

Financing Risk and Bubbles of Innovation*

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New ventures that commercialize radical technologies tend to cluster in time. Funding for new ventures also ebbs and flows, often synchronous with significant innovations. We suggest that financial market activity is not purely a response to novel technologies but rather, financial markets drive innovation bubbles. We show that financing risk is inherent to the funding of new ventures, but varies across sectors and time and alters the type of project funded. In equilibrium, more innovative projects are funded in ‘hot’ markets when financing risk is low. Thus, the financing environment for new ventures may create and magnify bubbles of innovation.

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Abstract

New ventures that commercialize radical technologies tend to cluster in time. Funding for new ventures also ebbs and flows, often synchronous with significant innovations. We suggest that financial market activity is not purely a response to novel technologies but rather, financial markets drive innovation bubbles. We show that financing risk is inherent to the funding of new ventures, but varies across sectors and time and alters the type of project funded. In equilibrium, more innovative projects are funded in 'hot' markets when financing risk is low. Thus, the financing environment for new ventures may create and magnify bubbles of innovation.

Introduction

Episodes of great innovative activity correspond with the formation of many new ventures. Recent revolutions in industries such as semiconductors, communications networking, the internet, biotechnology, clean technology as well as those in textiles, railways, motor cars and other new technologies have all involved an explosion of new ventures. These new ventures are thought to play a central role in Schumpeter's (1942) waves of creative destruction and to be a fundamental driver of productivity growth in the economy (Aghion and Howitt (1992), King and Levine (1993)).

There also seem to be times, industries and places in which financing to form new businesses seems either overly abundant or overly scarce. Prevailing wisdom suggests that the ebbs and flows of financing follow from the variation in innovation - money flows toward good ideas and away from bad, although possibly with some friction. Financial markets are thought to play a role to the extent that reducing frictions allows capital to flow more freely and improves economic growth (Levine and Zervos (1998)). Even work that focuses more directly on financial markets tends to explain pricing "bubbles", taking innovation as an exogenous event (Pastor and Veronesi (2009), Gompers et al. (2008), Hobbijn and Jovanovic (2001)). In this thinking, financing effects follow the underlying invention.

We suggest that financial markets could be driving innovative activity. We find that the funding to form new ventures around the pursuit of new inventions can in equilibrium be either abundant or scarce and jump between both. Furthermore, in the equilibrium with abundant funding, a different, more innovative type of firm is funded. The financial market equilibrium, therefore, plays a key role in driving the development and commercialization of new innovations. This is a radically different way to think about bubbles of innovation and leads to a number of new insights and empirical predictions relating to the creation of new ideas. It also suggests a much larger role for financial markets in the innovation process, even when the invention is exogenous, as the financial markets may naturally magnify any underlying real innovative phenomenon.

A key contribution of our paper is to show how characteristics of the financial market, even one with unlimited capital and all rational equally informed participants, can create equilibrium financing

risk. Financing risk is the risk that future funding will not be available even though the fundamental quality of the project has not changed. We demonstrate why financing risk occurs and how it can create and magnify bubbles of innovative activity. Thus, our work is related to a growing body of work that considers the role of financial intermediaries in the innovation process (see Kortum and Lerner (2000), Hellmann (2002), Lerner et al. (2011), Sorensen (2007), Tian and Wang (2011), Manso (2011), Hellmann and Puri (2000)).

The second important contribution of our paper is to highlight the mechanism through which equilibrium financing risk occurs. Understanding the mechanism behind financing risk allows us to understand when and where it may be most important. In our model, financing risk arises for investors who depend partially on the potential to be acquired to generate value. Investors who fear future financing risk have lower negotiating power in an acquisition - the acquirer faces a lower potential threat from the target and the target's stand alone option is less valuable. This lowers the present value of the project which lowers the probability that the investment can get funding. This in turn makes the fear of future financing risk rational.¹ There is the potential for a rational self fulfilling 'bad' equilibrium.

This equilibrium, however, does not apply to all firms whose value stems partially from potential acquisition. Investors who perceive the possibility of financing risk can reduce or eliminate the possibility by providing more funding up front. However, an important attribute of many new ventures is that their outcomes are highly uncertain.² A natural consequence of this uncertainty is that investors in such firms want to stage their investments, providing limited capital to the firm in each round, and learning more about the firm's potential at each stage in order to preserve the option to terminate their investments before providing more financing (see Gompers (1995), Bergemann and Hege (2005), Bergemann et al. (2008)).

Financing risk therefore creates a trade-off for investors in such firms. They can reduce financing

¹There are a number of other potential channels through which fear of future funding loss lowers the value of the project today - employees may leave or work less hard, customer may delay purchases, etc. These all work similarly in that the financing risk lowers the current value of the project to the point where in equilibrium it is rational for investors to withdraw financing. These will be discussed more in the body of the paper.

²For example, over 50% of venture backed startups are either liquidated or fail to receive follow-on funding, despite the extensive due diligence, help and support provided by the venture capitalists (Gompers and Lerner (2004)).

risk by giving a firm more money upfront, but this comes at a cost. A firm with more funding may spend some or all of the money even in the event of disappointing intermediate information. This cost is much more important for highly innovative firms where outcomes are uncertain and the real option to shut down the firm is most valuable. In equilibrium, financing risk therefore has the greatest impact on the projects with the most real option value who are also likely to be acquired.³

By demonstrating a potential channel for financing risk we are able to understand why financing risk is likely to create or magnify bubbles of innovative activity, as well as lead investors to fund a different types of firm at different times in the innovation cycle. The typical notion of what happens when financing is abundant is one of money chasing deals (Gompers and Lerner (2000)) - that when more money enters an area more “bad”, lower return, deals are done. Our idea is that simultaneously money *changes* deals. That is, during bubbles of activity, more innovative ideas are funded because financing risk falls, altering the NPV of innovative projects. Thus, not only do we suggest that financial markets may create bubbles of activity but that financiers in these bubbles may fund a fundamentally different type of activity. Stimulated by this theory Nanda and Rhodes-Kropf (2011) recently report empirical evidence that more innovative projects are funded when excess capital enters the venture capital market. This result supports the idea that when financing risk falls a more innovative project can get funding.

Our theory documents three types of projects. The first type is one that gets funded in both equilibria, because these projects are NPV positive even in the ‘bad’ funding equilibrium. The second type of project is never funded, as this type is NPV negative even in the ‘good’ funding equilibrium. We show that there is a third class of project where the extent to which a given investment is NPV positive or negative depends on financing risk – that is, they are funded in good times when financing risk is low, but not funded in bad times when financing risk is high. We expect this idea to stimulate other empirical research into the differences in the type of project funded across the cycle.

The third contribution of our work is that we don’t simply demonstrate the possibility that there

³As we will explore in the body of the paper, in a world of complete contracts financing risk could be eliminated with contracts that guaranteed future funding in the appropriate state. However, in a more realistic world with incomplete contracts across time financing risk is an important issue.

are multiple funding equilibria (and a channel through which this can occur), but we go further and endogenize the investors response to the possibility of financing risk. In a world where the equilibrium may jump, investors forecast the likelihood of the switch and provide more or less financing as insurance against the low funding equilibrium. For some firms, investors can provide enough ‘insurance’ to eliminate financing risk altogether. However, this insurance is much more expensive for innovative firms where the value generated by providing limited funding and waiting to learn more is the greatest. In equilibrium, less innovative sectors face less financing risk because they find it cheaper to insure against it, reducing the possibility that it occurs. It is the most innovative sectors and firms in the economy that need the abundant funding equilibrium to help invent and commercialize their radically new technologies.

Together, these contributions lead directly to a number of empirical predictions, the first of which is simply that financing risk should create variation in the funding of innovative new ventures. The literature on venture capital has documented the extreme variation in venture capital investment (Gompers and Lerner (2004)) and fund-raising (Gompers and Lerner (1998)), that are correlated with high market values, hot IPO markets or past returns (Kaplan and Schoar (2005)). Furthermore, technological revolutions seem to be associated with ‘hot’ financial markets (Perez (2002)). Prior work has suggested that these correlations could be overreaction by investors (Gompers and Lerner (1998)), rational reactions to fundamentals (Gompers et al. (2008), Pastor and Veronesi (2009)), herd behavior for reputational concerns (Scharfstein and Stein (1990)) or even reverse causality (Hobijn and Jovanovic (2001)). Our model suggests that at least part of the link between bubbles of innovation and periods of hot financial markets is because low financing risk leads investors to fund and hence discover and commercialize the most innovative ideas in the economy.

While some of the predictions of our model are similar to other explanations linking financial markets to innovation, others are not. For example, the second empirical implication of our model is that the mix of investors should change in periods of high financing risk, relative to periods of low financing risk. Early round investors of very innovative projects are subject to a greater amount of

financing risk as the option value at a projects earliest stages is higher. Their investing activity should be particularly impacted by hot and cold financial markets. Our model also predicts that the mix of investors should shift towards smaller investors (with less capital to deploy) in good times when financing risk is low, as the smaller and more frequent investments in periods of low financing risk are particularly well suited to smaller investors.

A third implication of accounting for financing risk is that any given investor should not rush to invest into *all* projects in a sector that is out of favor. Conventional wisdom (and most past work) suggests that when money leaves a sector it is a good time to invest, and when a lot of money enters it is just the time to leave. This intuition arises because the flood of money lowers the discipline of external finance and allows lower quality projects to get capital (Gompers and Lerner (2000); Nanda (2008)). However, accounting for financing risk makes it clear that investors cannot rush to invest into all projects in a sector that is out of favor. In particular, innovative projects have a low probability of receiving future funding and become NPV negative once financing risk is taken into account.

The fourth implication of our model is that some extremely novel but NPV positive technologies or projects may in fact *need* ‘hot’ financial markets to get through the initial period of discovery or diffusion, because otherwise the financing risk for them is too extreme. This provides a more positive interpretation to bubbles of financial activity and may also explain the historical link between the initial diffusion of many very novel technologies (e.g. canals, railways, telephones, motor cars, internet, clean technology) being associated with heated financial market activity (Perez (2002)).⁴ This implies that regulators should not always be concerned with popping bubbles, and furthermore, that those wishing to stimulate innovation should look for ways to concentrate investment in a sector or time or location in order to help create the coordination among investors that creates or magnifies innovation bubbles.

⁴Related to this, our model also provides a non-behavioral explanation for why asset prices in innovative sectors can fall precipitously after rising steadily for long periods, even when the fundamentals of a firm have not changed (Pastor and Veronesi (2009)). If a sector stays in the ‘good’ equilibrium longer than expected or if the expected probability of remaining in the ‘good’ equilibrium increases, then asset prices will rise and returns will be high, even if the fundamentals remain similar. When the ‘bad’ equilibrium eventually occurs, returns will be far lower than that predicted simply by looking at fundamentals since the low funding equilibrium implies a fall in NPV and hence asset prices, but no change in fundamentals.

Our final implication relates to direct measures of innovation such as patenting that occur in great waves of activity (see Griliches (1990)). There are many explanations for why innovative output might cluster in certain periods of time even though we expect ideas to occur at random. The classical explanations focus on sudden breakthroughs that lead to a cascade of follow-on inventions (e.g. Schumpeter (1939); Kuznets (1940); Kleinknecht (1987); Stein (1997)) or on changes in sales and profitability (or potential profitability) that stimulate investment in R&D and then drive concentrated periods of innovation (e.g. Schmookler (1966)).⁵ While these traditional explanations clearly have merit, combining our model of financing risk with the direct evidence on the link between financial market activity and innovation (Kortum and Lerner (2000), Mollica and Zingales (2007), Samila and Sorenson (2010)) suggests that financial markets may play a much larger and under-studied role in the creation and magnification of innovation waves in the real economy. Financial market “bubbles” of *activity* may create “bubbles” of innovation.

The remainder of the paper is organized as follows. Section I. discusses financing risk and the intuition behind it. Section II. outlines a simple model of investing and illuminates the existence of the two potential investing equilibria. Section III. expands the model to a general equilibrium and shows how accounting for the transition probabilities from one equilibrium to the other affects the funding strategy of investors. Section IV. allows complete, state contingent contracts and commitment among investors in an attempt to overcome financing risk, and shows why in a world of incomplete contracts it is the innovative projects in the economy that are most impacted by financing risk. Section V. summarizes the key implications and extensions of our model and Section VI. concludes.

I. Financing Risk

Participants in the new ventures market, such as entrepreneurs, CFOs and venture capitalists, seem very concerned about what they deem as financing risk - the risk that future funding will not be there even though the fundamental quality of the project has not changed. This concern leads to rules of

⁵See Stoneman (1979) for a discussion of the supply versus demand considerations.

thumb for new ventures such as “take all the money you can” and “take the money when you can get it.”⁶ We take the notion of financing risk seriously and ask both why it arises and what implications it has.

We emphasize that what we call financing risk is not just an unexplained exogenous shock to capital markets, but part of a rational equilibrium. All investors in our model will use an $NPV \geq 0$ investing rule and capital is unlimited. Despite this, we show how the simultaneous act of forecasting the actions of other future investors can actually *create* financing risk. Our model therefore shows why investments in highly exploratory technologies or business models are much more likely in ‘hot’ markets when investors forecast financing risk to be low and why ‘hot’ and ‘cold’ investing waves are a natural consequence of the need for staged financing and an inseparable part of the innovation cycle. We argue that financing risk may play more than just a supporting role in the innovative cycle - it may actually create and magnify the bubbles of innovation in the real economy.

The intuition behind endogenous financing risk is as follows: when a project requires multiple periods of investment that are cumulatively more than any individual investor has (or is willing to allocate), current investors need to rely on future investors to fund the project to cash flow positive and realize the benefits of their investment. If an investor forecasts limited funding for a sector in the next period, this implies that a firm in that sector that is not cash flow positive will have lower bargaining power in the event of an exit. Lower bargaining power comes from two effects. When funding is limited, constrained firms that are running out of cash have a lower outside option if they don’t sell. Furthermore, when funding is limited, incumbents, who are potential future acquirers, face a lower threat from an innovative startup.⁷ Thus, a forecast of limited funding lowers the NPV of a project today relative to a project with guaranteed funding.

To any one investor, financing risk is exogenous; however, in equilibrium it becomes endogenous. Each investor becomes less willing to make an investment because they are worried that others won’t

⁶One might suggest that taking extra money allows entrepreneurs to protect private benefits of control. However, entrepreneurs typically receive very low salaries and get large payoffs only when things go well. Thus, the incentives to stay with a project known to be poor are weak at best. Furthermore, venture capitalist themselves seem very concerned about financing risk and form large syndicates and provide funding to new ventures that often lasts for years.

⁷The bargaining is an important aspect of the model. Appendix A shows that the forecast of limited funding is not part of a rational equilibrium without a bargaining or similar effect.

support the investment in the future. Thus, like in a bank run, if current investors believe that future investors will withdraw financing from such a project, they should also withdraw their investment, even though all investors would be better off in the equilibrium in which everyone invests. This is not an irrational decision and furthermore, does not depend on information asymmetries. There are simply two equilibria – one in which everyone invests in a sector and one in which no one does.

This has some similarities to Shleifer (1986). In Shleifer's theory, inventions arrive randomly but are implemented simultaneously due to commonly shared expectations about the best time to bring out a new invention. This idea relies on a cautious and patient inventor who might wait years to unveil a new idea - a potentially difficult idea to square with entrepreneurial passions. In our paper we also suggest that the expectations of the actions of others matter but it is the cautious financier rather than the inventor who waits for the right time to fund the research or commercialization around new inventions. The rational financier with limited resources funds a project only when he rationally forecasts a high enough probability that a future investor will support the project. We show how this self fulfilling equilibrium is a natural part of innovative sectors.

Our work is also related to the literature on multiple equilibria (see Diamond (1982), Cooper and John (1988), Chatterjee et al. (1993)) and the big push literature (see Murphy et al. (1989), and a review by Matsuyama (1995)). In this work positive externalities between investments create the possibility of both a low and a high equilibrium that depends on the self-fulfilling expectations of investors.⁸ However, in our paper there are no externalities between investments and instead it is the required coordination between investors across time on the same investment that creates the two self-fulfilling equilibria. The key idea in our work is that nothing fundamental (externalities, novel inventions, investor risk aversion, information asymmetries, etc) is required to create financing risk and bubbles of activity and furthermore, the most innovative firms cannot be protected from financing risk. Thus, multiple equilibria are an inevitable part of the financing of new ventures.

Similar to the Keynesian 'beauty contest' in financial markets, each equilibrium is inherently

⁸An interesting paper in this literature, Rin and Thomas (2002), considers the ability of banks to move the economy from the low to high equilibrium. In our paper financial intermediaries actually create the problem and we show how on the most innovative end of the economy they cannot prevent it.

unstable as it depends on the beliefs of others. Even when investors are in the ‘good’ financing equilibrium, they realize that there is a potential to jump to the other equilibrium. In fact, financing risk is precisely the risk that the ‘good equilibrium’ switches after a given investor has funded a project but before returns can be realized. Investors thus estimate a transition probability that the state switches from the ‘good’ to the ‘bad’ financing equilibrium or vice versa. Investors who choose to be countercyclical therefore face an important cost: they need to protect against financing risk by giving the project more money upfront. However, this destroys the real option value of a project. Therefore, the higher the potential for financing risk, the more the project NPV falls.

Financing risk has the greatest impact on the most innovative projects in the economy, or ones that have the most real option value for investors and whose value is partially determined by a potential acquisition. Our model suggests that in times of heated financial market activity, when financing risk goes down, innovative projects with high real option value become NPV positive and get funding. This should shift the mix of projects that get funded towards the more innovative projects in the economy. It is not that frothy financial markets just fund projects that did not deserve funding (although they may do this), in frothy times financial markets fund a different type of project - a project with more real option value - which we think of as a more innovative project. Note that it is still true in our model that on average, ‘better’ projects are funded during a ‘bad’ funding equilibrium. This occurs because only the very best projects can attract financing even in bad times and hence are positive NPV even in the low funding equilibrium. However, our model also shows why fundamentally sound projects, particularly those with high real option value, can go unfunded in some periods but be funded in others.

We will now formally model financing risk and its impact in the simplest model that still contains equilibrium rational financing risk. It is important that we determine the driver of financing risk so that we can understand when and where it is most likely to have an impact.

II. A Model of Investment

The central goal of our model is to delineate the equilibrium impact of financing risk on investment decisions. Financing risk is the risk that future investors will not fund a firm at its next stage *even if the fundamentals of the project have not changed*, leading a viable firm with good fundamentals to go bankrupt. We emphasize that financing risk is part of a rational equilibrium and show why innovative projects are particularly susceptible to financing risk.

A. Setup

We model a single early stage project inside a broader economy. For simplicity, we equate this project with a firm. By early stage we aim to capture the idea that the firm does not have the cash flows to be self sufficient and hence requires outside investment to survive. A second aspect of early stage firms is that it is not yet clear that the project will ‘work’. That is, investment in an early stage firm may produce positive results, negative results or more research may be needed. Furthermore, even when the initial results are positive, more investment may be needed to get over the next hurdle. For example, a new biotech firm may do initial studies to determine how well a compound works in mice. Then, depending on the results, money may be spent to start primate trials, the project may be shut down, or more studies on mice may be needed.

Consider a firm that must get over hurdles in order to reach its potential expected payoff, V , which one can think of as the dividend stream from a cash flow positive firm. These hurdles could represent several rounds of technological uncertainty, or customer adoption risk, or scaling issues, etc. For simplicity we will examine a firm with just a single hurdle as this is enough to demonstrate the relevant issues.⁹ By spending $\$x$ the firm can attempt to get over the hurdle. We will refer to the NPV of a firm before it crosses the hurdle as Π_t , where the t subscript indicates the period.¹⁰

⁹The internet appendix demonstrates the model with two hurdles.

¹⁰Eventually the t subscript will be dropped due to the stationarity inherent in the model, i.e., the NPV of a project that has not yet crossed a hurdle is the same no matter how many times it has failed to cross the hurdle in the past - sunk costs are sunk. More realistically, work that resulted in neither clear success nor failure could still reveal some small amount of information (rather than no information as we have assumed) which could cause the value of the firm to drift up or down but would not fundamentally alter the value of the project. In which case the value of the project would depend on the number of periods of investment. This would increase the difficulty of the model but the key insights would remain.

With a probability γ_f the results are negative and the project fails, where the f subscript represents failure. Failure means that some information is learned about the firm that makes any new investment negative NPV regardless of the financing environment. It might be the case that its technology does not work, its new processes is not cost effective or estimates of the target market are smaller than initially hoped, etc. With probability γ_s the results are positive and there is initial success, where the s represents success. And with probability $1 - \gamma_f - \gamma_s$ there is neither success nor failure. When there is neither success nor failure then spending $\$x$ again gives the firm another attempt to get over the hurdle.

The ability of a firm with neither success nor failure to continue to attempt to get over the hurdle means that, in theory, a firm could continue for a nearly infinite number of periods. However, we would never expect to actually see this in the data. For example a firm with a 33% chance of neither succeeding or failing each period would only have an 11% chance of neither making it over the hurdle or failing after two periods and only a 0.4% chance after 5 periods. Thus, we might occasionally see a firm struggle on, never quite making it and never quite failing for 5 or even 10 periods/years. However, this would be extremely rare and anything much longer would essentially never occur. However, the notion that it may always be *possible* to try for one more period captures the idea that while at the start of a project we can be confident that the project will yield a positive or negative result within about 7 years we can never be sure when the project will end. Thus, conditional on a firm making it 7 years without failing or succeeding investors cannot be *sure* how much more investment will be needed to get an answer one way or the other.

We model the decision of investors willing to invest in early stage firms, which we call venture capitalists (VCs) although this is just short hand for all private investors.^{11,12} In each period, VCs choose whether or not to invest the $\$x$ to support the firm through the next period. Firms that do not receive capital go bankrupt. For simplicity, we assume that firms that go bankrupt are worth nothing. Initially we will consider VCs who can fund the firm for only one period, and later we consider

¹¹This is consistent with the view that VCs are thought to have a number of skills relating to the finding and nurturing new companies (Hsu (2004); Kaplan et al. (2009); Hellmann and Puri (2002); Sorensen (2007)).

¹²Even as early as 1900 Lamoreaux and Sokoloff (2007) show how private individuals acted as VCs and funded innovative new ventures.

larger investments.¹³ However, capital is never scarce in the model. Although each individual VC is capital constrained, we assume that there are enough VCs so that all positive NPV projects get done. Therefore, the entrepreneur captures any expected rent from the firm. These assumptions maximize the chance that the VC will invest as we want to make sure our results do not arise from any exogenous capital constraints.

VCs require an expected rate of return of, r .¹⁴ VCs are rational and use a positive NPV rule for investing and they expect other VCs to also rationally use a positive NPV rule. Since VCs compete away all rents leaving the entrepreneur with any positive NPV, a VC investing in period 1 gets a fraction $x/(\Pi_1 + x)$ of the firm. This fraction is then diluted down in the next period as the next investor gets a fraction of $x/(\Pi_2 + x)$. Of course, the present discounted value of the VCs fraction in each future period times the expected payoff in each future period exactly equals $\$x$.¹⁵ This ensures that the firm will get an investment as long as the firm is not NPV negative. Therefore, as we proceed, in order to determine if the VC will invest we will simply need to determine if the firm is not NPV negative.

To ensure that none of our results are driven by illiquidity we assume that the firm can be sold for its NPV at any time.¹⁶ In addition, however, we allow for strategic acquirers who value the firm more than it is worth to the current investors. We assume that these strategic acquirers are present in any period with a probability, $\alpha < 1$.¹⁷ The probability of arrival is less than one because it only

¹³VCs have limited pools of capital and are often further restricted by the contract with their limited partners to invest no more than a given percent in any one deal.

¹⁴Pastor and Veronesi (2009) provide an interesting explanation of asset price increases and decreases in innovative sectors based on changing discount rates. Our focus is on real activity in innovative sectors and the changing nature of that activity.

¹⁵For example consider a simple firm that requires an investment of \$1 but pays \$4 with a 50% probability or zero. If it pays zero then another \$1 investment will pay \$4 with a 50% probability and with a 50% probability the firm ends. The NPV of the firm, $Y = 0.5 * 4 + 0.5 * (0.5 * 4 - 1) - 1 = 2 + 0.5 - 1 = \1.50 , is captured by the entrepreneur. Therefore, the VC who invests the first \$1 gets $1/(Y + 1) = 1/2.5 = 2/5$. The NPV of the second investment \$1 is $Z = 0.5 * 4 - 1 = \$1$, so the VC who invests the second \$1 gets $1/(Z + 1) = 1/2$ of the firm. If the second investment occurs, then the first VC who originally owned $2/5^{th}$ of the company gets diluted down to $1/5^{th}$. Thus the first VC gets an expected payoff of $0.5 * 4(2/5) + 0.5 * 0.5 * 4(1/5) = 0.80 + .20 = \1 which is exactly what he invested. Therefore, the investment *including expected dilution* is NPV zero for the VCs. This ensures a VC will invest as long as the firm has $NPV \geq 0$, i.e., the fraction $x/(NPV + x)$ is less than or equal to 1.

¹⁶Allowing investors to sell the firm to other investors for the current NPV of the firm at any time changes nothing as the new investors face the same issues as the old. The probability discussed below relates to the probability that a potential acquirer arrives who values the firm *more* than the NPV in the hands of the current investors, which it is reasonable to assume is not always available.

¹⁷The potential for an acquirer to arrive and pay more than the investors NPV seems realistic given the large fraction of VC backed companies that are eventually sold to strategic acquirers.

includes the arrival of potential acquirers with values greater than the current NPV, and because a probability less than one captures the idea that it is costly and time consuming for potential acquirers to find and determine their value for a target (particularly small private targets), so potential acquirers only arrive in a given period with a less than 100% probability.¹⁸

Conditional on finding each other, the potential acquirer and target negotiate the price for the transaction. The negotiation, if consummated, results in the target receiving an amount Ω_{t+1} ($t + 1$ denotes the fact that the negotiation takes place one period after the investment in time t). To determine this amount we must decide on a model for negotiations. While many different choices for the model of negotiations will work for our purposes, the simplest is the Nash bargaining solution. In the Nash bargaining solution the transaction price will depend on the potential acquirer's value and the opportunity cost if each side walks away from the deal.

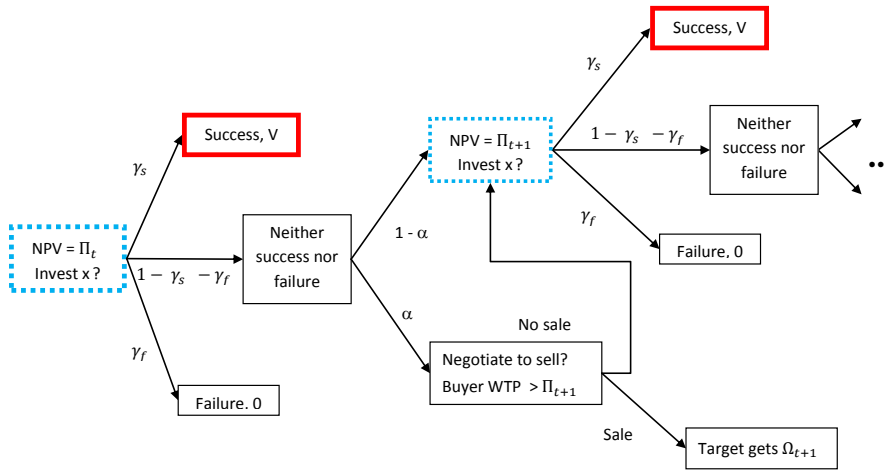
The potential strategic acquirer may value the target more than the target's stand alone value because of positive synergies such as cost savings or better sales channels, or from a greater probability of success, or a lower discount rate or potentially because they simply overvalue the firm. For simplicity we assume the potential acquirer's payoff conditional on success is $\hat{V} > V$ so the project is worth more to the potential acquirer in any period.

If the target walks away from the negotiation, the target is worth the NPV from continuing to look for investors, Π_t . If the potential acquirer does not purchase the target then either the target at some point fails, leaving the value of the potential acquirer unchanged, or the target succeeds (possibly after being purchased by someone else). If the target succeeds but is not purchased by the potential acquirer then it competes with the potential acquirer causing the potential acquirer's value to fall. In this case we assume that the potential acquirer suffers a loss that is proportional to the value of the firm, i.e., λ times the value of the project. This captures the idea that the profits a firm would earn are likely to come at least partially from incumbent competitors.¹⁹

The extensive form of the game is shown in figure 1.

¹⁸The idea that acquirers and targets must search for one another is fully developed in Rhodes-Kropf and Robinson (2008).

¹⁹To the extent this is not true then $\lambda = 0$.



WTP = willingness-to-pay

Figure 1: Extensive Form Representation of the Model

There is one last aspect of the model. Since investors have only enough money to support the firm for one (or limited) periods investors deciding whether or not to invest must determine whether or not they believe other investors will continue to support the firm in the future. Since all investors are rational and all investors know that other investors are rational it would seem that the need to forecast the actions of others would not matter. But we will see that this is not the case and financing risk will have an impact even though fundamentals do not change.

B. Forecasts

In this section, we will show how the NPV of the project and thus each VC's decision to invest depends on his rational forecast about the actions of future VCs. We hypothesize (and later confirm) that there are two symmetric pure strategy perfect public equilibria (Abreu et al. (1990)) - one in which VCs choose to fund a viable project and one in which they do not.²⁰ We will show that each equilibrium is inherently unstable as it depends on the beliefs of others. We assume that an exogenous signal causes investors to believe that the other investors are forecasting future funding or not. Since this common belief becomes self-fulfilling, the equilibrium will depend on this exogenous signal. This

²⁰This is not a perfect Bayesian equilibrium or a sequential equilibrium because no information is hidden so nothing is learned from off equilibrium path actions. An investor who sees a negative NPV investment made does not alter his belief in the probability of future investment because VCs are assumed equally informed and each VC is small enough that their effect of the aggregate probability of funding is negligible. Equilibria are thus invariant to small fluctuations in the behavior of any one player.

is similar to the “sunspot equilibria”, see Chatterjee et al. (1993).

Each investor maximizes their wealth given the signal. We will call the signal $I \in [0, 1]$ where $I = 1$ is the ‘Invest’ signal and $I = 0$ is the ‘No-Invest’ signal. Examples of such signals might relate to a key invention in a sector, future industry growth expectations, a government proposal to improve technology in an area, or alternatively a signal that some other sector is hot and thus money will head there. We think of these signals as relating to an industry or area of investing such as bio-tech, green-tech, or high-tech but they could also occur at a more or less granular level. For example, we would argue that part of the dramatic decline in venture capital investing that began in late 2008 is due to an equilibrium that is economy wide in which investors cannot invest because they do not believe others will be there to support the firms.²¹

In our model, the signal, and thus the state of the world has an exogenous transition probability $(1 - \theta)$ that an industry or sector shifts from the Invest to the No-Invest state and a probability ϕ that an industry transitions back to the Invest state.²² However, initially we will suppress the Markov chain ($\theta = 1$ and $\phi = 0$) to demonstrate the two equilibria in the simpler setting. In either case, for this to be a rational equilibrium all forecasts must be correct in expectation.

I is the signal and thus it also represents the rational forecast of the VCs. When $I = 1$ the forecast is that the next round VC will invest and when $I = 0$ the next round VC is forecasted not to invest.²³ Since all VCs are rational they will invest if the expected NPV of the project is positive. Let $\Pi_t|_{I=1}$ represent the NPV of the project when the forecast is ‘invest’ and let $\Pi_t|_{I=0}$ represent the NPV of the project when the forecast is for ‘no-investment’. And remember that for now each VC only has enough money to support the project for one period.

²¹The global games refinement proposed in Carlsson and van Damme (1993), and used in interesting papers such as Morris and Shin (1998), Goldstein and Pauzner (2005) and Goldstein and Pauzner (2004) might be an interesting extension. The refinement results in a unique equilibrium given the fundamentals rather than a unique equilibrium given a signal. This refinement may require some alteration to be applied in a model of investment across time because future investors know the actions of past investors and so there is no sense in which they are concerned about what action they may take or what signal they got. Our simpler set up is useful here as it allows us to endogenize the response of investors to the potential of multiple equilibria.

²²Economic logic dictates that $\theta > \phi$ since either state is more likely to occur in a subsequent period if investors are currently in that state.

²³We will see that when this forecast is accurate the forecast will also determine whether or not VCs will invest today so $I = 1$ or 0 will also represent the current ‘state’ of the world in equilibrium.

Conditional on a rational forecast of the VCs' actions in the future, the NPV of the project is

$$\Pi_t|_I = \frac{1 - \gamma_f - \gamma_s}{1 + r} [I(1 - \alpha)\Pi_{t+1}|_I + \alpha\Omega_{t+1}|_I] + \frac{\gamma_s}{1 + r}V - x \quad (1)$$

This equation depends on the signal, I , and forecasts the actions of VCs in the next period. Since a rational forecast must be correct in equilibrium, the next period NPV, $\Pi_{t+1}|_{I=1}$ must be greater than or equal to zero when the forecast is that the next period VC will invest, and $\Pi_{t+1}|_{I=0}$ must be less than zero when the forecast is that the next period VC will not invest. Since investors have limited liability, when $\Pi_{t+1}|_{I=0}$ is less than zero it drops out of the equation.

Exploiting the stationarity in the model we can drop the time subscripts and solve for equilibrium NPV.

$$\Pi|_I = \frac{(1 - \gamma_f - \gamma_s)\alpha\Omega|_I + \gamma_sV - (1 + r)x}{(1 + r) - (1 - \gamma_f - \gamma_s)I(1 - \alpha)} \quad (2)$$

The above equation demonstrates the effect on the current NPV of the forecast of the VC. Comparing equation (1) when $I = 1$ to the same equation when $I = 0$ we see that the NPV when the project is expected not to get funding is impacted in two ways. First, the project no longer accrues value from all future investments so $\Pi_{t+1}|_I$ falls out of the NPV equation. And second, the negotiations with a potential acquirer are affected ($\Omega_{t+1}|_{I=1}$ becomes $\Omega_{t+1}|_{I=0}$) because the outside opportunities of both the potential acquirer and the target change.

Thus to understand the impact of the different forecasts we need to understand how negotiations are impacted.

C. Negotiations

Acquisition negotiations under the Nash bargaining solution depend on the potential acquirer's value and the outside opportunities of each party.

As assumed above, the acquirer's payoff conditional on success is \widehat{V} , but, of course the acquirer still has to get the firm over any unmet hurdles. Therefore, at the point the potential acquirer is

negotiating with the target the NPV of the potential acquirer's expected gain is

$$\widehat{\Pi} = \frac{\gamma_s \widehat{V} - (1+r)x}{r + \gamma_f + \gamma_s} \quad (3)$$

if they buy the target, where $\widehat{\Pi}$ represents the NPV to an acquirer with payoff \widehat{V} (the *hat* will signify the acquirer throughout the paper).²⁴

If, however, the firm succeeds but was not purchased by the potential acquirer then the potential acquirer's value is reduced by $\lambda \widehat{V}$. Since this loss only occurs if the project succeeds, the expected loss depends on the company receiving enough financing to make it over the hurdles or to be sold to someone else in the future. Therefore, the potential acquirer expects to lose²⁵

$$\widehat{C} = -\frac{\gamma_s \lambda \widehat{V}}{r + \gamma_f + \gamma_s} \quad (4)$$

if they do not buy the target and the target receives enough funding to get to fruition, where \widehat{C} represents the present value of the expected cost of not buying the target. However, if the firm will not be funded next period, then the potential acquirer expects no costs if he does not acquire the target.

If the deal is not consummated then the project's value to the target shareholders is either $\Pi|_{I=1}$ or zero (since $\Pi|_{I=0} < 0$).²⁶ Thus, $I\Pi|_{I=1}$ represents the target's outside option or reservation value because if $I = 0$ then the target's outside option becomes zero.

Therefore, the set of possible acquisition agreements, $\widehat{\Omega}|_I$ for the acquirer and $\Omega|_I$ for the target is $\Omega = \{(\widehat{\Omega}|_I, \Omega|_I) : I\Pi|_I \leq \Omega|_I \leq \widehat{\Pi} - I\widehat{C} \text{ and } \widehat{\Omega}_t|_I = \widehat{\Pi} - I\widehat{C} - \Omega|_I\}$ where $I \in [0, 1]$. Note that since the potential acquire expects to lose some value if they face the target as a competitor, they are willing (but may not have to) pay more than $\widehat{\Pi}$ to acquire the target to prevent the loss.²⁷ Note

²⁴Note that it is implicitly assumed that the potential acquirer does not face financing risk because he has an asset that generates enough per period to support the project. Furthermore, it is also assumed that the potential acquirer will not sell the project before fruition. Neither assumption is required but they simplify the exposition.

²⁵These are the expected costs when $\theta = 1$. Equation (12) defines the costs more generally for $\theta \leq 1$.

²⁶Remember that if the NPV were positive then the VC would invest and it would not be rational to forecast the no-investment outcome.

²⁷i.e. \widehat{C} is a negative number.

also that the acquirer's expected loss, \widehat{C} is multiplied by I . This is because the potential acquirer only expects losses if the firm succeeds when it is not bought which can happen only if the firm gets funded.

Using the Nash bargaining solution, the equilibrium split is just the solution to²⁸

$$\max_{(\widehat{\Omega}|_I, \Omega|_I) \in \Omega} (\widehat{\Omega}|_I - I\widehat{C})(\Omega|_I - I\Pi|_I) \quad (5)$$

where $I \in [0, 1]$. The well known solution to the bargaining problem is presented in the following Lemma.

Lemma 1 *In equilibrium the resulting merger share for the target is*

$$\Omega|_I = \frac{1}{2}(\widehat{\Pi} - I\widehat{C} + I\Pi|_I) \quad (6)$$

where $\widehat{\Pi}$ is defined by equation (3), \widehat{C} is defined by equation (4), and $\Pi|_I$ is defined by equation (2).

Plugging this solution into equation (2) we find that

$$\Pi|_I = \frac{(1 - \gamma_f - \gamma_s) \left[\frac{\alpha}{2}(\widehat{\Pi} - I\widehat{C}) \right] + \gamma_s V - (1 + r)x}{(1 + r) - (1 - \gamma_f - \gamma_s)I(1 - \alpha/2)} \quad (7)$$

where

$$\widehat{\Pi} - I\widehat{C} = \frac{\gamma_s \widehat{V}(1 + I\lambda) - (1 + r)x}{r + \gamma_f + \gamma_s} \quad (8)$$

This leads directly to our understanding that there are potentially two equilibria²⁹

²⁸More generally one might expect that if the firm could not find future funding its bargaining position might be affected in ways other than just through the reservation values. In the generalized Nash bargaining solution, for example, one might think the bargaining power exponent parameters also shifted to favor the acquirer. This effect would magnify the results presented here.

²⁹When we say equilibria we mean symmetric pure strategy Nash equilibria as mix strategy equilibria have no economic meaning here since we have assumed there are an infinite number of investors in order to insure capital is always available and investors only earn their required return, and since our goal is to show that multiple equilibria are possible the potential for other asymmetric equilibria have no effect on our point.

D. Outcomes

Proposition 1 *There are some firms $\{V, \widehat{V}, x, \gamma_s, \gamma_f, \lambda, \alpha, r\}$ whose funding does not depend on the funding signal, I , (they either always get funding or never do). However, there are some firms for which there are two symmetric pure strategy perfect public equilibria - one in which the VCs invest (and they forecast other VCs will invest) and another in which VCs do not invest (and they forecast other VCs will not invest).*

Proof. See Appendix A.iii. ■

It is only rational for a VC to forecast that a future VC will invest if it is an NPV positive investment, $\Pi|_{I=1} \geq 0$. On the other hand, it is only rational to forecast other VCs will not invest if $\Pi|_{I=0} < 0$. However, for both equilibria to simultaneously hold for a firm at a given stage it must be the case that $\Pi|_{I=1} \geq 0$ and $\Pi|_{I=0} \leq 0$. Thus, for some parameters and stages a firm that is ‘good enough’ will get funded no matter what the signal is and a forecast of no funding is not rational. For other ‘weak’ firms the firm never receives funding as it is always NPV negative. But there are some firms where both equilibria are possible because when future funding is not expected the firm NPV drops from positive to negative.

The two equilibria have a similar flavor to the dual equilibria in the banking literature where depositors can ‘run’ on a bank as in Diamond and Dybvig (1983). Depositors leave money in the bank unless they believe others will withdraw. Once a depositor believes others will withdraw, the only rational response is to attempt to withdraw first. Depositors are better off in the ‘deposit’ equilibrium, but this equilibrium is inherently unstable, as anything that makes depositors think others will withdraw makes everyone withdraw and makes everyone worse off.³⁰ Our argument is that when investors must rely on other investors to fund projects, a similar phenomena can occur. That is, if investors believe that other *future* investors will not invest in the firm, then they themselves will not invest, leading to a self fulfilling equilibria in which everyone is worse off.³¹

³⁰The bank run equilibrium is possible because investors cannot coordinate their actions (if they could they would not run). It is also easy to believe that coordinating investors across time would be quite difficult although we consider some commitment mechanisms below.

³¹One difference between our model and a bank or currency run model is in the time delay between investor actions.

The central mechanism behind our theory is quite different than in a bank run. For the two equilibria to be possible the NPV of the project today must change from positive to negative NPV depending on whether or not the project is expected to get funding tomorrow. This is not as straightforward as it sounds. For example, if the ‘project’ is simply a series of NPV positive coin flips then failing to get funding tomorrow will reduce the total NPV of the project, *however, it will still be positive NPV*. Thus, the investor today should still pay to see the coin flipped, and so should the investor tomorrow thus making the forecast of no future financing incorrect. The only way for a ‘no-invest’ forecast to be correct even when fundamentals have not changed is if the forecast fundamentally alters today’s payoffs. Appendix A demonstrates why a simple model in which investors simply forecast the potential not to receive funding and thus lose the future project payoffs cannot rationally contain financing risk.

The channel we have chosen to use to demonstrate this effect is the sale to a strategic buyer. The negotiation to sell the company today is fundamentally altered by the ‘no-invest’ forecast because the target’s bargaining power is reduced *and* because the potential acquirer is less worried about the firm as a competitive threat when future funding is not available. This idea is supported by a recent working paper, Phillips and Zhdanov (2011), that finds evidence that the potential for future acquisition stimulates innovation. Limited acquisition potential or bargaining power in acquisitions lowers the incentive to innovate by lowering the NPV of innovation attempts.

We think the strategic sale channel is a fundamental and important force for venture investing. It is easy to believe that companies often wait to acquire startup firms, or at least become more aggressive, when they perceive the firm as a potential threat to their business. For example, the large oil companies and other energy firms have been slow to invest in alternative energies, while big pharma is a regular buyer of biotech start ups. We would argue that oil companies currently see little threat to their business, while big pharma’s business model depends crucially on their ability to find new drugs. We would also argue that this leads back to the VC community and leads to a great deal

In a bank or currency run model each player is concerned about the current actions of other players and furthermore, simultaneous actions are strategic complements. In our model investors in the future know the actions of investors in the past but are concerned about investors further into the future.

more capital flowing to biotech firms than alternative energy startups. In fact, the so called ‘valley of death’ that alternative energy firms must walk through to become successful may be partially a self fulfilling prophecy - since none of the firms can truly make it to a scale where they could compete with the big energy firms, the big energy firms feel no competitive threat and thus wont pay much for alternative energy companies. This in turn leads investors to not want to invest in alternative technologies particularly if it will take a lot of money to get it to scale (so more coordination of investors is required) thus the startup firms cant make it to scale and the big energy firms are not worried! But this equilibrium could flip at any time.

While we believe the sales channel is a central part of financing risk, this is not the only channel and we suggest that it is likely that forces work in concert to magnify financing risk. For example, another alternative channel is that employees today who forecast that financing wont be available in the next period leave or work less hard as they look for another job. This could fundamentally change the investment decision today making it rational to believe it won’t get financing. Furthermore, customers who do not think the company could get funding in the future may not want to buy a product today if it requires any future support. In fact, the balance sheet of most startups is a closely guarded secret and many firms hope for the statement from their auditor that they are a ‘going concern’ i.e., they have enough money to last for a year.³²

There are certainly even more channels but to create financing risk they must work the same way. They must change the project from positive to negative NPV by fundamentally altering the project in some way (negotiating power declines, customers hesitate, employees leave, etc.) based on a a forecast that the project will not get funding in the future. We will continue to use the acquisition channel as we go forward but we believe the other channels only magnify the results we present.

We have presented the main driver of financing risk. In the next section, we examine what happens if everyone expects the equilibrium to flip from the No-Invest to the Invest equilibrium at some point in the future (or vice versa). Then we consider the equilibrium investor response to the threat of

³²After the collapse in 2000 companies doing business with startups began asking about and only working with, entrepreneurial firms that were a going concern.

financing risk. We will see how these additional ideas allow us to establish that financing risk is more important for innovative projects.

III. Transitions from State to State

An important facet of this model is that each equilibrium is inherently unstable as it depends on the beliefs of others. Given this fact, VCs will also need to forecast the possibility of a jump to the other equilibrium and a jump back when calculating the NPV of their investment. VCs that forecast a possibility of the No-Invest equilibrium will prepare for it. And if a project does not need to survive an infinite No-Invest period, then more money may help prevent the No-Invest equilibrium from affecting the firm.

We assume the signal follows a Markov chain. The transition matrix for the signal I is

$$\begin{array}{c}
 I = 1 \quad I = 0 \\
 I = 1 \quad \begin{bmatrix} \theta & 1 - \theta \\ 1 - \phi & \phi \end{bmatrix} = S \\
 I = 0
 \end{array} \tag{9}$$

Given this transition matrix the NPV in period t can be written as

$$\Pi_t|_I = \frac{1 - \gamma_f - \gamma_s}{1 + r} [Z(1 - \alpha)\Pi_{t+1}|_{I=1} + Z\alpha\Omega_{t+1}|_{I=1} + Y\alpha\Omega_{t+1}|_{I=0}] + \frac{\gamma_s}{1 + r}V - x \tag{10}$$

where $Z = I\theta + (1 - I)(1 - \phi)$ and $Y = I(1 - \theta) + (1 - I)\phi$.

Note that this equation assumes that if investors enter the ‘No-Invest’ state then no further investment will occur. This must be confirmed in equilibrium. If it is not true then the NPV equation reduces to equation (2), as though the state is always in the invest equilibrium and the signal is meaningless.

Exploiting the stationarity in the model we can solve for equilibrium NPV

$$\begin{aligned} \Pi|_I &= \frac{\left[(1 - \gamma_f - \gamma_s) \frac{\alpha}{2} \widehat{\Pi} + \gamma_s V - (1 + r)x \right] (1 + (Z - \theta)(1 - \gamma_f - \gamma_s) \frac{(1 - \alpha/2)}{1 + r})}{(1 + r) - (1 - \gamma_f - \gamma_s)\theta(1 - \alpha/2)} \\ &\quad - \frac{(1 - \gamma_f - \gamma_s) Z \frac{\alpha}{2} \widehat{C}}{(1 + r) - (1 - \gamma_f - \gamma_s)\theta(1 - \alpha/2)} \end{aligned} \quad (11)$$

where $Z = I\theta + (1 - I)(1 - \phi)$ and \widehat{C} now equals

$$\widehat{C} = - \frac{\gamma_s \lambda \widehat{V}}{(1 + r) - \theta(1 - \gamma_f - \gamma_s)} \quad (12)$$

when $\theta \leq 1$.³³

The potential for the No-Invest state to end, improves the value of an investment in the No-Invest state and reduces financing risk. Thus the following proposition is similar to the first proposition, but must account for the probability that the No-Invest equilibrium might not last forever.

Proposition 2 *There are some firms $\{V, \widehat{V}, x, \gamma_s, \gamma_f, \lambda, \alpha, r\}$ whose funding does not depend on the funding signal, I , (they either always get funding or never do). However, as long as ϕ and θ are large enough, there are some firms for which there are two symmetric pure strategy perfect public equilibria - one in which the VCs invest (and they forecast other VCs will invest) and another in which VCs do not invest (and they forecast other VCs will not invest).*

Proof. See Appendix A.iv. ■

The intuition of the proof is straight forward. If the transition probability, ϕ , is one, then the No-Invest equilibrium, once entered, will last forever, and therefore the conditions for the No-Invest equilibrium to be an equilibrium are the same as in Proposition 1. Thus, for ϕ that is ϵ less than one the conditions for the No-Invest state to be an equilibrium still hold. Likewise, if the transition probability to the No-Invest state, $(1 - \theta)$ is zero, then the Invest equilibrium, once entered, will last forever. Therefore, for a $(1 - \theta)$ that is ϵ greater than zero, the conditions for the Invest state to be

³³ $\widehat{\Pi}$ is unaffected by the signal transition matrix because we have assumed the buyer does not face financing risk. The expected cost is reduced when $\theta < 1$ because the probability of the firm reaching fruition is reduced.

an equilibrium still hold.

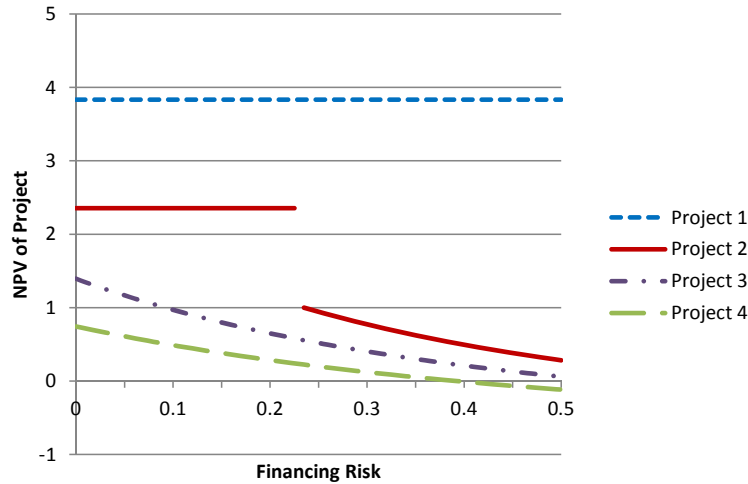


Figure 2: NPV and Financing Risk

In equilibrium different types of projects are impacted by financing risk in different ways. Figure 2 shows how the NPV of different projects changes as financing risk, $(1 - \theta)$, gets larger. Project 1 is unaffected by financing risk because it is so high quality that it can get funded in either equilibrium. Projects 3 and 4 become less valuable as financing risk increases, with project 4 actually becoming NPV negative if financing risk is high enough. Project 2 is interesting because for low levels of financing risk it is unaffected by a state shift but eventually for high enough financing risk its bargaining power falls to the point where it will not get funding in the bad state and so its NPV falls and continues to decrease with greater financing risk.

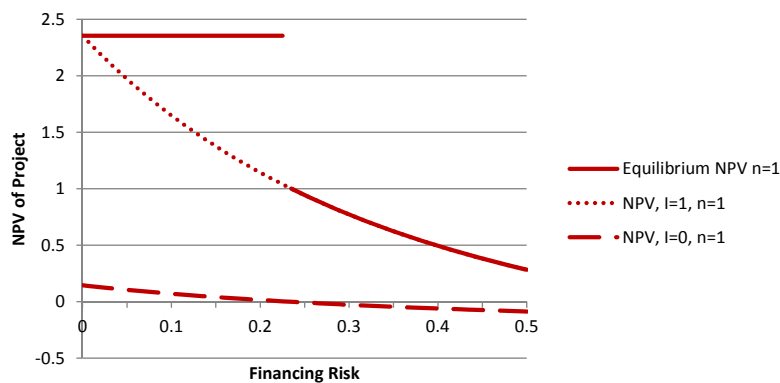


Figure 3: Equilibrium NPV

It is instructive to understand why the equilibrium NPV for project 2 jumps down. Figure 3 shows why. Remember that in equilibrium financing risk is only rational if the forecast that investors will

not invest in the bad state is rational. It is only rational for investors to not invest in the bad state if the project is NPV negative in that state. In Figure 3 the lowest line (large dashes) is the NPV of investing when the forecast is for no future funding ($I = 0$). However, for low enough financing risk the NPV is actually positive even when the investor believes (wrongly) that future funding is unlikely. In fact, since the NPV is positive whether or not the investor believes future investors will invest, the current investor should invest regardless of the signal and so will future investors. Thus, Project 2, is initially unaffected by financing risk. Eventually, however, the forecast that future investors will not invest if $I = 0$ becomes correct and the NPV, even in the good state, is impacted by the possibility of a future jump to the bad state, i.e., it is impacted by financing risk.

Overall, any project with a downward sloping NPV line will get funding in the good state and not get funding in the bad state. Thus, many projects are less valuable with greater financing risk and there are many projects that will only get funding in the good state.

IV. Investor response to financing risk.

Once we recognize that financing risk is a possible equilibrium outcome we must ask how investors respond to the potential risk. If it is the reliance on other investors that leads to the problem, the question arises as to whether a VC with more money or a syndicate of VCs can overcome the No-Invest equilibrium. We consider this first. Then we include the possibility that investors can write complete contracts that cover future financing needs. This demonstrates the optimal response in a perfect world. Then, we consider a more realistic world in which contracts are incomplete. It is this set up that demonstrates the trade-offs faced by investors attempting to fund innovative projects.

A. Can a wealthier investor overcome the ‘No Invest’ equilibrium?

Consider a VC who has enough capital to fund the investment for two periods. One might imagine that this VC faces less financing risk in the first period they invest because they can be sure to invest in the next period. In this case one might think that in the No-Invest equilibrium even with no chance

of jumping back to the invest equilibrium the expected value of the firm in the first period is

$$\Pi_t|_{I=0} = \frac{1 - \gamma_f - \gamma_s}{1 + r} \left[(1 - \alpha)\Pi_{t+1}|_{I=0} + \alpha\Omega_{t+1}|_{I=0}^{n=1} \right] + \frac{\gamma_s}{1 + r}V - x \quad (13)$$

which is the NPV from equation (1) with $I = 0$ but a guaranteed extra round of investment in spite of the No-Invest equilibrium (because of the second x held by the VC). This would suggest both that the investor gets $\Pi_{t+1}|_{I=0}$ instead of zero because of the extra funding (which is negative), and that negotiations are improved. With extra guaranteed funding we write $\Omega_{t+1}|_{I=0}^{n=1}$ with an $n = 1$ superscript to signify that the firm has the outside option of one more period of funding. If this were the case, then the NPV in period t might be greater for a VC with enough funding for two periods since the acquisition offer would be larger if it occurs and the funding is sure to come if no offer arrives.

However, equation (13) demonstrates the fallacy of this argument. In equation (13) the extra x is assumed to be invested even though the forecast is still that no other investors will invest. However, when the VC with $2x$ gets to the second period she will only have one x . At that point, if she invests, she knows that no other investor is forecasted to support the project. Therefore, she gets $\Pi_{t+1}|_{I=0}$ by investing. However, the No-Invest equilibrium is only rational if $\Pi_{t+1}|_{I=0} < 0$. Therefore, the VC will *not* invest their second x .

Of course, using backward induction, the VC will realize that they will not invest the second x and therefore, will reevaluate their decision to invest the first x . Since the second x will only be invested in the Invest equilibrium, the decision to invest the first x is the same for VCs with either x or $2x$.

This same logic applies even if we consider the possibility that the equilibrium might jump back to the Invest equilibrium before the investor runs out of money. To see this, note that the very last x that the VC has will only be spent if the equilibrium has jumped back to the Invest equilibrium. The VC knows this in the period before the last period and also knows that if the industry is still in the No-Invest equilibrium in the period just before this last period $1 - \phi$ is not large enough to make

investing the second to last $\$x$ a good idea (if it were large enough, it would cause VCs to behave as if the state occurs today and hence have caused the equilibrium to flip). Therefore, in the period just before this last period, the VC understands that the last $\$x$ will only be spent if the equilibrium jumps. So the second to last $\$x$ is not invested either. Continuing this backward induction eventually brings us back to the first $\$x$.

This backward induction tells us that (in the absence of commitment) only an investor with an amount of capital $X = \frac{x}{r}$ can break the No-Invest equilibrium. This suggests that only very large firms with cash generating assets greater than X will not face financing risk. Empirically, this implies that the variation in innovation inside large firms who generate internal cash for R&D should be much more stable across time than aggregate innovation produced by startup firms. Since the large firms face more limited financing risk the dual equilibria should not be evident. Furthermore, within the venture community the largest funds, that can potentially support a project to fruition without other future investors, also face less financing risk. This conveys a strong advantage to the largest funds that is most valuable and apparent in bad financing environments. Both of these predictions would be interesting to look for in the data.

In the absence of commitment, until an investor or a syndicate has more than $\$x/r$ (that can generate $\$x$ per period) more money will not break the No-Invest equilibrium. This is because future unrelated investors are already acting rationally so unless the investors can commit to an irrational action in the future they will act no differently from the market. However, we will see the importance of commitment in the next section.

B. The Benefits and Costs of Commitment

Increasing the dollars held by one investor or forming a syndicate does not help the company get over the No-Invest equilibrium because in each period the investment decision is made rationally and so a syndicate or even one investor with more money makes no decision differently than the market (until they have enough money that they never need the market again). After all, sunk costs are sunk. Therefore, if the market is rationally in the No-Invest equilibrium, then any investor would make the

same decision as the market.

However, we show that commitment to invest through a No-Invest equilibrium can change this result. We now allow an investor to commit to invest in the next period regardless of the equilibrium established by other investors. This increases the offer the firm will get in a sale during the No-Invest equilibrium due to the increase in bargaining power provided by the funding cushion.

Initially we will assume that contracts are *complete* and that there are no information asymmetries – so that the investor who has committed to invest in the second period does not invest if the project turns unviable (probability γ_f), but will invest if the project is viable and the equilibrium has jumped to the No-Invest equilibrium. Alternatively, an equivalent contract is a state contingent contract where investors give a project $\$2x$ or more in a period and the project commits to return any unused funds if the project becomes unviable but not if the state transitions to the No-Invest equilibrium.

Commitment trades off the potential increase in sale price with the potential loss from having to invest during the bad equilibrium. If an investor only invests a single $\$x$ then we know from above that the expected project NPV is equation (10).

If instead an investor or syndicate commits to invest in both the first and the second period then we will refer to the project NPV as $\Pi_t|_I^{n=2}$ where the $n = 2$ indicates two periods of commitment (one can think of all the NPVs above as having an implicit $n = 1$, although from here forward we will explicitly indicate the number of periods of commitment). The extra period of commitment ensures that the project will receive an investment in the next period even if the state, I , has changed to $I = 0$. This, in turn, alters the bargaining outcome of any sale so the negotiated outcomes will now be written with an n superscript to indicate the number of periods of future commitment, such as $\Omega_t|_{I=0}^{n=1}$ if there is one more period of money committed to the company.³⁴

Therefore, the expected project NPV when an investor commits to fund the project for two periods

³⁴Where $\Omega_{t+1}|_{I=1}^{n=1}$ is the same as $\Omega_t|_{I=1}$ because the extra period of funding would have occurred even without the commitment, but as we will see $\Omega_{t+1}|_{I=0}$ is altered by the extra commitment.

is

$$\begin{aligned} \Pi_t|_I^{n=2} = & \frac{1 - \gamma_f - \gamma_s}{1 + r} [Z(1 - \alpha)\Pi_{t+1}|_{I=1}^{n=1} + Y(1 - \alpha)\Pi_{t+1}|_{I=0}^{n=1} \\ & + Z\alpha\Omega_{t+1}|_{I=1}^{n=1} + Y\alpha\Omega_{t+1}|_{I=0}^{n=1}] + \frac{\gamma_s}{1 + r}V - x \end{aligned} \quad (14)$$

where $Z = I\theta + (1 - I)(1 - \phi)$ and $Y = I(1 - \theta) + (1 - I)\phi$ and $\Omega_{t+1}|_{I=0}^{n=1}$ is defined below. Appendix A.v. solves for the profits for any level of commitment $n = N$.

This equation differs from equation (10) in two ways. First, the bargaining outcome changes if the state transitions to $I = 0$ because funding is certain.³⁵ Second, the investor has agreed to provide financing in the bad state. Therefore, if the project doesn't sell and the bad state occurs, the investor makes an expected loss since $\Pi_{t+1}|_{I=0}^{n=1} < 0$.

Thus, the question of whether it is better to commit to a second round of investment is a question of whether profits with commitment are bigger than profits without. Subtracting the two profit equations, the question is reduced to whether $\Pi_t|_I^{n=2} - \Pi_t|_I^{n=1} > 0$ or

$$\Pi_t|_I^{n=2} - \Pi_t|_I^{n=1} = \frac{1 - \gamma_f - \gamma_s}{1 + r} Y [(1 - \alpha)\Pi_{t+1}|_{I=0}^{n=1} + \alpha\Omega_{t+1}|_{I=0}^{n=1} - \alpha\Omega_{t+1}|_{I=0}^{n=0}] > 0 ? \quad (15)$$

where $Y = I(1 - \theta) + (1 - I)\phi$. Thus, we can see that the question becomes one of whether or not the expected improvement in negotiating power, $\alpha\Omega_{t+1}|_{I=0}^{n=1} - \alpha\Omega_{t+1}|_{I=0}^{n=0}$, is worth the potential negative expected value investment if the state becomes $I = 0$ (because $\Pi_{t+1}|_{I=0}^{n=1} < 0$).

When there is no commitment, $n = 0$, then the negotiated outcome of a sale is as above, equation (6). Using Lemma 1 we can see that the negotiated outcome when $n = 1$ and $I = 0$ is $\Omega|_{I=0}^{n=1} = \frac{1}{2}(\widehat{\Pi} - \widehat{C}|_{I=0}^{n=1} + \Pi|_{I=0}^{n=1})$, where

$$\widehat{C}|_{I=0}^{n=1} = -(1 - \phi) \frac{\gamma_s \lambda \widehat{V}}{1 + r - \theta(1 - \gamma_s - \gamma_f)} - \phi \frac{\gamma_s \lambda \widehat{V}}{1 + r}. \quad (16)$$

Appendix A.ii. solves for $\widehat{C}|_{I=0}^{n=1}$ and appendix A.v. solves for the expected costs for any level of

³⁵We will see exactly how in a moment.

commitment $n = N$.

In general, the gain from committing more dollars to a project comes from a higher purchase price if a buyer arrives. The loss from committing more comes from the negative NPV from investing if no buyer has arrived nor success occurred but the state has transitioned.

The following proposition shows the impact of this trade-off.

Proposition 3 *If investors or syndicates can commit to invest in future periods and contracts are complete then for any project $\{V, \widehat{V}, x, \gamma_s, \gamma_f, \lambda, \alpha, r\}$ which faces financing risk, committing enough money increases the project NPV and eliminates the No-Invest equilibrium, i.e. the project no longer suffers from financing risk.*

Proof. See Appendix A.v. ■

By committing to one more period of investment the investor essentially ‘puts off’ having to make the negative NPV investment by one period. Simultaneously committing more improves the projects bargaining power for all previous periods of commitment. Thus, eventually by committing to fund the project for enough periods the bargaining improvement outweighs the ever more unlikely negative NPV investment. The negative NPV investment becomes less and less likely because a firm with more money is more likely to succeed or be bought before it runs out of money - and it is only when the project has little money that it *becomes* negative NPV.

Therefore, large investors and syndicates can actually increase the NPV of the projects they fund by giving them more dollars or implicitly or explicitly committing to fund them for longer. Enough committed dollars make the project NPV positive even in the No-Invest state. That is, if enough investors join together, then a large enough investment in the bad state becomes NPV positive. For these projects, the only equilibrium is the Invest equilibrium and commitment eliminates financing risk.

The logic above would seem to suggest that all projects should get significant up front funding. However, as noted above, we have so far assumed that an investor or syndicate that commits to fund a project can withdraw funding if the project becomes unviable, i.e. the commitment only relates to

the state of the world and not to the project quality.

The analogous venture capital contract is a tranching investment, in which the investors have committed to fund a project if certain milestones are reached. These type of contracts provide the investor with a real option, but we believe they are also an attempt to overcome financing risk as they commit the investor to invest if the company has done well even if the world has done poorly. However, they rarely cover more than one future financing, and for many projects (particularly innovative ones), it is very difficult to articulate and delineate a clear milestone. Thus, it is unrealistic to assume that complete state-contingent contracts can be written for all future funding dates at the start of a project. The next section explores the trade-offs under the more realistic scenario of incomplete contracts.

C. Incomplete Contracts and the Lost Real Option

Complete contracts are unrealistic as investors cannot contract on every future funding need at the start of a project. In this section, we assume that contracts are incomplete (a la Grossman and Hart (1986); Hart and Moore (1990)). We assume that it is not possible to either write down or verify all future states in which funding should or should not occur. For example, it might be the case that states of nature are observable by the investors but not verifiable by a court. Specifically we define an incomplete contract as follows.

Definition 1 *In an incomplete contract, investors cannot contract on actions that differ between the No-Invest equilibrium, $I = 0$ and project becoming unviable, (which happens with probability γ_f).*

We still assume that investors can commit or alternatively that it is costly for investors to renege on a commitment. Since one way to ‘commit’ to future funding is to provide extra funding today, the assumption that it is costly to renege on a commitment is the same as assuming that it is costly to shut down a project and return any unspent capital to investors. For simplicity we assume it is never optimal for the investor to fail to fund a contract. Effectively, this is the same as assuming commitment is enforceable.

Commitment was enforceable in the last section as well, but now, without complete contracts, project CEOs and investors are not able to write contracts that release the investor or return capital when bad firm specific information arrives. The CEO continues the project past when investors would want to shut down because his salary, options, equity and any private benefits of control are lost on shut down. Therefore, incomplete contracts create a world in which money given or committed to a firm is spent no matter what information arrives.³⁶

We will see that this drives the main trade-off faced by investors and the firm. If the money given to a firm will be spent, then giving more money to a firm destroys some of the value of the firm's real option to shut down in the event that intermediate information is not positive. On the other hand, more money better-protects the firm from the No-Invest equilibrium. Thus, it is those firms with more valuable real options for which protection from the No-Invest equilibrium is more costly.

In our model the real option value in a firm depends on the probability that a firm loses viability before it is sold. If $\gamma_f = 0$ the firm is always viable and there is no real option value in shutting the firm down (as it never needs to be shut down). However, for higher values of γ_f it becomes valuable to give the firm less up front funding (smaller commitment) and wait to learn that it is still a viable firm in the next period. So holding the NPV of a firm constant, a firm with a greater γ_f has more option value, i.e., it is more valuable to be able to abandon the project.

We can see the effect of incomplete contracts and real options on the profitability of committing extra dollars to a firm. In section IV., when we assumed complete contracts, the profit from committing to invest an extra $\$x$ was equation (14). With complete contracts if the project lost viability the investor would not lose the second committed $\$x$. Now, however, committing $\$2x$ requires the investor to lose the second $\$x$ if the project fails (i.e., it will be spent by the CEO). Thus, the expected

³⁶The main tradeoff is the same if only a fraction of the committed money would be spent after the arrival of bad news.

profit from committing $\$2x$ becomes

$$\begin{aligned} \Pi_t|_I^{n=2} &= \frac{1 - \gamma_f - \gamma_s}{1 + r} [Z(1 - \alpha)\Pi_{t+1}|_{I=1}^{n=1} + Y(1 - \alpha)\Pi_{t+1}|_{I=0}^{n=1} \\ &\quad + Z\alpha\Omega_{t+1}|_{I=1}^{n=1} + Y\alpha\Omega_{t+1}|_{I=0}^{n=1}] + \frac{\gamma_s}{1 + r}V - x(1 + \frac{\gamma_f}{1 + r}) \end{aligned} \quad (17)$$

where $Z = I\theta + (1 - I)(1 - \phi)$ and $Y = I(1 - \theta) + (1 - I)\phi$.

The difference between the profit function with complete contracts equation (14) and without is option value of potentially abandoning the project after one period instead of funding it for two. This equals $\frac{\gamma_f}{1+r}x$. Another way to say this is that it is the additional cost to commitment when the money will be spent even if the firm fails. This leads directly to our next proposition

Proposition 4 *Incomplete contracts reduce the value of committing more money and the reduction in value is larger for more innovative firms (firms with more real option value).*

Proof. See Appendix A.vi. ■

The central insight comes from comparing the profit equations with and without complete contracts. Note that if $\gamma_f = 0$ there is no real option value and no difference between the profit functions.³⁷ However, with incomplete contracts, holding the NPV constant, the larger γ_f becomes the more valuable it becomes to give the project only one period of funding to see if it fails. Thus, commitment becomes more and more costly. Investors who give a firm enough funding to get over the No-Invest equilibria lose the option to give the firm a little funding and wait to see how it performs to give it more. Therefore, it is more costly to overcome the No-Invest equilibrium for innovative projects with high real option value. The less innovative firms can be given a larger amount of up-front financing in order to avoid the No-Invest equilibrium. But the innovative firms cannot be given significant funding up front or the loss of the real options may change it to an NPV negative project. Therefore, more innovative firms should receive less funding up front and are more exposed to financing risk.

³⁷With $\gamma_f = 0$ there is no chance the project will fail so commitment only effects the No-Invest state of the world. Thus, when $\gamma_f = 0$ then just like in the last section, commitment trades off the cost of investing during the No-Invest equilibrium with the potential increase in sale price from doing so. But committing enough money always eliminates the No-Invest equilibrium.

For example, compare the funding of a local fast food store versus something novel that requires experimentation. It would make little sense to fund the building of one wall of the fast food store to wait and see how it looked before building the next wall. In fact, very little is probably learned about the overall viability of the new location during the building of the store. Thus, one would expect the fast food store to raise all the funding to build out the store before starting. However, an innovative project that requires experimentation would want to raise just enough for each experiment and use the results in the subsequent funding decision. Thus, the more a project has a wait-and-learn-more aspect to it the more it will face a trade-off between maximizing real option value and defending against financing risk.

We specifically model the option to abandon the project. However, our ideas and results relate to all type of real options, like an expansion option, where it is optimal to wait to provide more money. Any delay in fully funding all the potential project needs exposes the firm to financing risk and creates a trade-off between protecting against financing risk and maximizing real option value.

Thus, in a world with incomplete contracts, less innovative firms are not hurt as much by the prospect of financing risk. Instead it is the innovative end of the economy that is most impacted by waves of investor interest and disinterest in the sector. This does not require any behavioral explanation, although the effect could certainly be magnified by behavioral considerations. Rational investors know they face financing risk. They rationally try to mitigate that risk by forming syndicates and providing larger sums of money up-front. But for more innovative firms providing more money reduces the option value of the investment. Thus, innovative projects must be left exposed to the whims of the financial market.

V. Implications

A. Innovation Bubbles and Project Mix

A key implication of our model is that we should see bubbles of innovative activity that are endogenously driven by self fulfilling fluctuations in the capital markets. Investors attempt to protect their

firm from this effect by committing more money to a firm up front but the more innovative the firm the more likely early failure may occur and the more costly it is to provide significant money up front.

The likelihood that the world enters the ‘bad’ equilibrium depends on investors beliefs about other investors. We have assumed that some economic signal shifts investor beliefs. The likelihood of this shift, $(1 - \theta)$ in the model, could vary with time and by industry.

Any increase in financing risk³⁸ lowers the NPV of all firms that suffer from financing risk. If this occurs some firms will become NPV negative with their current level of commitment. At which point some of these firms will be unable to get funding while other firms may find it value enhancing to raise more money and thereby reduce the value of some of their real options but defend better against the potential No-Invest equilibrium.

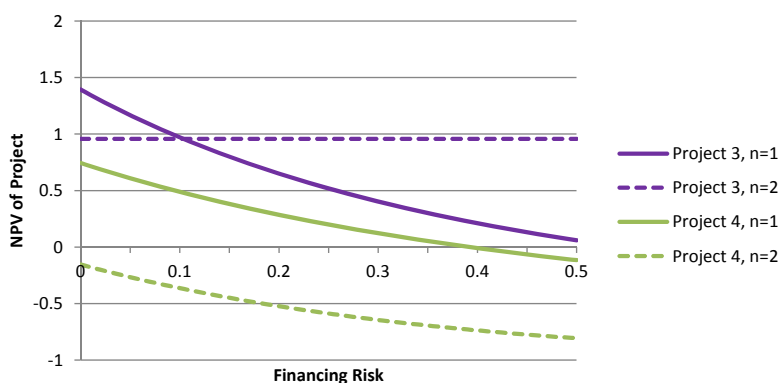


Figure 4: Financing Risk and Increased Funding

Consider projects 3 and 4 in figure 4. The only difference between them is that project 3 has a lower probability of failure. With no financing risk both projects have a higher NPV if they raise only one unit of financing (on the left side of the graph the solid lines are higher than the corresponding dashed lines). However, as financing risk increases eventually Project 3 creates more value by raising two units of capital, but project 4 is always worth less if it raises an extra unit of capital. This is because it is more costly for project 4 to raise an extra $\$x$ because it is more likely to fail, i.e., it is more valuable to take a wait-and-see approach even though this leaves the project exposed to more financing risk. Eventually, if financing risk gets high enough, project 4 will not raise even one $\$x$ because it becomes NPV negative.

³⁸A decrease in θ or an increase in ϕ .

Thus, the most innovative firms, firms where interim failure is high and a wait-and-see approach is important, will be the firms that cannot raise significant up-front financing because too much value is destroyed in the loss of their real options. So in bad times not only should fewer firms be financed but the mix of financed firms should become less innovative.

This theory has stimulated some empirical work examining the type of project that is funded at different points in the cycle. Nanda and Rhodes-Kropf (2011) report that while more failures occur when excess capital enters the venture capital market, it is also the case that the projects funded are also more innovative. Thus, while more ‘bad’ projects are done when capital is abundant (i.e., money chasing deals) it also seems to be the case that when financing risk falls a more innovative project can get funding.

B. Funding Levels and Bankruptcy

In equilibrium some firms will face financing risk and thus be unable to raise money in the bad state. Within this group some will raise extra money in advance. Even if this does not completely protect them from financing risk it may increase their value to do so. The least innovative firms will raise enough money to completely protect them from financing risk.

Interestingly some high quality but innovative projects may have the highest NPV if they take a different approach. In good times they may raise only one $\$x$ and then if the signal jumps they may be able to get out of the bad equilibrium by raising more money, where the amount depends on the extent of expected future financing risk (i.e. the probability the the signal stays in or returns to the No-Invest signal).

Consider the project in figure 5. As financing risk increases from zero the project’s NPV is initially maximized by raising $\$x$ in both good and bad states, then by raising $\$2x$ in both states and finally by raising $\$3x$ in both states. But these equilibria only allowed the firm to raise the same amount in both states. Relaxing this assumption we see that the NPV maximizing solution for a firm that can eliminate the No-Investment equilibrium is to raise little money in the good state and more money in the bad state. The idea is intuitive. As long as the good state persists the firm maximizes value

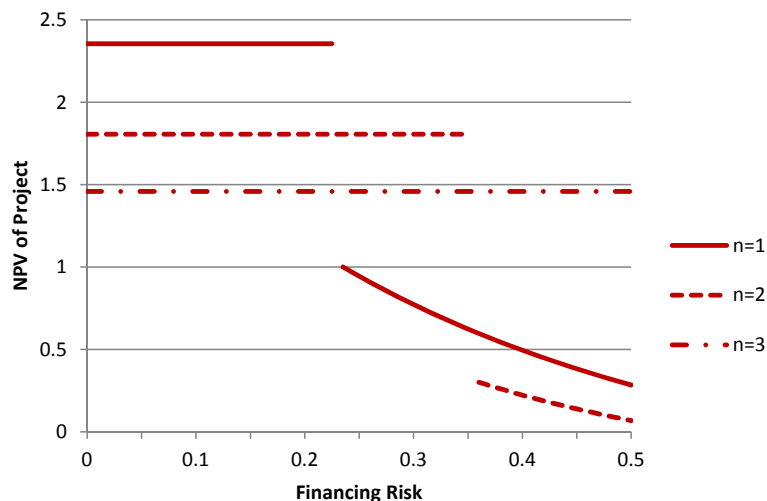


Figure 5: Increased Funding and the Elimination of Financing Risk

by raising only $\$x$ and preserving the option to shut down. However, when the bad state occurs the project may be NPV negative if it raises only $\$x$ (for higher levels of financing risk). However, figure 5 show us that the project would be NPV positive in the bad state *if* they raised $\$2x$ or $\$3x$ (depending on the level of financing risk).³⁹

This results in a surprising prediction that matches anecdotal evidence: when the bad state occurs, most innovative firms can no longer get financing but the few that do get funding actually get *more* funding in bad times. After the crash in 2008 investors anecdotally told high quality firms that they would only invest if they took an extra large amount of money to make sure they would not have to come back to the financial markets for an extended period. Thus, while many firms where finding it impossible to raise money others were being asked to take enough for multiple years.

Note the the above point is a statement about taking more funding relative to burn rate (amount spent per period) since it would also be logical that in bad times firms slowed down their burn rate.

Therefore, any surprising shock to the economy that results in a low financing equilibrium is likely to lead to the destruction of the most innovative firms in the economy first, but the few that survive may take even more money relative to their burn rate than in good times.

³⁹The fact that the equilibrium for higher levels of financing risk with $n = 2$ and $n = 3$ is not decreasing with with financing risk demonstrates that investing $\$2x$ or $\$3x$ must be NPV positive even in the bad state. Thus, the NPV maximizing investment strategy is to raise only $\$x$ in good times and then raise 1, 2, or 3 $\$x$ if the state jumps.

C. Firm Stage and Financing Risk

While a firm at any stage can face financing risk there are still some implications relating to the life cycle of a project.

The above model can be redone with multiple hurdles or stages. For example a project might have to first prove it works (hurdle A) and then prove it can be mass produced in a low cost fashion (hurdle B). Firms at different stages might need different amounts of funding, $\$x^A$ and $\$x^B$, and have different probabilities of success and failure, $\gamma_s^A, \gamma_f^A, \gamma_s^B$ and γ_f^B , where success at stage A means the ability to attempt stage B.⁴⁰

It is likely that for most projects the earlier the stage of the project the more likely it is to fail (i.e. $\gamma_f^A > \gamma_f^B$). Since Proposition 4 demonstrates that the value of committing more capital is lower for firms with larger probabilities of failure, it is the early stage firms, on average, that are less able to defend against financing risk and are more impacted by it. Therefore, early stage firms should get less funding (relative to burn rate) than later stage firms and should have to go back for financing sooner. Furthermore, as financing risk changes through time we should see a larger fluctuation in the birth of early stage firms and funding for early stage firms relative to later stage projects.

We can imagine projects with more than two stages. Projects with stages A, B, C, D, etc, face more financing risk than a project with only one stage (holding other parameters constant) and has more chances to fail. Essentially projects with only one stage can be more certain they have the money they need when they start relative to a project with many hurdles to jump. This suggests that projects that require more money spent over longer time frames face greater financing risk and is one explanation for why clean technology startups seem to face a more difficult funding environment than software startups.

D. Investor Mix

Lower financing risk lowers the amount of capital firms need and should therefore also allow smaller investors with more limited capital to invest. Our model therefore suggests that the mix of investors

⁴⁰This model has also been solved by the authors and all above propositions hold and is available upon request.

should shift towards smaller and more early stage investors in good times. In bad times, a large investor might be able to give a firm more support and break them out of the No-Invest equilibrium but small investors don't have this option and must therefore stop investing.

In making this point we are implicitly assuming some sort of coordination costs that prevent myriad little investors from simply joining to be a large investors. But this seems like a reasonable assumption.

Thus, in the low investing times small angel investors should virtually disappear from the market as the coordination costs to bring together enough of them is too high. Further, the only firms we should see getting funded should be funded by larger investors and actually given a *larger* fraction of total money needed. So while less total money will enter the sector and fewer firms will get funded, the few firms that get funded will be well funded *relative to burn rate*. Note we are not suggesting that firms will get more funding in bad times, only that those that get funded should have more funding relative to their expenses so they can better survive the funding drought.

E. 'Herd Behavior' in Innovative Investments

Conventional wisdom suggests that contrarian strategies might be good because following the crowd leads to a flood of capital in a sector and lowers returns. Our model implies that this is not true in every case. In our model, fully rational investors who only make NPV positive investments are optimally entering the market when prices are high (because the financing risk is low) and everyone else is also in the market. When financing risk is low, giving a firm less money and seeing how it does makes sense. Smaller investors who face greater hurdles to forming large pools of money can find valuable investments in high real option companies that need only a little money *but only during good times*. Making this same investment during the No-Invest equilibrium is NPV negative. Thus for innovative projects with high real option value, it may actually make sense to invest with the crowd.

The corollary to this view also provides a more positive interpretation to the bubbles of activity that are associated with the initial diffusion of very radical new technologies, such as railways, motor cars, internet or clean energy technologies. Our model implies that such technologies may in fact

need ‘hot’ financial markets, where financing risk is extremely low and many investors are in the market, to help with the initial diffusion of such technologies. Related to this, our model provides an understanding as to why asset prices in such times can steadily rise and then precipitously fall, even when the fundamentals of the firms have changed little. Since expectations of a low probability of a No-Invest equilibrium lead to high NPVs and hence high asset prices, a sudden change in the equilibrium will lead many firms to become negative NPV and lead asset prices to fall commensurately.

F. Stimulating Innovation.

To the extent that the bubbles that surround innovation contain mispricing then investment will be misdirected and result in inefficiencies in the economy. This leads many economists and regulators to believe that popping bubbles would improve outcomes and therefore should be attempted. However, our work suggests caution. We show how bubbles of activity can be completely rational and may actually be a necessary part of the creation and commercialization of new ideas. Thus, governments wishing to stimulate innovation may actually need to help *create* the coordination among investors that leads to high activity periods.

Our work suggests potential methods governments could use to help stimulate innovation. While one might expect governments to do a poor job of choosing which technology to back, our work shows that innovation can be stimulated though focusing on exits. Thus, tax incentives or financial support surrounding the purchase of new ventures should help innovation at its earliest stages. This effect should be larger than just a direct effect if it helps create the high funding equilibrium. Furthermore, incentives or money directed at the funding of later stage projects that are already shown to work will reduce financing risk and thus allow more early stage investment.

Our work also shows that encouraging innovation requires the stimulation of a simultaneous decision by many investors to begin investing. Thus, concentrating incentives on a particular sector or geographic location could help investors coordinate. This suggests that diffuse or broad based incentives to innovate are likely to be less effective than incentives concentrated in a location, sector or toward a particular goal.

In general, our work suggests the need to knock investors from a low investing equilibria to a high one. Thus, anything that can help create this simultaneous shift could radically change the level of innovation in a place or point in time.

VI. Conclusion

Startups have been associated with the initial diffusion of several technological revolutions (railway, semiconductors and computers, internet, motor cars, clean technology) and there is increasing evidence of the important role of startup firms in driving aggregate productivity growth in the economy (Foster et al. (2008)). This paper builds on the emerging research examining the role of the capital markets in driving innovation in the real economy (Kortum and Lerner (2000), Mollica and Zingales (2007), Samila and Sorenson (2010)) and provides a mechanism for why innovation in new firms might occur in waves of activity. We depart from the view that financial market activity is purely a response to novel technologies and suggest instead that financial markets drive innovation bubbles.

We argue that a particular feature of innovative startups is that they don't know how much investment will be required to get to the 'finish line'. Intermediate results may be equivocal, or additional investments may be required to get to cash flow positive. Any investor in such startups with limited resources must therefore also rely on other investors to bring innovative firms to fruition.

Because of this, such startups face two risks - fundamental risk (that the project gets an investment but turns out not to be viable) and financing risk (that the project needs more money to proceed but cannot get the financing even if it is fundamentally sound). Financing risk is typically ignored in the literature because all firms with positive fundamental NPV are assumed to get funded. This ignores the fact that investing requires coordination across time between investors with limited resources. Investors must, therefore, forecast the probability that other investors will be there to fund the firm in the future. When incumbent potential acquirers forecast a lower probability of future financing they offer less as they are less concerned about future competition. This in turn reduces the NPV of the project and makes the decision not to invest rational. Thus, financing risk is part of a rational

equilibrium.

The impact of financing risk on a firm can be reduced by giving the firm more funding. However, this comes at a cost. A firm with more funding may spend some or all of the money even in the event of disappointing intermediate information. This cost is greater for highly innovative firms where the real option to shut down the firm is most valuable. The more valuable the real option to shut down a firm, the less funding the firm should receive at a given time. Firms that receive less funding are more affected by a jump to the No-Invest equilibrium. Thus early round investors investing in innovative firms face an important trade-off between lowering financing risk and increasing real option value. The most innovative firms are thus most susceptible to financing risk as they are least able to acquire a ‘war chest’ to survive a down turn.

We show that financing risk is inherent to the funding of new ventures and demonstrate the channel through which it occurs. We argue that the most innovative firms, or those in the early period of a technology adoption, may need ‘hot’ financing environments to help with their initial financing or diffusion. This implies that a fundamentally different, more innovative type of project will be funded in ‘hot’ rather than ‘cold’ markets. By driving investment waves in innovative sectors financing risk may play a key role in creating and magnifying technological revolutions and bubbles of innovation in the economy.

References

- Abreu, Dilip, David Pearce, and Ennio Stachetti. "Toward a theory of discounted repeated games with imperfect monitoring." *Econometrica* 58 (1990), pp. 1041–1064.
- Aghion, Philippe, and Peter Howitt. "A Model of Growth through Creative Destruction." *Econometrica* 60 (1992), pp. 323–351.
- Bergemann, Dirk, and Ulrich Hege. "The Financing of Innovation: Learning and Stopping." *RAND Journal of Economics* 36 (2005), pp. 719–752.
- Bergemann, Dirk, Ulrich Hege, and Liang Peng. "Venture Capital and Sequential Investments." *Working Paper* (2008).
- Carlsson, Hans, and Eric van Damme. "Global Games and Equilibrium Selection." *Econometrica* 61 (1993), pp. 989–1018.
- Chatterjee, Satyajit, Russell Cooper, and B. Ravikumar. "Strategic Complementarity in Business Formation: Aggregate Fluctuations and Sunspot Equilibria." *The Review of Economic Studies* 60 (1993), pp. 795–811.
- Cooper, Russell W., and Andrew John. "Coordinating Coordination Failures in Keynesian Models." *Quarterly Journal of Economics* 103 (1988), pp. 441–464.
- Diamond, Douglas, and Philip Dybvig. "Bank Runs, Deposit Insurance and Liquidity." *Journal of Political Economy* 91 (1983), pp. 401–419.
- Diamond, Peter A. "Aggregate Demand Management in Search Equilibrium." *Journal of Political Economy* 90 (1982), pp. 881–894.
- Foster, Lucia, John Haltiwanger, and Chad Syverson. "Reallocation, Firm Turnover and Efficiency: Selection on Productivity or Profitability." *The American Economic Review* 98 (2008), pp. 394–425.

- Goldstein, Itay, and Ady Pauzner. “Contagion of self-fulfilling financial crises due to diversification of investment portfolios.” *Journal of Economic Theory* 119 (2004), pp. 151–183.
- Goldstein, Itay, and Ady Pauzner. “Demand-Deposit Contracts and the Probability of Bank Runs.” *Journal of Finance* LX (2005).
- Gompers, Paul. “Optimal Investment, Monitoring, and the Staging of Venture Capital.” *Journal of Finance* 50 (1995), pp. 1461–1489.
- Gompers, Paul, Anna Kovner, Josh Lerner, and David Scharfstein. “Venture capital investment cycles: The impact of public markets.” *Journal of Financial Economics* 87 (2008), pp. 1–23.
- Gompers, Paul, and Josh Lerner. “Venture capital distributions: short-run and long-run reactions.” *Journal of Finance* 53 (1998), pp. 2161–2183.
- Gompers, Paul, and Josh Lerner. “Money chasing deals? The impact of fund inflows on private equity valuations.” *Journal of Financial Economics* 55 (2000), pp. 281–325.
- Gompers, Paul, and Josh Lerner. *The Venture Capital Cycle*. Cambridge, MA and London: MIT Press (2004).
- Griliches, Zvi. “Patent Statistics as Economic Indicators: A Survey.” *Journal of Economic Literature* 28 (1990), pp. 1661–1707.
- Grossman, Sanford J., and Oliver D. Hart. “The costs and benefits of ownership: A theory of vertical and lateral integration.” *Journal of Political Economy* 94 (1986), pp. 691–719.
- Hart, Oliver D., and John Moore. “Property rights and the nature of the firm.” *Journal of Political Economy* 98 (1990), pp. 1119–1158.
- Hellmann, Thomas. “A Theory of Strategic Venture Investing.” *Journal of Financial Economics* 64 (2002), pp. 285–314.
- Hellmann, Thomas, and Manju Puri. “The Interaction between Product Market and Financing Strategy: The Role of Venture Capital.” *Review of Financial Studies* 13 (2000), pp. 959–984.

- Hellmann, Thomas, and Manju Puri. "Venture Capital and the Professionalization of Start-Up Firms: Empirical Evidence." *Journal of Finance* 57 (2002), pp. 169–197.
- Hobijn, Bart, and Boyan Jovanovic. "The Information-Technology Revolution and the Stock Market: Evidence." *The American Economic Review* 91 (2001), pp. 1203–1220.
- Hsu, David. "What Do Entrepreneurs Pay for Venture Capital Affiliation?" *Journal of Finance* 59 (2004), pp. 1805–1844.
- Kaplan, Steven, and Antoinette Schoar. "Private Equity Performance: Returns, Persistence and Capital Flows." *Journal of Finance* 60 (2005), pp. 1791–1823.
- Kaplan, Steven, Berk Sensoy, and Per Stromberg. "Should Investors Bet on the Jockey or the Horse? Evidence from the Evolution of Firms from Early Business Plans to Public Companies." *Journal of Finance* 64 (2009).
- King, Robert, and Ross Levine. "Finance and Growth - Schumpeter Might Be Right." *Quarterly Journal of Economics* 108 (1993), pp. 717–737.
- Kleinknecht, Arthur. *Innovation Patterns in Crisis and Prosperity*. McMillan (1987).
- Kortum, Samuel, and Josh Lerner. "Assessing the impact of venture capital on innovation." *Rand Journal of Economics* 31 (2000), pp. 674–92.
- Kuznets, Simon. "Schumpeter's Business Cycles." *American Economic Review* 30 (1940), pp. 257–271.
- Lamoreaux, Naomi R., and Kenneth L. Sokoloff. *Financing Invention in the United States, 1870 to the present*. Cambridge, MA: MIT Press (2007).
- Lerner, Josh, Morten Sorensen, and Per Stromberg. "Private Equity and Long-Run Investment: The Case of Innovation." *Journal of Finance* 66 (2011), pp. 445–477.
- Levine, Ross, and Sara Zervos. "Stock Markets, Banks, and Economic Growth." *American Economic Review* 88 (1998), pp. 537–558.

- Manso, Gustavo. “Motivating Innovation.” *forthcoming Journal of Finance* (2011).
- Matsuyama, Kiminori. “Association Complementarities and Cumulative Processes in Models of Monopolistic Competition.” *Journal of Economic Literature* 33 (1995), pp. 701–729.
- Mollica, Marcos, and Luigi Zingales. “The impact of venture capital on innovation and the creation of new businesses.” *University of Chicago working paper* (2007).
- Morris, Stephen, and Hyun Song Shin. “Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks.” *American Economic Review* 88 (1998), pp. 587–597.
- Murphy, Kevin M., Andrei Shleifer, and Provert W. Vishny. “Industrialization and the big push.” *Journal of Political Economy* 97 (1989), pp. 1003–1026.
- Nanda, Ramana. “Entrepreneurship and the Discipline of External Finance.” *Harvard University Working Paper* (2008).
- Nanda, Ramana, and Matthew Rhodes-Kropf. “Hot Markets and Startup Innovation.” *Harvard University Working Paper* (2011).
- Pastor, Lubos, and Pietro Veronesi. “Technological Revolutions and Stock Prices.” *American Economic Review* 99 (2009), pp. 1451–1483.
- Perez, Carlota. *Technological Revolutions and Financial Capital*. Edward Elgar (2002).
- Phillips, Gordon, and Alexei Zhdanov. “R&D and the Market for Acquisitions.” *University of Maryland and NBER working paper* (2011).
- Rhodes-Kropf, Matthew, and David Robinson. “Mergers and the Boundaries of the Firm.” *Journal of Finance* LXIII (2008), pp. 1169–1211.
- Rin, Marco Da, and Thomas. “Banks as Catalysts for Industrialization.” *Journal of Financial Intermediation* 11 (2002), pp. 366–397.

- Samila, Sampsa, and Olav Sorenson. "Venture capital, entrepreneurship and economic growth." *Review of Economics and Statistics*, forthcoming (2010).
- Scharfstein, David, and Jeremy Stein. "Herd behavior and investment." *American Economic Review* 80 (1990), pp. 465–479.
- Schmookler, Jacob. *Invention and Economic Growth*. Harvard University Press (1966).
- Schumpeter, Joseph. *Business Cycles: A Theoretical, Historical and Statistical Analysis of the Capitalist Process*. McGraw Hill (1939).
- Shleifer, Andrei. "Implementation Cycles." *Journal of Political Economy* 94 (1986), pp. 1163–1190.
- Sorensen, Morton. "How smart is smart money? An empirical two-sided matching model of venture capital." *Journal of Finance* 62 (2007), pp. 2725–62.
- Stein, Jeremy C. "Waves of creative destruction: Firm-specific learning-by-doing and the dynamics of innovation." *The Review of Economic Studies* 64 (1997), pp. 265–288.
- Stoneman, P. "Patenting Activity: A Re-evaluation of the Influence of Demand Pressures." *Journal of Industrial Economics* 27 (1979), pp. 385–401.
- Tian, Xuan, and Tracy Yue Wang. "Tolerance for Failure and Corporate Innovation." *working paper Indiana University* (2011).

A. Appendix

i. Proof that a simpler model wont have financing risk:

Some readers may feel that financing risk could be generated with a simpler model - after all ignoring any effects from sale negotiations, couldn't a project turn NPV negative if early investors forecast no future investors will support it? Yes, however, such a belief would be irrational and in the future investors would rationally support it. Thus, this kind of model would NOT have an endogenous equilibrium that contained financing risk. However, our implications and results do hold in a world with irrational financing risk, but our model helps demonstrate that no irrationality is needed and further we are able to show the channel that drives financing risk. This generates empirical predictions and the ability to test our model.

To demonstrate that a simpler model would not work consider the following model.

A project with NPV Π pays off V with probability p and with a probability $(1-p)$ it needs another $\$x$. Investing $\$x$ buys the investor a fraction q . A rational investor will invest as long as the fraction $q \geq \frac{x}{\Pi+x}$.

If investors forecast a No-Invest state then they will invest as long as $x \leq qpV$. This is because the present value of the project is pV so they invest if their fraction q of the PV is greater than the cost, $\$x$.

If investors forecast an Invest state for one more period followed by the no invest state then the an investor who invests $\$x$ for fraction q expects to receive $qpV + (1-p)(1-q)qpV$, that is, if it pays off in the first attempt (prob p) they get qV and if it pays off on the second attempt (prob $(1-p)p$) they get a fraction q of what doesn't go to the second investor $(1-q)V$. Therefore, the first investors would invest as long as $x \leq qpV + (1-p)(1-q)qpV$.

And if investors forecast infinite Invest states then the first investor would invest as long as $x \leq qpV + (1-p)(1-q)qpV + (1-p)(1-p)pq(1-q)(1-q)V + \dots = qpV \sum_{i=0}^{\infty} (1-p)^i (1-q)^i$ which equals $pV \frac{q}{1-(1-p)(1-q)}$

It might then seem that as long as $(1-p)(1-q)qpV > 0$ that some parameters would result in a world where $x \geq qpV$ but $x \leq qpV + (1-p)(1-q)qpV$ so investors would invest only if they forecasted future investment, and thus there would be financing risk. Or $x \geq qpV$ but $x \leq pV \frac{q}{1-(1-p)(1-q)}$, which would again result in financing risk.

However, this is not correct.

If $x \geq qpV$ the q endogenously increases until the first round investor invests ($x \leq q'pV$) or q or hits 1. That is, firms that are about to fail because the cant get funding increase the fraction they are willing to give to get funded. Thus, the first period investor will only not invest in equilibrium if $x \geq pV$. However, if this is true then $x \geq pV \frac{q}{1-(1-p)(1-q)}$ because $\frac{q}{1-(1-p)(1-q)} < 1 \forall 0 \leq q \leq 1$. Thus, if the first round investor will not invest in the No-Invest equilibrium then he wont invest in the Invest equilibrium, and vice versa.

Therefore, there is no financing risk in this model - investors either always invest or never do. In order to get endogenous rational financing risk the forecast of no financing tomorrow must fundamentally alter the outcome of the project in some way. This is what the model we present in this paper does. The channel we promote is bargaining power shifts but as noted in the paper, others are possible.

ii. Derivation of $\widehat{\Pi}$, \widehat{C} and $\widehat{C}|_{I=0}^{n=1}$:

The expected profit of the potential acquirer, $\widehat{\Pi}$, is the NPV from making the future investments . As long as the project is NPV positive the acquirer will invest until the project either succeeds or fails, i.e.

$$\widehat{\Pi} = \sum_{i=0}^{\infty} \left[\frac{\gamma_s \widehat{V}}{(1+r)^{i+1}} - x \right] (1 - \gamma_s - \gamma_f)^i = \left[\frac{\gamma_s \widehat{V}}{(1+r)} - x \right] \sum_{i=0}^{\infty} \left[\frac{1 - \gamma_s - \gamma_f}{1+r} \right]^i. \quad (\text{A-1})$$

Since this is a geometric series and $(1 - \gamma_s - \gamma_f)/(1 + r) < 1$ it converges to

$$\widehat{\Pi} = \left[\frac{\gamma_s \widehat{V}}{(1+r)} - x \right] \left[\frac{1+r}{r + \gamma_s + \gamma_f} \right] = \frac{\gamma_s \widehat{V} - (1+r)x}{r + \gamma_s + \gamma_f}. \quad (\text{A-2})$$

On the other hand if the potential acquirer does not acquire the target then the potential acquirer faces a cost if the project succeeds. That cost is proportional to the potential benefit, with proportion λ . If the target is not bought by the potential acquirer then the target continues to receive investment as long the invest equilibrium continues.⁴¹ Therefore, the potential acquirer's expected cost is

$$\widehat{C} = - \sum_{i=0}^{\infty} \frac{\gamma_s \lambda \widehat{V}}{(1+r)^{i+1}} (\theta(1 - \gamma_s - \gamma_f))^i = - \frac{\gamma_s \lambda \widehat{V}}{(1+r)} \sum_{i=0}^{\infty} \left[\frac{\theta(1 - \gamma_s - \gamma_f)}{1+r} \right]^i. \quad (\text{A-3})$$

Since this is a geometric series and $\theta(1 - \gamma_s - \gamma_f)/(1 + r) < 1$ it converges to⁴²

$$\widehat{C} = - \frac{\gamma_s \lambda \widehat{V}}{1+r - \theta(1 - \gamma_s - \gamma_f)}. \quad (\text{A-4})$$

The expected costs to the potential acquirer are zero in the no invest equilibrium unless there is committed money behind the project because the project will fail if it is not acquired. If there is another $\$x$ committed then the expected costs are

$$\widehat{C}|_{I=0}^{n=1} = -(1 - \phi) \sum_{i=0}^{\infty} \frac{\gamma_s \lambda \widehat{V}}{(1+r)^{i+1}} (\theta(1 - \gamma_s - \gamma_f))^i - \frac{\gamma_s \lambda \widehat{V}}{1+r} \quad (\text{A-5})$$

or

$$\widehat{C}|_{I=0}^{n=1} = -(1 - \phi) \frac{\gamma_s \lambda \widehat{V}}{1+r - \theta(1 - \gamma_s - \gamma_f)} - \phi \frac{\gamma_s \lambda \widehat{V}}{1+r}. \quad (\text{A-6})$$

iii. Proof of Proposition 1:

If the VCs forecast that other VCs will invest then the project NPV becomes

$$\Pi|_{I=1} = \frac{(1 - \gamma_f - \gamma_s) \left[\frac{\alpha}{2} (\widehat{\Pi} - \widehat{C}) \right] + \gamma_s V - (1+r)x}{(1+r) - (1 - \gamma_f - \gamma_s)(1 - \alpha/2)} \quad (\text{A-7})$$

where $\widehat{\Pi} - \widehat{C}$ is defined in equation (8), with I set equal to 1. It is only rational for a VC to forecast that a future VC will invest even if the project has not improved if $\Pi|_{I=1} \geq 0$.

If, on the other hand, VCs forecast that other VCs will not invest then the project NPV becomes

$$\Pi|_{I=0} = \frac{(1 - \gamma_f - \gamma_s) \frac{\alpha}{2} \widehat{\Pi} + \gamma_s V}{(1+r)} - x \quad (\text{A-8})$$

But it is only rational to forecast other VCs will not invest if $\Pi|_{I=0} < 0$.

It is, of course, possible that for some parameters $\Pi|_{I=1} \geq 0$, while for others $\Pi|_{I=0} < 0$. However, for both equilibria to simultaneously hold for a project it must be that case that

$$(1 - \gamma_f - \gamma_s) \frac{\alpha}{2} \widehat{C} \leq (1 - \gamma_f - \gamma_s) \frac{\alpha}{2} \widehat{\Pi} + \gamma_s V - x(1+r) < 0 \quad (\text{A-9})$$

⁴¹Since a potential acquirer is assumed to value the project more than the stand alone owners, $\widehat{V} > V$, the potential acquire always makes an acquisition in equilibrium. For simplicity we assume there is only one potential acquirer. Drooping this assumption simply increases the cost to failing to acquire as a future acquire would not face financing risk so success is more likely.

⁴²In equation (4) $\theta = 1$ because the Markov process has not yet been introduced at that point.

It is clearly possible for both inequalities to hold. When both hold the project's funding depends on the signal I . It is also possible that the first inequality in equation (A-9) does not hold but the second inequality does hold. In this case the project never receives funding as it is always NPV negative. It is also possible that the second inequality does not hold.⁴³ In which case the project always gets funding as it is always NPV positive so a 'no-invest' forecast is not rational.

Thus, there are only two possible symmetric pure strategy perfect public equilibria - the 'invest' equilibria in which each investor forecasts that the future VCs will invest or the 'no-invest' equilibria in which each investor forecasts that the future VCs will not invest. Q.E.D.

iv. Proof of Proposition 2:

It is only rational for a VC to forecast that a future VC will invest even if the project has not improved if $\Pi|_{I=1} \geq 0$.

$$\Pi|_{I=1} = \frac{(1 - \gamma_f - \gamma_s) \left[\frac{\alpha}{2} (\hat{\Pi} - \theta \hat{C}) \right] + \gamma_s V - (1 + r)x}{(1 + r) - (1 - \gamma_f - \gamma_s)\theta(1 - \alpha/2)} \quad (\text{A-10})$$

If, on the other hand, VCs forecast that other VCs will not invest then the project NPV becomes

$$\Pi|_{I=0} = \frac{(1 - \gamma_f - \gamma_s) \left[\frac{\alpha}{2} (\hat{\Pi} - (1 - \phi)\hat{C}) \right] + \gamma_s V - (1 + r)x}{(1 + r) - (1 - \gamma_f - \gamma_s)(1 - \phi)(1 - \alpha/2)} \quad (\text{A-11})$$

But it is only rational to forecast other VCs will not invest if $\Pi|_{I=0} < 0$.

In the limit as $\theta \rightarrow 1$ and $\phi \rightarrow 1$ equation (A-10) goes to equation (A-7), and equation (A-11) goes to equation (A-8). Thus, the same parameters that produce two equilibria in proposition 1 (when $\theta = 1$ and $\phi = 1$) continue to produce two equilibria as long as θ and ϕ are close enough to 1. Q.E.D.

v. Proof of Proposition 3:

We begin by solving for the profit functions for any level of commitment. This can be done using an iterative expansion process or by simply multiplying each potential outcome by the probability it occurs. For an investment with N periods of committed capital, $n = N$, there could be success, failure, a sale or the commitment could run out and the project would continue only if the state was $I = 1$ at the time the money ran out. So for any $N \geq 1$

$$\begin{aligned} \Pi|_I^{n=N} &= \left(\frac{\gamma_s V}{1 + r} - x \right) \sum_{i=0}^{N-1} \frac{(1 - \gamma_f - \gamma_s)^i (1 - \frac{\alpha}{2})^i}{(1 + r)^i} \\ &+ \Pi|_{I=1}^{n=0} \frac{(1 - \gamma_f - \gamma_s)^N (1 - \frac{\alpha}{2})^N}{(1 + r)^N} [Z \ Y] S^{N-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &+ \sum_{i=1}^N \left(\hat{\Pi} - [Z \ Y] S^{i-1} \begin{bmatrix} \hat{C}|_{I=1}^{N-i} \\ \hat{C}|_{I=0}^{N-i} \end{bmatrix} \right) \frac{\frac{\alpha}{2} (1 - \gamma_f - \gamma_s)^i (1 - \frac{\alpha}{2})^{i-1}}{(1 + r)^i} \end{aligned} \quad (\text{A-12})$$

where $Z = I\theta + (1 - I)(1 - \phi)$ and $Y = I(1 - \theta) + (1 - I)\phi$ and S is the transition matrix (equation (9)). The first term accounts for the possibility that the project might succeed in any period and investors must pay x until it succeeds, is sold, or fails. The second term accounts for the possibility that after N periods the firm has neither progressed nor been bought or failed. If the state is $I = 0$ then the project is worth zero at that point but if $I = 1$ then the project is worth $\Pi|_{I=1}^{n=0}$, which must be multiplied by the probability that the state is $I = 1$ which depends on the initial state. And the final term is the value that comes from a negotiation. This depends on when the negotiation happens because as the commitment runs out \hat{C} falls and the acquirer is willing to pay less for a less well funded project.

⁴³In which case the first inequality in each does hold by definition.

Furthermore, in $\Pi|_I^{n=N}$ for $n = N \geq 1$,

$$\widehat{C}|_I^{n=N} = -\frac{\gamma_s \lambda \widehat{V}}{(1+r)} \sum_{i=0}^{N-1} \frac{(1-\gamma_f - \gamma_s)^i}{(1+r)^i} + \widehat{C}|_{I=1}^{n=0} \frac{(1-\gamma_f - \gamma_s)^N}{(1+r)^N} [Z \ Y] S^{N-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (\text{A-13})$$

where $\widehat{C}|_{I=1}^{n=0}$ is defined in equation (16).

A project faces financing risk even with a commitment of $n = N$ as long as $\Pi|_{I=0}^{n=N} < 0$. Any particular level of commitment may not make the project NPV positive. However, the limit of $\Pi|_{I=0}^{n=N}$ as $N \rightarrow \infty$ is

$$\Pi|_{I=0}^{n=\infty} = \frac{(1-\gamma_f - \gamma_s) \left[\frac{\alpha}{2} (\widehat{\Pi} - \widehat{C}) \right] + \gamma_s V - (1+r)x}{(1+r) - (1-\gamma_f - \gamma_s)(1-\alpha/2)} \quad (\text{A-14})$$

which is just the value of the project without financing risk, and thus clearly NPV positive. Therefore, there is some $N < \infty$ such that $\Pi|_{I=0}^{n=N} > 0$. So investors will make an investment of Nx regardless of the signal I . So the project no longer has financing risk. Q.E.D.

vi. Proof of Proposition 4:

With complete contracts we know from Proposition 3 that the value of the project with commitment is equation (A-12).

With incomplete contracts the profit functions with commitment for $n = N \geq 2$ is

$$\begin{aligned} \Pi|_I^{n=N} &= \left(\frac{\gamma_s V}{1+r} - x \right) \sum_{i=0}^{N-1} \frac{(1-\gamma_f - \gamma_s)^i (1-\frac{\alpha}{2})^i}{(1+r)^i} \\ &+ \Pi|_{I=1}^{n=0} \frac{(1-\gamma_f - \gamma_s)^N (1-\frac{\alpha}{2})^N}{(1+r)^N} [Z \ Y] S^{N-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &+ \sum_{i=1}^N \left(\widehat{\Pi} - [Z \ Y] S^{i-1} \begin{bmatrix} \widehat{C}|_{I=1}^{N-i} \\ \widehat{C}|_{I=0}^{N-i} \end{bmatrix} \right) \frac{\frac{\alpha}{2} (1-\gamma_f - \gamma_s)^i (1-\frac{\alpha}{2})^{i-1}}{(1+r)^i} \\ &- \frac{x\gamma_f}{1+r} \sum_{i=0}^{N-1} \left[\sum_{j=0}^{N-1-i} \frac{1}{(1+r)^j} \frac{(1-\gamma_f - \gamma_s)^i (1-\frac{\alpha}{2})^i}{(1+r)^i} \right] \end{aligned} \quad (\text{A-15})$$

where $Z = I\theta + (1-I)(1-\phi)$ and $Y = I(1-\theta) + (1-I)\phi$ and S is the transition matrix (equation (9)). For $n < 2$ there is no difference between complete and incomplete contracts.

The difference between complete and incomplete contracts is (A-15) - (A-12)

$$-\frac{x\gamma_f}{1+r} \sum_{i=0}^{N-1} \left[\sum_{j=0}^{N-1-i} \frac{1}{(1+r)^j} \frac{(1-\gamma_f - \gamma_s)^i (1-\frac{\alpha}{2})^i}{(1+r)^i} \right] < 0 \quad (\text{A-16})$$

Thus, the value of committing and extra $\$x$ with incomplete contracts is less than with complete contracts. So incomplete contracts reduce the value of committing more money. Furthermore, the derivative of equation (A-15) with respect to γ_f is negative. Therefore, the reduction in value from incomplete contracts is larger for projects with larger γ_f . Thus, the reduction in value is larger for more ‘innovative’ firms - those where the option to shut down is more valuable. Q.E.D.