

# BEAR RAIDS AND SHORT SALE BANS: IS GOVERNMENT INTERVENTION JUSTIFIABLE?

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**ABSTRACT.** When important decision makers (such as firm managers, creditors, customers, suppliers, or other counterparties) base decisions partially on information gleaned from stock prices, short sellers may attempt to profit by manipulating prices down and causing bad decisions that permanently reduce firm value. However, we argue that an informed shareholder with an existing long position has a natural incentive to counter such manipulation by buying a sufficient quantity to ensure that prices are sending the right message. We find that while this holds for decisions with large expected impact on firm value, it does not necessarily hold for those with moderate impact if the informed shareholder's existing position is not large enough. This is because potential trading losses from countering the shorts' manipulation may overwhelm the expected increase in the value of his position from ensuring the right decision. Whether this justifies intervention depends critically on how much value is expected to be lost (which we quantify) compared to any (unmodelled) costs of intervention.

**Keywords:** speculation, short selling, regulation, manipulation, bear raids

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## 1. INTRODUCTION

In September 2008, the SEC temporarily banned short-sales on hundreds of financial institutions. The reason given in its press release dated September 19 was “it appears that unbridled short selling is contributing to the recent, sudden price declines in the securities of financial institutions unrelated to true market valuation.” The release goes on to say that such price declines are capable of causing a “crisis of confidence ... because they (institutions) depend on the confidence of their trading counterparties in the conduct of their core business.”<sup>1</sup> The ban has been heavily criticized by many who argue that short-sellers are being scape-goated by the very firms that took on extraordinary amounts of risk and leverage (some as high as 40 times capital) and were appropriately targeted once their excesses became known. In so doing shorts may have provided invaluable service by preventing stocks from being over-valued and, in the process, making the market more liquid.

In this paper we analytically investigate whether the SEC can be justified in banning short-selling in the face of such perceived bear raids. We argue that for such intervention to be justified, at least two conditions need to be met. First, consistent with prior work, there needs to be reverse causality from prices to firm value in that large price movements are expected to induce permanent changes in fundamental value through their impact on decisions affecting the firm.<sup>2</sup> Such real effects on firm value are most likely when decision makers like firm managers, creditors, suppliers, employees, customers or other counterparties depend on these prices to infer important information about firm prospects. In such situations, managers may react to price drops by, say, reducing (or not increasing) firm capacity, R&D, or business

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<sup>1</sup>For banks in particular, lower prices could result in runs, violation of statutory capital requirements, or in loss of faith by correspondent banks resulting in freezing of overnight lending markets and letter of credit based trade. All these would put further pressure on prices and so on.

<sup>2</sup>An extensive theoretical literature considers the relevance of such feedback effects, including Bernanke and Gertler (1989), Leland (1992), Khanna, Slezak, and Bradley (1994), Kiyotaki and Moore (1997), Dow and Gorton (1997), Subrahmanyam and Titman (2001), and Ozdenoren and Yuan (2008). Several recent papers specifically focus on how feedback effects may give rise to manipulation, including Khanna and Sonti (2004), Attari, Banerjee, and Noe (2006), and Goldstein and Guembel (2008), the last of which focuses on manipulative short selling. See page 5-6 for a full discussion of the relation of these papers to ours.

levels. Other decision makers similarly may be less willing to extend credit or enter into valuable relationships with the firm.<sup>3</sup> The damage is caused not so much by the initial price drop, but through its feedback effect on the real decisions of firms and their business partners, since that not only amplifies the price drop but makes it permanent. Without such a feedback/amplification effect from prices to fundamental value, it is harder to argue that price fluctuations caused by short selling are intrinsically bad.<sup>4</sup>

The second condition we believe is unique to our paper – that there must be a reason why informed, long-term shareholders are not effectively countering the speculators’ actions. For instance, if long-term shareholders suspect that speculators are attempting to destroy firm value by manipulating prices lower, they can counter by buying more shares to keep prices high even if they have to incur trading losses to do so. The larger these shareholders’ existing long positions, the stronger their incentives to ensure that prices are sending the right message. Thus, private markets should be able to handle value-destroying attempts by speculators without help from outside agencies or the government.

The question then is whether there are circumstances under which long-term investors are either unable and/or unwilling to fulfill this stabilizing role? Our analysis provides three key insights with respect to this question. First, we find that long-term shareholders will generally be willing to eliminate the possibility of manipulation when they perceive that short sellers may be able to affect a decision that is expected to have a large impact on firm value. Second, for decisions with more moderate impact, we find that bear raids may be successful at destroying value when informed long-term investors do not own a large enough stake in the firm (relative to the usual constraints on short selling) to justify the trading losses they may incur in fighting the speculators. Third, we highlight the role played by differences in the objectives of the firm’s shareholders and decision makers. In particular, if the relevant decision makers are relatively more risk averse, they are likely to demand a

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<sup>3</sup>See, e.g., Durnev, Morck, and Yeung (2005), Luo (2005), Sunder (2005), Bakke and Whited (2008), Chen, Goldstein, and Jiang (2007), and Edmans, Goldstein, and Jiang (2008) for evidence of managers, creditors, and other counterparties making decisions in part based on stock prices.

<sup>4</sup>Higher volatility at the macro level can increase required returns and thus affect investment decisions. However, the driver is not the fluctuation itself but its real impact. That precisely is the focus of our paper.

greater degree of certainty before undertaking a project or relationship shareholders believe to be good. Thus, long-term investors may be forced to buy a larger number of shares to send a stronger signal that the decision is indeed good, while opening themselves to the potential for larger trading losses. Consequently, they may require larger initial positions so that the expected gain in the value of their positions from a good project being accepted dominates the increase in potential ex-post trading losses.

These findings have a number of important empirical and regulatory implications. In particular, they imply that value-destroying manipulation is most likely when: (1) the decisions at issue may be perceived as having only a moderate impact on firm value; (2) long-term shareholders' stakes are small; (3) short sellers are relatively unconstrained; (4) decision makers are expected to behave in a risk-averse or constrained fashion; and (5) the market in the firm's stock is relatively illiquid (allowing the speculator to have a larger relative impact through its trades). Whether a confluence of these conditions justifies selective regulatory intervention then depends on the magnitude of value loss expected (keeping in mind that only moderately important projects are likely to be disrupted) versus other (unmodelled) costs of restricting short sales, such as reduced market quality (e.g., Diamond and Verrecchia, 1987). See Section 6 for a full discussion of these implications.

To capture these elements in a parsimonious model we study a firm with risk-neutral investors and a risk-averse decision maker. A long-term investor holds a long position in the firm's stock and possesses private information about the firm's prospects which is valuable to the decision maker. He attempts to profit from his information in two ways. One, he attempts to hide his information while trading strategically in a market where a risk neutral and wealth unconstrained market maker sets market clearing prices based on net order flows as in Glosten and Milgrom (1985), and Kyle (1985). Two, he trades in a way that he is able to communicate through prices (based on net order flows) enough of his private information to the decision maker to enable the latter to make a more informed decision and increase firm value. To do so he also needs to condition on the potential strategies of two other types of traders in the market: an uninformed speculator who could be either long or short in the firm's stock, and a noise trader who trades for liquidity reasons. If the speculator

has built up a short position, she has the incentive to counter the long's attempt to reveal good information by shorting more shares and keeping the price low (and vice versa if long). Aware of this possibility, if the long investor does possess good information he needs to overwhelm the short by buying a sufficiently large number of shares so that the market maker establishes a price that convinces the decision maker to accept the value increasing project.

We assume that the informed long investor's noisy private signal can be one of only three types: that the project is likely to be highly profitable (H), only marginally profitable (M), or likely to result in a loss (L). Since the investor is risk neutral, he would like the project to be accepted for the first two signals but rejected for the third. However, since the decision maker is risk averse (or perhaps not very precise at interpreting signals) he accepts only if his inference, based on the market clearing price set by the market maker, indicates the investor's signal has at least a reasonable probability of being H. This agency problem between the investor and the decision maker plays an important role in our paper. For instance, if the investor's signal is indeed H he can ensure that the project is taken by buying enough shares to unambiguously signal to the market maker that he has an H signal. However, then the market maker offers a price that reveals that information perfectly and the investor does not make any trading profits on his private information. He is better off if he can pretend his signal is M with positive probability (i.e., by buying fewer shares). This works if the resulting net order flow is such that the market maker optimally offers to buy at a lower (pooled) price which is still attractive enough for the decision maker to accept the project. Note, this not only endogenously imposes an upper bound on the number of shares the investor will buy after an H signal, but allows him to both profit on the trade and gain from the value increase of all his shares (ones he started with and the ones he bought).

However, this creates a problem for the investor if his signal is M. Since the decision maker will not accept the project if the investor honestly reveals his signal, he needs to pretend to have received an H signal by buying the same number of shares as he would after truly receiving an H. While this gets the project accepted and increases the value of his initial holding, since he knows his true signal is M he must buy the additional shares at a loss – the market maker sets a pooled price reflecting that there is a possibility of both

M and H signals.<sup>5</sup> This generates an endogenous constraint on the investor's willingness to trade sufficiently large quantities. If he does not have a large enough initial position, he will optimally choose not to buy (perhaps even sell), and a valuable project may be lost. However, with a large enough initial position he optimally manipulates the price above its expected value and incurs the trading loss, but makes it up through an increase in the value of his initial position.

Now consider the effect of the uninformed speculator who strategically attempts to hide her initial trades (to build up her position) behind noise trading, leaving the market unaware as to whether she holds a long or short initial position. When she trades against the long-term investor in the final round to take advantage of her hidden position, the effect is as if she is an additional noise trader for that round since she optimally buys if already long and sells if already short. Given the possibility that the speculator has developed a short position and will again sell short as much as possible (given existing constraints) in an attempt to manipulate the price down and induce the decision maker to reject a good project, the long-term investor is forced to buy an even larger number of shares to induce the decision maker to accept the project. Realizing that if he gets an M signal, he will incur bigger trading losses by buying a larger number of shares at the pooled price of M and H, the informed investor needs to start with an even larger position (relative to the case with no speculator) to recover this potential loss.<sup>6</sup>

This implies that the speculator's active presence creates an "efficiency gap" in that significantly larger shareholdings by informed long-term investors are required to ensure the

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<sup>5</sup>This problem does not go away completely if the agency problem is removed, ie if we assume the decision maker will accept the project even if he knows the signal is M. However, the agency problem does makes efficiency harder to achieve, while also making the model more tractable. See section 5 for further details.

<sup>6</sup>Since the speculator's actions effectively create additional noise, there are actually two related effects on the ability/incentive of the long-term investor to counter her actions. First, the additional noise means that the investor has to spread his signal-contingent trades further apart to help the decision maker separate L signals from M and H signals. Second, the additional noise makes the investor less willing to 'pool,' ie play the identical strategy when it has an H or M signal, since the noise makes deviations from pooling more difficult to detect. Both make it less likely the investor will have the proper incentives to ensure good decisions.

efficient outcome – if the actual holdings fall within this gap, manipulation likely reduces firm value (by causing some projects not to be accepted after an M signal) and generates profits for the speculator. Since we assume the existence of a level of natural constraints on short selling, the efficiency gap is measured *relative* to these existing constraints.<sup>7</sup> It is also important to note that even a relatively constrained speculator may be able to profitably manipulate in our setting because of the endogenous constraint on the long-term investor’s willingness to stabilize the price. However, it is important to note that any positive NPV projects that are prevented will always be those with an M signal, ie, those that have a more moderate impact on firm value.

This paper builds on Goldstein and Guembel (2008), who similarly model short sellers manipulating prices downwards to influence managers to take bad decisions and destroy firm value. As in our paper, prices are set by a risk neutral market maker on the basis of net order flows. However, unlike our paper they do not consider how the presence of a long-term investor and the size of his position affects the success of the short-seller’s strategy. Furthermore, their setting requires that the speculator have a reputation for sometimes being informed, while we show that under certain conditions even a speculator that is known to be uninformed can successfully manipulate in the presence of a feedback effect.<sup>8</sup> Our paper also builds on Khanna and Sonti (2004), who look at the problem from the side of the informed long-term investors who (like here) may manipulate prices upwards to influence managers to accept good projects and increase firm value. However, they do not consider the effect of a speculator on the trading strategies and success of the long investors’ strategy. Analyzing the strategies of both short-term speculators and long-term investors in a single unified

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<sup>7</sup>If there were no constraints on short selling at all, there would be no equilibrium in pure strategies since the speculator and an informed long-term investor with very positive information would engage in a “war of attrition,” each trying to unsuccessfully out-do the other.

<sup>8</sup>The fact that our speculator is uninformed about fundamentals may seem to imply that *any* agent could undertake the strategy we derive. However, our speculator does need to have the ability to recognize situations where the possibility of profitable speculation exists. That is, she needs to have some expertise in identifying both firms with the right characteristics and times at which important decisions can be affected by shifts in market prices.

model allows us to further understand how various agents' strategies interact to determine whether private markets are able to control short-sellers' attempts to destroy real value, and when there may be a need for outside intervention. Attari, Banerjee, and Noe (2006) also model value enhancing price manipulation, though around corporate control events. In their setting, institutional investors may strategically "dump" shares to induce relationship investors to buy and subsequently intervene in the firm's management. As in Khanna and Sonti (2004) and the present paper, the institutional holders' actions are motivated both by trading profits and by the desire to protect the value of their existing positions.

Earlier papers that model the feedback/amplification effect (though without directly modeling financial markets) include Bernanke and Gertler (1989), which shows that when an initial positive shock to the economy improves firm profits and retained earnings, it allows firms to invest more, further increasing profits and retained earnings and amplifying the upturn. Similarly, Kiyotaki and Moore (1997) show that a positive shock to land prices translates into increased borrowing capacity, allowing for additional investments. Papers that model the feedback effect of financial market prices on fundamentals but without strategic manipulation include Leland (1992), Khanna, Slezak, and Bradley (1994), Dow and Gorton (1997), Subrahmanyam and Titman (2001), and Ozdenoren and Yuan (2008). In many of these papers low price levels are particularly undesirable as they can result in firm or counterparty decisions that make values even lower.

Consistent with our assumptions, a number of empirical papers document that short-selling is more expensive and more constrained than taking long positions (see, e.g., D'Avolio, 2002, and Geczy, Musto and Reed, 2002). For example, proceeds from short-selling are generally not available to short-sellers, the interest paid on these proceeds is usually below market rates, Regulation T requires short-sellers to deposit additional collateral of 50% of the market value of the shorted shares, and there may be additional lending fees that owners charge short sellers for borrowing their shares (or a scarcity of shares available to borrow). While such constraints have been blamed for artificially high valuations and low subsequent returns for stocks that are expensive to short (as in Jones and Lamont, 2002, and Asquith

and Meulbroek, 1996)<sup>9</sup>, they serve a positive role in our paper in enabling long-term holders to neutralize the shorts' attempt to destroy value.

In our setting, large stockholders play an active stabilizing role to enhance firm value. This is related to Kyle and Vila (1991), Maug (1998), and Kahn and Winton (1998), which model a strategic trader directly taking an action that affects firm value. Other related papers tend to focus either on blockholders who exercise voice by directly intervening in the firms activities (Shleifer and Vishny (1986), Burkart, Gromb, and Panunzi (1997), Faure-Grimaud and Gromb(2004)), or those who use informed trading, also called exit, to improve stock price efficiency and encourage correct actions by managers (Admati and Peiderer (2006), Edmans (2008), Edmans and Manso (2008)).

Finally, our analysis is related to the general literature on stock market manipulation. For example, Bagnoli and Lipman (1996) and Vila (1989) both study manipulation involving direct actions such as a takeover bid. Manipulation based on price pressure or information alone has also been studied widely, such as by Jarrow (1992), Allen and Gale (1992), and Chakraborty and Yilmaz (2004).

The paper proceeds as follows. The base model is described in detail in Section 2. The equilibria of the base model are characterized in Section 3. In Section 4 we extend the model to endogenize the speculator's initial position. In Section 5 we show how the removal of the agency problem affects our results. Comparative statics, empirical implications, and regulatory implications are discussed in Section 6. Section 7 concludes. All proofs can be found in the Appendix.

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<sup>9</sup>These findings are generally at variance with Diamond and Verrecchia (1987) which argues that even with constraints on short-selling, prices should be unbiased since markets would adjust for the truncated bad news. Duffie, Garleanu, and Pedersen (2002) suggests that over-pricing may simply reflect the presence of lending fees. Given the possibility of earning these fees, the initial price of a security can be rationally pushed above its fundamental value. In the context of our model, it is possible that the fees simply reflect the possible damage that shorts are likely to do by preventing good projects. If so, then prices should again be an unbiased expectation of fundamental value as in Diamond and Verrecchia (1987).

## 2. THE BASE MODEL

We consider an economy with a single firm that has many indivisible equity shares outstanding. A risk-averse decision maker ( $D$ ) must make a project accept/reject decision that impacts the firm (as noted in the introduction, the decision maker could be a manager of the firm or any creditor or counterparty with a decision that will affect the firm). Firm value is \$1 per share if the project is not accepted. If the project is accepted and the future state of nature,  $\Theta \in \{B, G\}$ , is good ( $\Theta = G$ ), the project adds  $d \in (0, 1)$  per share to firm value. If it is accepted and the state of nature is bad ( $\Theta = B$ ), it reduces firm value by  $d - \epsilon$  per share,  $\epsilon \in (0, d)$ . The ex ante probability of  $\Theta = G$  is  $\frac{1}{2}$ .

There are (potentially) two strategic traders: a risk-neutral, informed long-term shareholder,  $I$ , and a risk-neutral, uninformed speculator,  $S$ .  $I$  enters the game with an exogenous long position in the stock equal to  $i > 0$ , which is consistent with the empirical regulatory that firms often have one or more long-term blockholders. For the base model, we assume that  $S$  either never arrives (the “no speculator” case), or arrives with an exogenous position that is long or short  $s$  shares with equal probability (the “active speculator” case). The arrival or non-arrival of the speculator is common knowledge, but the magnitude and direction of her position if she arrives are her private information. The initial position of the speculator is endogenized in an extended version of the model in Section 4, where we verify that the speculator’s overall strategy can be profitable. Note, however, that the assumption of an exogenous position is also useful because it captures scenarios where a speculator holds an *effective* position in a firm without owning that firm’s stock. For example, the speculator may hold the stock of a competitor or potential acquirer (generally an effective short interest) or a supplier or customer (generally an effective long interest).<sup>10</sup>

In the base model there is a single trading round. Before trading takes place,  $I$  receives a signal,  $\theta \in \{L, M, H\}$ , about the future state of nature, where H is high, M is marginal/medium, and L is low. The probability structure of the signals is such that  $Pr[\theta = H|\Theta = G] = Pr[\theta = L|\Theta = B] = \lambda$ ,  $Pr[\theta = H|\Theta = B] = Pr[\theta = L|\Theta = G] = \frac{1}{2} - \lambda$ ,

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<sup>10</sup>Kalay and Pant (2008) discuss many such possible “correlated” long and short positions that occur without directly trading the firm’s shares.

and  $Pr[\theta = M] = \frac{1}{2}$ .<sup>11</sup> We assume  $\lambda \in (\frac{1}{4}, \frac{1}{2})$  so that the H and L signals are informative in the correct direction (i.e., an H signal implies a higher probability of the good state). No other agents receive any signals regarding the state, and the only way for  $I$  to communicate its information to  $D$  is through his trading decisions.<sup>12</sup> While our assumption that  $I$  receives a private signal but  $D$  does not is standard in the feedback literature, all that we require is that  $I$  have access to some information that is incremental to  $D$ 's.

During the trading round, with probability  $\frac{1}{2}$  a noise trader places a market order to buy one share and with probability  $\frac{1}{2}$  it places an order to sell one share.  $I$  can place a market order for any integer quantity. The speculator can place a market order to buy or sell one share, or can choose not to trade. This limitation on the speculator's trades captures real life constraints on short selling as discussed in the introduction.<sup>13</sup> It should also be noted that limiting the speculator's trades endogenously determines how much  $I$  will choose to trade in equilibrium, implying that the interpretation of our results should always be *relative*. So if over some range of  $I$ 's initial position  $i$  the speculator is shown to reduce efficiency, we can say only that this is the case for such  $i$  measured *relative* to the existing constraint on short sales. Also, for analytical simplicity we do not formally restrict  $I$  from any level of short selling, however, it turns out that it is never necessary for  $I$  to sell more than 2 shares in any of the equilibria we derive. Thus, he never needs to sell more than 1 share short as long as his initial position is at least 1 share, and there is no effective asymmetry in the two players' ability to short sell.

After the players place their orders, a risk-neutral market maker sees only the net order flow,  $Q$ , and then prices the trades at the risk neutral expected value given his inference

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<sup>11</sup>Effectively, then,  $I$  is uninformed with probability  $\frac{1}{2}$ , which is similar to the information structure in Goldstein and Guembel (2008).

<sup>12</sup>There may be other ways to communicate the information to the manager that also induces him to act accordingly. However, if they do not permit  $I$  to make trading profits on his information,  $I$  is likely to prefer this particular route. Also, if  $I$  takes trading losses in an attempt to get the manager to take a particular decision it is more convincing.

<sup>13</sup>Note that it is easy to show that  $S$ 's willingness to *buy* additional shares would be endogenously limited by the extent of its long position. However, the short sale constraint is a binding one – a short speculator would often wish to sell additional shares if it could.

about  $I$ 's signal from observing  $Q$ . We represent this price as  $p(Q)$ . We assume that the market maker holds sufficient inventory to satisfy any relevant pattern of trades.

Next, the decision maker decides whether to implement the project (based on any information he can learn from the stock price, given that he knows the game being played). The risk neutral  $I$  would like  $D$  to accept the project as long as the signal is H or M, and not if the signal is L. However, we assume that  $D$  is risk averse to the extent that to implement the project it must believe there is at least a  $\frac{1}{3}$  probability that  $I$  received an H signal, and no chance it received an L signal.<sup>14</sup> This is equivalent to the assumption that  $D$  implements the project if its posterior after inferring  $I$ 's signal from the stock price is that the probability of the good state is at least  $\frac{1}{3} + \frac{2}{3}\lambda$  (this equivalence is derived below). The decision maker's risk aversion could arise either from its having an undiversified position in the firm (if it is a manager) or from credit constraints or career concerns due to negative consequences for entering an ex post bad business arrangement (if it is a creditor or counterparty to the firm).

After the project decision is made, the state of nature and resulting firm value are realized. Finally, all stock positions are closed out – long positions are paid the firm value per share, and short positions must be closed out by paying the firm value per share.

### 3. EQUILIBRIUM

We consider only pure strategy sequential equilibria. We also require that the posterior beliefs of  $D$  and the market maker about the probability of the good state be weakly increasing in net order flow for all possible order flows (including those that do not occur in equilibrium).<sup>15</sup> Where multiple equilibria may exist, we focus on the most efficient ones, where we measure efficiency from the perspective of the long term risk-neutral shareholder.

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<sup>14</sup>This captures a specific level of risk aversion (not modelled). Lowering or increasing the required probability that the signal is H would capture changes in the level of risk aversion of the decision maker – all that is required for our qualitative results is a minimum level of risk aversion. We discuss the case of a risk neutral decision maker in Section 5.

<sup>15</sup>This assumption rules out “perverse” equilibria, such as those in which  $I$  buys more shares after observing an  $L$  signal than after observing an  $H$  signal, which would mean that prices would actually *decrease* in net order flow over some range. Such equilibria are possible because of the discrete nature of our modeling assumptions. Our results could also be achieved by assuming a small carrying cost for  $I$  when it acquires

Given that the M signal is received with the same probability in the good and bad states, it is uninformative. Thus,  $I$ 's posterior after receiving the M signal is the same as the prior: a  $\frac{1}{2}$  probability of the good state. Since  $\epsilon > 0$ , the project is positive NPV given this posterior. The posterior after observing the H signal, using Bayes' rule, is

$$Pr[\Theta = G|\theta = H] = \frac{Pr[\theta = H|\Theta = G]}{Pr[\theta = H|\Theta = G] + Pr[\theta = H|\Theta = B]} = \frac{\lambda}{\lambda + (1 - \lambda - \frac{1}{2})} = 2\lambda > \frac{1}{2}.$$

Similarly, the posterior after observing an L signal is

$$Pr[\Theta = G|\theta = L] = \frac{Pr[\theta = L|\Theta = G]}{Pr[\theta = L|\Theta = G] + Pr[\theta = L|\Theta = B]} = \frac{1 - \lambda - \frac{1}{2}}{(1 - \lambda - \frac{1}{2}) + \lambda} = 1 - 2\lambda < \frac{1}{2}.$$

We also assume

$$V_L \equiv 1 + (1 - 2\lambda)d - 2\lambda(d - \epsilon) < 1,$$

that is, the project is negative NPV given an L signal. Thus, from  $I$ 's point of view a fully efficient equilibrium is one in which the project is always taken when the signal is H or M, but never taken when the signal is L.

It is useful to define other values analogously as follows:

$$V_M \equiv 1 + \frac{1}{2}d - \frac{1}{2}(d - \epsilon) = 1 + \frac{1}{2}\epsilon$$

is expected firm value per share if the project is accepted when  $\theta = M$ ; and

$$V_H \equiv 1 + 2\lambda d - (1 - 2\lambda)(d - \epsilon)$$

is expected firm value per share if the project is accepted when  $\theta = H$ . Finally, note that if an agent's posterior is that there is a  $\frac{1}{3}$  chance the signal is H and a  $\frac{2}{3}$  chance the signal is M then the posterior probability of the good state is

$$\frac{1}{3}(2\lambda) + \frac{2}{3}\left(\frac{1}{2}\right) = \frac{1}{3} + \frac{2}{3}\lambda.$$

This corresponds to the threshold posterior that we have assumed is necessary for  $D$  to accept the project. We thus define

$$V_P \equiv 1 + \left(\frac{1}{3} + \frac{2}{3}\lambda\right)d - \left(\frac{2}{3} - \frac{2}{3}\lambda\right)(d - \epsilon)$$

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additional shares and then eliminating equilibria that fail to satisfy the Intuitive Criterion of Cho and Kreps (1987), but that approach makes the analysis much more complicated with no additional insights.

as the expected firm value per share if the project is taken given that exact posterior.

We next define notation for the posterior beliefs of the market maker and  $D$  for different possible net order flows. Note that in equilibrium it does not matter whether  $D$  observes the net order flow or just the price (the one is as good as the other in terms of inferring signal probabilities), so we assume he can observe the net order flow. As such, the two agents' posterior beliefs are always equivalent. Let  $Q = q_S + q_I + q_N$  denote the net order flow realization given trading quantities of  $q_S$  for the speculator (if it arrives),  $q_I$  for the informed shareholder, and  $q_N$  for the noise trader. Throughout, for each possible equilibrium we also use the notation  $q_I^H$ ,  $q_I^M$ , and  $q_I^L$  for  $I$ 's *equilibrium* signal-contingent trades. We denote the posterior belief given  $Q$  as  $\mu(Q)$ .

Now consider the necessary characteristics of a fully efficient equilibrium, in which the project is always accepted after an H or M signal and always rejected after an L. The following requirements are immediate.

*Lemma 1.* Any fully efficient pure strategy equilibrium must be such that  $I$  plays the same strategy after an M or H signal ( $q_I^M = q_I^H$ ), and plays a sufficiently different strategy after an L signal so that no possible resulting order flows could arise from his equilibrium trade after an M or H signal.

If these conditions are violated, then there must be equilibrium order flows where the efficient decision is not taken. If  $I$  plays different pure strategies after H and M signals ( $q_I^M \neq q_I^H$ ), then some order flows occur only following an M, and  $D$  must consequently conclude upon seeing those order flows that the signal could not be H and not take the project. Similarly, if  $I$  plays a strategy after an L signal where the resulting order flow could also follow an M or H, when that order flow occurs either the project is sometimes taken after an L (if the relative probability of an H signal is high enough) or is sometimes not taken after an M or H.

We next determine when such fully efficient equilibria exist for both the no speculator and the active speculator cases. In the active speculator case the speculator's basic incentive is to trade in the direction of his initial position, ie, to buy if long and sell if short. This is because the main tension in the model is whether a medium project will be accepted, and

buying tends to reinforce  $I$ 's basic strategy of buying to signal that a medium project is good, while selling tends to work against that strategy. Thus, subject to its optimality, we assume the speculator buys a share if initially long and sells a share if initially short (we show in the proof of Proposition 1 in the Appendix that this behavior is, in fact, incentive compatible and individually rational in all of the equilibria we consider).<sup>16</sup>

For the no speculator case, consider the class of potential equilibria where  $I$  trades a quantity  $q_I^M = q_I^H = q_I^+$  after an M or H signal, and trades  $q_I^L \leq q_I^+ - 3$  after an L signal. The trades need to differ by at least 3 so that an L signal trade with a buy from the noise trader cannot be confused with an M or H signal trade with a sell from the noise trader (consistent with Lemma 1). The possible equilibrium order flows after an M or H signal are  $Q \in \{q_I^+ - 1, q_I^+ + 1\}$ , which occur with equal probability from  $I$ 's perspective (given the noise trader's probabilistic actions). After an L signal they are  $Q \in \{q_I^+ - 4, q_I^+ - 2\}$  if  $q_I^L = q_I^+ - 3$  (or less if  $q_I^L < q_I^+ - 3$ ), again with equal probability. This class of equilibria represents all possible pure strategy fully efficient equilibria in the no speculator case given our condition that beliefs must be monotonic in order flow (so  $q_I^L \leq q_I^M \leq q_I^H$  is required).

Any order flow that can follow an L signal, i.e.,  $Q \in \{q_I^+ - 4, q_I^+ - 2\}$  assuming  $q_I^L = q_I^+ - 3$ , must result in the belief that the signal was L. Using Bayes' rule, any order flow that can follow an M or H, ie,  $Q \in \{q_I^+ - 1, q_I^+ + 1\}$ , must result in the belief that there is a  $\frac{1}{3}$  probability that the signal was H, and  $\frac{2}{3}$  probability it was M. To see this, note that  $I$  is assumed to receive an M signal with probability  $\frac{1}{2}$ , and he receives an H signal with unconditional probability  $\frac{1}{4}$  (the state is good with probability  $\frac{1}{2}$  leading to an H signal with probability  $\lambda$ , and the state is bad with probability  $\frac{1}{2}$  leading to an H signal with probability  $\frac{1}{2} - \lambda$ , so the unconditional probability of an H signal equals  $\frac{1}{2}\lambda + \frac{1}{2}(\frac{1}{2} - \lambda) = \frac{1}{4}$ ). Thus, when  $D$  believes that  $I$  is pooling after M and H signals and it observes a corresponding order flow, it must conclude that the signal was H with probability  $\frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{2}} = \frac{1}{3}$ .

This posterior makes  $D$  indifferent over taking the project, and we assume it accepts when indifferent in equilibrium. Since the market maker believes that  $D$  will take the project, and

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<sup>16</sup>Note that it is possible for other strategies to be incentive compatible for the speculator in fully efficient equilibria, including perhaps not trading after arriving long, which yields qualitatively similar results. We choose to focus on the *most* active strategy for the speculator as this gives the clearest results.

has the same posterior belief about the probability of the good state, it sets the price at  $p(Q) = V_P$  for such order flows  $Q$  (from above, this value corresponds to the stated belief). However, since after an M signal  $I$  knows that the expected per share value is actually  $V_M$  if the project is taken, he expects to take a trading loss equal to  $q_I^+(V_M - V_P)$ . After an H signal, he analogously expects a trading gain equal to  $q_I^+(V_H - V_M)$ .

These trading gains and losses lead to two main effects that make it difficult to sustain fully efficient equilibria. First, following an M signal  $I$  may not be willing to suffer these trading losses, so may deviate downward to a smaller quantity. This will cause a loss with respect to the value of his initial position,  $i$ , since a desirable project is less likely to be taken, but will save (at least some of) the potential trading loss. This type of deviation will be more likely the smaller is his initial position  $i$ , i.e., the less  $I$  cares about the ultimate firm value. On the other hand,  $I$  may want to deviate upward to a larger quantity in order to maximize his trading gains following an H signal. The size of his initial position is less of an issue here since the project is taken at higher order flows (so he need not worry about an inefficient project decision if he deviates upward).

To determine when these deviations are profitable, we must specify out of equilibrium beliefs for  $D$  and the market maker. For all  $Q \leq q_I^+ - 2$  we assume a belief that the signal is L (this is pinned down by our belief monotonicity assumption when  $q_I^L = q_I^+ - 3$ ). The belief at  $Q = q_I^+$  is pinned down by our monotonicity assumption at a  $\frac{1}{3}$  probability of an H signal and  $\frac{2}{3}$  probability of an M signal. Finally, for all  $Q \geq q_I^+ + 2$  we assume a belief that the signal is H. Note that these assumed beliefs support each potential equilibrium in this class *as strongly as possible* since they make downward deviations after M signals and upward deviations after H signals as unattractive as possible (these beliefs minimize the probability of project acceptance following an M for downward deviations, and minimize potential trading profits following an H for upward deviations). Also note that these beliefs imply that for  $Q \geq q_I^+ + 2$  the project will be accepted and trades will be priced at  $V_H$ , for  $Q = q_I^+$  the project will be accepted and trades will be priced at  $V_P$ , and for  $Q \leq q_I^+ - 2$  the project will be rejected and trades will be priced at 1.

The structure of this potential equilibrium is illustrated in Figure 1 below, which shows the prescribed trading quantities for the different signals, the possible resulting net order flows at the ends of the arrows (with probabilities along the arrows determined by the noise trader's buying or selling 1 share with equal probability), and the resulting equilibrium (and assumed out of equilibrium) prices as described above. Equilibrium order flows and prices are in bold italics, and out of equilibrium quantities are in normal text.

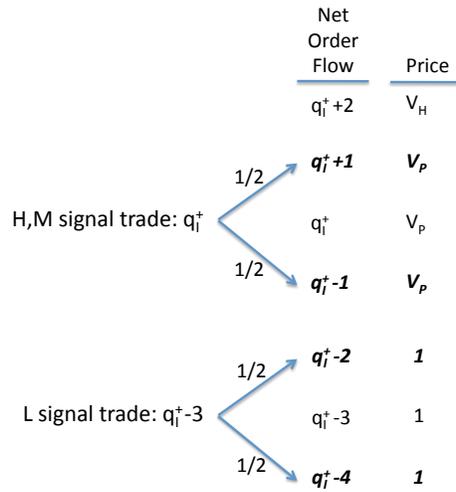


FIGURE 1. Proposed Equilibrium Orders for  $I$ , Resulting Net Order Flows, and Prices in the No Speculator Case

As noted above, the most relevant potential deviations are upward deviations after an H signal and downward deviations after an M signal. First consider an upward deviation by  $I$  after an H signal in which he places an order of  $q_I^+ + 2$  shares instead of  $q_I^+$  shares (see the proof of Proposition 1 in the Appendix for a proof that the deviations we consider in the text are the most relevant deviations). The resulting potential order flows are  $Q \in \{q_I^+ + 1, q_I^+ + 3\}$ . This potential deviation is illustrated in Figure 2 below, which lays out the possible order flows and prices after a deviation trade of  $q_I^+ + 2$ .

With this deviation,  $I$  expects the project to be accepted. With probability  $\frac{1}{2}$  the noise trader will sell and the price will be  $V_P$ , and with probability  $\frac{1}{2}$  the noise trader will buy and the price will be  $V_H$ . His expected trading profit is now  $\frac{1}{2}(q_I^+ + 2)(V_H - V_P)$ . Since he expects the project to be taken with certainty (and thus the value of his existing position to

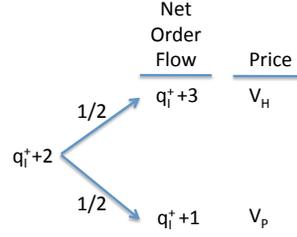


FIGURE 2. Possible Net Order Flows and Prices in the No Speculator Case Following a Deviation Trade of  $q_I^+ + 2$  Instead of the Expected  $q_I^+$  After an H Signal

be maximized with either trade), a comparison of this with his expected equilibrium trading profit suffices to test the optimality of the deviation. In particular, the deviation is profitable if  $\frac{1}{2}(q_I^+ + 2)(V_H - V_P) > q_I^+(V_H - V_M)$ , or, rearranging, if  $q_I^+ < 2$ . Thus, in the no speculator case, the existence of a fully efficient pure strategy equilibrium requires that  $I$  buy at least 2 shares following an M or H signal, so that he will not be able to increase his profits by deviating to a higher quantity after an H signal.

Now consider a downward deviation by  $I$  after an M signal to a trade of  $q_I^+ - 2$ . Note from Figure 1 that the possible resulting order flows are  $Q \in \{q_I^+ - 3, q_I^+ - 1\}$ , with corresponding prices 1 and  $V_P$ , respectively. With this deviation, the project is accepted only with probability  $\frac{1}{2}$  and the price is  $V_P$  (as in the equilibrium), and is rejected with probability  $\frac{1}{2}$  and the price is 1.  $I$ 's trading loss is therefore  $\frac{1}{2}(q_I^+ - 2)(V_M - V_P)$ . However, with the change in the project acceptance decision, the value of  $I$ 's initial position must also be considered to determine whether this deviation is profitable. Without the deviation the project is always accepted, so the value of the initial position is  $iV_M$ . When it is accepted with probability  $\frac{1}{2}$ , its value is  $i(\frac{1}{2}V_M + \frac{1}{2})$ . Thus, the deviation is profitable if  $i(\frac{1}{2}V_M + \frac{1}{2}) + \frac{1}{2}(q_I^+ - 2)(V_M - V_P) > iV_M + q_I^+(V_M - V_P)$ , or, rearranging, if  $i < \frac{(q_I^+ + 2)(V_P - V_M)}{V_M - 1}$ . Note that the right-hand side is increasing in  $q_I^+$ , and since  $q_I^+ \geq +2$  is required (from above) for this equilibrium to exist, the range of possible existence is  $i \geq \frac{4(V_P - V_M)}{V_M - 1}$ .

Next consider the active speculator case. To understand the role that the speculator plays, note that her trades essentially act as additional noise for the other players (though we later show that these actions are both incentive compatible and individually rational).

This has several effects. First of all, it means that  $I$  will have to spread his signal-contingent trades wider in order to fully separate his L signal trade from his M and H signal trade. In other words,  $I$  will either have to sell more after an L, buy more after an M or H, or both. Second, the additional noise impacts both of the deviation effects noted above in a way that makes fully efficient equilibria even harder to support. In particular, it makes both downward deviations after an M signal and upward deviations after an H signal *more* profitable because the deviations become harder to detect.

To see this, consider the class of equilibria where  $I$  trades  $q_I = q_I^+$  after an M or H signal (as above), but now trades  $q_I = q_I^L \leq q_I^+ - 5$  after an L signal to ensure full separation. The difference required for separation increases from three to five shares because the speculator's one-share trades expand the range of "noise" from two to four shares. The possible equilibrium order flows  $Q$  after an M or H signal are now  $Q \in \{q_I^+ - 2, q_I^+, q_I^+ + 2\}$ , with respective probabilities  $\frac{1}{4}$ ,  $\frac{1}{2}$ , and  $\frac{1}{4}$  reflecting the probabilistic actions of the noise trader and speculator. After an L signal they are  $Q \in \{q_I^+ - 7, q_I^+ - 5, q_I^+ - 3\}$  if  $q_I^L = q_I^+ - 5$  (or *less* if  $q_I^L < q_I^+ - 5$ ). Thus, the L signal is again fully separated as required by Lemma 1. As with the no speculator case above, this class of equilibria is the only possible class of pure strategy fully efficient equilibria in the active speculator case. We specify out of equilibrium beliefs analogously to the no speculator case: the signal is believed to be L for all  $Q \leq q_I^+ - 3$  and H for all  $Q \geq q_I^+ + 3$ , while for  $Q \in \{q_I^+ - 1, q_I^+ + 1\}$  the monotone beliefs assumption requires the belief that the signal is H with probability  $\frac{1}{3}$  and M with probability  $\frac{2}{3}$ . As above, these beliefs support the equilibrium as strongly as possible. The proposed equilibrium is illustrated in Figure 3 below. Again, equilibrium quantities are in bold italics, and out of equilibrium quantities are in normal text.

Now consider an upward deviation by  $I$  to a trade of  $q_I^+ + 2$  following an H signal. In the no speculator case, this deviation entailed giving up trading profits  $\frac{1}{2}$  of the time, but now, because of the extra noise created by the speculator,  $I$  must forego trading profits only  $\frac{1}{4}$  of the time for the same increase in trading quantity. See Figure 4 below for an illustration.

This means that expected trading profits are now  $\frac{3}{4}(q_I^+ + 2)(V_H - V_P)$ . Comparing this with the equilibrium trading profits of  $q_I^+(V_H - V_P)$  (again ignoring the value of  $I$ 's initial

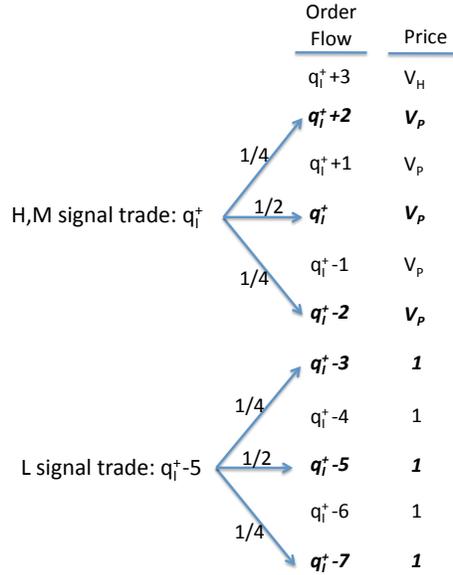


FIGURE 3. Proposed Equilibrium Orders for  $I$ , Resulting Net Order Flows, and Prices in the Active Speculator Case

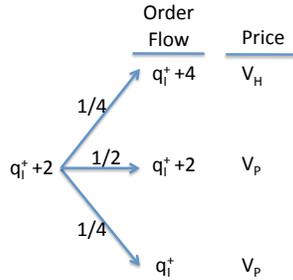


FIGURE 4. Possible Net Order Flows and Prices in the Active Speculator Case Following a Deviation Trade of  $q_I^+ + 2$  Instead of the Expected  $q_I^+$  After an H Signal

position since the project is always accepted either way), this deviation is profitable if  $\frac{3}{4}(q_I^+ + 2)(V_H - V_P) > q_I^+(V_H - V_P)$ , or, rearranging, if  $q_I^+ < 6$ . Thus, whereas with no speculator  $I$  had to buy at least 2 shares after an M or H signal to support the equilibrium, with an active speculator that requirement triples to 6 shares because of the greater ability to hide the deviation.

Finally, consider a downward deviation by  $I$  to  $q_I^+ - 2$  following an M signal. With no speculator, this deviation entailed losing the project  $\frac{1}{2}$  of the time, but now it is lost only

$\frac{1}{4}$  of the time. The possible order flows are  $Q \in \{q_I^+ - 4, q_I^+ - 2, q_I^+\}$ , and with reference to Figure 3 the project is rejected only at the lowest of the three. The expected payoff to this deviation is therefore  $i(\frac{3}{4}V_M + \frac{1}{4}) + \frac{3}{4}(q_I^+ - 2)(V_M - V_P)$ . Comparing this to the equilibrium payoff, the deviation is profitable if  $i(\frac{3}{4}V_M + \frac{1}{4}) + \frac{3}{4}(q_I^+ - 2)(V_M - V_P) > iV_M + q_I^+(V_M - V_P)$ , or, rearranging,  $i < \frac{(B+6)(V_P - V_M)}{V_M - 1}$ . As above, the right-hand side is increasing in  $q_I^+$ , and since  $q_I^+ \geq +6$  is required for this equilibrium to exist, the range of possible existence is  $i \geq \frac{12(V_P - V_M)}{V_M - 1}$ , or three times that with no speculator.

Proving the existence of these fully efficient equilibria over the prescribed ranges now requires showing that  $I$  will not deviate either up or down after an L signal, and will not deviate *downward* after an H signal or *upward* after an M signal. With respect to the L signal, note that  $I$  makes no trading profit or loss in equilibrium (the price is always correctly 1), and the value of his position  $i$  is maximized by non-acceptance since the project is negative NPV. The only possibility for a trading profit would be if  $I$  could *sell* some quantity for “too high” of a price and get a bad project accepted some of the time (buying and having the project accepted is never optimal because he would be buying at too high of a price, leading to a trading loss). But this is impossible given the results above since a sale of 1 share would result in a maximum order flow of  $Q = 0$  in the no speculator case and  $Q = +1$  in the active speculator case, which is never sufficient for project acceptance given  $q_I^+ \geq +2$  with no speculator and  $q_I^+ \geq +6$  with an active speculator. With respect to the H signal, note that deviating down will reduce the value of  $I$ 's initial position (the project is sometimes rejected) while also reducing his trading profits (there is no profit when the project is rejected). Similarly, after an M signal an upward deviation would leave the value of the initial position unchanged, but increase the trading loss since the price would sometimes be  $V_H$ .

The following result summarizes the analysis above.

**Proposition 1.** *In the no speculator case a fully efficient pure strategy equilibrium exists for all  $i > i^{*N} = \frac{4(V_P - V_M)}{V_M - 1}$ , and no such equilibria exist otherwise. In the active speculator case a fully efficient pure strategy equilibrium exists for all  $i > i^{*S} = \frac{12(V_P - V_M)}{V_M - 1}$ , and no such equilibria exist otherwise. Finally, we clearly have  $i^{*S} > i^{*N}$ .*

This result implies that there is a large range of the informed shareholder's initial position  $i$  for which no fully efficient equilibria exist with an active speculator, but do exist without (which is the "efficiency gap" discussed in the introduction).<sup>17</sup> Thus, the actions of the speculator are likely to reduce efficiency in this region. This occurs because the presence of the speculator means that  $I$  must buy more in equilibrium in order to ensure that the project is accepted, which does not create problems with an H signal but does with an M. With an M signal,  $I$  does not buy more shares because he would have to incur a larger trading loss and for this range of existing positions the trading loss dominates the gain from ensuring the right decision.

However, in the range where full efficiency exists, whether or not there is an active speculator has no impact. It is straightforward to show that, while an active trading strategy in a fully efficient equilibrium can be incentive compatible for the speculator, it will not generate any profits. It will be incentive compatible because, from the speculator's perspective, all of her trades are at zero profit or zero loss. The only other possible source of profit is an increase in the value of her initial position, but in a fully efficient equilibrium her presence does not affect overall firm value, so no profit occurs. However, if an active trading strategy turns out to be profitable for the speculator in a range where her presence means that no fully efficient equilibria exist, her actions may affect the efficiency of the firm's project accept/reject decision. To determine whether the speculator will ever profit from actively trading, we need to determine what type of equilibria may exist over ranges without fully efficient equilibria, and whether any such equilibria support profitable speculation.

We continue the strategy of first determining the most efficient possible equilibrium, and then checking for its existence. We assume for the active speculator case that the speculator optimally buys if initially long and sells if initially short. The conditions under which this is optimal for the derived equilibria are given in Proposition 3 below (and proven in the Appendix). One possible equilibrium (which exists everywhere) is a fully separating equilibrium where  $I$  trades a large positive amount after an H signal, and trades any amount after

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<sup>17</sup>Note that it is straightforward to show that the entire range of the efficiency gap,  $i \in [i^{*N}, i^{*S}]$ , always involves positions  $i$  in excess of one share (i.e.,  $i^{*N} > 1$  always holds), which is the technical minimum allowed since we have assumed indivisible shares.

an M or L that separates them from the trade following an H.<sup>18</sup> However, there are some intermediate equilibria that are both more efficient and allow for potential profits for the speculator. In particular, we characterize the existence of pure strategy “partial pooling” equilibria in which the project is always accepted after an H, never accepted after an L, and sometimes accepted and sometimes rejected after an M.

For now assume again that  $I$  is always willing to separate himself after an L signal to ensure that the project is not taken (which is verified in the proof of Proposition 2 in the Appendix). In order to have an equilibrium where the project is sometimes accepted after an M,  $I$ 's trades after M and H signals must be separated by a multiple of 2, ie after an M he must trade either 2 or 4 shares fewer than after an H (the monotone beliefs assumption requires that  $I$  trade fewer shares after an M than after an H). If they were not separated by multiples of 2, then the resulting order flows could never overlap (the strategy would always result in odd total order flows  $Q$  after one signal, and even total order flows after the other). Furthermore, the maximum combined trade of the noise trader and  $S$  is 2 shares in either direction, so if the M and H trades are more than 6 shares apart, they can never overlap. Analyzing the possible equilibria provides the following result.

*Lemma 2.* The most efficient possible pure strategy partial pooling equilibrium has: in the active speculator case, the project always taken after an H signal, the project taken with probability  $\frac{1}{4}$  after an M signal, and the project never taken after an L signal; in the no speculator case, the project always taken after an H signal, the project taken with probability  $\frac{1}{2}$  after an M signal, and the project never taken after an L signal.

When the speculator is active,  $I$  trades quantities that are either 2 shares or 4 shares apart after M and H signals. Each trade has three possible outcomes depending on whether  $S$  and the noise trader trade in the same direction up or down, or cancel each other out. It is more efficient if their trades are 4 shares apart. To see this, consider a potential equilibrium in

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<sup>18</sup>This results in the project being accepted only after an H, so  $I$  is indifferent over his equilibrium trading quantity after an M or L. To see this, note that all trades after an M or L are always correctly priced at  $p(Q) = 1$  as long as the resulting order flows could not arise from  $I$ 's equilibrium trade following an H, so there is no trading loss or gain.

which  $I$  is expected to buy 5 shares after an H signal, which results in possible net order flows of  $Q \in \{+3, +5, +7\}$  with corresponding probabilities  $\{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\}$ . If he buys 3 shares after an M signal, the net order flow possibilities are  $Q \in \{+1, +3, +5\}$ , again with corresponding probabilities  $\{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\}$ . Thus, at an order flow of  $Q = +3$ , the project will not be taken (there is too high of a probability this order flow resulted from an M signal rather than an H). Such a potential equilibrium is illustrated below in Figure 5 (note that the L signal has been left out for simplicity). The project is taken at order flows of  $Q = +4$  and higher, so overall it is

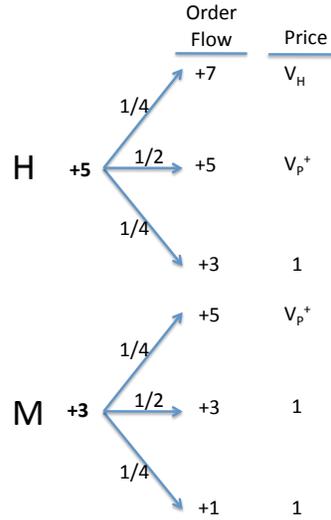


FIGURE 5. Proposed Equilibrium Orders for  $I$ , Resulting Net Order Flows, and Prices for a Partial Pooling Equilibrium with a 2-Share Trading Difference

taken with probability  $\frac{1}{4}$  after an M signal, but also it is *not* taken  $\frac{1}{4}$  of the time after an H. On the other hand, if  $I$  trades +1 after an M, the possible order flows are  $Q \in \{-1, +1, +3\}$ , again with corresponding probabilities  $\{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\}$ . This possibility is illustrated in Figure 6 below. Here, the project will be taken at all order flows  $Q \geq +3$ , implying, again, a  $\frac{1}{4}$  chance of acceptance after an M signal, but now ensuring that the project is *always* taken after an H, which is clearly more efficient. Note that in this example, since  $q_I^H - 2 = q_I^M + 2 = +3$  the equilibrium prices when the project is taken will be  $p(+3) = V_P$ ,  $p(+5) = V_H$ , and  $p(+7) = V_H$ .

In the no speculator case, since the noise trader's trade is either -1 or +1,  $I$ 's trades following M and H signals cannot be more than 2 shares apart, else there would be no

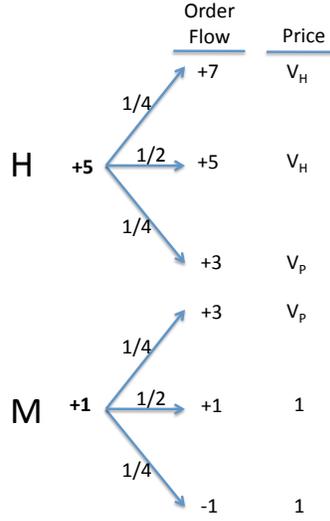


FIGURE 6. Proposed Equilibrium Orders for  $I$ , Resulting Net Order Flows, and Prices for a Partial Pooling Equilibrium with a 4-Share Trading Difference

potential for overlap. For example, if he buys 2 shares after an H, the resulting order flow can be  $Q \in \{+1, +3\}$  with probabilities  $\{\frac{1}{2}, \frac{1}{2}\}$ . Then if he does not trade after an M, the resulting order flow is  $Q \in \{-1, +1\}$ , again with equal probabilities. Thus, the project is accepted for all order flows  $Q \geq +1$ . Here, the project taken with probability  $\frac{1}{2}$  after an M and always taken after an H.

Analyzing such equilibria to determine when they exist provides the following result.

**Proposition 2.** *A pure strategy equilibrium with partial pooling between H and M signals, with the project always taken following an H signal and taken with probability  $\frac{1}{4}$  following an M signal, exists for all  $i \in \left[ \frac{2(V_P - V_M)}{V_M - 1}, i^*S \right]$  in the active speculator case. A pure strategy partial pooling equilibrium with the project always taken following an H signal and taken with probability  $\frac{1}{2}$  following an M signal, exists for all  $i \leq i^{*N}$  in the no speculator case.*

It turns out that these “partial pooling” equilibria give the speculator an opportunity to profit from manipulation in our model. Since  $S$  is uninformed, in order for her to profit she must be able to affect the firm’s real value. In the equilibrium described in the above result, the speculator knows that if the signal is M and she sells then the project will not be taken. However, if she buys, the project might be taken (if the noise trader also buys). This wedge

gives her the incentive to trade in the direction of her original position, and means that her trade can be profitable in that she increases the value of her initial position by causing a profitable project to be passed up (if she is short and sells) or helping a profitable project to be taken (if she is long and buys).

However, the trade itself will take place at a loss. Consider what can happen when the speculator sells. On the one hand the signal may be M or L, so the project will not be taken and the price will be  $p(Q) = 1$ , which is the correct price. On the other hand, the signal may be H. If the noise trader buys, this offsets  $S$ 's sell trade, the order flow is  $Q = q_I^H$ , and the price is, correctly,  $p(q_I^H) = V_H$ . If the noise trader sells, this reinforces  $S$ 's trade and we have  $Q = q_I^H - 2$  and  $p(q_I^H - 2) = V_P$  which is *too low* for  $S$  since she knows that such a net order flow can only result after an H signal. Thus,  $S$  is being forced to sell at too low of a price and faces a trading loss. A similar argument shows that if  $S$  buys she will either do so at zero trading profit, or a trading loss due to buying at too high of a price,  $V_P$ , when in fact  $S$  knows the order flow must have come from an M signal.

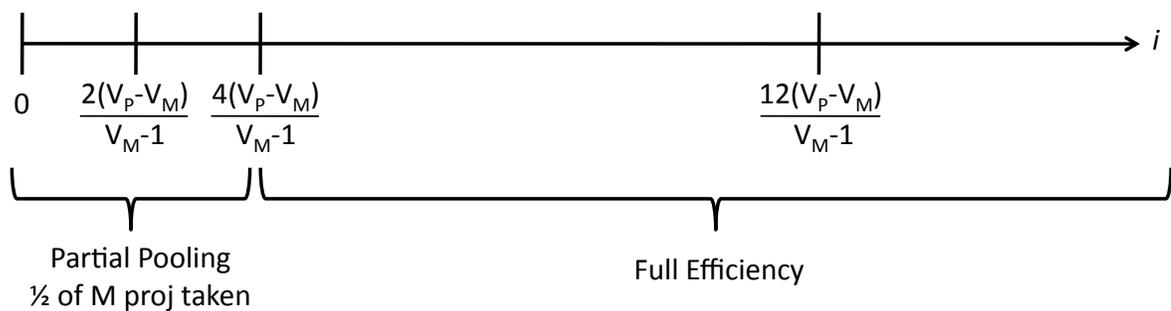
This implies that in order for active speculation to be incentive compatible, the speculator will have to have a sufficiently large initial position so that the gain on that position will overcome the potential loss from its trade. In fact, we have the following result.

**Proposition 3.** *In all of the pure strategy partial pooling equilibria derived above for the active speculator case, the speculator's equilibrium strategy is incentive compatible and individually rational as long as her long/short inventory is at least of magnitude  $s^* = \frac{2(V_P - V_M)}{V_M - 1}$ .*

This result implies that our above assumption about  $S$ 's actions in the partial pooling equilibria amounts to the assumption that  $s \geq s^*$ . This confirms that active speculation can be profitable for a speculator that has accumulated a "secret" long or short position in the stock (or an effective position based on correlated instruments such as the stock of a competitor, supplier, customer, counterparty, etc.). Note that the position size required to support active speculation in this class of equilibria is equal to one half of the cutoff below which no fully efficient equilibria exist in the no speculator case. Thus, it is small relative to the initial positions for  $I$  that we are focusing on in the region of interest.

The results thus far imply that a speculator's presence can reduce efficiency by causing the project to sometimes be rejected when it should not be. An informed long-term shareholder can prevent this loss, but at an endogenous cost that cannot be justified when his own initial position is not sufficiently large. These results are summarized in Figure 7 below, which plots the corresponding "most efficient" equilibrium as a function of  $I$ 's initial position  $i$ .

### No Speculator Case



### Active Speculator Case

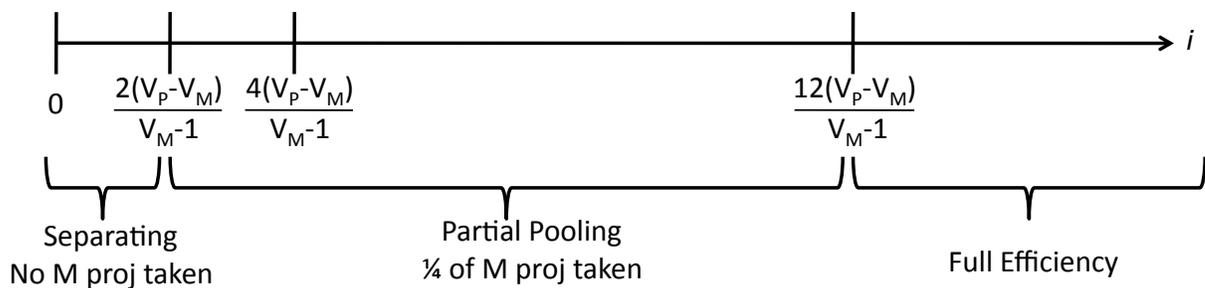


FIGURE 7. Equilibrium Map

The rightward arrow in each panel of the figure represents increasing values of  $I$ 's initial position,  $i$ . The labeled values correspond to the thresholds from propositions 1 and 2. For values of  $i$  above  $\frac{12(V_P - V_M)}{V_M - 1}$ , a fully efficient equilibrium exists with or without the

speculator. For values of  $i$  from  $\frac{4(V_P - V_M)}{V_M - 1}$ , or one-third of that level, to  $\frac{12(V_P - V_M)}{V_M - 1}$  a fully efficient equilibrium exists without the speculator, while the most efficient equilibrium with the speculator is the partial pooling equilibrium in which  $M$  projects are taken  $\frac{1}{4}$  of the time. Thus, in this range an active speculator can make profits if its initial position is at least  $\frac{2(V_P - V_M)}{V_M - 1}$ , and its trading activity can result in significant value loss for the firm.

When  $i$  is between  $\frac{2(V_P - V_M)}{V_M - 1}$  and  $\frac{4(V_P - V_M)}{V_M - 1}$ , the most efficient equilibrium both with and without the speculator is a partial pooling equilibrium, but again the best equilibrium with an active speculator is less efficient. Finally, for smaller  $i$  the existence of a partial pooling equilibrium is not guaranteed. Thus, in this region we cannot guarantee the existence of an equilibrium in which speculation is profitable (a fully separating equilibrium certainly exists, in which case the speculator's actions do not affect efficiency within the equilibrium, so  $S$  is at best indifferent to trading).

Recall that these results are best interpreted in relative terms since we have restricted the speculator to trading at most one share. In particular, the results indicate that uninformed speculation can (profitably) reduce firm value if informed long-term shareholders' stakes are not large enough *relative* to the existing level of binding short sale constraints.

#### 4. THE SPECULATOR'S INITIAL POSITION AND PROFITS

In this section we extend the base model and show how an uninformed speculator can profitably (and secretly) accumulate a sufficient initial position to make active speculation profitable. As noted previously, the speculator's trades in the base model are executed at an expected loss, so it is necessary that she be able to arrive at that stage with a sufficiently large (and secret) position. Here we show that the accumulation of such an ex ante position can be profitable.

To capture this we append to the base model a prior trading round, in which the noise trader buys  $N$  shares with probability  $\frac{1}{2}$  and sells  $N$  shares with probability  $\frac{1}{2}$ . We allow for  $N \neq 1$  to account for the fact that a speculator may accumulate a position over a longer time period than that over which the informed trading round takes place. The speculator arrives at this trading round with no position in the stock. It can buy or sell  $N$  shares, or choose

not to trade. As before, the market maker observes net order flow and sets the price at the risk neutral expected value (with full knowledge of the details of the future trading round). In this round the speculator clearly must play a mixed strategy (otherwise she would always arrive in the main trading round with a known position). In particular, we show that it is an equilibrium for  $S$  to buy  $N$  shares with probability  $\frac{1}{2}$  and sell  $N$  shares with probability  $\frac{1}{2}$ . This obviously corresponds to the base model's assumptions by mapping  $N = s$ , the latter being the magnitude of the speculator's position upon entering the main trading round.

If the speculator mixes in this way between buying and selling, her position will be hidden (secret) only  $\frac{1}{2}$  of the time - when the noise trader's trade goes in the opposite direction. When their trades are reinforcing, the speculator's trade will be revealed. Thus, in order to prove that the mixed strategy for  $S$  is part of an overall equilibrium, we must consider what happens in the main trading round when  $S$ 's position from the first round is common knowledge. We show that in that case, a pure strategy equilibrium exists in which  $S$  trades a single share in the same direction as her first round trade, that is, she buys if she went long in the first trading round and sells if she went short in the first trading round.

*Lemma 3.* If the sign of the speculator's initial position in the base model is common knowledge, there exists a pure strategy equilibrium in which  $S$  trades in the direction of her position, and  $I$  trades the same quantities as those in the full efficiency and partial pooling equilibria described above for the no speculator case. Furthermore,  $S$ 's trades in this equilibrium occur at zero profit or loss.

Essentially, if the speculator arrives with a known position and plays a pure strategy, the market maker and decision maker ignore her effect on the net order flow, and the equilibrium is essentially the same as with no speculator. Since  $S$ 's trades do not affect any outcomes, they are priced correctly (from her perspective) and entail no trading loss. From here forward we assume these equilibria form the subgame following outcomes where  $S$ 's first round trade is revealed by a reinforcing noise trade, whereas the most efficient available full efficiency and partial pooling equilibria derived in the previous section form the subgame following outcomes where  $S$ 's first round trade is hidden (i.e., the noise trader trades in the opposite

direction). Given these assumptions about the subgame, we have the following result for the first trading round.

**Proposition 4.** *If  $N > s^*$  and  $\frac{2(V_P - V_M)}{V_M - 1} \leq i \leq i^{*S}$ , an equilibrium for the first trading round exists in which  $S$  buys  $N$  shares with probability  $\frac{1}{2}$  and sells  $N$  shares with probability  $\frac{1}{2}$ . Furthermore,  $S$  makes an overall expected profit.*

This result shows that the speculator's ability to profit by trading in the first round is guaranteed as long as she can secretly trade more shares than are required to satisfy her second round incentive compatibility constraint (for the cases where her position remains secret), and the subgame equilibrium for the second trading round is a partial pooling equilibrium. Any additional amount she can trade,  $N - s^*$ , represents profit. The first round trade is profitable because, in states where her trade is hidden, the trade is priced at an average of the expected value of the firm with a long versus short speculator in the main trading round. There is a gap between these values since, in the future partial pooling equilibrium, the speculator will either induce an increase in firm value on average (if long) or induce a decrease firm value on average (if short). The speculator plays off this gap, capturing first round expected trading profits that exceed the expected second round trading losses. It is also worth noting that the quantity required for first round trading,  $s^*$ , is less than 1 for many relevant parameterizations of the model, i.e., it is not always necessary that the speculator be able to accumulate a position larger than the amount she can trade in the main trading round.

This strategy is only profitable when such a value gap can be generated. If  $i$  is high enough that full efficiency would prevail even in the presence of an active speculator, then  $S$  can at best break even. Her first and second round trades will both be at zero profit since she does not affect firm value in any way. Similarly, if  $i$  is low enough that no partial pooling equilibrium exists with an active speculator,  $S$  will not be able to profit since the overall project outcome will not depend on whether she ends up long or short in the first round (no partial pooling equilibria exist here).

## 5. THE CASE OF NO AGENCY PROBLEM

In this section we investigate how the model's results would change in the absence of the agency problem between  $I$  and  $D$ , that is, if  $D$  were willing to accept the project even if the signal were known to be M. In this setting, a fully separating equilibrium would be fully efficient. However, it generally does not exist. Since  $D$  will accept the project whenever the signal is perceived to be M or higher, after an H signal  $I$  will always want to “pool” the H and M signals to some degree since the project does not have to be sacrificed, and there will be trading profits if he can be seen to have an M signal with some probability. As such any feasible fully efficient equilibrium must involve at least some pooling between M and H signals. The important condition for full efficiency is then that  $I$  trade a sufficiently low quantity after an L signal so as to completely separate from the M and H signals.

One possible type of fully efficient equilibrium will be the same as the set of fully efficient equilibria derived above. In fact, it is straightforward to show that any fully efficient equilibrium that exists with the agency problem also exists without it. However, taking away the agency problem makes some additional fully efficient equilibria possible – those where the M and H signals are partially pooled, while the L signals are completely separated. Analyzing such possible equilibria yields the following result.

**Proposition 5.** *If there is no agency problem between  $I$  and  $D$ , a fully efficient equilibrium exists in the active speculator case for all  $i > i^{*\hat{S}}$ , where  $i^{*\hat{S}} \leq i^{*S}$ , and the inequality is strict for sufficiently small  $\epsilon$ .*

This result confirms that the agency problem tends to make efficiency more difficult to achieve, and creates additional room for harmful speculation by short sellers. However, taking away the agency problem does not completely solve the efficiency problem. Since  $I$  will always want to pool with the M signal after receiving an H, trading positive quantities is still costly for him after receiving an M – some trading losses will always be necessary if  $I$  is expected to buy shares after an M. One potential solution would then be for  $I$  to trade a very small quantity or not trade at all after an M signal. For example, a possible fully efficient equilibrium would be for  $I$  to buy 2 shares after an H signal, not trade after an

M signal, and sell 5 shares after an L signal. However, this creates a perverse incentive for  $I$  after receiving an L signal. If  $I$ 's initial position  $i$  is small, then after receiving an L he will perceive that if he sells fewer shares, the signal will sometimes be confused with an M signal, and the project will be accepted. This gives him an expected trading profit since he sells at “too high” of a price. Thus, a sufficiently large position  $i$  is required to ensure that he will sell 5 shares after an L. With the agency problem, this was never an issue because the project would not be accepted if the signal were perceived to be M. Thus, removing the agency problem actually makes the problem more difficult to solve.

The reason the inequality in the result is weak unless  $\epsilon$  is sufficiently small relates to the above mentioned incentive for  $I$  to deviate after an L signal. As  $\epsilon$  gets larger, the project's NPV with an L signal approaches zero. Thus, the decline in the value of  $I$ 's initial position from having a bad project accepted gets smaller. However, the trading profits do not shrink as  $\epsilon$  rises, which means the incentive to deviate can become very strong, so that deviation cannot be prevented with an initial position smaller than  $i^{*S}$ . However, it should be noted that cases with large  $\epsilon$  are not very economically relevant since  $I$ 's private information is not particularly valuable in that case (a policy of always accepting the project has very little cost). Thus, in the more economically relevant cases, the agency problem has a significant effect on efficiency and the scope for value-destroying manipulation.

## 6. IMPLICATIONS

**6.1. Empirical Implications.** Our model provides a number of new empirical implications. Most importantly, it implies that value-destroying speculation should be more likely (in terms of both frequency and success) when the holdings of informed, long-term shareholders are small relative to the feasible extent of short selling. It is also more likely when a significant agency problem exists between shareholders and decision makers (decision makers are likely to be more risk averse with respect to the firm's dealings) and when markets are less liquid (trading strategies have meaningful effects on prices). Finally, note that, in our specification, manipulation causes inefficient decisions only for moderately profitable projects – projects with M signals rather than H signals.

We can also derive comparative statics implications from the thresholds in Proposition 1:

**Proposition 6.** *The thresholds  $i^{*N}$  and  $i^{*S}$  are increasing in  $\lambda$  and  $d$ . However, if  $\epsilon = \gamma d$  for some proportion  $\gamma < 1$ , then the thresholds are independent of  $d$ .*

The result with respect to  $\lambda$  implies that the speculator will be more likely to find manipulation profitable if informed shareholders' information is relatively precise (when their signal is, in fact, informative). Intuitively, an increase in  $\lambda$  increases the wedge between the perceived NPV of the project with an H versus M signal - driving  $V_H$  up while leaving  $V_M$  unchanged. This raises the price  $V_P$  without increasing the incentive for  $I$  to make sure the project is taken after an M signal. As a result, it is harder to get him to pool - i.e., pooling requires a larger initial position  $i$ . This has direct empirical implications about which situations are more amenable to manipulation.

The result with respect to  $d$  is similar. Increasing  $d$  without changing  $\epsilon$  makes the project more profitable overall, which interacts with the better information under an H signal to make downward deviation more likely for  $I$  after an M. Thus, projects that ex ante look more profitable are more likely to encourage speculators to manipulate prices. On the other hand, if  $\epsilon$  and  $d$  are held in strict proportion, a change in project "scale" (an increase in  $d$  and  $\epsilon$  in lockstep) has no effect on the thresholds. This is because the increased size of the project affects the  $I$ 's incentive to make sure the project is taken after an M signal and the trading losses required to do so by the same proportion. Overall, these results imply that profitability matters more than scale in terms of predicting when manipulation is likely.

**6.2. Regulatory Implications.** Our model provides several useful implications for regulators considering both general and time-dependent restrictions on short-selling. Since we do not consider the potential positive efficiency effects of short selling by informed speculators, taken literally our model would suggest that a total ban on short sales would improve efficiency. There are, however, many positive aspects of short selling in normal times with regard to price efficiency and efficient resource allocation. Furthermore, in our model informed long-term shareholders are often able to prevent manipulation because their long-term positions give them the incentive to ensure that good decisions are taken. Even when they

do not completely prevent manipulation, they limit its impact so that it does not affect the most profitable projects.

However, our results provide several important clues for when restrictions on short selling may be justified. First, there must be a mismatch between the position sizes of long-term shareholders who are likely to be informed and the freedom with which speculators can short sell. If additional restrictions are to be considered, we would argue they should be targeted at firms with lower ownership concentration and more transitory blockholders, and with fewer “natural” restrictions on speculation (such restrictions may be proxied by volume, float, and availability of shares for borrowing).

Second, the problem of value-destroying manipulation is worse if there exists a significant agency problem between shareholders and important decision makers, be they managers, creditors, suppliers, customers, or governance activists. We believe such agency problems are likely strongly positively correlated with general economic conditions – decision makers are more likely to behave in a risk averse manner when there are significant credit constraints and other general market uncertainties. Thus, market disruptions may be times when additional short sale restrictions should be considered with respect to some firms.

Going outside the strict confines of the model, we can provide additional predictions with respect to which types of firms and situations are likely to become more vulnerable to attempted manipulation. First, our model implicitly assumes that project opportunities depreciate relatively quickly - i.e., if a project is not accepted the decision cannot be changed later. The problem could clearly be ameliorated if this were not the case and the value loss were less permanent. Second, stocks with lower liquidity in general are likely more vulnerable for two reasons. First, this allows the speculator’s trades to have a greater price impact, increasing her ability to affect outcomes. Second, the additional liquidity provided by the speculator’s trades is more likely to cause  $I$  to deviate from a pooling equilibrium – i.e., the speculator’s trades will have a greater impact on the informed shareholder’s willingness to trade sufficient amounts to counteract the potential speculative attack.

## 7. CONCLUSION

We show that manipulative short selling is likely to permanently erode value only when long-term shareholders' existing positions are small relative to speculators' short-selling ability and when agents with decision power over the firm are relatively risk averse. In such situations, the long-term holders' natural incentive to prevent negative manipulation via feedback effects by trading in the opposite direction is endogenously limited. When decision makers are highly risk-averse, informed investors must trade aggressively to convince them to take value-increasing actions, which exposes them to large potential ex post trading losses.

Our analysis raises several interesting questions that could be pursued in future work. First, there is the question of when informed, long-term shareholders will be willing to hold sufficiently large positions and commit themselves to prevent all possible manipulations. There are many reasons why the sizes of real world stakes are limited – diversification concerns, regulatory restrictions on institutional investors, etc. Modeling the endogenous choice of the size of these long positions will require incorporating such concerns, and is outside the scope of the current analysis, but should make for interesting future extensions. Second, while our model is suggestive that additional constraints should be considered only at certain times, it is not truly dynamic. A dynamic model with time-varying risk aversion for decision makers and time-varying natural constraints on short sales could provide additional important insights.

## APPENDIX

**Proof of Lemma 1:** Given any pure strategy for  $S$ , if  $q_I^H \neq q_I^M$  then there will be some equilibrium order flows  $Q$  after a trade of  $q_I^M$  such that only trades of  $q_I^M$  or  $q_I^L$  could result in those order flows. Since beliefs  $\mu(Q)$  must be consistent with Bayes' rule for any equilibrium order flow, the beliefs must place zero probability on an H signal at such order flows and the project will not be taken by  $D$ . With respect to  $I$ 's strategy after an L, if  $q_I^H = q_I^M$  while  $q_I^L$  is such that the resulting equilibrium order flows could not follow a trade of  $q_I^M$ , then all possible equilibrium order flows  $Q$  that *can* result after a trade of  $q_I^M$  will lead to beliefs  $\mu(Q) = \frac{1}{3} + \frac{2}{3}\lambda$ , which is just sufficient for the project to be taken. If instead  $q_I^L$  were such that any of the possible resulting order flows could also result from a trade of  $q_I^M$ , then by Bayes' rule the posterior would have to include some probability of an L signal, reducing the posterior so that the project would not be accepted by  $D$ . QED

**Proof of Proposition 1:** The remaining issues not proven in the text are: showing that the speculator's trades are incentive compatible and individually rational in the active speculator case; and showing that the deviations considered in the text are the most relevant deviations. First consider the speculator's trades. Note that given the equilibria under consideration,  $S$ 's trade cannot affect the project acceptance decision following any signal. Then denoting the expected value of the firm in equilibrium as  $E(V)$ ,  $S$ 's expected payoff is  $sE(V)$  no matter the quantity she trades since her trades are at zero expected profit or loss. To see this, note that the expected price of any of her trades is  $\frac{3}{4}V_P + \frac{1}{4}$ , while the expected value of the firm is  $\frac{1}{4}V_H + \frac{1}{2}V_M + \frac{1}{4}$ , which are equivalent (to see this, replace  $V_P$  with  $\frac{1}{3}V_H + \frac{2}{3}V_M$ ). Since a trade of zero is in the choice set, individual rationality is guaranteed.

We now show that we have focused on the relevant deviations for  $I$  in the text. First consider upward deviations after an H signal in the no speculator case. If  $I$  deviates up by 3 or more shares, the price is always  $V_H$ , so trading profits are eliminated. If  $I$  deviates up to  $q_I^+ + 1$ , the expected trading profit is  $\frac{1}{2}(q_I^+ + 1)(V_H - V_P)$ , which is lower than that derived for the 2 share deviation in the text. Next consider downward deviations after an M signal in the no speculator case. A downward deviation by 3 or more shares results in the project never being accepted, so the expected payoff is  $i$ . This is preferred to the equilibrium payoff

if  $i > iV_M + q_I^+(V_M - V_P)$ , or, rearranging, if  $i < \frac{q_I^+(V_P - V_M)}{V_M - 1}$ , which is always harder to satisfy than the condition for the 2 share deviation in the text. A downward deviation by 1 share yields an expected payoff of  $i(\frac{1}{2}V_M + \frac{1}{2}) + \frac{1}{2}(q_I^+ - 1)(V_M - V_P)$  since the project is accepted half of the time, just as with the 2 share deviation. Since the trading quantity is higher, this expected payoff is clearly always lower than that for the 2 share deviation in the text.

For the active speculator case, consider upward deviations after an H signal. An upward deviation by 1 share has the project still always accepted and yields an expected trading profit of  $\frac{3}{4}(q_I^+ + 1)(V_H - V_P)$ , which is clearly inferior to the 2-share deviation. A 3 share deviation again has the project always accepted, and an expected trading profit of  $\frac{1}{4}(q_I^+ + 3)(V_H - V_P)$ , while a 4 share deviation has expected trading profit of  $\frac{1}{4}(q_I^+ + 4)(V_H - V_P)$ , which is clearly superior. The 2 share deviation profit is even higher if  $\frac{3}{4}(q_I^+ + 2) > \frac{1}{4}(q_I^+ + 4)$ , which always holds for  $q_I^+ > -1$  and thus always holds in the ranges where the equilibria may exist given the analysis in the text. Deviations up by more than 4 yield no trading profits.

Now consider deviations downward after an M signal. Similar to the upward deviations, it is straightforward to show that a 2 share deviation is better than a 1 share deviation, and a 4 share deviation is better than a 3 share deviation (they have the same project acceptance probability and lower trading losses). A 4 share deviation has a  $\frac{1}{4}$  probability of project acceptance, leading to an expected payoff of  $i(\frac{1}{4}V_M + \frac{3}{4}) + \frac{1}{4}(q_I^+ - 4)(V_M - V_P)$ . Comparing this to the equilibrium payoff in the text, deviation is profitable if  $i < \frac{(q_I^+ + \frac{4}{3})(V_P - V_M)}{V_M - 1}$ , which is clearly harder to satisfy than the condition for the 2 share deviation in the text. A deviation by 5 or more shares has zero probability of project acceptance, and thus expected payoff of  $i$ . This is preferred to the equilibrium payoff if  $i < \frac{q_I^+(V_P - V_M)}{V_M - 1}$ , which is again harder to satisfy than the 2 share condition. QED

**Proof of Lemma 2:** Conditional on the assumption that the project not be taken after an L, the result follows from the discussion provided in the text just before and just after the result. To complete the proof, we show that no equilibrium in which the project is sometimes taken after an L can be more efficient. First note that in order for the project to be accepted in equilibrium after an L with some probability,  $D$  must believe that there is a significant probability that the signal was in fact H – a mixture between just M and L signals cannot

result in a sufficiently high posterior belief since an M signal by itself is insufficient. Next note that there cannot exist an equilibrium in which  $q_I^M = q_I^H$ , the project is always taken after such a trade, and  $q_I^L$  is such that any of the resulting order flows could also follow a trade of  $q_I^M$ . If  $q_I^M = q_I^H$  then the equilibrium posterior at the resulting order flows is no higher than  $\mu(Q) = \frac{1}{3} + \frac{2}{3}\lambda$ , which is the minimum required to have the project accepted. Thus, at any order flow where a trade of  $q_I^L$  by  $I$  is also possible, the posterior must be lower and the project will not be accepted.

Given our assumption of monotone beliefs, therefore, any equilibrium in which L signals are partially pooled with M or H signals must have  $I$  trading less with an M than with an H. As shown in the text, any such equilibrium has the project accepted after an M with at most  $\frac{1}{4}$  probability. QED

**Proof of Proposition 2:** The proof proceeds by construction. First consider an equilibrium in the no speculator case in which  $q_I^H = +2$ ,  $q_I^M = 0$ , and  $q_I^L = -2$ . At order flow  $Q = +3$ ,  $D$  and the market maker must infer that the signal is H. At order flow  $Q = +1$  their posterior is  $\mu(Q) = \frac{1}{3} + \frac{2}{3}\lambda$ , so we assume the project is accepted, which results in price  $V_P$ . At all equilibrium order flows  $Q \leq 0$  there is no chance of an H signal, so the project is not accepted and the price equals one. We assume out of equilibrium beliefs are such that at order flow  $Q = 0$  the signal is assumed to be M, that at all  $Q \geq +2$  the signal is assumed to be H, and that at all  $Q \leq -2$  the signal is assumed to be L.

First note that deviations by  $I$  following an L signal are not optimal. The initial position  $i$  has its value maximized when the project is not accepted (as it never is in equilibrium), and the only possibility of trading profits would be if  $I$  could sell a smaller number of shares and still have the project sometimes accepted. This is not possible since a sale of one share is not sufficient to ever have the project accepted (the maximum resulting order flow is zero). Next note that upward deviations by  $I$  after an H signal cannot be optimal. Any such deviation would have the project always accepted, as in equilibrium, and would have trading profits of zero since the price would always be  $V_H$ , so the equilibrium payoff is preferred.

Now consider downward deviations by  $I$  after an H signal. In equilibrium the project is always accepted after an H, maximizing the value of  $i$ , and  $I$  has an expected trading profit

of  $\frac{1}{2}(2)(V_H - V_P)$ . A deviation to +1 means that the project will be accepted only  $\frac{1}{2}$  of the time, and there are no trading profits (the trades are correctly priced at  $Q = 0$  and  $Q = +3$  given this deviation). A deviation to 0 has no trading profits and the project is also taken only  $\frac{1}{2}$  of the time, so this cannot be profitable. Similarly, upward deviations by  $I$  will not be profitable – the project is always taken at price  $V_H$ , so that trading profits are eliminated.

Finally,  $I$  has no incentive to deviate down after an M signal. In equilibrium the project is accepted  $\frac{1}{2}$  of the time, and there are no trading profits/losses, ie the expected payoff is  $i(\frac{1}{2} + \frac{1}{2}V_M)$ . After a downward deviation the project will never be accepted and there are still no trading losses in equilibrium. Finally, consider an upward deviation after an M. A deviation to +1 cannot be optimal - the project is still taken  $\frac{1}{2}$  of the time, but now trading losses occur when the project is taken. A deviation to +2 has the project always accepted – the expected payoff is  $iV_M - \frac{1}{2}(2)(V_P - V_M) - \frac{1}{2}(2)(V_H - V_M)$ . Comparing this to the equilibrium expected payoff shows that deviation is profitable if  $i > \frac{2(V_P - V_M) + 2(V_H - V_M)}{V_M - 1}$ , which equals  $2i^{*N}$ . Thus, this equilibrium exists for all  $i \in [0, 2i^{*N}]$ , which proves the result for the no speculator case.

Now consider the active speculator case, and an equilibrium in which  $q_I^M \geq +1$ ,  $q_I^H = q_I^M + 4$ , and  $q_I^L = q_I^M - 2$ . At the equilibrium order flows we have: if  $Q \in \{q_I^M + 4, q_I^M + 6\}$ , the project is taken and  $p(Q) = V_H$ ; if  $Q = q_I^M + 2$  the project is taken and  $p(Q) = V_P$ ; if  $Q \leq q_I^M$  the project is not taken and  $p(Q) = 1$ . For out of equilibrium beliefs we assume that for all  $Q \geq q_I^M + 3$  the signal is assumed to be H, for all  $Q \leq q_I^M - 3$  the signal is assumed to be L, for  $Q = q_I^M + 1$  the signal is assumed to be H with  $\frac{1}{3}$  probability and M otherwise, and at  $Q = q_I^M - 1$  the signal is assumed to be M or L with equal probability. For the out of equilibrium order flow  $Q = q_I^M + 1$  we have specified that  $D$  is indifferent over the project; we further specify that it would take the project with 50% probability.

Now consider possible deviations. There is no profitable deviation with an L since  $I$  cannot sell any quantity and have positive probability of acceptance (since  $q_I^M \geq +1$  – see above discussion for no speculator case).  $I$  will never optimally deviate upward with an H since all trading profits will be eliminated (the project will always be accepted at price  $V_H$ ). Consider downward deviations after an H. In equilibrium  $I$  has an expected payoff of

$iV_H + \frac{1}{4} * (q_I^M + 4)(V_H - V_P)$ . A deviation to  $q_I^M + 3$  still has the project taken  $\frac{3}{4}$  of the time at price  $V_H$ , and the remainder of the time there is a 50/50 chance of project acceptance at  $V_P$  or project rejection. Thus, this reduces both the value of  $i$  and the expected trading profits. It is straightforward to show that deviations to  $q_I^M + 1$  or less are similarly dominated by a deviation to  $q_I^M + 2$ . With a deviation to  $q_I^M + 2$  the project is either: accepted at  $Q = q_I^M + 4$  at price  $V_H$ , accepted at  $Q = q_I^M + 2$  at price  $V_P$ , or rejected at  $Q = q_I^M$ . The expected payoff is therefore  $i(\frac{1}{4} + \frac{3}{4}V_H) + \frac{1}{2}(q_I^M + 2)(V_H - V_P)$ . Comparing this to the equilibrium payoff, the deviation is profitable if  $i < \frac{q_I^M(V_H - V_P)}{V_H - 1}$ .

Next consider downward deviations after an M.  $I$ 's equilibrium expected payoff is  $i(\frac{3}{4} + \frac{1}{4}V_M) - \frac{1}{4}q_I^M(V_P - V_M)$ . If he deviates down by 1 share to  $q_I^M - 1$ , the project will be accepted with some probability only if  $Q = q_I^M + 1$ , and then with only  $\frac{1}{2}$  probability, which yields an expected payoff of  $i(\frac{7}{8} + \frac{1}{8}V_M) + \frac{1}{8}(q_I^M - 1)(V_M - V_P)$ . Comparing this to the equilibrium payoff the deviation is profitable if  $i < \frac{(q_I^M + 1)(V_P - V_M)}{V_M - 1}$ . Deviating down by more than 1 share results in the project never being accepted, and thus an expected payoff of  $i$ , which is preferable to the equilibrium payoff if  $i < \frac{q_I^M(V_P - V_M)}{V_M - 1}$ , which is clearly harder to satisfy, so the 1 share deviation is the relevant one to consider. Now compare the 1 share downward deviation condition after an M,  $i < \frac{(q_I^M + 1)(V_P - V_M)}{V_M - 1}$ , to the two share downward deviation condition after an H,  $i < \frac{q_I^M(V_H - V_P)}{V_H - 1}$ . By replacing the  $V$  terms with their algebraic definitions in terms of the model's primitives, it is straightforward to show that the former equals  $\frac{2(q_I^M + 1)Y}{3\epsilon}$  and that the latter equals  $\frac{q_I^M Y}{3\gamma}$ , where  $Y \equiv (2d - \epsilon)(2\lambda - \frac{1}{2})$  and  $\gamma \equiv (2d - \epsilon)\lambda + \frac{1}{2}(\epsilon - d)$ . Consider the ratio  $\frac{\gamma}{\epsilon}$ . Our assumption that  $V_L < 1$  implies  $\epsilon < \frac{d(4\lambda - 1)}{2\lambda}$ . We have  $\frac{\partial \gamma}{\partial \epsilon} = \frac{d(\frac{1}{2} - 2\lambda)}{\epsilon^2} < 0$ , and plugging for the maximum  $\epsilon$  we have  $\frac{\gamma}{\epsilon} = \frac{1}{2}$ , so  $\gamma \geq \frac{1}{2}\epsilon$  must always hold. Plugging this minimum  $\gamma$  into the expression for the downward deviation condition following an H yields  $\frac{q_I^M Y}{3\frac{1}{2}\epsilon} = \frac{2q_I^M}{3\epsilon}$ , so the downward deviation condition following an M is always larger and thus is the relevant downward cutoff for existence of the equilibrium.

Finally consider upward deviations after an M, in particular a deviation to  $q_I^M + 2$  (it is straightforward to show this is the relevant deviation by testing the other possibilities as above). This deviation yields an expected payoff of  $i(\frac{1}{4} + \frac{3}{4}V_M) - \frac{1}{4}(q_I^M + 2)(V_H - V_M) - \frac{1}{2}(q_I^M + 2)(V_P - V_M)$ . Comparing this to the equilibrium expected payoff the deviation is

profitable if  $i > \frac{(\frac{2}{3}q_I^M + \frac{5}{3})(V_H - V_M)}{V_M - 1}$ . Thus, the relevant range of existence for this equilibrium is  $i \in [\frac{(q_I^M + 1)(V_P - V_M)}{V_M - 1}, \frac{(\frac{2}{3}q_I^M + \frac{5}{3})(V_H - V_M)}{V_M - 1}]$ , or, replacing the V terms with their equivalents in terms of the primitives and simplifying,  $i \in [\frac{2(q_I^M + 1)Y}{3\epsilon}, \frac{2(2q_I^M + 5)Y}{3\epsilon}]$ , where  $Y \equiv (2d - \epsilon)(2\lambda - \frac{1}{2})$ .

Now note that at the minimum  $q_I^M$  we specified,  $q_I^M = +1$ , the lower boundary clearly corresponds to that given in the result. Also, as  $q_I^M$  is increased, both the upper and lower boundaries of existence for the equilibrium increase, and the upper boundary is clearly always greater. It is straightforward to show that  $i^{*S} = \frac{8Y}{\epsilon}$ , so the upper boundary exceeds  $i^{*S}$  at a value of  $q_I^M = 4$ . Finally, note that the new lower boundary lies below the old upper boundary each time  $q_I^M$  is increased by 1 (plugging  $q_I^M + 1$  into the lower boundary yields  $\frac{2(q_I^M + 2)Y}{3\epsilon} < \frac{2(2q_I^M + 5)Y}{3\epsilon}$ ), so considering each equilibrium as  $q_I^M$  increases by ones from +1 to +4 yields the result. QED

**Proof of Proposition 3:** Consider the equilibrium derived in the proof of Proposition 2 in which  $q_I^M \geq +1$ ,  $q_I^H = q_I^M + 4$ , and  $q_I^L = q_I^M - 2$ . The speculator enters the trading round with a position of magnitude  $s$ . First assume this is a short position,  $-s$ . Then if the speculator sells one share as the equilibrium requires, the possible equilibrium order flows are: if  $\theta = L$ ,  $Q \in \{q_I^M - 4, q_I^M - 2\}$  with equal probability (due to the noise trade); if  $\theta = M$ ,  $Q \in \{q_I^M - 2, q_I^M\}$ ; and if  $\theta = H$ ,  $Q \in \{q_I^M + 2, q_I^M + 4\}$ . Thus, the project will never be accepted after an M or L signal, and the price will always be 1 in those cases. The project will always be accepted after an H and the price is  $V_H$  or  $V_P$  with equal probability. L, M, and H signals arrive with ex ante unconditional probabilities of  $\frac{1}{4}$ ,  $\frac{1}{2}$ , and  $\frac{1}{4}$  respectively. Thus, the expected price to be paid is  $\frac{3}{4} + \frac{1}{8}V_H + \frac{1}{8}V_P$ . The expected value of the shares is  $\frac{3}{4} + \frac{1}{4}V_H$ . The speculator's expected payoff to the equilibrium strategy is therefore  $-s(\frac{3}{4} + \frac{1}{4}V_H) - \frac{1}{8}(V_H - V_P)$  (trading losses occur only when the project is accepted at price  $V_P$  after an H).

The only relevant deviation will be to not trade (buying will further reduce the value of  $s$  while also causing trading losses). With a deviation to zero, possible order flows are: if  $\theta = L$ ,  $Q \in \{q_I^M - 3, q_I^M - 1\}$ ; if  $\theta = M$ ,  $Q \in \{q_I^M - 1, q_I^M + 1\}$ ; and if  $\theta = H$ ,  $Q \in \{q_I^M + 3, q_I^M + 5\}$ . The only differences in outcomes are that the project is now accepted after an M signal  $\frac{1}{4}$  of the time at price  $V_P$  (noise buys  $\frac{1}{2}$  of the time, and then  $D$  accepts the project  $\frac{1}{2}$  of the time when that happens) while the price is always  $V_H$  after an H signal. The speculator's

expected payoff is therefore  $-s(\frac{5}{8} + \frac{1}{8}V_M + \frac{1}{4}V_H)$  since  $S$  is trading zero. Setting this equal to the equilibrium payoff and solving for  $s$  yields  $s = \frac{V_H - V_P}{V_M - 1} = \frac{2(V_P - V_M)}{V_M - 1}$ , which is the expression provided in the result.

For the case with a long position of  $s$ , we similarly must check the deviation to no trade. Following similar logic, the equilibrium expected payoff to buying one share is  $s(\frac{1}{2} + \frac{1}{4}V_M + \frac{1}{4}V_H) - \frac{1}{4}(V_P - V_M)$ . The expected payoff to not trading is  $s(\frac{5}{8} + \frac{1}{8}V_M + \frac{1}{4}V_H)$ . Setting these equal and solving for  $s$  again yields  $s = \frac{V_H - V_P}{V_M - 1} = \frac{2(V_P - V_M)}{V_M - 1}$ . QED

**Proof of Lemma 3:** The proof is again by construction. First consider the full efficiency equilibrium analog to the no speculator case in which  $q_I^M = q_I^H = q_I^+ \geq +2$  and  $q_I^L = q_I^+ - 3$ . Now assume the speculator arrives long  $s$  shares (which is common knowledge) and is prescribed to buy one share. Possible equilibrium order flows are  $Q \in \{q_I^+, q_I^+ + 2\}$  following H and M signals depending on whether the noise trader buys or sells. Thus, the project is taken at these order flows and the price is  $V_P$ . L signals result in  $Q \in \{q_I^+ - 3, q_I^+ - 1\}$ , so the project is rejected for all  $Q \leq q_I^+ - 1$  and the price is 1 (out of equilibrium beliefs must place all weight on L in that range). Our monotone beliefs assumption requires that at out of equilibrium node  $Q = q_I^+ + 1$ ,  $D$  be indifferent over project acceptance, so we assume it is accepted and the price is  $V_P$ . Checking the possible deviations by  $I$  proceeds as in the prior proofs (note that  $I$  still cannot deviate to a “sell” quantity after an L that gets the project accepted, as selling one share leads to a maximum order flow of  $q_I^+ - 1$  if  $S$  is long and buys, which is insufficient to get it accepted, and  $q_I^+ - 3$  if  $S$  is short and sells, which is again insufficient to get the project accepted – see below), and it is straightforward to show that  $I$ 's incentive to deviate downward to  $q_I^+ - 2$  after an M again limits the range of existence to  $i \geq i^{*N}$ . The only remaining deviations to check are deviations by  $S$ .

In equilibrium,  $S$ 's expected payoff is  $s(\frac{1}{4} + \frac{1}{2}V_M + \frac{1}{4}V_H)$  ( $S$ 's trade is at the true expected value, so there is no trading profit or loss). If  $S$  deviates to zero the M or H signal order flow becomes  $Q \in \{q_I^+ - 1, q_I^+ + 1\}$ , so the project will be taken only  $\frac{1}{2}$  of the time, reducing the expected payoff to  $s(\frac{5}{8} + \frac{1}{4}V_M + \frac{1}{8}V_H)$ . If  $S$  deviates to  $-1$ , the project will again be taken only  $\frac{1}{2}$  of the time, and there will again be no trading profit. Thus,  $S$  will not deviate. The proof for the case where  $S$  arrives short  $s$  shares is analogous.

Next consider the partial pooling equilibrium in which  $q_I^H = +2$ ,  $q_I^M = 0$ , and  $q_I^L = -2$ . Now assume the speculator arrives long  $s$  shares (which is common knowledge) and is prescribed to buy one share. The equilibrium order flow possibilities are: if the signal is H,  $Q \in \{+4, +2\}$ ; if the signal is M,  $Q \in \{+2, 0\}$ ; if the signal is L,  $Q \in \{-2, 0\}$ . Thus, the project is accepted at price  $V_H$  at  $Q = +4$ , accepted at price  $V_P$  at  $Q = +2$ , and is not accepted for lower  $Q$ . Out of equilibrium beliefs are such that the project is accepted at price  $V_H$  for all  $Q \geq +3$ , while the project is not accepted for any  $Q \leq +1$  (the signal is believed to be M or L). Again, checking for deviations by  $I$  proceeds as in prior proofs and shows that the equilibrium exists for the entire range of  $i \in [0, i^{*N}]$ .

Finally, consider deviations by  $S$ .  $S$ 's equilibrium payoff is  $s(\frac{1}{2} + \frac{1}{4}V_M + \frac{1}{4}V_H)$ . If  $S$  deviates to zero, the project will never be taken after an M and will be taken only  $\frac{1}{2}$  of the time after an H, so the equilibrium payoff is  $s(\frac{7}{8} + \frac{1}{8}V_H)$ . A deviation to  $-1$  will have the same expected firm value per share, but there will be a trading loss because when the project is accepted after an H it is accepted at price  $V_P$ , which is too low. The proof for the case where  $S$  arrives short  $s$  shares is again analogous. QED

**Proof of Proposition 4:** From Lemma 3 and the proceeding text we know that if the speculator and noise trader trade in the same direction, the speculator has zero profit/loss overall (both the first and second round trades occur at zero profit/loss). Note that all of the prices derived above for the active speculator case reflect a  $\frac{1}{2}$  probability that  $S$  will be long vs. short when its position entering the second trading round is unknown. Thus, using these prices it suffices to show that mixing with probability  $\frac{1}{2}$  between buying and selling  $N$  shares in the first trading round is incentive compatible and individually rational for  $S$ . If the speculator does not trade, she gets an overall expected payoff of zero.

First we derive the first round market price assuming that the speculator's trade is not discovered (ie, the noise trader trades in the opposite direction). The market maker perceives that there is a 50/50 probability of  $S$  having gone long or short. If the speculator is short, the expected per share firm value is  $\frac{3}{4} + \frac{1}{4}V_H$  (see the proof of Proposition 3). If the speculator is long, the expected per share firm value is  $\frac{1}{2} + \frac{1}{4}V_M + \frac{1}{4}V_H$  (again, see the proof of Proposition

3). The overall expected firm value places  $\frac{1}{2}$  weight on each, which yields  $\frac{5}{8} + \frac{1}{4}V_H + \frac{1}{8}V_M$ , so this is the first round price when  $S$ 's trade is hidden.

$S$ 's expected payoff equals the sum of the expected trading profits (losses) from each round. These both equal zero if the first round trade is revealed (the noise trader trades in the same direction). Thus, the overall expected payoff to  $S$  if it buys  $N$  shares can be expressed as  $\frac{1}{2} [N(\frac{1}{4}V_H + \frac{1}{4}V_M + \frac{1}{2} - \frac{5}{8} - \frac{1}{4}V_H - \frac{1}{8}V_M) - \frac{1}{4}(V_P - V_M)]$ , where the first term in the brackets is the expected first round trading profit, and the second term is the expected second round trading loss. This simplifies to  $\frac{1}{2} [\frac{1}{8}N(V_M - 1) - \frac{1}{4}(V_P - V_M)]$ . Similarly, the overall expected payoff if  $S$  sells  $N$  shares can be expressed as  $\frac{1}{2} [-N(\frac{3}{4} + \frac{1}{4}V_H - \frac{5}{8} - \frac{1}{4}V_H - \frac{1}{8}V_M) - \frac{1}{8}(V_H - V_P)]$ , which simplifies to  $\frac{1}{2} [-\frac{1}{8}N(V_M - 1) - \frac{1}{8}(V_H - V_P)]$ . It is straightforward to show that these are equivalent by plugging in the appropriate definitions for the various values from the text. This proves the strategy is incentive compatible.

To prove it is individually rational, we must simply show that these last expressions are weakly positive. Setting the last expression equal to zero and solving for  $N$  yields  $N = \frac{V_H - V_P}{V_M - 1} = \frac{2(V_P - V_M)}{V_M - 1}$ . QED

**Proof of Proposition 5:** As noted in the text, it is straightforward to show that the full efficiency equilibria derived for the base model also exist when the agency problem is removed, which gives the weak part of the inequality. We prove the strict case by construction. Consider an equilibrium with  $q_I^H = +5$ ,  $q_I^M = +3$ , and  $q_I^L = -2$ . Prices at equilibrium order flows where the project is accepted are as follows:  $p(+7) = V_H$ ,  $p(+5) = V_P^+$ ,  $p(+3) = V_P^-$ , and  $p(+1) = V_M$ , where  $V_P^+$  reflects a  $\frac{1}{2}$  chance of an H signal versus an M signal, and  $V_P^-$  reflects a  $\frac{1}{5}$  probability of an H signal versus an M signal. All order flows below +1 have a price of 1 and the project is not accepted. For out of equilibrium order flows of  $Q = +4, +2$  we assume the project is accepted with prices of  $V_P^+$  and  $V_P^-$ , respectively. For higher out of equilibrium order flows we assume the project is accepted and the price is  $V_H$ , and for lower ones we assume it is rejected, so the price is 1.

First consider deviations by  $I$  after receiving an H. It is straightforward to show that deviating to +7 is the most profitable possible upward deviation. Since the project is always accepted in either case, we focus on expected trading profits. In the equilibrium they are

$5(\frac{3}{4}V_H - \frac{1}{2}V_P^+ - \frac{1}{4}V_P^-)$ . The deviation to +7 yields  $7(\frac{1}{4}V_H - \frac{1}{4}V_P^+)$ , which is clearly lower. A deviation down to +3 is similarly the best possible downward deviation. It yields an expected trading profit of  $3(V_H - \frac{1}{4}V_P^+ - \frac{1}{2}V_P^- - \frac{1}{4}V_M)$ , which again is easily shown to be lower than the equilibrium payoff.

After an M,  $I$  will clearly never wish to deviate upward (the same project acceptance probability but more trading losses).  $I$ 's equilibrium expected payoff is  $iV_M - 3(\frac{1}{4}V_P^+ + \frac{1}{2}V_P^- - \frac{3}{4}V_M)$ . Consider a downward deviation to +1 (the best possible such deviation). This has an expected payoff of  $i(\frac{1}{4} + \frac{3}{4}V_M) - \frac{1}{4}(V_P^- - V_M)$ . Comparing this to the equilibrium payoff (and simplifying using the equalities  $V_P^+ = \frac{1}{2}V_H + \frac{1}{2}V_M$ ,  $V_P^- = \frac{1}{5}V_H + \frac{4}{5}V_M$ , and  $V_H = 3V_P - 2V_M$ ), deviation is profitable if  $i < \frac{\frac{24}{5}(V_P - V_M)}{V_M - 1} < i^{*S}$ .

Finally, consider deviations by  $I$  following an L. The equilibrium expected payoff is  $i$ . It is straightforward to show that the most profitable deviation is to  $-1$ , which yields  $i(\frac{1}{4}V_L + \frac{3}{4}) + \frac{1}{4}(V_M - V_L)$ . Comparing this to the equilibrium payoff, deviation is profitable if  $i < \frac{V_M - V_L}{1 - V_L}$ . The numerator and denominator are both weakly positive. As  $\epsilon$  approaches its maximum, which is constrained to keep  $V_L < 1$ , the expression goes to infinity. As  $\epsilon$  goes to zero, the numerator declines while the denominator rises, with a limiting value of 1 at  $\epsilon = 0$ , whereas  $i^{*S}$  goes to infinity as  $\epsilon$  goes to zero. This suffices to prove the result. QED

**Proof of Proposition 6:** Directly calculating  $\frac{\partial i^{*N}}{\partial \lambda}$  yields  $\frac{8(2d - \epsilon)}{3\epsilon} > 0$ . Directly calculating  $\frac{\partial i^{*N}}{\partial d}$  yields  $\frac{8(2\lambda - \frac{1}{2})}{3\epsilon} > 0$ . Substituting  $\gamma d$  for  $\epsilon$  in  $i^{*N}$  yields  $\frac{8(\lambda(2 - \gamma) + \frac{1}{2}(\gamma - 1))}{3}$ , which is clearly independent of  $d$ . The proofs for  $i^{*S}$  are analogous. QED

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