Abstract

The characteristics of both entrepreneurs and venture capitalists are jointly determined by the set of real investment opportunities. An improvement in these opportunities has two effects: 1) it encourages marginal, low-quality entrepreneurs to pool with high-quality entrepreneurs, and 2) it induces the entry of venture capitalists that are less efficient at screening new investments. The market for venture capital financing is therefore plagued by a double-sided asymmetric information problem. The severity of both frictions severities is increasing but concave in market heat, and this is shown to imply that supply and demand are more elastic in cold markets. The paradigm makes predictions regarding the evolution of venture capital partnership quality.
1 Introduction

The venture capital market is characterized by dramatic fluctuations in the overall level of activity. A major research agenda has been to empirically decompose these cycles into changes driven by demand-side factors and those driven by supply-side factors (Gompers and Lerner, 1998). This is a daunting task given the endogeneity of the actions of capital suppliers and demanders. For example, an across-the-board cut in tax rates affects both the incentive to engage in entrepreneurial activity and the profit from financing it (Poterba, 1989). Therefore without observing required returns — or making the heroic assumption that realized returns equal required returns — it is difficult to unambiguously attribute changing quantities to specific causal factors.

Instead, the approach has been to exploit natural experiments such as an unpredicted change to the interpretation of the Department of Labor’s Employee Retirement Income Security Act (ERISA). The 1979 ruling allowed that a moderate amount of pension fund investment could be allocated to private equity if invested with the care of a “prudent man.” Accordingly, it serves as an exogenous supply shock to capital in this market.

One emerging theme of this literature is that the supply of venture capital funds is “sticky” in the short-run. Private equity investing requires considerable skill and experience — attributes which are largely fixed in the short-run. During hot markets, general partners in established funds become the scarce resource in the market and earn significant economic rents (see also Ljungqvist and Richardson, 2003). In the long-run, supply adjusts as new VCs choose to enter the market, thereby competing away rents. However, this entry/exit reaction can be sluggish and imprecise, so that the market can sometimes go through episodes of over-investment followed by low returns (Lerner, 2002; Kaplan and Schoar, 2005) or under-investment with correspondingly high returns.

This evidence makes it clear that any formal analysis of the determinants of supply and demand for VC funds ought to account for the entry incentives
on both sides of the market. In other words, given an exogenous distribution of quality across potential entrepreneurs, which agents choose to enter the market and which do not? For an exogenous distribution of ability across potential venture capitalists, which agents choose to operate and which do not? The answers to these two questions determine 1) the severity of financing frictions in the venture capital market, 2) the equilibrium distribution of private firm value, and 3) the equilibrium quantity of funds in the market.

This paper takes a first step towards such a model using the paradigm of information asymmetry. As is well known, financing of startups entails potentially extreme adverse selection costs given the absent track record of the firms seeking capital, and given the risky nature of the industries in which many of them operate. Exacerbating the problem, this scenario often involves an innovator who has extensive technical knowledge (Denis, 2004) but lacks the reputation capital necessary to convey this information credibly.

In such an environment of asymmetric information, perhaps the most natural definition of venture capitalist “ability” is effectiveness in screening new investments.¹ VCs go through an extensive process of due diligence before investing (Kaplan and Stromberg, 2000) and the vast majority of funding proposals are rejected. Viewed more broadly, this skill has enormous social welfare consequences because it determines how efficiently capital is directed toward profitable projects.

The quantity of funds demanded depends upon economic conditions. In the model, agents choose to either start entrepreneurial firms (with VC financing)

¹“Extensive due diligence in the private equity market is needed because little, if any, information about issuers is publicly available and in most cases the partnership has had no relationship with the issuer. Thus, the partnership must rely heavily on information that it is able to produce de novo. Moreover, managers of the issuing firm typically know more than outsiders about many aspects of their business [but] the problem of adverse selection is mitigated by the extensive amount of due diligence conducted.”
– Fenn, Liang and Prowse (1995, pg. 30)
or continue in their current employment. In equilibrium, better quality agents are more likely to become entrepreneurs: their residual claim from doing so is relatively more valuable. Yet there is overinvestment, as some entrepreneurs with negative NPV projects pool with better entrepreneurs in an attempt to obtain the benefits of mispricing.

The capital supply side also has latent agents, i.e. information producers who can choose to enter the market as venture capitalists if market conditions warrant doing so. Obviously, this entry decision depends upon the agents’ privately-known information production ability. The decision must also take into consideration the distribution of entrepreneurial quality, because the heterogeneity therein dictates the severity of the information problems being faced. In turn, entrepreneurial entry decisions are affected by the equilibrium skill level of VCs. This is because the capital suppliers’ required returns depend upon their costs of information production. In addition, entrepreneurs face differential probabilities of being screened out (depending on their true quality), and these probabilities depend upon the skill level of VCs. All of these interdependencies underscore the importance of modeling these supply-side and demand-side frictions simultaneously.

These agents generate a supply curve of information production. In states of low capital demand (i.e., poor investment opportunities) only very efficient information producers are able to operate profitably. An improvement in real investment opportunities causes the market to “step up” the supply curve to progressively less efficient information production agents. Therefore, the average intrinsic quality of supply-side agents is countercyclical.

Like the supply-side, the demand-side informational friction is of procyclical severity. Consider a positive shock to the value of investment opportunities. With more profitable investments, competitive venture capitalists would be willing to hold smaller financial stakes in underlying firms, ceteris parabus. However, such an improvement in the terms of finance would induce the entry
of marginal, lower-quality entrepreneurs (see Yung, Colak and Wang (2008) for an analogous argument and evidence in the context of initial public offerings). Thus the capital demand-side asymmetric information problem is of procyclical severity.

The model has the following properties. In all states of the world, VC information production is able to partially, but not fully, mitigate the demand-side asymmetric information problem. In all states, within the set of active VCs, the lowest quality agents make zero profits. By contrast, high-quality VCs earn positive profits despite the fact that equilibrium profit-sharing rules are competitively determined. Their rents are increasing in the value of investment opportunities. On both the supply-side and the demand-side, the average quality of agents is lower during good times, and the dispersion of quality across agents is higher during good times.

While the paper does not explicitly model the relationship between general partners (GPs) and limited partners (LPs), the findings give some reason to be skeptical of partnerships founded during hot markets. This viewpoint seems consistent with Gomper, Kovner, Lerner and Sharfstein (2005), who conclude that experienced VCs react more accurately to changing market conditions. By contrast, a particularly encouraging scenario might involve a partnership operating during hot markets but founded during cold markets. The decision to operate in a cold market (and subsequent survival) suggests high innate ability, while the contemporaneous hot markets indicate attractive investment opportunities. A more formal analysis of these claims would need to account for the ability (and incentive) of agents to signal their quality to LPs via their choice of contract. Noting that the venture capital investment process emphasizes “staged” finance (i.e. capital is released over time, rather all at once) such an analysis hinges on dynamic choices such as when to raise new funds, and the role of intertemporal profit-sharing within a given fund. A full analysis of these contractual features is beyond the scope of the current focus.
Endogenous entry in the model also permits examination of supply and demand elasticity.\(^2\) When profitability is high, there are strong incentives to enter on both sides of the market. Consequently, marginal entry decisions are relatively insensitive to sharing rules, i.e., the division of the firm between VCs and entrepreneurs. This implies that both supply and demand are more elastic in cold markets. Section 3.3 illustrates these points graphically.

1.1 Comparison to the Agency Theoretic Approach

Separation between financing and real decision-making – a pillar of classical finance theory – breaks down in private markets. Venture capitalists have a myriad of non-financial roles, including the “professionalization” of the firm (Hellman and Puri, 2002), the decisions to liquidate or replace management (Gompers, 1995; Bergemann and Hege, 1997) and the form and timing of the exit strategy.

With both the entrepreneur and the VC having real inputs to firm value, the environment is one of double-sided moral hazard. This paradigm has important security design implications (Casamatta 2003; Repullo and Suarez, 1999; and Schmidt, 2003). Empirically, financial contracts cede to the VC a variety of state-dependent control rights (Kaplan and Stromberg, 2002), the terms of which are tailored to the circumstances of each individual firm. These contractual feature vary on a deal-by-deal, round-by-round basis (Kaplan and Stromberg, 2004).

By focusing on how these frictions vary with business conditions, two papers in the literature are particularly close to the spirit of the current model. Inderst and Muller (2004) model the process of search and bargaining between

\(^2\)Previous literature assumes that long-run supply is “likely to be quite flat” (Gompers and Lerner, 1998, pg. 7) but has not identified determinants of this elasticity. I am not aware of any attempt to characterize demand elasticity, although Poterba (1989) emphasizes the tax rate differential between wages and capital gains as a key determinant of the level of demand.
VCs and entrepreneurs. The relative supply and demand for funds determines the equilibrium stake held by VCs and the residual claim held by entrepreneurs, and in turn, the amount of effort chosen by each party. Total firm value can be nonmonotonic in the amount of capital market competition because at low (high) values of competition for funds the VC (entrepreneur) contributes too little effort.

In Inderst and Muller’s analysis, shocks to investment conditions have a dynamic effect because the supply of venture capital is sticky in the short-run. Therefore, given a positive shock, VCs temporarily earn positive profits. In the long-run, these profits are driven away by the entry of new VCs. (The severity of frictions can either go up or down, depending on the pre-shock sharing rule.)

Kanniainen And Keuschnigg (2003) take an alternative approach to modeling the effect of changing business conditions. They emphasize that VC monitoring is a scarce resource (see also Sorensen, 2007) to be divided over multiple, simultaneously-held portfolio firms. They demonstrate that when industry returns are high, VCs hold a greater number of portfolio firms and hence contribute less effort to each firm. Hence, as in the current model, there is a sense in which the average supply-side “quality” deteriorates during good times, although the notion of quality is quite different from the one employed here.

A broader question is how these agency theoretic approaches might be empirically distinguished from asymmetric information models. In corporate finance more generally, drawing such distinctions is not always straightforward. One difference is that in these agency models, VCs are homogeneous and all take the same actions. By contrast, an important property of the current model is that the distribution of venture capitalist quality changes

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3In the context of managerial compensation, Hagerty and Siegel (1988) point out that canonical formulations of the principal-agent model and of screening models (i.e., models of self-selection) are mathematically equivalent and therefore empirically indistinguishable.
over time. As an implication, the model makes testable implications regarding cross-sectional dispersion in returns; see Section 4. This heterogeneity has some parallels in the literature, as Kaplan and Schoar (2005) conclude “we think the most likely explanation for [our] results is a model of underlying heterogeneity in the skills of GPs.”

On the other hand, these agency models have insights and applications not shared by the current model. For example, they make predictions on the search time and bargaining power (in Inderst and Muller’s case) and the evolution of the number of portfolio firms (in Kannainen And Keuschnigg’s case). As emphasized by Kaplan and Stromberg (2004), agency considerations play a particularly important role in the governance of venture capital partnerships. Undoubtedly, both imperfections merit further study.

2 The Demand for VC Finance

There exists a continuum of entrepreneurs in the economy with assets-in-place worth $V$ in current use. A new project is available which redeploys existing assets at a cost $K$. Entrepreneurs have no available internal financing, so the project requires external finance.

At $T = 1$, if the new project is undertaken, the firm is worth 1 with probability $\pi_i$, or else 0. The subscript $i$ indicates the entrepreneur’s privately known type. Net present value is therefore $\pi_i - K - V$. Assume that $K + V \in (0, 1)$ and that success probabilities $\pi_i$ are uniformly distributed on $[0, 1]$. Hence, some entrepreneurs have positive NPV projects while others have negative NPV projects.

In this section, venture capitalists are homogenous information producers. They can evaluate entrepreneurs, obtaining a signal $s = G$ or $s = B$. This signal is associated with two costs. The VC incurs an information production
cost $C_{VC}$, and the entrepreneur pays a direct transaction cost $C_E$ (interpreted as the opportunity cost of meeting with VCs, preparing the pitch, etc.). The distinction between these two costs is that while the entrepreneur bears $C_E$ directly, he bears $C_{VC}$ only indirectly through its effect on the venture capitalist’s required stake. Assume that

$$\text{Prob} \{s = G \mid \pi_i\} = \pi_i$$  \hspace{1cm} (1)

Condition (1) implies that the signal is informative: higher quality firms are more likely to yield the signal $s = G$.

Some fraction of the potential entrepreneurs opt to remain private rather than seeking financing. Let $\pi_{MIN}$ denote the lowest quality entrepreneur that tries to obtain VC financing. This cutoff value will be determined later. The interval $[\pi_{MIN}, 1]$ then determines the number of active entrepreneurs. It is assumed that entrepreneurs are evenly distributed across VCs so that each VC has $n$ active entrepreneurs; see Fig 1 below.

Evenly distributing entrepreneurs across VCs in this way ignores an “integer” problem, i.e., the number of entrepreneurs is unlikely to be divisible by the number of VCs, so that each pool cannot literally be of exactly the same size. However, in equilibrium, VCs have no incentive to lure away entrepreneurs from others’ pools: each evaluation is associated with zero expected profit.
This section’s assumption is that VCs have sufficient capital to fund every entrepreneur in their pool (even though not all entrepreneurs will be funded). There is no economic cost to idle funds. Alternatively, in a large economy, it can be assumed that VCs only have sufficient capital to fund the number of entrepreneurs expected to have good signals in their pool. In that case, if a venture capitalist exhausts her funds before completing all of the evaluations in her pool, then any unevaluated entrepreneurs could approach another VC with remaining funds. Even under that alternative scenario, by the law of large numbers, the proportion of unevaluated entrepreneurs goes to zero as the economy gets large.

Assuming that entrepreneurial quality $\pi_i$ is drawn from a uniform distribution on $[0,1]$ it can be shown that the unconditional average quality of active entrepreneurs, and the quality condition on passing the VC screen, respectively, are

\[
E(\pi) = \frac{\pi_{MIN} + 1}{2}
\]

\[
E\{\pi | s = G\} = \frac{2}{3} \left( 1 - \frac{\pi_{MIN}^3}{\pi_{MIN}^2} \right)
\]

Naturally, because the signal $s$ is informative, $E\{\pi | s = G\} > E(\pi)$. Moreover, both the unconditional mean and the conditional mean are increasing in $\pi_{MIN}$. An exogenous improvement in the entrepreneurial pool implies a better expected quality conditional on passing the venture capitalist’s screen.

\footnote{It is assumed here that rejections are public information, so that an entrepreneur receiving a signal $s = B$ cannot re-enter another VC’s pool.}
2.1 The VC’s Problem

Since payoffs are binary, it is without loss of generality to describe the securities as equity. The VC takes the distribution of firm types as given and demands an equity stake $\alpha$ in exchange for supplying capital $K$. In a competitive market, these values satisfy

$$\left(\frac{\pi_{MIN} + 1}{2}\right) \cdot \underbrace{\frac{2\alpha}{3} \left(1 - \frac{\pi_{MIN}^3}{1 - \pi_{MIN}^2}\right) - K}_{\text{Expected Profit when } s=G} \geq C_{VC} \quad (4)$$

2.2 The Entrepreneur’s Problem

While it is defined as the value of assets-in-place, $V$ also serves as a reservation value for entrepreneurs. This is because if no additional funding is secured, then the NPV of the project is lost. Expected payoff from accepting the contract is

$$\pi_i \cdot \underbrace{\left[\pi_i(1 - \alpha) - C_E\right]}_{\text{Profit given funding}} + (1 - \pi_i) \cdot \underbrace{\left[V - C_E\right]}_{\text{Profit otherwise}} \quad (5)$$

The bracketed terms indicate the expected payoff if the project is funded or unfunded, respectively, while the leading terms reflect the probability of each outcome. Note that because entrepreneurs know their type, from their point of view the venture capitalist’s signal adds no value-relevant information. Rather, the signal only determines whether the funding proposal is accepted.

Entrepreneurs seek funding when (5) exceeds $V$. Since this expression is increasing in $\pi_i$, it follows that there is a cutoff quality $\pi_{MIN}$, i.e., an entrepreneur just indifferent between seeking funding and not doing so. In particular,

$$\pi_{MIN} = \frac{V + \sqrt{V^2 + 4(1 - \alpha)C_E}}{2(1 - \alpha)} \quad (6)$$

Putting these two participation constraints together yields the following result.
Theorem 1  Equilibrium consists of the pair \( \{ \pi_{MIN}, \alpha \} \) that jointly solve equations (4) and (6). This solution has the following comparative static properties:

\[
\frac{\partial \pi_{MIN}}{\partial K} > 0 \quad \frac{\partial \pi_{MIN}}{\partial V} > 0
\]  

Equation (7)

\[
\frac{\partial \alpha}{\partial K} > 0 \quad \frac{\partial \alpha}{\partial V} < 0
\]  

Equation (8)

Proof: Follows from the implicit function theorem; see the appendix.

Equation (7) indicates that as investment opportunities improve, lower quality entrepreneurs are drawn into the market. That is, as \( V \) and \( K \) shrink relative to terminal payoffs of a successful project, the cutoff quality \( \pi_{MIN} \) drops. In this sense the demand-side asymmetric information problem is of procyclical severity.

This result is closely related to Yung, Colacck and Wang’s (2008) analysis of the market for initial public offerings. In particular, their analysis focuses on the dispersion of quality within the set of IPOs in a given economic state. This dispersion is predicted to be procyclical. Taking the view that true quality will be revealed in the long-run, this result leads the prediction that cross-sectional variation in long-run returns should be higher within cohorts of firms that were funded during hot markets. Furthermore, this hypothesized expansion is more specific to the left side of the distribution rather than the right side. Consequently, financing waves will be associated with more portfolio firms going bankrupt ex-post.\(^5\) Equation (7) indicates that these results generalize to an environment with informed capital suppliers.

Equation (8) indicates how VC contracts respond to changing market conditions. Unsurprisingly, as the required capital contribution rises, VCs demand a larger stake. The result \( \frac{\partial \alpha}{\partial V} > 0 \) has more subtle intuition. An increase in

\(^5\)The mean return is not predicted to change. Capital suppliers in the model are rational; i.e. offer prices fully account for changing demand-side conditions.
the value of outside opportunities (relative to VC finance) leads to improvement in the quality of the entrepreneurial pool. Because of this reduction in the severity of the asymmetric information problem, VCs are satisfied with a smaller stake. Putting these two results together, the model is ambiguous regarding how $\alpha$ varies with the value of investment opportunities. If both $V$ and $K$ shrink relative to terminal payoffs, then there are competing effects on $\alpha$; this issue is explored again in a numerical example in Section 3.3.

The next result examines how the information production and transaction costs affect entrepreneurial entry and equilibrium financing terms.

**Corollary 1 (Effect of Costs)** The equilibrium values $\{\alpha, \pi_{MIN}\}$ satisfy

$$\frac{\partial \pi_{MIN}}{\partial C_E} > 0, \quad \frac{\partial \pi_{MIN}}{\partial C_{VC}} > 0 \quad (9)$$

$$\frac{\partial \alpha}{\partial C_E} < 0, \quad \frac{\partial \alpha}{\partial C_{VC}} > 0 \quad (10)$$

Proof: Similar to Theorem 1.

Equation (9) indicates that the two costs in the model have a symmetric effect on entrepreneurial entry: as either type of cost rises, marginal low quality entrepreneurs exit the market. This is true because both costs are ultimately borne by the entrepreneur.

As in Theorem 1, this symmetry is broken for $\alpha$. As venture capitalist costs $C_{VC}$ rise, the obvious direct effect is that the VCs demand larger stakes as compensation. On the other hand, as the entrepreneurial costs rise, low-quality entrepreneurs exit the market. Because of this reduction in the severity of the asymmetric information problem, VCs are satisfied with a smaller stake.
3 The Supply of VC Finance

In Section 2, venture capitalists are identical and perfectly competitive, which drives their profits to zero. This modeling assumption precludes analysis of incentives to enter the VC market. To address this issue, Section 3 introduces heterogeneity in VC skill.

3.1 Two Venture Capitalist Types

Assume that some VCs have information production cost $C_L$ and all others have cost $C_H > C_L$. Importantly, both types of venture capitalists obtain the same signal; it is only the costs of production that differ. Consequently, there is no post-financing difference between VCs and so, for a given contract, entrepreneurs have no strict preference regarding which VC supplies funding.

Further assume that there are some states in which aggregate capital demanded exceed the capital under control of the low-cost VCs, so that high-cost VCs must also be active to ensure market-clearing.\footnote{There are also equilibria in which only the low-cost VC are active. In particular, these are states of low demand for capital. These outcomes are economically equivalent to those of Section 2 (setting $C_{VC} = C_H$) and so are not considered further.}

The assumption begs the question of how high-cost VCs could attract funds in the first place. For example, why don’t low-cost VC simply raise larger funds? In the long-run, it is clear that capital will tend to flow to more efficient producers of information. The heterogeneity in this section requires only that this flow is not immediate and perfect.

There are several ways in which this imperfect flow can be motivated. Perhaps most obviously, it is difficult to detect abnormal performance even when managers have a fairly extensive track records. Kothari and Warner (2001) show that even in a mutual fund environment (where both the fund returns and the benchmarks are transparent and continuously observed) under rea-
sonable assumptions regarding the level of abnormal performance, it can take many years of return observations to reliably identify skill.

The inference problem in venture capital market is more difficult still. Gompers and Lerner (1998) observe that “returns from venture capital investments can only be observed many years after the original investments because private firms are valued at cost until they are sold or taken public many years later... [and] information on fund returns is closely guarded, and even the intermediaries who specialize in compiling this data do not have very comprehensive coverage.”

Second, the logically extreme counterassumption would lead to a single active fund manager — the one with the highest skill. But this outcome is problematic if agents have decreasing returns to scale in portfolio management.⁷ As a result, there is some generality in the assumption that even managers with less-than-perfect skill sets are allocated a positive amount of investment capital.

The participation constraint of venture capitalist i is

\[
\left( \frac{\pi_{MIN} + 1}{2} \right) \times \left[ \frac{2\alpha}{3} \left( \frac{1 - \pi_{MIN}^3}{1 - \pi_{MIN}^2} \right) - K \right] \geq C_i. \tag{11}
\]

Competition among high-cost VCs leads to equality in the participation constraint:

\[
\left( \frac{\pi_{MIN} + 1}{2} \right) \times \left[ \frac{2\alpha}{3} \left( \frac{1 - \pi_{MIN}^3}{1 - \pi_{MIN}^2} \right) - K \right] = C_H. \tag{12}
\]

⁷See Berk and Green (2003) for a model that exhibits this property in the context of active management of mutual funds. In the venture capital literature, Kaplan and Shoar (2005) find empirical support for decreasing returns to scale for active management.
Consequently the participation constraint of low-cost VCs does not bind:

\[
\left( \frac{\pi_{MIN} + 1}{2} \right) \cdot \frac{2\alpha \left( 1 - \frac{\pi^3_{MIN}}{1 - \pi^2_{MIN}} \right) - K}{\text{Prob}(s=G)} > C_L
\]  

(13)

This equilibrium has the following properties. High-cost VCs break even. Competition between them cannot reduce \( \alpha^* \) without violating equation (12). Low-cost VCs earn expected profits \( C_H - C_L > 0 \). They have no incentive to “separate” by reducing \( \alpha^* \) below the level offered by competing high-cost agents. Doing so would not increase business — their capital is exhausted in equilibrium — and would only serve to reduce profits.

Note that there is a sense in which entrepreneurs are apparently made worse off when low-quality venture capitalists enter the market, because this entry is associated with a discrete jump in the equilibrium \( \alpha \) (which takes the role of a “price” in this market). This statement is somewhat misleading, however. In any market with a downward-sloping demand curve and upward-sloping supply curve, a positive shock to demand causes consumers to move up the supply curve to higher marginal costs of production. It cannot be said that marginally more expensive additional supply hurts consumers, especially compared to the alternative of failing to meet demand at any price.
3.2 Continuum of VC Types

Section 2 demonstrates that venture capital funds demanded are a decreasing function of the cutoff quality $\pi_{MIN}$. When $\pi_{MIN}$ is high, few entrepreneurs are active and therefore aggregate capital demanded is low. More generally, denote this demand function $D = D(\pi_{MIN})$ where $D' < 0$.8

Next suppose that each potential VC is associated with an information production cost $C_i$ drawn from some continuous distribution. Following the logic of Section 3.1, there is a cutoff $C_{MAX}$ indicating the highest-cost VC active in equilibrium.

The VC with $C_i = C_{MAX}$ earns zero profit. All those with cost exceeding $C_{MAX}$ drop out of the market, while those with cost less than $C_{MAX}$ earn strictly positive profits. The measure of active VCs therefore generates a supply of venture capital finance $S = S(C_{MAX})$ where $S' > 0$.

Equilibrium consists of the triple $\{\pi_{MIN}, C_{MAX}, \alpha\}$ satisfying the following system of three equations.

\begin{align*}
\text{(DEMAND)} & \quad \pi_{MIN} [\pi_{MIN}(1 - \alpha) - C_E] + (1 - \pi_{MIN})(V - C_E) - V = 0 \\
\text{(SUPPLY)} & \quad C_{MAX} - \left( \frac{\pi_{MIN} + 1}{2} \right) \left[ \frac{2}{3} \left( \frac{1 - \pi_{MIN}^3}{1 - \pi_{MIN}} \right) - K \right] = 0 \\
\text{(MKT. CLEARING)} & \quad D(\pi_{MIN}) - S(C_{MAX}) = 0
\end{align*}

8Although unconditionally $\pi_i \sim Unif[0,1]$, so that active entrepreneurs are drawn from $Unif[\pi_{MIN},1]$, is not the case the demand is linear in $\pi_{MIN}$. As more (marginal) entrepreneurs are added to the pool during a hot market, the quantity of funded demanded goes up less than proportionally. The reason is that a smaller proportion of the added entrepreneurs receive the signal $s = G$. Hence, funds demanded are increasing and concave in market heat, or alternatively, decreasing and convex in $\pi_{MIN}$. This issue is developed numerically in Section 3.3.
The demand equation comes from equation (5). The left side of the equation is the profit of the worst entrepreneur active in equilibrium. By definition, $\pi_{\text{MIN}}$ is the entrepreneurial quality associated with zero profit. The supply equation comes from equation (4). That is, $C_{\text{MAX}}$ is the signal cost associated with zero profit. Finally, markets must clear, i.e., the aggregate supply must equal the aggregate demand.

**Theorem 2** The equilibrium triple $\{\pi_{\text{MIN}}, C_{\text{MAX}}, \alpha\}$ has the following comparative static properties.

a) (Effect of $K$)

\[
\frac{\partial \pi_{\text{MIN}}}{\partial K} > 0 \quad \frac{\partial C_{\text{MAX}}}{\partial K} < 0 \quad \frac{\partial \alpha}{\partial K} > 0
\]

b) (Effect of $V$)

\[
\frac{\partial \pi_{\text{MIN}}}{\partial V} > 0 \quad \frac{\partial C_{\text{MAX}}}{\partial V} < 0 \quad \frac{\partial \alpha}{\partial V} < 0
\]

b) (Effect of $C_E$)

\[
\frac{\partial \pi_{\text{MIN}}}{\partial C_E} > 0 \quad \frac{\partial C_{\text{MAX}}}{\partial C_E} < 0 \quad \frac{\partial \alpha}{\partial C_E} < 0
\]

Proof: Follows from the implicit function theorem; see the appendix.

Equation (14) formalizes the intuition laid out in the introduction, which emphasizes how the double-sided asymmetric information problem depends upon real investment opportunities. A positive shock to the economy (as indicated by a drop in $K$) has several implications. Most obviously, venture capitalists’ required stakes fall because they are being asked to provide less capital while terminal payoffs are unchanged.

More importantly, as emphasized in Section 2, a key property of this asymmetric information paradigm is that $\pi_{\text{MIN}}$ drops in response to a positive shock
to investment opportunities. Financing frictions worsen because as the set of real opportunities improves, low-quality entrepreneurs enter the market. Analogously on the supply side, $C_{MAX}$ deteriorates in response to this economic shock.

The model also makes a related observation regarding the rents to high-quality information producers. In Section 3.1, it was determined that the profit to high-quality VCs is zero when they are the only agents active (i.e., states of low demand) but is $C_H - C_L > 0$ in states of high demand. That is, high-quality agents’ profits depend upon the difference between their own quality and that of the worst active agent. Analogously here, since the quality of the worst agent is continuously decreasing in market heat, the profits of a fixed (active) VC are continuously increasing in market heat. As a result, total profits to the VC industry are also increasing in market heat.

Equations (15) and (16) have straightforward interpretations. As the relative costs to entrepreneurs of seeking VC finance increase, marginal entrepreneurs are forced out of the market — that is, $\pi_{MIN}$ grows. Though VCs do not bear these costs, there is an indirect effect on their required returns. Specifically, because the quality of pool improves, this enables VCs to demand smaller stakes.

The final indirect effect is that because demand falls, the equilibrium number of VCs operating must fall. Such a drop would be impossible if $\alpha$ rose, because in that case VCs would make higher profits even for a fixed level of asymmetric information. These higher profits would encourage entry, not exit.

### 3.3 An Example

Fig. 2 displays supply and demand curves for a particular numerical example. The supply and demand curves are governed by the equations on pg. 17. In this example the parameters are set to \{V = .2, K = .2, C_E = .1\}. It is
further assumed that supply is proportional to $C_{\text{MAX}}$, which is equivalent to the assumption that information production costs are drawn from a uniform distribution.

The horizontal axis indicates $\alpha$, the share of the firm offered to the venture capitalist. The entrepreneur’s residual claim is $1-\alpha$. The vertical axis indicates the quantity of funds. More specifically, it reflects the percentage of projects in the economy that receive funding in equilibrium. (Recall that some negative NPV projects are screened out by the VC whereas others self-select; the vertical axis here reflects the net effect of both types of removal.)

The demand curves in Fig. 2 have a downward slope. This is because a greater share $\alpha$ of the firm ceded to VCs implies that fewer entrepreneurs find it profitable to seek funding. Likewise, the supply curves have an upward slope because when a greater share of the firm is ceded to VCs, venture capital profits are higher (all else equal) and therefore more VCs enter the market.

![Fig. 2: Supply and Demand for VC Funds](image)

Fig. 2: Supply and Demand for VC Funds: The vertical axis indicates the proportion of funded projects in the economy. The horizontal axis indicates the share of the firm ceded to venture capitalists. The intersection of the solid lines depicts equilibrium under the assumptions $\{V=.2, K=.2, C_{\epsilon}=.1\}$ while the dotted lines correspond to $\{V=.1, K=.1, C_{\epsilon}=.1\}$.
Next, consider a positive shock to the value of investment opportunities. The dotted lines show the supply and demand curves when $V$ and $K$ fall to from .2 to .1. This drop in $V$ and $K$ is economically equivalent to an increase in the terminal payoffs relative to other parameters. However, Theorem 2 indicates (and as will be emphasized here) the effects of $V$ and $K$ are economically distinct. It is therefore pedagogically preferable to treat them separately.

The decrease in $V$ shifts the demand curve up. As entrepreneurs’ outside options become less valuable, they are more willing to enter the VC market. The drop in $K$ has no direct effect on demand. Rather, entrepreneurs bear $K$ only indirectly through the VCs’ required stake.

The decrease in $K$ shifts the supply curve up. This is a direct effect: for a given $\alpha$, reducing the required capital contribution would increase VC profitability. Consequently, more VCs would enter, increasing the willingness to supply capital.\footnote{This point also emphasizes why Fig 1 uses the proportion of projects funded on the vertical axis rather than (numeraire-denominated) funds supplied. Otherwise, shocks to $K$ trivially change the amount of capital supplied and demanded.} By contrast, the decrease in $V$ shifts the demand curve downward. This is an indirect effect. The adverse selection effect emphasized in Section 2 implies that the quality of the entrepreneurial pool deteriorates. This deterioration indirectly reduces the profitability to capital suppliers, and the resulting exit of marginal VCs causes supply to fall. Over the relevant range in the graph, the direct effect dominates and leads to higher supply.

In this numerical example, the effect of the demand shock dominates that of the supply shock, leading to a higher equilibrium venture capital stake $\alpha$. This comparative static result is not general, however, as Theorem 2 makes clear. That is, one can also construct shocks to $\{V, K\}$ such that the supply effect dominates the demand effect and hence equilibrium $\alpha$ is lower in hot markets.
It is also apparent in Fig 2 that both demand curves are less elastic (flatter) in hot markets. This property is driven by $V$. To see why this effect occurs, consider the extreme case when $V$ is very low compared to other parameters in the model. In that case, virtually every entrepreneur is active because no other choice is economically viable. Their collective entry decisions are therefore relatively invariant to $\alpha$. Hence the demand curve is relatively flat in $\alpha$.

A similar argument motivates why supply elasticity is a function of investment opportunities. In any economic state, increasing $\alpha$ increases VC profits both directly (they keep a larger share of the firm) and indirectly (because higher $\alpha$ improves the quality of the entrepreneurial pool). This indirect effect is most important in cold markets, however, when the incentives of all entrepreneurs to enter the market is weak. In that case, small changes in the sharing rule cause large changes in the average quality within the pool, and hence in VC profits.

**Fig. 3: Supply and Demand for VC Funds:** The vertical axis indicates the proportion of funded projects in the economy. The axis towards the front of the diagram indicates the share of the firm ceded to venture capitalists, and the final axis (going front to back) represents the reservation utility of entrepreneurs. The parametric assumptions are $K=.2$ and $C_e=.1$. 

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Fig. 3 generalizes figure Fig. 2. The axis toward the front of the picture is \( \alpha \), the share offered to VCs. The vertical axis is again the proportion of projects funded. The final axis, moving front to back in the picture, indicates entrepreneur’s reservation utility \( V \). Taking a slice of the figure along the \( V \)-axis one can see the supply and demand curves of the previous figure. Again it is clear that both supply and demand curves are more elastic when \( V \) is high. In this case, few entrepreneurs are active and so small changes rapidly alter the composition of the pool. Moving “backwards” in the picture toward lower \( V \) (interpreted as a positive shock to investment opportunities) causes an increase to both the equilibrium amount of capital supplied and to the stake of VCs.

4 Conclusions

The venture capital market is central to the financing of innovation but is subject to dramatic variation in the level of activity. Understanding the determinants of the underlying supply and demand curves is therefore a financial economic question of first-order importance.

This paper endogenizes this supply and demand in an environment of two-sided information asymmetry. Entrepreneurs have heterogeneous, private information about the quality of their projects. Venture capitalists have heterogeneous ability levels. In effect, these agents generate a supply curve of information production. Though all agents appear identical, there is equilibrium heterogeneity in abilities of active agents on both supply and demand sides of the market. Thus, there are two distributions of agent quality, both of which vary continuously in the heat of the market.

Both types of variation have some support in the literature. Regarding the phenomenon of wider pooling during hot markets, Lerner (2002) comments

“[F]unds appear to be deployed much less effectively during the
boom period. In particular, all too often these periods find venture capitalists funding firms that are too similar to one another.”

Yet even when firms appear similar ex-ante, there is evidence — at least in the IPO market (Yung, et. al, 2008) — that firms funded during booms exhibit heightened variability in long-run returns. These findings together suggest that, while the firms may be difficult to distinguish ex-ante, there is actually significant heterogeneity revealed in the long-run.

The scientific study of returns in the VC market is a relatively young field, and is somewhat hindered by data availability issues. As Gompers, Lerner, Kovner and Sharfstein (2005) (hereafter, GLKS) point out that Venture Economics “does not collect valuation information for all the companies that were merged or acquired and it is possible that these outcomes are not as lucrative as those where the company exited with a public offering.” Thus, for describing outcomes at the portfolio firm level, researchers have typically employed coarse measures such as whether a firm was liquidated, merged or went public.

Recent empirical papers circumvent the data problems alluded to by GLKS by using either proprietary databases (Ljungqvist and Richardson, 2003) or using fund-level cash flows (Jones and Rhodes-Kropf, 2004) rather than portfolio firm level cash flows. While this latter approach is useful for characterizing the returns of the asset class, it does not permit analysis of the demand-side heterogeneity posited by the current model.

Similarly, the model’s prediction of time-varying heterogeneity in the quality of supply side agents has not been subject to formal testing. Nevertheless, it has some parallels in Kaplan and Schoar’s (2005) findings. They conclude that a large fraction of fund inflows during hot markets go to new entrants, and that these entrants appear to have poor ex-post performance. Among possible explanations for these findings, Kaplan and Schoar list a variety of disadvantages faced by new entrants, including lack of access to deal flow and better bargaining power vis-a-vis entrepreneur. Perhaps their strongest word-
ing, however, is reminiscent of the current analysis: “we think the most likely explanation for [our] results is a model of underlying heterogeneity in the skills of GPs.”

Hochberg, Ljungqvist and Vissing-Jorgensen (2008) also model an environment of GP heterogeneity. As funds season, LPs acquire a form of inside information vis-a-vis outside investors. In particular, LPs may be able to distinguish luck from skill. The resulting informational hold-up leads to underadjustment of fees and performance persistence. There is also a possible efficiency gain, however, because LPs are sometimes willing to fund untested GPs owing to the inherent option value in doing so. Thus, the informational hold-up does not serve as a friction. Rather, it prevents a market failure.

The above story potentially interacts with the current analysis. Given a positive shock to investment opportunities, both incumbent GPs and their LPs earn economic rents on follow-up funds. In fact, the expected positive profits in hot markets accruing to LPs — who effectively share residual claimancy with GPs in Hochberg et al.’s story — could subsidize their losses in cold markets. This effect would tend to smooth out the supply of capital over the market cycle. The full range of implications (including time-varying incentives of GPs to signal quality) is left to future research.
5 References


6 Proofs

Proof of Theorem 1. I derive the comparative static result $\frac{\partial \pi_{MIN}}{\partial K} > 0$ here. The others are similar. The two first order conditions are:

$$F := \alpha \frac{2}{3} \left( \frac{1 - \pi_{MIN}^3}{1 - \pi_{MIN}^2} \right) - K \quad (17)$$

$$G := \pi_{MIN}^2 (1 - \alpha) - \pi_{MIN} V - C \quad (18)$$

Totally differentiating this system with respect to $K$, one obtains

$$\frac{\partial F}{\partial K} + \frac{\partial F}{\partial \pi_{MIN}} \frac{\partial \pi_{MIN}}{\partial K} + \frac{\partial F}{\partial \alpha} \frac{\partial \alpha}{\partial K} = 0 \quad (19)$$

$$\frac{\partial G}{\partial K} + \frac{\partial G}{\partial \pi_{MIN}} \frac{\partial \pi_{MIN}}{\partial K} + \frac{\partial G}{\partial \alpha} \frac{\partial \alpha}{\partial K} = 0 \quad (20)$$

The above is a system of two equations to be solved for two unknowns, $\frac{\partial \pi_{MIN}}{\partial K}$ and $\frac{\partial \alpha}{\partial K}$. The solution for $\frac{\partial \pi_{MIN}}{\partial K}$ is

$$\frac{\partial \pi_{MIN}}{\partial K} = \frac{\frac{\partial G}{\partial \alpha} \frac{\partial \pi_{MIN}}{\partial K} - \frac{\partial F}{\partial \alpha} \frac{\partial \pi_{MIN}}{\partial K}}{\frac{\partial F}{\partial \pi_{MIN}} \frac{\partial \pi_{MIN}}{\partial K} - \frac{\partial G}{\partial \pi_{MIN}} \frac{\partial \pi_{MIN}}{\partial K}} \quad (21)$$

Reporting only the signs of each of these partial derivatives, and noting that $\frac{\partial G}{\partial K} = 0$,

$$\text{sign} \left( \frac{\partial \pi_{MIN}}{\partial K} \right) = \bigoplus \bigoplus - \bigoplus 0 = \bigoplus = \bigoplus \quad (22)$$

Q.E.D.
Proof of Theorem 2. I derive the comparative static result \( \frac{\partial \pi_{\text{MIN}}}{\partial K} > 0 \) here. The others are similar. Denote \( H:=0, J:=0, M:=0 \) as the demand, supply and market clearing identities listed on page 16. Totally differentiating this system with respect to \( K \), one obtains

\[
\frac{\partial H}{\partial K} + \frac{\partial H}{\partial \pi_{\text{MIN}}} \frac{\partial \pi_{\text{MIN}}}{\partial K} + \frac{\partial H}{\partial C_{\text{MAX}}} \frac{\partial C_{\text{MAX}}}{\partial K} + \frac{\partial H}{\partial \alpha} \frac{\partial \alpha}{\partial K} = 0 \tag{23}
\]

\[
\frac{\partial J}{\partial K} + \frac{\partial J}{\partial \pi_{\text{MIN}}} \frac{\partial \pi_{\text{MIN}}}{\partial K} + \frac{\partial J}{\partial C_{\text{MAX}}} \frac{\partial C_{\text{MAX}}}{\partial K} + \frac{\partial J}{\partial \alpha} \frac{\partial \alpha}{\partial K} = 0 \tag{24}
\]

\[
\frac{\partial M}{\partial K} + \frac{\partial M}{\partial \pi_{\text{MIN}}} \frac{\partial \pi_{\text{MIN}}}{\partial K} + \frac{\partial M}{\partial C_{\text{MAX}}} \frac{\partial C_{\text{MAX}}}{\partial K} + \frac{\partial M}{\partial \alpha} \frac{\partial \alpha}{\partial K} = 0 \tag{25}
\]

Note that the above is a system of three equations to be solved for three unknowns \( \frac{\partial \pi_{\text{MIN}}}{\partial K} \), \( \frac{\partial C_{\text{MAX}}}{\partial K} \) and \( \frac{\partial \alpha}{\partial K} \). Solving the system, and using the fact that four of twelve leading terms are zero (e.g. \( \frac{\partial H}{\partial K} = 0 \) by inspection on \( H \)), one obtains the solution

\[
\frac{\partial \pi_{\text{MIN}}}{\partial K} = -\frac{\frac{\partial M}{\partial \alpha} \frac{\partial M}{\partial \pi_{\text{MIN}}} \frac{\partial J}{\partial K} + \frac{\partial J}{\partial C_{\text{MAX}}} \frac{\partial M}{\partial \pi_{\text{MIN}}} \frac{\partial H}{\partial K}}{\frac{\partial M}{\partial \alpha} \frac{\partial M}{\partial \pi_{\text{MIN}}} \frac{\partial J}{\partial K} + \frac{\partial J}{\partial C_{\text{MAX}}} \frac{\partial M}{\partial \pi_{\text{MIN}}} \frac{\partial H}{\partial K}} \tag{26}
\]

Reporting only the signs of each of these partial derivatives,

\[
\text{sign} \left( \frac{\partial \pi_{\text{MIN}}}{\partial K} \right) = \frac{\oplus \ominus \ominus}{\ominus [\ominus \ominus - \ominus \ominus] + \ominus \ominus \ominus} \tag{27}
\]

By inspection, both the numerator and denominator are positive. Therefore the fraction is positive.

Q.E.D.