Relational Contracts as a Foundation for Bonus Pools

Jonathan Glover

Carnegie Mellon University

(jglover@cmu.edu)

Hao Xue

New York University

(hxue@nyu.edu)

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Abstract

Much of our thinking about (and criticism of) the relationship between pay and performance seems to be based on a standard principal-agent model of explicit contracting and individual incentives. In this paper, we instead study implicit contracting, team incentives for mutual monitoring (tacit cooperation), and the interactions between team and individual incentives and implicit and explicit contracting. Bonus pools, which are optimal in the single-period version of our model, reward poor performance. We identify a broader role for such rewards. Paying for poor performance also has desirable incentive properties in motivating mutual monitoring and in mitigating tacit collusion. In an extension of our model, we study the seemingly contradictory incentives provided by bonus pools for both cooperation and competition.

1 Introduction

Discretion in awarding bonuses and other rewards is pervasive. Evaluators use discretion in determining individual rewards, the total reward to be paid out to all (or a subset of the) employees, and even in deviating from explicit bonus formulas (Murphy and Oyer, 2001; Gibbs et al., 2004). A common concern about discretionary rewards is that the evaluator must be trusted by evaluates (Anthony and Govindarajan, 1998).

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In a single-period model, bonus pools are a natural economic solution to the "trust" problem (Baiman and Rajan, 1995; Rajan and Reichelstein, 2006; 2009). When all rewards are discretionary (based on subjective assessments of individual performance), a single-period bonus pool rewards poor performance, since the total size of the bonus pool must be a constant in order to make the evaluator's promises credible. We identify a broader role for such rewards, which are often criticized (e.g., Bebchuk and Fried, 2006). Paying for poor performance also has desirable incentive properties in motivating mutual monitoring and in mitigating tacit collusion.

The relational contracting literature has explored the role repeated interactions can have in facilitating trust and discretionary rewards based on subjective/non-verifiable performance measures (e.g., Baker, Gibbons, and Murphy, 1994), but this literature has mostly confined attention to single-agent settings.[1] In this paper, we explore optimal discretionary rewards based on subjective/non-verifiable individual performance measures in a multi-period, principal-multi-agent model. The multi-period relationship creates the possibility of trust between the principal and the agents, since the agents can punish the principal for reneging behavior. At the same time, the multi-period relationship creates the possibility of trust between the agents and, hence, opportunities for both cooperation/mutual monitoring (beneficial to the principal) and collusion (harmful to the principal) between the agents.

In our model, all players share the same expected contracting horizon (discount rate). Nevertheless, the players may differ in their relative credibility because of other features of the model such as the loss to the principal of forgone productivity. In determining the optimal incentive arrangement, both the common discount rate and the relative credibility of the principal and the agents are important.

When the principal's ability to commit is strong, the optimal contract emphasizes team incentives (tacit cooperation) and joint performance (JPE). The principal uses the non-verifiable performance measures to set the stage for the agents to mutually monitor each other, as in exist-

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[1]One exception is Levin (2002), who examines the role that trilateral contracting can have in bolstering the principal's ability to commit—if the principal's reneging on a promise to any one agent means she will loose the trust of both agents, relational contracting is bolstered.

ing models with verifiable performance measures (e.g., Arya, Fellingham, and Glover, 1997; Che and Yoo, 2001).[2] As the principal's ability to commit becomes a binding constraint, rewarding (joint) poor performance can be a feature of the optimal compensation arrangement. The role of rewarding poor performance is that it enables the principal to keep incentives focused on mutual monitoring by using *JPE*. The alternative is to use relative performance evaluation (*RPE*) to partially substitute mutual monitoring incentives with individual incentives. When the agents' ability to commit is relatively strong, the benefit of mutual monitoring is so large that relying entirely on mutual monitoring for incentives is optimal. When the agents' ability to commit is instead relatively weak, mutual monitoring's advantage over individual incentives is small, and substituting individual incentives using *RPE* is optimal.

When the principal's ability to commit is weaker, motivating mutual monitoring is not optimal. As the principal's ability to commit vanishes and the problem converges to a single-period contracting problem, the solution converges to a static bonus pool. Static bonus pools reward agents for poor performance. Even when the principal's ability to commit is stronger, the optimal contract may reward poor performance. In this case, paying for poor performance allows the principal to create a strategic independence in the agents' payoffs that reduces their incentives to tacitly collude. If the principal did not have to prevent tacit collusion between the agents, she would instead use *RPE*. Here, the unappealing feature of *RPE* is that it creates a strategic substitutability in the agents' payoffs that encourages them to collude on an undesirable equilibrium that has them taking turns making each other look good—they alternate between (*work, shirk*) and (*shirk, work*).[3] Individual performance evaluation (*IPE*) can be seen as one of a class of incentive arrangements that create strategic independence. *IPE* is the only such arrangement that does not involve rewarding poor performance. Pay for poor performance and the strategic independence it facilitates is optimal when the agents' ability

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[2]There is an earlier related literature that assumes the agents can write explicit side-contracts with each other (e.g., Tirole, 1986; Itoh, 1993). Itoh's (1993) model of explicit side-contracting can be viewed as an abstraction of the implicit side-contracting that was later modeled by Arya, Fellingham, and Glover (1997) and Che and Yoo (2001). As Tirole (1992), writes: "[i]f, as is often the case, repeated interaction is indeed what enforces side contracts, the second approach [of modeling repeated interactions] is clearly preferable because it is more fundamentalist."

[3]Even in one-shot principal-multi-agent contracting relationships, the agents may have incentives to collude on an equilibrium that is harmful to the principal (Demski and Sappington, 1984; Mookherjee, 1984).

to commit is relatively strong and, hence, the cost of preventing collusion is relatively large. When the agents' ability to commit is instead relatively weak, RPE is optimal since the cost of preventing collusion is relatively small.

There is a puzzling (at least to us) aspect of observed bonus pools. Managers included in a particular bonus pool are being told that they are part of the same team and expected to cooperate with each other to generate a larger total bonus pool (Eccles and Crane, 1988). Those same managers are asked to compete with each other for a share of the total payout. We extend the model to include an objective/verifiable team-based performance measure. Productive complementarities in the objective team-based measure can make motivating cooperation among the agents optimal when it would not be in the absence of the objective measure. The productive complementarity also takes pressure off of the individual subjective measures, allowing for a greater degree of RPE (and less pay-for-bad performance) than would otherwise be possible. Put differently, the combination of rewarding a team for good performance but also asking agents to compete with each other for a share of the total reward is not inconsistent with motivating cooperation and mutual monitoring. Instead, such commonly observed schemes can be an optimal means of motivating cooperation and mutual monitoring when the principal's commitment is limited. The earlier theoretical literature on bonus pools did not develop this role for bonus pools because of their focus on static settings.

Like our main model, Kvaløy and Olsen (2006) study a multi-period, multi-agent model in which all performance measures are subjective. They allow for even more reneging that rules out pay for bad performance, which is the focus of our paper. Our extension is closely related to Baldenius and Glover (2012) on dynamic bonus pools. They take the form of the bonus pool as given. In particular, all of their bonus pools have the feature that the total payout does not depend on the subjective performance measures. In contrast, the focus of this paper is on optimal contracts, which incorporate discretion in determining the size of the bonus pool.

Baiman and Baldenius (2009) study the role of non-financial performance measures can have in encouraging cooperation by resolving hold-up problems. The empirical literature also provides evidence consistent with discretion being used to reward cooperation (e.g., Murphy

and Oyer, 2001; Gibbs et al., 2004). Our model is consistent with this view in that the discretionary rewards are used to motivate cooperation when possible. Our analysis highlights the importance of both the evaluator's and the evaluatees' reputation in sustaining cooperation through mutual monitoring.

The remainder of the paper is organized as follows. Section 2 presents the basic model. Section 3 studies implicit side-contracting between the agents. Section 4 studies the principal's problem. Section 5 characterizes the optimal overall contract. Section 6 studies an extension in which there are both individual subjective performance measures (as in the rest of the paper) and an objective team-based performance measure. Section 7 concludes.

2 Model

A principal contracts with two identical agents, $i = A, B$, to perform two independent and ex ante identical projects (one project for each agent) in an infinitely repeated relationship, where t is used to denote the period, $t = 1, 2, 3, ...$ Each agent chooses a personally costly effort $e^i \in \{0, 1\}$ in period t, i.e., the agent chooses either "*work*" ($e_t^i = 1$) or "*shirk*" ($e_t^i = 0$). Each agent's personal cost of *shirk* is normalized to be zero and of *work* is normalized to be 1. Agent i's performance measure in period t, denoted x_t^i, is assumed to be either high ($x_t^i = H > 0$) or low ($x_t^i = L = 0$) and is a (stochastic) function of only e_t^i. In particular, $q_1 = \Pr(x^i = H | e^i = 1)$, $q_0 = \Pr(x^i = H | e^i = 0)$, and $0 < q_0 < q_1 < 1$. (Whenever it does not cause confusion, we drop sub- and superscripts.) Notice that each agent's effort choice does not affect the other agent's probability of producing a high outcome. Throughout the paper, we assume each agent's effort is so valuable that the principal wants to induce both agents to *work* ($e_t^i = 1$) in every period. (Sufficient conditions are provided in an appendix.) The principal's problem is to design the contract that motives both agents to work in each and every period at the minimum cost.

Because of their close interactions, the agents observe each other's effort choice in each period. Communication from the agents to the principal is blocked—the outcome pair (x^i, x^j)

is the only signal on which the agents' wage contract can depend. Denote by w^i_{mn} the wage agent i receives if his outcome is m and his peer's outcome is n; $m, n \in \{H, L\}$. For tractability, we assume the ex ante identical agents are offered ex ante identical wage schemes, so we drop the i superscript. This is a restrictive assumption in that asymmetric contracts can be preferred by the principal, as in Demski and Sappington's (1984) single period model. As we will show shortly, restricting attention to symmetric contracts greatly simplifies our infinitely repeated contracting problem by reducing the set of collusion constraints to two. Without the restriction to symmetric contracts, we know of no way to simplify the set of collusion constraints into a tractable programming problem.

The wage contract provided to agent i is a vector $w \equiv \{w_{HH}, w_{HL}, w_{LH}, w_{LL}\}$. Given wage scheme w and assuming that agents i and j choose efforts level $k \in \{1, 0\}$ and $l \in \{1, 0\}$ respectively, agent i's expected wage is:

$$\pi(k, l; w) = q_k q_l w_{HH} + q_k(1 - q_l)w_{HL} + (1 - q_k)q_l w_{LH} + (1 - q_k)(1 - q_l)w_{LL}.$$

All parties in the model are risk neutral and share a common discount rate r, capturing the time value of money or the probability the contract relationship will end at each period (the contracting horizon). The agents are protected by limited liability—the wage transfer from the principal to each agent must be nonnegative:

$$w_{mn} \geq 0, \forall m, n \in \{H, L\} \qquad \text{(Non-negativity)}$$

Unlike Che and Yoo (2001), we assume the outcome (m, n) is unverifiable. The principal, by assumption, can commit to a contract form but cannot commit to reporting the unverifiable performance outcome (m, n) truthfully.[4] Denote the principal's report on the performance measures in period t by $\widehat{x}_t = (\widehat{x}^A_t, \widehat{x}^B_t)$. Like Che and Yoo (2001), the contract is short term in the sense that it specifies wages in each period as a function of only the current period's

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[4]In contrast, Kvaløy and Olsen (2006) assume the principal cannot commit to the contract, which makes it optimal to set wLL = 0. Our assumption that the principal can commit to the contract is intended to capture the idea that the contract and the principal's subjective performance rating of the agents' performance can be verified. It is only the underlying performance that cannot be verified.

performance.

Denote by H_t the history of all actions and outcomes before period t. For example $H_3 = (w, (e_1^A, e_1^B, x_1^A, x_1^B, \widehat{x}_t^A, \widehat{x}_t^B), (e_2^A, e_2^B, x_2^A, x_2^B, \widehat{x}_2^A, \widehat{x}_2^B))$. Denote by P_t the public profile at period t—those actions and outcomes that all parties observe, i.e., the history without the agents' actions. The principal's strategy is a wage scheme and reporting strategy, where the reporting strategy in period t can be conditioned on the public profile P_t up to that point. For the agents, their strategies map the entire history H_t to period t actions. The equilibrium concept is Perfect Bayesian Equilibrium (PBE) rather than Perfect Public Equilibrium, since the agents' strategies depend on more than the public profile. Among the large set of PBE's, we choose the one that is best for the principal subject to two constraints. First, the PBE must also be collusion-proof: there can be no other PBE that has only the agents changing their strategies and provides each agent with a higher payoff. Second, the PBE must motivate the principal to provide an honest evaluation of the agents—to report the performance measures truthfully.

There may be other PBEs that have the principal adopting a history-dependent lying strategy, but these would effectively mimic long-term explicit contracts. Since Che and Yoo restrict attention to short-term contracts, studying long-term contracts (or PBEs that mimic long-term contracts) in this paper would make our results difficult to compare to Che and Yoo's. It is unclear that such a PBE with a history-dependent lying strategy would provide a potential improvement to the principal, since the agents seem likely to shirk during the lying (punishment) phase. Such punishment phases typically have all parties reverting to the stage game equilibrium, and we have assumed the parameters are such that the principal always finds motivating high effort optimal.

Since the agents perfectly observe all actions, confining attention to trigger strategies for them is without loss of generality in the sense that whatever can be accomplished by a strategy that depends on a longer history can also be accomplished with a strategy that depends only on the last period. They will use these trigger strategies to motivate the principal to tell the truth and possibly also to motivate each other.

To motivate the principal to tell the truth, we consider the following trigger strategy played

by the agents: both agents behave as if the principal will honor the implicit contract until the principal lies about one of the performance measures, after which the agents punish the principal by choosing (*shirk, shirk*) in all future periods. This punishment is the severest punishment the agents can impose on the principal. The principal will not renege if:

$$\frac{2\left[q_1 H - \pi(1,1;w)\right] - 2q_0 H}{r} \geq \max\{w_{mn} + w_{nm} - (w_{m'n'} + w_{n'm'})\}. \qquad \text{(Principal's IC)}$$

This constraint assures the principal will not claim the output pair from the two agents as (m', n') if the true pair is (m, n). The left hand side is the cost of lying.[5] The agents choosing (*shirk, shirk*) and the principal paying zero to each agent is a stage-equilibrium. Therefore, the agents' threat is credible. The right hand side of this constraint is the principal's benefit of lying about the performance signal.

3 Implicit Contracting between the Agents

The fact that agents observe each other's effort choice, together with their multi-period relationship, gives rise to the possibility that they use implicit contracts to motivate each other to work as in Arya, Fellingham, and Glover (1997) and Che and Yoo (2001), as long as playing (*work, work*) Pareto-dominates all other possible strategy combinations. Consider the following trigger strategy used to enforce (*work, work*): both agents play work until one agent i deviates by choosing shirk; thereafter, agent j punishes i by choosing *shirk*:

$$\frac{1+r}{r}\left[\pi(1,1;w) - 1\right] \geq \pi(0,1;w) + \frac{1}{r}\pi(0,0;w) \qquad \text{(Mutual Monitoring)}$$

Such a mutual monitoring implicit contracting is beneficial for the principal and requires two conditions. First, each agent's expected payoff from playing (*work, work*) must be at least as high as from by playing the punishment strategy (*shirk, shirk*). In other words, (*work,*

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work) must Pareto dominate the punishment strategy from the agents' point of view in the stage game. Otherwise, (*shirk, shirk*) will not be a punishment at all:

$$\pi(1, 1; w) - 1 \geq \pi(0, 0; w) \qquad \text{(Pareto Dominance)}$$

Second, the punishment (*shirk, shirk*) must be self-enforcing. The following constraint ensures (*shirk, shirk*) will be a stage game Nash equilibrium:

$$\pi(0, 0; w) \geq \pi(1, 0; w) - 1 \qquad \text{(Self-Enforcing Shirk)}$$

The agents' observation of each other's effort choice and multi-period relationship also open the opportunity for the agents to collude with each other. A collusion strategy is an implicit contract between agents that is harmful for the principal— and we consider the following trigger strategy to support it: each agent sticks to the collusion strategy until any agent i deviates, in which case agent j would punish by choosing "*work*" indefinitely thereafter. Given the trigger strategy, any deviation from collusion will end up with (*work, work*)$^\infty$. This trigger strategy is self-enforcing because (*work, work*)$^\infty$ is the equilibrium that the principal induces and is assured by other constraints in equilibrium.

Given the infinitely repeated relationship, the space of potential collusions between the two agents is very rich: any strategy profile $e_0 = \{e_t^A, e_t^B\}_{t=0}^\infty, e_t^A, e_t^B \in \{1, 0\}$ can be a credible collusion, provided e_0 is self-enforcing. As it turns out, we can confine attention to two specific intuitive collusive strategies when constructing collusion-proof contracts. If we prevent these two types of collusion, all other possible collusion strategies are also upset.

First, the contract has to satisfy the following condition to prevent collusion on (*shirk, shirk*) in all periods:

$$\pi(1, 0; w) - 1 + \frac{\pi(1, 1; w) - 1}{r} \geq \frac{1 + r}{r}\pi(0, 0; w) \qquad \text{(No Joint Shirking)}$$

The left-hand side is the agent's expected payoff from unilaterally deviating from (*shirk, shirk*),

or "*Joint Shirking*," for one period by unilaterally choosing work and then being punished indefinitely by the other agent by playing the stage game equilibrium (*work, work*) in all future periods, while the right-hand side is his expected payoff from sticking to *Joint Shirking*.

Second, the following condition is needed to prevent agents from colluding by "Cycling," i.e., alternating between (*shirk, work*) and (*work, shirk*):

$$\frac{1+r}{r}\left[\pi(1,1;w)-1\right] \geq \frac{(1+r)^2}{r(2+r)}\pi(0,1;w) + \frac{(1+r)}{r(2+r)}\left[\pi(1,0;w)-1\right] \qquad \text{(No Cycling)}$$

The left hand side is the agent's expected payoff if he unilaterally deviates by choosing work when he is supposed to shirk and is then punished indefinitely with the stage game equilibrium of (*work, work*). The right hand side is the expected payoff if the agent instead sticks to the *Cycling* strategy.

Lemma 1 *A contract is collusion-proof if it satisfies No Joint Shirking and No Cycling conditions.*

Proof. All proofs are provided in an appendix. ∎

The intuition for Lemma 1 is that all other (less symmetric) potential collusive strategies can only provide some period t' shirker with a higher payoff than under *Joint Shirking* and Cycling if some other period t'' shirker has a lower continuation payoff than under *Joint Shirking* or Cycling. Hence, if the contract motivates all potential shirkers under *Joint Shirking* and Cycling to instead deviate to work, then so will the period t" shirker under the alternative strategy.

It is also helpful to distinguish three classes of contracts and point out how they interact with the two collusion-proof conditions above. The wage contract creates a *strategic complementarity* if $\pi(1,1)-\pi(0,1) > \pi(1,0)-\pi(0,0)$, which is equivalent to a payment complementarity $w_{HH} - w_{LH} > w_{HL} - w_{LL}$. Similarly, the contract creates a strategic substitutability if $\pi(1,1)-\pi(0,1) < \pi(1,0)-\pi(0,0)$, or equivalently $w_{HH}-w_{LH} < w_{HL}-w_{LL}$. The contract creates strategic independence if $\pi(1,1)-\pi(0,1) = \pi(1,0)-\pi(0,0)$, or $w_{HH}-w_{LH} = w_{HL}-w_{LL}$.

The reason a payoff and payment complementarity (substitutability) are equivalent is that the performance measures are uncorrelated. As noted in the following observation, this classification has implications for which collusion strategy is most profitable from the agents' point of view and, thus, more costly for the principal to upset.

Observation:

(1) *No Joint Shirking* implies *No Cycling* if the contract creates a strategic complementarity in the agents' payoffs.

(2) *No Cycling* implies *No Joint Shirking* if the contract creates a strategic substitutability in the agents' payoffs.

(3) *No Cycling* and *No Joint Shirking* are equivalent if the contract creates a strategic independence in the agents' payoffs.

Whether a contract exhibits a strategic complementarity or a strategic substitutability has a subtle impact on the nature of the agent-agent collusive strategy. Investigating when and why the principal purposely designs the contract with a strategic complementarity, substitutability, or independence is the focus of the remainder of the paper.

4   The Principal's Problem

The basic problem faced by the principal is to design a minimum expected cost wage contract $w = \{w_{HH}, w_{HL}, w_{LH}, w_{LL}\}$ that ensures (*work, work*) in every period is the equilibrium-path behavior of some collusion-proof equilibrium. The contract must satisfy the principal's reneging constraint, so that she will report her assessment of performance honestly. When designing the optimal contract, the principal can choose to explore the mutual monitoring between agents if it is worthwhile. Alternatively, he can implement a static Nash equilibrium subject to collusion proof constraints. The following integer programming summarizes the principal's problem.

$$\min_{\{w_{HH}, w_{HL}, w_{LH}, w_{LL}\}} \pi(1,1)$$

$s.t.$

No Cycling

Principal's IC

Non-negativity

$T \times$ Mutual Monitoring $\hspace{4cm} (IP)$

$T \times$ Pareto Dominance

$T \times$ Self-enforcing Shirk

$(1 - T) \times$ Static NE

$(1 - T) \times$ No Joint Shirking

$T \in \{0, 1\}$

The variable T (short for team incentives) takes a value of either zero or one, which makes the program an integer program. $T = 1$ means that the principal designs the contract in order to induce team incentives by motivating mutual monitoring between the agents. Since (*shirk, shirk*) is Pareto dominated by (*work, work*) from the agents' point of view as $T = 1$, the collusion strategy *Joint Shirking* considered in the previous section is not a concern here. Therefore, we know from Lemma 1 that the contract in this case is collusion-proof as long as it satisfies the *No Cycling* constraint.

While the agents' mutual monitoring is beneficial to the principal, implementing it can be prohibitively costly or even infeasible, in which case the principal would stick with implementing the stage game Nash equilibrium ($T = 0$), subject to the additional no collusion and principal's incentive constraints. The following condition assures that (*work, work*) as a stage Nash equilibrium.

$$\pi(1,1;w) - 1 \geq \pi(0,1;w) \qquad\qquad \text{(Static NE)}$$

We solve the integer program (IP) by the method of enumeration and complete the analysis in two steps. In Sections 4.1 and 4.2, we solve the solution of IP while restricting $T = 1$ and then $T = 0$, respectively. We then compare the solutions for each parameter region and optimize over the choice of T in Section 4.3.

4.1 Cooperation

We are now ready to characterize the optimal solution for cases in which mutual monitoring (team incentives) is optimal.

Proposition 1 *When team incentives are optimal, i.e., $T = 1$, the solution to IP is one of the following (with $w_{LH} = 0$ in all cases):*

(i) Pure JPE: $w_{HH} = \frac{1+r}{(q_1-q_0)(q_0+q_1+q_1 r)}$, $w_{HL} = w_{LL} = 0$ if and only if $r \in (0, \delta^A]$;

(ii) BP1^C: $w_{HH} = \frac{(q_1-q_0)^2(q_0+q_1-(1-q_1)r)H-(1+r)(q_1^2-1+r)}{(q_1-q_0)(q_0+q_1+(1-q_1)q_1 r-r^2)}$; $w_{HL} = 2w_{LL}$;

$w_{LL} = \frac{(q_1-q_0)^2(q_0+q_1+q_1 r)H-(1+r)(q_1^2+r)}{(q_1-q_0)(q_0+q_1+(1-q_1)q_1 r-r^2)}$ *if and only if $r \in (\delta^A, \min\{\tau^0, \delta^D\}]$;*

(iii) JPE: $w_{HH} = \frac{(1-q_1)q_1(1+r)+(q_1-q_0)^2(q_0-(1-q_1)(1+r))H}{(q_1-q_0)((q_1-1)r(1+r)+q_0(q_1+r))}$; $w_{HL} = \frac{\frac{(1+r)(q_1^2+r)}{q_1-q_0}-(q_1-q_0)(q_1+q_0+q_1 r)H}{(1-q_1)r(1+r)-q_0(q_1+r)}$;

$w_{LL} = 0$ *if and only if $r \in (\max\{\tau^0, \delta^A\}, \delta^C]$;*

(iv) BP2^C: $w_{HH} = \frac{q_1-q_0+q_1 q_0-q_1^2+q_0 r-(q_1-q_0)^2(1-q_1)H}{(q_1-q_0)((q_1-1)r+q_0(q_1+r-1))}$, $w_{HL} = \frac{q_0-2q_1 q_0+q_1^2+r-2q_0 r-(q_1-q_0)^3 H}{(q_1-q_0)((1-q_1)r-q_0(q_1+r-1))}$,

$w_{LL} = \frac{q_0(q_1-q_0)^2 H-q_0(q_1+r)}{(q_1-q_0)((1-q_1)r-q_0(q_1+r-1))}$ *if and only if $r \in (\max\{\tau^0, \delta^C\}, \delta^D]$;*

where $\tau^0 = \frac{q_1+q_0-1}{(1-q_1)^2}$, and δ^A, δ^C, and δ^D are increasing functions of H and are specified in the appendix.

It is easy to check that all the solutions create a strategic payoff complementarity (denoted by the C superscript). In order to optimally exploit mutual monitoring between agents, the contract ensures that each agent's benefit to free-riding is small.

When r is small ($r \in (0, \delta^A]$), the contract offered is the same as the "Joint Performance Evaluation" (*Pure JPE*) contract studied in Che and Yoo (2001), i.e., the agents are rewarded only if the outcomes from both agents are high. *Pure JPE* is optimal because it is the optimal

means of tying the agents together, so that they have both the incentive to mutually monitor each other and the means of punishing each other.

Starting from *Pure JPE*, as r increases, the principal has to increase w_{HH} because it becomes more difficult to motive the impatient agents to mutually monitor each other. However, the principal is also becoming less patient, and increasing w_{HH} will eventually become too expensive for the principal to truthfully report output (H, H), and she will have to choose between *JPE* and *BP1^C*. According to Proposition 1, *BP1^C* follows the full commitment contract *Pure JPE* if and only if $\delta^A < \tau^0$. This condition is equivalent to H not being too large, since δ^A increases in H while τ^0 is independent of H. Using a similar argument to the one given just after Proposition 1, we know H enhances the principal's ability to commit to the *JPE* solution. In fact, if H is extremely large, $r = \delta^A$ is so large that agent-agent side-contracting is already fragile, even before the principal's weakened commitment comes into play. Although increasing w_{HH} is most efficient in exploiting mutual monitoring between the agents, it comes at the cost that w_{LL} must also be increased. The principal uses *BP1^C* only if the agents are still somewhat patient so that the efficiency of w_{HH} in exploiting mutual monitoring among the agents dominates its cost.

We illustrate the intuition for why and when the *BP1^C* emerges sooner (relative to *JPE*) using a numerical example: $q_0 = 0.5$, $q_1 = 0.9$, and $H = 100$. In this example, *Pure JPE* violates the Principal's IC constraint whenever $r > \delta^A = 14.12$ (the origin of Figure 1), and therefore she has to either increase w_{HL} (*JPE*) or increase w_{LL} so that she can increase w_{HH} (*BP1^C*). The solid line in Figure 1 represents the relative cost of *JPE* compared to *BP1^C* calculated as $q_1(1 - q_1)(w_{HL}^{iii} - w_{HL}^{ii})$, and the dotted line is the relative benefit calculated as $q_1^2(w_{HH}^{ii} - w_{HH}^{ii}) + (1 - q_1)^2 w_{LL}^{ii}$ where the superscripts *ii* and *iii* refer the corresponding solutions. One can see the principal chooses *BP1^C* over *JPE* if the two agents are somewhat patient $(r < \tau^0 = 40)$ when the efficiency of w_{HH} in exploiting mutual monitoring is important enough. If one sets $H \geq 279.61$ instead, δ^A will exceed τ^0,and the region that used to be *BP1^C* $(14.12 < r < 40)$ is now replaced by *JPE* since the principal's ability to commit is greater. Moreover, whenever the principal loses her commitment at $r = \delta^A(H = 279.61) = 40$,

the agents are already so impatient that it is not worth increasing w_{HH} to induce mutual monitoring at the cost of increasing w_{LL}.
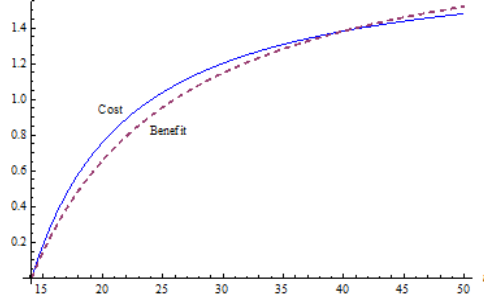

Figure 1: **Cost and Benefit of *JPE* relative to *BP1*^{C}**

A feature of Proposition 1 is that no feasible solution exists when r is large enough. There is a conflict between principal's desire to exploit the agents' mutual monitoring and her ability to commit to truthful reporting. Once r is sufficiently large, the intersection between the Mutual Monitoring constraint and the Principal's IC constraint is an empty set.

4.2   Collusion

Proposition 2 *When team incentives are not optimal, i.e., $T = 0$, the solution to the IP is one of the following ($w_{LH} = 0$ in all cases):*

(i) IPE: $w_{HH} = w_{HL} = \frac{1}{q_1 - q_0}, w_{LL} = 0$ if and only if $r \in (0, \delta^C]$;

(ii) BP^I: $w_{HH} = \frac{(q_1 - q_0)^2 H - (1+r)(q_1 + r - 1)}{(q_1 - q_0)(1 - r(q_1 + r - 1))}, \ w_{HL} = w_{HH} + w_{LL}$,

$w_{LL} = \frac{(q_1 - q_0)^2 (1+r) H - (1+r)(q_1 + r)}{(q_1 - q_0)(1 - r(q_1 + r - 1))}$ *if and only if $r \in (\delta^C, \tau^1]$;*

(iii) RPE: $w_{HH} = \frac{(1 - q_1) q_1 (1 + r) + (q_1 - q_0)^2 (q_1 (2 + r) - 1 - r) H}{(q_1 - q_0)(q_1^2 - r(1 + r) + q_1 r(2 + r))}, \ w_{HL} = \frac{(q_1 - q_0) q_1 (2 + r) H - \frac{(1+r)(q_1^2 + r)}{q_1 - q_0}}{q_1^2 - r(1 + r) + q_1 r(2 + r)}$,

$w_{LL} = 0$ *if and only if $r \in (\max(\delta^C, \tau^1), \delta^F]$;*

(iv) BP^S: $w_{HH} = \frac{(1+r)(r - (1 - q_1)^2) - (q_1 - q_0)^2 (1 - q_1)(2 + r) H}{(q_1 - q_0)(2q_1 + (3 - q_1) q_1 r + r^2 - 2(1 + r))}, \ w_{HL} = 2w_{HH}$,

$w_{LL} = \frac{(1+r)((2 - q_1) q_1 + r) + (q_1 - q_0)^2 (q_1 (2 + r) - 2(1 + r)) H}{(q_1 - q_0)(2q_1 + (3 - q_1) q_1 r + r^2 - 2(1 + r))}$ *if and only if $r > \max(\tau^1, \delta^F)$;*

where $\tau^1 = \frac{2q_1 - 1}{(1 - q_1)^2}$ and δ^C, δ^F are increasing functions of H, which are specified in the appendix.

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We use superscripts "S", and "I" to denote a (payoff) strategic strategic substitutability and strategic independence, respectively. In Proposition 2, individual performance evaluation (IPE) is optimal if both parties are patient enough ($r \leq \delta^C$). IPE is the benchmark solution——it is the optimal contract when the performance measures are verifiable (and mutual monitoring is not optimal), since the performance measures are uncorrelated.

As r increases, IPE is no longer feasible, because the impatient principal has incentive to lie when the output pair is (H, H). To see this, note that given the IPE contract, the right hand side of the Principal's IC constraint is $2 * w_{HH} = \frac{2}{q_1 - q_0}$ while the left hand side of the constraint is strictly decreasing in r. As the principal becomes less patient, eventually she has incentives to lie and report the output pair as (L, L) when it is actually (H, H). In particular, the Principal's IC constraint starts binding at $r = \delta^C$. As r increases further, the gap between w_{HH} and w_{LL} must be decreased in order to prevent the principal from misreporting.

The principal has two methods of decreasing the gap between w_{HH} and w_{LL}. First, she can decrease w_{HH} and increase w_{HL}—increasing her reliance on RPE. Second, she can increase w_{LL} so that the contract rewards bad performance in the sense that both agents are rewarded even though they both produce bad outcomes, corresponding to solution BP^I in Proposition 2. This second approach compromises the agents' incentive to *work* because increasing w_{LL} makes *shirk* more attractive to both agents; as a result, w_{HH} or w_{HL} needs to be increased even more to provide enough effort incentive to the agents. One may think that BP^I will never be preferred to RPE. In fact, the only reason that BP^I is optimal is that it is an efficient way of combatting agent-agent collusion on the *Cycling* strategy. RPE creates a strategic substitutability in the agents' payoffs that makes Cycling particularly appealing.

RPE relies on w_{HL} to provide incentives, creating a strategic substitutability in the agents' payoffs. Under RPE, each agent's high effort level has a negative externality on the other agent's effort choice, making the *Cycling* collusion more difficult (more expensive) to break up than the *Joint Shirking* collusion strategy. Since collusion is more costly to prevent for small r, the principal purposely designs the contract to create strategically independent payoffs. This intuition and the trade-off between the benefit and cost of RPE (increasing w_{HL}) relative to

BP^I (increasing w_{HH} and w_{LL}) is illustrated with a numerical example in Figure 2.

In the example, $q_0 = 0.5$, $q_1 = 0.7$, and $H = 100$. The principal uses IPE if she is sufficiently patient ($r \leq 3.33$), while she has to chooses between RPE and BP^I for $r > 3.33$ (the origin of Figure 2 at which both solutions are equally costly to the principal). The solid line represents the cost of RPE (Solution iii) relative to BP^I (Solution ii), calculated as $q_1(1 - q_1)(w_{HL}^{iii} - w_{HL}^{ii})$, where the superscripts iii and ii refer the corresponding solutions in Proposition 2. The dotted line measures the relative benefit of RPE, or equivalently the cost of increasing w_{LL} and w_{HH} under BP^I, calculated as $q_1^2(w_{HH}^{ii} - w_{HH}^{ii}) + (1 - q_1)^2 w_{LL}^{ii}$. The intersection of two lines determines the critical discount rate $\tau^1 = 4.4$ used to choose between the two solutions. If the agents are still patient enough ($r < \tau^1$), BP^I is preferred to RPE since the *No Cycling* constraint is the expensive one to satisfy.
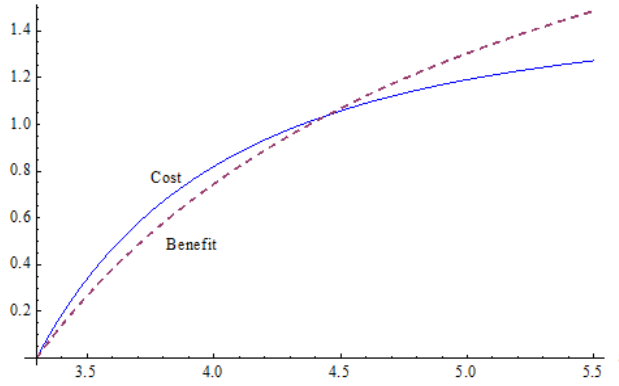

Figure 2: **Cost and Benefit of RPE relative to BP^I**

Under BP^I," pay without performance" emerges sooner than one might expect, since RPE is feasible and does not make payments for poor performance. When will this happen? Intuitively, BP^I is optimal when the agents' credibility to collude is relatively stronger than the principal's credibility to honor her promises. More precisely, from Proposition 2, we know BPI is optimal if and only if $\delta^C < \tau^1$, which is equivalent to restricting the high output of the project $H < H^*$ for some constant H^*, because δ^C increases in H while τ^1 is independent of H. To gain some intuition for the requirement $H < H^*$, notice that the principal's ability to commit to honoring the IPE contract is enhanced by a high value of H, since the punishment the

agents can bring to bear on the principal is more severe. Hence the region over which she can commit to the *IPE* contract becomes larger (δ^C becomes bigger) as H increases. For extremely large H ($H > H^*$), δ^C is so large that once the principal cannot commit to *IPE* ($r = \delta^C$), it has already passed the point (τ^1) that the two agents can enforce their own collusion. Since the only reason that BP^I is optimal is its efficiency in combatting agent-agent collusion on the Cycling strategy, the *RPE* contract instead of BP^I will be optimal if H is large (and thus $r > \tau^1$). Using the same numerical example as in Figure 1, if one sets $H \geq 128.61$ instead of 100, δ^C will exceed τ^1 and the region that used to be BP^I with $H = 100$ ($3.3 < r < 4.4$) now becomes *IPE* since the principal has a greater ability to commit with a higher H. Moreover, when the principal loses her commitment at $r = \delta^C = 4.4$, the agents are already so impatient that their Cycling collusion is of no concern, and the principal will offer *RPE* (instead of BP^I).

Solution BP^S emerges as r eventually becomes large enough (both parties become extremely impatient) and BP^S is similar to the static bonus pool. As pointed out in Levin (2003), "the variation in contingent payments is limited by the future gains from the relationship." The variation of wage payment is extremely limited under BP^S, because both parties are sufficiently impatient ($r > \max(\tau^1, \delta^F)$) and (thus) the future gains from the relationship are negligible. As a result, the principal has to set $w_{HL} = 2w_{HH}$ and also increase w_{LL} to make the contract self-enforcing. This coincides with the traditional view that bonus pools will eventually come into play because they are the only self-enforcing compensation form in such cases.

4.3   Overall optimal contract

The following proposition endogenizes the principal's choice of $T \in \{0, 1\}$ and characterizes the overall optimal contract.

Proposition 3 *The principal chooses to induce team incentives, i.e., $T = 1$ if and only if (i) $\tau^0 < \delta^C$ and $r < \delta^C$ or (ii) $\tau^0 \geq \delta^C$ and $r < \min\{\tau^0, \delta^D\}$.*

Not surprisingly, the key determinant of whether motivating cooperation is optimal is the discount rate. A longer expected contracting horizon gives the principal more flexibility in

the promises she can offer and enables the agents to make credible promises to each other to motivate cooperation. As a result, there is a solution to team incentives ($T = 1$) and a solution to not using team incentives ($T = 0$) that are never optimal in the overall solution. Of the solutions to T=0, *IPE* is never optimal overall. When the principal's ability to commit is strong enough that *IPE* is feasible, the principal is always be strictly better-off by offering a contract motivating cooperation (setting $T = 1$). Similarly, whenever the principal's and the agents' ability to commit is so limited that only $BP2^C$ can be used to motivate cooperation, cooperation is too expensive to implement and the principal will not choose to do so ($T = 0$).

The principal's and agents' relative credibility is key, as the following numerical examples illustrate. In the first example, the agents' ability to commit is always limited relative to that of the principal, and collusion is never the driving determinant of the form of the optimal compensation arrangement. Consider the example: $q_0 = 0.53$, $q_1 = 0.75$, and $H = 200$, which corresponds to Case (i) in Proposition 3 ($\tau^0 < \delta^C$). When the principal chooses not to induce team incentives, i.e., $T = 0$, *RPE* is used immediately and BP^I is never optimal.
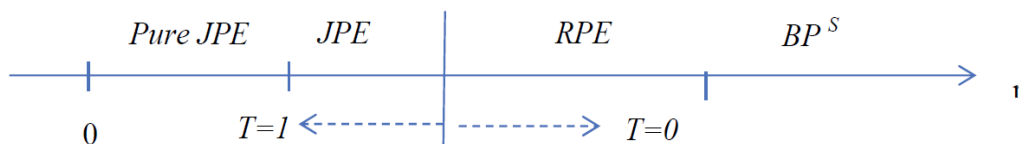

The second example also falls under Case (i) of Proposition 3: $q_0 = 0.47$, $q_1 = 0.72$, and $H = 100$. The principal's ability to commit is high relative to the agents', but the relative comparison is not as extreme. In this case, we move from BP^I to *RPE*, since the principal still has enough ability to commit to make *RPE* feasible after the discount rate is so large that the agents' collusion is not the key determinant of the form of the compensation contract (BP^I).
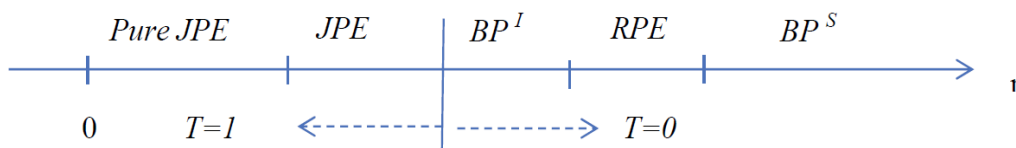
Next, consider a numerical example corresponding to Case (ii) in Proposition 3 ($\tau^0 \geq \delta^C$). $q_0 = 0.53$, $q_1 = 0.75$, and $H = 100$. In this case, the principal's ability to commit is low relative to the agents'. Once the discount rate is large enough that the agents' collusion is not the key

determinant of the form of the compensation contract, the principal's ability to commit is also quite limited and BP^S is the only feasible solution. The principal's low relative credibility also leads to $BP1^C$ being optimal in this example when JPE was in the previous two examples.
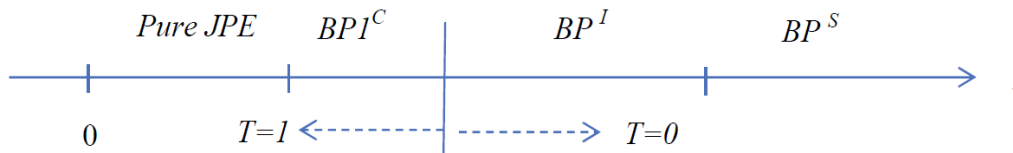
5   Incorporating an Objective Team-based Performance Measure

In a typical bonus pool arrangement, the size of the bonus pool is based, at least in part, on an objective team-based performance measure such as group or divisional earnings (Eccles and Crane, 1988). Suppose that such an objective measure y exists and define $p_1 = \Pr(y = H|e^A = e^B = 1)$, $p = \Pr(y = H|e^A = 1, e^B = 0) = \Pr(y = H|e^A = 0, e^B = 1)$, and $p_0 = \Pr(y = H|e^A = e^B = 0)$.

One interesting variation of the model is when the individual measures are noiseless. In this case, the optimal contract does not motivate mutual monitoring, since the principal's observation of the agents' efforts is already perfect. An optimal contract then makes the agents' payoffs strategically independent when the collusion constraints bind. The productive complementarity or substitutability in y is optimally combined with an offsetting payment effect to produce payoffs that are strategically independent.

Returning to imperfect individual performance measures, consider the following numerical example: $q_0 = 0.47$, $q_1 = 0.72$, $p_0 = 0.1$, $p = 0.8$, $p_1 = 0.9$, $r = 5$, and $H = 27$. In this example, the team-based performance measure y exhibits a large production substitutability[6] and cooperation is not feasible. If only subjective measures are used in contracting, the optimal wage scheme is $w = (w_{LL}, w_{LH}, w_{HL}, w_{HH}) = (4.27, 0, 8.97, 4.70)$, or BP^I. Once objective measure is incorporated into the contract, use the first subscript on the wage payment to

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[6]y exhibits a production substitutability if and only if $p_1 - p < p - p_0$.

denote the realization of the objective measure. For example, w_{Hmn} is the payment made to agent i when y is H, x^i is m, and x^j is n; $m, n \in \{L, H\}$. The optimal wage scheme is $w = (w_{LLL}, w_{LLH}, w_{LHL}, w_{LHH}, w_{HLL}, w_{HLH}, w_{HHL}, w_{HHH}) = (0,\ 0,\ 0,\ 0,\ 2.922,\ 0,\ 5.843,\ 3.675)$, which creates a strategic substitutability between the two agents' effort choice. There is a relatively small improvement in expected wages by introducing y: 9.19 without y and 5.96 with y. The objective measures are valuable because of their informativeness (Holmstrom, 1979) and because its verifiability nature compared to x, but the productive substitutability still precludes corporation from being optimal.

Continue with the same example, except now assume $p = 0.2$, which exhibits a large production complementarity ($p_1 - p > p - p_0$). Incorporating y into the optimal contract now facilitates cooperation when it would otherwise be infeasible. The optimal wage scheme is $w = (0, 0, 0, 0, 0.26, 0, 2.76, 1.38)$ and the expected total wages are 2.33, compared to 9.19 without incorporating the objective measure and 5.96 when the objective measure is incorporated but has a productive substitutability ($p = 0.8$). The optimal wage scheme can be viewed as a bonus pool of 0 when the verifiable team measure is low and a bonus pool of 2.76 when the team measure is high, with the principal given the discretion to reduce the reward to 0.26 if both agents are subjectively assessed to have contributed poorly. The substitutability in the subjectively determined wages reduces their cost to the principal, since the substitutability limits the amount of pay for bad performance that is required to maintain the principal's credibility. The productive complementarity facilitates both cooperation and a wage substitutability in the subjective measures. The example can be viewed as suggesting a new rationale for grouping employees whose actions are productive complements into a single bonus pool, cooperating through mutual monitoring to maximize their total reward but also competing for their share of the total reward.

6 Conclusion

A natural next step is to test some of the paper's empirical predictions. In particular, the model predicts that the form of the wage scheme will depend on (i) the expected contracting horizon, (ii) the relative ability of the principal and the agents to honor their promises, and (iii) the productive complementarity or substitutability of a team-based objective measure. A particularly strong prediction is that we should see bonus pool type incentive schemes that create strategic independence in the agents' payoffs in order to optimally prevent collusion. These particular bonus pool type arrangements should be observed when the agents' ability to collude is strong relative to the principal's ability to make credible promises and both are limited enough to be binding constraints. When the principal's credibility and the agents' credibility are both severely limited, we should instead observe bonus pool type arrangements that create a strategic substitutability in the agents' payoffs, since these allow for greater relative performance evaluation which is efficient absent collusion concerns. When instead the arrangement is used to motivate cooperation and mutual monitoring (which we suspect is more common), we should see productive and incentive arrangements that, when combined, create strategic complementarities.

Another avenue for empirical research is to re-examine settings in which mixed results have previously been obtained on team incentives/mutual monitoring and use this paper's limited commitment, productive complementarities, etc. to help refine those empirical predictions. Stepping back from the principal's commitment problem, the general theme of team-based vs. individual incentives seems to be under-explored. For example, the models of team-based incentives typically assume the agents are symmetric (e.g., Che and Yoo, 2001; Arya, Fellingham, and Glover, 1997). With agents that have different roles (e.g., a CEO and a CFO), static models predict the agents would be offered qualitatively different compensation contracts. Yet, in practice, the compensation of CEOs and CFOs are qualitatively similar, which seems to be consistent with a team-based model of dynamic incentives (with a low discount rate/long expected tenure).

If we apply the team-based model to thinking about screening, then we might expect to

see compensation contracts that screen agents for their potential productive complementarity with agents already in the firm's employ (or in a team of agents being hired at the same time), since productive complementarities reduce the cost of motivating mutual monitoring because the benefit to free-riding is small. A productive substitutability (e.g., hiring an agent similar to existing ones when there are overall decreasing returns to effort) is particularly unattractive, since the substitutability makes the cycling collusion constraint (the agents taking turns working) bind. We might also expect to see agents screened for their discount rates—that patient agents (those with long expected horizons with the firm) would be more attractive, since they are the ones best equipped to provide and receive mutual monitoring incentives. Are existing incentive arrangements such as employee stock options with time-based rather than performance-based vesting conditions designed, in part, to achieve such screening?

References

[1] Anthony, R. and V. Govindarajan, 1998. Management Control Systems, Mc-Graw Hill.

[2] Arya, A., J. Fellingham, and J. Glover, 1997. Teams, Repeated Tasks, and Implicit Incentives. *Journal of Accounting and Economics*, 23: 7-30.

[3] Arya, A., Glover, J., Mittendorf, B. and Zhang, L., 2004. The disciplining role of accounting in the long-run. *Review of Accounting Studies*, 9: 399-417.

[4] Baiman, S. and T. Baldenius, 2009. Nonfinancial Performance Measures as Coordination Devices. The Accounting Review, 84: 299-330.

[5] Baiman, S. and M. Rajan, 1995. The Informational Advantages of Discretionary Bonus Schemes. *The Accounting Review*, 70: 557-579.

[6] Baker, G., R. Gibbons, and K. Murphy, 1994. Subjective Performance Measures in Optimal Incentive Contracts. *Quarterly Journal of Economics*, 109: 1125-1156.

[7] Baldenius, T. and J. Glover, 2012. Relational Contracts With and Between Agents. Working Paper, Carnegie Mellon University and New York University.

[8] Bebchuk, L. and J. Fried, 2006. Pay Without Performance, Harvard University Press.

[9] Che, Y. and S. Yoo, 2001. Optimal Incentives for Teams. *American Economic Review*, June: 525-541.

[10] Demski, J. and D. Sappington, 1984. Optimal Incentive Contracts with Multiple Agents. *Journal of Economic Theory*, 33: 152-171.

[11] Eccles, R.G. and D.B. Crane, 1988. Doing Deals: Investment Banks at Work. Harvard Business School Press.

[12] Ederhof, M., 2010. Discretion in Bonus Plans. *The Accounting Review*, 85: 1921–1949.

[13] Ederhof, M., M. Rajan, and S. Reichelstein, 2011. Discretion in Managerial Bonus Pools. *Foundations and Trends in Accounting*, 54: 243-316.

[14] Gibbs, M., K. Merchant, W. Van der Stede, and M. Vargus, 2004. Determinants and Effects of Subjectivity in Incentives. T*he Accounting Review*, 79: 409-436.

[15] Glover, J., 2012. Explicit and Implicit Incentives for Multiple Agents. *Foundations and Trends in Accounting*, 7: 1-71.

[16] Holmstrom, B., 1979. Moral Hazard and Observability. *Bell Journal of Economics*, 10: 74-91.

[17] Kvaløy, Ola and Trond E. Olsen, 2006. Team Incentives in Relational Employment Contracts. *Journal of Labor Economics*, 24: 139-169.

[18] Levin, J., 2002. Multilateral Contracting and the Employment Relationship. *Quarterly Journal of Economics*, 117: 1075-1103.

[19] Levin, J., 2003. Relational Incentive Contracts. *American Economic Review*, 93: 835-847.

[20] Mookherjee, D., 1984. Optimal Incentive Schemes with Many Agents. *Review of Economic Studies*, 51: 433–446.

[21] Murphy, K., and P. Oyer, 2001. Discretion in Executive Incentive Contracts: Theory and Evidence. Working Paper, Stanford University.

[22] Rajan, M. and S. Reichelstein, 2006. Subjective Performance Indicators and Discretionary Bonus Pools. *Journal of Accounting Research*, 44: 525-541.

[23] Rajan, M. and S. Reichelstein, 2009. Objective versus Subjective Indicators of Managerial Performance. *The Accounting Review*, 84: 209-237.

[24] Tirole, J., 1986. Hierarchies and Bureaucracies: On the Role of Collusion in Organizations. *Journal of Law, Economics and Organization*, 2: 181-214.

[25] Tirole, J., 1992. Collusion and the theory of organizations. In J.-J. Laffont, ed., Advances in economic theory: Invited papers for the sixth world congress of the Econometric Society 2. Cambridge: Cambridge University Press.

Appendix

Proof of Lemma 1. If the payoff from (*work, work*) is higher for both agents than under (*shirk, shirk*), then we need only worry about strategies that have the agents using some combination of (*work, shirk*) and (*shirk, work*) over time. In this case, if there is some period t' shirker who has a higher continuation payoff than under Cycling, there will also be a period t'' shirker who has a lower payoff than under Cycling. Hence, if the contract upsets Cycling, it will also motivate the period t'' shirker to work instead of shirk, upsetting that potential equilibrium.

Now, consider the case that the payoff under (*work, work*) is lower for both agents than under (*shirk, shirk*). If the agents play only (*shirk, work*) and (*work, shirk*) in all future periods (whatever they are doing this period), then the previous argument applies. Consider 2 subcases, characterized by the total payoff—the sum of the payoffs to the two agents in the stage game.

Subcase 1: Suppose the total payoff under (*shirk, shirk*) is higher than under (*shirk, work*). At any (*shirk, shirk*) not followed by (*shirk, shirk*) forever, one of the two agents will have a lower continuation payoff. Hence, upsetting *Joint Shirking* means that any proposed collusion that falls under Subcase 1 will also be upset.

Subcase 2: Suppose the total payoff under (*shirk, work*) is higher than under (*shirk, shirk*). At any (*shirk, shirk*), if the total continuation payoff is as under Cycle, then the rest of the play must involve only (*shirk, work*) and (*work, shirk*), which is a case we have already dealt with. Otherwise, the total continuation payoff must be less at (*shirk, shirk*) than under Cycle. So, compare (*shirk, shirk*) under the proposed collusion to (*shirk, work*) under Cycling. At (*shirk, shirk*) under the proposed collusion, the continuation payoff is lower to either agent A or to agent B than under Cycling.

We have $\pi(0,0) > \pi(1,1) - 1$ and $\pi(1,1) - 1 \geq \pi(0,1)$ by the *Static NE* constraint. Since we are in Subcase 2, we also have $\pi(1,0) - 1 + \pi(0,1) > 2\pi(0,0)$. Combining these assumptions, we have $\pi(1,0) - 1 - \pi(0,0) > \pi(1,1) - 1 - \pi(0,1)$, i.e., the agents' actions are strategic substitutes.

Subcase 2a: If the continuation payoff is lower to A, then we are done, since the shirker will want to deviate to work given that we have strategic substitutes in the payoffs: moving from (*shirk, shirk*) to (*work, shirk*) has an even greater current period benefit than moving from (*shirk, work*) to (*work, work*). Since Cycling is upset, we also upset any collusion that falls under Subcase 2a.

Subcase 2b: If the continuation payoff is lower to B, then we use the observation that, under Cycling, the continuation payoff to the shirker is greatest. Then B's continuation payoff is lower than A's would be under Cycling, and the strategic substitutability in payoffs again ensures that moving from (*shirk, shirk*) to (*work, shirk*) in the current period is greater than under Cycling when we move from (*shirk, work*) to (*work, work*). So upsetting Cycling means we also upset any possible collusion that falls under Subcase 2b. ∎

The following parameters will be useful in the remaining proofs.

$$
\delta^A = \frac{1}{2}\left[\begin{array}{c} (q_0 - q_1)^2 q_1 H - 1 - q_1^2 + \\ \sqrt{((q_0 - q_1)^2 q_1 H - 1 - q_1^2)^2 + 4(q_0 - q_1)^2(q_0 + q_1)H - 4q_1^2} \end{array}\right],
$$

$$
\delta^C = (q_0 - q_1)^2 H - q_1,
$$

$$
\delta^D = (q_0 - q_1)^2 (q_0 + q_1)H - q_0 - q_1^2,
$$

$$
\delta^F = \frac{1}{2}\left[\begin{array}{c} (q_0 - q_1)^2(2 - q_1)H - q - 2q_1 + q_1^2 + \\ \sqrt{((q_0 - q_1)^2(2 - q_1)H - q - 2q_1 + q_1^2)^2 + 4(q_1 - 2)q_1 - 8(q_0 - q_1)^2(q_1 - 1)H} \end{array}\right],
$$

$$
\tau^0 = \frac{q_1 + q_0 - 1}{(1 - q_1)^2},
$$

$$
\tau^1 = \frac{2q_1 - 1}{(1 - q_1)^2}.
$$

Note that $\delta^A, \delta^C, \delta^D$, and δ^F increase in H, and we assume throughout the paper that H is larger enough to rank term by comparing the coefficient of the linear term of H. In particular, we obtain (*i*) $\delta^A < \delta^C < \delta^F$ and (*ii*) $\delta^C < \delta^D$ if and only if $q_0 + q_1 > 1$. The agent's effort is assumed to be valuable enough ($q_0 - q_1$ is not too small) such that $\delta^A > \sqrt{2}$ and $(q_0 - q_1)^2 H \geqslant \max\{1 + \frac{1}{q_1 - q_0}, \frac{q_1^2}{2q_1 - 1}, \frac{q_1(2 - q_1)}{2(1 - q_1)}, \frac{q_0 - (1 - q_1)q_1}{q_0 + q_1 - 1}\}$.

Proof of Proposition 1. The program can be written as follows.

$\min(1-q_1)^2 w_{LL} + (1-q_1)q_1 w_{LH} + (1-q_1)q_1 w_{HL} + q_1^2 w_{HH}$

$s.t$

$(2-q_0-q_1)w_{LL} + (q_0+q_1-1)w_{LH} + (q_0+q_1-1)w_{HL} - (q_0+q_1)w_{HH} \leqslant \frac{-1}{q_1-q_0}$ "Pareto Dominance"

(λ_1)

$(2-q_0-q_1+r(1-q_1))w_{LL} + (q_0+q_1+q_1 r - 1)w_{LH} + (q_0+(q_1-1)(1+r))w_{HL} - (q_0+q_1+q_1 r)w_{HH} \leqslant$

$\frac{-(1+r)}{q_1-q_0}$ "Mutual Monitoring" (λ_2)

$(1-q_1)(2+r)w_{LL} + (-1+q_1(2+r))w_{LH} + ((q_1-1)(1+r)+q_1)w_{HL} - q_1(2+r)w_{HH} \leqslant -\frac{1+r}{q_1-q_0}$ "No

Cycling"(λ_3)

$((1-q_1)^2 - r)w_{LL} + (1-q_1)q_1 w_{LH} + (1-q_1)q_1 w_{HL} + (q_1^2+r)w_{HH} \leqslant (q_1-q_0)H \quad ICP_{HH \succ LL}(\lambda_4)$

$((1-q_1)^2 - r)w_{LL} + (q_1-q_1^2+\frac{r}{2})w_{LH} + (q_1-q_1^2+\frac{r}{2})w_{HL} + q_1^2 w_{HH} \leqslant (q_1-q_0)H \quad ICP_{HL \succ LL}(\lambda_5)$

$(1-q_1)^2 w_{LL} + (q_1-q_1^2-\frac{r}{2})w_{LH} + (q_1-q_1^2-\frac{r}{2})w_{HL} + (q_1^2+r)w_{HH} \leqslant (q_1-q_0)H \quad ICP_{HH \succ HL}(\lambda_6)$

$(1-q_1)^2 w_{LL} + (q_1-q_1^2+\frac{r}{2})w_{LH} + (q_1-q_1^2+\frac{r}{2})w_{HL} + (q_1^2-r)w_{HH} \leqslant (q_1-q_0)H \quad ICP_{HL \succ HH}(\lambda_7)$

$((1-q_1)^2 + r)w_{LL} + (1-q_1)q_1 w_{LH} + (1-q_1)q_1 w_{HL} + (q_1^2-r)w_{HH} \leqslant (q_1-q_0)H \quad ICP_{LL \succ HH}(\lambda_8)$

$((1-q_1)^2 + r)w_{LL} + (q_1-q_1^2-\frac{r}{2})w_{LH} + (q_1-q_1^2-\frac{r}{2})w_{HL} + q_1^2 w_{HH} \leqslant (q_1-q_0)H \quad ICP_{LL \succ HL}(\lambda_9)$

$-w_{LL} \leqslant 0(\lambda_{10}); \quad -w_{HL} \leqslant 0(\lambda_{11}); \quad -w_{HH} \leqslant 0(\lambda_{12}); \quad -w_{LH} \leqslant 0(\lambda_{13});$

$(-1+q_0)w_{LL} - q_0 w_{LH} + (1-q_0)w_{HL} + q_0 w_{HH} \leqslant \frac{1}{q_1-q_0}$ "Self-Enforcing Shirk"(λ_{14})

We first solve a relaxed program without the "Self-Enforcing Shirk" constraint and then verify that solutions of the relaxed program satisfy the "Self-Enforcing Shirk" constraint.

Lemma 2: Setting $w_{LH} = 0$ is optimal in the relaxed program (without Self-Enforcing Shirk constraint.)

Proof: Suppose the optimal solution is $w = \{w_{HH}, w_{HL}, w_{LH}, w_{LL}\}$ with $w_{LH} > 0$. Consider the solution $w' = \{w'_{HH}, w'_{HL}, w'_{LH}, w'_{LL}\}$, where $w'_{LH} = 0$, $w'_{HL} = w_{HL}+w_{LH}$, $w'_{LL} = w_{LL}$ and $w'_{HH} = w_{HH}$. It is easy to see that w and w' generate the same objective function value. We show below that the constructed w' satisfies all the $ICP_{mn \succ m'n'}$ constraints and further relaxes the rest of the constraints (compared to the original contract w). Since w_{LH} and w_{HL} have the same coefficient in all the $ICP_{mn \succ m'n'}$ constraints, w' satisfies these constraints as long as w does. Denote the coefficient on w_{LH} as C_{LH} and the coefficient on w_{HL} as C_{HL} for

the "*Pareto Dominate*", "*Mutual Monitoring*", and "*No Cycling*" constraints. We can show that $C_{LH} - C_{HL} \geq 0$ holds for each of the three constraints. Given $C_{LH} \geq C_{HL}$, it is easy to show that w' will relax the three constraints compared to the solution w, which complete the proof of Lemma 2.

Denote the objective function by $f(w)$, the left-hand side less the right-hand side of the ith constraints by $g_i(w)$, and the Lagrangian Multiplier of the ith constraint by λ_i. The Lagrangian for the problem is $L = f(w) + \sum_{i=1}^{12} \lambda_i g_i(w)$. After setting $w_{LH} = 0$ (using Lemma 1) and simplifying the problem, FOCs of the relaxed program for the three wage payments (w_{LL}, w_{HL}, w_{HH}) are as follows:

$$1 - \lambda_{10} + 2\lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 - 2(1 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9)q_1$$

$$+ (1 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9)q_1^2 - \lambda_1(q_0 + q_1 - 2) +$$

$$(\lambda_3 - \lambda_4 - \lambda_5 + \lambda_8 + \lambda_9 - \lambda_3 q_1)r - \lambda_2(-2 + q_0 + q_1 + (-1 + q_1)r) = 0 \quad \text{(FOC-}w_{LL}\text{)}$$

$$q_1 + \lambda_1(q_0 + q_1 - 1) + q_1(2\lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 - (1 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9)q_1)$$

$$+ \frac{1}{2}(\lambda_5 - \lambda_6 + \lambda_7 - \lambda_9 + 2\lambda_3(-1 + q_1))r + \lambda_2(q_0 + (-1 + q_1)(1 + r)) - \lambda_{11} - \lambda_3 = 0$$

$$\text{(FOC-}w_{HL}\text{)}$$

$$q_1^2 + \lambda_5 q_1^2 + \lambda_9 q_1^2 - \lambda_1(q_0 + q_1) + \lambda_7(q_1^2 - r) + \lambda_8(q_1^2 - r) - \lambda_3 q_1(2 + r) +$$

$$\lambda_4(q_1^2 + r) + \lambda_6(q_1^2 + r) - \lambda_2(q_0 + q_1 + q_1 r) - \lambda_{12} = 0 \quad \text{(FOC-}w_{HH}\text{)}$$

The optimal solution is one that (i) satisfies all constraints, (ii) satisfies the three FOCs above, (iii) satisfies the complementary slackness conditions $\lambda_i g_i(w) = 0$, and (iv) has non-negative Lagrangian multipliers. For $r < \delta^A$, the solution listed below satisfies $(i) - (iv)$ and thus is optimal. This solution, denoted as *Pure JPE*, is obtained by solving the three binding

constraints: Mutual Monitoring, w_{HL}, and w_{LL}.

The *Pure JPE* solution is:

$$w_{LL} = 0, w_{HL} = 0, w_{HH} = \frac{1+r}{(q_1-q_0)(q_0+q_1+q_1r)};$$

$$\lambda_1 = q_1, \lambda_2 = \frac{q_1^2}{q_0+q_1+q_1r}, \lambda_3 = 0, \lambda_4 = 0,$$

$$\lambda_5 = 0, \lambda_6 = 0, \lambda_7 = 0, \lambda_8 = 0,$$

$$\lambda_9 = 0, \lambda_{10} = \frac{q_0-2q_0q_1+q_1(1+r-q_1r)}{q_0+q_1+q_1r}, \lambda_{11} = \frac{q_0q_1}{q_0+q_1+q_1r}, \lambda_{12} = 0.$$

Under *Pure JPE*, the $ICP_{HH\succ LL}$ constraint yields the upper bound on r. The optimal solution changes when $r > \delta^A$. For $\delta^A < r \leqslant \min(\tau^0, \delta^D)$, the solution listed below satisfies $(i) - (iv)$ and becomes optimal. This solution, denoted as BPI^C, is obtained by solving the following three binding constraints: Mutual Monitoring, $ICP_{HH\succ LL}$, and $ICP_{HH\succ HL}$.

The $BP1^C$ solution is:

$$w_{LL} = \frac{(q_1-q_0)^2(q_0+q_1+q_1r)H-(1+r)(q_1^2+r)}{(q_1-q_0)(q_0+q_1-(-1+q_1)q_1r-r^2)}, w_{HL} = 2*w_{LL},$$

$$w_{HH} = \frac{(q_1-q_0)^2(q_0+q_1+(-1+q_1)r)H-(1+r)(-1+q_1^2+r)}{(q_1-q_0)(q_0+q_1-(-1+q_1)q_1r-r^2)};$$

$$\lambda_1 = 0, \lambda_2 = \frac{-r}{q_0+q_1+q_1r-q_1^2r-r^2}, \lambda_3 = 0, \lambda_4 = \frac{q_0-(q_1-2)((q_1-q)r-1)}{q_0+q_1+q_1r-q_1^2r-r^2},$$

$$\lambda_5 = 0, \lambda_6 = \frac{2(q_0-(q_1-1)((q_1-1)r-1))}{q_0+q_1+q_1r-q_1^2r-r^2}, \lambda_7 = 0, \lambda_8 = 0,$$

$$\lambda_9 = 0, \lambda_{10} = 0, \lambda_{11} = 0, \lambda_{12} = 0.$$

Under $BP1^C$, the non-negativity of w_{LL} and w_{HL} requires $r > \delta^A$, while the *Pareto Dominant* constraint and $\lambda_6 \geqslant 0$ impose upper bounds δ^D and τ^0 respectively. If $r > \tau^0$, the solution changes because otherwise $\lambda_6 < 0$. For $\min\{\tau^0, \delta^A\} < r \leqslant \delta^C$, the solution is listed below. This solution, denoted as JPE, is obtained by solving the following three binding constraints: Mutual Monitoring, $ICP_{HH\succ LL}$, and w_{LL}.

The *JPE* solution is:

$$w_{LL} = 0, w_{HL} = \frac{\frac{(1+r)(q_1^2+r)}{q_1-q_0}-(q_1-q_0)(q_0+q_1+q_1r)H}{(1-q_1)r(1+r)-q_0(q_1+r)},$$

$$w_{HH} = \frac{(1-q_1)q_1(1+r)+(q_1-q_0)^2(q_0+(-1+q_1)(q+r))H}{(q_1-q_0)((-1+q_1)r(1+r)+q_0(q_1+r))};$$

$$\lambda_1 = 0, \lambda_2 = \frac{(q_1-1)q_1r}{(q_1-1)r(1+r)+q_0(q_1+r)}, \lambda_3 = 0, \lambda_4 = \frac{-q_0q_1}{(q_1-1)r(1+r)+q_0(q_1+r)},$$

$$\lambda_5 = 0, \lambda_6 = 0, \lambda_7 = 0, \lambda_8 = 0,$$

$$\lambda_9 = 0, \lambda_{10} = \frac{r(q_0-(q_1-1)((q_1-q)r-1))}{(q_1-1)r(1+r)+q_0(q_1+r)}, \lambda_{11} = 0, \lambda_{12} = 0.$$

Under *JPE*, the non-negativity of w_{HL} requires $r > \delta^A$ and $\lambda_{10} \geqslant 0$ yields another lower bound

τ^0 on r. The non-negativity of w_{HH} and w_{HL} also requires $r > s' \equiv \frac{q_0+q_1-1+\sqrt{(q_0+q_1-1)^2+4(1-q_1)q_0q_1}}{2(1-q_1)}$.

In addition, both the Pareto Dominance and No Cycle constraints require $r < \delta^C$. We claim $(\max\{s', \delta^A, \tau^0\}, \delta^C] = (\max\{\delta^A, \tau^0\}, \delta^C]$. The claim is trivial if $s' \leqslant \delta^A$ and therefore consider the case where $s' > \delta^A$. Since δ^A increases in H while s' is independent of H, one can show $s' > \delta^A$ is equivalent to $H < H'$ for a unique positive H'. Meanwhile, algebra shows that $\delta^c < \delta^A$ for $H < H'$. Therefore $s' > \delta^A$ implies $\delta^c < \delta^A$, in which case $(\max\{s', \delta^A, \tau^0\}, \delta^C] = (\max\{\delta^A, \tau^0\}, \delta^C] = \varnothing$. For $\max\{\tau^0, \delta^C\} < r \leqslant \delta^D$ and $q_1 + q_0 \geqslant 1$, the optimal solution is as follows. This solution, denoted as $BP2^C$, is obtained by solving the three binding constraints: Mutual Monitoring, Pareto Dominance, and $ICP_{HH \succ LL}$.

The $BP2^C$ solution is:

$w_{LL} = \frac{q_0(q_1-q_0)^2 H - q_0(q_1+r)}{(q_1-q_0)((1-q_1)r-q_0(-1+q_1+r))}, w_{HL} = \frac{(q_1-q_0)^2+r-2q_0 r-(q_1-q_0)^3 H}{(q_1-q_0)((1-q_1)r-q_0(-1+q_1+r))}$,

$w_{HH} = \frac{(q_0+q_1)(q_1-1)+q_0 r+(q_1-q_0)^2(-1+q_1)H}{(q_1-q_0)((1-q_1)r-q_0(-1+q_1+r))}$;

$\lambda_1 = \frac{-q_0+(-1+q_1)(-1+(-1+q_1)r)}{(q_1-1)r+q_0(-1+q_1+r)}, \lambda_2 = \frac{q_0+q_1-1}{(q_1-1)r+q_0(-1+q_1+r)}, \lambda_3 = 0, \lambda_4 = \frac{q_0(1-q_1)}{(q_1-1)r+q_0(-1+q_1+r)}$,

$\lambda_5 = 0, \lambda_6 = 0, \lambda_7 = 0, \lambda_8 = 0$,

$\lambda_9 = 0, \lambda_{10} = 0, \lambda_{11} = 0, \lambda_{12} = 0$

Under $BP2^C$, the non-negativity of λ_1 requires $q_1 + q_0 \geqslant 1$. Given $q_1 + q_0 \geqslant 1$, the non-negativity of w_{HH} and w_{HL} together yield $r > \delta^C$ and $r > s'' \equiv \frac{(1-q_1)q_0}{q_0+q_1-1}$. The other lower bound τ^0 on r is generated by intersecting requirements for $\lambda \geqslant 0$ and for the non-negativity of w_{HH} and w_{HL}. The $ICP_{HH \succ HL}$ constraint yields the upper bound on r, i.e. $r \leqslant \delta^D$. We claim $(\max\{s'', \delta^C, \tau^0\}, \delta^D] = (\max\{\tau^0, \delta^C\}, \delta^D]$. Subtracting q_1 from both sides of $\delta^C \leqslant \delta^D$ and collecting terms, one obtains $s'' \leqslant \delta^C$ which means $\delta^C \leqslant \delta^D$ if and only if $s'' \leqslant \delta^C$. Therefore $(\max\{s'', \delta^C, \tau^0\}, \delta^D] = (\max\{\tau^0, \delta^C\}, \delta^D]$ is verified. As r becomes even larger, the problem (LP-2) becomes infeasible because the intersection of the Mutual Monitoring constraint and the Pricipal's IC constraint(s) is an empty set.

Finally, tedious algebra verifies that the solutions characterized above satisfy the "Self-Enforcing Shirk" constraint that we left out in solving the problem. Therefore adding this constraint back does not affect the optimal objective value. ∎

Proof of Proposition 2.

Lemma 3: Setting $w_{LH} = 0$ is optimal.

Proof of Lemma 3 is similar to Lemma 2 and therefore omitted.

After setting $w_{LH} = 0$, rewrite the program as

$$\min(1 - q_1)^2 w_{LL} + (1 - q_1)q_1 w_{HL} + q_1^2 w_{HH}$$

s.t

$$(1 - q_1)w_{LL} - (1 - q_1)w_{HL} - q_1 w_{HH} \leqslant \frac{-1}{q_1 - q_0} \text{"Stage NE"}(\lambda_1)$$

$$((2 - q_1 - q_0) + (1 - q_0)r)w_{LL} + ((q_0 - 1)(1 + r) + q_1)w_{HL} - (rq_0 + q_0 + q_1)w_{HH} \leqslant \frac{-(1+r)}{q_1 - q_0} \text{"No}$$

Joint Shirking"(λ_2)

$$(1 - q_1)(2 + r)w_{LL} + ((q_1 - 1)(1 + r) + q_1)w_{HL} - q_1(2 + r)w_{HH} \leqslant -\frac{1+r}{q_1 - q_0} \text{"No Cycling"}(\lambda_3)$$

$$((1 - q_1)^2 - r)w_{LL} + (1 - q_1)q_1 w_{HL} + (q_1^2 + r)w_{HH} \leqslant (q_1 - q_0)H \quad ICP_{HH \succ LL}(\lambda_4)$$

$$((1 - q_1)^2 - r)w_{LL} + (q_1 - q_1^2 + \frac{r}{2})w_{HL} + q_1^2 w_{HH} \leqslant (q_1 - q_0)H \quad ICP_{HL \succ LL}(\lambda_5)$$

$$(1 - q_1)^2 w_{LL} + (q_1 - q_1^2 - \frac{r}{2})w_{HL} + (q_1^2 + r)w_{HH} \leqslant (q_1 - q_0)H \quad ICP_{HH \succ HL}(\lambda_6)$$

$$(1 - q_1)^2 w_{LL} + (q_1 - q_1^2 + \frac{r}{2})w_{HL} + (q_1^2 - r)w_{HH} \leqslant (q_1 - q_0)H \quad ICP_{HL \succ HH}(\lambda_7)$$

$$((1 - q_1)^2 + r)w_{LL} + (1 - q_1)q_1 w_{HL} + (q_1^2 - r)w_{HH} \leqslant (q_1 - q_0)H \quad ICP_{LL \succ HH}(\lambda_8)$$

$$((1 - q_1)^2 + r)w_{LL} + (q_1 - q_1^2 - \frac{r}{2})w_{HL} + q_1^2 w_{HH} \leqslant (q_1 - q_0)H \quad ICP_{LL \succ HL}(\lambda_9)$$

$$-w_{LL} \leqslant 0(\lambda_{10}); \quad -w_{HL} \leqslant 0(\lambda_{11}); \quad -w_{HH} \leqslant 0(\lambda_{12}).$$

Denote the objective function of by $f(w)$, the left-hand side less the right-hand side of the ith constraints by $g_i(w)$, and the Lagrangian Multiplier of the ith constraint by λ_i, then the Lagrangian for the problem is $L = f(w) + \sum_{i=1}^{12} \lambda_i g_i(w)$. The first-order-conditions (FOCs) of the Lagrangian with respect to the three wage (w_{LL}, w_{HL}, w_{HH}) are as follows:

$$1 + \lambda_1 - \lambda_{10} + 2\lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 - \lambda_1 q_1$$

$$- 2(1 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9)q_1 + (1 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9)q_1^2$$

$$+ (\lambda_3 - \lambda_4 - \lambda_5 + \lambda_8 + \lambda_9 - \lambda_3 q_1)r - \lambda_2(-2 + q_0 + q_1 + (-1 + q_0)r) = 0; \quad (\text{FOC} - w_{LL})$$

$$\lambda_1(-1+q_1) + q_1 + q_1(2\lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9$$

$$- (1 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9)q_1) + \frac{1}{2}(\lambda_5 - \lambda_6 + \lambda_7 - \lambda_9)$$

$$+ 2\lambda_3(-1+q_1))r + \lambda_2(-1 + q_0 + q_1 + (-1+q_0)r) - \lambda_{11} - \lambda_3 = 0; \quad (\text{FOC}-w_{HL})$$

$$(1 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9)q_1^2 + (\lambda_4 + \lambda_6 - \lambda_7 - \lambda_8)r$$

$$- \lambda_3 q_1(2+r) - \lambda_2(q_0 + q_1 + q_0 r) - \lambda_{12} - \lambda_1 q_1 = 0 \quad (\text{FOC}-w_{HH})$$

The optimal solution is one that (i) satisfies all 12 constraints, (ii) satisfies the three FOC above, (iii) satisfies the 12 complementary slackness conditions $\lambda_i g_i(w) = 0$, and (iv) all the Lagrangian multipliers are non-negative, i.e. $\lambda_i \geqslant 0$.

For $r \leqslant \delta^c$, the solution listed below satisfies $(i) - (iv)$ and thus is optimal. Under this solution, denoted as IPE, the wage payments are derived by solving the following three binding constraints in (LP-1): Stage NE, *No Joint Shirking*, and w_{LL}. (*No Cycling* is also binding, and the Lagrangian multipliers under this solution are not unique due to the degeneracy of the problem. However finding one set of λ satisfying $(ii) - (iv)$ is enough to show the optimality.)

The IPE solution is:

$w_{LL} = 0$, $w_{HL} = w_{HH} = \frac{1}{q_1 - q_0}$

$\lambda_1 = q_1$, $\lambda_2 = 0$, $\lambda_3 = 0$, $\lambda_4 = 0$,

$\lambda_5 = 0$, $\lambda_6 = 0$, $\lambda_7 = 0$, $\lambda_8 = 0$,

$\lambda_9 = 0$, $\lambda_{10} = 1 - q_1$, $\lambda_{11} = 0$, $\lambda_{12} = 0$.

Under IPE, the $ICP_{HH \succ LL}$ constraint imposes the upper bound δ^c on r. The optimal solution changes when $r \geqslant \delta^c$. For $\delta^c < r < \tau^1$, the solution is listed below. This solution, denoted as BP^I, is obtained by solving the following three constraints: $ICP_{HH \succ LL}$, *No Joint Shirking*, and *No Cycling*.

The BP^I solution is:

$w_{LL} = \frac{(q_1 - q_0)^2(1+r)H - (1+r)(q_1 + r)}{(q_1 - q_0)(q - r(-1 + q_1 + r))}$, $w_{HL} = w_{HH} + w_{LL}$,

$w_{HH} = \frac{(q_1 - q_0)^2 H - (1+r)(-1 + q_1 + r)}{(q_1 - q_0)(q - r(-1 + q_1 + r))}$;

$\lambda_1 = 0$, $\lambda_2 = \frac{r(1+r+q_1^2 r - 2q_1(q+r))}{(q_1-q_0)(1+r)(-1+(-1+q_1)r+r^2)}$, $\lambda_3 = \frac{r(-1+q_0+q_1-r+q_0 r - q_1^2 r)}{(q_0-q_1)(1+r)(-q+(-1+q_1)r+r^2)}$, $\lambda_4 = \frac{1+r-q_1 r}{-1-(1-q_1)r+r^2}$,

$\lambda_5 = 0$, $\lambda_6 = 0$, $\lambda_7 = 0$, $\lambda_8 = 0$,

$\lambda_9 = 0$, $\lambda_{10} = 0$, $\lambda_{11} = 0$, $\lambda_{12} = 0$.

Under BP^I, both the non-negativity of w_{LL} and the Stage NE constraints require $r > \delta^c$ and $\lambda_2 \geqslant 0$ requires $r < \tau^1$. The optimal solution changes if $r > \tau^1$. For $\max\{\tau^1, \delta^c\} < r \leqslant \delta^F$, the optimal solution is listed below. The solution, denoted as RPE, is obtained by solving the following three constraints: $ICP_{HH \succ LL}$, No Cycling, and w_{LL}.

The RPE solution is:

$w_{LL} = 0$, $w_{HL} = \frac{(q_1-q_0)q_1(2+r)H - \frac{(1+r)(q_1^2+r)}{q_1-q_0}}{q_1^2 - r(1+r) + q_1 r(2+r)}$,

$w_{HH} = \frac{(1-q_1)q_1(1+r) + (q_1-q_0)^2(-1-r+q_1 r(2+r))H}{(q_1-q_0)(q_1^2 - r(1+r) + q_1 r(2+r))}$,

$\lambda_1 = 0$, $\lambda_2 = 0$, $\lambda_3 = \frac{(1-q_1)q_1 r}{r(1+r) - q_1^2 - q_1 r(2+r)}$, $\lambda_4 = \frac{q_1^2}{r(1+r) - q_1^2 - q_1 r(2+r)}$,

$\lambda_5 = 0$, $\lambda_6 = 0$, $\lambda_7 = 0$, $\lambda_8 = 0$,

$\lambda_9 = 0$, $\lambda_{10} = \frac{r(1+r+q_1^2 r - 2q_1(1+r))}{r(1+r) - q_1^2 - q_1 r(2+r)}$, $\lambda_{11} = 0$, $\lambda_{12} = 0$.

Under RPE, the Stage NE constraint and $\lambda_{10} \geqslant 0$ yields two lower bounds δ^C and τ^1 on r. $ICP_{HL \succ LL}$ and the non-negativity of w_{HH} and w_{HL} together require $r \leqslant \delta^F$. $w_{HH} \geqslant 0$ also requires $r > s \equiv \frac{2q_1 - 1 + \sqrt{(2q_1-1)^2 + 4(1-q_1)q_1^2}}{2(1-q_1)}$, and we claim $(\max\{s, \delta^C, \tau^1\}, \delta^F] = (\max\{\delta^C, \tau^1\}, \delta^F]$. Consider the case where $s > \delta^C$ (as the claim is trivial if instead $s \leqslant \delta^C$). Since δ^C increases in H while s is independent of H, one can show $s > \delta^C$ is equivalent to $H < H^*$ for a unique positive H^*. Algebra shows that $\delta^F < \delta^C$ for $H < H^*$. Therefore $s > \delta^C$ implies $\delta^F < \delta^C$, in which case both $(\max\{s, \delta^C, \tau^1\}, \delta^F]$ and $(\max\{\delta^C, \tau^1\}, \delta^F]$ are empty sets. For $r > \max\{\delta^F, \tau^1\}$, the optimal solution is listed below and denoted as BP^S. Under BP^s, the optimal payment is obtained by solving the following three constraints: $ICP_{HH \succ LL}$, $ICP_{HL \succ LL}$, and No Cycle.

The BP^S solution is:

$w_{LL} = \frac{(1+r)((2-q_1)q_1 + r) + (q_1-q_0)^2(-2(1+r) + q_1(2+r))H}{(q_1-q_0)(2q_1 + (3-q_1)q_1 r + r^2 - 2(1+r))}$, $w_{HL} = 2w_{HH}$,

$w_{HH} = \frac{(1+r)(-(1-q_1)^2 + r) - (q_1-q_0)^2(1-q_1)(2+r)H}{(q_1-q_0)(2q_1 + (3-q_1)q_1 r + r^2 - 2(1+r))}$,

$\lambda_1 = q_1$, $\lambda_2 = 0$, $\lambda_3 = \frac{r}{-2-2r-q_1^2 r + r^2 + q_1(2+3r)}$, $\lambda_4 = \frac{q_1(-2+(-1+q_1)r)}{2+2r+q_1^2 r - r^2 - q_1(2+3r)}$,

$\lambda_5 = \frac{2(1+r+q_1^2 r - 2q_1(1+r))}{-2-2r-q_1^2 r + r^2 + q_1(2+3r)}$, $\lambda_6 = 0$, $\lambda_7 = 0$, $\lambda_8 = 0$,

$\lambda_9 = 0$, $\lambda_{10} = 1 - q_1$, $\lambda_{11} = 0$, $\lambda_{12} = 0$.

Where the two lower bound δ^F and τ^1 on r are derived from the non-negativity constraint of w_{LL} and λ_5. Collecting conditions verifies the proposition. ■

Proof of Proposition 3. The proposition is proved by showing a sequence of claims.

Claim 1: LP-1 is optimal for $r > \max\{\delta^C, \delta^D\}$.

Claim2: $BP2^C$ of LP-2 is never the overall optimal contract.

Claim 3: JPE^1 of LP2, if feasible, is the overall optimal contract.

Claim 4: JPE^2 of LP2, if feasible, is the overall optimal contract.

Claim 5: $BP1^C$ of LP2, if feasible, is the overall optimal contract.

Claim 6: $\min\{\tau^0, \delta^D\} > \delta^C$ if and only if $\tau^0 > \delta^C$.

Using Claims 1 - 5, one can verify the following statement: when $\min\{\tau^0, \delta^D\} \leqslant \delta^C$, LP-2 is optimal if and only if $r < \delta^C$; otherwise for $\min\{\tau^0, \delta^D\} > \delta^C$, LP-2 is optimal if and only if $r < \min\{\tau^0, \delta^D\}$. Claim 6 shows that condition $\min\{\tau^0, \delta^D\} > \delta^C$ is equivalent to $\tau^0 > \delta^C$ and, thus, is equivalent to the statement in the proposition.

Proof of Claim 1: The claim is trivial as we know from Proposition 2 that LP-2 does not have feasible solution on the region.

Proof of Claim 2: Recall that $BP2^C$ of LP-2 is obtained by solving the following three binding constraints: Mutual Monitoring, Pareto Dominance, and $ICP_{HH \succ LL}$. It is easy to see that $\pi(0, 0; w) = \pi(0, 1; w)$ when both Mutual Monitoring constraint and the Pareto Dominance constraint are binding, in which case the Mutual Monitoring constraint can be re-written as follows:

$$\frac{1+r}{r}[\pi(1, 1; w) - 1] \geqslant \pi(0, 1; w) + \frac{1}{r}\pi(0, 1; w)$$

Note this is same as the "Stage NE" constraint in LP-1 and therefore all the constraints in (LP-1) are implied by those in (LP-2) under the $BP2^C$ solution. In this case, (LP-2) has a smaller feasible set, so it can never do strictly better than (LP-1).

Proof of Claim 3: We know from Proposition 2 that $JPE1$ is the optimal solution of LP2 for $r \in (0, \delta^A]$, over which the optimal solution of LP1 is IPE (Proposition 1). Substituting the corresponding solution into the principal's objective function, we obtain $obj_{JPE1} = \frac{q_1^2(1+r)}{(q_1-q_0)(q_0+q_1+q_1 r)}$ and $obj_{JPE} = \frac{q_1}{q_1-q_0}$. Algebra shows $obj_{JPE} - obj_{JPE1} = \frac{q_0 q_1}{(q_1-q_0)(q_0+q_1+q_1 r)} > 0$, which verifies the

claim.

Proof of Claim 4: $JPE2$ is the solution of LP2 for $r \in (\max\{\tau^0, \delta^A\}, \delta^C]$, over which IPE is the corresponding solution of LP1. Algebra shows that $obj_{JPE2} = \frac{(q_1-1)q_1 r(1+r) + q_0(q_0-q_1)^2 q_1 H}{(q_0-q_1)((q_1-1)r(1+r) + q_0(q_1+r))}$, $obj_{IPE} = \frac{q_1}{q_1-q_0}$, and $obj_{JPE2} - obj_{IPE} \leqslant 0$ if and only if $\frac{q_0+q_1-1+\sqrt{(q_0+q_1-1)^2+4(1-q_1)q_0q_1}}{2(1-q_1)} \leqslant r \leqslant \delta^C$ (with equality on the boundary). The claim is true if $\max\{\frac{q_0+q_1-1+\sqrt{(q_0+q_1-1)^2+4(1-q_1)q_0q_1}}{2(1-q_1)}, \tau^0, \delta^A\} \leqslant r \leqslant \delta^C$, which was shown in the proof of Proposition 2 to be equivalent to $r \in (\max\{\tau^0, \delta^A\}, \delta^C]$. Therefore, $JPE2$ is the overall optimal contract whenever it is feasible.

Proof of Claim 5: We know that $BP1^C$ is the solution of LP2 if $r \in (\delta^A, \min\{\tau^0, \delta^D\}]$. In this region, IPE and BP^I are potential solutions in LP1 because the other two solutions (RPE and BP^S) require $r \geqslant \tau^1 > \tau^0$. Let us compare first $BP1^C$ of LP2 and BP^I of LP1. It is easy to show $obj_{BP1^C} = \frac{r(1+r)-(q_0-q_1)^2(q_0+q_1(1+r-q_1 r))H}{(q_0-q_1)(q_0+q_1-(-1+q_1)q_1 r - r^2)}$ and $obj_{BP^I} = \frac{r(1+r)+(q_0-q_1)^2(r(q_1-1)-1)H}{(q_0-q_1)(r(q_1+r-1)-1)}$. Tedious algebra verifies $obj_{BP1^C} < obj_{BP^I}$ for $\delta^C < r \leqslant \min\{\tau^0, \delta^D\}$ where both solutions are feasible.

Showing $BP1^C$ is always more cost efficient than the IPE solution is more involved and is presented in two steps. We first derive the sufficient condition for this to be true and then show that the sufficient condition holds whenever both solutions are optimal in their corresponding program, namely $\delta^A < r \leqslant \min\{\tau^0, \delta^C, \delta^D\}$. Given obj_{BP1^C} and obj_{IPE} defined above, one can show that $obj_{BP1^C} < obj_{IPE} \Leftrightarrow r < \delta$, where

$$\delta = \frac{1}{2(1-q_1)}[((q_1-q_0)^2 H - q_1)q_1(1-q_1) - 1 +$$
$$\sqrt{(((q_1-q_0)^2 H - q_1)q_1(1-q_1) - 1)^2 + 4(1-q_1)((q_1-q_0)^2 H - q_1)(q_1+q_0)}]$$

Note that if $\delta \geqslant \delta^C$, then $r < \delta$ (thus $obj_{BP1^C} < obj_{IPE}$) is satisfied trivially for $\delta^A < r \leqslant \min\{\tau^0, \delta^C, \delta^D\}$. Consider the opposite case in which $\delta < \delta^C$. For $q_0 \in [0, q_1)$, one can show that $\delta < \delta^C$ corresponds to either $r < \frac{1+\sqrt{1+4(1-q_1)^2(-1+q_1(3+(q_1-2)q_1))H}}{2(1-q_1)^2 H}$ or $q_1 - \sqrt{\frac{q_1}{H}} < r < q_1$. Since the latter condition contradicts the maintained assumption that $(q_1-q_0)^2 H > q_1$, we consider $r < \frac{1+\sqrt{1+4(1-q_1)^2(-1+q_1(3+(q_1-2)q_1))H}}{2(1-q_1)^2 H}$ only. Given $r < \frac{1+\sqrt{1+4(1-q_1)^2(-1+q_1(3+(q_1-2)q_1))H}}{2(1-q_1)^2 H}$, one can show $\delta > \tau^0$ for any $q_0 \in [0, q_1)$. Therefore, under the maintained assumption $(q_1-q_0)^2 H > q_1$,

$\delta < \delta^C$ implies $\tau^0 < \delta$. If the choice is between $BP1^C$ and IPE, $r \leqslant \min\{\tau^0, \delta^C, \delta^D\}$. Then $\tau^0 < \delta$ implies $r < \delta$. $r < \delta$ implies $obj_{BP1^C} < obj_{IPE}$ whenever both are feasible (which is in the region $\delta^A < r \leqslant \min\{\tau^0, \delta^C, \delta^D\}$).

Proof of Claim 6: The "only if" direction is trivial. To show the "if" direction, note that if $\tau^0 > \delta^C$, we know $q_1 + q_0 > 1$ as otherwise $\tau^0 < 0 < \delta^C$. Under the maintained assumption on H, $q_1 + q_0 > 1$ implies $\delta^D > \delta^C$. Therefore, $\min\{\tau^0, \delta^D\} > \delta^C$ if and only if $\tau^0 > \delta^C$. ∎