Optimal Dynamic Relative Performance Evaluation

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Abstract

The theoretical prediction of a negative coefficient on positively correlated peer performance that underlies much of the empirical literature on relative performance evaluation, is commonly obtained from the special case where variance-covariance matrix of the performance measures is exogenously restricted to be independent of the evaluatee’s action. Using the dynamic approach of Holmström and Milgrom (1987), I study the properties of contracts that optimally condition an agent’s compensation both on his own performance and on how well he fares relative to a peer (group) when this restriction is not imposed. I show that if the covariance is non-zero, the optimal contract is linear in own and peer performance as well as the correlation between own and peer performance while, in line with the preponderance of the empirical evidence, in it’s simplest and perhaps most reasonable form the model predicts that the expected coefficient on peer performance is exactly zero.
1 Introduction

Relative performance evaluation (RPE hereafter) permeates almost all aspects of human endeavour. In sports, in personal life, politics, academe and in business it is more or less a truism that “achieving” and/or “performing” is at least partially judged in relative terms. Winning the Indy 500, keeping up with the Joneses, being the first man on the moon, beating an earnings forecast, losing a presidential debate or, in general, outperforming your competitors on some desirable dimension are purely relative in nature and key to how we assign credit. Surely, lap-times matters in racing, doing well financially is of value even if the Joneses are doing slightly better, playing golf on the moon is cool if it is not a first and delivering solid profits is desirable even if other managers win the profitability contest. It is hard to dispute, however, that when humans evaluate the performance others, in particular those who are stewards of our well being (such as the managers of companies in which we are stakeholders), comparisons virtually always play a key part.

Nearly three decades have passed since the publication of the pioneering work of Antle and Smith (1986) opened the empirical inquiry into the prevalence of relative performance evaluation for managers of public corporations. Based on Holmström (1979, 1982) they proposed and tested several versions of what was perceived to be the central (one shot) agency theory prediction with respect to relative performance evaluation: economy wide shocks are optimally (partially) removed from the performance related compensation of firm managers via a negative coefficient on the performance of their peers.\(^1\) Despite the thoroughness of their study, Antle and Smith (1986) found only what can be characterized as quite modest indications that some form of relative performance evaluation might have been at work for some part of their sample. This disappointing message, in turn, was the seed that gave rise

\(^1\)The theoretical references used to motivate the specific empirical relative performance evaluation hypotheses/tests have evolved somewhat over time. The so-called strong-form predictions are typically obtained from the standard properties of the normal distribution, either relying on Banker and Datar (1989) or (more recently) the so-called LEN model, which is motivated by the work of Holmström and Milgrom (1987, 1991), See for example Albuquerque (2011) and Dikolli et. al. (2011b). The general prediction has, however, remained fundamentally unchanged.
to the relative performance evaluation puzzle that persists in the literature to this day.

The relative performance evaluation puzzle can perhaps best be described as the lack of cooperation with the above mentioned prediction by almost any set of data examined following Antle and Smith’s (1986). Indeed, the robustness of Antle and Smith’s (1986) discouraging empirical results has persisted despite the (loosely speaking) countless studies aimed at refining the empirical approaches used to test particular versions of this prediction.\(^2\)

Rather than continuing down that path, in this paper I revisit some of the key assumptions of the underlying models on which the standard empirical prediction is based. Specifically, that the covariance of own and peer performance is exogenously determined and thus independent of the choices of the managers, and while perhaps more subtle, the assumption that performance, own and peer, is measured just once, being at the end of some fixed length period typically taken to be the entire contracting horizon. My motivation for revisiting these assumptions derive from economic rather than technical concerns. That is, while both are arguably “measure zero” type of restriction from a technical perspective, more importantly they seem highly implausible from an economic perspective as well.

Consider for example seemingly close peers such as McDonald’s and Burger King. By locating on opposite street corners of the same intersection, next to the same malls or freeway ramps, clearly increases their exposure to factors driving local demand. Conversely, by locating in suburban area malls away from the inner cities where many Sears stores traditionally were to be found, a retailer such as Target reduces its exposure some of the demographic-specific uncontrollable events Sears is facing. Similarly in many non-business situations that involves competing peers. Take Tour de France for example. A cyclist working hard to constantly keep within close distance to his main peers will be exposed to the exact same weather and congestion conditions as the peer at the same points of the route, while others that don’t, won’t. People that choose chicken will generally be exposed to food borne illnesses at different times than those who chose beef instead, etc. etc. The list of examples

\(^2\)Because of the vastness of the relative performance evaluation literature, I’ll refrain from attempting to do all the idiosyncratic contributions justice here.
is endless.

Separate but related, using a single shot, standard Principal-Agent model as the theoretical vehicle to study the use of RPE for many such real world situations seems at a minimum somewhat less appropriate than for many other issues. When the cyclist in Tour de France competes he is provided very frequent, if not continuous, feedback about own and peer performance, and can obviously choose to alter his actions in response. McDonald’s and Burger King can monitor one another’s pricing, product mix, promotional actions, store openings etc. and casual empiricism certainly suggests that they do follow each others’ actions closely. Moreover, if the performance measure in focus is stock returns, which is indeed the central focus of the empirical RPE literature, it is clear that own and peer stock-returns provide evalees with more or less continuous feedback on which they can condition their actions. Frequent feedback seems almost part and parcel to many real world situations where RPE is relevant, and results obtained from a single shot model likely fail to account for the implications hereof.

The alternative approach I take in this paper intended to address these issues related to the standard approach, is based on the dynamic approach of Holmström and Milgrom (1987). Specifically, I extend the basic version of their one-agent model where the agent’s optimal contract is guaranteed to be linear in normal distributed aggregate returns for the entire period in question to the case of multiple each controlling the returns of their own firms in real time. Then, rather than relying on the “ad hoc principle” limit the optimal contracts to be linear functions of normal distributed (aggregate) performance measures and to impose an exogenous covariance matrix,3 I instead examine the underlying model directly and derive these properties as the feedback frequency grows large and my model thus converges to one of continuous time.

Several key insights are obtained. First, what drives the formation of peer groups for the propose of performance evaluation is not simply determined by the sign of the (equilib-

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3The latter is central to the justification for the former. See Holmström and Milgrom (1987, pp 323-25).
rium) peer covariance with own performance. Instead, groups of peers are optimally formed based on the marginal effect of the agent’s second-best action on the correlation of his own performance with that of his peer(s). Second, while the optimal contract for any agent remains linear in aggregate own performance, it is not a simple linear function of aggregate peer performance as per the ad-hoc principle. Rather, the relative performance component is reflected in the optimal contract via the correlation between realized own and peer performance over the entire horizon in question. Specifically, for a group of peers where a marginal increase in the agent’s (second best) action increases (decreases) the covariation with the performance of his peers, the agent’s compensation increases (decreases) linearly in the coefficient of correlation.

At first, this result may seem somewhat counter-intuitive as it is in sharp contrast to the conventional insights produced by the standard model(s). However, when considering that a key feature of the basic dynamic model I analyze is that the covariance matrix inevitably does depend on the agents’ actions the intuition is straightforward: a measure of the covariance becomes informative about effort and such a measure obtained from the entire history of own and peer return realizations generally then contain more information than the aggregate of these histories can. And while aggregate peer performance does enter linearly into the optimal contract as well, for random samples within either of the above mentioned groups of peers, compensation is actually predicted to be independent of (aggregate) peer performance under natural/neutral distributional assumptions.

The remainder of this paper is dedicated to introducing the model, deriving the core properties of $RPE$ under broad conditions where a simple aggregate performance measure won’t suffice but where knowing relative performance in addition to own performance actually matters. I’ll then derive the empirical predictions of this model and show that while they differ fundamentally from those obtained in the special case where aggregates are sufficient, they match the bulk of the empirical evidence accumulated to date perfectly. I will go on to discuss the economic intuition of this model’s predictions relative to the intuition of those
models common in the literature and end with a few concluding remarks.

2 Model

This section develops a simple two-firm/two-agent model where again i) performance feedback is frequent, ii) the focal agent’s own aggregate performance and aggregate peer performance are distributed joint normal, and iii) the effect of the agent’s action on the variance-covariance matrix is allowed to arise endogenously and iv) absent covariation between own and peer performance the optimal contract for either agent is linear in aggregate own performance.\footnote{Restricting attention to just two firms is purely for simplicity and without loss of generality. I’ll return to address this issue directly towards the end of the analysis part of this paper.}

My approach is based on that developed in the seminal paper by Holmström and Milgrom (1987).\footnote{Using their approach to develop the predictions also allow me to make the point as starkly as possible, that the predictions one would derive using the LEN model are fundamentally different from, and in inherent conflict with those obtain here.} Accordingly, rather than assuming that the focal agent acts only once at the start of the period in question and that nothing happens until at the very end where performance then appears, effort is here assumed to be supplied, and (stock price) performance is assumed to evolve continuously, over the period covered by the model.\footnote{Of course, the particular length of the period and the confinement of attention to just one period is completely inconsequential for the results using this model set-up.} As in Holmström and Milgrom (1987), I start by sub-dividing the fixed-length contracting horizon into $m$ identical sub-periods where $m$ is a positive integer. Normalizing the length of the contracting horizon to “one,” the length of each of the $m$ sub-periods I then denote by

$$\theta = 1/m.$$ 

In each sub-period of length $\theta$, I assume performance follows a multinomial distribution. Specifically, to make sure that in the absence of a correlated peer performance measure the agent’s optimal compensation contract becomes linear in aggregate own performance as $m \to \infty$, I restrict attention to a case where the measure of the focal agent’s performance,
\( \omega_{\theta} \), in any sub-period of length \( \theta \) can be either positive or negative so that:\(^7\)

\[
\omega_{\theta} = \omega_{\theta}^i, \quad i \in \{+, - \}.
\]

In each sub-period the agent influences his own performance through his action choices, which is here taken to be the choice of the probability \( p_{\tau} \) of a positive outcome for each sub-period \( \tau \in \{1, \ldots, m\} \). The history of realized own performance from the first to the last sub-period is denoted \( \overrightarrow{\omega} \), while aggregate realized own performance for the entire \( m \) sub-periods is denoted \( \Omega \).

For the correlated peer performance measure, I assume that it evolves in the same fashion as own performance, but independently of the actions of the focal agent. Thus, for each sub-period of length \( \theta \) let peer performance also be either positive or negative such that:

\[
\pi_{\theta} = \pi_{\theta}^i, \quad i \in \{+, - \}.
\]

I use \( q \) to denote the probability that peer performance in each of the sub-periods is \( \pi_{\theta}^+ \) which, again, does not depend on the focal agent’s actions. Following Holmström (1982), \( q \) is pre-determined as part of a Nash Equilibrium and can thus be treated as "fixed" in the analysis of the focal agent. Also, following Holmström and Milgrom (1987) equilibrium \( q \) is constant over time and I therefore suppress the time sub-script on this variable. The history of realized peer performance from the first to the last sub-period is denoted \( \overrightarrow{\pi} \), while aggregate realized peer performance for the entire \( m \) sub-period horizon is denoted \( \Pi \).

Of course, in any set-up like this, the use of peer performance in the evaluation of an agent is optimal only if the sufficient statistic condition is violated which implies that the covariance between peer performance and own performance, here denoted \( \sigma_{\omega\pi} \), is non zero. In the spirit of the literature on RPE, the covariance is supposed to be the result of firms’ exposure to a “common risk” component such as economy wide conditions: when the economy is good

\(^7\)Notice that with more outcomes per sub-period, the agent’s optimal contract is not linear in own performance in the limit. Rather it is linear in the Holmstrom and Milgrom (1987)- type account balances which is something entirely different. I will return to the accounts in the next section.
all firms are more likely to do well while when general conditions are poor, firms are more likely to perform poorly.

To introduce covariance into the model I rely on $\gamma^i_\tau \in [0, 1]$ to denote the conditional probability $pr_\tau(\pi^i_\theta|\omega^i_\theta)$ where, again, $i \in \{+, -\}$. Then,

$$q = p_\tau \gamma^+_\tau + (1 - p_\tau)(1 - \gamma^-_\tau).$$

Because, as in Holmström and Milgrom (1987) the production technology is assumed time-invariant, for a given $q$, $\gamma^i_\tau = \gamma^i_\tau'$, whenever $p_\tau = p'$. Furthermore, for this structure to be consistent with the idea that covariance between own and peer performance originating from exposure to a common factor, it must be the case that upon the observation of any realization of own performance the updated probability of the same level of peer performance either increases (in the case of positive covariance) or decreases (in the case of negative covariance). Formally, this implies that $\text{sgn}(\gamma^+ - p) = \text{sgn}(\gamma^- + p - 1)$. Moreover, it must be the case that if the focal agent’s action does impact the covariance, it does so by either increasing or decreasing the relative importance of the common component. This in turn implies for this model structure that for the derivatives of $\gamma^i_\tau$ w.r.t. $p$, hereafter denoted $\gamma^i_p$, $\text{sgn}(\gamma^+_p) = \text{sgn}(\gamma^-_p)$.

Finally, for simplicity and also wlog, I assume the principal to be risk-neutral and normalize the agent’s coefficient of absolute risk-aversion to one so that the agent’s utility function takes the form

$$u(S, c(p)) = -e^{-[S - \Sigma_{r=1}^m c_r(p_r)]},$$

where $c_\tau(\cdot) \equiv \theta c(\cdot)$ is the agent’s personal cost of implementing $p_\tau$ in one of the $m$ sub-periods.\textsuperscript{8} $c'$ is used to represent the (positive) first derivative of $c(\cdot)$ and $S$ represents the aggregate payment under the optimal contract. The payments associated with the possible

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\textsuperscript{8}Technically, the agent’s choice can differ across the sub-periods, but since by Holmström and Milgrom (1987) Theorem 5, in equilibrium $p^*$ is constant over time, the simpler cost representation used here will suffice.
performance measure realizations in sub-period $\tau$ are denoted by $s_{ij}^\tau$, $i, j \in \{-, +\}$, where index $i$ is associated with the realization of own performance and index $j$ the realization of peer performance.

3 Optimal Relative Performance Evaluation

Under the assumptions introduced in the prior section Holmström and Milgrom (1987) shows that it is optimal to implement the same $p^*$ in each of the sub-periods and that the optimal incremental incentive payments in period $\tau$ are identical to those in period $\tau' \neq \tau$. To identify the optimal properties of the incremental incentive payments generated by the realizations of the two available performance measures it is thus sufficient to identify the contract that implements $p^*$ in any given period at the lowest possible expected compensation cost subject to an (arbitrary) IR-constraint. There is therefore no need to carry the $\tau$ subscripts on any of the variables either and the properties of the principal’s contract design problem can therefore be summarized as:

$$
\min_{s_{ij}^\tau} p^* \gamma^+ s^{++} + p^* (1 - \gamma^+) s^{+-} + (1 - p^*) \gamma^- s^{-+} + (1 - p^*) (1 - \gamma^-) s^{-+} \\
\tilde{\lambda}_0 \left\{ p^* \gamma^+ u (s^{++}) + p^* (1 - \gamma^+) u (s^{+-}) + (1 - p^*) \gamma^- u (s^{-+}) + (1 - p^*) (1 - \gamma^-) u (s^{-+}) \right\} \\
+ \mu_0 \left\{ \theta c' p^* \gamma^+ u (s^{++}) + \theta c' p^* (1 - \gamma^+) u (s^{+-}) \right. \\
+ \theta c' (1 - p^*) \gamma^- u (s^{-+}) + \theta c' (1 - p^*) (1 - \gamma^-) u (s^{-+}) \\
+ u \left( s^{++} \right) \left. \frac{d (p \gamma^+)}{dp} \right|_{p=p^*} + u \left( s^{+-} \right) \left. \frac{d (p (1 - \gamma^+))}{dp} \right|_{p=p^*} \\
+ u \left( s^{-+} \right) \left. \frac{d ((1 - p) \gamma^-)}{dp} \right|_{p=p^*} + u \left( s^{-+} \right) \left. \frac{d ((1 - p) (1 - \gamma^-))}{dp} \right|_{p=p^*} \right\}.
$$

The corresponding optimal contract is summarized by the following lemma.
Lemma 1. The contract that solves the above program satisfies

\[
\frac{1}{-u(s_\theta^+)} = \hat{\lambda}_\theta + \mu_\theta \left\{ \theta c' + \frac{d(\rho \gamma^+)}{dp}\bigg|_{p=p^*} \right\} \equiv \lambda_\theta + \mu_\theta L^{++}, \quad (1)
\]

\[
\frac{1}{-u(s_\theta^+)} = \hat{\lambda}_\theta + \mu_\theta \left\{ \theta c' + \frac{d(\rho \gamma^-)}{dp}\bigg|_{p=p^*} \right\} \equiv \lambda_\theta + \mu_\theta L^{+-}, \quad (2)
\]

\[
\frac{1}{-u(s_\theta^-)} = \hat{\lambda}_\theta + \mu_\theta \left\{ \theta c' + \frac{d(1-p \gamma^-)}{dp}\bigg|_{p=p^*} \right\} \equiv \lambda_\theta + \mu_\theta L^{-+}, \quad (3)
\]

\[
\frac{1}{-u(s_\theta^-)} = \hat{\lambda}_\theta + \mu_\theta \left\{ \theta c' + \frac{d(1-p)(1-\gamma^-)}{dp}\bigg|_{p=p^*} \right\} \equiv \lambda_\theta + \mu_\theta L^{--}. \quad (4)
\]

Proof of Lemma 1. These are standard first-order conditions which here easily can be verified to be both necessary and sufficient.

Based on Lemma 1 it is also straight-forward, using the (single) IR-constraint and one of the \( m \) identical IC-constraints, to verify that both \( \lambda_\theta \) and \( \mu_\theta \) are decreasing in \( m \) and approach zero from above as \( m \to \infty \).

Before proceeding it is worth noting that there is nothing surprising or novel about the solution to the simple one (sub-) period P-A problem as identified in Lemma 1. What is simply reaffirmed here is the standard insight, that when a positively (negatively) correlated performance measure is available and the Sufficient Statistic Condition thus does not hold for the agent’s own performance, the optimal use of this correlated performance measure is to condition the agent’s pay on matching realizations. Clearly, this role is not limited to simply “taking out (some of) the common risk” and the optimal contract is thus not necessarily going to conform to the standard form underlying much of the empirical RPE literature.
To gain formal insights into the optimal use of correlated peer performance in the limiting case where there are infinitely many identical subsequent periods or, more appropriately, instants and where the aggregate performance measures thus becomes normal, first introduce the "\(N - 1\) accounts" as in Holmström and Milgrom (1987), here corresponding to each of the three different "\(s_{ij} - s_{-} - 0\)" events that can take place in each of the \(m\) sub-periods. That is an account for the event \(ij = ++\), an account for the event \(ij = +-\) and one for \(ij = --\). In each sub-period, then, credit the "++ account" with \(\theta\) if event ++ occurs and with zero if that event does not occur and similarly for the two other accounts. Let \(A^{++}, A^{+-}\) and \(A^{--}\) denote the respective account balances of these three accounts at the end of the contracting horizon. Then:

**Lemma 2** The agent’s optimal RPE contract can be written as

\[
S(\omega, \pi) = m \times \left( s_{-}^{--} + A^{-} \times \left[ s_{-}^{+-} - s_{-}^{--} \right] + A^{+-} \times \left[ s_{-}^{++} - s_{-}^{--} \right] + A^{++} \times \left[ s_{+}^{++} - s_{-}^{--} \right] \right).
\]

(5)

The contract presented in the above Lemma simply summarizes the key insight from Holmström and Milgrom (1987) that the optimal contract for a \(m\) sub-periods problem can be obtained by solving the contract for one of the sub-periods and then simply multiply each of these \(1\) sub-period payments by the number of times each uniquely compensated event occurs. Again, the contract as here presented reveals little if anything about the role of peer performance and the determinants hereof. The following Theorem, however, provides the basis for gaining such insights:

**Theorem 1** Let \(m \to \infty\) so the accounts become continuous on \([0, 1]\). The optimal contract then takes the form

\[
\tilde{S}(\omega, \pi) = m \times \left[ \lambda_{\theta} + \tilde{\mu}_{\theta}\left\{ \left( 1 - A^{-} - A^{+-} - A^{++} \right) \times L^{-} \right. \right.
\]

\[
+ A^{+-} \times L^{+-} + A^{++} \times L^{++} + A^{++} \times L^{++} \right\},
\]

(6)

where \(\tilde{\mu}_{\theta} > 0\) and \(\left( 1 - A^{-} - A^{+-} - A^{++} \right)\) is the aggregate account balance for the "missing" \(N^{th}\) account, \(A^{--}\).
Proof of Theorem 1. From the derivations underlying Lemma 3, the change in the agent’s compensation associated with a change in $A^{ij}$ is simply
\[
\ln \left( \lambda_\theta + \mu_\theta L^{--} + \mu_\theta \left( L^{ij} - L^{--} \right) \right)
\]
Since in the limit $s^{ij} - s^{--}$ becomes infinitessimal small as $m$ grows large, the above expression converges to
\[
s^{--} + \frac{\mu_\theta (L^{ij} - L^{--})}{\lambda_\theta + \mu_\theta L^{--}},
\]
where it follows from Theorem 1 that both $\mu_\theta$ and the denominator are strictly positive. Then, since $s^{--}$ can always be expressed as $\frac{\mu_\theta L^{--}}{\lambda_\theta + \mu_\theta L^{--}}$ the result follows. ■

The basic message here is that when the information-flow about own and peer performance is frequent and timely as in the case of stock returns, the properties of the optimal RPE contract are exclusively determined by the likelihood ratios here. That the natural logs imbedded in (5) vanishes in the limit simplifies matters significantly. The objective at hand now is to convert (6) into a function of own and peer performance. Theorem 2 provides specifics.

Theorem 2 Let $m \to \infty$ so the accounts become continuous on $[0, 1]$. The optimal dynamic contract (6) then can be re-expressed as
\[
\tilde{S} (\overrightarrow{\omega}, \overrightarrow{\pi}) = \beta_0 + \beta_\Omega \Omega + \beta_\Pi \Pi + \beta_\rho \rho,
\]
where $\rho$ is the rank-correlation between $\overrightarrow{\omega}$ and $\overrightarrow{\pi}$.

Proof of Theorem 2. Define
\[
A^\Omega \equiv A^{++} + A^{+-}
\]
and
\[
A^\Pi \equiv A^{++} + A^{-+}
\]
so that $A^\Omega$ is the “account balance” for aggregate own positive performance regardless of peer performance and $A^\Pi$ is the “account balance” for aggregate positive peer performance regardless of own performance. $A^\Omega = 0$ ($= 1$) then corresponds to the lowest (highest) possible realization of $\Omega$ with $\Omega$ increasing linearly in $A^\Omega$ while $A^\Pi = 0$ ($= 1$) then corresponds to the lowest (highest) possible realization of $\Pi$ with $\Pi$ increasing linearly in $A^\Pi$. Accordingly, if $\tilde{S} (\overrightarrow{\omega}, \overrightarrow{\pi})$ can be expressed as some function of $\Omega$ and $\Pi$, it can also be expressed as a function of $A^\Omega$ and $A^\Pi$. Finally, let for simplicity $b^{ij}$, denote the total payment received by the agent if $A^{ij} = 1$.

Since here generally there are four different values of $b^{ij}$, to determine $\tilde{S} (\overrightarrow{\omega}, \overrightarrow{\pi})$ requires all four account balances. It is then straight forward to verify that $A^\Omega$ and $A^\Pi$ are generally
not sufficient to determine aggregate compensation since the system (8), (9) together with
the “summing-up” restriction

\[ 1 = A^{++} + A^{+-} + A^{-+} + A^{--} \] (10)

does not uniquely identify the four account balances.
Since the rank correlation between realized own and peer performance, \(\rho\), is determined as

\[ \rho = A^{++} - A^{+-} - A^{-+} + A^{--} \] (11)

and since it is easily verified the determinant of the system (8), (9), (10) and (11) is non-zero the result follows.

Clearly, the use of peer performance in the optimal dynamic contract derived here is more complex and conceptually different from the common suggestion in the RPE literature. However while a simple linear combination of aggregate own and peer performance does not generally suffice to determine compensation does not imply that the optimal contract is much more complicated of difficult to administer. There is no need for keeping track of how the return paths match up at various points of time. In the optimal contract the rank correlation between own and peer performance over the entire horizon is a sufficient statistic for the information about relative paths missing in the aggregate own and peer performance measures.

That in general optimal compensation depends on the evolution of own and peer returns as captured by \(\rho\) does, of course, not rule out that, under broad and/or reasonable conditions the optimal weight on \(\rho\) would be (close to) zero while the optimal weight on peer performance would be “significantly different from zero” with a sign opposite of the (equilibrium) covariance between own and peer performance. This is ruled out by Theorem 3.

**Theorem 3** For the model specified in section 2, if the set \(\{\Omega, \Pi\}\) is a sufficient statistic for \(\{\Omega, \Pi, \rho\}\) with respect to \(p\), then \(\Omega\) is a sufficient statistic for \(\{\Omega, \Pi, \rho\}\) with respect to \(p\).

**Proof of Theorem 3.** First, it follows directly from Theorem 1 that for \(\tilde{S}(\overline{\omega}, \overline{\pi}) = \beta_0 + \beta_\Omega \Omega + \beta_\Pi \Pi\),

\[ L^{++} - L^{+-} = L^{-+} - L^{--}. \]

Since

\[ q = p\gamma^+ + (1 - p)(1 - \gamma^-) \]
and thus
\[ \gamma^+ + \gamma^- - 1 + p\gamma^+_p - (1 - p)\gamma^-_p = 0, \]
this requirement is easily verified to be equivalent to requiring that
\[ \left( \frac{p}{1-p} \right)^2 = \frac{\gamma^+(1-\gamma^+)}{\gamma^-(1-\gamma^-)} \]
(12)
which in turn implies that
\[ \gamma^+_p (1 - 2\gamma^+)_p = -\gamma^-_p (1 - 2\gamma^-). \]

Since (12) together with the requirement that \( sgn(\gamma^+ - p) = sgn(\gamma^- + p - 1) \) implies that \( sgn(1 - 2\gamma^+) = sgn(1 - 2\gamma^-) \), given the requirement that \( sgn(\gamma^+_p) = sgn(\gamma^-_p) \) this last equality is satisfied if and only iff \( \gamma^+_p = \gamma^-_p = 0 \). This, in turn, implies \( L^{++} = L^{+-} \) and \( L^{-+} = L^{--} \) and thus that the optimal contract is independent of \( \Pi \).

The main take-away from this result is that the properties arising endogenously in the dynamic model are almost certainly different from those obtained from one-shot models as well as from the ad-hoc restricted LEN specification. The general case here is clearly one where the contract is not simply linear in aggregate own and peer performance. Whenever peer performance is optimally referenced in the optimal contract, relative performance history, as captured by the realized correlation between own and peer performance over the entire horizon, is as well. More significantly, but arguably not too surprising, the converse is not the case. As a matter of fact, for the arguably "neutral" benchmark case of \( \gamma^+ = \gamma^- (\equiv \gamma) \) peer performance plays no role in the optimal RPE contract beyond determining the correlation, \( \rho \). Theorem 4 provides specifics.

**Theorem 4** Let \( \gamma^+ = \gamma^- \ \forall p \). In the limiting case where \( m \to \infty \), the optimal contract given by (6) can be written on the form
\[
\tilde{S}(\vec{\omega}, \vec{\pi}) = \beta_0 + \beta_\Omega \Omega + \beta_\rho \rho.
\]

**Proof of Theorem 4.** First note that when \( \gamma^+ = \gamma^- \ \forall p \),
\[
L^{++} - L^{--} = L^{+-} - L^{-+}
\]
and let for simplicity \( b^{ij} \), denote the total payment received by the agent if \( A^{ij} = 1 \). From (6) it is then also easily verified that \( b^{--} < b^{-+} < b^{--} < b^{++} \).
Next, fix again own performance at some level \( A^\Omega \), but instead of fixing peer performance fix instead the realized correlation coefficient of own and peer performance, \( \rho \), at some level, say \( \overline{\rho} \). Since \( A^\Omega \equiv A^{++} + A^{+-} \) and \( \rho \equiv A^{++} + A^{+-} - A^{+-} - A^{++} \), clearly there are infinitely many account balance combinations that corresponds to any (interior) \( A^\Omega, \overline{\rho} \) pair. However, for every fixed \( A^\Omega, \overline{\rho} \), using their respective definitions (and the fact that \( A^{++} + A^{+-} + A^{+-} + A^{++} = 1 \)), we can re-express \( \overline{\rho} \) as

\[
\overline{\rho} = 1 - 2A^{+-} - 2A^{++}.
\]

Consider then continuously changing \( A^{+-} \) from the lowest to the highest value consistent with \( A^\Omega, \overline{\rho} \). From the above it is then straightforward to verify that 

\[
\frac{dA^{++}}{dA^{+-}} \bigg|_{A^\Omega, \overline{\rho}} = \frac{dA^{-+}}{dA^{+-}} \bigg|_{A^\Omega, \overline{\rho}} = -\frac{dA^{+-}}{dA^{+-}} \bigg|_{A^\Omega, \overline{\rho}} = -1.
\]

Then it follows directly from (the corollary to) Lemma 2 and Theorem 1 that

\[
\frac{dS(\omega, \pi)}{dA^{+-}} \bigg|_{A^\Omega, \overline{\rho}} = 0
\]

and thus the optimal compensation for a given level of \( \Omega \) and \( \rho \) is independent of the specific combination of the associated account balances.

Due to this property of the optimal contract it is always possible to rewrite the optimal contract as some linear function of \( A^{++}, A^{+-} \) and \( A^{+-} \) hereafter denoted \( \psi \left(A^\Omega, A^{++}, A^{+-}\right) \). Then, for \( A^\Omega = 1, \rho = 1 \) if \( A^{+-} = 0 \) and \( \rho = -1 \) if \( A^{+-} = 1 \) so that for \( A^\Omega = 1, \rho = 2 ((1 - A^{+-}) - \frac{1}{2}) \). For \( A^\Omega = 0, \rho = 1 \) for \( A^{+-} = 0 \) and \( \rho = -1 \) for \( A^{+-} = 1 \) so that for \( A^\Omega = 0, \rho = 2 ((1 - A^{+-}) - \frac{1}{2}) \). Then, again due to the "linearity in accounts" the optimal compensation corresponding to a particular \( \rho \), say \( \overline{\rho} \), is given as

\[
A^\Omega \times \psi \left(1, \frac{1 - \overline{\rho}}{2}, 0\right) + (1 - A^\Omega) \times \psi \left(0, 0, \frac{1 - \overline{\rho}}{2}\right)
\]

\[
= \psi \left(0, 0, \frac{1 - \overline{\rho}}{2}\right) + A^\Omega \left[ \psi \left(1, \frac{1 - \overline{\rho}}{2}, 0\right) - \psi \left(0, 0, \frac{1 - \overline{\rho}}{2}\right) \right].
\]

Further note that

\[
\psi \left(1, \frac{1 - \overline{\rho}}{2}, 0\right) - \psi \left(0, 0, \frac{1 - \overline{\rho}}{2}\right)
\]

\[
= b^{+-} + \left(1 - \frac{1 - \overline{\rho}}{2}\right) (b^{++} - b^{+-}) - \left(b^{+-} + \frac{1 - \overline{\rho}}{2} (b^{++} - b^{+-})\right) = b^{+-} - b^{--} (> 0),
\]

and therefore the reward for own performance is independent of \( \rho \).

Now also note that since the optimal compensation is linear in \( \Omega \) and \( dS/d\Omega \) is independent of \( \rho \), \( dS/d\rho \) is also independent of \( \Omega \). Then, given that

\[
\frac{dS}{d\rho} \bigg|_{A^{\Omega}=0} = -\frac{b^{+-} - b^{--}}{2} = \frac{b^{++} - b^{+-}}{2} = \frac{dS}{d\rho} \bigg|_{A^{\Omega}=1},
\]

\[
\frac{dS}{d\rho} = \frac{b^{++} - b^{+-}}{2}
\]
independent of own performance, \( \Omega \). Since for any \( \{\Omega, \Pi\} \) pair, the feasible combinations of \( A^{+-} \) and \( A^{-+} \) correspond uniquely to a particular level of \( \rho \), this concludes the proof.

The intuition for Theorem 4 can be gleaned from the plots in Figures 1, 2 and 3. Figure 1 plots of the potential values of \( \mathcal{S}_2(\overline{w}, \overline{\pi}) \) as a function of the account balances. Both plots are done for positive (expected) covariance between own and peer performance, which here implies \( \gamma^i > .5 \). Panel A represents the case where \( q > p^* \), while Panel B represents the case where \( q < p^* \). Panel A can easily be verified to correspond to the cases where the covariance between own and peer performance is increasing in \( p \) at \( p^* \), while Panel B corresponds to the case where it is decreasing. And while in both cases own performance is rewarded \textit{in expectation}, in the case of Panel A mismatched performance is penalized while in Panel B mismatched performance is instead rewarded. What these panels reveal is thus by no means earth shattering: they simply reaffirm the basic insights about the optimal use of correlated performance measures in standard P-A models.

Figure 2 collapses Figure 1 panel A into two two-dimensional plots, the first in the "own" dimension (Panel A) and the second in the "peer" direction (Panel B). The bold lines through the plotted areas in Panel B of both figures are the mean-squared-error-minimizing (\textit{OLS}) lines for either "sample" taking into account the underlying joint density. Accordingly, the slope of these lines represent the \textit{OLS} regression-coefficients one would expect to obtain from a (standard \textit{RPE}) regression. Figure 3 does the same for Panel B of Figure 1 as Figure 2 does for Panel A of Figure 1. The key take-away is that in the benchmark case where \( \gamma^+ = \gamma^- \), \( \beta_{\Pi} \) is predicted to be zero whether the covariance is increasing or decreasing in own effort. The skeptic can easily verify the same would be true if the correlation between own and peer performance had been negative. That \( \beta_{\Omega} \) is always positive here independent of the sign of the covariance is also straight-forward to verify.
4 Empirical Implications and Predictions

So far the analysis has focused on a just a single peer firm. It is, however, straightforward to extend the model to multiple such peers. In the case where there are more than one peer, the optimal contract is simply extended by adding two additional terms for each additional peer: one for total performance of the peer and one for the correlation between own and peer performance. Of course, the standard empirical approach, following Antle and Smith (1982), has not been to estimate a separate coefficient on each peer but rather rely on the (appropriately weighted) average performance of a group of peers also exposed to the common shock. This was initially justified based on Holmstrom (1982), but relying on a peer group performance index rather than the performance of the individual peers also have support in Banker and Datar (1988) and causes no further loss of generality in LEN models.

The key to forming peer groups in the standard models of RPE is of course that the effect of the common shock should have the same sign on all peers in the same group. Empirically, only peers with positive covariance with the focal firm are considered. From a theoretical vantage point, however, a group of peers all with negative correlated performance with the focal firm would of course be equally valid albeit the predicted coefficient for that group is positive rater than the standard prediction of a negative coefficient on positively correlated peers.

The dynamic approach I offer in this paper lends itself equally well to relying on weighted-average peer group performance measures due to the simple linear additive form of the optimal contract. The basis for assigning firms to a specific peer group differ fundamentally from that of the traditional static models, however. Rather than doing the assignment based on the sign of the covariance, the assignment should be based on the sign of the effect of the agent’s actions on the correlation. The final issues I will provide some answers to are the empirical predictions of my dynamic model for the standard RPE regression of the focal agent’s compensation on aggregate own and a peer performance index for peers presumed to have similar exposure to the common shock as well as the predictions for the regression
motivated by the analysis in this paper and where the peer groups are formed based on the effect of the focal agent’s effort on the correlation with the peers.

While it follows directly from the analysis that the optimal weight on aggregate own performance is always positive regardless of peer performance and that the weight on the correlation coefficient is non-zero with the sign being the same as the effect of effort on the correlation when peer groups are formed based on that sign, what remains to be addressed is then the predictions for the coefficient on the aggregate peer performance indices in either regression. The answers to those questions inevitably depend on the statistical properties of the firms making up the population and to be able to say anything thus requires some assumption about the population. I propose and utilize the seemingly very weak and natural, at least for a reasonably sized sample, assumption of “symmetry” of the joint parameter distribution defined as follows:

Definition 1 The joint parameter distribution for the population of all firms is said to be symmetric if

\[ \begin{array}{ll}
& i) \quad pr(p = \hat{p}) = pr(p = 1 - \hat{p})|p \in (0, 1), \\
& ii) \quad pr[\gamma^+] = \gamma|p = \hat{p}, sgn(\gamma^+ - p) = i] = \\
& pr[\gamma^-] = \gamma|p = 1 - \hat{p}, sgn(\gamma^- - (1 - p)) = i], \hat{p} \in (0, 1), i \in \{-, +\} \\
& iii) \quad pr[\gamma^+_p] = \gamma|p = \hat{p}, sgn(\gamma^+_p) = i] = \\
& pr[\gamma^-_p] = \gamma|p = 1 - \hat{p}, sgn(\gamma^-_p) = i], \hat{p} \in (0, 1), i \in \{-, +\} \\
\end{array} \]

With this I have

Theorem 5 Suppose that the joint parameter distribution in the population of firms is symmetric. The OLS estimate of the coefficient on average peer performance $\hat{\beta}_{11}$ in a regression of own compensation on own and either i) average peer performance where the peer group is formed based on the sign of the covariance or ii) correlation between own and peer performance and average peer performance where the peer group is formed based on the sign of the effect of effort on the covariance, is zero.

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Proof of Theorem 4. To be provided □

5 Discussion

The results obtained in the previous section differ fundamentally from the standard predictions in the \textit{RPE} literature. A key difference is clearly (but in my mind entirely unsurprisingly) that in the general case, it is not the sign of the covariance that drives optimal \textit{RPE}; it is what the agent can do about the covariance that matters! From an empirical vantage point that is, at first glance, somewhat unfortunate as it moves the predictor from being an equilibrium property that can be easily estimated (covariance) to an out-of-equilibrium one (change in covariance) than cannot, at least not directly. If one steps back from the technical aspects of the \textit{RPE} results, both the common ones in the existing aggregate performance evaluation literature and those presented here, and instead considers the general economic intuition behind them I think the ones presented here make a lot more sense; both in terms of what they say about identifying the relevant peers and in terms of the observable variables that should be able to explain observed compensation.

To see why this is, consider again the Holmström (1982) Theorem 9 case. While insightful and elegant, I don’t see much enthusiasm in the literature for the case that the predicted weight on peer performance should be minus one.\textsuperscript{9} The reason, I suspect, is that as discussed in the introduction it corresponds to a case where only relative performance, not own performance, matters. Generally, I think, most people’s experience with \textit{RPE} is less extreme. Winning is not enough, for example. If you played well typically matters too. Doing slightly better than Enron is not likely to elicit much applause, having the fastest car on the block is not really that cool if it is a ’87 Yugo GV, having the fewest publications in the department is a bit different if fewest means 20 than if it means zero and so on.\textsuperscript{10} In other words, a

\begin{footnotesize}
\textsuperscript{9} Allocation of fixed bonus pools or total raises available would fit as an example, but is in my mind not the kind of \textit{RPE} we expect for executives that are typically the subjects of study in this literature.
\end{footnotesize}

\begin{footnotesize}
\textsuperscript{10} Furthermore, while pundits for many years have called for, for example, indexing Executive Stock Options thus effectively placing a negative weight of one on peer performance, that idea appears to have gained very
\end{footnotesize}
negative weight of one doesn’t correspond to the way we intuitively think about RPE and it shouldn’t be surprising either, that it isn’t what the data suggest.

I would argue that the same type of reality check on the standard (strong form) prediction in the existing RPE literature does not actually correspond to most people’s intuition or experiences either. I cannot think of (m)any good economic examples where independent of whether the individual did better or worse than his peers, individual performance is simply adjusted (down) by some fraction of average peer performance. Alternatively, I don’t think that people exposed to RPE are ever satisfied just to be provided with some summary statistic without being able to discern how they actually performed relatively to their peers either. Telling students that they got a B+ is not going to be sufficient without also telling them the mean for their peers. Or telling a faculty member that their raise is 3% is not going to work that well, unless it is accompanied with information about what the peers got and why. However, if the conditions that underlie the standard “strong form” prediction are met, providing anything beyond the final aggregate should really be of no significance other than perhaps to verify that the aggregation was done correctly. And again the data agrees with the intuition/casual empiricism here: it is just not how RPE appears to be done in general.

The key insights generated by my analysis seem, at least to me, fare a lot better in this gut-check contest. First of all, unlike in the case of Holmström’s (1982) Theorem 9 both own and peer performance matter. Second, relative performance matters too, because a simple linear aggregate as in Banker and Datar (1989) is not sufficient in general to communicate the implications of own and peer performance. More importantly perhaps, the notion that it is the change in the covariance that is key appears intuitively consistent with how we go about RPE in all kinds of standard situations. Think, for example, about a parent trying to encourage his/her child to excel in High School. How to best do this may, of course, differ by the nature of the school, culture, etc., but I don’t believe the typical approach is little traction as a practical matter.
that the parent compares the kid with the average student in the school or deducting some fraction of average test scores from their own child’s scores as part of metering out a reward (or punishment). Rather, it is to identify peers that the student should try to be more like (think honor roll) and/or peers that the student should try and be less like (I’ll leave identification of this group to the reader’s own experience and imagination).

This approach to identify groups of peers suggested by my analysis seems so much more in line with how I believe we think about \( RPE \) in general. For a cell phone maker, for example, my guess is that the benchmark used for \( RPE \) is not based on a weighted average of Apple and RIM. Rather, \( RPE \) is used to encourage managers at this point in time to make their firms more like the Apples of the world and less like the RIMs. Politicians argue that we should be less like Greece and more like whatever country they view as having desirable properties to be emulated. People have heros and role models they aspire and strive to be like and similarly reference groups they actively try and disassociate themselves from. The examples are endless and I’d therefore argue that this kind of “good peer, bad peer” \( RPE \) suggested by my very basic analysis is the norm while the use of an “average peer” is at best a marginal and, as the evidence suggests, clearly an empirically irrelevant exception.

Furthermore, the task of identifying the relevant peer groups I think is actually quite straightforward. Becoming more like an ideal group implies as far as I am concerned not simply boosting ones performance relative to this group, but more importantly increasing ones covariance with it. Becoming less like another group of peers have the same relative consequences but also implies reducing the covariance with that group. And as I have shown in the previous section, reward structures that provide incentives to become more like some group differ fundamentally from reward structures that provide incentives to distance oneself from another group. Moreover, in neither case does the optimal reward structure disintegrate into simply placing a positive weight on own performance and a negative weight on average aggregate performance peers similarly exposed to a common shock. Instead, it adjusts own performance with relative performance as captured by the realized correlation between own
and peer performance.

6 Conclusion

I develop a set of agency theory based predictions about $RPE$ using the basic framework of Holmström and Milgrom (1987). As I show, that generally the covariance between jointly normal own and peer performance is not independent of the agent’s effort in this type of model. Accordingly, as pointed out by Banker and Datar (1989), linearly aggregated performance measures are therefore not sufficient and a contract written on own minus scaled peer performance is not optimal. What is optimal depends on how the agent’s effort impacts the covariance between own and peer performance. If it increases the covariance the optimal contract, while increasing in own performance, rewards "similar" performance with peers. If the covariance is decreasing, the optimal contract instead rewards "dissimilar" performance. As a result, the optimal contract varies in the correlation between own and peer performance but, consistent with the empirical evidence, optimal $RPE$ is likely to be independent of aggregate peer performance.
References


7 Figures

Figure 1. $\bar{S}(\omega', \pi')$ for $\frac{d\gamma}{dp} > 0$ (Panel A) and $\frac{d\gamma}{dp} < 0$ (Panel B).
Figure 2. Compensation from Figure 1, Panel A levels plotted against own (Panel A) and peer (Panel B) performance.
Figure 3. Compensation from Figure 1, Panel B levels plotted against own (Panel A) and peer (Panel B) performance.