-1}$

In estimation I adopt the alternate form

- $\Delta Y_{t+1} = \begin{bmatrix} p_\infty \\ 0 \end{bmatrix} + \text{diag}(\lambda^{\mathbb{Q}})Y_t + D\Sigma_X \epsilon^{\mathbb{Q}}_{t+1}$
- $p_t = \iota \cdot Y_t$
- $A_Y = \begin{bmatrix} p_\infty + \frac{1}{2}\iota' \Sigma_{\mathcal{P}} \Sigma'_{\mathcal{P}} \iota \\ \vdots \\ A_{Y,J-1} + B_{Y,J-1} \begin{bmatrix} p_\infty \\ 0 \end{bmatrix} + \frac{1}{2} B_{Y,J-1} \Sigma_{\mathcal{P}} \Sigma'_{\mathcal{P}} B'_{Y,J-1} \end{bmatrix}$
- $K_1^{\mathbb{Q}} = WB_Y \text{diag}(\lambda^{\mathbb{Q}})(WB_Y)^{-1}$, $K_0^{\mathbb{Q}} = WB_Y \begin{bmatrix} p_\infty \\ 0 \end{bmatrix} - K_1^{\mathbb{Q}} W A_Y$
- $\rho_0 = -\iota \cdot (WB_Y)^{-1} W A_Y$, $\rho_1' = \iota \cdot (WB_Y)^{-1}$

which is numerically stable when $\lambda^{\mathbb{Q}}(1) \to 0$. See the online supplement to JSZ 2011.

B.2 Uniqueness

We consider two models of the form (10) with parameters θ and $\hat{\theta} = (\hat{\lambda}^{\mathbb{Q}},\ \hat{p}_\infty,\ \hat{\Sigma}_Z,\ \hat{K}_0^{\mathbb{P}},\ \hat{K}_1^{\mathbb{P}})$ that are observationally equivalent and show that this implies $\theta = \hat{\theta}$.

Since $Z_t = \begin{bmatrix} \mathcal{P}_t^L \\ M_t \end{bmatrix}$ are all observed, $\{\Sigma_Z,\ K_0^{\mathbb{P}},\ K_1^{\mathbb{P}}\} = \{\hat{\Sigma}_Z,\ \hat{K}_0^{\mathbb{P}},\ \hat{K}_1^{\mathbb{P}}\}$.

Since $f_t = A + BZ_t$ are observed, $A(\theta) = A(\hat{\theta})$, $B(\theta) = B(\hat{\theta})$.

Suppose $\lambda^{\mathbb{Q}} \neq \hat{\lambda}^{\mathbb{Q}}$. Then by the uniqueness of the ordered eigenvalue decomposition,

$$B_Y^j(\lambda) \neq B_Y^j(\hat{\lambda})\,\forall j$$

$$\Rightarrow WB_Y(\lambda) \neq WB_Y(\hat{\lambda}) \;\Rightarrow (WB_Y(\lambda))^{-1} \neq (WB_Y(\hat{\lambda}))^{-1}$$

$$\Rightarrow \rho_1(\lambda) \neq \rho_1(\hat{\lambda}) \Rightarrow B(\lambda) \neq B(\hat{\lambda})$$

, a contradiction. Hence $\lambda^{\mathbb{Q}} = \hat{\lambda}^{\mathbb{Q}}$. Then $A(\lambda^{\mathbb{Q}}, p^{\infty}) = A(\hat{\lambda}^{\mathbb{Q}}, \hat{p}^{\infty}) \Rightarrow p^{\infty} = \hat{p}^{\infty}$.

C Estimation

Given the futures prices and macroeconomic time series $\{f_t, M_t\}_{t=1,\ldots,T}$ and the set of portfolio weights W that define the pricing factors:

$$\mathcal{P}_t = W f_t$$

we need to estimate the minimal parameters $\theta = (\lambda^{\mathbb{Q}}, \ p_{\infty}, \ \Sigma_Z, \ K_0^{\mathbb{P}}, \ K_1^{\mathbb{P}})$ in the JPS form. The estimation is carried out by maximum likelihood (ML). If no restrictions are imposed (i.e. we are estimating the canonical model (11)), then $K_0^{\mathbb{P}}$, $K_1^{\mathbb{P}}$ do not affect futures pricing and are estimated consistently via OLS. Otherwise $K_0^{\mathbb{P}}$, $K_1^{\mathbb{P}}$ are obtained by GLS taking the restrictions into account. The OLS estimate of Σ_Z is used as a starting value, and the starting value for p_{∞} is the unconditional average of the nearest-maturity log futures price. Both were always close to their ML value. Finally I search over a wide range of values for the eigenvalues $\lambda^{\mathbb{Q}}$, and the ML values were always of reasonable magnitude, real valued, and distinct.

After the ML estimate of the model in the JPS form is found, I rotate and translate the spanned factors from $\mathcal{P}_t^1$, $\mathcal{P}_t^2$ to s_t, c_t as described in A.2. I rotate and translate UM_t to M_t, so that the estimate reports the behavior of the actual macro time series M_t instead of their subcomponents:

$$\begin{bmatrix} s_t \\ c_t \\ M_t \end{bmatrix} = \begin{bmatrix} \rho_0 \\ \frac{1}{2}\sigma_s^2 + \rho_1 K_0^{\mathbb{Q}} \\ \alpha_{M\mathcal{P}} \end{bmatrix} + \begin{bmatrix} \rho_1 & 0_{1\times N_M} \\ \rho_1 K_1^{\mathbb{Q}} & 0_{1\times N_M} \\ 0_{N_M\times 1} & \beta_{M\mathcal{P}} \end{bmatrix} \begin{bmatrix} \mathcal{P}_t \\ UM_t \end{bmatrix}$$

where

$$M_t = \alpha_{M\mathcal{P}} + \beta_{M\mathcal{P}}\mathcal{P}_t + UM_t$$

D Comparison with other Futures Models

The model (7) is a canonical affine Gaussian model, so any affine Gaussian model is a special case. For example, discretized, the Gibson and Schwartz (1990); Schwartz (1997); Schwartz and Smith (2000) two factor model can be written

$$\begin{bmatrix} \Delta s_{t+1} \\ \Delta \delta_{t+1} \end{bmatrix} = \begin{bmatrix} \mu \\ \kappa\alpha \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 0 & -\kappa \end{bmatrix} \begin{bmatrix} s_t \\ \delta_t \end{bmatrix} + \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}^{1/2} \epsilon^{\mathbb{P}}_{t+1} \tag{13}$$

$$\begin{bmatrix} \Delta s_{t+1} \\ \Delta \delta_{t+1} \end{bmatrix} = \begin{bmatrix} r \\ \kappa\alpha - \lambda \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 0 & -\kappa \end{bmatrix} \begin{bmatrix} s_t \\ \delta_t \end{bmatrix} + \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}^{1/2} \epsilon^{\mathbb{Q}}_{t+1} \tag{14}$$

which is clearly a special case of (7).

The Casassus and Collin-Dufresne (2005) model, discretized, is:

$$\begin{bmatrix} \Delta X_{t+1} \\ \Delta \hat{\delta}_{t+1} \\ \Delta r_{t+1} \end{bmatrix} = \begin{bmatrix} \kappa^P_X \theta^P_X + \kappa^P_{Xr}\theta^P_r + \kappa^P_{X\hat{\delta}}\theta^P_{\hat{\delta}} \\ \kappa^P_{\hat{\delta}}\theta^P_{\hat{\delta}} \\ \kappa^P_r \theta^P_r \end{bmatrix} + \begin{bmatrix} -\kappa^P_X & -\kappa^P_{X\hat{\delta}} & -\kappa^P_{Xr} \\ 0 & -\kappa^P_{\hat{\delta}} & 0 \\ 0 & 0 & -\kappa^P_r \end{bmatrix} \begin{bmatrix} X_t \\ \hat{\delta}_t \\ r_t \end{bmatrix} + \begin{bmatrix} \sigma_X & 0 & 0 \\ 0 & \sigma_{\hat{\delta}} & 0 \\ 0 & 0 & \sigma_r \end{bmatrix} \begin{bmatrix} 1 & & \\ \rho_{X\delta} & 1 & \\ \rho_{Xr} & \rho_{\delta r} & 1 \end{bmatrix}^{1/2} \epsilon^{\mathbb{P}}_{t+1} \tag{15}$$

$$\begin{bmatrix} \Delta X_{t+1} \\ \Delta \hat{\delta}_{t+1} \\ \Delta r_{t+1} \end{bmatrix} = \begin{bmatrix} \alpha_X \theta^Q_X + (\alpha_r - 1)\theta^Q_r + \theta^Q_{\hat{\delta}} \\ \kappa^Q_{\hat{\delta}}\theta^Q_{\hat{\delta}} \\ \kappa^Q_r \theta^Q_r \end{bmatrix} + \begin{bmatrix} -\alpha_X & -1 & 1 - \alpha_r \\ 0 & -\kappa^Q_{\hat{\delta}} & 0 \\ 0 & 0 & -\kappa^Q_r \end{bmatrix} \begin{bmatrix} X_t \\ \hat{\delta}_t \\ r_t \end{bmatrix} + \begin{bmatrix} \sigma_X & 0 & 0 \\ 0 & \sigma_{\hat{\delta}} & 0 \\ 0 & 0 & \sigma_r \end{bmatrix} \begin{bmatrix} 1 & & \\ \rho_{X\delta} & 1 & \\ \rho_{Xr} & \rho_{\delta r} & 1 \end{bmatrix}^{1/2} \epsilon^{\mathbb{Q}}_{t+1} \tag{16}$$

(see their formulas 7, 12, 13 and 27, 28, 30) which is the Schwartz three factor model with more flexible risk premiums.

Table 6: Parameters of the model for the purpose of real option valuation

	$K_0^{\mathbb{P}}$	$K_1^{\mathbb{P}}$		
		s_t	c_t	GRO_t
s_{t+1}	0.00	1.00	0.083	0.03
c_{t+1}	0.00	0.00	0.90	0.00
GRO_{t+1}	0.00	-0.10	0.00	0.60

	Σ		
	s	c	GRO
s	0.10		
c	-0.08	0.06	
GRO	0	0	0.50

	$K_0^{\mathbb{Q}}$	$K_1^{\mathbb{Q}}$		
		s_t	c_t	GRO_t
s_{t+1}	0.00	1.00	0.08	0.00
c_{t+1}	0.00	0.00	0.90	0.00
GRO_{t+1}	$-\lambda$	-0.10	0.00	0.60

E Real Option Valuation

The lifting cost is

$$l_t = \kappa_l + 0.1 s_t + 0.01 GRO_t + \epsilon_t^l,\ \epsilon_t^l \sim N(0, \sigma_l)$$

, that is, l_t varies with both s_t and GRO_t as well as having an i.i.d idiosyncratic component.

Notice the third row of $K_1^{\mathbb{Q}}$, which was not there in the previous estimates. When we consider assets with payoffs that depend on M_t, the risk neutral dynamics of M_t are material. In principle one can estimate them with a mimicking portfolio for GRO (Lamont 2001), but here I simply assume that exposure to GRO carries a non time varying risk premium equal to λ.

I compute option values for different starting values of $L_0 = exp(l_0)$, with $S_0 = exp(s_0)$ equal to \$80 per barrel and $c_0 = 0$. This is meant to mimic an oil firm evaluating wells that differ in their cost and difficulty of extraction, conditional on today's oil prices.

F Robustness Checks

A natural question from Figure 1 is whether my results are driven by the huge swings in oil prices and real activity during 2008-2009. Table 7 presents the model estimate on a subsample from January 1986 to December 2007. Overall the subsample estimate is very similar to the full-sample estimate, and the key coefficients of GRO_{t+1} on s_t, s_{t+1} on GRO, and INV_{t+1} on c_t are virtually unchanged and remain statistically significant at the 5% level. The most notable differences relative to the full sample results are that in the subsample, c_t and INV_t appear to have significant forecasting power for future values of GRO.

Table 8 shows that the regression results using the principal components of log prices are also very similar, and indeed the incremental predictability from the macro factors appears stronger when we omit the financial crisis from the data.

Table 7: Maximum likelihood (ML) estimate of the macro-finance model for Nymex crude oil futures, using data from 1/1986 to 12/2007. s, c are the spot price and annualized cost of carry respectively. GRO is the monthly Chicago Fed National Activity Index. INV is the log of the private U.S. crude oil inventory as reported by the EIA. The coefficients are over a monthly horizon, and the state variables are de-meaned. ML standard errors are in parentheses. Coefficients in bold are significant at the 5% level.

Historical ($\mathbb{P}$) Measure

	$K_0^{\mathbb{P}}$	$K_1^{\mathbb{P}}$			
		s_t	c_t	GRO_t	INV_t
s_{t+1}	0.011	**0.996**	0.058	**0.029**	-0.006
	(0.007)	(0.014)	(0.036)	(0.011)	(0.107)
c_{t+1}	-0.013	0.026	**0.864**	-0.012	0.031
	(0.007)	(0.015)	(0.037)	(0.011)	(0.111)
GRO_{t+1}	0.056	**-0.171**	**0.633**	**0.427**	**-1.571**
	(0.035)	(0.070)	(0.178)	(0.054)	(0.533)
INV_{t+1}	0.002	**-0.008**	**0.031**	-0.003	**0.855**
	(0.002)	(0.004)	(0.010)	(0.003)	(0.031)

Risk Neutral ($\mathbb{Q}$) Measure

	$K_0^{\mathbb{Q}}$	$K_1^{\mathbb{Q}}$	
		s_t	c_t
s_{t+1}	-0.003	**1.000**	**0.083**
	(0.007)	(0.005)	(0.011)
c_{t+1}	0.000	-0.008	**0.885**
	(0.012)	(0.013)	(0.031)

Shock Volatilities
[off-diagonal = % correlations]

	s	c	GRO	INV
s	0.102			
c	-85%	0.057		
GRO	7%	0%	0.500	
INV	-23%	31%	4%	0.028

Table 8: Panel A shows the results of forecasting returns to oil futures. Panel B shows the results of forecasting changes in the level and slope, $PC1$ and $PC2$ respectively, of the oil futures curve. The data are monthly from from 1/1986 to 12/2007. The forecasting variables are the first five principal components of log futures prices, PC^{1-5}, and the residuals of the Chicago Fed National Activity Index and U.S. oil inventory projected on PC^{1-5}. The standard errors are Hansen-Hodrick. $^{*}: p < 0.10$, $^{**}: p < 0.05$, $^{***}: p < 0.01$.

Panel A: Forecasting Returns to Futures

$$r_{t\to t+horizon} = \alpha + \beta_{1-5} PC_t^{1-5} + \beta_{UGRO,UINV} UM_t + \epsilon_t$$

Horizon	1 month		6 months		12 months	
Futures maturity	1 m	12 m	1 m	12 m	1 m	12 m
β_{UGRO}	0.028**	0.022***	0.034	0.034	0.015	0.026
	(0.010)	(0.006)	(0.036)	(0.023)	(0.068)	(0.045)
β_{UINV}	-0.027	-0.001	-0.727^{*}	-0.58^{**}	-1.161	-1.12^{**}
	(0.119)	(0.072)	(0.414)	(0.27)	(0.800)	(0.55)
Adjusted $R^2(PC^{1-5})$	-0.2%	0.7%	4.1%	2.6%	6.8%	2.9%
Adj. $R^2(PC^{1-5} + UM_t)$	2.1%	4.1%	7.6%	8.9%	10.0%	10.8%
F-ratio	7.9***	5.6***	5.7***	9.7***	11.2***	11.8***

Panel B: Forecasting Changes in Level and Slope

$$\Delta PC^{(n)}_{t\to t+horizon} = \alpha + \beta_{1-5} PC_t^{1-5} + \beta_{UGRO,UINV} UM_t + \epsilon_t$$

Horizon	1 month		6 months		12 months	
Principal Component	Level	Slope	Level	Slope	Level	Slope
β_{UGRO}	0.082***	-0.007	0.104	0.007	0.059	0.023
	(0.027)	(0.006)	(0.085)	(0.012)	(0.151)	(0.020)
β_{UINV}	-0.011	0.012	-2.04^{*}	0.106	-3.54	0.127
	(0.302)	(0.071)	(1.06)	(0.187)	(1.92)	(0.182)
Adjusted $R^2(PC^{1-5})$	-0.3%	8.5%	1.6%	35.0%	0.3%	51.2%
Adj. $R^2(PC^{1-5} + UM_t)$	2.5%	8.3%	6.7%	34.8%	6.7%	51.8%
F-ratio	9.5***	0.7	15.8***	0.6	19.7***	2.7*